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Structural Synthesis of 3-DoF Spatial Fully Parallel Manipulators

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Abstract In this paper, the architectures of three degrees of freedom (3-DoF) spatial, fully parallel manipulators (PMs), whose limbs are structurally identical, are obtained systematically. To do this, the methodology followed makes use of the concepts of the displacement group theory of rigid body motion. This theory works with so-called 'motion generators'. That is, every limb is a kinematic chain that produces a certain type of displacement in the mobile platform or end-effector. The laws of group algebra will determine the actual motion pattern of the end-effector. The structural synthesis is a combinatorial process of different kinematic chains' topologies employed in order to get all of the 3-DoF motion pattern possibilities in the end-effector of the fully parallel manipulator.

Keywords Parallel Manipulator, Structural Synthesis, Group of Displacement, Kinematic Bond, Motion Pattern

1. Introduction

The first step in the design of a parallel manipulator is to identify the requirements of the operation, focusing on the customer's specifications for a specific application. One of these requirements is the so-called 'motion pattern' of the platform [1]. The motion pattern gives the designer information about the number and type of degrees of freedom of the end-effector - this is, whether they are rotational or translational, and instantaneous or permanent, including also their directions. Based on this, a structural synthesis (also called 'morphological synthesis') can be performed. Its objective is to determine the topology of the PM so that its mobile platform possesses a specific motion pattern. The 3-DoF PMs are included in the so-called 'set' of lower-mobility PMs.

In general, the design of lower-mobility parallel manipulators is more complex. In order to make the structural synthesis of such mechanisms, several approaches already exist. One of them is the theory of groups of displacements, which uses the mathematical properties of Lie Groups of rigid body displacements.. These mathematical concepts were not introduced into the field of the structural synthesis of mechanisms until the seminal work of Hervé [2]. In this way, the kinematics of a rigid body arises from the general group of displacements with dimension six and its 12 subgroups. One of the concepts used is the kinematic bond $L(i,j)$ between two elements i and j, which is defined as the relative displacement between these elements. In this work, the element *i* is the fixed frame and the element *j* is the end-effector. A kinematic bond can be materialized by a set of connected elements and pairs constituting a kinematic chain or limb of a PM. Every limb is a motion generator, in the sense that it produces a particular displacement in the end-effector. Furthermore, the kinematic pairs included in a limb are, at the same time, motion generator subgroups. The motion generators follow the rules of group algebra under the operations of union (product) and intersection. In this way, the displacement generated in the end-effector results from the product of the pair-motion generator subgroups in the kinematic chain. Each of the kinematic chains constituting every limb of the PM generates a displacement in the end-effector, which must be compatible with the motion pattern required. To obtain the end-effector motion pattern in the assembled manipulator, the motion generators of all the limbs have to be intersected. Then, only the common type of limb-DoFs remains in the PM mobile platform. In conclusion, the determination of the different kinematic bonds, as well as a definition of their possible materializations through kinematic joints, is a fundamental aspect in the structural synthesis.

Two additional concepts will appear during the development of this work: redundant constraint and redundant kinematic pairs. From a kinematical point of view, a mechanism is a set of geometrical constraints among the elements that compose it. Sometimes, the geometrical particularities of a mechanism can ensure that two different geometrical constraints initially behave as just one. In this case, we are considering a redundant constraint. The Grübler formula does not work in mechanisms with redundant constraints. Some of the limbs and complete architectures used in this work will include redundant constraints, so we will have to be especially careful when checking the number and type of DoFs in these cases.

When a kinematic chain (or a mechanism in general) has redundant pairs (or DoFs) it is because the end-effector has fewer DoFs than the kinematic chain (or the mechanism). In this circumstance, it could be said that it is a waste of the capacity of the mechanism unless another function is sought, such as, for example, to move in an environment with obstacles.

In this paper, we will obtain - in a systematic manner - all the 3-DoF spatial parallel manipulator topologies. The methodology that follows has its preliminary bases in [3]. In this paper, consideration will only be given to fully parallel manipulators whose limbs are structurally equal, being located in a – preferably - symmetrical form. This work will not consider planar PMs because they have been sufficiently studied in the bibliography. The authors presented the fundamentals of the synthesis procedure in [4], where they focused mainly on the synthesis of 3-DoF PMs with a 3T motion pattern to prove the methodology used. In the present paper, the research is extended and completed by obtaining the families of a 3-DoF parallel architectures with 3T, 2T1R, 1T2R and 3R motion patterns (where *T* and *R* refer to the character of translation and rotation of the DoF). As a result of this process, the obtained architectures will be referred to in the corresponding previous works in case they already exist. Otherwise, they will be novel cases that could be a source of new designs.

2. Bases of the synthesis

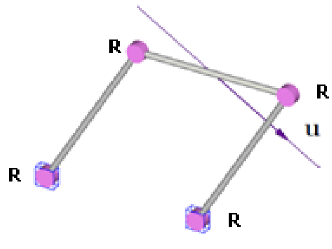
The following fundamentals are going to systematize the structural study of a 3-DoF PM,

1. Each kinematic chain must have at least the same number and type of DoFs as the terminal element of the PM. The kinematic chains with 6-DoFs are excluded. This is due to the fact that the intersection of their motion patterns does not cancel any particular DoF because they include all the components of the rigid body motion group. Therefore, the number of DoFs at the kinematic chains will be a minimum of three and a maximum of five.
2. The structures of the kinematic chains are composed by combining those kinematic pairs needed to obtain the required number and type of DoFs. Specifically, the kinematic pairs to be used will be: 1-DoF (Revolute *R*, Prismatic *P*, Parallelogram *Pa*), 2-DoF (Cylindrical *C*, Universal Joint *U*, Double Parallelogram Pa^2) and 3-DoF (Spherical *S*). It is interesting to note that the translation motion of the end-effector using kinematic chains with *Pa* and Pa^2 depends on the posture of the robot. In Figure 1, some configurations including *Pa* and Pa^2 joints are depicted.
3. The motion pattern at the end-effector of the chain is the product of the subgroups of displacement that compose the kinematic bonds of that chain - in other words, the product of the transformation matrices that they represent. If there are no redundant pairs in the chain, the DoF of the end-effector is the sum of the DoF of the kinematic pairs that compose the chain.
4. Geometric aspects and nomenclature. It is necessary to bear in mind the geometry arrangement of the kinematic chains. For example, if the chain *RRR* has three parallel axes, the generated displacement is *2T1R*; if only two are parallel, the displacement will be *1T2R*. A *RRRRR* chain with all its axes parallel generates, in the end-effector, only 3-DoF with a displacement *2T1R*. Nevertheless, if there is a set of three *R* parallel pairs and another set of two *R* parallel pairs independent of the previous one, the displacement is *3T2R*. For this reason, regarding

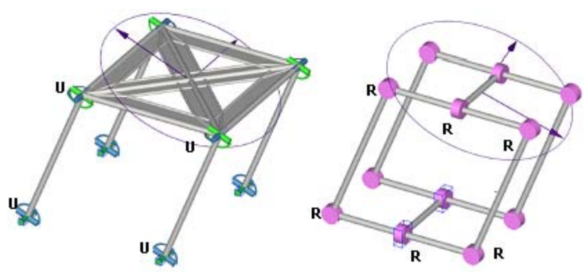
naming the chain for its pairs, if the axes of the pairs are parallel then they will be indicated by an underline or an overline (e.g., $\underline{RRR}\overline{RR}$ in the previous case). To indicate that the axes meet at one point, they will be put into parentheses; for example, the displacement of the $\underline{RR}(RRR)$ chain is *2T3R*. Finally, the actuated pairs are also underlined (double-underlined in the case of coinciding with another underline).

5. Manipulators built using limbs with kinematic bonds of dimension three should have six redundant constraints to get the desired mobility in the platform. The ones formed with the limbs with kinematic bonds of dimension four should have three redundant constraints to guarantee the mobility desired in the platform. Those formed with limbs with kinematic bonds of dimension five should not have redundant constraints so as to guarantee the desired mobility in the platform. All of this can be seen clearly if the criteria of Grübler are applied to the different examples.
6. Finally, in this work consideration will be given only to kinematic chains whose dimension equals that of its corresponding kinematic bond. That is to say, that redundant pairs will not be considered as in the case of the $\underline{RRRR}$ redundant chain corresponding to the motion pattern *2T1R*.

In the following figures of this paper, the motion patterns of the platform will be represented according to the following nomenclature: double-headed arrows represent the directions of the rotational DoFs, and single arrows represent the translation directions. Ellipses circumscribing two arrows represent the planes where rotational or translational motions are possible.



a) Planar parallelogram *Pa*



b) Spatial parallelogram Pa^2

Figure 1. Non-conventional *Pa* and Pa^2 kinematic pairs

3. Synthesis of the families of 3-DoF spatial parallel manipulators

The structural synthesis is arranged step by step in the next sequence:

1. Definition of all the end-effector motion patterns. In this case, they are: *3T0R*, *2T1R*, *1T2R* and *0T3R*.
2. Obtaining all the serial kinematic chains of dimensions three, four and five, according to every motion pattern identified in the first step.
3. Selection of the kinematic chains that include at least the motion pattern considered for the study, indicating the position and orientation conditions of the pair axes, if applicable.
4. Configuration of three identical kinematic chains with a geometrical disposition, such that the intersection of the motion pattern of the three chains is the answer sought.

3.1 PMs with 3T0R displacements

Only kinematic bonds with displacements containing at least three translations can be used at each limb to obtain this type of PM. This type of PM is known as a 'Translational Parallel Manipulator' (TPM). For the sake of the clarity of the figures depicted later on, *Pa* and Pa^2 pairs have not been used in the manipulators generated by the structural synthesis process.

3.1.1 Kinematic chains with three-dimensional bonds

In this case, the kinematic chains are *PPP*, *PPPa*, PPa^2, Pa^2Pa and their permutations, requiring that the three directions of translation are linearly independent. A typical case is that of the *3-PPP* PM [5]. In general, any geometrical configuration of three limbs with prismatic pairs will generate six redundant constraints and so they make the movement of the manipulator possible.

3.1.2 Kinematic chains with four-dimensional bonds

The only kinematic bond of dimension four, which includes three translations, is the generator of the displacement group *3T1R*, known as 'Schönflies motion' or 'SCARA'. The kinematic chains to be considered are:

a) Chains *PPPR*, *PPPaR*, PPa^2R, Pa^2PaR, *PPC*, *PPaC*, Pa^2C and permutations, being the three directions of translation that are linearly independent.
b) Chains *PPRR*, *PPaRR*, Pa^2RR, *PCR*, *PaCR* and permutations. The revolute pairs have to be of parallel axes that are not coincidental, and the resulting generated translation has to be linearly independent of the directions of the other two independent translations.
c) Chains *PRRR*, *PaRRR*, *CRR* and permutations. The revolute pairs have to be of parallel axes and not coincidental, and the direction of translation of *P*, *Pa*

or C cannot be parallel to a plane perpendicular to the axes of the R pairs.

The PM originating from chains in α) does not contribute any advantage with regard to the $3-\underline{P}PP$; only the $3-\underline{P}PC$ (see Figure 2a) and its combinations with pairs Pa seem useful. More interesting configurations originate from β); we can highlight the $3-\overline{R}P\overline{C}$ [6], the $3-\underline{P}\overline{RC}$ [7] and the $3-\underline{P}P\overline{RR}$ in Figure 2b. Within the chains in ◎) we can highlight the $3-\overline{\underline{C}RR}$ robot developed by Kong and Gosselin [8] and the $3-\overline{\underline{R}R}Pa\overline{R}$ University of Maryland manipulator developed by Tsai and Stamper [9], this latter design being a variant of the well known Delta parallel robot [10].

3.1.3 Kinematic chains with five-dimensional bonds

The only kinematic bond of dimension five which includes three translations, is the generator of displacement *3T2R*. The kinematic chains to consider are:

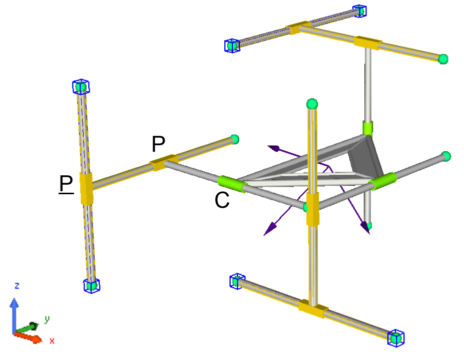
a) Chains *PPPRR*, *PPPaRR*, *PPa*²RR, *Pa*²*PaRR*, *PPCR*, *PPaCR*, *Pa*²*CR*, *PPPU*, *PPPaU*, *PPa*²*U*, *Pa*²*PaU*, *CCP*, *PaCC* and permutations, being the three directions of translation that are linearly independent and the axes of the two rotations which are also independent

b) Chains *PPRRR*, *PaPRRR*, *Pa*²*RRR*, *PPUR*, *PaPUR*, *Pa*²*UR*, *PCRR*, *PaCRR*, *PCU*, *PaCU*, *CCR* and permutations. Two translations have to be linearly independent, and two of the axes of rotation must be parallel but not coincidental, as well as independent of the two translations and the third rotation.

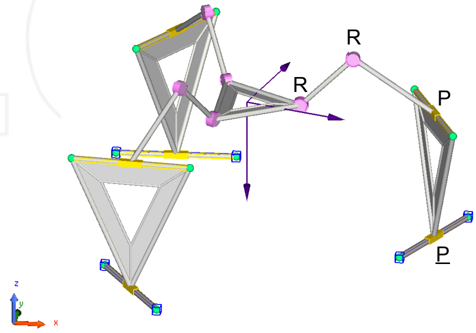
c) Chains *PRRRR*, *PaRRRR*, *PRRU*, *PaRRU*, *PUU*, *PaUU*, *CRRR*, *CRU* and permutations. Three of the axes of rotation have to be parallel to each other and not coincidental, and the direction of translation cannot be perpendicular to them. Other feasible configurations have two sets of parallel axes not coincidental and not perpendicular to the direction of translation.

d) Chains *RRRRR*, *RRRU*, *RUU* and permutations. A set of three rotation axes have to be parallel to each other and not coincidental, another set with the other two axes has to be of the same architecture, being both groups linearly independent of each other.

The kinematic chains in α) do not contribute anything with regard to the $3-\underline{P}PP$. In any case, to obtain three translations exclusively, three equal kinematic chains must be positioned in a manner such that the two rotations are cancelled. This is the case of the $3-\underline{P}PCR$ in Figure 3. The configurations in b) do not provide anything extra to that already studied, and having a greater number of kinematic pairs and elements increases its complexity. The 3-UPU presented in [11] and geometrically optimized in [12] is an example of the use of this kind of chain. Worth mentioning apart are the configurations in d). They offer a good alternative for chains with prismatic pairs, as, for example with the $3-\underline{RRR}\overline{RR}$ and some other designs that can be found in [13-15].



a) $3-\underline{P}PC$



b) $3-\underline{P}P\overline{RR}$

Figure 2. *3T0R* displacement generators with four-dimensional bonds

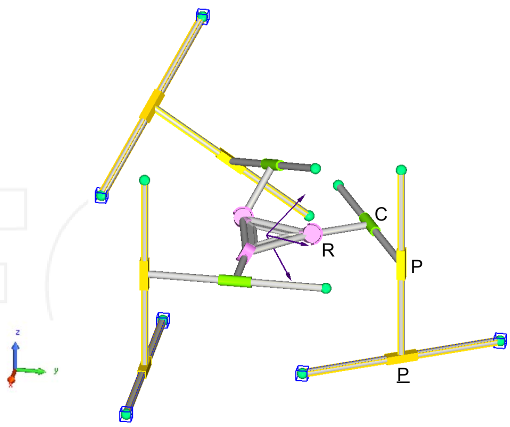


Figure 3. *3T0R* displacement generator with five-dimensional bonds ($3-\underline{P}PCR$)

3.2 PMs with 2T1R displacements

Only the kinematic bonds containing at least two translations and one rotation will produce parallel manipulators with displacements *2T1R*. Bonds with three translation generators must not be taken into account because the intersection of the three limbs will not eliminate any of the translations. Taking into account these considerations, no architecture with a motion pattern different from the planar motion has been found.

	3T0R displacements
3 dim. kin. bonds	• *PPP, PPPa, PPa^2, Pa^{2}Pa.*
4 dim. kin. bonds	• *PPPR, PPPaR, PPa^{2}R, Pa^{2}PaR, PPC, PPaC, Pa^{2}C.* • *PPRR, PPaRR, Pa^{2}RR, PCR, PaCR.* • *PRRR, PaRRR, CRR.*
5 dim. kin. bonds	• *PPPRR, PPPaRR, PPa^{2}RR, Pa^{2}PaRR, PPCR, PPaCR, Pa^{2}CR, PPPU, PPPaU, PPa^{2}U, Pa^{2}PaU, CCP, PaCC.* • *PPRRR, PaPRRR, Pa^{2}RRR, PPUR, PaPUR, Pa^{2}UR, PCRR, PaCRR, PCU, PaCU, CCR.* • *PRRRR, PaRRRR, PRRU, PaRRU, PUU, PaUU, CRRR, CRU.* • *RRRRR, RRRU, RUU.*

Table 1. *3T0R* kinematic chains (and permutations). The geometrical conditions to be fulfilled are detailed in the text.

3.3 PMs with 1T2R displacements

As before, only kinematic bonds with displacements containing at least one translation and two rotations will be able to produce parallel manipulators with displacements *1T2R*. Bonds with three translation generators must not be taken into account, as explained on section 3.2.

3.3.1 Kinematic chains with three-dimensional bonds

In this case, the only kinematic bonds to be considered are those with displacements 1T2R, that is:

a) Chains *PRR*, *PaRR*, *PU*, *PaU*, *CR* and permutations. The rotation axes have to cut or cross between them.

b) Chains *RRR*, RU and permutations. Two rotation axes have to be parallel to each other or the three axes contained in a plane intersected by pairs.

The displacements imposed by each chain have to be identical in order to maintain the 1T2R motion pattern after the intersection of the three motion generators. Consequently, no structures with practical use can be obtained.

3.3.2 Kinematic chains with four-dimensional bonds

In this case, the kinematic bonds to be considered are those corresponding to the displacements *2T2R* and *1T3R*.

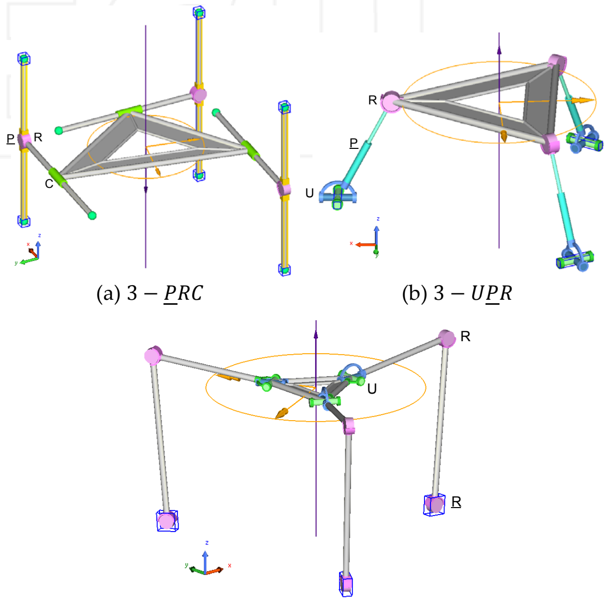
a) Displacements *2T2R*

- Chains *PPRR*, *PaPRR*, *Pa^{2}RR*, *PPU*, *PaPU*, *Pa^{2}U*, *PCR*, *PaCR*, *CC* and permutations. The two directions of translation have to be linearly independent, and the same applies to the two rotations.
- Chains *PRRR*, *PaRRR*, *PRU*, *PaRU*, *CRR*, *CU* and permutations. Two of their three *R* axes have to be parallel, while the translation generated by them must be independent of the other translation.
- Chains *RRRR*, *RRU*, *UU* and permutations. Three of the four rotations have to be parallel. Consequently, chain *UU* is not usable.

b) Displacements *1T3R*

- Chains *PRRR*, *PaRRR*, *PUR*, *PaUR*, *PS*, *PaS*, *CRR*, *CU* and their permutations. The axes of rotation generators have to be linearly independent.
- Chains *RRRR*, *RRU*, *RS*, *UU* and their permutations. The axes of the four rotation generators must define three independent directions.

Some of the architectures using kinematic chains in a) are as follows: $3-\underline{P}RC$ (Figure 4a), $3-U\underline{P}R$ (Figure 4b) and $3-\overline{\underline{R}}RU$ (Figure 4c). The problem is that all of them have *1T2R* instantaneous mobility. It can achieve a *1T1R* permanent mobility by restricting the inputs to a linear dependency among them. Using the chains in b), it has not been possible to obtain any useful manipulator.



(a) $3-\underline{P}RC$ (b) $3-U\underline{P}R$

(c) $3-\overline{\underline{R}}RU$

Figure 4. 1T2R Displacement generator with four-dimensional bonds

3.3.3 Kinematic chains with five-dimensional bonds

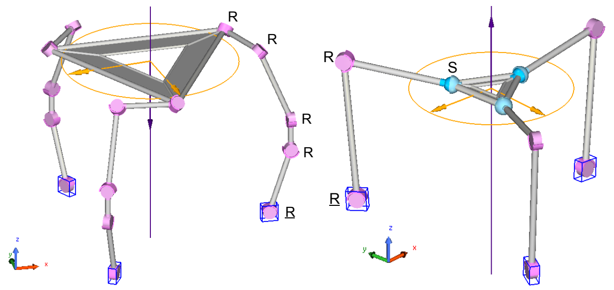
For the same reason above, we discard the kinematic bond *3T2R* and only study those with displacements *2T3R*:

a) Chains *PPRRR*, *PaPRRR*, *Pa^{2}RRR*, *PPRU*, *PaPRU*, *Pa^{2}RU*, *PCRR*, *PaCRR*, *PCU*, *PaCU*, *CCR*, *PPS*, *PaPS*, *Pa^{2}S* and permutations. The two directions of translation have to be linearly independent, as do the three *R* axes.

b) Chains *PRRRR*, *PaRRRR*, *PURR*, *PaURR*, *PUU*, *PaUU*, *CRRR*, *CRU*, *PRS*, *PaRS* and permutations. The axes of the three rotations have to be linearly independent while the remaining *R* axis can be parallel to any of them (or not). If it is parallel, the resulting translation has to be independent of the direction of the prismatic pair

c) Chains *RRRRR*, *RRRU*, *RUU*, *RRS*, *US* and permutations. As in the previous case, the axes of at least three revolute joints have to be linearly independent. The remaining two rotations will not generate additional rotations in the end-effector; hence, they will produce the remaining two translations.

To this group belongs the well-known $3-R\underline{P}S$, studied, for example, in [16-17]. Other manipulators, like $3-\underline{RR}PU$ and $3-P\underline{RR}U$ are studied in [18]. The manipulators $3-\overline{\underline{R}R}(RRR)$ and $3-\overline{\underline{R}R}S$, represented in Figures 5a and 5b, respectively, include kinematic chains with five axes of rotation. Although in this case there is no need to define redundant constraints in order to get 3-DoFs in the platform, and given that kinematic bonds are of dimension five, there is a need to impose a geometric condition in order to convert one of the rotations into one translation. We must bear in mind that the intersection of movements in the platform eliminates the translations first, except in two cases. The first one is when the limbs have three translations, in which case there is no elimination of any translation. The second one is when an imposed geometrical requirement forces the elimination of one rotation instead of one translation, as in the case of the PMs in Figure 5. Every limb has a vertical plane of translations that intersects with that of the other two limbs in a vertical line.



(a) $3-\overline{\underline{R}R}(RRR)$ (b) $3-\overline{\underline{R}R}S$

Figure 5. 1T2R Displacement generator with five-dimensional bonds

3.4 PMs with 0T3R displacements

Only the kinematic bonds containing at least three rotations will produce PMs with displacements *0T3R*.

3.4.1 Kinematic chains with three-dimensional bonds

In this case, only kinematic bonds with *0T3R* displacements are possible. Consequently, the kinematic chains to be considered are *RRR*, *RU*, *S* and their permutations. The directions of the three *R* axes have to be linearly independent.

In this group, a PM with *0T3R* displacements can only be generated using kinematic chains whose axes meet permanently at a point, and in an arrangement such that this point is the same for all the three chains. Consequently, the *RU* and *S* chains are not able to accomplish these conditions, and only architectures with *RRR* chains are useful. In this latter case, and when such conditions are fulfilled, the platform points will move in concentric spherical surfaces with the centre at that point. This solution corresponds to the spherical $3-(\underline{R}RR)$, being the *Agile Eye* [19] application of this configuration.

	1T2R displacements
3 dim. kin. bonds	No structures with practical use.
4 dim. kin. bonds	Only PMs with instantaneous mobility.
5 dim. kin. bonds	• *PPRRR*, *PaPRRR*, *Pa²RRR*, *PPRU*, *PaPRU*, *Pa²RU*, *PCRR*, *PaCRR*, *PCU*, *PaCU*, *CCR*, *PPS*, *PaPS*, *Pa²S*. • *PRRRR*, *PaRRRR*, *PURR*, *PaURR*, *PUU*, *PaUU*, *CRRR*, *CRU*, *PRS*, *PaRS*. • *RRRRR*, *RRRU*, *RUU*, *RRS*, *US*.

Table 2. *1T2R* kinematic chains (and permutations). The geometrical conditions to be fulfilled are detailed in the text.

3.4.2 Kinematic chains with four-dimensional bonds

Kinematic bonds of this type with at least three rotations are only those with *1T3R* displacements:

a) Chains *PRRR*, *PaRRR*, *PUR*, *PaUR*, *PS*, *PaS*, *CRR*, *CU* and permutations. The axes of the rotation generators have to be linearly independent.
b) Chains *RRRR*, *RRU*, *RS*, *UU* and permutations. The axes of the four rotation generators have to define a basis of dimension three.

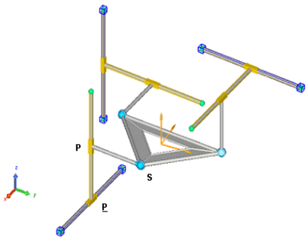
In α), assuming that the prismatic pair is located at the end of the limb, and supressed the translation in the platform due to the intersection of the limb displacements, the only possibility is the spherical motion of the $3-(\underline{R}RR)$. If the prismatic pair is at the beginning of the limb, the platform's spherical motion will only be obtained if the *P*-pair positions ensure that the rotation axes of the limbs intersect at a unique point. In a similar way, case β) leads us to the spherical motion. However, to achieve this kind of motion, three-dimensional bonds are preferred.

3.4.3 Kinematic chains with five-dimensional bonds

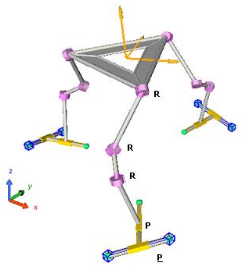
As the kinematics bonds must have at least three rotations, only those with *2T3R* motion will be used,

a) Chains *PPRRR*, *PaPRRR*, *Pa²RRR*, *PPRU*, *PaPRU*, *Pa²RU*, *PCRR*, *PaCRR*, *PCU*, *PaCU*, *CCR*, *PPS*, *PaPS*, *Pa²S* and permutations. The two directions of translation have to be linearly independent and the same applies to the three rotation axes.
b) Chains *PRRRR*, *PaRRRR*, *PURR*, *PaURR*, *PUU*, *PaUU*, *CRRR*, *CRU*, *PRS*, *PaRS* and permutations. The axes of the four rotation generators must define three independent directions.
c) Chains *RRRRR*, *RRRU*, *RUU*, *RRS*, *US* and their permutations. The axes of the five rotation generators must define three independent directions.

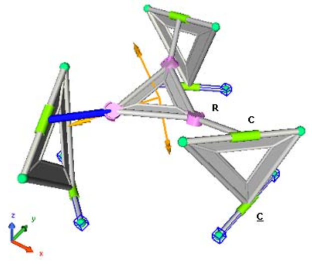
Using these chains, a wide variety of PMs can be generated. Related to case a) are $3-\underline{P}PS$ (Figure 6a), $3-\underline{P}PRRR$ (Figure 6b), $3-\underline{C}CR$ (Figure 6c) and $3-\underline{P}CU$ (Figure 6d). Related to chains of case b), we have $3-\underline{C}S$ (Figure 7a), $3-\underline{R}PS$ (Figure 7b) and $3-\underline{P}RRRR$ (Figure 7c).



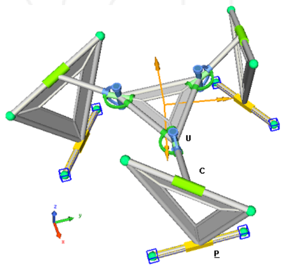
(a) $3-\underline{P}PS$



(b) $3-\underline{P}PRRR$

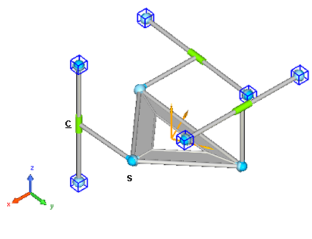


(c) $3-\underline{C}CR$

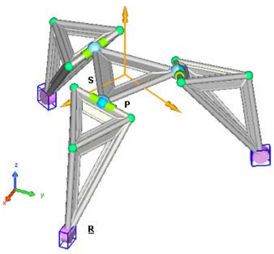


(d) $3-\underline{P}CU$

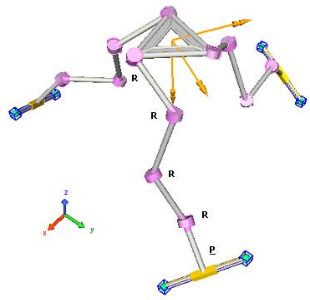
Figure 6. 0T3R Displacement generator with five-dimensional bonds (case a)



(a) $3-\underline{C}S$



(b) $3-\underline{R}PS$

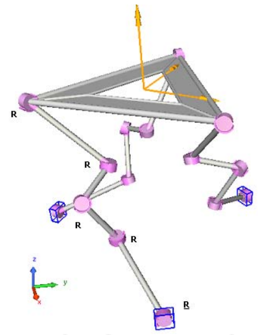


(c) $3-\underline{P}RRRR$

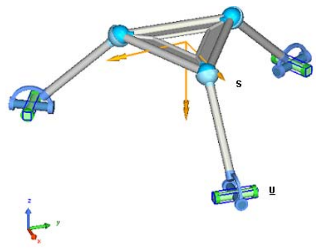
Figure 7. 0T3R Displacement generator with five-dimensional bonds (case b)

Finally, for case c), we obtain $3-\underline{R}RRRR$ (Figure 8a) and $3-\underline{U}S$ (Figure 8b).

Other configurations are $3-\underline{R}RS$ and $3-\underline{R}UU$ [20] and the 3-URU optimized by Huda et al. [21]. As the intersection of the displacements of the kinematic chains eliminates the translations, initially *0T3R* motion is achieved in the mobile platform. Note that none of these manipulators have any geometrical requirements. This is because redundant constraints are not necessary using five-dimensional bonds. This kind of manipulator is characterized because manufacturing tolerances and component assembly errors are factors that are not as important as in over-constrained manipulators



(a) $3-\underline{R}RRRR$



(b) $3-\underline{U}S$

Figure 8. 0T3R Displacement generator with five-dimensional bonds (case c)

4. Conclusions

Using the topological synthesis approach presented in this paper systematically, families of architectures of spatial 3-DoF parallel manipulators have been obtained. These architectures present the following motion patterns: *3T0R*, *2T1R*, *1T2R* and *0T3R*. The approach used is based on the theory of groups of displacements. One of the contributions of the paper has been to establish the bases and premises of the synthesis that appears in Section 2 of this article. They constitute a hierarchical doctrine in a series of rules that arrange the kinematic structure of the mechanism, as well as certain conditionings on the geometric characteristics of the elements and kinematic pairs that compose the limbs of the parallel manipulator. The result of the process has been the generation of families of PMs, some of which were already referenced in the literature, but also others which are novel and which might be a source of new designs.

	0T3R displacements
3 dim. kin. bonds	• *RRR*. Spherical motion.
4 dim. kin. bonds	• *PRRR, PaRRR, PUR, PaUR, PS, PaS, CRR, CU.* • *RRRR, RRU, RS, UU.*
5 dim. kin. bonds	• *PPRRR, PaPRRR, Pa²RRR, PPRU, PaPRU, Pa²RU, PCRR, PaCRR, PCU, PaCU, CCR, PPS, PaPS, Pa²S.* • *PRRRR, PaRRRR, PURR, PaURR, PUU, PaUU, CRRR, CRU, PRS, PaRS.* • *RRRRR, RRRU, RUU, RRS, US.*

Table 3. *0T3R* kinematic chains (and permutations). The geometrical conditions to be fulfilled are detailed in the text.

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