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Addressing students' difficulties with Faraday's law: A guided problem solving approach

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In traditional teaching, the fundamental concepts of electromagnetic induction are usually quickly analyzed, spending most of the time solving problems in a more or less rote manner. However, physics education research has shown that the fundamental concepts of the electromagnetic induction theory are barely understood by students. This article proposes an interactive teaching sequence introducing the topic of electromagnetic induction. The sequence has been designed based on contributions from physics education research. Particular attention is paid to the relationship between experimental findings (macroscopic level) and theoretical interpretation (microscopic level). An example of the activities that have been designed will also be presented, describing the implementation context and the corresponding findings. Since implementing the sequence, a considerable number of students have a more satisfactory grasp of the electromagnetic induction explicative model. However, difficulties are manifested in aspects that require a multilevel explanation, referring to deep structures where the system description is better defined.

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I. INTRODUCTION

Electromagnetic induction (EMI) is an important part of physics instruction at many different levels. Students learn simple electromagnetic induction phenomena at higher secondary education, and gradually integrate more complex models into these simple ideas throughout their education. In university introductory physics courses, students continue to learn more detailed models for describing EMI in classic electromagnetism.

EMI is a topic in which different fundamental quantities of electricity and magnetism are involved, such as magnetic field, magnetic flux, nonconservative electric field, etc. EMI is challenging to teach, in part because these key quantities are independent yet closely related [1]. Within the framework of Maxwell's theory, electromagnetic induction is a phenomenon associated with two different facts, the time variation of a magnetic field, and the movement of a conductor in a magnetic field, or a combination of the two [2]. When analyzing different EMI phenomena, it may be challenging for students to select the appropriate interpretation. It is necessary to provide many activities, exercises, or examples for students to overcome the complexity of the topic [3]. In particular, Chabay and Sherwood [4] state that "Faraday's law is usually difficult for students. Moreover, the integral form involves the concept of flux, traditionally introduced at the start of the course in the context of Gauss's law and not mentioned again until Faraday's law is introduced" (p. 333).

As is shown in the next section, interpreting Faraday's law has been a challenge in the physics community and most students experience learning difficulties when attempting to understand EMI phenomena and Faraday's law. The fact that these difficulties were encountered after traditional instruction led us to design a teaching-learning sequence to help improve students' understanding. This paper describes the design, implementation, and evaluation of an EMI teaching-learning sequence for university introductory physics courses.

II. PREVIOUS RESEARCH

Studies performed on teaching and learning EMI and Faraday's law are not encouraging. The Faraday's law of induction statement seems very simple: whenever the total flux varies through a closed circuit, there is an induced electromotive force whose magnitude is proportional to the rate of flux change through the integral path. This law is expressed by bringing together the idea of magnetic flux and the time interval during which it changes and can be expressed as

$$\varepsilon = -\frac{d\Phi_B}{dt}, \tag{1}$$

where Φ is the magnetic flux through the circuit with area S,

$$\Phi = \int_S \vec{B} \cdot d\vec{S}. \tag{2}$$

Despite widespread teaching of Faraday's law in introductory physics courses, interpreting electromagnetism continues to pose challenges. As the surface over which the flux is calculated may, in general, be any surface whose boundary is formed by the closed integration path, it is not simple to explain flux, posing a challenge for students and teachers of introductory courses [3].

Controversy over the validity of Faraday's law has been generating a buzz since the late 1960s. In 1964, Pugh [5] stated that "While concepts concerning electromagnetism as covered by Maxwell's equations have been well established for some time concerning certain phenomena, some confusion still exists in the minds of many who should have mastered the subject. This fact has been impressed upon me by discussion with colleagues and with graduate and undergraduate students. For example, analyzing EMF produced by homopolar generators, Faraday disks, and similar devices still causes considerable difficulty" (p. 879). Tilley [6] states that "The rule (Faraday's flux rule) breaks down in situations where the circuit material changes (the 'circuit' is taken to be placed wherever the current is" (p. 458). Tilley presents an example featuring a wide flux variation through the circuit but no induced electromotive force is produced. The author states that this example shows that Faraday's "flux rule" has exceptions as already shown in the examples proposed by Feynman [2] (p. 3–17).

The discussion as to whether Faraday's law is valid for all cases of electromagnetic induction or whether it is merely a "rule" with exceptions raged among experts during the 1970s and 1980s. Nussbaum [7] states that "Some unusual circuits have been devised which appear to produce a flux change without generating a corresponding induced potential difference, thus violating Faraday's law. What has been generated is a large amount of controversy and this article intends to show the cause of the dispute and its resolution" (p. 231). In the same article, Nussbaum demonstrates the validity of Faraday's law for any situation as long as "the time rate of change of flux equals the introduced potential difference only when work is done in producing change" (p. 231).

Over the last decade of the 20th century and the first of the 21st century, discussions continued on the validity of Faraday's law, centered on electromagnetic induction cases where the circuit is not well defined for a finite time interval. These cases were the focus of previous discussions on exceptions to Faraday's law. There was particular discussion around the induction phenomena, referring to lengthy conductors or to points of contact in motion [8,9]. Studies over the last decade converge, indicating that there are no exceptions to Faraday's law of electromagnetic induction [Eqs. (1) and (2)] if it is interpreted that Faraday's law considers the flux integration surface as the area formed when the circuit or conductor is in motion [10,11]. Munley [3] demonstrates in his study that Faraday's law can be used in any situation where the Lorentz force can be used. See also Lorrain, Corson, and Lorrain [1] (Chap. 18) and Cheng [12] (Chap. 6) who demonstrate, with detailed mathematical calculations, that Faraday's law can be applied in any situation where the flow changes due to the fact that the electric current varies or the surface area on which the flux integral is calculated is modified.

Faraday's law has not only been a source of discussion in the physics community but research into teaching physics provides evidence that teaching and learning can be problematic. Internationally, there have been a modest number of studies into problems associated with learning induced electromotive force and Faraday's law. Of those performed, some have looked into general problems of interpreting electromagnetic induction phenomena, while others have only studied learning Faraday's law.

Different studies focusing on comprehension difficulties among secondary and first year university students regarding concepts implicated in the theory of electromagnetic induction and Faraday's law, such as field and magnetic flux variation or induced nonconservative electric field, indicate that the main learning difficulties center on the following:

(a) The vast majority of secondary students and a significant proportion of first year university students do not recognize electromagnetic induction in phenomena traditionally taught in the curriculum. A significant number use explanations based on transmitting a "force" or "contact with the field." Most students do not distinguish between the empirical level (voltmeter and ammeter measurements) and the interpretative level that uses concepts such as fields and electromotive force [13,14].

(b) Many students interpret that the magnetic field produces electromagnetic induction [15,16].

(c) Many students understand magnetic flux as "flowing" from the field or they confuse it with the field itself. Most university students use Faraday's law without any physical meaning [13,14,17–19].

(d) When students apply Faraday's law, most of them tend to confuse the circuit area with the integration area in Faraday's law [10,11,15].

(e) Many students are not capable of recognizing electromagnetic induction when there is no induced current [14,16].

(f) Students have a major tendency to explain induction phenomena using a field model, even in situations where reasoning based on Lorentz's force considerably facilitates induction analysis. Most students do not understand the equivalence of the explanation based on a field model and on Lorentz's force for all induction phenomena [16].

Many physics teachers would agree that, for higher secondary and first year university teaching levels, Faraday's law is mainly used like an algorithm to provide the value of the induced electromotive force. However, we have seen in this introduction that it is necessary to introduce conceptual understanding.

While most introductory university physics textbooks provide a quantitative treatment of Faraday's law, the development of a coherent conceptual framework in classical physics that allows students to consider EMI phenomena without resorting to solving equations is typically not a goal of instruction [4,16]. Research shows that avoiding discussion of topics likely to lead to misconceptions does not work. It is much more effective to explicitly address the problems that students are likely to encounter. This approach is especially important when discussing instruction on EMI at the university level. Students learn aspects of EMI in high school, so they begin introductory university physics courses with preconceived ideas about electromagnetism. In this research, taking into account the aforementioned characteristics, we use a combination of the problem-based learning (PBL) approach and instruction in problem solving. PBL is an approach to learning and instruction with the following characteristics: (1) the use of problems as the starting point for learning, (2) small-group collaboration, and (3) flexible guidance from a teacher. Since problems steer the learning in this type of curriculum, (4) the number of lectures is limited. The latter is in line with the principle that (5) learning ought to be student initiated and that (6) ample time for self-study should be available [20,21]. These characteristics lead to an explicit instruction in problem solving. This is consistent with much of the advice given by proponents of problem-based learning. While practice is crucial for mastering skills such as problem solving, greater gains are realized through explicit instruction in problem-solving skills [22]. However, traditional science and engineering courses do not generally teach problem-solving skills explicitly. The combination of a PBL approach and problem-solving methodology is called the guided problem solving (GPS) approach [23].

III. THE STUDY

To address the mentioned difficulties this study propose a teaching sequence, which aims to help students to understand that Faradays' law involves any electromagnetic induction phenomena and that quantifying electromagnetic induction can be used either Faraday's law or the Lorentz force in any situation.

The context of the study is a transformed calculus-based physics course for first year engineering degree students at the University of the Basque Country (UPV-EHU), taught by the authors over three years (2009–10, 2010–11, 2011–12), and using the newly designed teaching sequence. At UPV-EHU, the electromagnetism curriculum is taught during the second semester (Sp10, Sp11, Sp12) of the course. The traditional course format is two hours per week of full scale lecture classes, with an enrollment of 60–90 students, and one and one-half hours per week problem sessions with half of the students from the large lecture class. The magnetic induction syllabus incorporates elements included in the course textbook [24]: (1) magnetic flux, (2) induced EMF and Faraday's law, (3) Lenz's law, (4) motional EMF, and (5) eddy currents. In the traditional courses, students do not normally have the opportunity to participate actively and are limited to taking notes from the teacher's explanations, both in lectures and in problem sessions.

The syllabus is the same in the transformed course format and did not require additional time or resources. The teaching material for the transformed course is available on the Internet at University of the Basque Country Open Courseware [25] for the students at our university studying in Spanish. However, it provides an interactive learning context through the problem sessions where the teacher develops a GPS approach [23,26]. The purpose of the teaching sequence is to explain the design intentions for part of the physics curriculum, explaining why particular decisions have been made. The teaching sequence promotes a highly interactive environment during the discussion sessions. Much of the course experience involves a cooperative learning model for students [27]. Students were organized in groups of three or four. They were asked to take the role of speaker, scribe, or timekeeper. They changed roles for each chapter in the syllabus. The students were presented with conceptual questions or problems and were asked to discuss them for a fixed time. This approach puts students in a position where they are able to extend their knowledge and abilities in a certain direction, which helps to solve the problem. When the students have finished their work, there is a round table discussion, directed by the teacher. During the discussion, each group's spokesperson must justify their answers. Ultimately, there will be one or several explanations for each problem. In the transformed course the format is the same as that in the traditional course. That is, there is one and one-half hours per week with half of the students from the large class. In these sessions, students discuss the problems or conceptual questions. In the weekly two-hour full class (plenary) session per week, each group's spokesperson presents the group consensus; all ways of solving the task are discussed, guided by the teacher, and a classroom summary is formulated. The main difference between the small and large scale classes is that in the former, the teacher encourages the students to work on problem-solving skills. To do this, the teacher will select the most appropriate problems for the teaching and learning of the sequence. For example, in the first task on Faraday's law, students examine the relationship between the magnetic field, the area of a surface, and the relative orientation of both. During the task students received information on four magnetic induction experiments where the task variables are measurements. In the next class, each group has to propose an answer and the teacher-guided class discussion starts.

The teacher guides students to propose a qualitative approach to the problems and to generate hypotheses as possible solutions based on scientific arguments. Then, the teacher guides the discussion of the groups' results. The teacher asks the students to test their results with the proposed hypothesis or with the theory. They ask about the logical coherence of the results. Finally, the teacher summarizes the results of the groups of tasks that have the same objective and explains, if necessary, the theory. The time allocated to the chapter on induction is the same (2 weeks: 7 hours) for the transformed course and the traditional course.

IV. TEACHING SEQUENCE ON FARADAY'S LAW

The teaching sequence was based on the results of previous research into students' difficulties understanding electromagnetic induction (Sec. II). These results led us to believe that students need additional guidance to understand the following: (a) Faraday's law groups together phenomena associated with two different facts, time variation of a magnetic field or the movement of a conductor in a magnetic field or a combination of the two; (b) all EMI phenomena can be explained by means of the field model or the Lorentz force model, using the appropriate integral surface of Faraday's law [11,28]. Therefore, we designed a new EMI teaching sequence with the aim of eliciting and effectively resolving these difficulties. We use the learning demands analysis tool [29,30] for addressing differences between everyday and scientific ways of thinking and talking. The sequence design involves successive refinement of the designed educational interventions, the purpose of which is to test and systematically improve the fitness for purpose of a designed artifact [31]. For this reason, after the first year of implementation (Sp 10), changes were made to the wording of the activities—some of them were removed and new activities were also added. The final sequence was implemented during the second and third years (Sp 11 and Sp 12). The data shown in this work belong to Sp 11 with a group of 75 students and Sp 12 with a group of 73 students.

Students must have basic familiarization with EMI phenomena [13–15]. So, before the study of a quantitative interpretation of EMI phenomena, students worked on a task where they analyzed that electromagnetic induction is a phenomenon associated with two different facts: time variation of a magnetic field or the movement of a conductor in a magnetic field or a combination of the two. It is important that explicit tasks are given on this matter. These tasks are provided in the first part of the EMI sequence and they are guided by the first problem (see Table I, Sec. I). For more information about the first part of the sequence see Zuza and Guisasola [32]. Some of the tasks used in the sequence were influenced by the textbook "Matters and Interactions Vol. 2" [33]. In this paper we will focus on the part of the EMI sequence which involves Faraday's law and the Lorentz force. The treatment of Faraday's law and the Lorentz force in the teaching material is outlined in Table I (Secs. II and III), guided by a second and third problem.

The sequence presents the students with three general problems (see Table I, first column) which the students discuss in groups and in which they must work out, using PBL methodology, what they need to know in order to resolve the general problem. For each problem the students are set a series of tasks (see Table I, fourth column) that will guide them in learning the new concepts they need to resolve the general problem. These tasks are designed specifically to get the students to work on problem-solving skills. For example, in order to solve general problem No. 2, the students work through 9 tasks and for general

TABLE I. Teaching sequence on electromagnetic induction.

Problem sequence	Procedures regarding science to be learned by the students	Explanations to be understood by the students	Implementation (Activities)
1. When do EMI phenomena occur?	Understand the usefulness of the study on electromagnetic induction and its applications in S-T-S relationships. **Become familiar with the experimental phenomena** of electromagnetic induction.	Distinguish between the empirical level (use of multimeters and measurements) and the interpreting level that uses concepts such as magnetic and electrical fields that vary over time, Lorentz force, magnetic flux, and electromotive force.	A.1 to A.6
2. How can EMI be quantified?	Science must be able to measure the phenomena that are observed and **give quantitative answers using scientific procedures** such us relating variables and the phenomena that are observed.	**Field model explanation** of EMI (Faraday's law)- Magnetic flux and Induced EMF.- Nonconservative electric field.- Energy conservation law (Lenz's law).	A.7 to A.16
3. Is there another way of measuring the induced EMF?	Science can solve the same problem using different laws and points of view. A problem can be solved with **different procedures and obtain the same results**.	**Force model explanation** of EMI (Lorentz force).**Relationship** between field model and Lorentz force model with the electromagnetic field.	A.17 to A.25

problem No. 3 they have to complete 8 tasks. Table I, column 2 shows the problem-solving skills that the students have to develop in the tasks, and column 3 shows the learning outcomes which they are expected to achieve by working on each general problem and its associated tasks. These procedures and general objectives are informed by previous research in physics education mentioned earlier in Sec. II and a GPS approach is the basis of the learning cycle.

Examples about activities of the second part of the teaching sequences, and on how they were implemented are shown in Appendix A.

The examination on this part of the course and the final course examination included questions on the difference between the concepts of magnetic and electrical fields that vary over time, Lorentz force, magnetic flux, induced electromotive force, and Faraday's law. Students' answers are analyzed below.

V. ASSESSMENT OF THE EFFECTIVENESS OF THE TEACHING SEQUENCE

The teaching sequence we have described has been used at the University of the Basque Country. To assess its effectiveness, we gave pretest and post-test questions to students in the groups who have worked on the sequence (Sp10, Sp11, Sp12), and five groups of three students were interviewed. After the first year of implementation (Sp 10), changes were made to the activities' wording; some were removed and others added. The final sequence was implemented during the second and third years (Sp 11 and Sp 12). The teaching sequence was implemented in Sp 11 with a group of 75 students and in Sp 12 with a group of 73 students (148 students in total). The pretest and post-test had similar but not identical situations so that students tackled each question once. In previous years (Sp 08 and Sp 09), we gave the same post-test to 147 students that followed the traditional course. In the data shown below these groups appear as the "comparison group."

All students take two physics courses involving topics on electromagnetism during postcompulsory education (16–18 years old), and they must pass an exam to enter the Engineering School at the University. The students are randomly distributed among the first year engineering groups. Every year, to ascertain the students' initial knowledge of electricity and magnetism, we give a sample of students from the comparison and experimental groups the questionnaire entitled brief electricity and magnetism assessment (BEMA) which has been shown to be a reliable assessment tool [34]. The results obtained show that students' knowledge of the area can be described as memory-based learning of concepts, laws, rules, and procedures, which can be useful for them to solve standard problems and examination exercises, but does not give them sufficient comprehension to apply these concepts to different contexts and phenomena. There were no significant differences in correct answers between the sample groups from the Sp 08 to Sp12 (an ANOVA test of BEMA scores for students at the beginning of the course found no significant differences for any group). We can therefore conclude that all groups had approximately the same level of academic competence.

To see how much students had improved their understanding of electromagnetic induction and Faraday's law we used two types of evaluation. For evaluating the learning progression of students of the experimental groups, we used the Hake index between pretest and post-test in the experimental groups [35]. An index under 0.10 (a gain of 10%) would imply that the improvement was not substantial and that students who had no teaching sequence significantly improved their learning on electromagnetic induction. The post-test was given to students from experimental groups and comparison group students in exam conditions and the result was included as a part of the final mark for the subject unit. The scores for each group's category of answer were compared. To decide whether there were any significant differences between the experimental and comparison groups, the statistical chi square was used for the usual level of confidence of 5% or less [36]. The results tables group together the experimental and comparison data, as there are no significant differences between them in accordance with the chi squared statistical results.

Some of the post-test questions have been discussed in previous papers on student difficulties concerning EMI [11,16]. Regarding the validity of the questions and their relevance for the study goals, five faculty members from the department of Applied Physics from the University of the Basque Country confirmed that the content of the questionnaire was appropriate for any student who had taken the Introductory Physics course. Additionally, a pilot study was conducted with a small student sample. This confirmed that students generally had no problem understanding the meaning of the questions.

Students' answers were analyzed by one researcher. The same researcher then reread the student answers and tentatively allocated each answer to one of the draft categories. The intrarater reliability kappa coefficient was calculated for this researcher three weeks later, obtaining a value of 0.88, on average for all questions, which is satisfactory for a level of confidence of 95%. Then, the other researchers carried out the categorization of answers independently. They used the draft categories as reference and they can establish new categories. Once the answers had been classified, answer allocations made by all the researchers were compared (kappa Cohen interrater reliability coefficient was 0.85). Any disagreements about category description or answer allocations were resolved by referring to the answers as the only evidence of student understanding. The focus was on the students' understanding, taking the students' answers as a whole, rather

TABLE II. Relations between questions and objectives of the teaching sequence.

Question	Objectives from Table I that are assessed.
Q1	Field model explanation: varying magnetic field, varying magnetic flux.
Q2	Field model explanation: varying magnetic flux, non-Coulombian nature of induced electric field.
Q3	Force model explanation: motional EMF, magnetic forces produced by a conductor moving in a magnetic field.
Q4, Q5	It can be explained by using both the field model and the force model. Relationship between the two models.

than on the occurrence of particular statements corresponding to a specific category of description. An iterative process was used to produce final category descriptions that reflected similar understanding among answers allocated to each category and the differences between the categories.

The answers to the questions were grouped into the following categories:

A.1. Answers that explicitly state that the magnetic flux variation produces the electromagnetic induction and that correctly use Faraday's law.

A.2. Answers that correctly explain electromagnetic induction using the Lorentz Force exerted on moving charges in the magnetic field.

B. Alternative explanations to the scientific model:

B.1. The magnetic field or the electric current produce EMI.

B.2. Applying Faraday's law, misunderstand between the circuit surface area and the integration surface area, coming to the wrong conclusions.

B.3. "*Ad hoc*" explanations that are limited to describing the induction phenomenon without explaining it or that use remembered rote learned without logical consistency.

B.4. Not considering the induced electric field's nonconservative nature and/or the forces that are acting on the charges.

C. No answer.

Interviews took place about 1 month after the end of the Sp12 course in which the written tests were given. The 15 students included in the five groups were volunteers from the Sp12 course. Each group contained a student who did very well on the midterm exams but had some trouble on the final exam, another student who was a weak student, and the third student who did well in the midterm exams and improved on the final exam. The interviews lasted about 40 min. each.

The interview consisted of questions Q2, Q3, and Q5 from the post-test. All of them were transcribed and the transcripts subjected to the same analysis, which is described above. Interviewers attempted to encourage the students to give full explanations of their understanding by nondirective questions such as "What do you mean by that?", "Could you explain that further?", "Do you want to say anything else about this question?"

A. Written tests

The questions Q1-5 are shown at the end of the paper. The questions are related to the general objectives and procedures of Table I. We summarize this relation in Table II.

1. A time-varying magnetic field induces electromotive force and Faraday's law

The first two questions (see Appendix B) dealt with electromagnetic induction situations associated with a time-variable magnetic field. These questions required students to recognize that variation in a magnetic field brings about a variation in magnetic flux through the chosen surface (Faraday's law) and, they should also know that the time-varying magnetic field induces a nonconservative electric field that is responsible for the induced current if there is a circuit.

Question Q1 comprises a circuit that is connected and located beside another without a battery. This problem is similar to the textbook example of induced current in a circuit [37]. The students had to explain why ammeter G registers a current. A correct example from one of the students is given below:

> *"When the top circuit is closed, current starts to circulate, increasing until it becomes stationary. During this time, there is a variable electric current that generates a variable magnetic field. Therefore, the lower circuit is crossed by a variable magnetic flow that will generate an induced electric current. The induced electromotive force in the lower circuit can be calculated using Faraday's law."*

Question Q2 is asked as an example of an induced current in a circuit within a variable magnetic field in many textbooks for introductory physics courses [24,38]. In the question, students were told that the loop had an induced electric current due to a force and were asked to explain the origin of this force. In order to reply correctly students had to know that a nonconservative electrical field is produced by a time-varying magnetic field and that this is responsible for the electric force, which acts on the electrons producing the induced EMF and the movement of charges in the loop. The results of these two questions are shown in

TABLE III. Results for questions Q1 and Q2: (*) Correct answer. Experimental groups in Spring 11 and Spring 12 (E-11-12). Comparison groups in Spring 08 and Spring 09 (C-08-09).

	Q1			Q2		
	Percentage of answers in category type			Percentage of answers in category type		
	E-11-12		C-08-09	E-11-12		C-08-09
Categories	Pre	Post	Post	Pre	Post	Post
A.1*	6.0	57.0	49.0	4.0	57.0	18.0
B.1	51.0	28.5	38.0	21.0	14.0	24.0
B.3	28.0	11.0	9.0	24.5	10.0	10.0
B.4					10.0	40.0
C	15.0	3.5	4.0	50.5	9.0	12

Table III and Fig. 1 shows the frequency of correct answers for the questions.

In the experimental group, correct answers were given more frequently than in the comparison group. Regarding the comparison group, a significant difference was obtained for question Q2, with a level of confidence below 1% (χ^2 chi-squared test $\chi^2 = 1.22 \times 10^{-8}$, $p \ll 0.0001$). However, in question Q1 there is no significant difference ($\chi^2 = 0.257$, $p = 0.38$). In this category, the students correctly explain the relationship between the variable magnetic field over time and the induced electromotive force, in accordance with Faraday's law. A standard example of this type of answer for question Q2 is as follows:

> *"Around a circuit there is a variable magnetic field and this generates a non-Coulombian electric field so we can say that a variable magnetic field generates an electric field and this field, in turn, will generate an electrical force and a current. It can be concluded that the I is generated by the non-Coulombian electric field."*

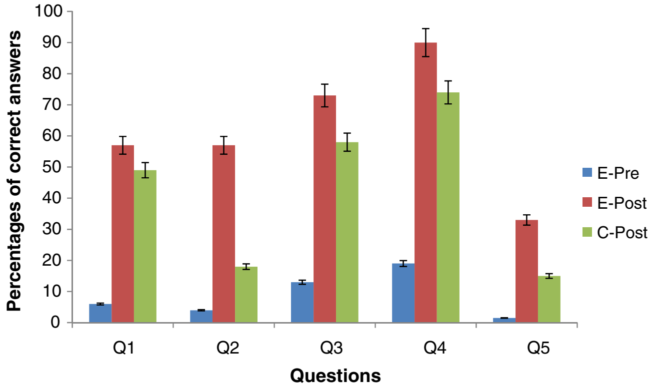

FIG. 1. The percentages of correct answers are shown in the experimental pretest (E-Pre) and post-test (E-Post) and in the comparison groups' post-test (C-Post).

The percentage of students from the experimental group who continued to explain that induction is caused by a magnetic field or an electric current (category B.1) stands at around a quarter of the answers. However, in the comparison groups the percentage of this alternative explanation is around a third of the answers. These answers coincide with previous studies that indicate that this alternative idea is difficult to change [14,39]. One example of this type of answer is as follows:

> *"On closing the circuit, a magnetic field is produced that influences the electrons in the top circuit and induces electrical current" (question Q1)*
> *"When the magnet comes close to the coil, the magnet's magnetic field influences the coil's charge and produces electrical current" (question Q2)*

The students' tendency to look for *ad hoc* explanations that are not consistent overall with the theory and the students' difficulty to use arguments based on scientific methodology were shown by other research results [40]. In the experimental groups, after instruction, a minority of students do not use arguments to justify the statements they make (category B.3). In category B.3 there are explanations that might be correct but are not justified and do not indicate understanding of the phenomenon being analyzed. For example,

> *"On closing the top circuit, a current is induced in the lower circuit. The induction is due to a variation in magnetic flux" (question Q1)*

Another type of answer appearing after instruction is grouped together in category B.4. In this category, the students explain the induction phenomenon but they do not mention the nature of the induced electric field or the forces that act on the charges. A standard example from category B.4 is as follows:

> *"By bringing the magnet close to the coil, a magnetic flux variation occurs through the areas bound by the coil as the magnetic field is variable and, a current is induced in the coil" (question Q2)*

In the comparison group, almost half of the responses (40% in Q2) failed to mention the non-Coulombian nature of the electric field even though question Q2 explicitly asks about the nature of the inducted field. In the experimental groups, the percentage falls to 10% after instruction.

2. Motional electromotive force and field model and force model

The other three questions involved electromagnetic induction phenomena caused by the movement of a circuit or part thereof in a time-constant magnetic field (see Appendix B). The questions are familiar to students in the academic context and are usually mentioned in textbooks as examples of electromagnetic induction phenomena.

In questions Q3, Q4, and Q5, students have to explain why the galvanometer registers a current. The students can justify the induced current by means of two different, although equally valid, explanations: (a) due to movement of free charges (in the metal) within a magnetic field, a magnetic force appears on these charges. This magnetic force is responsible for initiating the charge movement process; (b) a variation in magnetic flux occurs due to the area swept by the coil as it moves in the magnetic field, in a time interval. This variation in flux produces an induced EMF (Faraday's law). This double description in terms of field or of actions exerted by the field on matter was the learning goal for the third part of the teaching sequence (see Sec. 3 of Table I). Results are shown in Table IV, and Fig. 1 shows the frequency of correct answers for the questions for experimental and comparison groups.

More students from the experimental group were able to explain correctly the questions Q3, Q4, and Q5 than students from the traditional teaching group. The differences are statistically significant below 0.05 (χ^2 test, $p < 0.05$) in all the questions. Considering categories A.1 and A.2 as correct answers, in question Q3 ($\chi^2 = 3.40 \times 10^{-3}, p = 0.045$), in question Q4 ($\chi^2 = 3.23 \times 10^{-3}, p = 0.045$), and in question Q5 ($\chi^2 = 2.88 \times 10^{-3}$, $p = 0.042$) the differences between the experimental and comparison groups are significant.

In questions Q3 and Q4, students had to know that when a conductor moves within a magnetic field this exerts a magnetic force on the conductor's charges. The correct answer may be explained using Lorentz's law, which the students had repeatedly practiced in the preceding section on magnetic fields (category A.2). A standard example of this type of answer is as follows:

> *"A magnetic field exerts forces on the charges of a moving conductor. Therefore, the magnetic force produces the movement of the electrons in the coil and the induced current." (question Q4)*

In the experimental and comparison groups, a greater percentage of correct answers explain the phenomenon using the field model and Faraday's law (category A.1). For example,

> *"When the coil is tilted, the magnetic flux through it changes. Consequently, an electromotive force and an electric current are induced. The force that moves the charges will be the force of the induced electric field" (question Q3)*

In question Q5, which involves Faraday's unipolar generator, students were asked to state whether there was an induced electric current and to justify their answers. The students could use Lorentz's law which explains the movement of charges in the copper disk due to the magnetic force exerted on the electrons by the uniform magnetic force. The students could also explain the question by using Faraday's law and the fact that the induced EMF is associated with the magnetic flux variation when the area changes. Quite a few papers have correctly shown the explanation of Q5 by using Faraday's law (category A.1). In experimental groups, one-third of the explanations using Faraday's law confuse the circuit surface (the entire disk) with the integral surface between two points on the disk (category B.2). This confusion is shown by previous studies [10,11]. One example of category B.2 is shown below:

> *"The disk turns around its axis and there is no displacement. Therefore, there is no area variation. In addition, as the magnetic field is stationary, there is no flow variation and therefore there is no current induced in the disk."*

TABLE IV. Results for the Questions Q3, Q4, and Q5: (*) Correct answer. Experimental groups in Spring 11 and Spring 12 (E-11-12). Comparison groups in Spring 08 and Spring 09 (C-08-09).

	Q3			Q4			Q5		
	Percentage of answers			Percentage of answers			Percentage of answers		
	E-11-12		C-08-09	E-11-12		C-08-09	E-11-12		C-08-09
	Pre	Post	Post	Pre	Post	Post	Pre	Post	Post
A.1*	9.0	51.5	49.0	14.0	85.0	74.0	0.0	8.5	0.0
A.2*	4.5	21.5	4.0	5.5	5.0		1.5	24.5	15.0
B.1	24.5	3.0	18.0	26.0	4.0	14.0	10.0	3.0	11.0
B.2							11.5	30.0	58.0
B.3	24.0	12.0	12.0	23.5		8.0	24.0	18.0	6.0
B.4									
C	38.0	12.0	17.0	31.0	6.0	4.0	53.0	16.0	10.0

In the experimental group, the category B.2 represents around 10% before instruction and increases after instruction to one-third of answers. If we take into account that before instruction half the students did not answer question Q5, the result from category B.2 can be interpreted in that some students have made progress in their understanding of induction using Faraday's model but that their learning is incomplete and they are using Faraday's law incorrectly. In the comparison group, the percentage of answers with the confusion of category B.2 is almost twice (58%) that in experimental groups.

Although the explanation based on the force model is simple and straightforward to justify the induction, in the experimental groups only a quarter of the explanations, and in the comparison groups 15% of the explanations, use Lorentz's law to account for the fact that the metal disk charges move due to the magnetic force exerted by the magnetic field on the moving disk (category A.2).

The difference between the correct answer percentages for the five questions in the pretest and post-test and the differences between the experimental and comparison groups are shown in Fig. 1.

In the experimental groups, the difference between the correct answer percentages for the five questions in the pretest and post-test is more than 0.1, implying a gain in the Hake index (0.54 in Q1, 0.55 in Q2, 0.69 in Q3, 0.88 in Q4, and 0.32 in Q5). As seen before, the percentages of correct answers in the experimental groups are statistically different (chi-squared statistic) from the comparison groups for questions Q2, Q3, Q4, and Q5. Similarly, there were no significant differences from one experimental group to another, and likewise for the control group. All statistics were calculated using the statistical chi square with the hypothesis that the approach would lead to an increase in the conceptual understanding of Faraday's law in the context of magnetic induction. In questions like these, which are routine in the traditional curriculum on electromagnetism, such as questions Q1, Q3, and Q4, the differences (although they are statistically significant for questions Q3 and Q4) are not as large as they are for questions Q2 and Q5. For Q5, a highly popular example in the teaching of electromagnetic induction but one that has been little analyzed, the differences between the control and experimental groups are significant. The question calls for a sound knowledge of the model of field and force for an understanding of electromagnetic induction. The most significant differences were found for Q2. Furthermore, the reduction in alternative interpretations to the accepted scientific accounts is considerable from the pretest to the post-test experimental groups (see Fig. 2).

In the experimental groups, the percentage of category B.1 that involves the explanations based on the alternative that a magnetic field produces induction decreases considerably after instruction. The percentage stands at around 10% in the experimental groups. In the comparison groups, the percentage is twice that in experimental groups (see Fig. 2).

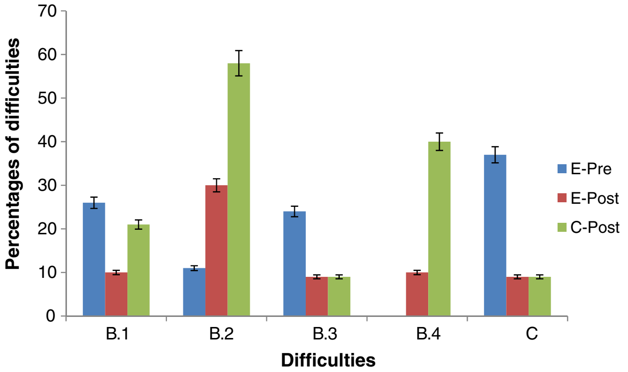

FIG. 2. The average percentage of students expressing each of the incorrect answer categories. For example, difficulty B.1 is relevant to all five questions and the percentage of students expressing this difficulty averaged over all five questions is shown. Ideally, all the bars would be at 0% after instruction, indicating that students expressed none of the difficulties.

In the experimental groups there continues to be a significant percentage of answers in category B.2 (mixing up the circuit surface area and the integration surface area), but the percentage of responses in this category among the comparison group is greater (the difference is statistically significant). The same occurs for the percentage of responses in category B.4. In the comparison group, about 40% of the responses do not take into account the nonconservative nature of an induced electric field, while among the experimental group the figure is 10%. In the experimental groups, there is a drop in the students who do not explain their conclusions with proper argumentation (category B.3).

B. Student interviews in experimental groups

The interview session includes questions Q2, Q3, and Q5 from the questionnaire with the same objectives. Regarding question Q2, the entire group concludes that it could calculate the current induced by the magnetic flux variation due to the variation in the magnetic field. One of the standard answers is as follows:

(1) Interviewer (I): In question Q2 there is a magnet moving towards a coil. Will the ammeter show a current?
(2) Student 1 (S1): Yes.
(3) Student 2 (S2): Yes.
(4) I: All three of you agree. Good, and so why does it happen?
(5) S1: Because a magnetic field is moving towards it and because it wants to maintain the situation it was in before, a current will be generated in the coil.
(6) I: Do all three of you agree? Yes. And how would you calculate the induced EMF?

(7) S3: By calculating the flux variation. When the magnet comes closer, there is a variation in the value of the magnetic field crossing the coil and so flux variation.
(8) S2: The induced EMF is calculated using Faraday's law.

However, there was discussion in all the groups on the nature of the induced electrical field although, in the end, a consensus was reached in four out of the five groups. The following is a transcription of an example of the consensus reached:

30. I: Let us move on to another question from question Q2. We have studied in class that the current induced in the coil is the consequence of an induced electrical field. What is the nature of this electric field?
31. S1: The induced electric field is produced by the magnet's movement, isn't it?
32. S2: No, I believe it is due to the flux variation.
33. S3: But it asks why the electric field is generated, not the EMF.
34. S1: The EMF is produced by the flux variation. We know this thanks to Faraday's law.
35. S2: Well I understand that the question is about the induced electric field, not about calculation of induced EMF.
36. S3: The induced electric field is produced by the magnet's variable magnetic field. So …it has to be non-Coulombian.
37. S1: What is it that generates this non-Coulombian electric field?
38. S3: A variable magnetic field. Do you all agree?
39. S1: Yes, but I didn't put that in the test. I did not write anything about the nature of induced electric field.
40. S3: It doesn't matter if we all understand it now.
41. S2: Yes, I understand that the electric field here is not like the electric field that we studied in electrostatic. Here it is produced by a variable magnetic flux.

Two dimensions of activities can be identified in the raw transcript, a focus on the physics learn and the interactions between group members. Focusing on the first dimension, it is noticed that students are interacting with the question of the nature of the induced electric field (S3 line 33). Two of the students have problems identifying the question (S1 line 31and 34; S2 line 35). During the discussion, S3 student tries to focus the discussion on the question and she tries to justify her answer to her colleagues (S3 line 33 and line 36). In the end, S1 and S2 agree with S3 (S1 line 39 and S2 line 41).

Regarding the second dimension, this short sequence shows that the three students individualize the situation in their own way and, if they want to interact, they have to negotiate what they perceive as salient. This negotiation influences their behavior and the course of their individual cognition.

The difficulty in explaining the nature of the field or forces acting when there is an induced electric field does not occur when we look at magnetic forces produced by variable or stationary magnetic fields. For example, in question Q3 all students groups explain the following:

1. I: In question Q3 we have a coil that is moving in a magnetic field. Will there be an electrical current in the loop?
2. S7: Yes, while there is movement.
3. I: While it is moving. What about if the loop was at rest?
4. S8: There would not induction.
5. I: Why?
6. S7: Because then there would be no flux variation and no induction.
7. S9: I agree. According to Faraday's law, flux variation is required to calculate the induced electromotive force and the current intensity.
8. I: What are the types of forces that act on the coil charges and that produce the current?
9. S9: They are magnetic.
10. S7: They are the forces that act on the charges in the coil because it is moving.
11. S8: If there is a conductor moving in a magnetic field, a magnetic force occurs on the conductor charges.

This excerpt illustrates the successions of "ideas" which characterize students' processes of situated cognition development through the described situation. First, they justify based on the scientific model the induction produced by the movement of the loop (S8 line 4, S7 line 6, and S9 line 7). All the students have no difficulty in interpreting the magnetic nature of the force and they justify by the scientific model studied in the sequence (S7 line 10 and S8 line 11).

In relation to the difficulty of choosing the appropriate integration surface area to calculate the electromotive force using Faraday's law (difficulty B.2), in questions Q2 and Q3, all groups calculate Faraday's law properly as the integration surface area coincides with the circuit surface area. A standard answer to question Q3 is as follows:

1. I: How would you calculate the electromotive force induced in the ring?
2. S10: Using Faraday's law.
3. S11: The flux variation is calculated through the loop. You have to calculate the angle between the coil's surface area and the magnetic field direction.
4. S3: Yes. The flux variation is calculated.
5. I: And which area would you take to calculate the flux?
6. S3: The coil's area.
7. S11: The coil's area.

However, difficulty B.2 detected in the questionnaires clearly appears in question Q5. At the start of the interview, two ways of analyzing the question emerged in all groups. The majority of the students (12 out of the 15 in total) analyze the questions by means of whether flux variation exists or not. They reach the incorrect conclusion that there is no flux variation and that therefore there is no induced current. However, three students, each in a different group, analyze the question using the magnetic force that the stationary field exerts on the moving copper disk's charges. The three students come to the correct conclusion that there is an induced current. The start of the discussion in one of the groups is presented as an example.

> *1. I: Let us go on to question Q5, would there be an induced current?*
> *2. S10: I think they wouldn't be a current. Because there is no variation in the area and the flux will remain constant.*
> *3. S12: I agree. There is no flux variation. Neither the magnetic field nor the disk surface area varies.*
> *4. S11: I do not agree. I think there is.*
> *5. I: And on what basis do you say this?*
> *6. S11: We've got a conducting disk that is moving in a magnetic field. A magnetic force would appear that would act on the disk charges and will move them. A current is produced.*
> *7. S10: Someone is wrong. The phenomenon can only have one result: either a current is induced or it isn't.*

The short sequence shows that students S10 and S12 try to find out what happened in a different way than student S11 did (S10 line 2, S12 line 3, and S11 line 4 and line 6). A discussion began in the group based on empirical evidences (S10 line 7).

In the two groups where there was a consensus stating that there was no electric current, the discussion was caused by the interviewer. In the end, all groups concluded that a current was induced and they resorted to arguments worked on in class. Let us look at the following example:

> *18. I: When S10 calculated using Faraday's law, the electromotive force came out as zero. How would you calculate for this electromotive force to be other than zero?*
> *19. S11: The area that you take into consideration is like the bar magnet and the amperemeter we looked at in class. You just have to consider a "moving" area, the area that it leaves free due to the movement. At the end, there is a variation of area there and a flux variation.*
> *20. S12: You (student S11) then relate it to the problem that we did in class on the bar magnet, taking into account the area left behind due to the movement. But this case is more difficult. It is difficult to imagine a "small" area.*
> *21. S10: Let us see if I understand. You take (student S11) the area that is moved, not the whole copper disk. In addition, it is true that magnetic forces are acting and therefore there is an induced current.*
> *22. S12: Okay, our calculations for the integral might not be correct because there is an induced current because there is a magnetic force and charge displacement.*

The excerpt shows that initially student S11 tries to demonstrate to other students that he uses the correct way of applying Faraday's law (S11 line 19). He uses an analogy with other activity done in the classroom (see activity A.19 in this paper). Student S12 follows the sequence of S11's reasoning (S12 line 20) and at the end they accept the argument based on empirical evidence (S12 line 22).

Before the instruction, the data from the questionnaire show that a significant number of students (category B.3) propose explanations that are limited to describing the phenomena or that use remembered concepts without logical consistency. In contrast, for the five groups interviewed, the data show that students, after the instruction, use empirical data and scientific arguments based on the scientific model used in the instruction for justifying their statements and to convince others. The students' reasoning complexity increases from the description of phenomena to linking these phenomena to the interpretative concepts and laws used in the scientific model studied in instruction [41].

VI. CONCLUSION

To help students understand EMI, we introduced a teaching intervention involving a series of problems and activities. The positive learning outcomes may be due to three features of the intervention. First, the intervention integrates experimental phenomena via activities, with the explanatory theory models, such as understanding the Faraday's law or an explanatory model of EMI. This connection helped to understand the connections between the qualitative model of description of EMI, in terms of variables at macroscopic level, and the processes described by the models at microscopic level [14]. Second, after students finished the activity or a group of activities, there was a round table discussion where the instructor gave verbal clarifications with theoretical and mathematical derivations from the explanatory models. Laws are used as tools to enhance the consistency of qualitative explanations for the interpretative models (see section 2 and 3 of Table I). Third, while the teaching strategy aims to engage students in the essential characteristics of scientific methodology such as creating a hypothesis, empirical verification, capacity for prediction, and being universal, the teacher's guidance was not overlooked [40,42]. When the activities were completed, students were asked to argue their statements using the characteristics of scientific work.

Since implementing the sequence, a considerable number of students have a more satisfactory grasp of the EMI explanatory model. The achievement of learning is better in the experimental groups than in the comparison groups in terms of correct answers and in the percentage of alternative conceptions of categories B (see Figs. 1 and 2). This seems to confirm that the aspects highlighted in the sequence are relevant to the defined aims. However, the percentage of experimental students that explain and use the explanatory field or force model correctly is around 60%. This percentage might seem to be barely satisfactory but it must be taken into account that after instruction, in the experimental groups, the number of answers with alternative conceptions also drops (see Fig. 2).

Category B.1, related to the notion that the magnetic field produces induction, falls dramatically in the experimental groups for questions Q1 and Q2 and does not appear in even 10% of answers to questions Q3, Q4, and Q5. However, in the comparison group, after instruction, it stays at about 30% of answers in Q1 and Q2 issues and, for questions Q3, Q4, and Q5, the response rate is greater than 10%. In addition, in the experimental groups "intermediate" categories appear that explain without the required depth of understanding (B.4) or without correct analysis of the phenomena (B.2) of EMI, but that do indicate some progress compared to initial knowledge. In the case of difficulty B.2 (see Fig. 2), it might come as a surprise that it increases in post-test answers. However, in the pretest, the vast majority of students did not answer question Q5 and in the post-test there is a significant increase in the number of answers, both correct (see Fig. 1) and those that mistakenly chose the integration area in Faraday's law (see Fig. 2). This seems to indicate that progress has been made in learning towards greater capacity to explain electromagnetic induction. Moreover, in the comparison groups, the percentage of answers included in different B categories which involve alternative ideas about electromagnetic induction is greater than the percentages from experimental groups (see Fig. 2).

Another result from the experimental groups that supports this reflection on "learning progress" is the steep reduction in students who did not respond to questions. In addition, the number of answers that use scientific arguments increases considerably compared to the number of *ad hoc* answers or answers based on "common reasoning" included in difficulty B.3 [43].

The correct answers to the questionnaire demand a multilevel explanation, referring to deep structures where the system description is better defined. These explanations correlate two descriptions of the same system at two different levels of analysis with different set of variables [41,44]. In the case of EMI, students have to correlate the macroscopic measurements (current, potential difference, and EMF) with the explanatory concepts (magnetic flux, induced electric field and acting forces). Maybe more time would be necessary to analyze and enhance multilevel reasoning on EMI issues. We were restricted by the established curriculum and our challenge was to make changes to teaching strategies within this context. Moreover, this study evaluated a teaching sequence designed for a specific topic and it would be necessary to introduce this kind of macro-micro analysis for previous topics in electricity and magnetism.

Although the study results are positive, the teaching design feasibility and outcomes may vary in different contexts. In our experience, continuous modifications by instructors based on evaluations of prior implementations will be necessary. Working examples of teaching sequences that bridge the gap between general educational research results and classroom practice constitute an important goal for continuing research in physics education.

ACKNOWLEDGMENTS

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APPENDIX A: EXAMPLES OF ACTIVITIES AND THEIR IMPLEMENTATION

Examples of activities and their implementation:

In the second part of the sequence of Table I, to understand and successfully apply Faraday's law, students require extra information that has occasionally been mentioned in previous topics but needs to be reviewed now. One of these concepts is the magnetic flux worked on in a previous chapter, although also worked on here [19]. For example,

A.7. Let us suppose that we vary current I_1 that flows through a very long solenoid with radius r_1 and that, using an ammeter, we can measure the induced current I_2 in the outer circuit with resistance R (see Fig. 3). If we carry out two experiments,

E1: When the current I_1 is increased in the solenoid, the ammeter measures a negative current, meaning that I_2 circulates clockwise.

E2: If we use a solenoid that creates the same magnetic field inside as in E1, but with double the cross section, we can see that I_2 has doubled.

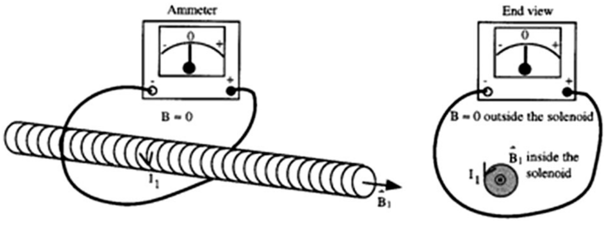

FIG. 3. Experimental setup in the laboratory of activity A.7.

Answer and explain the following questions for each experiment:

(a) Describe the map of magnetic field lines.
(b) Which surface should be used for measuring the flux?
(c) What will the variation of the flux be over time?
(d) What will the value of the induced electromotive force be?

In this task, students first have to remember the key points about a magnetic field produced by a current through a very long solenoid (B into the solenoid is constant and B is zero outside the solenoid) and they have to take into account the influence of the area of the solenoid's cross section, which is proportional to an induced non-Coulomb electric field (experiment E2). Second, students have to work out the entire surface and calculate the magnetic flux through it. In this task, intentionally, the area through which the flux must be calculated does not match the area bounded by the circuit [11]:

$$\Phi_B = \int_S \vec{B} \cdot d\vec{S} = B\pi r^2$$

Third, students will analyze the variation of flux over time and calculate Faraday's law. In this case,

$$\frac{d\Phi}{dt} = \pi r^2 \frac{dB}{dt}$$

and

$$\frac{dB}{dt}\uparrow \Rightarrow \frac{d\Phi_B}{dt}\uparrow \quad \frac{dB}{dt} = 0 \Rightarrow \frac{d\Phi_B}{dt} = 0 \quad r\uparrow \Rightarrow \frac{d\Phi_B}{dt}\uparrow$$

Each group's spokesperson presents the group's consensus; all ways of solving the task are discussed, guided by the teacher and the classroom summary finds that the induced electromotive force ε is quantified by the rate of the change of the magnetic flux (through the surface bound by the integral): $|\varepsilon| = |d\Phi_B/dt|$.

Classic electromagnetism theory shows that EMI can be explained from two points of view: field acting and force acting [1]. These two equivalent interpretations are addressed in the second part of the sequence. Students should have opportunities to work from both points of view and also be able to relate them both in cases where induction is associated with the movement of a conductor in a magnetic field and where electromagnetic induction is associated with the temporary variation of a magnetic field. The sequence develops some activities to work out the value of EMF from both points of view. For example,

A.19. An iron magnet with an internal field B is linked to an open circuit containing a voltammeter as shown in the Fig. 4. The magnetic material has fairly high conductivity. So the circuit drawn closes around the magnet using two point contacts at A and B (the circuit wire is isolated). When the wire is moved, electrical contact is maintained on the magnet and the closed circuit never breaks. The field outside the magnet is essentially vanishing. When the wire is moved with constant velocity v, will the ammeter G measure induced current?

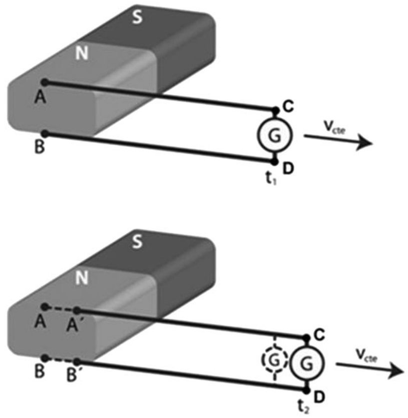

FIG. 4. Experimental setup in the laboratory of activity A.19.

Concerning the field model interpretation, students have to define the integral surface for Faraday's law, that is, the moving surface of the displacement. If they calculate the flux change rate due to the swept surface looking at the displacement of AB to $A'B'$, there is no flux change so there is no induced electromotive force. Moreover, the same result is obtained when applying Lorentz force to the system, meaning that the resulting force is null because there is no magnetic field in the CD part of the circuit.

Another activity that shows the equivalence between the field model and force model again is the following:

A.21. The magnetic field inside a solenoid increases at constant speed $dB/dt = \alpha$. If we put a conducting ring inside the solenoid with radius r (see Fig. 5) and resistance R concentric with it, (a) we evaluate the magnitude and direction of the induced current in the ring, if there is one, and (b) find what is the force that acts on the charges in conducting ring for induced current to take place?

Students are asked to calculate the induced EMF and the force acting on moving charges. Students arrive at their answers through discussion in each group and with the teacher. Students have to make hypotheses and analyze variables to justify their answers. General discussion

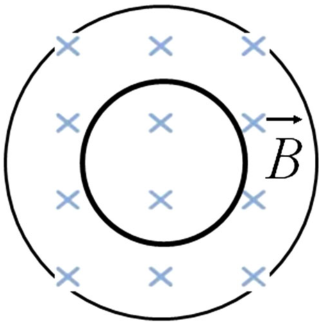

FIG. 5. The small circle represents the conducting ring inside the solenoid.

summarizes that when using Faraday's law, the EMF value is $\varepsilon = -d\Phi_B/dt = -\pi r^2 dB/dt = -\pi r^2\alpha$ and that the induced current through the conductor is $I_{\mathrm{ind}} = \varepsilon/R = \pi r^2\alpha/R$. However, students have to think about an induced non-Coulombian electric field and how it relates to the Lorentz force to calculate the force that acts on a mobile charge in the ring. In other words, students have to relate both explanatory models. The majority of the groups answered correctly and a general discussion took place in the class. The conclusion was the calculation of the force

$$\emptyset_B = \oint \vec{B}\cdot d\vec{S} = B\pi r^2,$$

$$\frac{d\Phi_B}{dt} = \pi r^2\frac{dB}{dt}, \qquad \oint \vec{E}_{NC}\cdot d\vec{l} = E_{NC}\cdot 2\pi r = \left|\frac{d\emptyset_B}{dt}\right|,$$

$$E_{NC}.2\pi r = \pi r^2\frac{dB}{dt} \rightarrow E_{NC} = \frac{\pi r^2\alpha}{2\pi r} = \frac{I_{\mathrm{ind}}R}{2\pi r},$$

$$F = qE_{NC} = \frac{qI_{\mathrm{ind}}R}{2\pi r} = \frac{qr\alpha}{2}.$$

APPENDIX B: QUESTIONNAIRE

Q1.- When the switch is closed in the top circuit of Fig. 6, it can be proven experimentally that ammeter *G* in the lower circuit registers a current. Explain in detail why a current appears in the lower circuit.

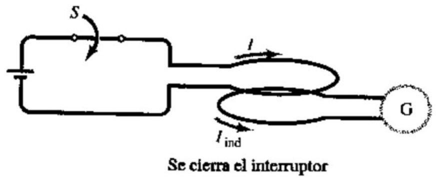

FIG. 6. The switch *S* of the circuit with a battery is closing.

Q2.- A magnet is moving towards a conducting coil at rest in terms of our observation (see Fig. 7); at any given moment as it moves closer, the ammeter registers a current passing through the conducting coil. As you have studied, the electric current in the conducting coil is due to an electric force associated with an electric field, explain how this electric field appears in the coil and its nature.

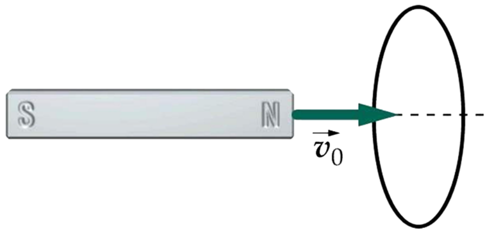

FIG. 7. A magnet is moving towards a conducting coil which is at rest from our observation.

Q3.- When the direction of the coil is changed as shown in Fig. 8, it can be proven experimentally that ammeter *G* will register a current passing through it. Explain where the forces come from that move the charges in the circuit and their nature.

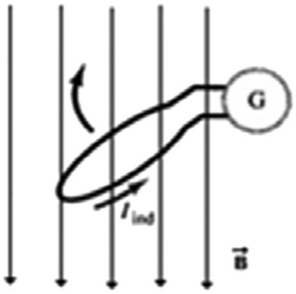

FIG. 8. The coil is rotating in a stationary magnetic field.

Q4.- A coil is moving with velocity v as shown in the Fig. 9. The ammeter *G* registers a current. Explain in detail why a current appears in the coil.

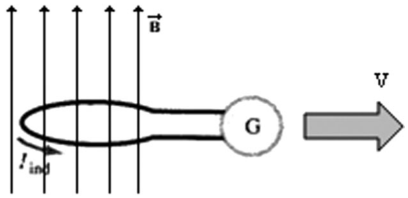

FIG. 9. The coil is coming out of a stationary magnetic field.

Q5.- A copper disk is turning in a magnetic field perpendicular to it (see figure 10). We want to know if electromagnetic induction will occur in this situation and we will use an ammeter to do this. We have put one of the terminals in the centre of the disk and the second rubs up against the outside of the revolving disk. Will the ammeter show a current is passing through?

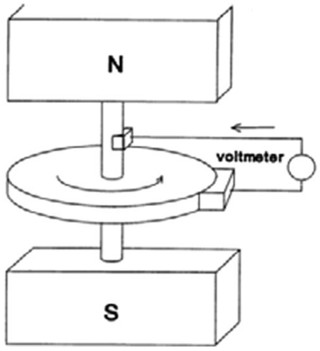

FIG. 10. Faraday's generator.

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