Sliding Mode Control Strategy for Variable Speed Wind Turbines

Oscar Barambones and Jose Maria Gonzalez de Durana
Automatic Control and System Engineering Departament
University of the Basque Country
EUI de Vitoria.
Nieves cano 12. 1006 Vitoria. (SPAIN)
oscar.barambones@ehu.es

Abstract

The efficiency of the wind power conversions systems can be greatly improved using an appropriate control algorithm. In this work, a robust control for variable speed wind power generation that incorporates a doubly feed induction generator is described. The electrical systems incorporates a wound rotor induction machine with back-to-back three phase power converter bridges between its rotor and the grid. In the presented design it is applied the so called vector control theory. The proposed control scheme uses stator flux-oriented control for the rotor side converter bridge control and grid voltage vector control for the grid side converter bridge control. The proposed robust control law is based on a sliding mode control theory, that, as it is well known, presents a good performance under system uncertainties.

The stability analysis of the proposed controller under disturbances and parameter uncertainties is provided using the Lyapunov stability theory. Finally simulated results show, on the one hand that the proposed controller provides high-performance dynamic characteristics, and on the other hand that this scheme is robust with respect to the uncertainties that usually appear in the real systems.

1. Introduction

Wind power has experienced a rapid global growth since the late 1990s. In the year 1997, there was only 7480 MW of installed capacity worldwide, this capacity was increased by about 2000 MW per year. However this anual growth rate has been increased until reach a rate of 27261 MW that were added in 2008. In this sense the worldwide wind power installed capacity reaches 121188 MW in 2008, and it is expected an annual growth rate of 31000 MW in 2009 and 38000 MW in 2010 [16].

The high growth rate of wind power capacity is explained by the cost reduction as well as by new public government subsides in many countries; linked to efforts to increase the use of renewable power production and reduce CO2 emissions, further cost reduction is anticipated. Wind power production is highly dependant on the wind resources at the local site.

All wind turbines installed by the end of 2008 worldwide are generating 260 TWh per annum, equalling more than 1.5 % of the global electricity consumption, and it is expected that this percentage will increase in the following years.

Based on the experience and growth rates of the past years, World Wind Energy Association (WWEA) expects that wind energy will continue its dynamic development also in the coming years. Although the short term impacts of the current finance crisis makes short-term predictions rather difficult, it can be expected that in the mid-term wind energy will rather attract more investors due to its low risk character and the need for clean and reliable energy sources. More and more governments understand the manifold benefits of wind energy and are setting up favourable policies, including those that are stimulation decentralized investment by independent power producers, small and medium sized enterprises and community based projects, all of which will be main drivers for a more sustainable energy system also in the future

Carefully calculating and taking into account some insecurity factors, wind energy will be able to contribute in the year 2020 at least 12% of global electricity consumption. By the year 2020, at least 1500000 MW can be expected to be installed globally.

Nowadays, the major market area for wind power is the European Union with over 66 GW of installed wind generation capacity, the second one is North America with over 27 GW, and the third one is Asia with over 24 GW. However in terms of continental distribution, a continuous diversification process can be watched as well. In general, the focus of the wind sector moves away from Europe to Asia and North America. Europe decreased its share in total installed capacity from 65,5% in 2006 to 61% in the year 2007 further down to 54,6% in 2008. Only four years ago Europe dominated the world market with 70,7% of the new capacity. In 2008 the continent lost this position and, for the first time, Europe (32,8%), North America (32,6%) and Asia (31,5%) account for almost similar shares in new capacity. However, Europe is still the strongest continent while North America and Asia are increasing rapidly their shares.

Doubly Feed Induction Generator (DFIG), with vector control applied, is widely used in variable speed wind turbine control system owing to their ability to maximize wind power extraction. In these DFIG wind turbines the control system should be designed in order to to achieve the following objectives: regulating the DFIG rotor speed for maximum wind power capture, maintaining the DFIG stator output voltage frequency constant and controlling the DFIG reactive power.

One of the main task of the the controller is to carry the turbine rotor speed into the desired optimum speed, in spite of system uncertainties, in order to extract the maximum active power from the wind. This paper investigates a new robust speed control method for variable speed wind turbines, in order to obtain the maximum wind power capture in spite of system uncertainties [6], [9].

Basically, the proposed robust design uses the sliding mode control algorithm to regulate both the rotor-side converter (RSC) and the grid-side converter (GSC). In the design it is used a vector oriented control theory in order to decouple the torque and the flux of the induction machine. This control scheme leads to obtain the maximum power extraction from the different wind speeds that appear along time.

Finally, test of the proposed method based on a two-bladed horizontal axis wind turbine is conducted using the Matlab/Simulink software. In this test, several operating conditions are simulated and satisfactory results are obtained.

2. Problem statement

There are three fundamental modes of operation in order to extract effectively wind power while at the same time maintaining safe operation [10]. In this modes, illustrated in figure 1 the wind turbine should be driven according to:

Mode 1 Operation at variable speed/optimum tip-speed ratio: $v_c \leq v \leq v_b$

Mode 2 Operation at constant speed/variable tip-speed ratio: $v_b \leq v \leq v_r$

Mode 3 Operating at variable speed/constant power: $v_r \leq v \leq v_f$

where v is the wind speed, v_c is the cut-in wind speed, v_b denotes the wind speed at which the maximum allowable rotor speed is reached, v_r is the rated wind speed and v_f is the furling wind speed at which the turbine needs to be shut down for protection.

It is seen that, if high-power efficiency is to be achieved at lower wind speeds, the rotor speed of the wind turbine must be adjusted continuously against wind speed.

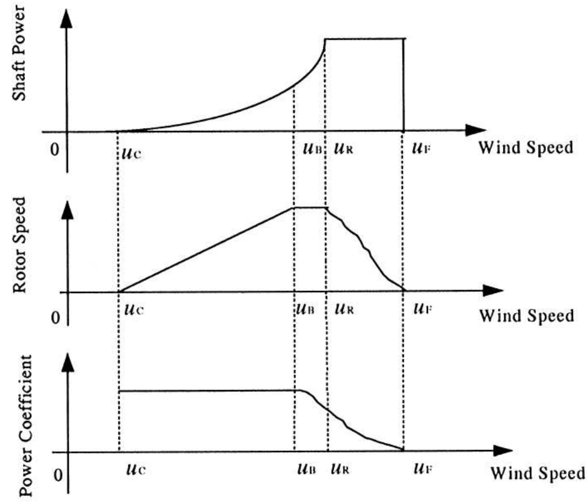

Figure 1. Turbine speed opreration modes

A common practice in addressing the control problem of wind turbines is to use linearization approach. This method allows the linear system theory to be applied in control design and analysis. However, due to the stochastic operating conditions and the inevitable uncertainties inherent in the system, such a control method comes at the price of poor system performance and low reliability [10].

In this work we present a method for variable speed control of wind turbines. The objective is to make the rotor speed track the desired speed that is specified according to the three fundamental operating modes as described earlier. This is achieved regulating the rotor currents of the double feed induction generator (DFIG) through the developed nonlinear and robust control algorithms. Such a control scheme leads to more energy output without involving additional mechanical complexity to the system. Test of the proposed method based on a two-bladed horizontal axis wind turbine is conducted. Several operating conditions are simulated and satisfactory results are obtained.

3. System modelling

The power extraction of wind turbine is a function of three main factors: the wind power available, the power curve of the machine and the ability of the machine to respond to wind fluctuation. The expression for power produced by the wind is given by [1], [15]:

$$P_m(v) = \frac{1}{2} C_p(\lambda, \beta)\rho\pi R^2 v^3 \tag{1}$$

where ρ is air density, R is radius of rotor, v is wind speed, C_p denotes power coefficient of wind turbine, λ is the tip-speed ratio and β represents pitch angle. The tip-speed ratio is defined as:

$$\lambda = \frac{R w}{v} \tag{2}$$

where w is the turbine rotor speed. Therefore, if the rotor speed is kept constant, then any change in the wind speed will change the tip-speed ratio, leading to the change of power coefficient C_p, as well as the generated power output of the wind turbine. However, if the rotor speed is adjusted according to the wind speed variation, then the tip-speed ratio can be maintained at an optimal point, which could yield maximum power output from the system.

For a typical wind power generation system, the following simplified elements are used to illustrate the fundamental work principle. The system primarily consists of an aeroturbine, which converts wind energy into mechanical energy, a gearbox, which serves to increase the speed and decrease the torque and a generator to convert mechanical energy into electrical energy.

Driving by the input wind torque T_m, the rotor of the wind turbine runs at the speed w. The transmission output torque T_t is then fed to the generator, which produces a shaft torque of T_e at generator angular velocity of w_e. Note that the rotor speed and generator speed are not the same in general, due to the use of the gearbox.

The mechanical equations of the system can be characterized by [10]:

$$\begin{aligned} J_m\dot{w} + B_m w &= T_m + T & (3)\\ J_e\dot{w}_e + B_e w_e &= T_t + T_e & (4)\\ T_t w_e &= -Tw & (5) \end{aligned}$$

where J_m and J_e are the moment of inertia of the turbine and the generator, B_m and B_e are the viscous friction coefficient of the the turbine and the generator, T_m is the wind generated torque in the turbine, T is the torque in the transmission shaft before gear box, T_f is the torque in the transmission shaft after gear box, and T_e is the the generator torque, w is the angular velocity of the turbine shaft and w_e is the angular velocity of the generator rotor.

The relation between the angular velocity of the turbine w and the angular velocity of the generator w_e is given by the gear ratio γ:

$$\gamma = \frac{w_e}{w} \quad (6)$$

Then, using equations 3, 4, 5 and 6 it is obtained:

$$J\dot{w} + Bw = T_m - \gamma T_e \quad (7)$$

with

$$\begin{aligned} J &= J_m + \gamma^2 J_e & (8)\\ B &= B_m + \gamma^2 B_e & (9) \end{aligned}$$

From equations 1 and 2 it is deduced that the input wind torque is:

$$T_m(v) = \frac{P_m(v)}{w} = \frac{P_m(v)}{\frac{\lambda v}{R}} = k_v \cdot v^2 \quad (10)$$

where

$$k_v = \frac{1}{2}\, C_p\, \rho\, \pi \frac{R^3}{\lambda} \quad (11)$$

Now we are going to consider the system electrical equations. In this work it is used a double feed induction generator (DFIG). This induction machine is feed from both stator and rotor sides. The stator is directly connected to the grid while the rotor is fed through a variable frequency converter (VFC). In order to produce electrical active power at constant voltage and frequency to the utility grid, over a wide operation range (from subsynchronous to supersynchronous speed), the active power flow between the rotor circuit and the grid must be controlled both in magnitude and in direction. Therefore, the VFC consists of two four-quadrant IGBT PWM converters (rotor-side converter (RSC) and grid-side converter (GSC)) connected back-to-back by a dc-link capacitor [8], [9].

The DFIG can be regarded as a traditional induction generator with a nonzero rotor voltage. The dynamic equation of a thee-phase DFIG can be written in a synchronously rotating d-q reference frame as [7].

$$\begin{aligned} v_{ds} &= r_s i_{ds} - w_s \psi_{qs} + \frac{d\psi_{ds}}{dt} & (12)\\ v_{qs} &= r_s i_{qs} + w_s \psi_{ds} + \frac{d\psi_{qs}}{dt} & (13)\\ v_{dr} &= r_r i_{dr} - s\, w_s \psi_{qr} + \frac{d\psi_{dr}}{dt} & (14)\\ v_{qs} &= r_r i_{qr} + s\, w_s \psi_{dr} + \frac{d\psi_{qr}}{dt} & (15) \end{aligned}$$

where w_s is the rotational speed of the synchronous reference frame, $sw_s = w_s - w_e$ is the slip frequency and s is the slip, w_e is the generator rotor speed and the flux linkages are given by:

$$\begin{aligned} \psi_{ds} &= L_s i_{ds} + L_m i_{dr} & (16)\\ \psi_{qs} &= L_s i_{qs} + L_m i_{qr} & (17)\\ \psi_{dr} &= L_r i_{dr} + L_m i_{ds} & (18)\\ \psi_{qr} &= L_r i_{qr} + L_m i_{qs} & (19) \end{aligned}$$

where L_s, L_r, and L_m are the stator inductance, rotor inductance and mutual inductances, respectively. The electrical torque equation of the DFIG is given by:

$$T_e = \frac{3p}{4} L_m (i_{qs} i_{dr} - i_{ds} i_{qr}) \quad (20)$$

where p is the pole numbers.

Neglecting the power loses associated with the stator resistances, the active and reactive stator powers are:

$$\begin{aligned} P_s &= \frac{3}{2}(v_{ds} i_{ds} + v_{qs} i_{qs}) & (21)\\ Q_s &= \frac{3}{2}(v_{qs} i_{ds} - v_{ds} i_{qs}) & (22) \end{aligned}$$

Similarly, the rotor power (also called slip power) can be calculated as:

$$\begin{aligned} P_r &= \frac{3}{2}(v_{dr} i_{dr} + v_{qr} i_{qr}) & (23)\\ Q_r &= \frac{3}{2}(v_{qr} i_{dr} - v_{dr} i_{qr}) & (24) \end{aligned}$$

Then, when the power losses in the converters are neglected, the total real power P_e injected into the main network equals to the sum of the stator power P_s and the rotor power P_r. In the same way, the reactive power Q_e exchanged with the grid equals to the sum of stator reactive power Q_s and the rotor reactive power Q_r.

4 Wind turbine control scheme

The DFIG wind turbine control system should be designed in order to to achieve the following objectives:

1. Regulating the DFIG rotor speed for maximum wind power capture.
2. Maintaining the DFIG stator output voltage frequency constant.
3. Controlling the DFIG reactive power.

In order to achieve these objectives the DFIG wind turbine control system are generally composed of two parts: the electrical control on the DFIG and the mechanical control on the wind turbine blade pitch angle [2]. Control of the DFIG is achieved controlling the variable frequency converter (VFC), which includes control of the rotor-side converter (RSC) and control of the grid-side converter (GSC) [4]. The objective of the RSC is to govern both the stator-side active and reactive powers independently; while the objective of the GSC is to keep the dc-link voltage constant regardless of the magnitude and direction of the rotor power. The GSC control scheme can also be designed to regulate the reactive power or the stator terminal voltage of the DFIG. A typical scheme of a DFIG equipped wind turbine is shown in Figure 2.

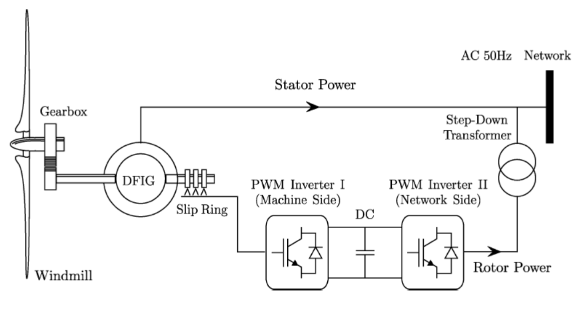

Figure 2. Scheme of a DFIG equipped wind turbine

When the wind turbine generator (WTG) operates in the variable-speed mode, in order to extract the maximum active power from the wind, the shaft speed of the WTG must be adjusted to achieve an optimal tip-speed ratio λ_{opt}, which yields the maximum power coefficient $C_{p_{max}}$, and therefore the maximum power [5]. In other words, given a particular wind speed, there is a unique wind turbine speed required to achieve the goal of maximum wind power extraction. The value of the λ_{opt} can be calculated from the maximum of the power coefficient curves versus tip-speed ratio, which depends of the modeling turbine characteristics.

The power coefficient C_p, can be approximated by equation 25 based on the modeling turbine characteristics [11]:

$$C_p(\lambda, \beta) = c_1 \left(\frac{c_2}{\lambda_i} - c_3\beta - c_4 \right) e^{\frac{-c_5}{\lambda_i}} + c_6\lambda \quad (25)$$

where the coefficients c_1 to c_6 depends on the wind turbine design characteristics, and λ_i is defined as

$$\frac{1}{\lambda_i} = \frac{1}{\lambda + 0.08\beta} - \frac{0.035}{\beta^3 + 1} \quad (26)$$

The value of λ_{opt} can be calculated from the roots of the derivative of the equation 25. Then, based on the wind speed, the corresponding optimal generator speed command for maximum wind power tracking is determined by:

$$w^* = \frac{\lambda_{opt} \cdot v}{R} \quad (27)$$

5 Rotor Side Converter Control

The RSC control scheme should be designed in order to regulate the wind turbine speed for maximum wind power capture. Therefore, a suitably designed speed controller is essential to track the optimal wind turbine speed reference w^* for maximum wind power extraction. The design of such a speed controller should take into account the dynamics of the WTG shaft system.

In the DFIG-based wind generation system, these objective are commonly achieved by electrical generator rotor current regulation on the stator-flux oriented reference frame [7].

In the stator-flux oriented reference frame, the d-axis is aligned with the stator flux linkage vector ψ_s, and then, ψ_{ds}=ψ_s and ψ_{qs}=0 . This yields the following relationships [4]:

$$i_{qs} = \frac{L_m i_{qr}}{L_s} \quad (28)$$

$$i_{ds} = \frac{L_m(i_{ms} - i_{dr})}{L_s} \quad (29)$$

$$T_e = \frac{-L_m i_{ms} i_{qr}}{L_s} \quad (30)$$

$$Q_s = \frac{3}{2} \frac{w_s L_m^2 i_{ms}(i_{ms} - i_{dr})}{L_s} \quad (31)$$

$$v_{dr} = r_r i_{dr} + \sigma L_r \frac{di_{qr}}{dt} - s w_s \sigma L_r i_{qr} \quad (32)$$

$$v_{qr} = r_r i_{qr} + \sigma L_r \frac{di_{qr}}{dt} \quad (33)$$

$$+ s w_s \left(\frac{\sigma L_r i_{dr} + L_m^2 i_{ms}}{L_s} \right) \quad (34)$$

where

$$i_{ms} = \frac{v_{qs} - r_s i_{qs}}{w_s L_m} \quad (35)$$

$$\sigma = 1 - \frac{L_m^2}{L_s L_r} \quad (36)$$

Since the stator is conectad to the grid, and the influence of the stator resistance is small, the stator magnetizing current (i_{ms}) can be considered constant [8]. Therefore, the electromagnetic torque can be defined as follows:

$$T_e = -K_T i_{qr} \quad (37)$$

where K_T is a torque constant, and is defined as follows:

$$K_T = \frac{L_m i_{ms}}{L_s} \quad (38)$$

Then, from equations 7 and 37 it is deduced that the wind turbine speed can be controlled by regulating the q-axis rotor current components (i_{qr}) while equation 31 indicates that the stator reactive power (Q_s) can be controlled by regulating the d-axis rotor current components, (i_{ds}). Consequently, the reference values of i_{qr} and i_{dr} can be determined directly from w_r and Q_s references.

From equations 7 and 37 it is obtained the following dynamic equation for the system speed:

$$\dot{w} = \frac{1}{J}\left(T_m - \gamma K_T i_{qr} - Bw\right) \quad (39)$$

$$= -aw + f - bi_{qr} \quad (40)$$

where the parameters are defined as:

$$a = \frac{B}{J}, \quad b = \frac{\gamma K_T}{J}, \quad f = \frac{T_m}{J}; \quad (41)$$

Now, we are going to consider the previous dynamic equation (40) with uncertainties as follows:

$$\dot{w} = -(a + \triangle a)w + (f + \triangle f) - (b + \triangle b)i_{qs}^e \quad (42)$$

where the terms $\triangle a$, $\triangle b$ and $\triangle f$ represents the uncertainties of the terms a, b and f respectively.

Let us define define the speed tracking error as follows:

$$e(t) = w(t) - w^*(t) \quad (43)$$

where w^* is the rotor speed command.

Taking the derivative of the previous equation with respect to time yields:

$$\dot{e}(t) = \dot{w} - \dot{w}^* = -a\, e(t) + u(t) + d(t) \quad (44)$$

where the following terms have been collected in the signal $u(t)$,

$$u(t) = f(t) - b\, i_{qr}(t) - a\, w^*(t) - \dot{w}^*(t) \quad (45)$$

and the uncertainty terms have been collected in the signal $d(t)$,

$$d(t) = -\triangle a\, w(t) + \triangle f(t) - \triangle b\; i_{qr}(t) \quad (46)$$

To compensate for the above described uncertainties that are present in the system, it is proposed a sliding control scheme. In the sliding control theory, the switching gain must be constructed so as to attain the sliding condition [14]. In order to meet this condition a suitable choice of the sliding gain should be made to compensate for the uncertainties.

Now, we are going to define the sliding variable $S(t)$ with an integral component as:

$$S(t) = e(t) + \int_0^t (k + a)e(\tau)\, d\tau \quad (47)$$

where k is a constant gain.

Then the sliding surface is defined as:

$$S(t) = e(t) + \int_0^t (a + k)e(\tau)\, d\tau = 0 \quad (48)$$

Now, we are going to design a variable structure speed controller in order to control the wind turbine speed.

$$u(t) = -k\, e(t) - \beta\, \mathrm{sgn}(S) \quad (49)$$

where the k is the constant gain defined previously, β is the switching gain, S is the sliding variable defined in eqn. (47) and $\mathrm{sgn}(\cdot)$ is the signum function.

In order to obtain the speed trajectory tracking, the following assumptions should be formulated:

($\mathcal{A}\,1$) The gain k must be chosen so that the term $(k + a)$ is strictly positive, therefore the constant k should be $k > -a$.

($\mathcal{A}\,2$) The gain β must be chosen so that $\beta \geq |d(t)|$ for all time.

Note that this condition only implies that the uncertainties of the system are bounded magnitudes.

Theorem 1 *Consider the induction motor given by equation (42). Then, if assumptions ($\mathcal{A}\,1$) and ($\mathcal{A}\,2$) are verified, the control law (49) leads the wind turbine speed $w(t)$, so that the speed tracking error $e(t) = w(t) - w^*(t)$ tends to zero as the time tends to infinity.*

The proof of this theorem will be carried out using the Lyapunov stability theory.

Proof : Define the Lyapunov function candidate:

$$V(t) = \frac{1}{2}S(t)S(t) \quad (50)$$

Its time derivative is calculated as:

$$\begin{aligned}\dot{V}(t) = \;& S(t)\dot{S}(t)\\ &= S\cdot[\dot{e}+(k+a)e]\\ &= S\cdot[(-a\,e+u+d)+(k\,e+a\,e)]\\ &= S\cdot[u+d+k\,e]\\ &= S\cdot[-k\,e-\beta\,\mathrm{sgn}(S)+d+k\,e]\\ &= S\cdot[d-\beta\,\mathrm{sgn}(S)]\\ &\le -(\beta-|d|)|S|\\ &\le 0\end{aligned} \quad (51)$$

It should be noted that the eqns. (47), (44) and (49) and the assumption $(\mathcal{A}\,2)$ have been used in the proof.

Using the Lyapunov's direct method, since $V(t)$ is clearly positive-definite, $\dot{V}(t)$ is negative definite and $V(t)$ tends to infinity as $S(t)$ tends to infinity, then the equilibrium at the origin $S(t)=0$ is globally asymptotically stable. Therefore $S(t)$ tends to zero as the time tends to infinity. Moreover, all trajectories starting off the sliding surface $S=0$ must reach it in finite time and then will remain on this surface. This system's behavior once on the sliding surface is usually called *sliding mode* [14].

When the sliding mode occurs on the sliding surface (48), then $S(t)=\dot{S}(t)=0$, and therefore the dynamic behavior of the tracking problem (44) is equivalently governed by the following equation:

$$\dot{S}(t)=0 \quad \Rightarrow \quad \dot{e}(t)=-(k+a)e(t) \quad (52)$$

Then, under assumption $(\mathcal{A}\,1)$, the tracking error $e(t)$ converges to zero exponentially.

It should be noted that, a typical motion under sliding mode control consists of a *reaching phase* during which trajectories starting off the sliding surface $S=0$ move toward it and reach it in finite time, followed by *sliding phase* during which the motion will be confined to this surface and the system tracking error will be represented by the reduced-order model (eqn. 52), where the tracking error tends to zero.

Finally, the torque current command, $i_{qr}^{*}(t)$, can be obtained from equations (49) and (45):

$$i_{qr}^{*}(t)=\frac{1}{b}\left[k\,e+\beta\,\mathrm{sgn}(S)-a\,w^{*}-\dot{w}^{*}+f\right] \quad (53)$$

Therefore, the proposed variable structure speed control resolves the wind turbine speed tracking problem for variable speed wind turbines in the presence of uncertainties. This wind turbine speed tracking let us obtain the maximum wind power extraction for all wind speeds.

6 Grid Side Converter Control

The objective of the GSC control is to keep the dc link voltage constant regardless of the direction of rotor power flow. In order to achieve this objective, a vector control approach is used, with a reference frame oriented along the stator (or supply) voltage vector position. In such scheme, direct axis current is controlled to keep the dc link voltage constant, and quadrature axis current component can be used to regulate the reactive power flow between the supply side converter and the supply. In this vector control scheme, all voltage and current quantities are transformed to a reference frame that rotates at the same speed as the supply voltage space phase with the real axis (d-axis) of the reference frame aligned to the supply voltage vector. At steady state, the reference frame speed equals the synchronous speed.

The scheme makes use of the supply voltage angle determined dynamically to map the the supply voltage, the converter terminal voltage and the phase currents onto the new reference frame.

In the stator voltage oriented reference frame, the d-axis is aligned with the supply voltage phasor V_s, and then $v_d=V_s$ and $v_q=0$. Hence, the powers between the grig side converter and the grid are:

$$P = \frac{3}{2}(v_d i_d + v_q i_q) = \frac{3}{2} v_d i_d \quad (54)$$

$$Q = \frac{3}{2}(v_q i_d - v_d i_q) = -\frac{3}{2} v_d i_q \quad (55)$$

where v_d and v_q are the direct and quadrature components of the supply voltages, and i_d and i_q are the direct and quadrature components of the stator side converter input currents.

From the previous equations it is observed that the active and reactive power flow between supply side converter and the supply, will be proportional to i_d and i_q respectively.

The dc power change has to be equal to the active power flowing between the grid and the grid side converter. Thus,

$$E i_{0s} = \frac{3}{2} v_d i_d \quad (56)$$

$$C\frac{dE}{dt} = i_{0s} - i_{0r} \quad (57)$$

where E is the dc link voltage, i_{0s} is the current between the dc link and the rotor and i_{0r} is the current between the dc link and the stator.

From eqn. 56 and 57 it is obtained:

$$\dot{E} = \frac{1}{C}\left(\frac{3}{2}\frac{v_d}{E} i_d - i_{0r}\right) \quad (58)$$

$$\dot{E} = g(t) i_d - \frac{1}{C} i_{0r} \quad (59)$$

where the function $g(t)$ is defined as:

$$g(t) = \frac{1}{C}\frac{3}{2}\frac{v_d}{E} \quad (60)$$

The function $g(t)$ can be split up into two parts:

$$g(t) = g_0 + \triangle g(t) \tag{61}$$

where

$$g_0 = \frac{1}{C}\frac{3}{2}\frac{v_d}{E^*} \tag{62}$$

where the term E^* represents the reference value of E, and the term $\triangle g(t)$ takes into account the deviations from the reference value.

It should be noted that if the controller works appropriately, the term $\triangle g(t)$ will be a small value, because the dc link voltage will be roughly constant.

Then eqn. 59 can be put as,

$$\dot{E} = (g_0 + \triangle g)i_d - \frac{1}{C}i_{0r} \tag{63}$$

$$\dot{E} = g_0 i_d + \frac{1}{C}i_{0r} + d(t) \tag{64}$$

where $d(t) = \triangle g i_d$ is the uncertainty term.

Let us define the dc link voltage error as follows,

$$e(t) = E(t) - E^* \tag{65}$$

Taking the derivative of the previous equation with respect to time yields,

$$\dot{e}(t) = \dot{E}(t) - 0 = g_0 i_d + \frac{1}{C}i_{0r} + d(t) \tag{66}$$

$$= u(t) + d(t) \tag{67}$$

where

$$u(t) = g_0 i_d - \frac{1}{C}i_{0r} \tag{68}$$

Now the sliding variable $S(t)$ is defined with an integral component as:

$$S(t) = e(t) + \int_0^t \lambda\, e(\tau)\, d\tau \tag{69}$$

where λ is a positive constant

Then the sliding surface is defined as:

$$S(t) = e(t) + \int_0^t \lambda\, e(\tau)\, d\tau = 0 \tag{70}$$

Now, we are going to design a variable structure speed controller in order to regulate the dc link,

$$u(t) = -\lambda\, e(t) - \gamma\, \mathrm{sgn}(S) \tag{71}$$

where the λ is the constant gain defined previously, γ is the switching gain, S is the sliding variable defined in eqn. (69) and $\mathrm{sgn}(\cdot)$ is the signum function.

As in the case of the wind turbine speed controller, the following assumptions should be formulated in order to regulate the dc link:

($\mathcal{A}$1) The gain λ must be a positive constant.

($\mathcal{A}$2) The gain γ must be chosen so that $\gamma \geq |d(t)|$ for all time.

Note that this condition only implies that the uncertainty term $d(t)$ is a bounded magnitude.

Theorem 2 *Consider the dc-link voltage dynamic equation (64). Then, if assumptions* ($\mathcal{A}$1) *and* ($\mathcal{A}$2) *are verified, the control law (71) leads the dc-link voltage* $E(t)$*, so that the voltage regulation error* $e(t) = E(t) - E^*(t)$ *tends to zero as the time tends to infinity.*

The proof of this theorem will be carried out using the Lyapunov stability theory.

Proof : Define the Lyapunov function candidate:

$$V(t) = \frac{1}{2}S(t)S(t) \tag{72}$$

Its time derivative is calculated as:

$$\begin{aligned}\dot{V}(t) =\; & S(t)\dot{S}(t)\\ & = S\cdot(\dot{e} + \lambda e)\\ & = S\cdot(u + d + \lambda\, e))\\ & = S\cdot(-\lambda\, e - \gamma\, \mathrm{sgn}(S) + d + \lambda\, e)\\ & = S\cdot(d - \gamma\, \mathrm{sgn}(S))\\ & \leq -(\gamma - |d|)|S|\\ & \leq 0\end{aligned} \tag{73}$$

Using the Lyapunov's direct method, since $V(t)$ is clearly positive-definite, $\dot{V}(t)$ is negative definite and $V(t)$ tends to infinity as $S(t)$ tends to infinity, then the equilibrium at the origin $S(t) = 0$ is globally asymptotically stable. Therefore $S(t)$ tends to zero as the time tends to infinity. Moreover, all trajectories starting off the sliding surface $S = 0$ must reach it in finite time and then will remain on this surface.

When the sliding mode occurs on the sliding surface (70), then $S(t) = \dot{S}(t) = 0$, and therefore the dynamic behavior of the regulation problem (67) is equivalently governed by the following equation:

$$\dot{S}(t) = 0 \quad \Rightarrow \quad \dot{e}(t) = -\lambda e(t) \tag{74}$$

Then, under assumption ($\mathcal{A}$1), the regulation error $e(t)$ converges to zero exponentially.

Finally, the direct component of the supply voltage command, $i_d^*(t)$, can be obtained from equations (71) and (68):

$$i_d^*(t) = \frac{1}{g_0}\left[\frac{1}{C}\, i_{0r} - \lambda e - \gamma\, \mathrm{sgn}(S)\right] \tag{75}$$

Therefore, the proposed variable structure voltage control resolves the dc link voltage regulation.

7 Simulation Results

In this section we will study the variable speed wind turbine regulation performance using the proposed sliding-mode field oriented control scheme. The objective of this regulation is to maximize the wind power extraction in order to obtain the maximum electrical power. In this sense, the wind turbine speed must be adjusted continuously against wind speed.

The simulation are carried out using the Matlab/Simulink software and the turbine model is the one provided in the SimPowerSystems library [12].

In this example simulation it is used a variable speed wind farm with a rated power of 9 MW. The farm consists of six 1.5 MW wind turbines connected to a 575 V bus line. The wind turbines use a doubly-fed induction generator (DFIG) consisting of a wound rotor induction generator and an AC/DC/AC IGBT-based PWM converter. The stator winding is connected directly to the 60 Hz grid while the rotor is fed at variable frequency through the AC/DC/AC converter.

The system has the following mechanical parameters. The combined generator and turbine inertia constant is $J = 5.04s$ expressed in seconds, the combined viscous friction factor $B = 0.01pu$ in pu based on the generator rating and there are three pole pairs [12].

In this simulation examples it is assumed that there is an uncertainty around 20 % in the system parameters, that will be overcome by the proposed sliding control.

Finally, the following values have been chosen for the controller parameters, $k = 100$, $\beta = 30$, $\lambda = 70$ and $\gamma = 25$.

In this case study, the rotor is running at subsynchronous speed for wind speeds lower than 10 m/s and it is running at a super-synchronous speed for higher wind speeds. In figure 3, the turbine mechanical power as function of turbine speed is displayed in, for wind speeds ranging from 5 m/s to 16.2 m/s.

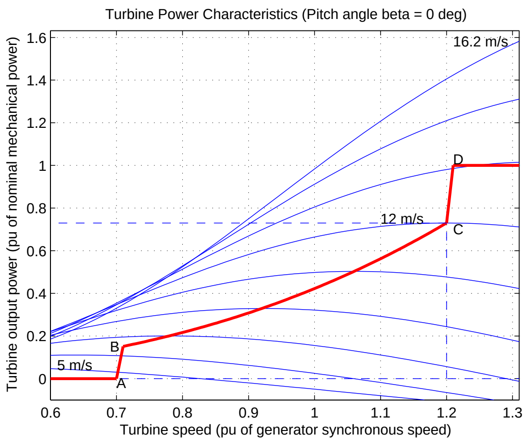

Figure 3. Turbine Power Characteristics

In this simulation it is used a variable wind speed, and figure 4 shows the wind speed used in this simulation. As it can be seen in the figure, the wind speed varies between $0m/s$ and $20m/s$.

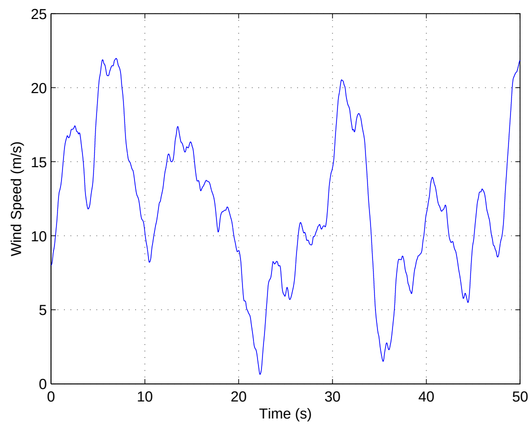

Figure 4. Wind speed

Figure 5 show the reference (dashed line) and the real rotor speed (solid line). As it may be observed, after a transitory time in which the sliding mode is reached, the rotor speed tracks the desired speed in spite of system uncertainties. In this figure, the speed is expressed in the per unit system (pu), that is based in the generator synchronous speed $w_s = 125.60rad/s$.

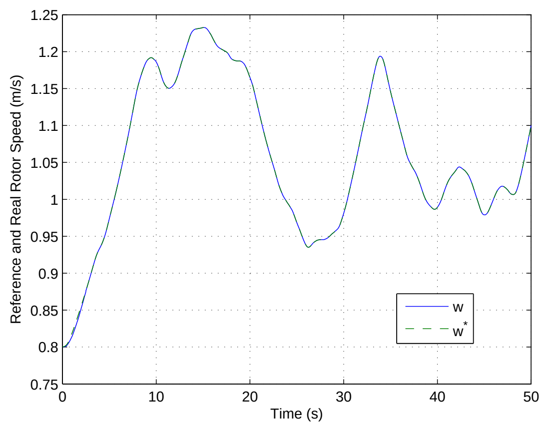

Figure 5. Reference and real rotor speed

Figure 6 shows the generated active power, whose value is maximized by our proposed sliding mode control scheme. As it can be observed in this figure, at time $12.1s$ the mechanical power (and therefore the generated active power) should be limited by the pitch angle so as not to exceed the rated power of this system.

8 Conclusion

In this paper a sliding mode vector control for a doubly feed induction generator drive, used in variable speed wind power generation is described. It is proposed a new

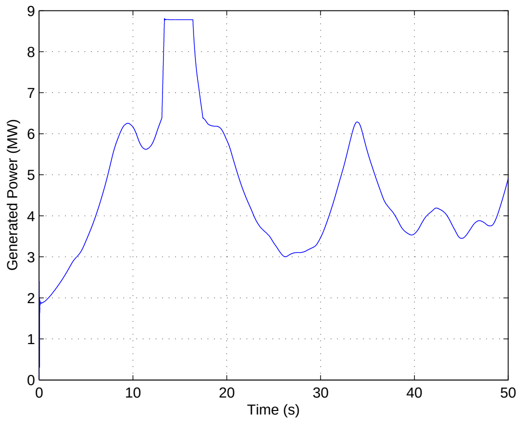

Figure 6. Generated active power

variable structure control which has an integral sliding surface to relax the requirement of the acceleration signal, that is usual in conventional sliding mode speed control techniques. Due to the nature of the sliding control this control scheme is robust under uncertainties that appear in the real systems. The closed loop stability of the presented design has been proved through Lyapunov stability theory.

The implemented control method allows the wind turbine to operate with the optimum power efficiency over a wide range of wind speed.

The simulations show that the control method successfully controls the variable speed wind turbine efficiently, within a range of normal operational conditions.

At wind speeds less than the rated wind speed, the speed controller seeks to maximize the power according to the maximum coefficient curve. As result, the variation of the generator speed follows the slow variation in the wind speed. At large wind speeds, the power limitation controller sets the blade angle to maintain rated power.

Finally, by means of simulation examples, it has been shown that the proposed control scheme performs reasonably well in practice, and that the speed tracking objective is achieved in order maintain the maximum power extraction under system uncertainties.

Acknowledgment

The authors are very grateful to the Basque Government by the support of this work through the project S-PE07UN04, to the UPV/EHU by its support through project GUI07/08 and to MEC by its support through projects DPI2006-01677 and DPI2006-00714.

References

[1] FERNANDO D. BIANCHI, HRNAN DE BATTISTA AND RICARDO J. MANTZ 2007 *Wind Turbine Control Systems* Advances in Industrial Control Series. Springer.

[2] GAGNON, R. SYBILLE, G. ET ALL 2005 Modelling and Real-Time Simulation of a Doubly-Fed Induction Generator Driven by a Wind Turbine *Int. Conf. on Power Systems Transients.* Montreal, Canada. Paper No. IPST05-162.

[3] D.J. LEITH AND W.E. LEITHEAD 1997, Implementation of wind turbine controllers, *Int. J. Control.*, **66**, pp. 349-380.

[4] YAZHOU LEI, ALAN MULLANE, GORDON LIGHTBODY, AND ROBERT YACAMINI 2006, Modeling of the Wind Turbine With a Doubly Fed Induction Generator for Grid Integration Studies, *IEEE Trans. on Energy Conversion*, vol. 21, no. 1, pp. 257-264.

[5] G.M. JOSELIN HERBERT, S. INIYAN, E. SREEVALSAN, S. RAJAPANDIAN 2007, A review of wind energy technologies, *Renewable and Sustainable Energy Reviews.*, **11**, pp. 11171145.

[6] MULLER, S., DEICKE, M., DE DONCKER, R.W., 2002, Doubly fed induction generator system for wind turbines, *IEEE Industry Application Magazine.*, **8**, pp. 26-33.

[7] D. W. NOVOTNY AND T. A. LIPO, 2000 *Vector Control and Dynamics of AC Drives*. Oxford, U.K.: Oxford Univ. Press, 2000.

[8] R. PENA, J. C. CLARE, AND G. M. ASHER 1996 Doubly fed induction generator using back-to-back PWM converters and its application to variablespeed wind-energy generation *Proc. Inst. Elect. Eng.*, vol. 143, no. 3, pp. 231241.

[9] WEI QIAO, WEI ZHOU, JOS M. ALLER, AND RONALD G. HARLEY 2008, Wind Speed Estimation Based Sensorless Output Maximization Control for a Wind Turbine Driving a DFIG *IEEE Trans. on Power Electronics*, vol. 23, no. 3, pp. 1156-1169.

[10] Y.D. SONG, B. DHINAKARAN, X.Y. BAO, 2000, Variable speed control of wind turbines using nonlinear and adaptive algorithms, *Journal of Wind Engineering and Industrial Aerodynamics.*, **85**, pp. 293-308.

[11] SIEGFRIED HEIER 1998 *Grid Integration of Wind Energy Conversion Systems* John Wiley & Sons Ltd, 1998, ISBN 0-471-97143-X.

[12] *SimPowerSystems 5. Users Guide*, The MathWorks

[13] SUN, T., CHEN, Z., BLAABJERG, F., 2005, Transient stability of DFIG wind turbines at an external short-circuit fault, *Wind Energy.*, **8**, pp. 345-360.

[14] UTKIN V.I., 1993, Sliding mode control design principles and applications to electric drives, *IEEE Trans. Indus. Electro.*, **40**, 26-36.

[15] VLADISLAV AKHMATOV, 2002, Variable-Speed Wind Turbines with Doubly-Fed Induction Generators,*Wind Engineering*, vol. 26, n. 2, pp. 85108.

[16] WORLD WIND ENERGY ASSOCIATION 2009 *World Wind Energy Report 2008* WWEA Head Office. Bonn, Germany.