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On the Benefits of Including Age-structure in Harvest Control Rules

On the Benefits of Including Age-structure in Harvest Control Rules *

José-María Da-Rocha† Rosa Mato-Amboage‡

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Abstract

This paper explores the benefits of including age-structure in the control rule (HCR) when decision makers regard their (age-structured) models as approximations. We find that introducing age structure into the HCR reduces both the volatility of the spawning biomass and the yield. Although at a fairly imprecise level the benefits are lower, there are still major advantages for actual assessment precision of the case study. Moreover, we find that when age-structure is included in the HCR the relative ranking of different policies in terms of variance in biomass and yield does not differ. These results are shown both theoretically and numerically by applying the model to the Southern Hake fishery.

Keywords: Management Strategy Evaluation, Harvest Control Rules, Reference Points, Robustness, Risk

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1 Introduction

Fishery biologists have long called for economic models based on more complex, more accurate biological models. A crucial extension of biological complexity is the inclusion of age-structure. Biomass based models have been criticized often for their poor performance. For example Hilborn and Walters [23] claim that age structure models should be used, and biomass based models should be considered only if data are not available. Wilen [37] also notes that the biomass approach is too simplistic for empirical applications and policy making. Similar remarks are found in dynamic optimization studies, such as Tahvonen [33], [32].

Yet, there has been an upsurge of bioeconomic models that incorporate age into the biological dimension[1]. However Harvesting Control Rules biomass based continue to be widely used (Daroba [16]). Most of the criticism comes from the additional data and knowledge required to correctly estimate age structure. Moreover, it is commonly assumed that when it is not possible to estimate age-structure with near-perfect accuracy a robust approach implies the use of simple rules.

The contribution of this paper is to show the benefits of introducing age structure into harvest control rules even when it is not possible to estimate that structure with near-perfect accuracy. The gains from adding age structure depend on the accuracy of the estimates. As the imprecision increases, the benefits of introducing age structure relative to using a simple biomass based rule decrease. Moreover, we show that, unlike simpler rules, including age-structure in the control rule reduces stock volatility and generates a positive correlation between variances in biomass and yield.

We develop a theoretical model where the goal of the manager is to stabilize the resource close to a target point. Fishery managers usually use harvest control rules based on the use of target reference points which are indicators of a stock status which is a desirable target for management [2]. In our case, this reference point is exogenously given, and the manager must avoid the risk of the stock dropping below a limit point.

We assume that decision makers estimate age-structure with non-perfect accuracy (see Cope and Punt,[9]). Thus, at each point in time and for all age levels, the manager makes an estimation error when applying the optimal harvest control. Moreover, we are implicitly assuming that there exists model uncertainty on the Stock Recruitment relationship that can be backed by having uncertainty in the stock recruitment process.

[1]See Da Rocha [15], [11],[12], [13], [14], Dichmont [18], Diekert[19], Grafton [20], Kompas [28], Skonhoft [31], Tahvonen [35] and Voss [34].

[2]These are usually related to the size of the stock biomass and the fishing mortality associated with this stock size, see [7])

We show the results in two steps. Firstly, we show that the benefits of including age-structure in the control rule clearly hold for the case of stochastic recruitment. We find that in HCRs that account for the age structure of the biomass the volatility of the stock decreases. We also find that the optimal fishing mortality decreases as the correlation of the stochastic recruitment process increases. The intuition of this result is that a higher correlation implies higher persistence and smoother future recruitment levels. Thus, if the target level is far from being met now it is very likely to remain that way in the following periods. To avoid increasing the gap still further it is optimal to lower F. The opposite case is when recruitments are independent over time, i.e. correlation is zero, which means that coming close to the reference point in the future is independent of the recruitments made in the current period. Thus, there are incentives to exploit the stock further for the same recruitment level.

The HCR also depends on the relative age composition of spawners. This result asserts that biomass models that ignore age-structure underestimate the optimal fishing mortality and increase the risk of the stock dropping below a threshold point. We find that the higher the relative contribution of juveniles to the total spawner biomass is, the lower the fishing mortality associated with optimal control is. Therefore biomass-based HCR which do not account for age structure result in an underestimation of the optimal fishing mortality.

Secondly we show the benefits of including age-structure when it is estimated with some degree of imprecision by using numerical simulations. Even under incomplete knowledge of the dynamics, we find that if the goal is to stabilize the resource so that it does not drop below a limit point, it is advantageous to include age-structure.

These results have direct implications for policy making. First, we show that using biomass-based HCRs that ignore age-structure increases the gap between the biomass and the target level. Second, constant-effort harvesting control rules increase stock volatility and overestimate the risk status of stocks.

Our results contribute to two areas. First, we extend previous findings of the benefits of introducing age-structure in fisheries management to a stochastic environment. Second, fishery researchers and international agencies have developed different methods to take into account the risk carried by different HCRs. In recent years, management strategy evaluation (MSE) has attempted to deal with this issue (Dichmont [17], Francis and Mace [29], Parma [30]). The goal is to understand the trade offs and limitations of a set of feasible management options, rather than obtaining the best or optimal solution. Such evaluations are commonly conducted using Monte Carlo simulation methods to assess the different strategies. While there is literature on the technical aspects and the practical experience of this method (Kell [26], [27]), to the best of our knowledge there is no theoretical framework that evaluates the implications of different strategies for the management of an age-structured population model and derives theoretical conditions from among the managerial tools.

The rest of the paper is organized as follows. Section 2 draws up a simplified model with two age classes. From this simplified model it is possible to characterize the relationship between harvesting control rules (HCR), reference points and the risk of the stock dropping below a limit point. Section 3 evaluates the impact of using biomass-based models in inherently age-structured models. Finally, Section 4 generalizes the model to any number of age classes and applies the model the Southern Hake Fishery to conduct numerical experiments that exemplify the benefits of introducing age structure into harvest control rules even when it is not possible to estimate that structure with near-perfect accuracy.

2 The Case of Stochastic Recruitment

A useful simplification of the biological structure of a fishery is to consider a model with only two age classes: juveniles and adults. This simplification enables analytic conclusions to be drawn about the relationship between the risk of the stock dropping below the limit reference point and the target reference point.

2.1 Stock Dynamics

Consider a stochastic version of Hannesson's [21] fishery based in the Beverton-Holt model with two age classes, juveniles and adults. Let N_t^1, and N_t^2 be the population of juveniles and adults in period t, respectively. Each year, t, a stochastic exogenous number of juvenile fish are born

$$N_{t,1} = exp(z_t)$$

where z_t follows an AR(1) process

$$z_{t+1} = \rho z_t + \varepsilon_{t+1},$$

with zero mean, $E\varepsilon_{t+1} = 0$, and variance σ_z.

The parameter ρ, the correlation coefficient, defines the relationship between the number of recruitments today and tomorrow. Note that we are assuming that the number of juveniles is independent of the spawning biomass of the system. Although we are assuming stochastic recruitment, we care about spawners in order to stabilize the entire population around the reference point. We are implicitly assuming that there exists uncertainty on the Stock Recruitment relationship. Therefore, an stochastic recruitment is considered a better model choice if spawner's biomass levels are not far from the target point [3].

Additionally, it is assumed that only a part of the juveniles survive to become adults next period. The dynamics of the second age group are then:

$$N_{t+1,2} = N_{t,1}e^{-pF_t-m},$$

[3]In order for the dynamics not to explode, we assume that $\rho < 1$

where m is the natural mortality rate and p is the selectivity parameter that indicates how the fishing effort affects the fishing mortality of juveniles. Thus, the first term in the exponential represents fishing impacts, while the second refers to mortality due to natural causes.[4]

For simplicity, we can do a change of variables, and define the dynamics in logarithm terms. Thus, the stock dynamics become:

$$\log N_1 = z$$

and

$$\log N_2 = x$$

So that the population can be represented as follows:

		Time (t)			
age classes		t	$t+1$	$t+2$	$t+3$
log Juveniles	z	z_t			
log Adults	x		$z_t - pF_t - m$		

Finally, the total biomass of the fishery can be defined as:

$$B_t = \log N_{t,2},$$

This equation implies that the spawning stock biomass is an increasing function of the number of adults in the population and that only a non constant fraction of adults are spawners.[5]

2.2 Management of the Fishery

Assume that the fishery is managed to reach an exogenous target reference point. This target point may be, for example, the B_{msy} and the corresponding F_{msy}, or any other target that represents the objectives of the manager [6].

Formally, the manager's objective is to minimize the distance between the fishing mortality, F_t, and the biomass, B_t to the target reference point, (B_{tar}, F_{tar}) subject to the stock dynamics, where the expectation term is associated with the recruitment stochastic process, and $\lambda > 0$ weights the importance of biomass versus effort-oriented

[4] One problem of this type of stock dynamics is how to deal with the last age group. For the purposes of this exercise, we can assume that all adults who are not caught, they die, simulating a maximum surviving age.

[5] Note that only a fraction of adults are spawners given that $\log N_{t,2} < N_{t,2}$ and that this fraction $\frac{\log N_{t,2}}{N_{t,2}}$ is decreasing.

[6] In this paper we use the Maximum Sustainable Yield, but any other reference point can be applied [17]

objectives; in other words, it characterizes the trade offs between the two target objectives.

We can plot the target in a 2D graph:

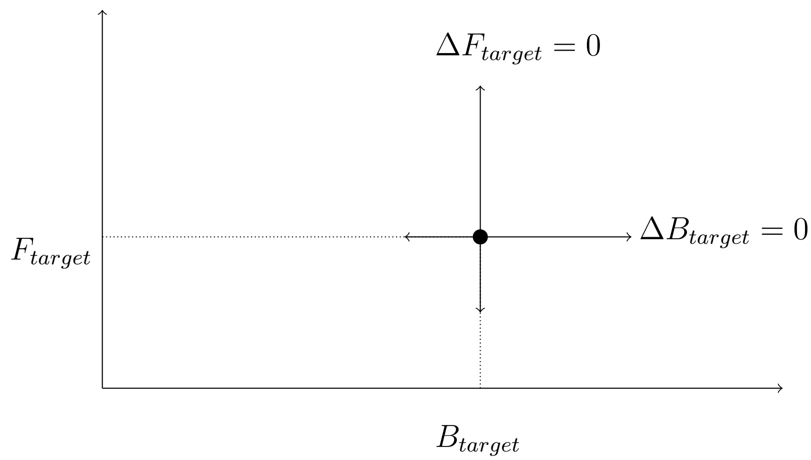

Figure 1: Biomass and Fishing mortality target point and its isolines

Then choose the desired gap between the state of the stock and the target. This gap is given by the weight that the manager has on reaching the effort reference point relative to the biomass target, thus determining whether the fishery is to be managed by effort-focused control rules or biomass control rules.

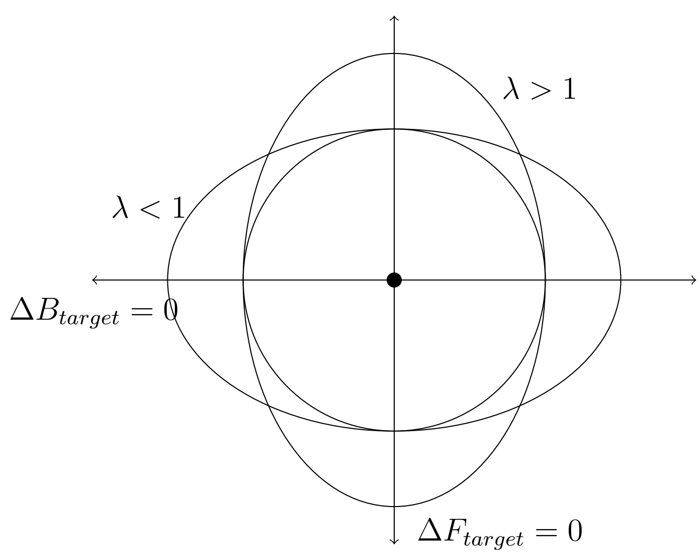

Figure 2: Defining the HCR: relating the value of λ and distances to the target point

In other words, the HCR for stochastic age structured models can be defined as the optimal feedback policy" that minimizes the weighted sum of squares between the stock assessment outputs and a given "biological reference point". Then, given this HCR, it is possible to explore the "dynamics" of the fishery and the implications of the three managerial tools on reaching the target.

The problem can be stated as follows:

$$\max_{F_t, B_{t+1}} E_0 \quad \sum_{t=0}^{\infty} -\beta^t \left\{ (F_t - F_{tar})^2 + \lambda (B_t - B_{tar})^2 \right\}$$

$$s.t. \quad \begin{cases} B_{t+1} = z_t - pF_t - m \\ \\ z_{t+1} = \rho z_t + \varepsilon_{t+1}. \end{cases}$$

Note that the state variables of this problem are $z_t = \log(N_{1,t})$ and $B_t = \log(N_{2,t})$, and F_t is the control variable.

One novelty of this approach is that the harvest control rules are characterized in terms of distances to a target point. The objective is to stabilize the resource around a desired point. Moreover, the different HCR are given in terms of the value of a single parameter, λ. Later, it is shown how we can relate different commonly used HCR to the rules obtained adjusting λ.

The problem can be simplified with the following change of variables: $\Delta F = F_t - F_{tar}$, $\Delta z_t = z_t - z_{tar}$ and $\Delta B_t = B_t - B_{tar}$. Now the minimization problem can be rewritten as:

$$\max_{\Delta F_t, \Delta B_{t+1}} E_0 \quad \sum_{t=0}^{\infty} -\beta^t \left\{ \Delta F_t^2 + \lambda \Delta B_t^2 \right\}$$

$$s.t. \quad \begin{cases} \Delta B_{t+1} = \Delta z_t - p\Delta F_t \\ \\ \Delta z_{t+1} = \rho \Delta z_t + \varepsilon_{t+1}. \end{cases}$$

Note, that the dynamics of the stock are independent of the natural mortality parameter m.[7] The natural mortality of a population, m, is rarely known; it is one of the largest sources of uncertainty in the biological dynamics of the stock. Thus, one advantage of using an HCR that minimizes the distance to a given target reference point is that the HCR is robust with respect to this uncertainty.

The problem can easily be converted into an unconstrained deterministic optimization problem. The solution of the unconstrained problem must verify:

$$\max_{\Delta F_t} - \left\{ \Delta F_t^2 + \lambda \Delta B_t^2 \right\} - \beta \left\{ \Delta F_{t+1}^2 + \lambda (\Delta z_t - p\Delta F_t)^2 \right\}.$$

[7] This occurs because when taking differences, $B_{t+1} - B_{tar} = (z_t - pF_t - m) - (z_{tar} - pF_{tar} - m)$.

The first order conditions is

$$\Delta F_t - p\beta\lambda(\Delta z_t - p\Delta F_t) = 0.$$

Solving for the HCR, ΔF_t, we have

$$\Delta F_t = \frac{p\beta\lambda}{1 + p^2\beta\lambda}\Delta z_t,$$

which is linear in the state variable Δz_t.[8] Combining the HCR with the dynamics of the stock, $\Delta B_{t+1} = \Delta z_t - p\Delta F_t$, we have

$$\Delta B_{t+1} = \frac{1}{1 + p^2\beta\lambda}\Delta z_t.$$

Already from this simple model it is possible to start drawing conclusions on the impact of recruitments and possible implications of the model parameters for the design of HCR. The first thing to note is that good recruitments imply higher fishing mortality. While this relation makes intuitive sense, it is important to note that it will hold independently of any starting point. That is, whatever the spawner biomass level is, good recruitment in the last year implies higher fishing mortality, even if the biomass level is lower than B_{tar}. This result is related to Tahvonen [32], [33].

2.3 HCRs and the Role of λ

A second implication of the results above concerns the role of the parameter λ. Note that the higher λ is, the more weight is given by managers to achieving the target biomass in the future. Thus, a high λ reduces the possible deviations from the target biomass. The optimal rule sets that the higher λ is, the higher the variations in fishing mortality are and the smaller the gap is between B_{t+1} and the target. Therefore, by adjusting the parameter λ, it is possible to parameterize a continuum of HCRs.

Formally, the value of λ determines the slope of the HCR

$$\frac{\Delta F_t}{\Delta B_{t+1}} = p\beta\lambda$$

One of the advantages of the model is that the distance control rules can be related to the common rules used in the relevant literature. By changing the value of λ we can replicate some of the most widely used rules in real world fisheries (Deroba and Bence [16]). If $\lambda < 0$, the HCR generates a negative relationship between fishing mortality and biomass, similar to that of a constant catch rule. If $\lambda = 0$, the HCR reproduces a constant fishing mortality rule. If $\lambda > 0$ the HCR reproduces a biomass-based rule. Finally, if $\lambda \to \infty$, the HCR reproduces a constant or fixed escapement rule.

[8] In this simplified case, the optimal harvest rule is independent of the age-structure of the biological population. This is due to the simple dynamics imposed on the problem, that only adults are spawners.

2.4 HCR and Risk

Many fisheries worldwide use threshold reference points as limit points beyond which harvesting is decreased or ceased in order to allow for stock rebuilding [16]. Thus an important question when choosing the optimal HCR and target reference point is to identify the likelihood of the managed population falling below a given limit (see Francis and Mace, 2005). As Beddington Agnew and Clark [3] point out *it is important to avoid situations where the stock is at or below this level. Accordingly, management should aim to target a level of stock size that carries a low risk (allowing for scientific uncertainty) of the stock dropping below the limit reference point.*

Therefore, in this section we develop the theoretical conditions for designing harvest control rules for stochastic age-structured models when management wants to avoid the risk of the stock dropping below a limit point.

By iterating the stochastic process in the stock dynamics

$$\Delta B_{t+1} = \frac{1}{1+p^2\beta\lambda}\Delta z_t = \frac{1}{1+p^2\beta\lambda}\left(\rho\Delta z_{t-1} + \varepsilon_t\right)$$

Solving for the total biomass next period we have:

$$B_{t+1} = B_{tar} + \left(\frac{\rho}{1+p^2\beta\lambda}\right)^N \Delta z_{t-k-1} + \sum_{k=1}^{N}\left(\frac{\rho}{1+p^2\beta\lambda}\right)^k \varepsilon_{t-k}$$

Taking limits, as T goes to infinity

$$\lim_{T\to\infty} B_{t+1} = B_{tar} + \sum_{k=1}^{\infty}\left(\frac{\rho}{1+p^2\beta\lambda}\right)^k \varepsilon_{t-k}$$

Assuming that ε_t is a Gaussian process, then B_{t+1} also follows a Gaussian distribution. Thus, we can easily obtain the biomass moments: the expected value, μ_B, and its variance, σ_B, the mean and variance are given by

$$\mu_B = B_{tar}$$

and

$$\sigma_B = \sigma_z \sum_{k=0}^{\infty}\left(\frac{\rho}{1+p^2\beta\lambda}\right)^k = \frac{(1+p^2\beta\lambda)^2}{(1+p^2\beta\lambda)^2-\rho^2}\sigma_z$$

It is now possible to relate the design of target reference points, in a stochastic environment, given a predetermined risk level that is to be avoided. Thus, to calculate the value of λ for which $Pr(B \leq B_{lim}) = v$, the cumulative distribution can be used:

$$Pr(B \leq B_{lim}) = \frac{1}{2}\left[1 + \text{erf}\left(\frac{B_{lim}-\mu_B}{\sigma_B\sqrt{2}}\right)\right] = v$$

where erf is the Gaussian error function. Rearranging terms, we have

$$\lambda_v = \frac{\sqrt{\dfrac{\rho^2(B_{lim}-B_{tar})}{(B_{lim}-B_{tar})-\sigma_z \mathrm{erf}^{-1}(2v-1)\sqrt{2}}}-1}{p^2\beta}$$

λ_v is the HCR for which v is the probability that the stock is below a given threshold B_{lim}.

The feasible set of HCR compatible with an uncertainty σ can now be characterized. Formally we have[9]

$$\lambda_v \begin{cases} \leq 0 & \text{if} \quad \sigma_z \leq \dfrac{(1-\rho^2)(B_{tar}-B_{lim})}{-\mathrm{erf}^{-1}(2v-1)\sqrt{2}} \\ \\ \to \infty & \text{if} \quad \sigma_z \to \dfrac{||B_{tar}-B_{lim}||}{||\mathrm{erf}^{-1}(2v-1)\sqrt{2}||} \end{cases}$$

Figure 4 shows the link between biomass, risk as measured by variance and HCR. We show how the three elements are related to one another. Thus, not all combinations of reference points and HCR carry an acceptable risk level. For example, if it is wished to decrease risk while still maintaining a target point of MSY, the HCR must be changed to a more biomass-based catch or even constant escapement. Similarly, if the target reference point is B_{msy} and the manager switches from constant escapement to constant effort controls, the risk of overfishing increases automatically.

Thus,the feasible set in the state space of uncertainty and target reference points can be characterized by computing the following iso-HCR lines:

$$\left.\frac{dB_{tar}}{d\sigma_z}\right|_{\lambda_v=0} = \frac{-\mathrm{erf}^{-1}(2v-1))\sqrt{2}}{(1-\rho^2)} > 0$$

and

$$\left.\frac{dB_{tar}}{d\sigma_z}\right|_{\lambda_v\to\infty} = -\mathrm{erf}^{-1}(2v-1))\sqrt{2} > 0$$

In Figure 4 we can see that for a given a target risk level, $Pr(B \leq B_{lim}) = v$, $\sigma_z(\lambda_v = 0, B_{msy})$ is the maximum uncertainty compatible with sustaining MSY as the target reference point. If the uncertainty is between $\sigma_z(\lambda_v = 0, B_{msy})$ and $\sigma_z(\lambda_v \to \infty, B_{msy})$, MSY is feasible only if we use a biomass based control rule, that is $\lambda > 0$. Therefore, if uncertainty is higher than $\sigma_z(\lambda_v = 0, B_{msy})$, the target level of fishing mortality must provide stock sizes above B_{msy}.

[9] Remember that $\mathrm{erf}^{-1}(2v-1) < 0$ if $v < 50\%$.

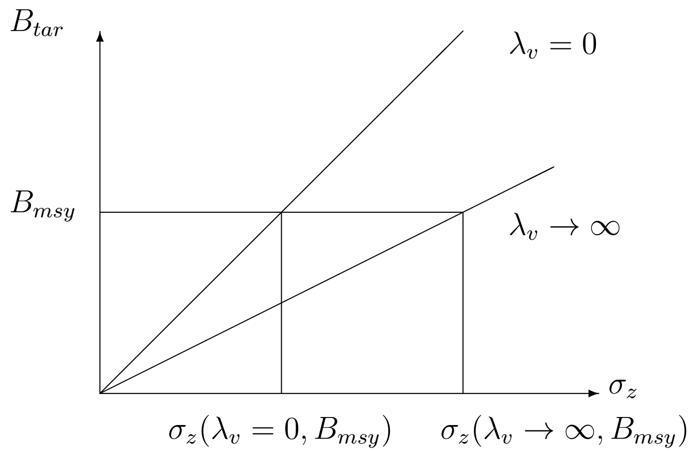

Figure 3: Feasible HCR set in the uncertainty and target reference points space for a given risk v.

Let v be the probability that the stock is below a given threshold B_{lim}. Then, if the uncertainty is such that

$$\sigma_z > \frac{||B_{msy} - B_{lim}||}{||\text{erf}^{-1}(2v-1)\sqrt{2}||}$$

the optimal reference biomass level, B_{tar}, must be higher than B_{msy}. This result that B_{msy} should be more of a limit point, rather than the target has become appealing, given that MSY does not account for stochastic processes. Thus, by moving the target point above the MSY the risk of overfishing can be reduced.

3 Stochastic Properties of Heuristic HCR

As mentioned in the introduction, while there have been some advances in the introduction of biological structure in bioeconomic models ([33], [15]), there is still much need for further understanding about the implications of using biomass-based models in age structured populations. The aim of this section is to evaluate the impact of applying a heuristic harvesting control rule to a structured population.

3.1 Optimal HCR

Consider the Hannesson fishery [21] with three age-classes, juveniles, pre-adults, and adults. Let $N_{t,1}$ be the population of juveniles, and $N_{t,2}$ and $N_{t,3}$ be the population of pre-adults and adults in period t, respectively. Assume also that there is perfect selectivity on juveniles and the first adult group, i.e. $p_z = 0 = p_{x_1} = 0$ and denote the second adult selectivity p_{x_2} as p, also known as knife-edge selectivity. Thus, the dynamics of the two adult groups are given by:

$$N_{t+1,2} = N_{t,2}e^{-m}$$

$$N_{t+1,3} = N_{t,3}e^{-pF_t - m}$$

where m is the natural mortality rate and p is the selectivity parameter.

Finally, the total biomass of the fishery can be defined as:

$$B_t = \sum_{a=0}^{2} \mu_a N_{a,t} = \mu_z e^{z_t} + \mu_{x_1} e^{x_{1,t}} + \mu_{x_2} e^{x_{2,t}}$$

Let us assume that the fishery is managed to reach an exogenously given target reference point.

$$\max_{F_t, x_{t+1}} E_0 \quad \sum_{t=0}^{\infty} -\beta^t \left\{ (F_t - F_{tar})^2 + \lambda (B_t - B_{tar})^2 \right\}$$

$$s.t. \quad \begin{bmatrix} 1 \\ z_{t+1} \\ x_{1,t+1} \\ x_{2,t+1} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \rho & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ z_t \\ x_{1,t} \\ x_{2,t} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ -p \end{bmatrix} F_t + \begin{bmatrix} 0 \\ \varepsilon_{t+1} \\ 0 \\ 0 \end{bmatrix}$$

The problem can be simplified with the following change of variables: $\Delta F = F_t - F_{tar}$, $\Delta z_t = z_t - z_{tar}$ and $\Delta B_t = B_t - B_{tar}$. Now we can rewrite the minimization problem as:

$$\max_{\Delta F_t, \Delta B_{t+1}} E_0 \quad \sum_{t=0}^{\infty} -\beta^t \left\{ \Delta F_t^2 + \lambda \Delta B_t^2 \right\}$$

$$s.t. \quad \begin{cases} \Delta z_{t+1} = \rho \Delta z_t + \varepsilon_{t+1} \\ \Delta x_{t+1,1} = \Delta x_{t,1} \\ \Delta x_{t+1,2} = \Delta x_{t,2} - p\Delta F_t \end{cases}$$

Note that the objective function is not a quadratic function of the state, given that

$$B_t - B_{tar} = \mu_z e^{z_t} + \mu_{x_1} e^{x_{1,t}} + \mu_{x_2} e^{x_{2,t}} - \mu_z e^{z_{tar}} - \mu_{x_1} e^{x_{1,tar}} - \mu_{x_2} e^{x_{2,tar}}.$$

It is possible to approximate the objective function using a Taylor expansion of order two around the target (B_{tar}, F_{tar}). After the approximation, a standard LQ problem results.

$$\max_{\mathbf{y}_t} E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \mathbf{x}_t^T \mathbf{R} \mathbf{x}_t + 2\mathbf{y}_t^T \mathbf{W} \mathbf{x}_t + \mathbf{y}_t^T \mathbf{Q} \mathbf{y}_t \right\}$$

$$s.t. \qquad \mathbf{x}_{t+1} = \mathbf{A}\mathbf{x}_t + \mathbf{B}\mathbf{y}_t + \varepsilon_{t+1} \qquad \varepsilon_{t+1} \sim (0, \Sigma)$$

where $\mathbf{y} = F_t - F_{tar}$, is the control and $\mathbf{x}^T = [1 \quad z_t \quad x_{1,t} - x_{1,tar} \quad x_{2,t} - x_{2,tar}]$ are the state variables. In this notation, ^{T} is used to denote the transpose of a matrix. The following matrices describe the stock dynamic parameters. $\mathbf{R}$ is a matrix of relative effects; $\mathbf{A}$ is a transitional matrix of each age group; $\mathbf{B}$ is a vector of the proportions of stock removed from each age group at each time period, and Σ is the variance-covariance matrix. In our model $\mathbf{W}$ and $\mathbf{Q}$ are defined as $\mathbf{W}_{1\times4} = \begin{bmatrix} 0 & 0 & 0 & 0 \end{bmatrix}$ and $\mathbf{Q}_{1\times1} = [-1]$, and $\mathbf{R}$, $\mathbf{A}$ and $\mathbf{B}$ are defined as

$$\mathbf{R}_{4\times4} = \lambda \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & -\mu_z^2 e^{2z_{tar}} & -\mu_z \mu_{x^1} e^{z_{tar}} e^{x_{1,tar}} & -\mu_z \mu_{x_2} e^{z_{tar}} e^{x_{2,tar}} \\ 0 & -\mu_{x^1} \mu_z e^{z_{tar}} e^{x_{1,tar}} & -\mu_{x^1}^2 e^{2x_{1,tar}} & -\mu_{x^1} \mu_{x_2} e^{x_{1,tar}} e^{x_{2,tar}} \\ 0 & -\mu_{x_2} \mu_z e^{z_{tar}} e^{x_{2,tar}} & -\mu_{x^1} \mu_{x_2} e^{x_{1,tar}} e^{x_{2,tar}} & -\mu_{x_2}^2 e^{2x_{2,tar}} \end{bmatrix}$$

$$\mathbf{A}_{4\times4} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \rho & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \qquad \mathbf{B}_{4\times1} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ -p \end{bmatrix}$$

Problems of this type have a special structure which can be exploited to derive the optimal solution using the Ricatti equation. The optimality condition in LQ is given by

$$G = -\left(\mathbf{Q} + \mathbf{B}^T \mathbf{P} \mathbf{B}\right)^{-1} \mathbf{B}^T \mathbf{P} \mathbf{A}$$

where $\mathbf{P}$ verifies the Riccati equation

$$\mathbf{P} = \mathbf{R} + \mathbf{A}^T \mathbf{P} \mathbf{A} - \mathbf{A}^T \mathbf{P} \mathbf{B} \left(\mathbf{Q} + \mathbf{B}^T \mathbf{P} \mathbf{B}\right)^{-1} \mathbf{B}^T \mathbf{P} \mathbf{A}$$

After substituting for the corresponding matrices, the solution of the maximization problem for the three age class model is given by the following HCR:

$$\Delta F = F - F_{tar} = \frac{\Theta}{p\Theta + 1} \left[\left(\frac{\mu_{x_1} e^{x_{1,tar}}}{\mu_{x_2} e^{x_{2,tar}}} + \rho \frac{\mu_z e^{z_{tar}}}{\mu_{x_2} e^{x_{2,tar}}} \right) z + \Delta x_1 \right]$$

where $\Theta = \beta \lambda p \mu_{x_2}^2 e^{2x_{2,tar}}$. Given the optimal HCR, we introduce the stochastic shocks to generate the optimal trajectories. Thus, the dynamics of each age class are given by:

$$\begin{aligned} \Delta z_{t+1} &= \rho \Delta z_t + \varepsilon_{t+1} \\ \Delta x_{t+1}^1 &= \Delta z_t \\ \Delta x_{t+1}^2 &= \frac{1}{p\Theta + 1} \left[-\left(\frac{\mu_{x_1} e^{x_{1,tar}}}{\mu_{x_2} e^{x_{2,tar}}} + \rho \frac{\mu_z e^{z_{tar}}}{\mu_{x_2} e^{x_{2,tar}}} \right) \Theta \Delta z_t + \Delta z_{t-1} \right] \end{aligned}$$

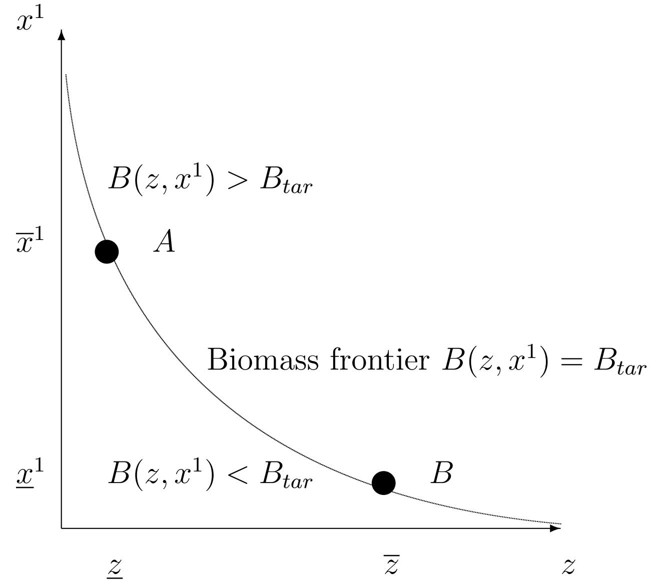

Figure 4: Two populations, A and B, with the same total biomass $B(z, x^1) = B_{tar}$, but with different age class compositions

It can be seen right away that the optimal HCR and the corresponding stock dynamics are different than the optimal solutions obtained under biomass-based models. Moreover, the solutions depend on the age structure of the model. In the following section we explore the policy implications of these results, and evaluate the effects of using biomass based rules in age structured stocks.

3.2 Biomass-Based Catch: Does it Work?

We now evaluate the impact of applying a biomass harvest control rule to a structured population. This section looks at the effects of using a biomass-based approach to an age structured model.

As shown above, optimal solutions obtained using LQ methods on the age model are different from the optimal HCR of biomass models. With a three age class structured model, the composition plays an important role when determining the HRC. Note that the policy function accounts for the relative contribution of recruitments over the total spawning biomass, given by $\left(\frac{\mu_{x_1} e^{x_{1,tar}}}{\mu_{x_2} e^{x_{2,tar}}} + \rho \frac{\mu_z e^{z_{tar}}}{\mu_{x_2} e^{x_{2,tar}}}\right)$. The higher the contribution of z and x_1 to the total biomass level is, the lower the optimal fishing mortality. Thus the optimal HCR takes into account the importance of future age specific contributions to the stock level.

Moreover, by allowing there to be a spawning population in both age classes it is possible to infer the importance of the correlation coefficient of the stock recuitment process. The correlation parameter, ρ, is the persistence of the system, and it is related to having smooth recruitments over time. A higher ρ implies that fishing mortality should be decreased and stock level x increased. Therefore, if the system is far off target today, it will probably continue to be off target in future periods. Thus, to decrease the distance to the reference point, F is decreased. The other extreme case is if there is no correlation between spawners. In this case, getting closer to the target point is independent of recruitments today. Thus, since the system only behaves randomly, high recruitment implies higher fishing mortality in the HCR.

Even though there has been an upsurge of fishery models that incorporate the age structure of the biological system, in many cases biomass-based models are till used in management [16]. What are the implications of the omission of the biological structure? Consider that we have two populations with a different age composition but both with the same biomass level, (points A and B in Figure 4). Moreover, assume that in both populations, the second adult group is at the target point, that is $\Delta x^2 = 0$. A biomass-based HCR determines the optimal control rule as $F = F_{tar}$, for both populations. On the contrary, the optimal age LQ HCR sets:

$$\begin{aligned}
\Delta F = F - F_{tar} = \quad & \frac{\Theta}{p\Theta + 1}\left[\left(\frac{\mu_{x_1} e^{x_{1,tar}}}{\mu_{x_2} e^{x_{2,tar}}} + \rho \frac{\mu_z e^{z_{tar}}}{\mu_{x_2} e^{x_{2,tar}}}\right)\Delta z + \log\left(\frac{B_{tar} - \hat{\mu}_z e^z}{\hat{\mu}_x^1}\right)\right] \\
= \quad & \frac{\Theta}{p\Theta + 1}\left[\left(\frac{\hat{\mu}_x^1}{\hat{\mu}_x^2} + \rho \frac{\hat{\mu}_z}{\hat{\mu}_x^2}\right)\Delta z + \log\left(1 + \frac{\hat{\mu}_z}{\hat{\mu}_x^1}\left(1 - e^z\right)\right)\right]
\end{aligned}$$

where we use the fact that we can write the target Biomass as:

$$B_{tar} = \hat{\mu}_z + \hat{\mu}_x^1 + \hat{\mu}_x^2 = \hat{\mu}_z e^{\underline{z}} + \hat{\mu}_x^1 e^{\overline{x}} + \hat{\mu}_x^2 = \hat{\mu}_z e^{\overline{z}} + \hat{\mu}_x^1 e^{B_{lim}} + \hat{\mu}_x^2$$

where $\hat{\mu}_j = \mu_j e^{j_{tar}}$ $j = z, x_1, x_2$.

Proposition 1. *Let $\hat{\mu}_z \to 0$. Then heuristic HCR based on biomass underestimates fishing mortality under high recruitments.*

Proof See Appendix.

Therefore, neglecting the biological structure of the resource results in an underestimation of the optimal fishing mortality. This result is in line with that obtained by Tahvonen [33], about the importance of taking age structure into account when designing and implementing optimal control rules.

3.3 Constant Effort HCR Amplify Risk

As mentioned in the introduction, historically, one the most common target points is B_{msy} and its associated fishing mortality F_{msy}. International fisheries agencies have been using heuristic harvesting control rules to attain this target point. Thus, constant effort policies are quite common. For example, ICES commonly uses HCR as follows: F should be F_{msy} when the stock is greater than B_{msy} and F should be reduced when the stock is lower than some trigger biomass lower than B_{msy}, to allow the stock to rebuild [3]. In this section we evaluate the implications of constant effort rules, $\lambda = 0$, and risk.

In order to obtain analytical results, now assume that $\rho = 0$ and z are uniformly distributed in the interval $[-\sigma, \sigma]$. Then, we can calculate the variance of the Biomass of the population as:

$$
\begin{aligned}
Var(B) &= E\left[\left(\hat{\mu}_z e^{\varepsilon_{t+1}} + \hat{\mu}_x^1 e^{\varepsilon_t} + \hat{\mu}_x^2 e^{\theta_1 \varepsilon_t + \theta_2 \varepsilon_{t-1}} - E(B)\right)^2\right] \\
&= \frac{1}{8\sigma^3} \int_{-\sigma}^{+\sigma} \int_{-\sigma}^{+\sigma} \int_{-\sigma}^{+\sigma} \left(\hat{\mu}_z e^{\varepsilon_{t+1}} + \hat{\mu}_x^1 e^{\varepsilon_t} + \hat{\mu}_x^2 e^{\theta_1 \varepsilon_t + \theta_2 \varepsilon_{t-1}}\right)^2 d\varepsilon_{t+1} d\varepsilon_t d\varepsilon_{t-1} - E(B)
\end{aligned}
$$

where we use the fact that $B = \hat{\mu}_z e^{\Delta z} + \hat{\mu}_x^1 e^{\Delta x^1} + \hat{\mu}_x^2 e^{\Delta x^2}$ and

$$
\Delta x_{t+1}^2 = \theta_1 \Delta z_t + \theta_2 \Delta z_{t-1}
$$

with

$$
\begin{aligned}
\theta_1 &= \frac{-\Theta}{p\Theta + 1}\left(\frac{\mu_{x_1} e^{x_{1,tar}}}{\mu_{x_2} e^{x_{2,tar}}} + \rho \frac{\mu_z e^{z_{tar}}}{\mu_{x_2} e^{x_{2,tar}}}\right) < 0 \\
\theta_2 &= \frac{1}{p\Theta + 1} \in (0, 1).
\end{aligned}
$$

Now consider a constant effort HCR which implements F_{tar} for all possible states, such as $F = F_{msy}$. The following proposition shows that for plausible values of recruitment volatility, an optimal HCR with positive biomass weight, $\lambda > 0$, always reduces the biomass volatility.

Proposition 2. *Let $F = F_{tar}$ for all possible states, i.e. $\lambda = 0$. If $\sigma < sinh^{-1}(1)/2$, Biomass volatility can be reduced by applying a HCR with a positive λ.*

Proof See Appendix.

This proposition has direct implications in policy design. If fisheries need to be managed to diminish the risk of the stock collapsing, the aim should be to design policies that assign more weight to biomass goals, $\lambda > 0$, such as biomass based catch control rules or constant escapement rules. Thus, implementing constant effort rules amplifies future risk by increasing the biomass volatility of the stock. Since the risk is higher, other control policies, such as total allowable catches and quotas are lower than optimal.

4 HCR Evaluation under Non-perfect Accuracy Estimation

The objective of this section is to illustrate with a simple numerical example of a hake fishery the propositions and implications of the results described throughout

Table 1: **Biological Data from the Southern Hake Recovery Plan**

Age	N	weight	maturity	m_a	p_a
0	78856.7	0.00	0.0181	0.4	0.076
1	49006.5	0.05	0.1194	0.4	0.317
2	23915.4	0.33	0.5000	0.4	0.559
3	9164.5	0.90	0.8806	0.4	0.623
4	3293.3	1.71	0.9819	0.4	0.633
5	1171.9	2.70	0.9975	0.4	0.635
6	416.4	3.79	0.9997	0.4	0.635
7	148.0	4.93	1.0000	0.4	0.635
8	52.6	6.06	1.0000	0.4	0.635
9	18.7	7.14	1.0000	0.4	0.635
10	6.6	8.16	1.0000	0.4	0.635
11	2.4	9.09	1.0000	0.4	0.635
12	0.8	9.94	1.0000	0.4	0.635
13	0.3	10.70	1.0000	0.4	0.635

the paper. To that end, we extend the model to a general case. With any number of age classes the population dynamics is a simple extension of the previous three age classes model. Thus, the models is analogous to the one in Section 3. Each year t, an exogenous number of juvenile fishes are born.

$$N_{t,1} = exp(z_t)$$

where z_t follows an AR(1) process

$$z_{t+1} = \rho z_t + \sigma_z \varepsilon_{t+1}$$

with zero mean and variance σ_z. Moreover, population dynamics are given by:

$$N_{t+1,a+1} = N_{t,a} e^{-p_a F_t - m},$$

Also, we consider that only a fraction μ_a of each age class a are spawners. That is

$$B_t = \sum_{a=1}^{A} \mu_a N_{t,a}.$$

The objective is to minimize the following loose function

$$\sum_{t=0}^{\infty} -\beta^t \left\{ (F_t - F_{tar})^2 + \lambda \left(\sum_{a=1}^{A} \mu_a e^{x_{t,a}} - \sum_{a=1}^{A} e^{x_{msy,a}} \right)^2 \right\}$$

In order to solve the minimization problem we use the same tools used in Section 2, first we perform a second order Taylor approximation, and re-write the problem in standard LQ matrix form. The LQ problem can be written as:

$$\max_{\mathbf{y}_t} E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \mathbf{x}_t^T \mathbf{R} \mathbf{x}_t + 2\mathbf{y}_t^T \mathbf{W} \mathbf{x}_t + \mathbf{y}_t^T \mathbf{Q} \mathbf{y}_t \right\}$$

$$s.t. \begin{bmatrix} 1 \\ z_{t+1} \\ x_{t+1,2} \\ \ldots \\ x_{t+1,A} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & \ldots & 0 & 0 \\ \rho & 0 & 0 & \ldots & 0 & 0 \\ -0 & 1 & 0 & \ldots & 0 & 0 \\ \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\ -0 & 0 & 0 & \ldots & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ z_t \\ x_{t,2} \\ \ldots \\ x_{t,A} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ -p_1 \\ \ldots \\ -p_{A-1} \end{bmatrix} F_t + \begin{bmatrix} 0 \\ \varepsilon_{t+1} \\ \epsilon_{t+1} \\ \ldots \\ \epsilon_{t+1} \end{bmatrix}$$

where $\mathbf{y} = F_t - F_{tar}$, $\mathbf{x}^T = [1 \;\; z_t \;\; x_{t,2} - x_{msy,2} \;\; \ldots \;\; x_{t,A} - x_{msy,A}]$ with $\mathbf{W}_{1\times A+1} = \mathbf{0}_{1\times A+1}$, $\mathbf{Q}_{1\times 1} = 1$ and

$$\mathbf{R}_{A+1\times A+1} = -\lambda \begin{bmatrix} 0 & 0 & 0 & \ldots & 0 \\ 0 & (\mu_1 e^{z_{tar}})^2 & \mu_1\mu_2 e^{z_{tar}} e^{x_{2,tar}} & \ldots & \mu_1\mu_A e^{z_{tar}} e^{x_{A,tar}} \\ 0 & \mu_2\mu_1 e^{z_{tar}} e^{x_{2,tar}} & (\mu_2 e^{x_{2,tar}})^2 & \ldots & \mu_2\mu_A e^{x_{2,tar}} e^{x_{A,tar}} \\ \ldots. & \ldots & \ldots & \ldots & \ldots \\ 0 & \mu_A\mu_1 e^{z_{tar}} e^{x_{A,tar}} & \mu_A\mu_2 e^{x_{2,tar}} e^{x_{A,tar}} & \ldots & (\mu_A e^{x_{A,tar}})^2 \end{bmatrix}$$

The model is applied to the Southern Hake Fishery [10]. Data from Table 1 consist of the main biological parameters of the population used for projections in the Report of the Southern Hake Recovery Plan. The final data used in the calculations were obtained by averaging out the data for the last 3 years.

4.1 HCR, reference points, and risk

In order to exemplify the behavior of the LQ HCR, we designed a number of experiments to assess the quantitative role of the reference point and the weighting parameter λ in the stock volatility, and apply them to a hake fishery. In particular, in this first experiment we are concerned with evaluating how much does a reference point lower than F_{max} decreases the volatility. Our results suggest that the answer depends on the type of HCR used.

We chose two target reference points, F_{max} and $2/3F_{max}$. In a sense, this experiment compares the implications of setting the target point against using a lower, more preventive point. For these two target points, we then implement four different HCR, thus characterizing the continuum of feasible HCR. For the experiments, we consider $\lambda = [-1, 0, 0.2, 1]$. These values were chosen to mimic each of the standard HCRs [16]. Thus, $\lambda = -1$ replicates a constant catch rule. A $\lambda = 0$ is equivalent to a constant effort. $\lambda = 0.2$ simulates a biomass-based catch. And finally, a constant escapement rule is design by setting $\lambda = 1$. The effect of each HCR is simulated 10000 times for each experiment and each simulation is run over 100 seasons.

Table 2: **Numerical Experiment to evaluate the implications between HCR, reference points, and risk.**

	$Ftarget = Fmax$				$Ftarget = (2/3)Fmax$			
λ	-0.1000	0.0000	0.2000	1.0000	-0.1000	0.0000	0.2000	1.0000
F	0.9959	1.0000	1.0072	1.0265	0.6598	0.6667	0.6773	0.7011
std (F)	0.0284	0.0000	0.0496	0.1853	0.0468	0.0000	0.0733	0.2306
SSB	1.1166	1.1001	1.0759	1.0355	1.5851	1.5401	1.4877	1.4390
std (SSB)	0.3094	0.2769	0.2241	0.0972	0.4683	0.3823	0.2629	0.0670
$Yield$	2.2638	2.2548	2.2362	2.1700	2.3655	2.3733	2.3573	2.2271
std ($Yield$)	0.5854	0.5641	0.5262	0.4349	0.5882	0.5853	0.5720	0.5968
corr(SSB, F)	-0.8734	0.0000	0.8719	0.7756	-0.8652	0.0000	0.8571	0.0717
corr($Yield, SSB$)	0.9996	0.9991	0.9950	0.9205	0.9958	0.9994	0.9839	0.1517

Table 2 summarizes the results of the experiments. The table reports the first and second moments of the fishing mortality, biomass and yield for different HCR analyzed under each reference point. The yield for year t, was computed by using Baranov's equation [2]

$$Y_t = \sum_{a=0}^{A} pr^a \omega^a \frac{p^a F_t}{m + p^a F_t} \left[1 - e^{-(p^a F_t + m)}\right] e^{x_{t,a}}$$

where ω^a is the weight of the a-age class. The table also displays the correlations. The main results of the simulations can be summarized as:

1. Stock Volatility and Target Points: Comparing the results obtained for the SSB volatility for both target points, we can see that for HCR that carry a high weigth on biomass, such as constant escapement rules, stock volatility is reduced when setting $2/3F_{max}$ (0.067) instead of F_{max} (0.097). Thus, reducing the target point from F_{max} to $2/3F_{max}$ reduces the volatility of the stock spawning biomass.

2. Stock Volatility and HCR: Moreover, for both reference points, a higher biomass weight decreases the volatility. That is, the higher the λ, the lower the volatility. The values of the variations in SSB go from 0.468 for a constant catch rule, to 0.067 for a constant escapement rule, for the $2/3F_{max}$ reference point. Similar results are obtained for the F_{max} target: the volatility decreases from 0.309 to 0.097. These results exemplify the theoretical results obtained in Proposition 2.

3. Target Points and Yield: Setting all periods $F = F_{max}$ it can be the case that we are not maximizing the yield ([15]). In particular, Da Rocha et al. [15] show that F_{max} is not the optimal solution to maximization problem whenever the

following inequality holds

$$H = \sum_{a=1}^{n} \left[\frac{\partial^2 y_{ss}^a}{\partial F_{ss}^2} + y_{ss}^a \left(\sum_{j=1}^{a-1} \beta^{a-j} \left(-p^j\right)^2 \right) \right] \phi_{ss}^a \geq 0. \tag{1}$$

with $\phi_{ss}^a = \prod_{i=1}^{a-1} e^{-(p^i F_{ss} - m^i)}$ For our case study, $H = 0.0616 > 0$. In the numerical simulations we find precisely these results. For target point F_{max}, yield ranges from 2.264 from constant catch rule to 2.170 of the constant escapement. However, when the target point is lowered, $2/3F_{max}$, the yield obtained is higher, it ranges from 2.365 to 2.227.

4.2 HCR under misestimation

The next experiment consists on a series of simulations intended to provide an understanding of the implications of omitting age structure, even if the stock cannot be estimated with perfect accuracy. The results obtained with the HCR obtained using LQ are compared to those from a simple biomass-based rule. To simulate inaccurate estimates of the population, in each point in time and for all age structures, the population is hit with a shock when calculating the optimal stock dynamics.

For the case of the LQ age structured methods, it is assumed that first an estimate of the population ($\mathbf{x}$) is drawn up, then the corresponding optimal HCR is obtained using $F = Gx$ and the control rule is applied, the state of the stock is updated with the rule and a measurement error.

$$x(t+1) = (A + BG)x + error$$

For the biomass-based rule, the SSB is obtained by adding up the optimal age states ($\mathbf{x}$). Then, a rule based on biomass is applied. For this exercise, the following is used: if the SSB is above a limit point $K * SSB$, then $F = F_{max}$ is used. If the SSB for that period is below the limit point, then there is a proportional harvest.

$$F = \begin{cases} F_{max} & if \quad SSB > KSSBmax \\ \\ F_{max} \times \frac{SSB(t)}{KSSB_{max}} & if \quad SSB < KSSB_{max} \end{cases}$$

The corresponding F is thus obtained, the corresponding dynamics are again calculated and the measurement errors are added.

$$xt(:, it+1) = A * xt(:, it) + B. * (Ft(it) - Fss) + error$$

In order to be able to compare the results effectively, the errors used in both simulations are the same. We chose $\rho = 0.95$ and an error of mean zero and standard deviation $\epsilon = [0.2]$. For these two precision levels, we then implement four different HCR ($\lambda = [0, 0.5, .75, 1]$). Table 3 and 4 summarizes the results of the experiments.

Table 3: **Comparison of results under age-structured HCR and biomass based rule under perfect accuracy** ($\rho = 0.95$ $\epsilon = 0$)

	LQ				SSB			
λ	0.0000	0.5000	0.7500	1.0000	0.0000	0.5000	0.7500	1.0000
F	1.0000	1.0018	1.0024	1.0030	1.0000	0.9970	0.9673	0.8913
std (F)	0.0000	0.1065	0.1473	0.1822	0.0000	0.0130	0.0833	0.1620
SSB	1.0652	1.0344	1.0318	1.0338	1.0652	1.0668	1.0903	1.1737
std (SSB)	0.2724	0.1532	0.1116	0.0799	0.2724	0.2705	0.2490	0.2086
$Yield$	2.1821	2.1482	2.1359	2.1251	2.1821	2.1829	2.1949	2.2358
std ($Yield$)	0.5546	0.4553	0.4221	0.3973	0.5546	0.5543	0.5575	0.5878
corr(SSB, F)	NaN	0.8502	0.7927	0.6501	NaN	0.1436	0.3356	0.4884
corr($Yield, SSB$)	0.9991	0.9801	0.9469	0.8429	0.9991	0.9986	0.9871	0.9519

Table 4: **Comparison of results under age-structured HCR and biomass based rule under non-perfect accuracy** ($\rho = 0.95$, $\epsilon = 0.2$)

	LQ				SSB			
λ	0.0000	0.5000	0.7500	1.0000	0.0000	0.5000	0.7500	1.0000
F	1.0000	1.0009	1.0013	1.0016	1.0000	0.9959	0.9640	0.8880
std (F)	0.0000	0.1083	0.1498	0.1853	0.0000	0.0197	0.0966	0.1724
SSB	1.1038	1.0704	1.0679	1.0705	1.1038	1.1059	1.1315	1.2158
std (SSB)	0.3437	0.2267	0.1909	0.1670	0.3437	0.3417	0.3262	0.3010
$Yield$	2.2671	2.2289	2.2157	2.2043	2.2671	2.2680	2.2802	2.3228
std ($Yield$)	0.6887	0.5833	0.5481	0.5218	0.6887	0.6896	0.7033	0.7507
corr(SSB, F)	NaN	0.7076	0.5950	0.4172	NaN	0.1526	0.3309	0.4904
corr($Yield, SSB$)	0.9977	0.9677	0.9192	0.8152	0.9977	0.9972	0.9885	0.9658

A comparison of the volatility of both yield and SSB for the age structure rule relative to the biomass rule under measurement error shows that including age structure decreases the deviations of the measurements. Morevoer, this result holds for different values of λ and K.

We now look at the benefits of including age structure but for a range of values of measurement errors ($\epsilon = [0, 0.1, 0.2, 0.3]$) and for different values of the correlation parameter ρ. We repeat the exercise for Yield and SSB. The results are displayed in the tables below.

We find that under perfect precision ($\epsilon = 0$), the advantage of introducing age structure in the HCR clearly holds. As the imprecission level increases, the model has larger difficulties to efficiently incorporate that information. However, it is still beneficial to account for age structure. A second finding deals with the robustness of the value of the persistence parameter. For both SSB and Yield, as the persistence parameter increases, the relative volatility tends to decrease, for all error levels. Thus, highly persistent biological cases tend to carry lower risk.

Lastly, an interesting result is that, counter to what is assumed as common knowledge, the HCR using age structure prescribes the same policy rankings using yield volatility and SSB volatility. As can be seen, there is a positive correlation between the two measures, for different levels of the persistence parameter, and also for different levels of imprecision levels.

Table 5: **Biomass benefits of including age structure for different correlation and misestimation errors (relative std (SSB))**

$\lambda = 0.5$				
	ρ			
ϵ	0	0.2	0.5	.95
0	0.6952	0.6800	0.6570	0.5663
0.1	0.8487	0.8254	0.7829	0.5957
0.2	0.8739	0.8660	0.8478	0.6636
0.3	0.8757	0.8729	0.8658	0.7420
$\lambda = 0.75$				
	ρ			
ϵ	0	0.2	0.5	.95
0	0.5832	0.5634	0.5331	0.4483
0.1	0.7952	0.7654	0.7115	0.4934
0.2	0.8261	0.8155	0.7923	0.5853
0.3	0.8359	0.8332	0.8262	0.6890

5 Conclusions

In this paper we show that, in contrast to pessimistic views regarding the knowledge of stock dynamics to implement more complex management rules, age-structured fishery models can be analytically tractable and can reduce the volatility of biomass and yield. From a management perspective, empirical estimates of the gains from optimal harvesting compared to currently applied biomass based rules can be obtained.

Table 6: **Yield benefits of including age structure for different correlation and misestimation errors (relative std ($Yield$))**

$\lambda = 0.5$				
	ρ			
ϵ	0	0.2	0.5	.95
0	0.8164	0.8132	0.8127	0.8215
0.1	0.9137	0.8991	0.8774	0.8280
0.2	0.9324	0.9271	0.9164	0.8458
0.3	0.9165	0.9145	0.9100	0.8563
$\lambda = 0.75$				
	ρ			
ϵ	0	0.2	0.5	.95
0	0.7556	0.7505	0.7489	0.7570
0.1	0.8828	0.8637	0.8338	0.7654
0.2	0.8964	0.8847	0.8632	0.7794
0.3	0.8802	0.8769	0.8701	0.8026

We find that with more than two age classes, the HCR depends on the age composition of the spawning stock biomass. Also, HCR based on biomass underestimates fishing mortality under high recruitments. This result is in line with that obtained by the literature on deterministic age structure dynamic optimization models concerning the importance of taking age structure into account when designing and implementing optimal control rules.

The benefits of including age structure are considerable, even if the state of the stock cannot be estimated with perfect accuracy. For this particular case scenario we find that data inaccuracy is not excuse for ignoring the biological complexity of the resource. Moreover, when age structure is used the prescription of HCR in terms of yield and SSB is the same.

Lastly, it is relevant to point that the use of Linear Quadratic in fishery management has large potential benefits. This methodology is flexible while still allowing for incorporating complex biological structure. Here we look at age structure, but the model can be easily extended to capture relations in a mixed fishery. Moreover, it would be of great policy interest to evaluate the economic implications of including uncertainty in prices and the potential economic trade offs among different harvest control rules.

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A Appendix

A.1 HCR and stock dynamics in the three age class model

The steps used for the three age model are equivalent to the ones described in Appendix **??**. We explicitly define the matrices that solve for the optimal HCR and the stock dynamics. By solving the following system of equations given by the Riccati equation, we obtain the solution stated in Section 3.

The HCR is obtained by solving the following Riccati equation

$$\mathbf{P} = \begin{bmatrix} P_{11} & P_{12} & P_{13} & P_{14} \\ P_{12} & P_{22} & P_{23} & P_{24} \\ P_{13} & P_{23} & P_{33} & P_{34} \\ P_{14} & P_{24} & P_{34} & P_{44} \end{bmatrix} =$$

$$\begin{bmatrix} \beta P_{11} - \frac{(beta^2 P_{14}^2 s2^2)}{(\beta P_{44} s2^2 - 1)} & \beta(P_{13} + P_{12}\rho) - \frac{(\beta^2 P_{14} s2^2 (P_{34} + P_{24}\rho))}{(\beta P_{44} s2^2 - 1)} & -\frac{(\beta P_{14})}{(\beta P_{44} s2^2 - 1)} & 0 \\ \beta(P_{13} + P_{12}\rho) - \frac{(\beta^2 P_{14} s2^2 (P_{34} + P_{24} rho))}{(\beta P_{44} s2^2 - 1)} & \beta P_{33} - hxx11\lambda + \beta P_{22}\rho^2 + 2\beta P_{23}\rho - \frac{(\beta^2 s2^2 (P_{34} + P_{24} rho)^2)}{(\beta P_{44} s2^2 - 1)} & -hxx12\lambda - \frac{(\beta(P_{34} + P_{24}\rho))}{(\beta P_{44} s2^2 - 1)} & -hxx13\lambda \\ -\frac{(\beta P_{14})}{(\beta P_{44} s2^2 - 1)} & -hxx12\lambda - \frac{(\beta(P_{34} + P_{24}\rho))}{(\beta P_{44} s2^2 - 1)} & -hxx22\lambda - \frac{(\beta P_{44})}{(\beta P_{44} s2^2 - 1} & -hxx23\lambda \\ 0 & -hxx13\lambda & -hxx23\lambda & -hxx33\lambda \end{bmatrix}$$

By solve the equation by using a symbolic Matlab code. Given $\mathbf{P}$ we compute $G = -\left(\mathbf{Q} + \mathbf{B}^T\mathbf{P}\mathbf{B}\right)^{-1}\mathbf{B}^T\mathbf{P}\mathbf{A}$ to obtain the optimal harvesting control rule. is thus that

$$\Delta F = \mathbf{Gx} = \begin{bmatrix} 0 & \frac{\beta\lambda p_2(\mu_{x_1} e^{x_{1,tar}} \mu_{x_2} e^{x_{2,tar}} + \mu_z e^{z_{tar}} \mu_{x_2} e^{x_{2,tar}} \rho)}{\beta\lambda\mu_{x_2}^2 e^{2x_{2,tar}} p_2^2 + 1} & \frac{\beta\lambda\mu_{x_2}^2 e^{2x_{2,tar}} p_2}{\beta\lambda\mu_{x_2}^2 e^{2x_{2,tar}} p_2^2 + 1} & 0 \end{bmatrix} \begin{bmatrix} 1 \\ z_t \\ \Delta x_{1,t} \\ \Delta x_{2,t} \end{bmatrix}$$

Given G, we introduce the stochastic shocks to generate the optimal trajectories by using $\mathbf{x}' = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{G}\mathbf{x} + \varepsilon' = \mathbf{D}\mathbf{x} + \varepsilon'$ where

$$\mathbf{D} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \rho & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & -\frac{\beta\lambda p_2(\mu_{x_1} e^{x_{1,tar}} \mu_{x_2} e^{x_{2,tar}} + \mu_z e^{z_{tar}} \mu_{x_2} e^{x_{2,tar}} \rho)}{\beta\lambda\mu_{x_2}^2 e^{2x_{2,tar}} p_2^2 + 1} & \frac{1}{\beta\lambda\mu_{x_2}^2 e^{2x_{2,tar}} p_2^2 + 1} & 0 \end{bmatrix}$$

A.2 Proof of Proposition 1

From the expression for the optimal LQ HCR, if $z = 0$, then the optimal fishing mortality and stock level is the same as in the heuristic case, with $B_{tar} = B$ and $F = F_{tar}$. However, if we account for recruitment structure, $z > 0$, then $\Delta F = F - F_{tar}$. Therefore, the two optimal policies are no longer equivalent. Additionally assume that $\hat{\mu}_z$ is a low value, $\hat{\mu}_z \to 0$, then the optimal LQ HCR sets a higher fishing mortality, meaning that heuristic HCR underestimate fishing mortality. Then, If $\hat{\mu}_z \to 0$ then $\Delta F = F - F_{tar} = \dfrac{\Theta}{p\Theta + 1}\left(\dfrac{\hat{\mu}_x^1}{\hat{\mu}_x^2}\right) z$
■.

A.3 Proof of Proposition 2

Using the standard formula for calculating the variance of a random variable, the Biomass volatility is given by:

$$
\begin{aligned}
Var(B) \quad = \quad & \frac{1}{8\sigma^3}\Bigg[4\sigma^2 \sinh(2\sigma)\left[\hat{\mu}_{x^1}^2 + \hat{\mu}_z^2\right] + 16\hat{\mu}_{x^1}\hat{\mu}_z\sigma\sinh(\sigma)^2 + \frac{2\hat{\mu}_{x^2}^2\sigma\sinh(2\sigma\theta_1)\sinh(2\sigma\theta_2)}{\theta_1\theta_2} + \\
& \frac{16\hat{\mu}_{x^2}\hat{\mu}_z\sinh(\sigma\theta_1)\sinh(\sigma\theta_2)\sinh(\sigma)}{\theta_1\theta_2} + \frac{8\hat{\mu}_{x^1}\hat{\mu}_{x^2}\sigma\sinh(\sigma\theta_2)(e^{2\sigma+2\sigma\theta_1} - 1)}{\theta_2(\theta_1+1)e^{\sigma+\sigma\theta_1}}\Bigg] - E(B)
\end{aligned}
$$

where $\sinh(a) = \frac{e^a - e^{-a}}{2}$ is the hyperbolic sine function. Now consider a HCR that implements F_{tar} for all possible states, such as a constant catch rule. In our model it is equivalent to calculating the solution of the optimal HCR when $\lambda = 0$. In that case $\theta_1 = 0$ and $\theta_2 = 1$. Then

$$
\begin{aligned}
Var(B|\lambda = 0) \quad = \quad & \frac{1}{8\sigma^3}\Bigg[4\sigma^2 \sinh(2\sigma)\left[\hat{\mu}_{x^1}^2 + \hat{\mu}_z^2\right] + 16\hat{\mu}_{x^1}\hat{\mu}_z\sigma\sinh(\sigma)^2 + \frac{8\hat{\mu}_{x^1}\hat{\mu}_{x^2}\sigma\sinh(\sigma)(e^{2\sigma} - 1)}{e^{\sigma}} + \\
& \lim_{\theta_1 \to 0}\left(\frac{2\hat{\mu}_{x^2}^2\sigma\sinh(2\sigma\theta_1)\sinh(2\sigma)}{\theta_1} + \frac{16\hat{\mu}_{x^2}\hat{\mu}_z\sinh(\sigma\theta_1)\sinh(\sigma)^2}{\theta_1}\right)\Bigg] - E(B)
\end{aligned}
$$

Applying l'Hopital Rule, we have

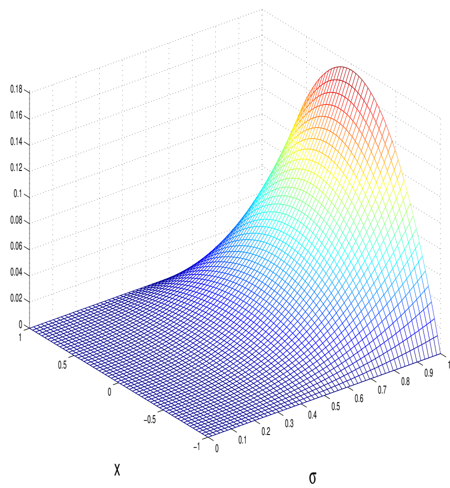
$$
\begin{aligned}
Var(B|\lambda = 0) \quad = \quad & \frac{1}{8\sigma^3}\Bigg[4\sigma^2 \sinh(2\sigma)\left[\hat{\mu}_{x^1}^2 + \hat{\mu}_z^2\right] + 16\hat{\mu}_{x^1}\hat{\mu}_z\sigma\sinh(\sigma)^2 + \frac{8\hat{\mu}_{x^1}\hat{\mu}_{x^2}\sigma\sinh(\sigma)(e^{2\sigma} - 1)}{e^{\sigma}} + \\
& 2\hat{\mu}_{x^2}^2\sigma\cosh(0)\sinh(2\sigma) + 16\hat{\mu}_{x^2}\hat{\mu}_z\cosh(0)\sinh(\sigma)^2\Bigg] - E(B)
\end{aligned}
$$

Then

$$
\begin{aligned}
Var(B|\lambda = 0) - Var(B) \quad = \quad & \frac{1}{8\sigma^3}\Bigg[8\hat{\mu}_{x^1}\hat{\mu}_{x^2}\sigma\left(\frac{\sinh(\sigma)(e^{2\sigma} - 1)}{e^{\sigma}} - \frac{\sinh(\sigma\theta_2)(e^{2\sigma+2\sigma\theta_1} - 1)}{\theta_2(\theta_1+1)e^{\sigma+\sigma\theta_1}}\right) + \\
& 2\hat{\mu}_{x^2}^2\sigma\left(\cosh(0)\sinh(2\sigma) - \frac{\sinh(2\sigma\theta_1)\sinh(2\sigma\theta_2)}{\theta_1\theta_2}\right) + \\
& 16\hat{\mu}_{x^2}\hat{\mu}_z\left(\cosh(0)\sinh(\sigma)^2 - \frac{\sinh(\sigma\theta_1)\sinh(\sigma\theta_2)\sinh(\sigma)}{\theta_1\theta_2}\right)\Bigg]
\end{aligned}
$$

Given that $\forall x \in [-1, 1]$, $\sinh(\sigma) - \frac{\sinh(\sigma x)}{x}$ and $\frac{(e^{2\sigma}-1)}{e^{\sigma}} > \frac{(e^{2\sigma+2\sigma\theta_1}-1)}{(\theta_1+1)e^{\sigma+\sigma\theta_1}}$ (see Figure A.3)

$$
\begin{aligned}
Var(B|\lambda = 0) - Var(B) \quad > \quad & \frac{1}{8\sigma^3}\left\{2\hat{\mu}_{x^2}^2\sigma\sinh(2\sigma)\left[\cosh(0) - \sinh(2\sigma)\right] + \right. \\
& \left. 16\hat{\mu}_{x^2}\hat{\mu}_z\sinh(\sigma)^2\left[\cosh(0) - \sinh(\sigma)\right]\right\}
\end{aligned}
$$



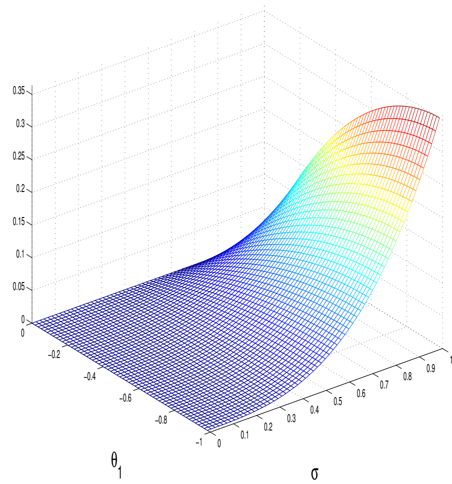


Figure 5: Hyperbolic sin function properties. Left hand side shows that $\forall x \in [-1, 1]$, $\sinh(\sigma) - \frac{\sinh(\sigma x)}{x} > 0$. Right hand side shows that $\forall \theta_1 \in [-1, 0]$, $\frac{(e^{2\sigma}-1)}{e^{\sigma}} - \frac{(e^{2\sigma+2\sigma\theta_1}-1)}{(\theta_1+1)e^{\sigma+\sigma\theta_1}} > 0$.

Note that the value of the limiting variance is positive if the hyperbolic cosine function in zero is equal to 1. Then $Var(B|\lambda = 0) - Var(B) > 0$ if $\sigma \leq \dfrac{\sinh^{-1}(1)}{2} = 0.4407$ ■