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GHOST SEATS IN THE BASQUE PARLIAMENT

by

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Ghost seats in the Basque Parliament[*]

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Abstract

In elections voters have generally four options: to abstain, to cast a blank vote, to cast a null vote, or to vote for a candidate or party. This last option is a positive expression of support, while the other three options reflect lack of interest, or dissatisfaction with the parties or the political system. However only votes for parties or candidates are taken into account in the apportionment method. In particular the number of seats allocated to parties remains constant even if the number of non votes (i.e. blank votes, null votes or abstention) is very large. This paper proposes to treat the non votes as a party in the apportionment method and to leave empty the corresponding seats. These empty seats are referred to as "ghost seats". How this would affect the decision-making is quantified in terms of power indices. We apply this proposal to a case study: the regional Parliament of the Basque Autonomous Community (Spain) from 1980 till 2012.

1   Introduction

Electoral turnout has decreased in most democratic countries these last decades. Since 1988 the average turnout in the EU members states has been around 78% while it was almost 84% before 1987. The average hides significant differences between countries. In Belgium, Luxembourg and Italy, around 90% of electors usually vote, while in Ireland, France and Portugal the turnout does not reach 75% of the electorate (IDEA, 2004). The 2014 elections to the European Parliament exhibit the lowest turnout in record:

only 42.54% of the Europeans voted, with a turnout that did not even reach 20% in the Czech Republic or Slovakia. As put by Muxel (2009) the party of abstainers would be the first political group in the European Parliament.

Part of the abstention may be circumstantial (illness, absence from home, registration problems) while the rest is voluntary. Blondel et al. (1997) propose the following reasons to explain voluntary abstention in the European Parliament elections: lack of interest, lack of knowledge, distrust or dissatisfaction with politics and politicians, distrust or dissatisfaction with the electoral process. According to Delwit (2013) the growth of abstention is a sign of indifference toward or distrust of politicians and politics.

Among the "non votes" (that is, abstention, blank votes or null votes) abstention has attracted the lion's share in the literature. See however Power and Roberts (1995), Power and Garand (2007), Uggla (2008) or Troumpounis (2011). One reason of this little attention is that these other two forms of expression[1] are notoriously difficult to interpret. As put by Damore et al. (2012) the "non votes" is part of the signal that the election sends to the political system, although the signal is not clear. Non votes may arise from motivation varying from alienation to boredom or confusion. Another reason may be that null and blank votes represent a very low percentage of the electorate. Nevertheless the phenomenon has become increasingly common, as well as the vote for extra parliamentary parties (Uggla, 2008). For instance in France the percentage of blank votes increased from 2.5% in 1990 to 5% in 2000 (Zulfikarpasic, 2001). It even reached between 9 and 20% in districts where the competition in the second round was between two candidates from the same side of the electoral axis (two right-wing candidates or two left-wing candidates). Power and Garand (2007) point out that these percentages may be not negligible in some countries, especially in Latin America (with an average of 11% of invalid voting, the variation being between 2-3% in some countries to 20-30% in others).

A blank vote is usually intentional, although it may sometimes be a form of hidden abstention. It may express dissatisfaction with politics or politicians or impossibility to choose a candidate (Zulfikarpasic, 2001). Teixidor (2012) distinguishes between the null votes due to inexperience or error from those which are intentionally null (to show political disagreement). The respective proportions of intentional and unintentional null votes cannot be evaluated. The following examples illustrate clear signals of disagreement or dissatisfaction sent by electors through non votes. In 1928 in Antwerp (Belgium) the by-election to substitute a deceased liberal member of Parliament was transformed into a plebiscite on the nation language politics. Some parties called for a boycott of the election and as a result 31.3% of the ballots were spoiled[2] (van Goethem, 2010, p 153-154). In the 1969 French presidential election the defeated communist candidate in the first round called his supporter to abstain in the second round. As a result

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[1] As pointed out by Teixidor (2012) some countries do not recognize the blank vote option. In these cases blank votes are considered as null votes.

[2] We thank Luc Bovens for attracting our attention on this example.

the abstention raised from 21.8% in the first round to 30.9%, and in parallel the blank votes raised from 1% to 4.5% (Zulfikarpasic, 2001). In the 2001 Argentinian elections 20% of the votes cast were either blank or null with the clear objective of casting an "angry vote" (Uggla, 2008). In the 2009 elections to the Basque Parliament the illegalized party Batasuna called its supported to cast a null vote and 5.7% of the electorate did so. Note that disagreement or dissatisfaction can also be hidden behind vote for an alternative candidate. In the second round of the 2002 French presidential elections, the right wing candidate Chirac was chosen by 82.21% of the voters. Among the votes in favor of Chirac many were against his opponent, the extreme right candidate Le Pen.

The "None of the Above" (NOTA) option has the advantage of offering a non ambiguous means to signal dissatisfaction. Voters bear the cost of electoral participation in order to send this signal. This option is however used in few countries. A well-known example is the state of Nevada where it obtained an average percentage of 10.98% of the electorate during the 1976-2010 period (Damore et al., 2012). The NOTA option was introduced in Russia in the 1993 electoral reform and suppressed in 2006 in spite of its success. For instance in 2003 this option was the second most voted "party" with nearly 13% of the votes. In some constituencies it achieved more votes than the largest party (Mc Allister and White, 2008). In 2013 the NOTA option was introduced in India by the Supreme court. So far (that is, in the first general elections held afterwards, in 2014) its score has been modest, with 1.1% of the votes (Diwakar, 2015).

In France there has been some movement in order to have the blank option recognized. Since 2014 blank and null votes are separately counted even if both options are not considered as valid and do not count in the tally.[3] Zulfikarpasic (2001) reports that in 1992 a party whose objective was to represent blank votes obtained one seat in regional elections in Brittany (and got 5% of the votes). This is also the case of Spain where a party ("Blank Seats") has as programme to reform the electoral law in order to achieve that blank votes are transformed into empty seats in the Parliament. Meanwhile their elected candidates will not sit in the Parliament. They achieved a total of four seats in the 2011 local elections in Catalonia[4] (two out of seven in Foixá, one out of thirteen in Gironella, and one of thirteen in Santa Maria de Palautordera).

The objective of this paper is to propose some way to evaluate the potential effect of such a proposal. More generally we intend to measure the impact that each category of non votes could have in Parliaments if it counted as a party in the apportionment method. We proceed by applying the apportionment method to the list of competing parties to which we add a fictitious party whose number of votes is equal either to the number of blank votes, either to the number of abstentions, or to the number of null votes. Three alternative distributions of seats are obtained, each one is compared with the actual distribution (that is, the one obtained when the apportionment method is

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[3]By contrast in other countries blank votes are considered as valid and do count in computing the threshold for a party to obtain representation while null votes are not.

[4]See the web page of the Spanish Ministery of the Interior: www.infoelectoral.mir.es

exclusively applied to the competing parties). In particular we quantify the number of "ghost seats", i.e. the seats that would correspond to the votes of the additional fictitious party. We then analyze how the three alternative apportionments would affect the decision-making in the Parliament (i.e. the weighted majorities among the parties if the "ghost seats" are left empty).

There exists a vast literature to assess decision-making in Parliament. Many power indices have been proposed to quantify the parties' capacity to influence the outcome. The most popular ones are the Shapley-Shubik (1954), Banzhaf (1965), Rae (1969), or Coleman (1971, 1986) indices, among others. Here we follow the probabilistic approach proposed by Laruelle and Valenciano (2005, 2008) and measure the respective probabilities of acceptance of a proposal and probabilities of obtaining the preferred outcome. As the objective is a *normative* comparison of the decision-making imputable to the different apportionments, we leave out of our model elements that should be incorporated, if the objective were an accurate descriptive account of the Parliament. Thus, we ignore, as inconsistent with the normative approach, the information concerning party location in the political space. The only relevant information is the distribution of seats.

The amplitude of the effect on a Parliament has to be evaluated on a case study. In this paper we focus on the regional Parliament of the Basque Autonomous Community (Spain) from 1980 till 2012. The apportionment rule is the D'Hondt method with a threshold. In each of the three districts (one per province), one third of the seats is allocated. For all years we use the same apportionment method (with the same threshold) but adding either the blank votes, the null votes or the abstention to the list of competing parties. We compare then the three resulting apportionments with the actual one. The results show that neither the blank votes nor the null votes (with the exception of the year 2009) would have an effect on the distribution of seats. By contrast the potential effect of the abstention would be very important. More than 20% of the seats of the Parliament would be ghost seats. The presence of ghost seats has the effect of increasing the quota in relative terms. This decreases the probability of adopting a proposal. This also decreases the party's probability of getting the proposal adopted when it favors it. Nevertheless the probability of getting the proposal rejected when the party is against the proposal increases. Moreover in each legislature there exists at least a party which can act as a vetoer. The global effect is that a party's probability of getting its preferred outcome generally decreases. We expect that the potential effect of the abstention would be as important in all countries that exhibit similar rates of abstention (33.6% of the electorate).

The limits of the exercise have to be kept in mind. First it is difficult to predict what voters would really do. Through the whole exercise we assume that the modification of the apportionment method would not alter the numbers of blank votes, null votes or abstentions. We also assume that all voters who vote for a given party would continue voting for it. Second the behavior of the parties is another unknown. It is clear that emptying the seats that correspond to non votes would be a clear incentive

for politicians to pay more attention to non voters. Third the objective of the exercise is normative: the probabilities obtained cannot capture the parties similarities and differences on the political space and thus do not provide a descriptive analysis.

The rest of the paper is organized as follows: Section 2 details the way to quantify the effect of the abstention, the null and blank votes. Section 3 analyses the case of the Basque Parliament. Section 4 concludes with some remarks. The Appendix contains a description of the Basque party system.

2 Quantifying the effect of non votes

In electoral year t the electorate (denoted e_t) can be decomposed into the number of abstainers (that we denote a_t), the number of blank votes (that we denote b_t), the number of null votes (that we denote c_t) and the votes for the different parties. Let m_t denote the number of competing parties, and M_t their set. If party $i \in M_t$ has obtained $v_t(i)$ votes, we have

$$e_t = a_t + b_t + c_t + \sum_{i \in M_t} v_t(i).$$

An apportionment method allocates the k_t seats of the Parliament (or the k_t seats of the district) among parties on the basis of the votes for the different parties. Formally it is a function F that associates to any vector of votes $\vec{V}_t^0 = (v_t(1), ..., v_t(m_t))$ a vector of seats $\vec{Y}_t^0 = (y_t^0(1), ..., y_t^0(m_t))$: $\vec{Y}_t^0 = F(\vec{V}_t^0)$ with

$$y_t^0(i) \geq 0 \text{ for any } i \in M_t, \text{ and } \sum_{i \in M_t} y_t^0(i) = k_t.$$

The apportionment method satisfies the monotonicity property: a party with more votes than another should not have fewer seats

$$\text{if } v_t(i) < v_t(j) \text{ then } y_t^0(i) \leq y_t^0(j). \tag{1}$$

For an analysis of apportionment methods, see Balinski and Young (1982).

Let N_t^0 be the set of parties with representation in the Parliament in apportionment $\vec{Y}_t^0$. That is, $N_t^0 = \{i \in M_t \mid y_t^0(i) \neq 0\}$ is the set of parties with a non null number of seats in apportionment $\vec{Y}_t^0$. The number of parties with representation is denoted n_t^0 and the corresponding distribution of seats in the Parliament is given by $\vec{W}_t^0 = (w_t^0(1), ..., w_t^0(n_t^0))$ with $w_t^0(i) = y_t^0(i)$ for any $i \in N_t^0$.

The apportionment method only takes into account the votes for parties. That is, it is as if the non votes were not part of the electorate. Why are these votes not included in the apportionment? How many seats would correspond to abstention (A)? blank votes (B)? null votes (C)? How would the distribution of seats among the parties be affected?

For $Z = A, B, C$, let $\vec{V}_t^Z$ denote the vectors of votes for the different parties if we add a fictitious party whose number of votes is equal to the votes received by category Z of non votes. That is,

$$\vec{V}_t^Z = (v_t(1), ..., v_t(m_t), z_t) \text{ with } z_t = a_t \text{ if } Z = A, \ z_t = b_t \text{ if } Z = B, \ z_t = c_t \text{ if } Z = C.$$

We denote by $\vec{Y}_t^Z = (y_t^Z(1), ..., y_t^Z(m_t + 1))$ the resulting distribution of seats: $\vec{Y}_t^Z = F(\vec{V}_t^Z)$. The number of seats of party i $(i = 1, ..., m_t)$ is given by $y_t^Z(i)$, while the number of seats that would correspond to Z is given by $y_t^Z(m_t + 1)$. These seats are referred to as *ghost seats* because these are seats that are not occupied in the Parliament.

The set (resp. number) of parties with representation associated to apportionment $\vec{Y}_t^Z$ is denoted N_t^Z (resp. n_t^Z). That is, $N_t^Z = \{i \in M_t \mid y_t^Z(i) \neq 0\}$. Let $\vec{W}_t^Z$ denote the corresponding vector of seats: $\vec{W}_t^Z = (w_t^Z(1), ..., w_t^Z(n_t^Z))$ with $w_t^Z(i) = y_t^Z(i)$ for any $i \in N_t^Z$.

The objective is to quantify the effect of substituting $\vec{V}_t^0$ by $\vec{V}_t^Z$ for $Z = A, B, C$ in the Parliament. If $y_t^Z(m_t + 1) = 0$ then $y_t^Z(i) = y_t^0(i)$ for $i = 1, ..., m_t$ and $\vec{W}_t^Z = \vec{W}_t^0$ as well as $N_t^Z = N_t^0$. In this case category Z of the non votes has no effect. By contrast if $y_t^Z(m_t + 1) \neq 0$ then $\vec{W}_t^Z \neq \vec{W}_t^0$ and the number of seats is reduced: $w_t^Z(i) \leq w_t^0(i)$ for any party with representation in the Parliament $(i \in N_t^Z)$. The total number of seats occupied by parties is smaller in $\vec{W}_t^Z$ than in $\vec{W}_t^0$:

$$\sum_{i \in N_t^0} w_t^0(i) = k_t \text{ while } \sum_{i \in N_t^Z} w_t^Z(i) = k_t - y_t^Z(m_t + 1).$$

We also have $N_t^Z \subseteq N_t^0$ (and thus $n_t^Z \leq n_t^0$). If the inclusion is strict at least one party loses its representation in the Parliament.[5]

If we assume that there is a perfect discipline of party we can go a step further in the analysis and model the decision-making in the Parliament as weighted majorities among the parties. The weights are the respective number of seats of the different parties and the quota is equal to half the total number of seats (i.e. $k_t/2$). Then the questions that can be addressed are how the alternative apportionments modify the ease to adopt proposals, and whether a party will obtain more easily its preferred outcome. Some definitions are necessary to answer these questions.

A *voting rule* is a well-specified procedure to make decisions among n voters[6]. Let N denote the set of voters. Once a proposal is submitted, voters cast votes. A *vote configuration* is a possible or conceivable result of a vote, that lists the votes cast by all voters. If all non 'yes'-votes are assimilated to 'no'-votes, there are 2^n possible configurations of votes. Each configuration can be represented by the set of 'yes'-voters. So, we refer as the vote configuration S to the result of a vote where only the voters in S vote 'yes', while those in $N \backslash S$, vote (or are assimilated to) 'no'.

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[5] These are the $n_t^0 - n_t^A$ smallest parties of N_t^0.

[6] Here the voters will be the parties with representation in the Parliament.

A voting rule is fully specified by the set of vote configurations that would lead to the passage of a proposal. These configurations will be called *winning configurations*. In what follows $\mathcal{W}$ will denote the set of winning configurations representing a voting rule. It will be assumed that a voting rule satisfies the following requirements: (i) $N \in \mathcal{W}$, (ii) $\emptyset \notin \mathcal{W}$, (iii) If $S \in \mathcal{W}$, then $T \in \mathcal{W}$ for any T containing S, (iv) if $S \in \mathcal{W}$ then $N \backslash S \notin \mathcal{W}$.

Symmetric voters are interchangeable in the voting rule. Voters i and j are symmetric in $\mathcal{W}$ if for any vote configuration S such that $i, j \notin S$,

$$S \cup i \in \mathcal{W} \Leftrightarrow S \cup j \in \mathcal{W}.$$

A *vetoer* can prevent the passage of a proposal: if the vote from a vetoer is 'no' then the proposal is rejected. That is, i is a vetoer in $\mathcal{W}$ if

$$i \notin S \Rightarrow S \notin \mathcal{W}.$$

A *null voter* is a voter such that her vote never makes a difference. In other words the vote of the other voters determine the outcome irrespective of this voter's vote. That is, i is a null voter in $\mathcal{W}$ if

$$S \in \mathcal{W} \Leftrightarrow S \setminus i \in \mathcal{W}.$$

In a *weighted majority rule* a vector of *weights* $\vec{W} = (w(1), .., w(n))$ is associated to voters, so that the final result is 'yes' if the sum of the weights in favor of the proposal is larger than a given quota Q, with $0 < Q < \sum_{i \in N} w(i)$:

$$\mathcal{W} = \{S \subseteq N : \sum_{i \in S} w(i) > Q\}.$$

Of course if $w(i) = w(j)$ then i and j are symmetric but voters may be symmetric with different weights. In the *unanimity* rule the proposal is accepted if there is a unanimous support:

$$\mathcal{W} = \{N\}.$$

Several features can be evaluated in a voting situation, as the probability of a proposal being accepted or a voter obtaining her preferred outcome. These features require a second input: a probability distribution over the set of vote configurations. Let $p(S)$ denote the probability of S being the vote configuration. For normative purposes we assume that all vote configurations are equally probable $p(S) = \frac{1}{2^n}$ for any S. For a discussion of this assumption, see Laruelle and Valenciano (2005, 2008).

The probability of a proposal being accepted in rule $\mathcal{W}$ is given by

$$\alpha(\mathcal{W}) = \sum_{S \in \mathcal{W}} \frac{1}{2^n}. \tag{2}$$

Voter i's probability of having the result voter i voted for in rule $\mathcal{W}$ is denoted $\Omega(i, \mathcal{W})$. It is given by

$$\Omega(i, \mathcal{W}) = \sum_{\substack{S: \\ i \in S \in \mathcal{W}}} \frac{1}{2^n} + \sum_{\substack{S: \\ i \notin S \notin \mathcal{W}}} \frac{1}{2^n}. \tag{3}$$

We will also deal with 'interim' evaluations (i.e., conditional expectations updated with the private information of each voter's own vote). Let $\Omega^+(i, \mathcal{W})$ denote voter i's probability of getting the proposal accepted given that voter i favors the proposal, and $\Omega^-(i, \mathcal{W})$ denote voter i's probability of getting the proposal rejected given that voter i is against the proposal. These conditional probabilities are given by

$$\Omega^+(i, \mathcal{W}) = \sum_{\substack{S: \\ i \in S \in \mathcal{W}}} \frac{1}{2^{n-1}} \text{ and } \Omega^-(i, \mathcal{W}) = \sum_{\substack{S: \\ i \notin S \notin \mathcal{W}}} \frac{1}{2^{n-1}}. \tag{4}$$

In the following we will use the following notation: $\vec{\Omega}(\mathcal{W}) = (\Omega(1, \mathcal{W}), ..., \Omega(n, \mathcal{W}))$, $\vec{\Omega}^+(\mathcal{W}) = (\Omega^+(1, \mathcal{W}), ..., \Omega^+(n, \mathcal{W}))$ and $\vec{\Omega}^-(\mathcal{W}) = (\Omega^-(1, \mathcal{W}), ..., \Omega^-(n, \mathcal{W}))$. Some properties are worth mentioning (we omit their proof as obvious[7]).

For any rule $\mathcal{W}$,

$$\alpha(\mathcal{W}) \leq \frac{1}{2}, \text{ and } \Omega(i, \mathcal{W}) = \frac{\Omega^+(i, \mathcal{W}) + \Omega^-(i, \mathcal{W})}{2} \text{ for any } i \in N. \tag{5}$$

In the weighted majority $\mathcal{W}$ with $\sum_{i \in N} w(i) = 2Q + 1$ the following equalities hold:

$$\alpha(\mathcal{W}) = \frac{1}{2} \text{ and } \Omega^+(i, \mathcal{W}) = \Omega^-(i, \mathcal{W}) = \Omega(i, \mathcal{W}) \text{ for any } i \in N. \tag{6}$$

If i is a vetoer in $\mathcal{W}$, then

$$\Omega^-(i, \mathcal{W}) = 1. \tag{7}$$

If i and j are symmetric in $\mathcal{W}$, then

$$\Omega^+(i, \mathcal{W}) = \Omega^+(j, \mathcal{W}), \ \Omega^-(i, \mathcal{W}) = \Omega^-(j, \mathcal{W}), \ \Omega(i, \mathcal{W}) = \Omega(j, \mathcal{W}). \tag{8}$$

If i is a null voter in $\mathcal{W}$, then

$$\Omega^+(i, \mathcal{W}) = \alpha(\mathcal{W}), \ \Omega^-(i, \mathcal{W}) = 1 - \alpha(\mathcal{W}), \text{ and } \Omega(i, \mathcal{W}) = \frac{1}{2}. \tag{9}$$

If $\mathcal{W} = \{N\}$, then

$$\alpha(\mathcal{W}) = \frac{1}{2^n}, \ \Omega^+(i, \mathcal{W}) = \frac{1}{2^{n-1}}, \ \Omega^-(i, \mathcal{W}) = 1, \text{ and } \Omega(i, \mathcal{W}) = \frac{1 + 2^{n-1}}{2^n} \text{ for any } i \in N. \tag{10}$$

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[7]To obtain (6) note that in such a weighted majority $S \in \mathcal{W}$ if and only if $N \backslash S \notin \mathcal{W}$ on the one hand and $i \in S$ if and only if $i \notin N \backslash S$ on the other hand. Property (9) is a direct corollary of Proposition 2 in Laruelle et al. (2006).

On the basis of the vectors of distribution of seats $\vec{W}_t^0$, $\vec{W}_t^A$, $\vec{W}_t^B$, and $\vec{W}_t^C$, we can derive the weighted majorities $\mathcal{W}_t^0$, $\mathcal{W}_t^A$, $\mathcal{W}_t^B$, and $\mathcal{W}_t^C$, all of them with an identical quota equals to half the number of seats in the Parliament: $Q_t = k_t/2$. We then compare $\mathcal{W}_t^0$ with $\mathcal{W}_t^Z$ for $Z = A, B, C$ by computing (2), (3) and (4) for $\mathcal{W}_t^0$ and $\mathcal{W}_t^Z$. In the next section we study in detail the case of the Basque Parliament.

3   Application to the Basque Parliament

We focus on the Parliament of the Basque Autonomous Community, one of the 17 regions in Spain. The party system is described in the Appendix. The web page of the Department of Security of the Basque Government[8] provides all electoral results since the first elections in 1977, including the number of abstentions, null and blank votes.

Ten elections were held for the Basque Parliament between 1977 and 2015: $t = 1980, 1984, 1986, 1990, 1994, 1998, 2001, 2005, 2009$ and 2012. The number of seats in the Basque Parliament (k_t) has been 75 since 1984, it was 60 in the 1980 election. There exist three electoral districts corresponding to the three provinces: Biscay, Gipuzcoa and Alava. In each district one third[9] of the total number of seats is allocated according to the D'Hondt method with a threshold of 3% in all electoral years but in 1986 and 1998 when it was 5%.

By contrast with other European countries the Spanish party system is characterized by the existence of many non state-wide parties (Pallares et al., 1997). This explains the large number of competing parties in elections (m_t), given in Table 1. It varies between eight and nineteen, the average is twelve.

t	1980	1984	1986	1990	1994	1998	2001	2005	2009	2012
m_t	16	10	13	19	8	10	10	12	14	19

Table 1: Number of competing parties in the Basque Parliament

For each electoral year the actual apportionment $(\vec{Y}_t^0)$ in the Parliament is obtained by summing the apportionments in the different districts. The set of represented parties (N_t^0) is obtained by restricting the set of competing parties to those with at least one seat. It is given by (21) in the Appendix. Table 2 summarizes the results of the different elections in terms of electorate (e_t), abstention (a_t), blank votes (b_t) and null votes (c_t), as well as the number of parties with representation in Parliament (n_t^0). In the last two columns are the number of votes obtained by the party with the largest number of seats $(v_t(1))$ and the smallest party with a representation in the Parliament

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[8]http://www.euskadi.net/
[9]Each province elects the same number of representatives in spite of an average ratio of populations among the three provinces equals to (4.2: 2.5: 1).

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$(v_t(n_t^0))$.

t	e_t	a_t	b_t	c_t	n_t^0	$v_t(1)$	$v_t(n_t^0)$
1980	1,554,527	625,476	3,570	9,206	7	349,102	36,845
1984	1,584,540	499,236	5,029	6,247	5	451,178	85,671
1986	1,660,143	504,328	5,003	6,737	7	252,233	40,445
1990	1,687,936	658,479	7,580	5,163	7	289,701	14,351
1994	1,749,250	705,165	18,080	6,184	7	304,346	27,797
1998	1,821,608	546,600	17,641	6,802	7	350,322	15,738
2001	1,813,356	381,360	11,508	6,219	5	604,222	78,862
2005	1,799,500	575,866	9,001	4,035	6	468,117	28,180
2009	1,776,059	627,362	11,562	100,939	7	399,600	22,233
2012	1,775,351	639,783	14,640	9,168	5	384,766	21,539

Table 2: Electoral results in the Basque Parliament

As can be seen from Table 2, the number of abstainers is always important, with an average rate of 33.6% of the electorate. The trend has been decreasing in the eighties, starting from its highest level in 1980 (40.23%). Then it has been increasing in the first part of the nineties, and decreasing in the second part to reach its lowest level in 2001 (21.03%). Again it has increased since 2001. The number of abstainers is always larger than the number of votes for the largest party with an exception in 2001. It can thus be expected that the number of seats allocated to the abstention would be large if these votes were taken into account. The percentage of blank votes has always been low, between 0.22% and 1% of the total electorate. The number of blank votes is always smaller than the number of votes of the smallest party with a representation in the Parliament. There is however an exception in 1998, when it obtained more votes than the smallest party that got two seats. The percentage of null votes has also been low, between 0.2% and 0.6% of the electorate. There is an exception in 2009 when it obtained 100,939 votes and reached 5.7% of the total electorate. The number of represented parties (n_t^0) varies between five and seven, while the actual distribution of seats $(\vec{W}_t^0)$ is the following:

$$
\begin{aligned}
\vec{W}_{1980}^0 &= (25, \quad 11, \quad 9, \quad 6, \quad 6, \quad 2, \quad 1) \\
\vec{W}_{1984}^0 &= (32, \quad 19, \quad 11, \quad 7, \quad 6) \\
\vec{W}_{1986}^0 &= (19, \quad 17, \quad 13, \quad 13, \quad 9, \quad 2, \quad 2) \\
\vec{W}_{1990}^0 &= (22, \quad 16, \quad 13, \quad 9, \quad 6, \quad 6, \quad 3) \\
\vec{W}_{1994}^0 &= (22, \quad 12, \quad 11, \quad 11, \quad 8, \quad 6, \quad 5) \\
\vec{W}_{1998}^0 &= (21, \quad 16, \quad 14, \quad 14, \quad 6, \quad 2, \quad 2) \\
\vec{W}_{2001}^0 &= (33, \quad 19, \quad 13, \quad 7, \quad 3) \\
\vec{W}_{2005}^0 &= (29, \quad 18, \quad 15, \quad 9, \quad 3, \quad 1) \\
\vec{W}_{2009}^0 &= (30, \quad 25, \quad 13, \quad 4, \quad 1, \quad 1, \quad 1) \\
\vec{W}_{2012}^0 &= (27, \quad 21, \quad 16, \quad 10, \quad 1)
\end{aligned}
\tag{11}
$$

Whenever there is a single district the apportionment method satisfies the monotonicity condition (1). This property may be violated when there are several districts. This happens in three electoral years out of ten. In 1986 the most voted party (271,208 votes) obtained fewer seats (17) than did the second most voted party (19 seats for 252,233 votes). The reason is that the most voted party got more votes in one district, where it got two more seats while in the other two districts the second most voted party got more votes and two more seats in both. In 1990 the smallest party with representation in Parliament mainly concentrated its 14,351 votes in one district and obtained three seats while no seat was allocated to the party that obtained 14,440 votes. In 2012 the largest party without representation in the Parliament obtained 30,318 votes while the smallest party with representation obtained one seat with 21,539 votes.

For each electoral year t the vector of weight $\vec{W}_t^0$ and the quota Q_t (with $Q_{1980} = 30$ and $Q_t = 37.5$ for $t \neq 1980$) define the actual voting rule among the parties that we denote $\mathcal{W}_t^0$. Analyzing them we can observe classical properties in weighted majorities: (i) many parties appear to be symmetric in spite of different numbers of seats and (ii) some parties are null voters in spite of having a strictly positive number of seats.

(i) In $\mathcal{W}_{1980}^0$ the second, the third, the fourth and the fifth largest parties are symmetric with 11, 9, 6, and 6 seats, respectively. It is also the case for the two smallest parties with 2 and 1 seats, respectively. In $\mathcal{W}_{1984}^0$ all parties (but the largest party) are symmetric with 19, 11, 7, and 6 seats, respectively. In $\mathcal{W}_{1986}^0$ the two largest parties (with 19 and 17 seats, respectively), and the third, fourth and fifth parties (with 13, 13, and 9 seats, respectively) are symmetric. In $\mathcal{W}_{1990}^0$ the three smallest parties are symmetric (with 6, 6, and 3 seats, respectively). In $\mathcal{W}_{1994}^0$ the second, the third and the fourth largest parties (with 12, 11, and 11 seats, respectively), and the three smallest parties (with 8, 6, and 5 seats, respectively) are symmetric. In $\mathcal{W}_{2001}^0$ the second, third and fourth largest parties (with 19, 13 and 7 seats, respectively) are symmetric. In $\mathcal{W}_{2005}^0$ the second, third and fourth parties are symmetric (with 18, 15 and 9 seats, respectively), as well as the two smallest parties (with 3 and 1 seats, respectively). In $\mathcal{W}_{2009}^0$ the three largest parties are symmetric (with 30, 25, and 13 seats, respectively), so are the four smallest parties (with 4, 1, 1, and 1 seats, respectively). In $\mathcal{W}_{2012}^0$ the second and third largest are symmetric (with 21 and 16 seats, respectively), as well as the two smallest parties (with 10 and 1 seats, respectively).

(ii) Existence of null voters in spite of strictly positive weights. In $\mathcal{W}_{1980}^0$ the two smallest parties null voters (with 2 and 1 seats, respectively). In $\mathcal{W}_{2001}^0$ the smallest party is a null voter (with 3 seats). In $\mathcal{W}_{2005}^0$ the two smallest parties are null voters (with 3 and 1 seats, respectively). In $\mathcal{W}_{2009}^0$ the four smallest parties are null voters (with 4, 1, 1, and 1 seats, respectively).

The difference of seats between symmetric parties can be large (as large as 17 seats in $\mathcal{W}_{2009}^0$). A party with 4 seats can be a null voter (in $\mathcal{W}_{2009}^0$). Note however that in $\mathcal{W}_{2012}^0$ the smallest party with 1 seat is not a null voter.

We continue the analysis of $\mathcal{W}_t^0$ by computing the normative probabilities. First note that as a direct application of (6) we have[10] for any t

$$\alpha(\mathcal{W}_t^0) = \frac{1}{2} \text{ and } \Omega(i, \mathcal{W}_t^0) = \Omega^+(i, \mathcal{W}_t^0) = \Omega^-(i, \mathcal{W}_t^0) \text{ for any } i \in N_t^0. \quad (12)$$

Second symmetric parties will have equal probabilities by (8), and equalities (9) hold for null voters. As equalities (12) hold only the probabilities of the parties getting their preferred are given:

$$
\begin{aligned}
\vec{\Omega}(\mathcal{W}_{1980}^0) &= (0.94, \quad 0.56, \quad 0.56, \quad 0.56, \quad 0.56, \quad 0.50, \quad 0.50) \\
\vec{\Omega}(\mathcal{W}_{1984}^0) &= (0.94, \quad 0.56, \quad 0.56, \quad 0.56, \quad 0.56) \\
\vec{\Omega}(\mathcal{W}_{1986}^0) &= (0.73, \quad 0.73, \quad 0.64, \quad 0.64, \quad 0.64, \quad 0.52, \quad 0.52) \\
\vec{\Omega}(\mathcal{W}_{1990}^0) &= (0.80, \quad 0.70, \quad 0.67, \quad 0.58, \quad 0.55, \quad 0.55, \quad 0.55) \\
\vec{\Omega}(\mathcal{W}_{1994}^0) &= (0.84, \quad 0.63, \quad 0.63, \quad 0.63, \quad 0.56, \quad 0.56, \quad 0.56) \\
\vec{\Omega}(\mathcal{W}_{1998}^0) &= (0.77, \quad 0.70, \quad 0.64, \quad 0.64, \quad 0.61, \quad 0.55, \quad 0.55) \\
\vec{\Omega}(\mathcal{W}_{2001}^0) &= (0.88, \quad 0.63, \quad 0.63, \quad 0.63, \quad 0.50) \\
\vec{\Omega}(\mathcal{W}_{2005}^0) &= (0.88, \quad 0.63, \quad 0.63, \quad 0.63, \quad 0.50, \quad 0.50) \\
\vec{\Omega}(\mathcal{W}_{2009}^0) &= (0.75, \quad 0.75, \quad 0.75, \quad 0.50, \quad 0.50, \quad 0.50, \quad 0.50) \\
\vec{\Omega}(\mathcal{W}_{2012}^0) &= (0.81, \quad 0.69, \quad 0.69, \quad 0.56, \quad 0.56)
\end{aligned}
\quad (13)
$$

The largest party has a probability of obtaining its preferred outcome between 0.73 and 0.94, with an average of 0.83 on the ten electoral years. For the smallest party the probability varies between 0.5 (when it is a null voter) and 0.56, with an average of 0.53.

We then apply the same apportionment method to the vectors of votes adding a fictitious party whose number of votes is equal to one category of the non votes to obtain $\vec{Y}_t^Z$ for $Z = A, B, C$ and $t = 1980, ..., 2012$. As we will see the blank seats would have no effect, the null seats would have an effect in 2009, and the abstention would always have an effect.

For any t, we have $y_t^B(m_t + 1) = 0$: the number of blank votes is never sufficient to obtain a seat. Recall (from Table 2) that the number of blank votes is always smaller than the number of votes of the smallest party with representation in the Parliament (except in 1998). In 1998 no seat would correspond to the blank votes because they are relatively equally distributed in the three districts, while the smallest party with representation in the Parliament concentrates its votes in one district managing to obtain two seats there. A question that arises is then how many votes would be necessary to obtain a ghost seat. Table 3 answers this question, province by province:

---

[10]Although the total number of seat is even for $t = 1980$ there is no configuration of votes such that the total weight is exactly 30 (and thus we have $S \in \mathcal{W}$ if and only if $N \backslash S \notin \mathcal{W}$).

Year	Biscay	Gipuzcoa	Alava
1980	21,069	12,380	4,521
1984	21,826	13,035	4,498
1986	33,733	19,688	7,408
1990	19,291	11,410	4,264
1994	20,184	11,522	4,980
1998	36,462	21,612	9,030
2001	27,996	16,954	6,971
2005	22,970	22,970	6,174
2009	20,145	10,982	5,281
2012	20,928	12,717	5,060

Table 3: Minimum number of votes to achieve one ghost seat

The table shows that in 1990 only 4,264 votes would be sufficient to obtain a ghost seat as long as these votes were concentrated in the smallest province, Alava.

With the exception of the year 2009, the number of null votes is never sufficient to obtain one seat either. As it had been observed in Table 2 the number of null votes is always smaller than the number of votes of the smallest party with representation in the Parliament (except in 2009). We obtain $y_t^C(m_t + 1) = 0$ for $t \neq 2009$ and

$$y_{2009}^C(m_t + 1) = 7, \quad n_{2009}^C = 6, \quad \vec{W}_{2009}^C = (28, \quad 23, \quad 11, \quad 4, \quad 1, \quad 1). \tag{14}$$

Seven seats would correspond to the null votes if they were treated as a party. Recalling that $n_{2009}^0 = 7$, $n_{2009}^C = 6$ means that the smallest party with representation in the actual Parliament would lose its representation. If we compare (11) with (14) it can be seen that $w_{2009}^C(i) < w_{2009}^0(i)$ holds for any $i = 1, 2, 3$ while $w_{2009}^C(i) = w_{2009}^0(i)$ for $i = 4, 5, 6$. The three largest parties lose seats, the smallest party loses its representation while the others keep the same number of seats. Rule $\mathcal{W}_{2009}^C$ is the weighted majority with weights (14) and quota Q_t. This rule exhibits the following properties: (i) the second and the third largest parties are symmetric (with 23 and 11 seats, respectively), (ii) the two smallest parties (with 1 seat) are null voters. Computing (2), (3) and (4) for $\mathcal{W}_{2009}^C$ we respectively obtain:

$$
\begin{aligned}
\alpha(\mathcal{W}_{2009}^C) &= \quad 0.44, \\
\vec{\Omega}^+(\mathcal{W}_{2009}^C) &= (0.75, \quad 0.63, \quad 0.63, \quad 0.5, \quad 0.44, \quad 0.44) \\
\vec{\Omega}^-(\mathcal{W}_{2009}^C) &= (0.88, \quad 0.75, \quad 0.75, \quad 0.63, \quad 0.56, \quad 0.56) \\
\vec{\Omega}(\mathcal{W}_{2009}^C) &= (0.81, \quad 0.69, \quad 0.69, \quad 0.56, \quad 0.5, \quad 0.5)
\end{aligned}
\tag{15}
$$

If we compare (15) with (12) and (13) we can observe that the following inequalities hold:

$$\alpha(\mathcal{W}_{2009}^C) < \alpha(\mathcal{W}_{2009}^0), \text{ and } \Omega^+(i, \mathcal{W}_{2009}^C) \leq \Omega^+(i, \mathcal{W}_{2009}^0) = \Omega^-(i, \mathcal{W}_{2009}^0) \leq \Omega^-(i, \mathcal{W}_{2009}^C)$$

for any $i \in N_{2009}^C$. It is more difficult to adopt a proposal with $\mathcal{W}_{2009}^C$ than with $\mathcal{W}_{2009}^0$. Any party has a larger probability to obtain its preferred outcome in $\mathcal{W}_{2009}^C$ given that it is against the proposal, but a smaller probability if it is in favor of the proposal. The global effect is not identical for all parties as we have $\Omega(i, \mathcal{W}_{2009}^C) > \Omega(i, \mathcal{W}_{2009}^0)$ for $i = 1, 4$ while $\Omega(i, \mathcal{W}_{2009}^C) < \Omega(i, \mathcal{W}_{2009}^0)$ holds for $i = 2, 3, 5, 6$.

By contrast the abstention would have an important effect on the decision-making if it counted as a party in the apportionment method. As it has been seen in Table 2 the number of abstainers is always larger than the number of votes received by the largest party (except in 2001). Table 4 gives the number of seats that would correspond to the abstention if it were treated as a party and the number of parties with representation in Parliament for each electoral year.

t	1980	1984	1986	1990	1994	1998	2001	2005	2009	2012
$y_t^A(m_t + 1)$	29	26	26	33	34	26	16	26	32	32
n_t^A	5	5	6	7	7	6	5	5	4	4

Table 4: Number of ghost seats corresponding to the abstention and number of parties.

The number of ghost seats is between 16 and 34 with an average of 28 seats out of 75 between 1984 and 2012. It always represents more than 20% of the total number of seats in the Parliament, even reaching more than 48% (in 1980, with 29 seats out of 60). If we compare Tables 2 and 4 it can be observed that in the majority of the electoral years at least one party would lose its representation in the Parliament.

The distribution of seats among the parties are modified as follows:

$$
\begin{aligned}
\vec{W}_{1980}^A &= (14, \quad 6, \quad 4, \quad 4, \quad 3) \\
\vec{W}_{1984}^A &= (22, \quad 12, \quad 7, \quad 5, \quad 3) \\
\vec{W}_{1986}^A &= (12, \quad 12, \quad 9, \quad 9, \quad 6, \quad 1) \\
\vec{W}_{1990}^A &= (13, \quad 9, \quad 8, \quad 5, \quad 3, \quad 3, \quad 1) \\
\vec{W}_{1994}^A &= (13, \quad 6, \quad 6, \quad 6, \quad 4, \quad 3, \quad 3) \\
\vec{W}_{1998}^A &= (14, \quad 13, \quad 10, \quad 9, \quad 2, \quad 1) \\
\vec{W}_{2001}^A &= (25, \quad 15, \quad 11, \quad 5, \quad 3) \\
\vec{W}_{2005}^A &= (20, \quad 12, \quad 10, \quad 5, \quad 2) \\
\vec{W}_{2009}^A &= (19, \quad 16, \quad 7, \quad 1) \\
\vec{W}_{2012}^A &= (16, \quad 12, \quad 9, \quad 6)
\end{aligned}
\tag{16}
$$

Comparing (11) and (16) we observe that the numbers of seats lost by the largest parties are in general smaller in percentage than the number of seats lost by the smallest parties. For instance in 1980, the two largest parties would lose less than 50% of the seats (from 25 seats to 14 or from 11 to 6) while the two smallest parties would lose their representation and the third smallest party would lose exactly 50% (from 6 seats to 3 seats).

In electoral year t, rule $\mathcal{W}_t^A$ is the weighted majority with weights $\vec{W}_t^A$ given in (16) and quota Q_t. Analyzing the voting rules we obtain again the classical results on (i)

symmetric parties and (ii) null voters. The new property observed in $\mathcal{W}_t^A$ is that (iii) the largest party is a vetoer in all electoral years.

(i) In $\mathcal{W}_{1980}^A$ all parties are symmetric (with 14, 6, 4, 4 and 3 seats, respectively). In $\mathcal{W}_{1984}^A$ the two largest parties (with 22 and 12 seats, respectively) and the third and fourth largest parties (with 7 and 5 seats, respectively) are symmetric. In $\mathcal{W}_{1986}^A$ the third, fourth and fifth largest parties are symmetric (with 9, 9 and 6 seats, respectively). In $\mathcal{W}_{1990}^A$ the four largest parties (with 13, 9, 8 and 5 seats, respectively) are symmetric. In $\mathcal{W}_{1994}^A$ the five largest parties are symmetric (with 13, 6, 6, 6 and 4 seats, respectively) are symmetric. In $\mathcal{W}_{1998}^A$ the two largest parties are symmetric (with 14 and 13 seats, respectively). This is also the case for fourth and fifth largest parties (with 9 and 2 seats, respectively). In $\mathcal{W}_{2001}^A$ the two smallest parties are symmetric (with 5 and 3 seats, respectively). In $\mathcal{W}_{2005}^A$ the two largest parties (with 20 and 12 seats, respectively) and the two smallest parties (with 5 and 2 seats, respectively) are symmetric. In $\mathcal{W}_{2009}^A$ the three largest parties are symmetric (with 19, 16 and 7 seats, respectively). In $\mathcal{W}_{2012}^A$ all parties are symmetric (with 16, 12, 9 and 6 seats, respectively).

(ii) In $\mathcal{W}_{1984}^A$, $\mathcal{W}_{1986}^A$, $\mathcal{W}_{1990}^A$, $\mathcal{W}_{2001}^A$ and $\mathcal{W}_{2009}^A$ the smallest party is a null voter (with 3, 1, 1, 3 or 1 seats, respectively).

(iii) In $\mathcal{W}_{2001}^A$ the largest party is a vetoer. In $\mathcal{W}_{1984}^A$, $\mathcal{W}_{1986}^A$, $\mathcal{W}_{1998}^A$ and $\mathcal{W}_{2005}^A$ the two largest parties are vetoers. In $\mathcal{W}_{2009}^A$ the three largest parties are vetoers. In $\mathcal{W}_{1990}^A$ the four largest parties are vetoers. In $\mathcal{W}_{1994}^A$ the five largest parties are vetoers. Note that $\mathcal{W}_{1980}^A$ and $\mathcal{W}_{2012}^A$ are unanimities: all parties are vetoers.

t	1980	1984	1986	1990	1994	1998	2001	2005	2009	2012
$\alpha(\mathcal{W}_t^A)$	0.03	0.19	0.13	0.05	0.02	0.14	0.34	0.16	0.13	0.06

Table 5: probabilities of adopting a proposal in $\mathcal{W}_t^A$.

Table 5 presents the results of computing (2) for $\mathcal{W}_t^A$. As it can be observed from the table, the probability of adopting a proposal varies between 0.02 and 0.34. It is very low, nearly always below 0.2. This represents an important decrease compared to $\alpha(\mathcal{W}_t^0) = 0.5$. The average on the ten electoral years is 0.13. Note that the probability of adopting a proposal follows the opposite path of the abstention: when the level of abstention decreases the probability of adopting a proposal in $\mathcal{W}_t^A$ increases.

Computing (4) for $\mathcal{W}_t^A$ we obtain:

$$
\begin{aligned}
\vec{\Omega}^+(\mathcal{W}_{1980}^A) &= (0.07, \quad 0.07, \quad 0.07, \quad 0.07, \quad 0.07) \\
\vec{\Omega}^+(\mathcal{W}_{1984}^A) &= (0.38, \quad 0.38, \quad 0.25, \quad 0.25, \quad 0.19) \\
\vec{\Omega}^+(\mathcal{W}_{1986}^A) &= (0.25, \quad 0.25, \quad 0.19, \quad 0.19, \quad 0.19, \quad 0.13) \\
\vec{\Omega}^+(\mathcal{W}_{1990}^A) &= (0.09, \quad 0.09, \quad 0.09, \quad 0.09, \quad 0.06, \quad 0.06, \quad 0.05) \\
\vec{\Omega}^+(\mathcal{W}_{1994}^A) &= (0.05, \quad 0.05, \quad 0.05, \quad 0.05, \quad 0.05, \quad 0.03, \quad 0.03) \\
\vec{\Omega}^+(\mathcal{W}_{1998}^A) &= (0.28, \quad 0.28, \quad 0.19, \quad 0.22, \quad 0.19, \quad 0.16) \\
\vec{\Omega}^+(\mathcal{W}_{2001}^A) &= (0.69, \quad 0.5, \quad 0.44, \quad 0.38, \quad 0.38) \\
\vec{\Omega}^+(\mathcal{W}_{2005}^A) &= (0.31, \quad 0.31, \quad 0.25, \quad 0.19, \quad 0.19) \\
\vec{\Omega}^+(\mathcal{W}_{2009}^A) &= (0.25, \quad 0.25, \quad 0.25, \quad 0.13) \\
\vec{\Omega}^+(\mathcal{W}_{2012}^A) &= (0.13, \quad 0.13, \quad 0.13, \quad 0.13)
\end{aligned}
\tag{17}
$$

The probabilities are very small in $\mathcal{W}_{1980}^A$, $\mathcal{W}_{1990}^A$, $\mathcal{W}_{1994}^A$, and $\mathcal{W}_{2012}^A$: the values do not reach 0.1 for the largest party. The average probability is 0.25 for the largest party, and is above 0.5 only in $\mathcal{W}_{2001}^A$. For the other parties the values are always below (or in one case equal to) 0.5.

Computing (4) for $\mathcal{W}_t^A$ we obtain:

$$
\begin{aligned}
\vec{\Omega}^-(\mathcal{W}_{1980}^A) &= (1, \quad 1, \quad 1, \quad 1, \quad 1) \\
\vec{\Omega}^-(\mathcal{W}_{1984}^A) &= (1, \quad 1, \quad 0.88, \quad 0.88, \quad 0.81) \\
\vec{\Omega}^-(\mathcal{W}_{1986}^A) &= (1, \quad 1, \quad 0.94, \quad 0.94, \quad 0.94 \quad 0.88) \\
\vec{\Omega}^-(\mathcal{W}_{1990}^A) &= (1, \quad 1, \quad 1, \quad 1, \quad 0.97, \quad 0.97, \quad 0.95) \\
\vec{\Omega}^-(\mathcal{W}_{1994}^A) &= (1, \quad 1, \quad 1, \quad 1, \quad 1, \quad 0.98, \quad 0.98) \\
\vec{\Omega}^-(\mathcal{W}_{1998}^A) &= (1, \quad 1, \quad 0.91, \quad 0.94, \quad 0.91, \quad 0.88) \\
\vec{\Omega}^-(\mathcal{W}_{2001}^A) &= (1, \quad 0.81, \quad 0.75, \quad 0.69, \quad 0.69) \\
\vec{\Omega}^-(\mathcal{W}_{2005}^A) &= (1, \quad 1, \quad 0.94, \quad 0.88, \quad 0.88) \\
\vec{\Omega}^-(\mathcal{W}_{2009}^A) &= (1, \quad 1, \quad 1, \quad 0.88) \\
\vec{\Omega}^-(\mathcal{W}_{2012}^A) &= (1, \quad 1, \quad 1, \quad 1)
\end{aligned}
\tag{18}
$$

The probability of getting the preferred outcome given that the party is against the proposal is very large: it is always above 0.8 for all parties (with the exception of $\mathcal{W}_{2001}^A$ where the values are between 0.69 and 0.75 for the three smallest parties). There are many vetoers (whose probability is equal to 1).

Computing (3) for $\mathcal{W}_t^A$ we obtain:

$$
\begin{aligned}
\vec{\Omega}(\mathcal{W}_{1980}^A) &= (0.53, \quad 0.53, \quad 0.53, \quad 0.53, \quad 0.53) \\
\vec{\Omega}(\mathcal{W}_{1984}^A) &= (0.69, \quad 0.69, \quad 0.56, \quad 0.56, \quad 0.5) \\
\vec{\Omega}(\mathcal{W}_{1986}^A) &= (0.63, \quad 0.63, \quad 0.56, \quad 0.56, \quad 0.56, \quad 0.5) \\
\vec{\Omega}(\mathcal{W}_{1990}^A) &= (0.55, \quad 0.55, \quad 0.55, \quad 0.55, \quad 0.52, \quad 0.52, \quad 0.5) \\
\vec{\Omega}(\mathcal{W}_{1994}^A) &= (0.52, \quad 0.52, \quad 0.52, \quad 0.52, \quad 0.52, \quad 0.51, \quad 0.51) \\
\vec{\Omega}(\mathcal{W}_{1998}^A) &= (0.64, \quad 0.64, \quad 0.55, \quad 0.58, \quad 0.55, \quad 0.52) \\
\vec{\Omega}(\mathcal{W}_{2001}^A) &= (0.84, \quad 0.65, \quad 0.59, \quad 0.53, \quad 0.53) \\
\vec{\Omega}(\mathcal{W}_{2005}^A) &= (0.66, \quad 0.66, \quad 0.59, \quad 0.53, \quad 0.53) \\
\vec{\Omega}(\mathcal{W}_{2009}^A) &= (0.63, \quad 0.63, \quad 0.63, \quad 0.5) \\
\vec{\Omega}(\mathcal{W}_{2012}^A) &= (0.56, \quad 0.56, \quad 0.56, \quad 0.56)
\end{aligned}
\tag{19}
$$

Comparing (13), (17), (18) and (19) we observe that

$$
\Omega^+(i, \mathcal{W}_t^A) < \Omega^+(i, \mathcal{W}_t^0) = \Omega^-(i, \mathcal{W}_t^0) < \Omega^-(i, \mathcal{W}_t^A) \text{ for any } i \in N_t^A. \tag{20}
$$

Note that the difference between $\Omega_i^+(\mathcal{W}_t^A)$ and $\Omega_i^+(\mathcal{W}_t^0)$ and between $\Omega_i^-(\mathcal{W}_t^0)$ and $\Omega_i^-(\mathcal{W}_t^A)$ are large. If abstention were treated as a party in the apportionment method the probability of getting the preferred outcome given that the party favors (resp. rejects) the proposal would substantially decrease (resp. increase) in the Parliament. Comparing (13) and (19) we can observe that in general $\Omega(i, \mathcal{W}_t^A) \leq \Omega(i, \mathcal{W}_t^0)$ (the three exceptions being $i = 2$ for $t = 1984$ and $i = 5$ for $t = 2001, 2005$). For any party rule $\mathcal{W}_t^0$ generally gives a higher probability of getting its preferred outcome than rule $\mathcal{W}_t^A$ and therefore should be preferred (unless the party attaches the greatest importance to the possibility of blocking proposal).

4   Discussion

The potential impact of non voters is usually measured by simulating complete turnout. Lutz and Marsh (2007) review the studies on the impact of turnout on the electoral outcome. They conclude that turnout does not matter a great deal, no matter what method, dataset or period of time the authors consider. Kohler (2011) studies the effect on the government formation in Germany from 1949 to 2009. Non voters are assumed to behave like voters with the same social characteristics. He arrives to a similar conclusion: the non voters would most likely not have had a big influence on government formation in Germany. Here we depart from the above mentioned assumption on the non voters' preferences. We basically assume that non voters choose not to vote to express dissatisfaction and allow the institutional change that permits to represent them via empty seats. The potential impact of non voters would then be substantial. Our results complement the previous ones as part of the non voters

are not interested in politics while others are dissatisfied with the political system as illustrated by the recent creation of some parties in Italy, Greece or Spain.

The institutional change suggested in this paper brings some insight to the discussion on compulsory voting. Lijpart (1997) supports compulsory voting as it would boost participation and discusses two arguments against it. First if it forces to the polls people with little political interest, it may also serve as an incentive to become more informed. Second although individual freedom is reduced, there is no duty to cast a valid vote. Nevertheless only valid votes really count in the apportionment method. This paper suggests that additional options could be offered to voters. For instance on top of being allowed to vote for candidates, voters could cast an indifference vote or a vote of dissatisfaction. Votes of dissatisfaction could be computed as a party in the apportionment method and the corresponding seats would be left empty, while indifferent votes would be considered as the current blank votes. This additional options would allow dissatisfied citizens to be "represented" in the Parliament by ghost seats. Lijpart (1997) mentions the opposition of conservative parties as an especially big obstacle to the adoption of compulsary voting. The opposition of parties to the introduction of ghost seats may even be stronger.

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5 Appendix: Party system in the Basque Parliament

The parties with representation in Parliament are the following:

$$
\begin{aligned}
N^0_{1980} &= \{EAJ\text{-}PNV, HB, PSE\text{-}PSOE, UCD, EE, AP, PCE\text{-}EPK\} \\
N^0_{1984} &= \{EAJ\text{-}PNV, PSE\text{-}PSOE, HB, AP\text{-}PDP\text{-}UL, EE\} \\
N^0_{1986} &= \{PSE\text{-}PSOE, EAJ\text{-}PNV, HB, EA, EE, CDS, AP\text{-}PL\} \\
N^0_{1990} &= \{EAJ\text{-}PNV, PSE\text{-}PSOE, HB, EA, PP, EE, U.AL\} \\
N^0_{1994} &= \{EAJ\text{-}PNV, PSE\text{-}EE/PSOE, HB, PP, EA, EB\text{-}B, U.AL\} \\
N^0_{1998} &= \{EAJ\text{-}PNV, PP, EH, PSE\text{-}EE/PSOE, EA, U.AL, EB\text{-}B\} \\
N^0_{2001} &= \{EAJ\text{-}PNV/EA, PP, PSE\text{-}EE/PSOE, EH, EB\text{-}B\} \\
N^0_{2005} &= \{EAJ\text{-}PNV/EA, PSE\text{-}EE/PSOE, PP, PCTV\text{-}EHAK, EB\text{-}B, Aralar\} \\
N^0_{2009} &= \{EAJ\text{-}PNV, PSE\text{-}EE/PSOE, PP, Aralar, EA, UPyD, EB\text{-}B\} \\
N^0_{2012} &= \{EAJ\text{-}PNV, EH\text{-}Bildu, PSE\text{-}EE/PSOE, PP, UPyD\}. \quad\quad (21)
\end{aligned}
$$

They are ordered by their number of seats. Out of the four largest parties, two are non state-wide parties, and two are state-wide parties. The political project of the non state-wide parties concerns the concept of nation and identity. The largest party in terms of votes is the Basque Nationalist party (EAJ-PNV), a (non state-wide) nationalist right-wing party. The other nationalist party represents what is known as the Patriotic Left ("Izquierda Abertzale"). It competed in elections under different names and sometimes in coalitions (HB from 1980 till 1994, EH in 1998 and 2001, Batasuna from 2001 till 2011, and EH-Bildu since 2012). Note that Batasuna was outlawed in 2003 for its connection with the ETA terrorist group. In the 2005 elections Batasuna's supporters mainly voted for the Communist party of the Basque Lands (PCTV-EHAK). See Pallares et al. (2007). After this last party was outlawed in 2008 Batasuna called its supporters to cast a null vote in the 2009 election (and as a result the null votes reached 5.7% of the total electorate). The other two main parties are state-wide parties divided along the left-right cleavage: the right-wing Popular party (PP, formally AP, AP-PDP-UL, or AP-PL) and the left-wing Socialist party (PSE-PSOE, or currently PSE-EE/PSOE after the integration of the EE party).

A part from these four main parties, three others parties have obtained a representation in half the elections or more. The Basque Solidarity party (EA) was born by the splitting of the Basque Nationalist party. It has been represented since 1986 till nowadays, alone or in coalition (in 2001 and 2005 with the EAJ-PNV, in 2012 within the EH-Bildu coalition). The Basque Left (EE) was present in the first four legislature before it joined the Socialist party. The United Left and Green party (EB-B) is a Communist party, partner of the state-wide United Left party (IU). It obtained seats in the five legislatures between 1994 and 2009.

Three parties obtained seats in two or three legislatures. Aralar is a party that results from the split from Batasuna when this party was banned: it was present in the 2005 and 2009 elections before joining the EH-Bildu coalition for the 2012 elections. When the popular party was founded in 1989 part of the members did not agree with its programme regarding the territory division of the Basque Country and formed the party of Alavesan Unity (UAL). This party obtained seats in three consecutive legislatures (from 1990 till 1998) and was dissolved in 2005. The party of Union, Progress and Democracy (UPyD) was founded by a former socialist leader in 2007 as an anti-nationalist centrist party.

Finally some parties have obtained some parliamentary representation in some isolated elections: the state-wide centrist parties Union of Democratic Centre (UCD) and Democratic Social Centre (CDS) (respectively present in the 1980 and 1986 elections), or the communist party (PCE-EPK in the 1980 elections). For more details, see Leonisio (2012), Strijbis and Leonisio (2012) or Gomez and Cabeza (2013).