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Aspects and Some Results on Passivity and Positivity of Dynamic Systems

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Abstract. This paper is devoted to discuss certain aspects of passivity results in dynamic systems and the characterization of the regenerative systems counterparts. In particular, the various concepts of passivity as standard passivity, strict input passivity, strict output passivity and very strict passivity (i.e. joint strict input and output passivity) are given and related to the existence of a storage function and a dissipation function. Later on, the obtained results are related to external positivity of systems and positivity or strict positivity of the transfer matrices and transfer functions in the time-invariant case. On the other hand, it is discussed how to achieve or how eventually to increase the passivity effects via linear feedback by the synthesis of the appropriate feed-forward or feedback controllers or, simply, by adding a positive parallel direct input-output matrix interconnection gain.

1. Introduction

This paper is devoted to discuss certain aspects of passivity results in dynamic systems and the characterization of the regenerative versus passive systems counterparts. In particular, the various concepts of passivity as standard passivity, strict input passivity, strict output passivity and very strict passivity (i.e. joint strict input and output passivity) are given and related to the existence of a storage function and a dissipation function. Basic previous background concepts on passivity are given in [1-4] , [10-12] and some related references therein. Later on, the obtained results are related to external positivity of systems and positivity or strict positivity of the transfer matrices and transfer functions in the time- invariant case. On the other hand, it is discussed and formalized how to achieve in case of passivity failing or how eventually to increase the passivity effects via linear feedback by the synthesis of the appropriate feed-forward or feedback controllers or, simply, by adding a positive parallel direct input-output matrix interconnection gain having a minimum positive lower-bounding threshold gain which is also an useful idea for asymptotic hyperstability of parallel disposals of systems, [10] . For the performed analysis, the concept of relative passivity index which is applicable for both passive and non-passive systems is addressed and modified to a lower number by the use of appropriate feedback or feed-forward compensators. Finally, the concept of passivity is discussed for switched systems which can have both passive and non-passive configurations which become active governed by switching functions. The passivity property is guaranteed by the switching law under a minimum residence time at passive active configurations provided that t he first active configuration of the switched disposal is active and that there are no two consecutive active non-passive configurations in operation.

Notation

- $\boldsymbol{R}_{0+} = \boldsymbol{R}_{+} \cup \{0\}$, where $\boldsymbol{R}_{+} = \{r \in \boldsymbol{R} : r > 0\}$, $\bar{p} = \{1,2,....,p\}$,
- $D \succ 0$ denotes that the real matrix D is positive definite while $D \succeq 0$ denotes that it is positive semidefinite,
- $\lambda_{min}(.)$ and $\lambda_{max}(.)$ denote, respectively, the minimum and maximum eigenvalues of the real symmetric $(.)$-matrix,
- $\hat{G} \in \{PR\}$ denotes that the transfer matrix $\hat{G}(s)$ of a linear time-invariant system is positive real, i.e. $Re\,\hat{G}(s) \ge 0$ for all $Re\ s > 0$ and $\hat{G} \in \{SPR\}$ denotes that it is strictly positive real, i.e. $Re\,\hat{G}(s) > 0$ for all $Re\ s \ge 0$,
- A dynamic system is positive (respectively, externally positive) if all the state components (respectively, if all the output components) are non-negative for all time $t \ge 0$ for any given non-negative initial conditions and non-negative input ,
- $\boldsymbol{i} = \sqrt{-1}$ is the complex unity,
- I_m is the m-th identity matrix,
- the superscript T stands for matrix transposition,
- $\boldsymbol{H}_\infty$ is the Hardy space of all complex-valued functions $F(s)$ of a complex variable s which are analytic and bounded in the open right half-plane $Re\,s > 0$ of norm $\| F \|_\infty = sup\{|F(s)| : Re\,s > 0\} = sup\{|F(\boldsymbol{i}\omega)| : \omega \in \boldsymbol{R}\}$(by the maximum modulus theorem) and $\boldsymbol{RH}_\infty$ is the sub set of real-rational functions of $\boldsymbol{H}_\infty$.

2. Passivity and positivity

Consider a dynamic system $G : \mathsf{H}_e \to \mathsf{H}_e$ with state $x \in \boldsymbol{R}^n$, input $u \in \boldsymbol{R}^m$ and output $y \in \boldsymbol{R}^m$, where H_e is the extended space of the Hilbert space H endowed with the inner product $\langle .,. \rangle$ from $\mathsf{H}_e \times \mathsf{H}_e$ to $\boldsymbol{R}$ consisting of the truncated functions $u_t(\tau) = u(\tau)$ for $\tau \in [0, t]$ and $u_t(\tau) = 0$; $\forall t, \tau(> t) \in \boldsymbol{R}_{0+}$ and $u : \boldsymbol{R}_{0+} \to \boldsymbol{R}^m$. If $u \in \mathsf{H}_e$ then

$$\| u_t \|_2^2 = \langle u, u \rangle_t = \langle u_t, u_t \rangle = \int_0^t u^T(\tau) u(\tau) d\tau = \int_0^\infty u_t^T(\tau) u_t(\tau) d\tau \ ; \ \forall t \ge 0 .$$

Definitions [2]. The above dynamic system is:. L_2*-stable* if $u \in L_2^m$ implies $Gu \in L_2^m$.

2. *Nonexpansive* if $\exists \lambda$ and $\exists \gamma > 0$ s. t. for all $u \in \mathsf{H}_e$

$$\int_0^t (Gu)^T(\tau) u(\tau) d\tau \le \lambda + \gamma^2 \int_0^t u^T(\tau) u(\tau) d\tau \ ; \ \forall t \ge 0 .$$

3. *Passive* if $\exists \varepsilon \ge 0$ such that $\int_0^t y^T(\tau) u(\tau) d\tau \ge -\varepsilon$; $\forall t \ge 0$.

4. *Strictly- input passive* if $\exists \varepsilon \ge 0$ and $\exists \varepsilon_u > 0$ s. t.

$$\int_0^t y^T(\tau) u(\tau) d\tau \ge -\varepsilon + \varepsilon_u \int_0^t y^T(\tau) u(\tau) d\tau \ ; \ \forall t \ge 0 .$$

5. *Strictly- output passive* if $\exists \beta \ge 0$ and $\exists \varepsilon_y > 0$ s. t.

$$\int_0^t y^T(\tau) u(\tau) d\tau \ge -\varepsilon + \varepsilon_y \int_0^t y^T(\tau) y(\tau) d\tau \ ; \ \forall t \ge 0 .$$

6. *Strictly input/output passive* (or *very strictly passive*) if $\exists \beta \ge 0$, $\exists \varepsilon_u > 0$ and $\exists \varepsilon_y > 0$ s. t.

$$\int_0^t y^T(\tau) u(\tau) d\tau \ge -\varepsilon + \varepsilon_u \int_0^t y^T(\tau) u(\tau) d\tau + \varepsilon_y \int_0^t y^T(\tau) y(\tau) d\tau \ ; \ \forall t \ge 0 .$$

The constants ε, ε_u and ε_y are respectively referred to as the passivity, input passivity and output passivity constants.

The following two results are relevant to mutually relate the time and frequency domains descriptions concerned with the passivity and positivity properties:

Theorem 1: Consider a linear time-invariant SISO (i.e. m=1) system whose transfer function $\hat{G} \in \{PR\}$. Then, the following properties hold:

(i) $\int_0^t y(\tau)u(\tau)d\tau \geq 0$ and $y(t)u(t) \geq 0$; $\forall t \geq 0$ and, furthermore, if $u \in L_2$ then $y \in L_2$. Then, the system is passive.

(ii) Assume, in addition, that $\hat{G} \in \{SPR\}$. Then $\gamma \geq \int_0^t y(\tau)u(\tau)d\tau \geq \varepsilon_u \int_0^t |u(\tau)|^2 d\tau - \varepsilon$ for any $t \in (0, \infty]$ and some $\gamma, \varepsilon \in \boldsymbol{R}_{0+}$.

(iii) If, furthermore, the system is externally positive then $\gamma \geq \int_0^t y(\tau)u(\tau)d\tau > 0$; $\forall t \geq 0$ for any given non-negative initial conditions and non-negative input.

(iv) Define $R_{\hat{G}} = \left\| \frac{1-\hat{G}(\boldsymbol{i}\omega)}{1+\hat{G}(\boldsymbol{i}\omega)} \right\|_\infty = \sup_{\omega \in \boldsymbol{R}_{0+}} \left| \frac{1-\hat{G}(\boldsymbol{i}\omega)}{1+\hat{G}(\boldsymbol{i}\omega)} \right|$ as the relative passivity index of the transfer function $\hat{G}(s) = \frac{\hat{N}(s)}{\hat{D}(s)} \in \boldsymbol{RH}_\infty$ ($\hat{N}(s)$ and $\hat{D}(s)$ being the numerator and denominator polynomials of $\hat{G}(s)$). Then, the constraint $a_G \leq R_{\hat{G}} = \left\| \frac{\hat{D}_G(\boldsymbol{i}\omega) - \hat{N}_G(\boldsymbol{i}\omega)}{\hat{D}_G(\boldsymbol{i}\omega) + \hat{N}_G(\boldsymbol{i}\omega)} \right\|_\infty \leq b_G$ is guaranteed for some $a_G, b_G (\geq a_G) \in \boldsymbol{R}_{0+}$ if

$$\frac{1-a_G^2}{2\left(1+a_G^2\right)}\left(Re^2 \ \hat{D}_G(\boldsymbol{i}\omega) + Re^2 \ \hat{N}_G(\boldsymbol{i}\omega)\right) \geq \frac{1-b_G^2}{2\left(1+b_G^2\right)}\left(Re^2 \ \hat{D}_G(\boldsymbol{i}\omega) + Re^2 \ \hat{N}_G(\boldsymbol{i}\omega)\right) ; \ \forall \omega \in \boldsymbol{R}_{0+}$$

If $b_G \leq 1$ (respectively, $b_G < 1$) then $\hat{G} \in \{PR\}$ (respectively, $\hat{G} \in \{SPR\}$).

Note that positivity is a very important property in some dynamic systems related to biological or epidemic-type models . See, for instance, [5-9] and references therein The generalization of Theorem 1 to the multi input multi-output (MIMO) case (i.e. $m > 1$) is direct by replacing the instantaneous power $y(t)u(t)$ by the scalar product $y^T(t)u(t)$ in the corresponding expressions. In particular, the subsequent two results discuss how the basic passivity property can become a stronger property as, for instance, strict-input passivity or very strict passivity, by incorporating to the input-output operator a suitable parallel static input-output interconnection structure.

Theorem 2. Consider a class $[G]_{\rho D}$ of dynamic systems $G(\rho, D): \mathsf{H}_e \to \mathsf{H}_e$, defined as $G(\rho, D) = G_0 + \rho D$ for given $\rho \in \boldsymbol{R}$, $D\left(\in \boldsymbol{R}^{m \times m}\right) \succeq 0$ and $G_0 : \mathsf{H}_e \to \mathsf{H}_e$, such that $G(\alpha, D) \in [G]_{\rho D}$ for any $\alpha \in [0, \rho]$. The following properties hold:

(i) Assume that $G(\rho, D)$ is very strictly passive, $D \succ 0$ and $\varepsilon_{u\rho} > \frac{\rho \lambda_{max}\left(D + D^T\right)}{2}$, where $\varepsilon_{u\rho}$ is the input passivity constant for $G(\rho, D)$.Then, $G(\alpha, D)$ is very strictly passive for all $\alpha \in [0, \rho]$. If

$\varepsilon_{u\rho} = \frac{\rho\lambda_{max}\left(D + D^T\right)}{2}$ then $G(\alpha, D)$ is very strictly passive for all $\alpha \in (0, \rho]$ while G_0 is strictly-output passive.

(ii) If G_0 is passive (respectively, strictly output passive) then $G(\alpha, D)$ is strictly-input passive (respectively, very strictly passive) if $D \succ 0$ for any $\alpha \in \boldsymbol{R}_+$.

Assume that G_0 is passive and non-expansive. Then:
(iii) $G_\rho = G_0 + \rho D$ is L_2 -stable and strictly-input passive if $\rho \in \boldsymbol{R}_+$,

(iv) G_0 is L_2 -stable if $\rho > -\frac{2\left(\lambda_0 + \gamma_0^2\right)}{\lambda_{max}\left(D + D^T\right) - \lambda_{min}\left(D + D^T\right)}$ if $D(\neq I_m, 0) \succeq 0$,

(v) G_0 is L_2 -stable if $D = I_m$ for any given $\rho \in \boldsymbol{R}$.

It turns out through simple mathematical derivations that Theorem 2 still holds with the replacement $D \to G_1$, where $G_1 : \mathsf{H}_e \to \mathsf{H}_e$ is passive with associated constant $\varepsilon_1 \leq 0$ for the properties to be extended from the case that $D \succeq 0$ and strictly-input passive for those extended from the case when $D \succ 0$.

3. Control compensators

It is now discussed how the passivity properties can be improved via feedback with respect to an external reference input signal. Consider the following linear time-invariant SISO cases:

- The controlled plant transfer function $\hat{G}(s)$, whose relative passivity index [Theorem 1 (iv)] is $R_{\hat{G}} = \sup_{\omega \in \boldsymbol{R}_{0+}} \left| \frac{1 - \hat{G}(\boldsymbol{i}\omega)}{1 + \hat{G}(\boldsymbol{i}\omega)} \right|$, is controlled by a feedback controller of transfer function $\hat{K}_1(s)$ so that $\hat{M}_1(s) = \frac{1 - \hat{T}_1(s)}{1 + \hat{T}_1(s)} = \frac{1 + \hat{G}(s)\left(\hat{K}_1(s) - 1\right)}{1 + \hat{G}(s)\left(\hat{K}_1(s) + 1\right)}$, where $\hat{T}_1(s) = \frac{\hat{G}(s)}{1 + \hat{G}(s)\hat{K}_1(s)}$ is the resulting closed-loop transfer function. The closed-loop relative passivity index is $R_{\hat{T}_1} = \sup_{\omega \in \boldsymbol{R}_{0+}} \left| \frac{1 - \hat{T}_1(\boldsymbol{i}\omega)}{1 + \hat{T}_1(\boldsymbol{i}\omega)} \right| = \sup_{\omega \in \boldsymbol{R}_{0+}} \left| \hat{M}_1(\boldsymbol{i}\omega) \right|$. For any given $\hat{T}_1(s)$ and associated $\hat{M}_1(s)$, the controller transfer function is:

$$\hat{K}_1(s) = \frac{\left(2\hat{G}(s) - 1\right)\hat{T}_1(s) + 1}{2\hat{G}(s)\hat{T}_1(s)} = \frac{\hat{G}(s)\left(1 + \hat{M}_1(s)\right) + \hat{M}_1(s)}{\hat{G}(s)\left(1 - \hat{M}_1(s)\right)} .$$

- The controlled plant transfer function $\hat{G}(s)$ is controlled by a feed-forward controller of transfer function $\hat{K}_2(s)$ so that $\hat{M}_2(s) = \frac{1 - \hat{T}_2(s)}{1 + \hat{T}_2(s)} = \frac{1}{1 + 2\hat{G}(s)\hat{K}_2(s)}$,where $\hat{T}_2(s) = \frac{\hat{G}(s)\hat{K}_2(s)}{1 + \hat{G}(s)\hat{K}_2(s)}$ is the resulting closed-loop transfer function. The closed-loop relative passivity index is $R_{\hat{T}_2} = \sup_{\omega \in \boldsymbol{R}_{0+}} \left| \frac{1 - \hat{T}_2(\boldsymbol{i}\omega)}{1 + \hat{T}_2(\boldsymbol{i}\omega)} \right| = \sup_{\omega \in \boldsymbol{R}_{0+}} \left| \hat{M}_2(\boldsymbol{i}\omega) \right|$. For any given $\hat{T}_1(s)$ and associated $\hat{M}_1(s)$, the controller transfer function is:

$$\hat{K}_2(s)=\frac{\hat{T}_2(s)}{\hat{G}(s)\left(1-\hat{T}_2(s)\right)}=\frac{1-\hat{M}_2(s)}{2\hat{G}(s)\hat{M}_2(s)} \ .$$

The subsequent result is concerned with the fact that a positive real transfer function can be designed by using feedback or feed-forward control laws for the case when the plant transfer function is inversely stable even if it is not either positive real or stable.

Theorem 3. Assume that $\hat{G}(s)$ is inversely stable with relative degree 0 or 1 while non-necessarily in $\{PR\}$ (or even non-necessarily in $\boldsymbol{RH}_\infty$). Then, the following properties hold:

(i) A non-unique (state-space) realizable closed-loop transfer function $\hat{T}_1 \in \{PR\}$, or respectively $\hat{T}_1 \in \{SPR\}$, may be designed via a stable feedback controller of transfer function $\hat{K}_1(s)$ Eq. (14) which is realizable if $\hat{G}(s)$ and $\hat{T}_1(s)$ have respective zero relative degrees. In the above cases, $\hat{T}_1^{-1} \in \{PR\}$, or respectively, $\hat{T}_1^{-1} \in \{SPR\}$.

(ii) A non-unique realizable closed-loop transfer function $\hat{T}_2 \in \{PR\}$, or $\hat{T}_2 \in \{SPR\}$, may be designed via a feed-forward controller of transfer function $\hat{K}_2(s)$ via (16) which is realizable if the relative degree of the closed-loop transfer function $\hat{T}_2(s)$ is non less than that of the plant $\hat{G}(s)$. In the above cases, $\hat{T}_2^{-1} \in \{PR\}$, or respectively, $\hat{T}_2^{-1} \in \{SPR\}$.

4. Non-passive and passive systems

Note that passive systems are intrinsically stable and either consume or dissipate energy for all time. However unstable systems are always non-passive although some stable systems are also non-passive. Looking at Definition 3, we can give the next one:

Definition 7. A dynamic system is *Non-passive* (or *Active* or, so-called, *Regenerative*) if $\int_0^{t_i} y^T(\tau)u(\tau)d\tau+\varepsilon_{t_i}<0$ for some unbounded sequences $E=\{\varepsilon_{t_i}\}\subseteq \boldsymbol{R}_{0+}$, $T=\{t_i\}\subseteq \boldsymbol{R}_{0+}$ which satisfy the conditions:

a) $0<\delta_{i-1}\le t_{i+1}-t_i\le \delta_i<\infty$; $\forall i \in \boldsymbol{Z}_{0+}$ for some positive bounded sequence $\Delta=\{\delta_i\}$,

b) $0<\theta_{i-1}\le \tilde{\varepsilon}_{t_i}=\varepsilon_{t_{i+1}}-\varepsilon_{t_i}\le \theta_i<\infty$; $\forall i \in \boldsymbol{Z}_{0+}$ for some positive bounded sequence $\Theta=\{\delta_i\}$,

c) $\varepsilon_i \ , t_i \to +\infty$ as $i\to +\infty$.

The following result follows for a non-passive system:

Theorem 4. If a dynamic system is non-passive then $\lim\limits_{t\to\infty} \int_0^t y^T(\tau)u(\tau)d\tau=-\infty$

Proof: Define $\tilde{\varepsilon}_{t_i}\in \boldsymbol{R}_+$ such that $\int_0^{t_i} y^T(\tau)u(\tau)d\tau=-\varepsilon_{t_i}-\tilde{\varepsilon}_{t_i}<-\varepsilon_{t_i}$. Thus,

$$\int_{t_i}^{t_{i+1}} y^T(\tau)u(\tau)d\tau=-\varepsilon_{t_{i+1}}-\tilde{\varepsilon}_{t_{i+1}}-\int_0^{t_i} y^T(\tau)u(\tau)d\tau=\varepsilon_{t_i}+\tilde{\varepsilon}_{t_i}-\left(\varepsilon_{t_{i+1}}+\tilde{\varepsilon}_{t_{i+1}}\right)$$

$$\int_0^t y^T(\tau)u(\tau)d\tau=\int_0^{t_i} y^T(\tau)u(\tau)d\tau+\int_{t_i}^t y^T(\tau)u(\tau)d\tau=-\varepsilon_{t_i}-\tilde{\varepsilon}_{t_i}+\int_{t_i}^t y^T(\tau)u(\tau)d\tau$$

$$\int_0^t y^T(\tau)u(\tau)d\tau=\int_0^{t_{i+1}} y^T(\tau)u(\tau)d\tau-\int_t^{t_{i+1}} y^T(\tau)u(\tau)d\tau=-\varepsilon_{t_{i+1}}-\tilde{\varepsilon}_{t_{i+1}}-\int_t^{t_{i+1}} y^T(\tau)u(\tau)d\tau$$

Subtracting the two above ones:

$$\begin{aligned}\varepsilon_{t_{i+1}}-\varepsilon_{t_i}+\left|\tilde{\varepsilon}_{t_{i+1}}-\tilde{\varepsilon}_{t_i}\right| &\ge \left|\int_{t_i}^{t_{i+1}} y^T(\tau)u(\tau)d\tau\right| \\ &\ge 2\,max\left(\left|\int_t^{t_{i+1}} y^T(\tau)u(\tau)d\tau\right|,\left|\int_{t_i}^t y^T(\tau)u(\tau)d\tau\right|\right)\end{aligned}$$

and

$$\left|\varepsilon_{t_i}+\tilde{\varepsilon}_{t_i}-\varepsilon_{t_{i+1}}+\varepsilon_{t_i}-\left|\tilde{\varepsilon}_{t_{i+1}}-\tilde{\varepsilon}_{t_i}\right|\right| \le \left|\varepsilon_{t_i}+\tilde{\varepsilon}_{t_i}-\left|\int_{t_i}^{t} y^T(\tau)u(\tau)d\tau\right|\right|$$
$$\le \left|\int_0^t y^T(\tau)u(\tau)d\tau\right| \le -\varepsilon_{t_i}-\tilde{\varepsilon}_{t_i}+\left|\int_{t_i}^{t} y^T(\tau)u(\tau)d\tau\right| \le -\left(\varepsilon_{t_i}+\tilde{\varepsilon}_{t_i}\right)+1/2\left(\varepsilon_{t_{i+1}}-\varepsilon_{t_i}+\left|\tilde{\varepsilon}_{t_{i+1}}-\tilde{\varepsilon}_{t_i}\right|\right)$$

since $\{\varepsilon_{t_i}\}$ is unbounded but its associated incremental sequence $\{\tilde{\varepsilon}_{t_i}\}$ is bounded , $\{2\varepsilon_{t_i}-\varepsilon_{t_{i+1}}\}\to\infty$ as $t_i\to\infty$ then $\left|\int_0^\infty y^T(\tau)u(\tau)d\tau\right|<+\infty$ contradicts the above relations. □

Note that a non-passive system can reach an absolute infinity energy measure in finite time under certain atypical inputs as, for instance, a second-order impulsive Dirac input of appropriate component signs at some time instant $t_1<\infty$ with $u(t)=0$ for $t>t_1$. Then, $\int_0^{t_1} y^T(\tau)u(\tau)d\tau = \lim_{t\to\infty}\int_0^t y^T(\tau)u(\tau)d\tau = -\infty$.

Theorem 5. The following properties hold:
(i) A passive system cannot be non-passive in any time sub-interval. A non-passive system in some time interval cannot be a passive system.
(ii) A passive system is always stable and also dissipative (i.e. the dissipative energy function takes non-negative values for all time) including the conservative particular case implying identically zero dissipation through time.
(ii) A non-passive system can be stable or unstable (so, stable systems are non- necessarily passive).

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