Cosmological constrains on fast transition Unified Dark Matter models

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Abstract.   In this work we show the results obtained applying a Unified Dark Matter (UDM) model with a fast transition to a set of cosmological data. Two different functions to model the transition are tested, and the feasibility of both models is explored using CMB shift data from Planck [1], Galaxy Clustering data from [2] and [3], and Union2.1 SNe Ia [4]. These new models are also statistically compared with the ΛCDM and quiessence models using Bayes factor through evidence. Bayesian inference does not discard the UDM models in favor of ΛCDM.

1. Introduction

In Unified Dark Matter (UDM) models the role of the dark energy and dark matter are played by a single fluid. The Equation of State $w = p/\rho$ changes through time, usually from a matter like era in the past ($w \simeq 0$) to a cosmological constant form ($w \simeq -1$) or to an effective dark energy content ($w < -1/3$). The best known example is the so-called Chaplygin gas [5][6].

Among the different UDM models, the ones with fast transitions are very interesting. In principle these models can be clearly distinguished from a standard ΛCDM without being ruled out by observational data. Whereas this is not the case for the generalized Chaplygin gas [7]. Thus, UDM models with fast transitions can provide an alternative explanation of the accelerated expansion of the universe [8], as they can fit the experimental data quite well, while they provide interesting and different new features. Besides, fast transition UDM models with scalar fields are also compatible with observational data [9].

A simple set-up parametrizing directly the energy density term [10], instead of the equation of state parameter w_{UDM} [8], is computationally lighter in likelihood calculations because most of its important variables required for the numerical calculations are expressed analytically (less integrals are involved). Here we present two parametrizations for this UDM set-ups with fast transition and constrain them using CMB, Galaxy Clustering and Supernovae Ia data. Then we discuss whether they are statistically favoured or not as compared to ΛCDM.

2. UDM Models

The background geometry considered for the phenomenological UDM models in this case is a flat Friedman-Lemaître-Robertson-Walker (FLRW) metric, $ds^2 = -dt^2 + a^2(t)\delta_{ij}dx_i dx_j$, where $a(t)$ is the scale factor as a function of the cosmic time t. We will consider perfect fluids with fractional densities Ω_i as sources; and taking $8\pi G = c = 1$, the Friedman equation takes the form,

$$E^2(a) = H^2/H_0^2 = \sum_i \Omega_i(a), \tag{1}$$

where $H = \dot{a}/a$ is the Hubble function with the dot denoting differentiation with respect to the cosmic time and H_0 being the Hubble constant today.

Being more specific about the sources, we need to clarify the role and form of our proposed UDM fluid: we want an UDM fluid which exhibit a fast transition from the dark matter stage to a scenario that resembles a ΛCDM. We propose the analytical form

$$\Omega_{UDM} = \Omega_t \left(\frac{a_t}{a}\right)^3 + \Omega_\Lambda \left[1 - \left(\frac{a_t}{a}\right)^3\right]\Theta(a - a_t), \tag{2}$$

being $\Theta(a - a_t)$ a Heaviside function and a_t the value of the scale factor at which the transition happens. We can easily see that for $a < a_t$ the fluid behaves like a pure dark matter fluid, with energy density $\Omega_t \left(\frac{a_t}{a}\right)^3$. For $a > a_t$, the fluid will behave like $(\Omega_t - \Omega_\Lambda)\left(\frac{a_t}{a}\right)^3 + \Omega_\Lambda$, resembling a ΛCDM scenario.

Given the properties of the Heaviside function, one can identify the usual dark matter density Ω_c with the term $\Omega_t a_t^3$. Thus, the total matter component will be $\Omega_m = \Omega_t a_t^3 + \Omega_b$ when the baryonic matter term is also considered. In principle, there would be a degeneracy between the Ω_c and Ω_b terms if we were using only low redshift observational data. As long as we are going to use high redshift CMB data, this degeneracy is broken. Moreover, using CMB data makes necessary the introduction of a radiation term too, $\Omega_r(a)$, which has no influences on the late-time expansion, but is fundamental in the early stages of the Universe history. Resuming, the Friedmann equation 1 can be finally written as

$$E^2(a) = \Omega_c a^{-3} + \Omega_\Lambda \left[1 - \left(\frac{a_t}{a}\right)^3\right]\Theta(a - a_t) + \Omega_b a^{-3} + \Omega_r a^{-4} . \tag{3}$$

The last ingredient missing to model this UDM transition is what Heaviside-like functions to consider. We will consider two different functions; the first model for the transition will be :

$$\Theta(a - a_t) = \frac{1}{2} + \frac{1}{\pi}\arctan\left(\beta\pi(a - a_t)\right) . \tag{4}$$

The second transition function considered will be:

$$\Theta(a - a_t) = \frac{1}{2}\left[1 + \tanh\left(2\beta(a - a_t)\right)\right] . \tag{5}$$

In both cases, the parameter β mainly controls the velocity of the transition, being exactly the value of the first derivative with respect to the scale factor of the transition function, evaluated at the transition point a_t.

3. Observational data

In this section we specify the observational data sets we have used for our analysis, and the analytical expression of the χ^2 we are going to minimize in order to perform our statistical analysis.

3.1. CMB data

CMB data are taken from [1], where distance priors are derived from Planck first release data. The CMB shift parameters are the scaled distance to photon-decoupling surface and angular scale of the sound horizon at the photon-decoupling epoch

$$R \equiv \sqrt{\Omega_m H_0^2} \frac{r(z_*)}{c} \ , \quad l_a \equiv \pi \frac{r(z_*)}{r_s(z_*)} \ , \tag{6}$$

where $r(z_*)$ is the comoving distance

$$r(z_*) = \frac{c}{H_0} \int_0^{z_*} \frac{dz'}{E(z')} \tag{7}$$

and $r_s(z_*)$ is the comoving sound horizon

$$r_s(z_*) \ = \ \frac{c}{H_0} \int_{z_*}^{\infty} dz' \frac{c_s}{E(z')} \equiv \frac{c}{H_0} \int_0^{a_*} \frac{da'}{\sqrt{3(1 + \overline{R_b}a')a'^4 E^2(a')}}, \tag{8}$$

where $\overline{R_b}a = 3\rho_b/(4\rho_\gamma) = 31500\Omega_b h^2 (T_{CMB}/2.7K)^{-4}a$ and $E(a)$ is given by eq. 3. Both shift parameters are evaluated at photon-decoupling epoch z_*. The last shift parameter is the baryon density $\Omega_b h^2$.

The mean values for these shift parameters, their standard deviations and the corresponding normalized covariance matrix are obtained by [1], from where is possible to construct the covariance matrix $\mathbf{C_{CMB}}$.

In order to write the CMB contribution to the χ^2, we first define

$$\mathbf{X_{CMB}} = \begin{pmatrix} l_a - \langle l_a \rangle \\ R - \langle R \rangle \\ \Omega_b h^2 - \langle \Omega_b h^2 \rangle \end{pmatrix} \ , \tag{9}$$

and using the inverse of the covariance matrix $\mathbf{C_{CMB}}$, the CMB contribution to the χ^2 term is

$$\chi_{CMB}^2 = \mathbf{X_{CMB}^T} \mathbf{C_{CMB}^{-1}} \mathbf{X_{CMB}} \ . \tag{10}$$

3.2. GC data

The Galaxy Clustering (GC) data we use are the measurements of $H(z)r_s(z_d)/c$ and $D_A(z)/r_s(z_d)$ from the two dimensional two-point correlation function measured at $z = 0.35$ by [2] using a SDSS DR7 Luminous Red Galaxies sample, and at $z = 0.57$ by [3] using the CMASS galaxy sample from BOSS. Here, $H(z)$ is the hubble parameter; $D_A(z)$ is the angular diameter distance

$$D_A(z) = \frac{c}{1+z} \int_0^z \frac{dz'}{H(z')} \ ; \tag{11}$$

and the comoving sound horizon $r_s(z_d)$ is evaluated at the drag epoch.

The Galaxy Clustering contribution is calculated independently for each redshift, $\chi_{GC}^2 = \chi_{GC1}^2 + \chi_{GC2}^2$, where in each point the term is

$$\chi_{GCi}^2 = \frac{1}{1 - r_i^2} \left(\frac{X1_{GCi}^2}{\sigma_{1i}^2} + \frac{X2_{GCi}^2}{\sigma_{2i}^2} - 2r_i \frac{X1_{GCi}}{\sigma_{1i}} \frac{X2_{GCi}}{\sigma_{2i}} \right) \ , \tag{12}$$

where r_i is the correlation between the two functions at each redshift, and

$$X1_{GCi} = \frac{H(z_i)r_s(z_d)}{c} - \langle \frac{H(z_i)r_s(z_d)}{c} \rangle, \quad X2_{GCi} = \frac{D_A(z_i)}{r_s(z_d)} - \langle \frac{D_A(z_i)}{r_s(z_d)} \rangle \ . \tag{13}$$

3.3. SNe Ia data

The SNe Ia dataset used is the Union2.1 compilation, made of 580 Type Ia Supernovae distributed in the redshift interval $0.015 < z < 1.414$. The dataset provides the distance modulus $\mu(z_i)$ for each supernovae and the full statistical plus systematics covariance matrix. The distance modulus $\mu(z) = 5 \log_{10} d_L(z) + \mu_0$ is defined using the dimensionless luminosity distance, which is given by

$$d_L(z) = (1 + z) \int_0^z \frac{dz'}{E(z')}, \tag{14}$$

and μ_0 is a nuisance parameter involving the value of the Hubble constant H_0 and the SNe Ia absolute magnitude. We will marginalize over μ_0 (see ref. [11]). Defining the difference vector between the model and the observed magnitudes

$$\mathbf{X_{SN}} = \begin{pmatrix} \mu(z_1) - \mu_{obs}(z_1) \\ \cdots \\ \mu(z_\mathcal{N}) - \mu_{obs}(z_\mathcal{N}) \end{pmatrix}, \tag{15}$$

and the covariance matrix $\mathbf{C}$ given by [4], we construct the terms $a \equiv \mathbf{X_{SN}}^T \cdot \mathbf{C}^{-1} \cdot \mathbf{X_{SN}}$, $b \equiv \mathbf{X_{SN}}^T \cdot \mathbf{C}^{-1} \cdot \mathbf{1}$, and $d \equiv \mathbf{1}^T \cdot \mathbf{C}^{-1} \cdot \mathbf{1}$, being $\mathbf{1}$ the unitary vector. With these terms, the SNe Ia contribution to the χ^2 marginalizing over μ_0 is

$$\chi^2_{SN} = a + \log \frac{d}{2\pi} - \frac{b^2}{d}. \tag{16}$$

4. Discussion

The statistical analysis will be performed by minimizing the χ^2 function using Markov Chain Monte Carlo (MCMC) Method [12][13][14]. The statistical convergence of each MCMC case has been tested using the method described in [15]. We have tested a fast transition UDM model, with two type of Heaviside-like functions. In order to state the effective statistical weight and validity of our models, we have also analysed the ΛCDM model and the quiessence model as to be compared with.

The priors for the free parameters are: a positive matter density $\Omega_m > 0$ (or $\Omega_c > 0$ for UDM models), proper baryonic bound $0 < \Omega_b < 1$, a positive Hubble function $E(a) > 0$, and $0 < a_t < 1$ because we want the transition have happened. A gaussian prior for the Hubble constant is taken around $H_0 = 100 \, h \, km \, s^{-1} Mpc^{-1} = 69 \, km \, s^{-1} Mpc^{-1}$. In order to show the same parameters in all the models, the parameter Ω_c is used in both ΛCDM and quiessence models, simply transforming $\Omega_c = \Omega_m - \Omega_b$.

Table 1. Median values for the free parameters using CMB, GC and SN data. The median value for the χ^2_{red} and evidence in favour of ΛCDM is also shown.

Model	Ω_c	Ω_b	parameter ♯3	χ^2_{red}	$\ln B_{i\Lambda}$
$\arctan(\beta = 500)$	$0.2432^{+0.0067}_{-0.0064}$	$0.04602^{+0.00088}_{-0.00090}$	$a_t = 0.161^{+0.074}_{-0.094}$	0.9525	+0.010
$\tanh(\beta = 500)$	$0.2433^{+0.0070}_{-0.0065}$	$0.04606^{+0.00081}_{-0.00095}$	$a_t = 0.159^{+0.078}_{-0.094}$	0.9522	−0.015
ΛCDM	$0.246^{+0.010}_{-0.010}$	$0.04696^{+0.00097}_{-0.00097}$	−	0.9472	0
Quiessence	$0.242^{+0.013}_{-0.013}$	$0.0460^{+0.0024}_{-0.0023}$	$w = -1.028^{+0.064}_{-0.067}$	0.9488	−0.330
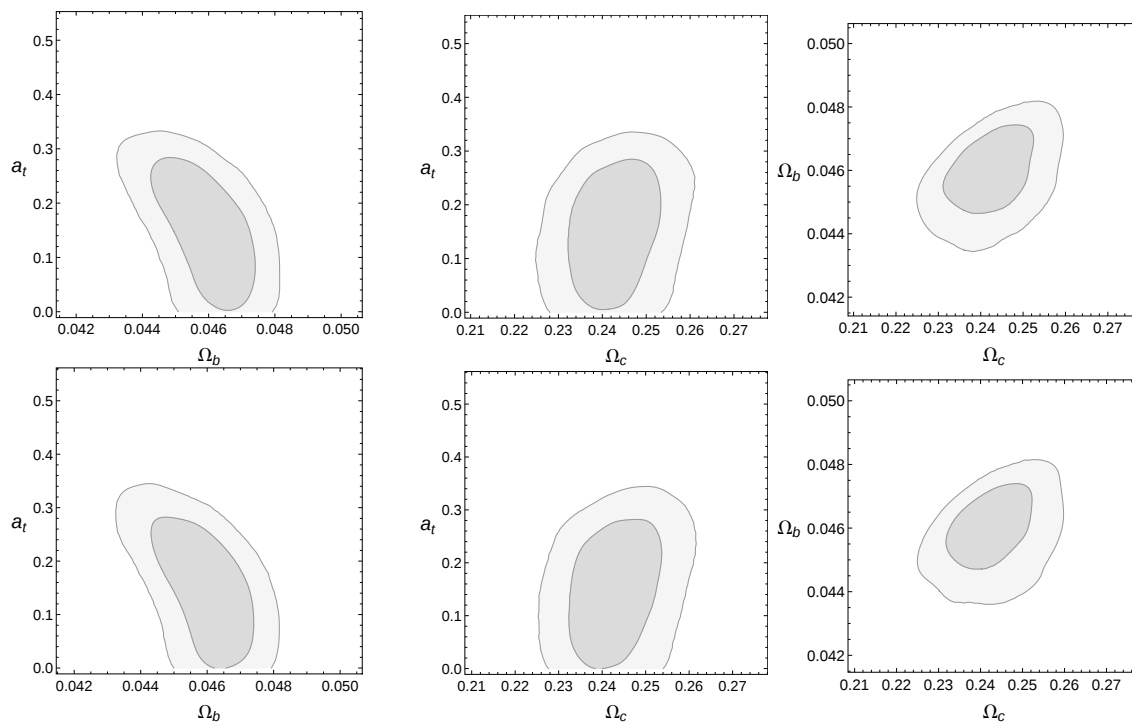
Figure 1. Confidence regions for the arctan (upper row) and tanh (lower row) models, dark grey areas are 1σ region and light grey areas are 2σ region.

Preliminary fits have been carried out using MCMC chains in which the tuning parameter β was left free, but convergence issues occurred. In the end, in order to get reasonable results, it was fixed around a possible minimum. The value of around $\beta = 500$ for both models were hinted by those runs, which is also supported theoretically by [10].

Figures 1 show the constrains of the free parameters of the UDM models, and figure 2 for quiessence and ΛCDM models. The table 1 shows the mean value of the constrained parameters.

A more robust conclusions for model selection can be made with the Bayes factors using the evidence. For the computation of the evidence a nested sampling algorithm has been used [16]. With the resulting evidence, the Bayes factor is obtained comparing the models to the ΛCDM, where the results are shown in the last column of table 1.

All the Bayesian approach shows that the proposed UDM model is worth of consideration, as it can be seen in the table 1 where all the models get a very similar evidence and the difference, according to the so-called "Jeffreys' scale", is inconclusive for all of them. The Arctan model gets even a slightly better evidence than ΛCDM model.

In this work we have analysed two possible functions for the Heaviside function of the fast transition UDM model, both with a quite good results. Due to these results, it could be interesting to analyse deeper other possible Heaviside-like functions in a further research. In any way, the results show that UDM models could be interesting.
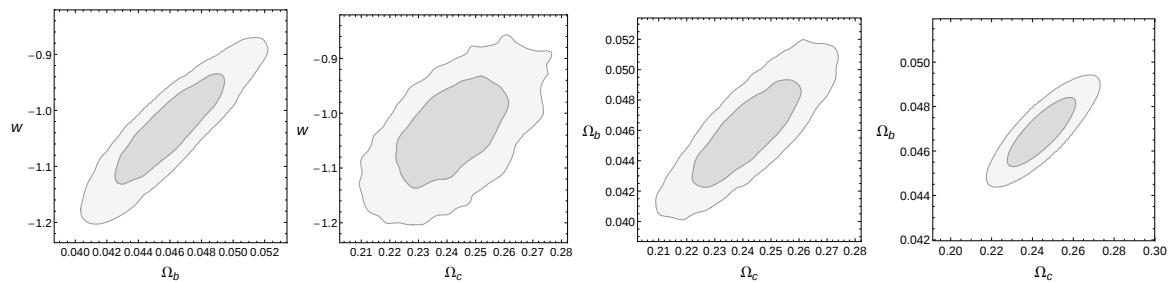
Figure 2. Contour plots for the quiessence (first 3 images) and ΛCDM (last) models, dark grey areas are 1σ region and light grey areas are 2σ region.

References
[1] Wang Y and Wang S 2013 *Phys.Rev.* **D88** 043522 (*Preprint* `1304.4514`)
[2] Chuang C H and Wang Y 2012 *Mon.Not.Roy.Astron.Soc.* **426** 226 (*Preprint* `1102.2251`)
[3] Chuang C H, Prada F, Cuesta A J, Eisenstein D J, Kazin E *et al.* 2013 (*Preprint* `1303.4486`)
[4] Suzuki N, Rubin D, Lidman C, Aldering G, Amanullah R *et al.* 2012 *Astrophys.J.* **746** 85 (*Preprint* `1105.3470`)
[5] Kamenshchik A Y, Moschella U and Pasquier V 2001 *Phys.Lett.* **B511** 265–268 (*Preprint* `gr-qc/0103004`)
[6] Bilic N, Tupper G B and Viollier R D 2002 *Phys.Lett.* **B535** 17–21 (*Preprint* `astro-ph/0111325`)
[7] Bilic N, Lindebaum R J, Tupper G B and Viollier R D 2004 *JCAP* **0411** 008 (*Preprint* `astro-ph/0307214`)
[8] Piattella O F, Bertacca D, Bruni M and Pietrobon D 2010 *JCAP* **1001** 014 (*Preprint* `0911.2664`)
[9] Bertacca D, Bruni M, Piattella O F and Pietrobon D 2011 *JCAP* **1102** 018 (*Preprint* `1011.6669`)
[10] Bruni M, Lazkoz R and Rozas-Fernandez A 2013 *Mon.Not.Roy.Astron.Soc.* **431** 2907–2916 (*Preprint* `1210.1880`)
[11] Conley A *et al.* (SNLS Collaboration) 2011 *Astrophys.J.Suppl.* **192** 1 (*Preprint* `1104.1443`)
[12] Christensen N, Meyer R, Knox L and Luey B 2001 *Class.Quant.Grav.* **18** 2677 (*Preprint* `astro-ph/0103134`)
[13] Lewis A and Bridle S 2002 *Phys.Rev.* **D66** 103511 (*Preprint* `astro-ph/0205436`)
[14] Trotta R and Durrer R 2004 (*Preprint* `astro-ph/0410115`)
[15] Dunkley J, Bucher M, Ferreira P G, Moodley K and Skordis C 2005 *Mon.Not.Roy.Astron.Soc.* **356** 925–936 (*Preprint* `astro-ph/0405462`)
[16] Mukherjee P, Parkinson D and Liddle A R 2006 *Astrophys.J.* **638** L51–L54 (*Preprint* `astro-ph/0508461`)