CRANFIELD UNIVERSITY

ANDER ALBERTO AIZPURU DOMINGUEZ

ENERGY PRODUCTION AND CONSUMPTION SCHEDULING IN SMART BUILDINGS

SCHOOL OF ENERGY, ENVIRONMENT AND AGRIFOOD
Renewable Energy Engineering (REE)

MSc
Academic Year: 2015 - 2016

Supervisor: Dr. Giorgos Kopanos
September 2016

CRANFIELD UNIVERSITY

SCHOOL OF ENERGY, ENVIRONMENT AND AGRIFOOD
Renewable Energy Engineering (REE)

MSc

Academic Year 2015 - 2016

ANDER ALBERTO AIZPURU DOMINGUEZ

ENERGY PRODUCTION AND CONSUMPTION SCHEDULING IN SMART BUILDINGS

Supervisor: Dr. Giorgos Kopanos
September 2016

This thesis is submitted in partial fulfilment of the requirements for the degree of Renewable Energy Engineering (REE)

ABSTRACT

Energy scheduling is a promising solution to alleviate the economic and environmental stresses society places on the power system. Moreover, it provides multiple benefits to users and suppliers. Therefore, there is a lot of interest in the topic and many researchers are working to develop novel configurations that will further increase the advantages of scheduling. In this work, we propose a Mixed Integer Linear Programming model that will manage the production of several Distributed Energy Resources and the consumption of a number of variable and fixed loads with the help of electrical and thermal storage units. The main novelty of this work is the combination of the above with the variable heat to electricity generation of the Combined Heat and Power to minimize costs. The result proves the effectiveness of the scheduling in reducing the cost and the peak energy demands of consumers. It also shows the efficacy in shifting loads under variable pricing policies. All in all, this work shows the potential of energy scheduling in smart buildings to solve the challenges in the power system today and in the future.

Keywords:

Combined Heat and Power, Variable Heat to Electricity Ratio, Price Scheme, Microgrid, Optimization, Heat Demand, Operational Planning

Word count: 7027

ACKNOWLEDGEMENTS

First and foremost, I would like to express my gratitude to my supervisor Dr. Giorgos Kopanos for proposing this interesting topic and for his continuous support.

I would also like to give my most sincere gratitude to Gema Gerena, who helped me by giving hints and providing me with very valuable assistance.

Additionally, I would also like to thank all my friends here in Cranfield who made this year the best it could ever be, giving me very valuable support and motivation to do this work.

Thank you to Javier Silvente for his readiness to help me.

Last but not least, I would like to thank all my family and especially my parents for their invaluable support and love that made this experience a smooth and comfortable journey.

TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

LIST OF EQUATIONS

LIST OF ABBREVIATIONS

CHP	Combined Heat and Power
CPES	Cyber-Physical Energy Systems
CPP	Critical Peak Pricing
DER	Distributed Energy Resource
DSM	Demand Side Management
EMS	Energy Management Systems
FIT	Feed in Tariff
IT	Information Technology
RTP	Real Time Pricing
GHG	Green House Gas
MILP	Mixed Integer Linear Programming
PV	Photovoltaic
DHW	Domestic Hot Water
HST	Heat Storage Tank

1 INTRODUCTION

The human energy demand is continuously increasing [1] and studies [1], [2] predict this trend to continue for the next decades. This means that the already scarce fossil fuels will be unable to meet the forecasted demand [3], [4]. Moreover, the use of these non-renewable energies is affecting the global climate and is risking our way of life on earth as it is known today. Therefore, governments are taking measures to make a transition to a more environmentally friendly system [5], [6].

Many of these actions affect the three main energy consuming sectors: households, industry and transport which together consume 82% of the total energy used in the UK [7]. This means that their shift from fossil fuels to electricity to reduce carbon emissions, will have a huge impact. The shift of the electricity generation sector to renewables will also enhance the effect on the environment.

These improvements face difficult challenges such as the dramatic increase in electricity demand [2], as well as the pressure on renewable energies. To achieve these aims, more renewable plants will have to be built, which will increase the energy price significantly. Also, in order to meet the electricity demand, the electric system is oversized due to the unpredictable nature of the power plants, [4], [8], [9], further increasing electricity prices and complicating the management for electricity operators [10].

To prevent higher energy bills, it is important to reduce the peak electricity demand, and thus, the amount of new constructed power plants. This can be done by distributing households' concentrated energy needs [11], [12]. There are several ways to move the energy demand in households: from penalised energy consumption in periods where demand is highest, to giving incentives to customers who allow the distribution companies to schedule their energy needs [13]. This last method also helps electric network operators to manage the electrical system.

Furthermore, Distributed Energy Resources (DERs) which can be installed in dwellings [14] can also reduce the need for big power plants. It can help the switch to renewables and improve the efficiency by reducing the amount of grid transportation losses [15], [14], [16].

Household space heating is another large contributor of carbon emissions. The heating for a particular household depends heavily on the location and the habits of the users [17], but overall, the energy used for space heating in the UK is much bigger than the electricity demand [18]. As a result, big energy savings can be achieved by improving the house’s insulation [19]. Further energy savings can be achieved by using heat pumps, which are electrically driven devices that reach very high efficiencies [20] and could help shift the heating, based on fossil fuels, to electricity. However, this would increase electricity demand even more, hence electricity prices would go up. Combined Heat and Power (CHP) units (internal and external combustion engines and fuel cells) on the other hand, are the best systems as the have very high efficiency, reaching values of up to 90%, and they generate electricity as well as, so they can also help reduce the need for new power plants. Moreover, CHP units are more effective in reducing Green House Gas (GHG) emissions than all the current electricity and heat generation supply installations together.

Further peak demand reductions and energy savings can be achieved by electrically interconnecting several houses [11], [17] or sharing heat as well [21]. By doing this, the groups of houses are able to become more independent as they generate and consume energy within the group. This configuration also reduces losses, as the transportation distance of the heat and electricity is very short.

Some of the measures mentioned above such as load shifting, DERs installation, heat generation with CHP units, and house connections can be optimized with a scheduler in order to reduce cost and become more efficient. Therefore, in this paper, the energy scheduling for a single and a group of houses electrically interconnected will be analysed to demonstrate the potential of this method. The Mixed Integer Linear Programing (MILP) model will contain a CHP unit that will satisfy the heat demand and will help, together with a Photovoltaic (PV) panel, with the electric needs. Moreover, variable and fixed electrical loads are included to check the effectiveness of the scheduler at distributing the loads. The system will also contain heat and electricity storage units for regulation.

To focus the project, an aim and objectives have been defined. The general aim for this thesis is:

- To analyse the effectiveness of scheduling in reducing electric peak demand, energy costs and finding the best CHP working mode.

In order to achieve this, the following objectives were set:

- Design a model which can schedule energy production and consumption to reduce energy costs.
- Carry out research to obtain realistic data which is then applied to the case studies.
- Analyse a case study of a single house comparing variable heat to electricity ratio of generation as well as different pricing schemes.
- Analyse a case study of a group of four buildings electrically interconnected compared to ones which are isolated.
- Evaluate the case studies to conclude which scenario is most effective at reducing energy costs.

2 LITERATURE REVIEW

The energy scheduling model proposed in this work requires a grid that enables the bidirectional flow of power and information without compromising the reliability of the network. These characteristics together with DERs are common features in the current smart grids, but further improvements such as enhanced reliability, self-healing capabilities and efficient energy management are prospective characteristics for the future smart grids [22]. The need to reduce the carbon footprint with more DERs, particularly renewables and reduce the electric-peak loads with demand side management (DSM), requires new energy management systems (EMS). [23] Summarises aspects that will influence the development of future EMS.

Some works such as [24], [25] have developed multi-agent systems that are able to work in islanded modes in order to improve the reliability of the network. [24], analyses a multi-agent system that is able to disconnect the microgrid from the traditional grid in less than half a period. Moreover, once working in islanded mode, the microgrid manages critical loads (loads that cannot be disconnected) and non-critical loads with the DERs in order to control the voltage, power and current. Besides, the work gives a software base alternative to the hardware based zonal protection allowing more flexible redefinition of zonal boundaries by fly. Another paper, [25], proposes another scheduling model which optimizes the cost with three steps. First, satisfies the internal demand, second, finds the best bids to export energy to the market and finally schedules the microgrid taking into account the internal demand and the sales. The model bases the result on the ability to predict the market prices. Although both works apply their case to the power network, the model can be scaled down to be applicable to smart buildings, as in [26].

Scheduling in smart building has the potential to reduce costs for customers and distributors, therefore, it is the main driver to research this topic. [27] uses a scheduling algorithm which changes the power mode of the appliances under real-time pricing, which allows the model to cut costs by 25.6% on average. [28] uses variable frequency drive as well as capacity-limited storage to cut down costs 41% compared to a traditional scheduling scheme. [29] uses a self-learning software, based on the usage patterns of the inhabitants of the smart building, which schedules the appliances without any action of the users. This last paper, proves the profitability of the algorithm by reporting savings of 10.92% in a single house compared with the case that does not schedule the loads.

Peak reduction is also another important feature of the scheduling technique. To do this, the scheduler distributes the loads during the time. This feature might become even more important once electric cars double the electric demand of the houses [11]. [30] uses Cyber-Physical Energy Systems (CPES), which are integrated embedded processing devices that control the smart home appliances. This, enables the loads to be controlled automatically. However, the controllability of the loads depends on their nature (fixed on time, variable power, preemptive, etc). It is important to keep in mind that the nature of the loads will affect the computational time of the problem [11]. The same author proposes a heuristic scheduling algorithm aiming to reduce the peak load. The results show a 23.1% reduction on the peak load of a single building compared to the earliest starting time. This benefit increases further as the amount of loads increases. Moreover, the paper compares the peak reduction when several houses are put together without any global scheduler that would manage the different houses. The result shows a decrease of up to 16% on the total peak demand. [31] also proves peak demand reduction by proposing a scheduling problem with CHP, Boiler, thermal and electrical storage and DERs.

The environmental effect is also present in some scheduling works, however, having focus only in the environment and forgetting the costs is not realistic, hence, most of the cases use a multi-objective function. [32] proposes MILP to minimise the costs as well as the CO2 emissions where the resulted Pareto curves state the trade-off between the economic and the environment goals. The model also analyses the effect of different pricing methods such as Real Time Pricing (RTP) and Critical Peak Pricing (CPP). The result shows the effectivity of the method by distributing the loads under the price schemes that charge the peak demands.

DERs also play a key role in the reduction of the demand as well as diversifying the cost for further power capacity installation [16]. This paper analyses the benefits of the DERs from several points of view and evaluates different network operation systems with their respective influence in the future of the network.

One system that can strongly affect the DERs of the network are the CHP units. The UK government is committed to help this technology due to their flexibility, efficiency and reliability by assisting them with the Feed In Tariff (FIT). [33] analyses a system with CHP and storage units and compares them with a system with only a boiler, showing the effectivity of this technology. [34] also proves the effectivity of a CHP systems by reducing up to 30% houses' electrical demand, cutting the cost to a quarter and reducing the emissions of CO2, NOx and SO2. A more recent work, [21] proposes a novel configuration of a network of houses with shared heat between them, which increases the generation of electricity from the CHP units. This, reduces the amount of imported electricity, thus, the cost. Other works analyse the combined effect of the storage with different CHP technologies, proving the non-linear relation between the storage capacity and the thermal power of the prime mover. It also proves that the increase of the storage reduces the total purchase of electricity because the CHP unit works more time, reducing the cost. As stated by [35], CHP units generate heat as a by-product of the generated electricity, however, when the electricity is required, the heat is not required and vice versa. This lack of heat and electricity synchronisation, makes storing energy a wise choice. However, energy storage has losses that decrease the efficiency of the system. As a result, detailed focus needs to be addressed to them to make the system optimal.

Although many of the tasks of a building can be predicted and therefore scheduled, there are some other tasks which are randomly activated (lights, TV, etc.). Similarly, the electricity prices or the generation of certain DERs have an unknown variability in their output. These uncertainties can have a severe effect in the outcome of the problem. As a result, many researches have presented works to account for those unpredictable effects. [28] introduces in their algorithms an energy consumption adaptation variable, which is used to model the stochastic energy consumption patterns for various household appliances. The paper also introduces another parameter, the trip rate, which is a user defined parameter that accounts for the probability that a house goes over its contracted power, and thus, cuts the power. [36] on the other hand, deals with the uncertainty by developing a rolling horizon, which updates the results for the unexpected events. [37] focuses only on the price uncertainty from the point of view of the customer. The case studies studied in that paper show that statistical information about future prices can be highly beneficial. Moreover, having high power and short time tasks increases further the economic savings.

The literature review shows how much work has been done in the energy scheduling and management of either buildings or smart grids. Special focus has been drawn to the reduction of the cost and peak demand. However, the environmental effect of the scheduling is not so thorough.

DERs are also a well-researched topic, with special focus on CHP system. There are different types which have advantages and disadvantages depending on the application, but there is not a lot of investigation about CHP systems that are able to change the heat to electricity ratio generation. Other DERs such as wind mills or solar panels are also included in some of the works, but there is not so detailed research on the effect of the variability of the output of these systems.

The information found from literature on heat demand is based on previous surveys which were carried out, however to the best of my knowledge there are very few works done which introduce the heat transfer coefficient and the outside and inside temperatures of the building.

All in all, the present work proposes a model which takes into account the heat-transfer coefficients of the houses to generate a heat demand satisfied by a CHP unit working in two different working modes. The second working mode, is not common in the literature and uses a variable heat to electricity ratio generation This work also integrates the heat and electricity demand combined scheduling which is not so common in the literature and schedules .

3 PROBLEM STATEMENT

This paper proposes a network of houses, interconnected electrically between each other and to the power grid in order to buy and sell electricity. In Figure one, the used model structure for the case studies is shown:

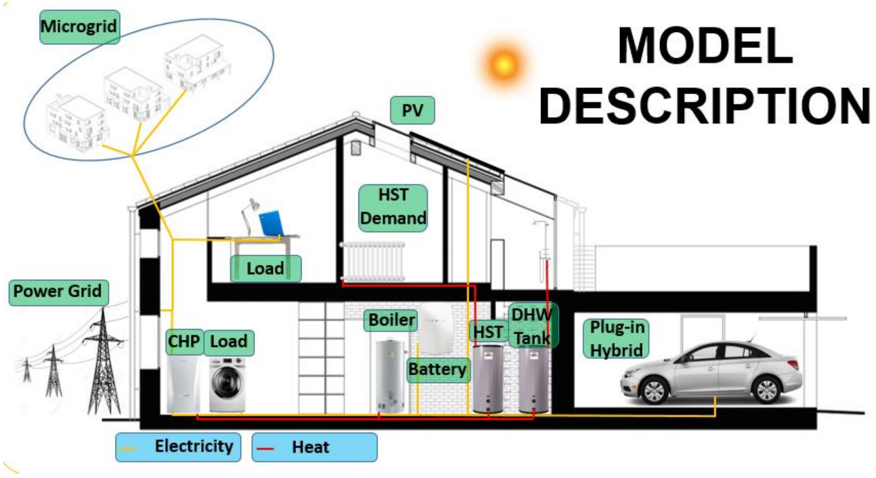

Figure 1: Structure of the energy structure of the house

The houses produce heat from the CHP and the boiler and store this heat in two different tanks which satisfy the house heat demand. Moreover, the dwellings generate electricity with the CHP and the PV panels. This electricity will be used by the appliances, stored in batteries or given to the microgrid shared between the houses of the model. When the electricity in the microgrid cannot be consumed, then, this energy will be sold to the power grid.

Further details about the reasons to build such model are specified in Appendix A.

4 MATHEMATICAL MODEL DESCRIPTION

The CHP system has been modelled to account for its heat and electricity generation limits. Besides, losses for the start-up and shut-down have also been modelled. The CHP has two working modes that have been introduced in the model.

The other heat generator system, the boiler, also has limits and have been introduced mathematically.

The thermal demand is composed by the thermal losses of the generation and storage units, the thermal losses from the difference between the inner and outer temperatures of the house, the ventilation rate, the house temperature change for each time interval, the heat generated by the people and the appliances in the house and the domestic hot water demand.

Similarly, the electricity demand is composed by the schedulable and non-schedulable loads, the electricity sent to the batteries, microgrid and the power network and the losses in the charge/discharge of the batteries.

The schedulable appliances can only be used once each day and cannot stop working once they have started. They have an earliest time to start operating and the latest to finish. In that time period, the smart house will decide the best moment to activate the load.

The detailed mathematical model and the input data are specified in Appendix B.

5 CASE STUDIES

5.1 Case Study 1: Single home

In the first case study, the effect of the variable and fixed heat to electricity ratio generation and the different pricing policies will be analysed for a single home.

5.1.1 Fix heat to electricity ratio vs Variable heat to electricity ratio generation

In this study, a single house with a CHP unit that has a fixed heat to electricity ratio equal to six will be compared to a CHP with a variable heat to electricity ratio from 3.2 to 6.0.

5.1.1.1 Results

In the Figure two, the electricity balance for the case with fixed heat to electricity ratio:

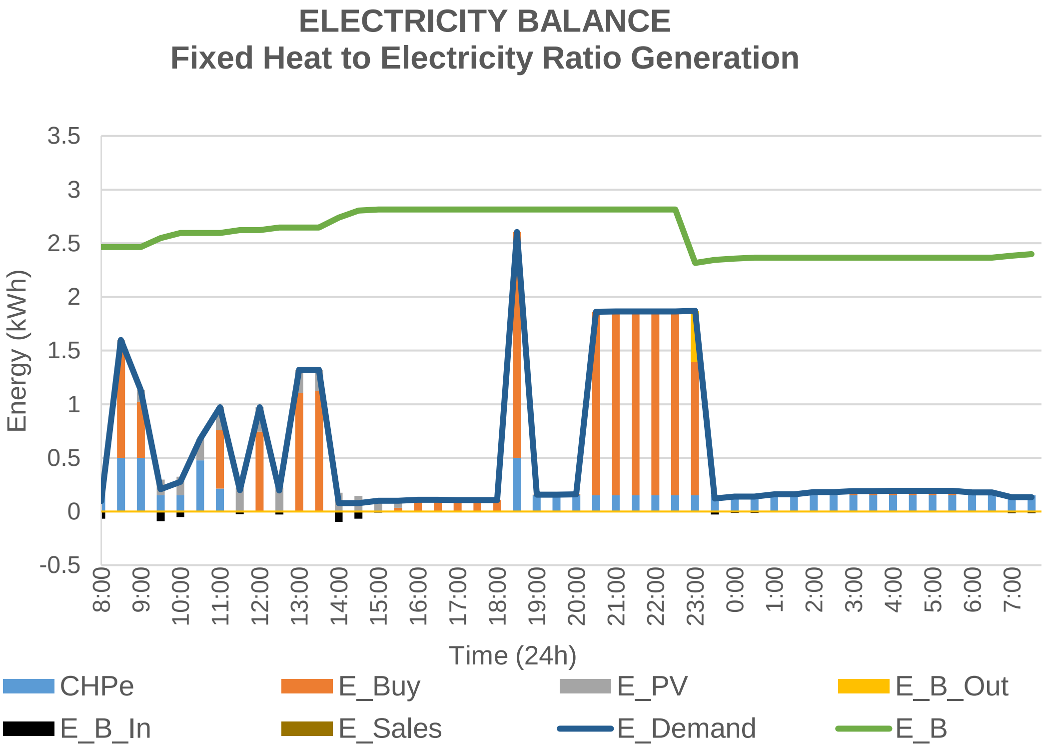

Figure 2: Electricity balance for the case with fixed heat to electricity ratio

In the following graph, the electricity balance for the case with variable heat to electricity ratio is shown:

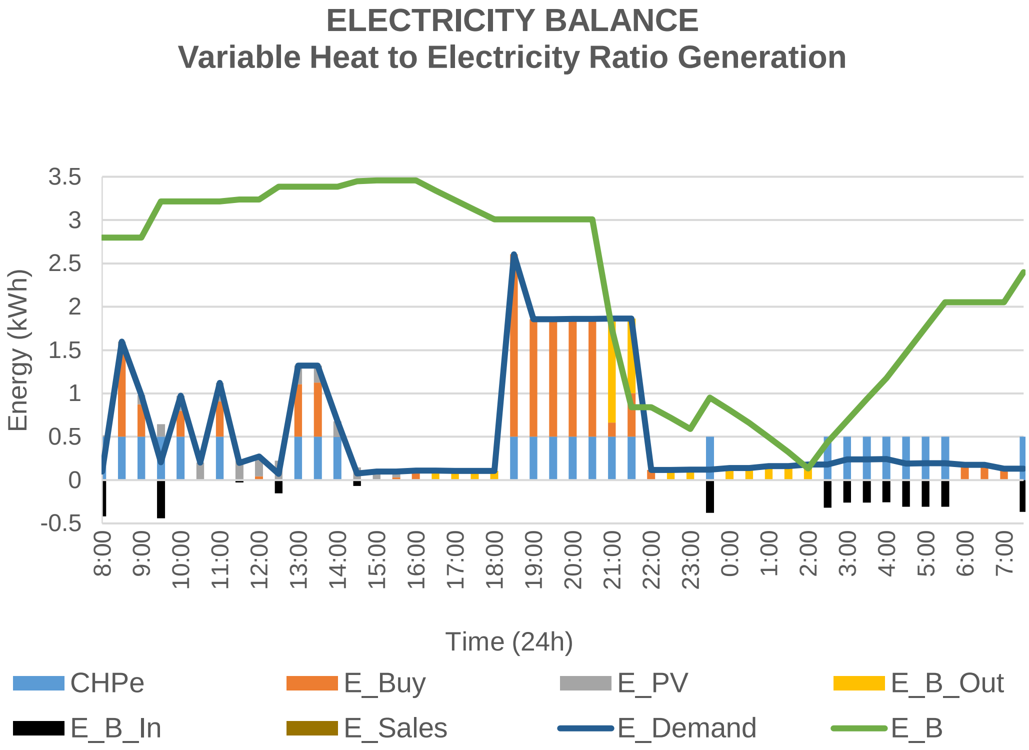

Figure 3: Electricity balance for the case with variable heat to electricity ratio

The Figure three shows how the model with variable heat to electricity ratio is able to produce more electricity due to its capacity to lower the value of this ratio. Therefore, more electricity is generated enabling the system to better regulate the produced energy using the batteries. This means that the electricity purchased for the case with fixed heat to electricity ratio amounts to 17.96kWh while the variable ratio purchased 12.33kWh.

In total, the system with the fixed parameter produces 6.54 kWh of electricity with the CHP, whereas, with the variable parameter it produces 12.5 kWh. This is possible because the parameter stays constant at 3.2 during the whole simulation. As a result, the cost for the second case is cheaper by £1.97 pounds, compared to the other case which is £2.70 pounds.

Another interesting feature of the model is the amount of energy sold. In both cases it is zero, because the amount of generated electricity is very small compared to the demand which is 26.98kWh. Besides, the selling price for the electricity is very low: £0.0486 per kWh. As a result, it is always more convenient to use that energy in the appliances or to store it in the battery.

Focusing on the battery, it can be seen that it is only charged when the electricity demand is below the amount supplied supply. Therefore, only in a few cases is the battery is charged. However, when the heat to electricity coefficient is variable, the amount of energy produced is higher, thus, the probability of the supply exceeding the demand is higher. This is the reason why more energy flow can be seen in the batteries in Figure three.

Regarding heat balance, the system is able to satisfy the demand of the house as it has a maximum heat generation of 12kW when the maximum heat demand is lower. Besides, the system has heat storage units that can supply extra heat if the heat generation units are not working or when the units cannot generate as much energy due to their capacity. This is why only one heat generation unit (Boiler or CHP) is able to satisfy the heat demand. Moreover, the CHP is more efficient and the FIT of the government awards each kWh of generated electricity with £0.1345 [38]. Hence, the boiler is never in operation.

For the heat storage units, it is important to realize how the Domestic Heat Water (DHW) tank's average specific heat is greater than the Heat Storage Tank (HST). This is because of the geometry of the tank, which enables the DHW storage unit to have fewer losses than the HST when the temperatures of both tanks are the same. This can be seen in Figure 4:

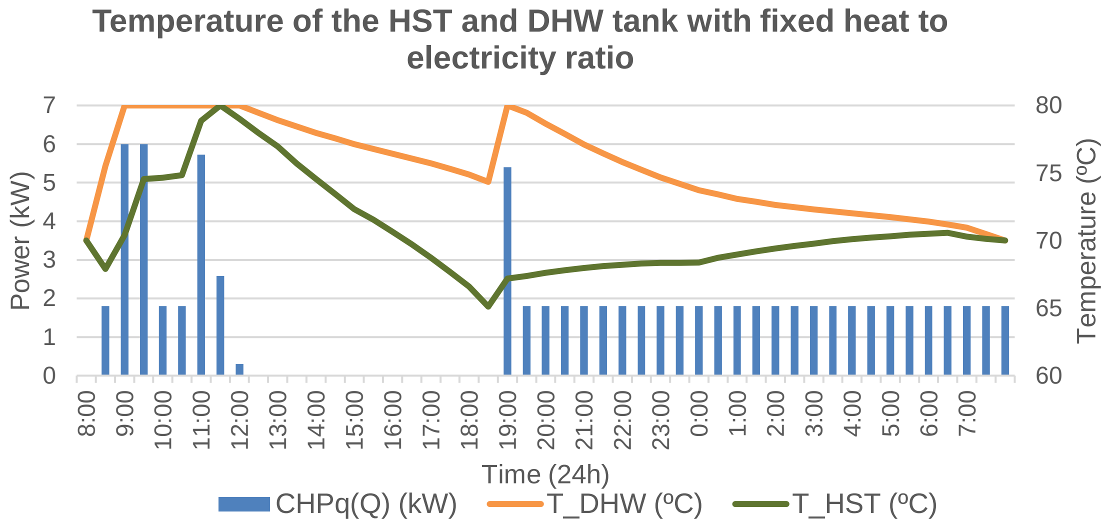

Figure 4: Heat limitation

Finally, when the HST gets to its maximum capacity, the CHP needs to stop, because the heat cannot be discarded, so the system is limited by the heat demand and its storage units. This is seen in the Figure four. When the heat to electricity coefficient is fixed, in order to reduce the amount of generated heat, the amount of electricity produced must be reduced as well. However, when the coefficient is variable, the produced heat can stay at its minimum (3.2kW) and the electricity production can still be maintained at its maximum (1kW). Therefore, the case with variable heat to electricity ratio is able to produce more electricity while producing less heat, obtaining lower costs.

5.1.1.2 Discussion

As stated before, the low heat to electricity demand of the houses compared to the higher heat to electricity generation of the CHP leads to the model not being optimised, but it ensures that the heat demand will at any moment be satisfied. Besides, the electricity can always be purchased from the grid. In the Figure five, the heat generated from the CHP and heat requirements from the house are shown. It can be seen that when the CHP is working, the unit is always able to supply more heat than the amount required at any time.

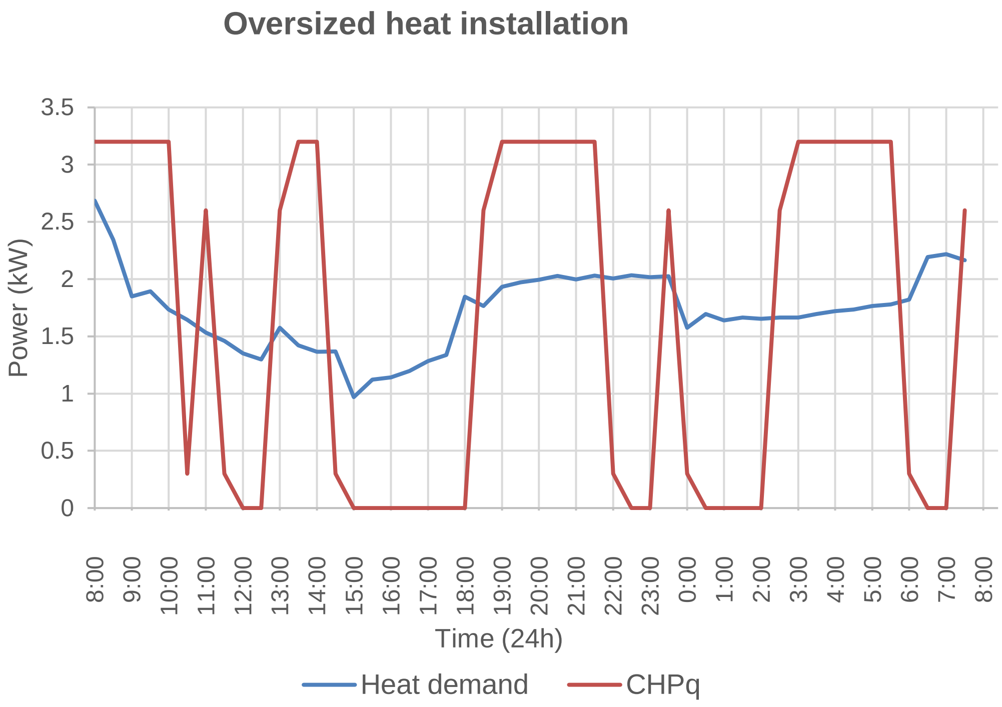

Figure 5: Oversized heat installation

Low gas prices compared to electricity leads to a system that is prone to heat losses in order to achieve maximum cost effectiveness. The current system is prone to produce heat energy losses because the price for gas is very cheap compared with the electricity one. In the present case study, the gas price for the CHP is £0.025 per kWh and the electricity price is £0.1558 per kWh. Taking into account that the fixed heat to electricity ratio produces one kWh of electricity for every six kWh of heat, this means that producing one kWh with the CHP costs £0.15, which is lower than the price of the electricity. This means that even for someone who doesn’t need heat, it is cheaper to produce electricity with the CHP than buying it. Finally, the government benefits these systems with FIT by paying them £0,1345 for every kWh of electricity produced. This will increase the benefit of discarding heat, which is self-defeating because the government is trying to increase the efficiency of the energy production system.

Figure six shows that, the temperature of the house is always 0.5ºC higher than the target temperature because the house will have more thermal losses, thus, it will have to produce more with the CHP, generating more electricity and reducing the cost.

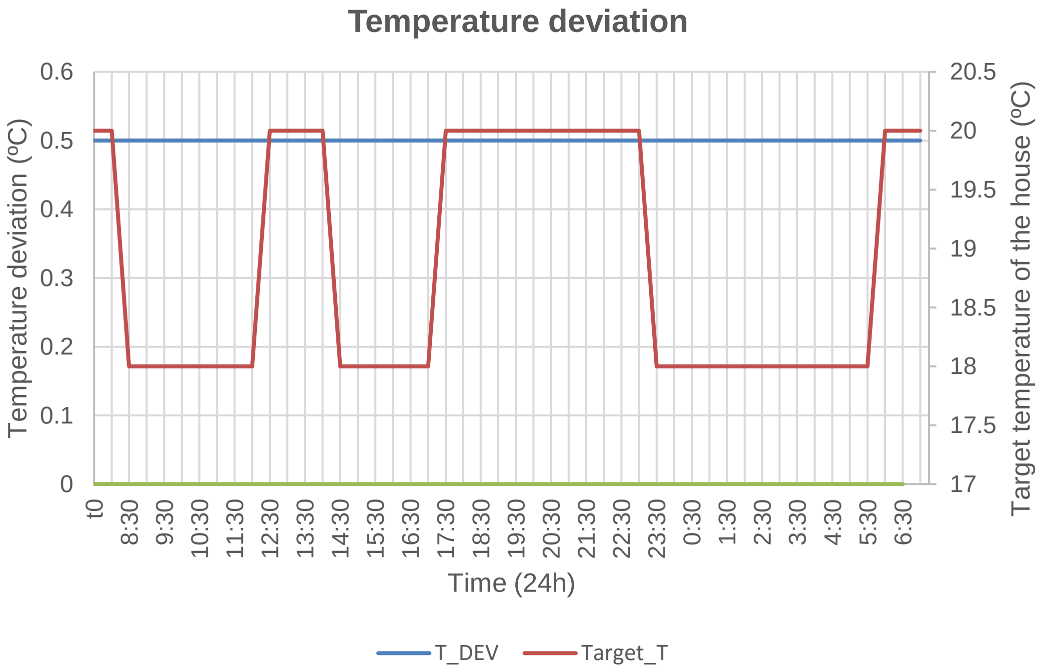

Figure 6: Temperature deviation in the house

5.1.2 Different pricing policies

For this study, a single house with variable heat to electricity ratio generation will be analysed under two pricing schemes. One with a constant value during the whole day and another with a lower price during 7 hours of the night. The latter is called Economy 7.

5.1.2.1 Results

As proven before, the HST and the low heat demand are the limitations of the system to improve the cost of the operations, therefore, it is interesting to see how they behave together with the CHP units, which are the ones in charge of filling them. This can be seen in Figure seven.

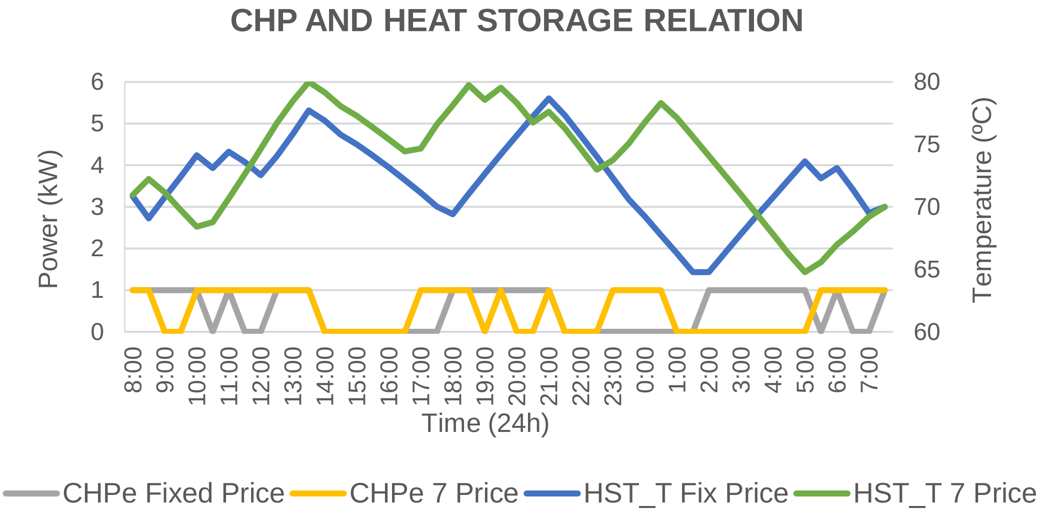

Figure 7: CHP and Heat Storage relation

It can be seen in the Figure seven, that for Economy 7 price, the CHP works until the heat storage unit is filled up. Then, because the heat demand is too low and the heat storage units cannot take more heat, the CHP must stop. There are only a couple of exceptions where the CHP unit can stop without having the storage at its maximum. This is when the electricity demand is lower than the PV generation; or when the electricity demand is below 1kW. When this happens, part of the generated electricity must go to the batteries. By doing this, some electrical losses will occur that will have to be compensated with electricity from the grid because the CHP production is limited. This means that more electricity will have to be purchased, increasing the cost.

For the case with fixed pricing the same thing happens, but the CHP stops more often, thus, the HST does not reach the maximum level as many times as with Economy 7. The reason for this is the purchased electricity price. In the fixed price, the average purchased electricity cost is more expensive than in the Economy 7 price scheme, making the losses of the electric storage more costly.

Regarding the performance of the battery, it can be seen how the case with two price zones discharges the battery until the price decreases, and it starts charging. This is because the system tries to avoid purchasing energy from the grid when it is most expensive. However, there are some particular cases where the battery is charged. As explained before, these cases happen when the generation units produce more energy than the demand and the cost of the losses of storing electricity are less than purchasing that stored energy from the grid, when it is required. The charging of the battery at the end is just a process to leave the battery at the same state as it was at the beginning of the day. This can be seen in Figure eight.

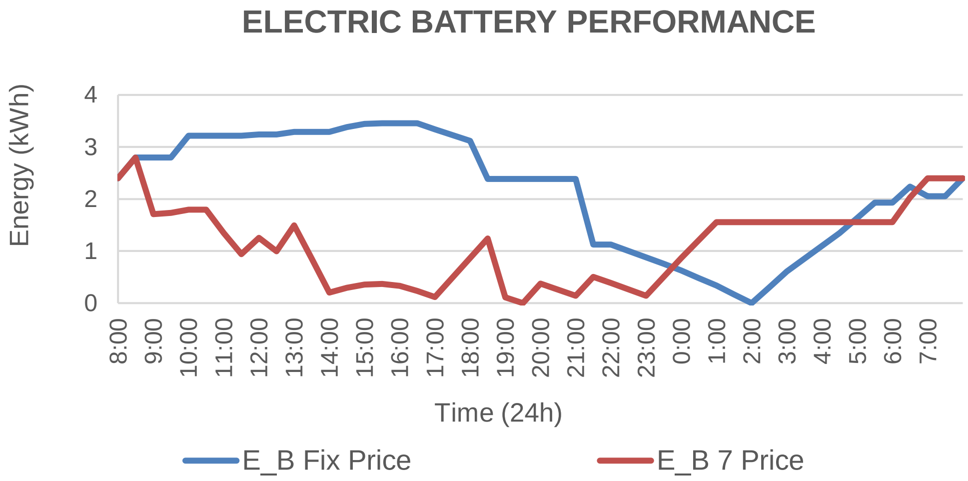

Figure 8: Electric battery performance

Finally, one of the aims to use different time pricing is the distribution of the loads. The case study shows the affectivity of the model in doing so. Figure eight shows that the loads shift to the times when the cost of electricity is cheaper. However, some loads are very restricted in time, therefore, they cannot be moved.

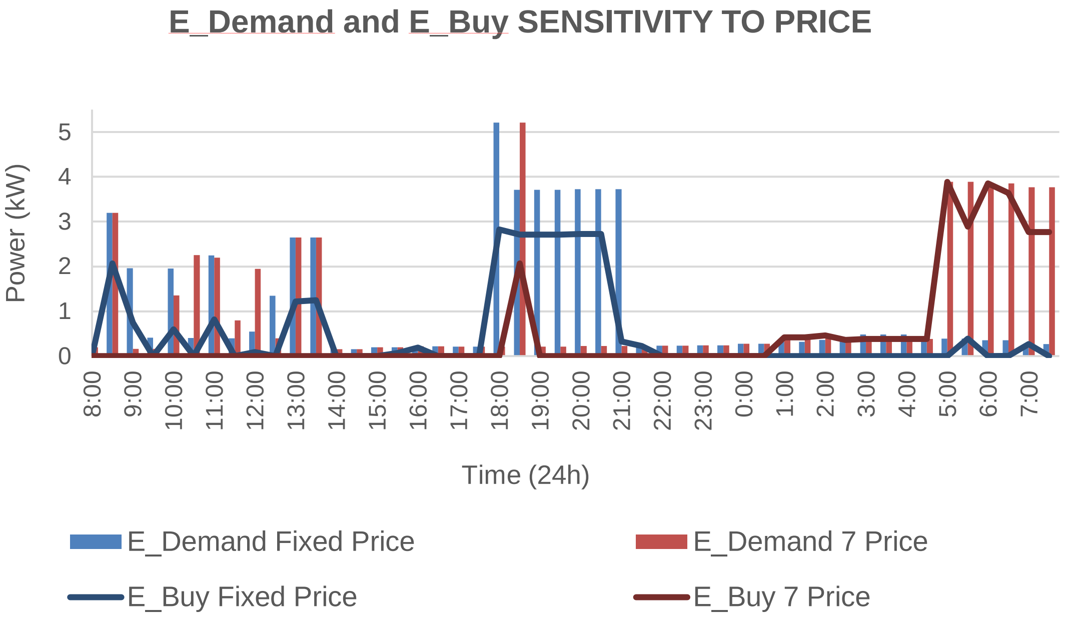

Figure 9: Load sensitivity to price

5.1.2.2 Discussion

One of the aims of different pricing policies is to reduce the cost for the suppliers, who have to purchase the electricity from the wholesale market and the price is more expensive when the demand is higher. By doing this, they manage to charge more the customers, who use the energy during peak demand, so suppliers cut costs, however, it is proven that someone with more flexible habits can also benefit from this kind of scheme. The model shows a reduction in cost for the Economy 7 of 53.6%.

For battery usage, this simulation shows that the battery is used more when the electricity prices vary, since they take advantage of the different pricing by charging the battery during peak periods with cheap prices and discharging when prices are high. This shows that with extreme prices it makes more sense to purchase a battery. It also shows that when the demand is much higher than the power generation under constant electricity prices, it does not make a lot of sense to use a battery because is rare the case when the electricity production is lower than the demand

5.2 Case Study 2: Four houses

In the second case study, the effect of sharing and not sharing electricity among a group of 4 houses will be analysed. The four houses included in the model have been explained in the Appendix B.1.4 Input Data.

5.2.1 Electricity sharing vs Isolated

In this study, 4 isolated houses will be compared with 4 houses which share electricity for free when each house has an excess of electricity.

5.2.1.1 Results

In the case where the four houses are isolated from each other, the battery usage of each of the houses is greater compared to the houses which are interconnected between each other, because the excess electricity that in the isolated case goes to the battery, can instead go to another house. This way, the case with electrical connexions between the houses uses the battery a lot less. The reason why the model prefers to share electricity is because the smart grid is more efficient than the batteries, so less losses are produced. In Figure ten, the sum of the stored energy in the batteries of the houses is displayed:

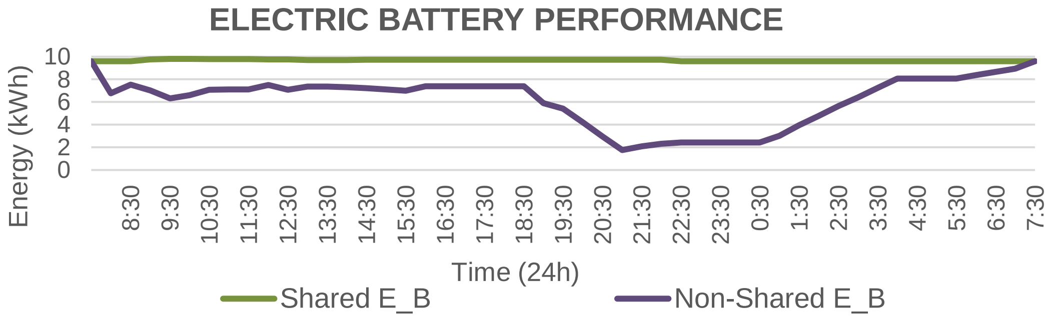

Figure 10: Electric battery performance

The peak electricity demand of the group of buildings is also reduced in the case with shared electricity. The peak demand of each of the individual houses is maintained constant, as the task that produces the peak is the cooker oven and its power consumption cannot be divided or reduced. When the houses are isolated, each house tries to minimize the peak demand by using the cooker oven when fixed power consumption is lowest and power generation is highest. Because the houses are very similar, this occurs at the same time for all four houses, when they are isolated. However, when the electricity is shared, the four cooker ovens will be distributed in order to generate the minimum peak between the 4 houses. This is why the peak demand is reduced

The comparison between the two demands is shown in the Figure 11:

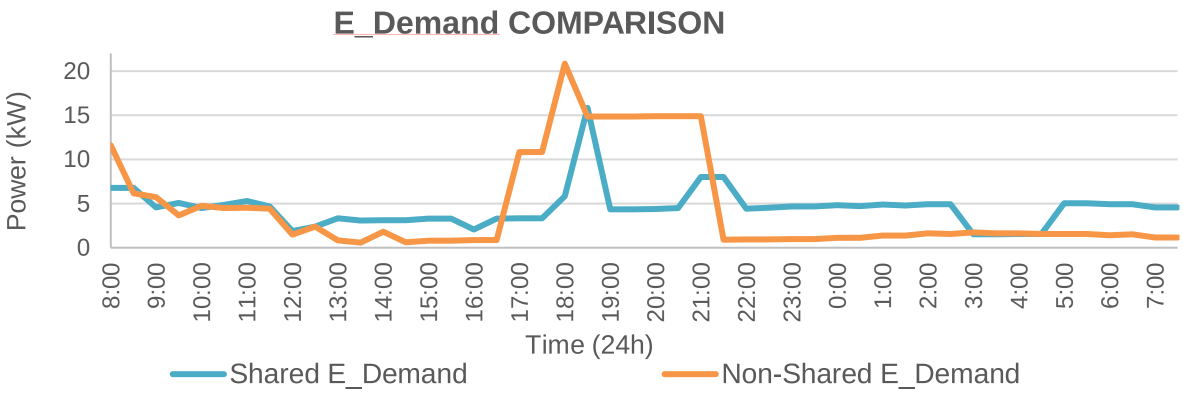

Figure 11: Electricity demand

Finally, this distribution of the loads reduces the peak purchase of electricity, which is lower for the case where electricity is shared, as seen in Figure 12:

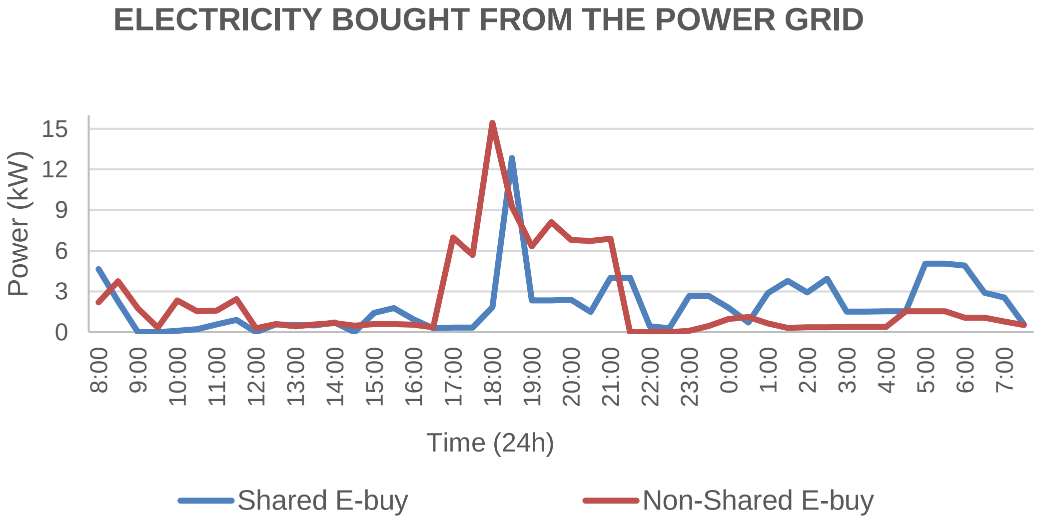

Figure 12: Electricity bought from the grid

Moreover, because the electricity peak is lower, the DERs are able to supply a greater percentage of the demand, therefore, reducing the total amount of purchased electricity. For the case with no electricity sharing, the amount of purchased electricity is 53.4kWh whereas for the case with electricity sharing it is 48.34kWh.

There is one more difference between the two cases: the amount of generated electricity with the CHP unit which is higher for the case with shared electricity at 49.5kWh compared to no electricity sharing, at 45.5kWh. The reason for this is the cost to store or share the excess electricity of each of the houses. While the isolated houses have to store the energy in the batteries, the houses with electricity sharing can give the electricity to the microgrid between the four houses with higher efficiency and at no cost. So when the isolated houses stop the CHP because of the cost that storing electricity incurs, the other case can still produce electricity because sharing the electricity in the microgrid has a lower cost.

5.2.1.2 Discussion

This second case study proves that it is more profitable to share electricity between houses than isolating them. This profitability is not only better for the household owners who save 8.33% of the costs, but for the network operators and the distributors as well. The distributors will have to pay less for the electricity that they purchase from the wholesale market due to the reduction of the peak demand. Moreover, having lower peaks will put fewer constrains on the power grid, resulting in more options for the network operators, thus, easing their work.

Sharing electricity, however, creates some issues. In reality, the house appliances will not be exactly the same. This will make the demand of some buildings bigger than others. If the DERs are not the same either, there might be cases in which one house will have oversized DERs, so it will give electricity for free. This would be unfair and it would cause the house-owners to always install fewer DERs than required. To prevent this, a price should be applied to the electricity shared in the microgrid. This price should also be cheaper than the price offered by the suppliers, otherwise buildings would always prefer the electricity from the power grid, making the system more complex and instable.

6 CONCLUSION

The designed model shows a clear limitation on the heat balance which prevents the CHP from working more time, thus, to generate more electricity and reduce the cost. This issue can be solved by changing the CHP system to a model with a much lower heat to electricity ratio, such as, a fuel cell. It can also be solved by reducing the electricity demand, but this is not feasible, since in the last decade the electricity demand has gone up. Finally, the last option is to increase the heat demand, which is the least efficient one, but also the less costly. It only requires opening a window. This situation is generated because of the difference in price between electricity and gas. The FIT of the government can further increase this effect.

The DERs help to significantly reduce the electricity demand of the household, reducing the need to purchase electricity by around a 50%. This means that even if the electricity demand rises in the future, this measurement can prevent huge installations of new power plants. If these systems become quite popular quickly, they can even help in the closing of most polluting coal plants, significantly reducing carbon emissions. Besides, there is no extra gas consumption, because the CHP is a heat demand driven appliance, meaning, it will only work when the heat demand requires it to.

Looking into the future, this reduction in electricity consumption of households could be used to provide electricity to the automotive sector which is being electrified. This change, which could cause the electricity consumption to skyrocket, would be significantly reduced, avoiding huge investments in new power plants.

The variable price scheme has proven that it can help in shifting loads from the peak demands to the valleys, flattening the electricity demand, therefore, avoiding the use of utilities that only supply power during peaks. This would make the electrical system more efficient. Moreover, this measure would benefit customers with lower bills as well as the suppliers with lower costs and reduced peaks.

With scheduling, consumers are able to better control some of their loads and benefit by consuming when their CHP or PV generates. This scheduling can be taken one step further if the batteries take a more active role. In this new approach, the batteries would supply energy for the non-schedulable loads, such as lights, TV, etc. and the forecasted errors in the PV's electricity generation. When the house is able to schedule with a very high confidence, the energy required from the power grid can be scheduled in advanced avoiding the need for costly peak reserve power plants. Besides, houses will be able to group together and become their own suppliers, obtaining much lower energy prices. This is possible with arrangements such as the UK's License Lite [39].

Based on the built model, further work can be carried out by including uncertainties in the PV's electricity generation, prices and carried out the non-schedulable loads. This will make the model more realistic. Besides, a statistical prediction can be added to forecast the previously mentioned uncertainties that can be updated with a rolling horizon. With this, the uncertainty in the energy consumption of buildings could be mitigated.

REFERENCES

[1] Eia, “World energy demand and economic outlook EIA’s handling of non-U.S. policies in the International Energy Outlook,” *U.S. Energy Inf. Adm.*, 2016.

[2] Iea, “World Energy Outlook 2015,” Paris, 2015.

[3] S. Shafiee and E. Topal, “When will fossil fuel reserves be diminished?,” 2008.

[4] “The End Of Fossil Fuels,” 2015. [Online]. Available: https://www.ecotricity.co.uk/our-green-energy/energy-independence/the-end-of-fossil-fuels. [Accessed: 04-Jul-2016].

[5] “Climate Change.” [Online]. Available: https://www.gov.uk/government/topics/climate-change. [Accessed: 04-Jul-2016].

[6] “UN CLIMATE CHANGE CONFERENCE,” 2015. [Online]. Available: http://www.cop21.gouv.fr/en/. [Accessed: 04-Jul-2016].

[7] L. Waters, E. Wilkes, and V. Goodright, “Energy Consumption in the UK (2015) Chapter 1: Overall energy consumption in the UK since 1970,” *Department of Energy & Climate Change*, 2015. [Online]. Available: https://www.gov.uk/government/uploads/system/uploads/attachment_data/file/449102/ECUK_Chapter_1_-_Overall_factsheet.pdf. [Accessed: 04-Jul-2016].

[8] K. Chia, T. Gleeson, J. Lippiatt, and P. Simshauser, “Challenges Facing the Renewable Energy Industry.”

[9] “Renewable Energy and Electricity,” *WORLD NUCLEAR ASSOCIATION*, 2016. [Online]. Available: http://www.world-nuclear.org/information-library/energy-and-the-environment/renewable-energy-and-electricity.aspx. [Accessed: 04-Jul-2016].

[10] I. J. Pérez-Arriaga, “Managing large scale penetration of intermitent

renewables,” 2011.

[11] M. K. Junghoon Lee, Hye-Jin Kim, Gyung-Leen Park, “Energy consumption scheduler for demand response systems in the smart grid,” *J. Inf. Sci. Eng.*, vol. 27, no. 27, pp. 197–211, 2011.

[12] DECC, “Demand Side Response in the domestic sector - a literature review of major trials,” no. 1, pp. 1–155, 2012.

[13] L. Merkert, I. Harjunkoski, A. Isaksson, S. Säynevirta, A. Saarela, and G. Sand, “Scheduling and energy - Industrial challenges and opportunities,” *Comput. Chem. Eng.*, vol. 72, pp. 183–198, 2015.

[14] M. F. Akorede, H. Hizam, and E. Pouresmaeil, “Distributed energy resources and benefits to the environment,” *Renew. Sustain. Energy Rev.*, vol. 14, no. 2, pp. 724–734, 2010.

[15] “Benefits of distributed Generation.” [Online]. Available: http://www.dg.history.vt.edu/ch1/benefits.html. [Accessed: 04-Jul-2016].

[16] A. Alarcon-Rodriguez, G. Ault, and S. Galloway, “Multi-objective planning of distributed energy resources: A review of the state-of-the-art,” *Renew. Sustain. Energy Rev.*, vol. 14, no. 5, pp. 1353–1366, 2010.

[17] H. Brohus, P. Heiselberg, A. Simonsen, K. C. Sørensen, and C. Engineering, “UNCERTAINTY OF ENERGY CONSUMPTION ASSESSMENT OF DOMESTIC BUILDINGS,” pp. 1022–1029, 2009.

[18] “Energy Consumption in the UK (ECUK) Domestic data tables 2015.” Department of Energy & Climate Chage.

[19] International Energy Agency, *Transition to Sustainable Buildings - Strategies and opportunities to 2050*. 2013.

[20] “Heat pump systems.” [Online]. Available: http://energy.gov/energysaver/heat-pump-systems. [Accessed: 04-Jul-2016].

[21] G. M. Kopanos, M. C. Georgiadis, and E. N. Pistikopoulos, “Energy

production planning of a network of micro combined heat and power generators," *Appl. Energy*, vol. 102, pp. 1522–1534, 2013.

[22] R. E. Brown, "Impact of Smart Grid on Distribution System design," *IEEE Power Energy Soc. 2008 Gen. Meet. Convers. Deliv. Electr. Energy 21st Century, PES*, 2008.

[23] Y. Zhang, M. Mao, M. Ding, L. Chang, and S. Member, "Study of energy management system for distributed generation systems," *2008 Third Int. Conf. Electr. Util. Deregul. Restruct. Power Technol.*, no. April, pp. 2465–2469, 2008.

[24] M. Pipattanasomporn, H. Feroze, and S. Rahman, "Multi-agent systems in a distributed smart grid: Design and implementation," *IEEE/PES Power Syst. Conf. Expo. 2009. PSCE '09.*, pp. 1–8, 2009.

[25] T. Logenthiran, D. Srinivasan, and A. M. Khambadkone, "Multi-agent system for energy resource scheduling of integrated microgrids in a distributed system," *Electr. Power Syst. Res.*, vol. 81, no. 1, pp. 138–148, 2011.

[26] T. G. Stavropoulos, E. S. Rigas, E. Kontopoulos, N. Bassiliades, and I. Vlahavas, "A multi-agent coordination framework for smart building energy management," *Proc. - Int. Work. Database Expert Syst. Appl. DEXA*, vol. 200056, pp. 126–130, 2014.

[27] E. Lee and H. Bahn, "Electricity usage scheduling in smart building environments using smart devices," *Sci. World J.*, vol. 2013, 2013.

[28] X. Chen, T. Wei, and S. Hu, "Uncertainty-aware household appliance scheduling considering dynamic electricity pricing in smart home," *IEEE Trans. Smart Grid*, vol. 4, no. 2, pp. 932–941, 2013.

[29] C. O. Adika and L. Wang, "Autonomous appliance scheduling for household energy management," *IEEE Trans. Smart Grid*, vol. 5, no. 2, pp. 673–682, 2014.

[30] T. Facchinetti, E. Bini, and M. Bertogna, “Reducing the Peak Power through Real-Time Scheduling Techniques in Cyber-Physical Energy Systems,” *Proc. 1st Int. Work. Energy Aware Des. Anal. Cyber Phys. Syst.*, no. August 2016, pp. 1–8, 2010.

[31] D. Zhang and L. Papageorgiou, “Optimal scheduling of smart homes energy consumption with microgrid,” *ENERGY 2011, First …*, no. c, pp. 70–75, 2011.

[32] D. Zhang, S. Evangelisti, P. Lettieri, and L. G. Papageorgiou, “Economic and environmental scheduling of smart homes with microgrid: Der operation and electrical tasks,” *Energy Convers. Manag.*, vol. 110, pp. 113–124, 2016.

[33] A. Collazos, F. Maréchal, and C. Gähler, “Predictive optimal management method for the control of polygeneration systems,” *Comput. Chem. Eng.*, vol. 33, no. 10, pp. 1584–1592, 2009.

[34] J. M. Pearce, F. Starr, D. W. Auckland, and B. a. T. Al Zahawi, “Electricity generation in the home: evaluation of single-house domestic combined heat and power,” *IEE Proc. - Sci. Meas. Technol.*, vol. 143, no. 6, pp. 345–350, 1996.

[35] E. S. Barbieri, F. Melino, and M. Morini, “Influence of the thermal energy storage on the profitability of micro-CHP systems for residential building applications,” *Appl. Energy*, vol. 97, pp. 714–722, 2012.

[36] J. Silvente, G. M. Kopanos, E. N. Pistikopoulos, and A. Espuña, “A rolling horizon optimization framework for the simultaneous energy supply and demand planning in microgrids,” *Appl. Energy*, vol. 155, pp. 485–501, 2015.

[37] T. T. Kim and H. V. Poor, “Scheduling Power Consumption With Price Uncertainty,” *IEEE Trans. Smart Grids*, vol. 2, no. 3, pp. 519–527, 2011.

[38] M. W. Hydro, M. W. Solar, M. W. Solar, M. W. Higher, M. Lower, H. Middle, L. Higher, and M. Lower, “Feed-in Tariff (FIT) Generation & Export

Payment Rate Table," no. April 2016, pp. 32–33, 2017.

[39] Ofgem, "'Licence Lite': SLC 11.3 operating guidance," no. April, pp. 1–22, 2015.

[40] N. Standard, E. Equivalent, K. Washing, M. Fridge, P. Lcd, M. Desktop, P. C. Notes, and S. Watts, "TV and Video How these results are calculated :," no. Thomson 160, pp. 1–2.

[41] T. Lineham, H. Oxford, W. Brown, J. Griggs, G. Surveys, L. Harding, and A. E. A. Technology, "Client : Household Electricity Survey A study of domestic electrical product usage," 1908.

[42] E. M. Company, E. S. Trust, S. Energy, P. Division, and R. Affairs, "Measurement of Domestic Hot Water Consumption in Dwellings," 2008.

[43] B. R. E. Garston and X. X. Enquiries, "SAP 2012 The Government ' s Standard Assessment Procedure for Energy Rating of Dwellings," no. October 2013, 2014.

[44] "Average variable unit costs and fixed costs for electricity for UK regions 2015." Department of Energy & Climate Chage.

APPENDICES

Appendix A JUSTIFICATION OF THE SYSTEMS INTRODUCED IN THE MODEL STRUCTURE

In the following section, the justification for the selection of each of the systems in the model will be explained:

- **PV Panel:** Solar energy is an unlimited energy resource that will play a very important role in the future. Besides, the manufacturing price reduction is making this system more economical and it is also very simple to install and maintain. Therefore, it is ideal for household application. It also sets the base for further analysis of the model that might want to add uncertainty in the energy production. The economic support of the government to install PV panels also indicates an increase in the use of this DER, thus, will make this work useful for more people. The type of PV panel used in the model is monocrystalline, which reports very high efficiencies. Besides, this materials are very stable, making the panels last over 20 years with very small efficiency reductions.

- **CHP unit:** This systems are the best economic and environmental solution for providing heat to a household. The FIT of the UK government makes them very attractive economically and the high efficiencies that they obtain due to the combination of heat and electricity generation make them very environmental as well. There are several types of CHP units, which report several power ranges, heat to electricity ratio and other characteristics. However, for home application the external combustion Stirling engine is the best solution. This is because the heat to electricity ratio in the average UK houses is similar to the one of the

Stirling engine. Besides, the power that it creates is within the limits of the FIT, which is up to 2kW.

- **Battery Storage:** Battery storage is crucial if there is excess of electricity at any time. Furthermore, the battery is useful as a regulator. This can be used for further developments of the model that aim to purchase electricity in a planned manner. This means, that the purchased electricity will always be known in advance and the battery will supply energy to the charges that are not planned (lights, TV…). This energy usage control will easy a lot network operators and can report lower energy prices. Therefore, the battery sets the base for further analysis. The electrical storage also increases the usefulness of the CHP because it allows the storage of electricity that otherwise would be discarded doe to the mismatch on the heat and electricity demand.

- **Heat Storage:** The heat storage is a very important system that not only allows the reduction of the size of the heat generation unit, but it also increases the comfort in the house by always supplying instant heated water. This way, the heat generation system does not have to work every time there is heat demand in the house, so the unit works more constantly, witching on and off less frequently, thereby, increasing the life of the CHP unit.

- **Single House:** The model uses one single house in the first case study because the optimization process is much faster. Besides, it experiments with measures that can be applied to either a single or a group of houses

- **Group of houses:** The group of houses used in the second case study has the aim to prove the advantage of being together, rather than separate. The amount of houses used is big enough to see the difference between isolated and interconnected houses and small enough to simulate the model in a reasonable amount of time.

- **Boiler:** The boiler is an auxiliary system that has been installed to prevent the system from being unable to supply the heat demand. However, it has been proved that it is not necessary.

- **Loads:** The proposed loads for the model are fixed and variable on the time. This means that some loads can be schedulable (variable loads) and some others not (fixed loads). The loads that are fixed are the lights, a fridge and a desktop. In reality this loads cannot be scheduled because they are used randomly. For future works, the energy requirements of these loads can be supplied by the battery, avoiding buying energy to the grid randomly. This would decrease a lot the uncertainty in the grid.

Appendix B MATHEMATICAL MODEL

In this section, the description of the equations and expressions used in the model are shown. The equations contain unknown variables whereas the expressions parameters, so are independent of the solution. The data used in the problem is also explained:

B.1.1 Equations

The following expressions have been introduced in GAMS to obtain the solution of the problem.

Objective function

In order to asses more accurately the wear of the CHP, in the start-up and the shut-down, a cost has been introduced to each of them. Other cost such as fuel consumption, comfort penalties, heat dissipated penalty, and cost of purchased electricity have been included as well to account for the operating costs. The benefits reported to the model come from the FET applied to the PV and CHP and from the energy sold to the power grid. The improvements introduced in the objective function compared to the optimization equation by Gema et al. is the electricity scheduling, the battery storage and PV generation.

$$\sum_{i \in l} \sum_{t \in T} \theta_i^S S_{it} + \sum_{i \in l} \sum_{t \in T} \theta_i^F F_{it} + \sum_{i \in I} \sum_{t \in T} \xi_{it}^{GB} Q_{it}^{GB} \Delta t$$

$$+ \sum_{i \in l} \sum_{t \in T} \xi_{it} Q_{it}^{s} \Delta t (if\ \rho Opt < 2) + \sum_{i \in I} \sum_{t \in T} \xi_{it}\, \rho_i^{max} E_{it} \Delta t (if\ \rho Opt = 2)$$

$$+ \sum_{i \in I} \sum_{t \in T} ComfPen_{it} Tdev_{it} \Delta t + \sum_{i \in I} \sum_{t \in T} DispPen_i\, Qdisp_{it} \Delta t$$

$$\sum_{i \in I} \sum_{t \in T} \psi_t E_{it}^{buy} \Delta t - \sum_{i \in I} \sum_{t \in T} v_t E_{it}^{sales} \Delta t - \sum_{i \in I} \sum_{t \in T} \pi_{CHP} E_{it} \Delta t$$

$$+\sum_{i\in I}\sum_{t\in T}\pi_{PV}E\_PV_{it}\Delta t \qquad \textbf{(B-1)}$$

$$\forall\, i \in I, t \in T$$

CHP constrains

To prevent the CHP from starting-up or shutting-down more than once in the same time interval, some binary variables have been introduced. This constrain would simulate the realistic behaviour of a similar system, which would never start and stop very fast due to the shortened of the system lifetime as well as efficiency losses.

$$S_{it} - F_{it} = X_{it} - X_{it-1} \qquad \forall\, i \in I, t \in T \qquad \textbf{(B-2)}$$

$$S_{it} + F_{it} \ \le 1 \qquad \forall\, i \in I, t \in T \qquad \textbf{(B-3)}$$

$$X_{it} \ge \sum_{t'=\max\left[1,t-\delta_{it}^{on}+1\right]}^{t} S_{it'} \qquad \textbf{(B-4)}$$

$$\forall\, i \in I, t \in T, \delta_{it}^{on} > 1$$

$$1 - X_{it} \ge \sum_{t'=\max\left[1,t-\delta_{it}^{off}+1\right]}^{t} F_{it'} \qquad \textbf{(B-5)}$$

$$\forall\, i \in I, t \in T, \delta_{it}^{off} > 1$$

CHP Generator

As in any heat generation system, the heat production is not instant. Similarly, when the system shuts-down a residual heat is released. The generated heat will be stored in the HST and DHW tanks.

$$Q_{it} = Q_{it}^{s} - \sum_{k=1}^{\alpha_i^-} \lambda_{ik}^{-} S_{it-(k-1)} + \sum_{k=1}^{\alpha_i^+} \lambda_{ik}^{+} F_{it-(k-1)} \qquad \textbf{(B-6)}$$

$$\forall\, i \in I, \forall\, t \in T, \forall k \in K$$

There are two working modes in the model for the CHP which enables to generate heat and electricity with different ratios:

<u>Operating Mode 1</u>

The heat to electricity ratio that the CHP produces is fixed.

$$Q_{it}^{s} = \rho_i E_{it} \qquad \forall\, i \in I, t \in T \qquad \textbf{(B-7)}$$

<u>Operating Mode 2</u>

The second mode of operation has variable heat to electricity ratio. This is produced by discarding some heat in the generation.

$$\rho_i^{min} E_{it} \le Q_{it}^{s} \le \rho_i^{max} E_{it} \qquad \forall\, i \in I, t \in T \qquad \textbf{(B-8)}$$

The electricity produced by the CHP generator is limited by the following boundary:

$$\varepsilon_t^{min} \le E_{it} \le \varepsilon_t^{max} \qquad \forall\, i \in I, t \in T \qquad \textbf{(B-9)}$$

Backup Boiler

The back-up boiler has been model as an auxiliary system than will only work if the CHP system is unable to meet the heat demand. The limits for the heat generation of the Backup Boiler have been modelled as shown:

$$\gamma_i^{min} X_{it}^{GB} \leq Q_{it}^{GB} \leq \gamma_i^{max} X_{it}^{GB} \qquad \forall\, i \in I, t \in T \tag{B - 10}$$

Energy ramp between two time intervals

Similarly to other real system, the CHP units cannot start and stop immediately, so a ramp constrain is introduced.

$$E_{it} - E_{it-1} \leq ramp_i^{up} \ \ \forall\, i \in I, t \in T \tag{B - 11}$$

$$E_{it-1} - E_{it} \leq ramp_i^{down} \ \ \forall\, i \in I, t \in T \tag{B - 12}$$

Electrical balance

The electricity produced by the household through the CHP and the PV panels has to match the electrical demand from the house and the battery.

$$E_{it} + E\_PV_{it} = E_{DEM}{}_{it} + E\_B\_In_{it} \ \ \forall\, i \in I, t \in T \tag{B - 13}$$

Similarly, the required electricity from the house, the electricity that goes out of the battery and the purchased electricity are used to satisfy the demand of the house appliances, or is sold to the power grid.

$$E_{DEM}{}_{it} + E\_B\_Out_{it} + E_{it}^{buy} = Demand\_E_{it} + E_{it}^{sales} \tag{B - 14}$$

$$\forall\, i \in I, t \in T$$

Electrical storage balance

The electricity stored in the battery is the equal to the previous storage state plus the electricity charged minus the electricity discharged.

$$E\_B_{it} = E\_B_{it-1} + E\_B\_In_{it}\Delta t + E\_B\_Out_{it}\Delta t \tag{B - 15}$$

$$\forall\, i \in I, t \in T$$

Electrical storage limits

The battery is limited by its capacity:

$$\beta^{min} \leq E\_B_{it} \leq \beta^{max} \quad \forall\, i \in I, t \in T \tag{B - 16}$$

The battery is also limited by the amount of the power that can give:

$$E\_B\_In_{it} \leq C^{min} \quad \forall\, i \in I, t \in T \tag{B - 17}$$

$$E\_B\_Out_{it} \leq C^{max} \quad \forall\, i \in I, t \in T \tag{B - 18}$$

$$E\_B_{i0} = E\_B_{i48} \tag{B - 19}$$

$$\forall\, i \in I, t \in T$$

Time window

The time window is a binary variable which prevents a schedulable load to be activated more than once in one single day.

$$\sum_{t=t\_es}^{t\_ls} W_{ijt} = 1 \quad \forall\, i \in I, j \in J, t \in T \tag{B - 20}$$

Electrical demand

The electrical demand is the sum of the constant and the schedulable demands for each time interval.

$$Demand\_E_{it} = El\_Con_{it} + Cons_{it} * W_{ijt} \tag{B - 21}$$

$$\forall\, i \in I, j \in J, t \in T$$

Heat balance

The heat generation systems (CHP and Backup boiler) satisfy the demanding heat by the DHW and the heating of the house:

$$Q_{it} + Q_{it}^{GB} = Q_{it}^{DHW\_IN} + Q_{it}^{HS\_IN} \quad \forall\, i \in I, t \in T \tag{B - 22}$$

DHW tank

The total heat stored in the tanks is the sum of the heat stored in the previous time slot plus the heat supplied by the heat generators minus the domestic heat water demand at that present time.

$$B_{it}^{DHW} = B_{it-1}^{DHW} + Q_{it}^{DHW\_IN} - Q_{it}^{DHW\_loss} - \zeta_{it}^{DHW} \qquad \textbf{(B - 23)}$$

$$\forall\, i \in I, t \in T$$

In the following formulas, the heat losses of the tank are calculated, for which the temperature of the tank must be known. Besides, the storage capacity is limited which has been calculated taking into account the maximum and minimum temperature of the tanks: 80ºC and 60ºC respectively:

$$Q_{it}^{DHW\_loss} = U_i^{DHW} A_i^{DHW} \left(T_{it}^{DHW} - T_{it}^{DHW\_room}\right) \quad \forall\, i \in I, t \in T \qquad \textbf{(B - 24)}$$

$$T_{it}^{DHW} = \frac{B_{it}^{DHW}}{c_p^{water} \delta_{water} v_i^{DHW}} + T_i^{DHWmin} \qquad \textbf{(B - 25)}$$

$$\forall\, i \in I, t \in T$$

$$\beta_{it}^{DHWmin} \leq B_{it}^{DHW} \leq \beta_{it}^{DHWmax} \qquad \textbf{(B - 26)}$$

$$\forall\, i \in I, t \in T$$

$$B_{i0}^{DHW} = B_{i48}^{DHW} \qquad \textbf{(B - 27)}$$

$$\forall\, i \in I, t \in T$$

Heat storage (HS) tank

The characteristics of the heat storage tank are calculated in the same way.

$$B_{it}^{HS} = B_{it-1}^{HS} - Q_{it}^{HS\_loss} + Q_{it}^{HS\_IN} - Q_{it}^{HS\_OUT} \qquad \textbf{(B - 28)}$$

$$\forall\, i \in I, t \in T$$

$$\beta_{it}^{HSmin} \leq B_{it}^{HS} \leq \beta_{it}^{HSmax} \qquad \forall\, i \in I, t \in T \tag{B - 29}$$

$$Q_{it}^{HS\_loss} = U_i^{HS} A_i^{HS} \left(T_{it}^{HS} - T_{it}^{HS\_room}\right) \qquad \forall\, i \in I, t \in T \tag{B - 30}$$

$$B_{it}^{HS} = c_p^{water} \delta_{water} v_i^{HS} \left(T_{it}^{HS} - T_i^{HSmin}\right) \tag{B - 31}$$

$$\forall\, i \in I, t \in T$$

$$B_{i0}^{HS} = B_{i48}^{HS} \tag{B - 32}$$

Heat model of the house

The house heat balance is composed by heat losses and gains of the house, the heat requirements to set the house temperature at the desired level and the heat supplied by the heat storage tank. There is one more term in the equation, called Qdisp, which simulates the action of an open window. This will only happen when the model cannot release heat in any other way, because its cost is very high.

$$Q_{it}^{HS\_OUT} + Q_{it}^{gains} = Q_{it}^{HOUSE\_loss} + Q_{it}^{\Delta T} + Qdisp_{it} \tag{B - 33}$$

$$\forall\, i \in I, t \in T$$

Among the heat losses, the model has introduced the ones due to the ventilation rates, calculated according to (SAP 2012) and the heat lost through the surfaces of the house. The surface includes walls, glazing, roof and floor.

$$Q_{it}^{HOUSE\_loss} = Q_{it}^{fabric\_l} + Q_{it}^{vent\_l}$$

$$= \left(\sum_n U_i^n A_i^n + c_p^{air} \delta_{air} N_i v_i^{house}\right)\left(T_{it}^{house} - T_t^{target}\right) \tag{A - 34}$$

$$\forall\, i \in I, t \in T, n \in N$$

Similarly, heat gains are composed by the solar gains and the gains due to the living people of the house and the appliances.

$$Q_{it}^{gains} = Q_{it}^{solar\_g} + Q_{it}^{internal\_g} + Q_{it}^{fabric\_g} \qquad \textbf{(A - 35)}$$

$$\forall\, i \in I, t \in T$$

The heat requirement is the heat required to warm up or cool down the house in order to maintain the comfort temperature at any time.

$$Q_{DT_{it}} = c_p^{air} \delta_{air} v_i^{house} * \left(T_{it}^{house} - T_{it-1}^{house}\right) \qquad \textbf{(A - 36)}$$

$$\forall\, i \in I, t \in T$$

Comfort penalty

The house must be able to reach the target temperature, which is what the user wants. However, it might happen that the economic benefit to leave the house temperature a bit away from the target is very big. Therefore, an economic value must be introduced to the objective function that will account for this temperature offset.

$$T_t^{target} - T_i^{comf} \leq T_{it}^{house} \leq T_t^{target} + T_i^{comf} \qquad \textbf{(A - 37)}$$

$$\forall\, i \in I, t \in T$$

$$Tdev_{it} \geq \left|T_{it}^{house} - T_t^{target}\right| \quad \forall\, i \in I, t \in T \qquad \textbf{(A - 38)}$$

Electricity share balance

When the four houses are put together and they share electricity, other formulas must be added to interconnect the buildings.

In the following formulas, all the electricity that is sent to the microgrid by each of the houses must be consumed by other dwellings. No energy can be stored in the microgird.

$$T\_MicroGrid\_In_t - T\_Microgrid\_Out_t = 0 \quad t \in T \qquad \textbf{(A - 39)}$$

$$T\_MicroGrid\_In_t = \sum_i MicroGrid\_In_{it}$$ **(A - 40)**

$$\forall\ i \in I, t \in T$$

$$T\_Microgrid\_Out_t = \sum_i MicroGrid\_Out_{it}$$ **(A - 41)**

$$\forall\ i \in I, t \in T$$

B.1.2 Nomenclature

The terms used in the equations and expressions are explained in this section:

Sets

$i \in I$	Households
$t \in T$	Time intervals
$n \in N$	Surfaces of the house (floor, roof, glazing and wall)
$j \in J$	Flexible power consumption task
$k \in K$	Start-up or shutdown periods

Parameters

θ_i^S	Start-up cost of CHP
θ_i^F	Shut down cost of CHP
ρ_i	Heat to electricity ratio (option 1)
ρ_i^{max}	Maximum heat to electricity ratio (option 2)
ρ_i^{min}	Minimum heat to electricity ratio (option 2)
$DispPen_i$	Dispend penalty
α_i^-	Number of start-up periods of CHP
α_i^+	Number of shutdown periods of CHP
U_i^n	Heat transfer coefficient of surface n of the i house
A_i^n	Area of the n surface of the i house
γ_i^{min}	Back-up boiler minimum heat generation
γ_i^{max}	Back-up boiler maximum heat generation
$ramp_i^{up}$	Ramp up limitation for CHP electricity generation
$ramp_i^{down}$	Ramp down limitation for CHP electricity generation
U_i^{DHW}	DHW overall heat transfer coefficient
U_i^{HS}	HS overall heat transfer coefficient
A_i^{DHW}	DHW surface area
A_i^{HS}	HS surface area
v_i^{DHW}	DHW tank volume
v_i^{HS}	HS tank volume
v_i^{house}	House volume
T_i^{DHWmin}	Minimum temperature allowed inside DHW tank
T_i^{HSmin}	Minimum temperature allowed inside HST

T_i^{comf}	Temperature tolerance range
T_i^{HSmax}	Maximum temperature of the water that the HS tank can have
T_i^{DHWmax}	Maximum temperature of the water that the DHW tank can have
N_i	Number of air changes for ventilation
ψ_t	Electricity purchase price
v_t	Tariff for electricity exported to micro grid
π_{CHP}	Tariff for electricity production by CHP
π_{PV}	Tariff for electricity production by PV
ε_t^{min}	Minimum electricity production by CHP
ε_t^{max}	Maximum electricity production by CHP
T_t^{target}	Target temperature of the house
ξ_{it}^{GB}	Back-up boiler fuel cost
ξ_{it}	CHP fuel cost
$ComfPen_{it}$	Comfort penalty

δ_{it}^{on}	CHP minimum running period
E_PV_{it}	PV electricity production
δ_{it}^{off}	CHP minimum shutdown period
ζ_{it}^{DHW}	Domestic hot water demand
β_i^{DHWmin}	Minimum DHW heat capacity
β_i^{DHWmax}	Maximum DHW heat capacity
β_i^{HSmin}	Minimum HS heat capacity
β_i^{HSmax}	Maximum HS heat capacity
λ_{ik}^{-}	CHP start-up heat generation loss
λ_{ik}^{+}	CHP shutdown heat generation surplus
ρOpt	2 choices for heat to electricity ratio pattern
c_p^{water}	Water specific heat capacity
c_p^{air}	Air specific heat capacity
δ_{water}	Water density
δ_{air}	Air density

Positive variables

Q_{it}^{GB}	Back-up boiler heat production
Q_{it}^{s}	CHP real heat production
Q_{it}	CHP heat production including loss or extra generation

$Q_{it}^{DHW\_IN}$	Heat supplied to DHW tank
$Q_{it}^{HS\_IN}$	Heat supplied to HS tank
$Q_{it}^{HS\_OUT}$	Heat supplied by HS tank
$Q_{it}^{DHW\_loss}$	DHW tank heat losses
$Q_{it}^{HS\_loss}$	HS tank heat losses
Q_{it}^{gains}	House heat gains excluding from heat generators
$Q_{it}^{HOUSE\_loss}$	House heat loss
$Q_{it}^{\Delta T}$	Heat required to maintain the temperature inside the house
$Qdisp_{it}$	Dispend heat
$Q_{it}^{solar\_g}$	Heat gains from solar energy
$Q_{it}^{internal\_g}$	Heat gains from internal electrical use
$Q_{it}^{fabric\_g}$	Heat gains from fabric
$Q_{it}^{fabric\_l}$	Heat loss from fabric
$Q_{it}^{vent\_l}$	Heat loss from ventilation

E_{it}	CHP electricity production
$E_{DEM_{it}}$	Electricity required that it is not stored in the battery
$E\_B\_Out_{it}$	Electricity discharged from the battery
$E\_B\_In_{it}$	Electricity charged to the battery
$Demand\_E_{it}$	Electric demand of the house appliances
El_Con_{it}	Electric demand of the house appliances which are not schedulable
$Cons_{it}$	Electric demand of the house appliances which are schedulable
E_B_{it}	Battery energy level
Δt	Time interval duration in hours
β^{min}	Minimum stored energy of the battery
β^{max}	Maximum stored energy of the battery
C^{min}	Minimum discharge rate of the battery
C^{max}	Maximum discharge rate of the battery
B_{it}^{DHW}	DHW tank heat storage level
B_{it}^{HS}	HS tank heat storage level
$Tdev_{it}$	Absolute difference between room

	temperature and target temperature
T_{it}^{DHW}	DHW water temperature
T_{it}^{HS}	HS water temperature
$T_{it}^{DHW\_room}$	Room temperature where DHW sits
$T_{it}^{HS\_room}$	Room temperature where HS sits
T_{it}^{house}	House temperature
E_{it}^{buy}	Electricity bought from the house
E_{it}^{sales}	Electricity exported to the power grid from each house
$MicroGrid\_Out_{it}$	Electricity exported to the micro grid from each house
$MicroGrid\_In_{it}$	Electricity imported from the micro grid to each house
$T\_MicroGrid\_Out_{t}$	Total Electricity exported to the micro grid from each of the houses
$T\_MicroGrid\_In_{t}$	Total Electricity imported from the micro grid to each of the houses

Binary variables

S_{it}	1 if CHP starts up at time t
F_{it}	1 if CHP shuts down at time t
W_{ijt}	1 if task j is being done at time t
X_{it}	1 if CHP is running at time t
X_{it}^{GB}	1 is back-up boiler is running at time t

B.1.3 Expressions

In this section, the mathematical formulations composed by parameters are shown:

PV

The photovoltaic generation depends not only on the irradiance of the sun, the area and the type of PV panel, but also in the efficiency of the systems that transforms the current from DC to AC. In the following case, the efficiency of the solar panel, the efficiency due to the outside temperatures, the efficiency of the inverter and cables and the efficiency of the dirt and the reflection is taken into account as in the following expression:

$$E_{PV_{it}} = rad_t * a * \eta_{panel} * \eta_{system} * \eta_{reflection} * \eta_{temperatures}$$

Heat storage tank capacity

The heat stored in the tanks is not given directly by the manufacturer as it depends on the temperature. Maximum and minimum temperatures are given by the system where the storage tanks are installed. But always within the limits of the manufacturer. With these temperatures, the capacity of the storage tanks is determined as follows:

$$\beta_i^{HSmax} = c_p^{water} \delta_{water} v_i (T^{HSmax} - T^{HSmin})$$

$$\beta_i^{DHWmax} = c_p^{water} \delta_{water} v_i (T^{DHWmax} - T^{DHWmin})$$

B.1.4 Input data

The values used to solved the problem are explained here:

Ventilation rate

It is the rate at which the air volume of the house is completely renovated. This value depends of the typology of the house. The whole procedure for the calculation of this parameter is explained in the section two of SAP 2012.

Internal gains

The internal gains depend on the average surface of the floor of the house, which is used as an estimator for the internal heat generated by the appliances and the people living inside. The calculations have been proceeded as in the section 5 of SAP 2012.

Appliances

In order to build a realistic model, the appliances that have been considered have been researched in several papers [26], [32] and reports [40], [41]. Finally, 8 schedulable appliances and a series of non-schedulable ones have been selected, which are the most common in the previously mentioned references. The following table resumes the main characteristics of the schedulable appliances:

	Earliest start time (h)	Latest finishing time (h)	Duration (h)	Power consumption (kW)
Dishwasher	8:00	16:30	2	1.8-0.25
Washing Machine	8:00	12:30	2	2.1-0.25
Spin Dryer	13:00	17:30	1	2.5

Cooker Hob	8:00	9:30	0.5	3
Cooker Oven	17:30	19:00	0.5	5
Laptop	18:00	07:30	1.5	0.1
Vacuum Cleaner	9:00	17:30	0.5	1.2
Hybrid car	18:00	07:30	3	3.5

Table 1: Characteristics of schedulable appliances

Target temperatures of the house and outside temperatures

After researching usual house target temperatures in several reports, [42], [43], [41] the temperatures selected for the inside of house are between 20ºC and 18ºC. The temperature will vary between those ranges depending on the occupancy of the house. It has been estimated that the house will be occupied in the following time ranges: 6:30-8:30, 14:00-15:00 and 18:00-00:00. Therefore, at that time the house temperature will be at 20ºC. The rest of the time 18ºC to save energy.

Another parameter has been introduced, “Comfort temperature Allowance” which allows the target temperature to deviate. This comfort temperature allowance has been assigned a value of 0.5ºC.
Outside temperatures also haven been researched through historical average values. The location determined for the selected temperatures is Milton Keynes and the month for which the temperatures have been selected is January.

This means that the model works for average values and not for extreme temperatures. However, the models heat demand is oversized, so there is no risk to obtain low temperatures inside the house.

DHW consumption

The DHW needs has been taken from the following survey, [42], however, other sources [41], [43] have been checked to verify the correctness of this data.

HST and DHW tanks

The maximum and minimum temperatures for both storage units have been determined by reviewing several documents: From surveys [41], [42], to procedures [43] and it has been determined that the most usual temperatures in which the storage units work are between 60ºC and 80ºC.

Besides, the same sources give information about the sizes of the storage units, which have been determined as $0.45m^3$ for the HST and $0.12m^3$ for the DHW tank.

Based on that volume, the dimensions of the tanks have been designed by assuming cylindrical tanks and with accordance to the manufacturers. Therefore, the HST has a height of 1.9 meters and a radius of 0.275m. On the other side, the DHW tank has a height of 1.2 m and a radius of 0.35m.

The initial state of the tanks is at the middle, since this is the most probable capacity at which the tanks will be: 5.234kWh for the HST and 1.395kWh for the DHW tank.

Another important characteristic for the heat storage tanks are the heat transfer coefficient. Many houses in UK do not have a proper DHW tank insulator, therefore, there is the option to install a jacket that will insulate better the tank. However, for this model, the tanks selected are brand new, so they usually have a good insulator. The values selected for the model are $0.01kW/m^2K$ for the HST and $0.0033kW/m^2K$ for the DHW tank.

Electric battery characteristics

The capacity used for the battery is the similar to the home application batteries that Tesla or BMW produce: 4.8kWh. There are bigger battery capacities in the market, but the aim of them differs from the objective of this built model. In the current case, the battery regulates the electricity produced by the CHP and the PV panel to avoid selling electricity.

The efficiency of the battery is 95% because there are thermal losses in the transformation from AC to DC and vice versa. Besides, this efficiency loss makes the model consider whether it should charge the battery or not, because there is a cost. The cost of the lost electricity.

The charge and discharge maximum power is defined by c, the discharge rate, which is equivalent to the capacity of the battery. For the current case, c=0.5, the battery is able to discharge half of the capacity in hone hour. Batteries are manufactured with very variable discharge capacities, however, long life expectancy batteries have low discharge values. Thus, the selected value aims to have a long life battery.

CHP characteristics

The start-up of the CHP has an extra wear, which will affect the life expectancy of the system. Assigning a cost to this wear is very complicated as it depends on the number of times that the unit switches on. However, this cost will also affect to the number of times that the CHP unit starts. Therefore, the value of 0.01 pounds has been assigned after running the problem several times with different values and deciding that the present value generates the closest CHP operation, to the reality.

As any real heat generation unit cannot produce heat instantly, heat losses have been introduced in the model. The value of those, as in the previous case, has been iterative, starting with reasonable values obtained from the literature. As a result, 0.15kWh heat is generated once the system is not working and 0.3kWh heat is lost when starting the unit.

The limits for the heat and electricity production as well as the heat to electricity ratio have been take from the Baxi Ecogen model. It is able to vary the heat to electricity ratio from 6 to 3.2, thus, can produce from 6kW to 3.2kW of heat while generating up to 1kW of electricity.

The government benefits the generation of electricity from this unit with 0.1345 pounds per kWh energy. This benefit is supposed to stay this way until 2019 [38].

PV formula data

The electricity generation from the radiation of the sun goes through various processes where the energy is lost. Those values, after researching several sources has been determined as: 19% the efficiency of the PV monocrystalline panel, 92% the thermal efficiency, 86% the system efficiency, which include the AC & DC transformation, and 97% the reflection efficiency.

The radiation values used in the model have been obtained for the location of Milton Keynes, for the day January 19th 2016.

The FIT that benefits the production of the PV is of 4.25 pence per kWh. This value will change in the following months reducing until having a benefit of 3.55 pence until 2019.

Cost of electricity

The cost of the electricity has been reviewed in the daily whole-sale market, however, those prices do not match the prices that the distributors offer to their customers. Therefore, [18] document has been used for the price. For the variable price however, [44] has been used.

Cost of gas

The price of the gas has been obtained from averaging several prices from the main gas suppliers offered to customers. This price: 0.0225 pounds per kWh has been then modified to account for the efficiency differences of the CHP and the boiler. This makes the cost of the CHP to be cheaper, with a cost of 0.025 pounds per kWh against the 0.032143 pounds per kWh of the boiler. To obtain this values, the gas price has been divided by the efficiency of each system. Thus, 0.9 for the CHP and 0.7 for the boiler.

Heat transfer coefficient of the house

Thermal conductivity of the house has been asses by evaluating several values of different documents. The average heat transfer coefficient in the UK is very high due to the amount of old houses that exist. However, that value, 3.15W/m^{2}K [18] is very high and it is not realistic for the average houses that are built today. Therefore, the values according to [43] have been taken for the model. Thereby, the roof has 0.13 kW/m^{2}K, the glazing 1.4 kW/m^{2}K, the walls 0.18 kW/m^{2}K and the floor 0.13 kW/m^{2}K.

Dimensions of the house

Two types of houses have been selected for the present project, which are in accordance with the usual dimensions of the houses of UK.

House Type A:

Floor: 76.2 m^2

Roof: 76.2 m^2

Wall: 167.6 m^2

Glazing: 16.76 m^2

House Type B:

Floor: 76.9 m^2

Roof: 76.9 m^2

Wall: 168.37 m^2

Glazing: 16.84 m^2