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Solitons and black hole in shift symmetric scalar-tensor gravity with cosmological constant

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Abstract: We demonstrate the existence of static, spherically symmetric *globally regular*, i.e. solitonic solutions of a shift-symmetric scalar-tensor gravity model with negative cosmological constant. The norm of the Noether current associated to the shift symmetry is finite in the full space-time. We also discuss the corresponding black hole solutions and demonstrate that the interplay between the scalar-tensor coupling and the cosmological constant leads to the existence of new branches of solutions. To linear order in the scalar-tensor coupling, the asymptotic space-time corresponds to an Anti-de Sitter space-time with a non-trivial scalar field on its conformal boundary. This allows the interpretation of our solutions in the context of the AdS/CFT correspondence. Finally, we demonstrate that — for physically relevant, small values of the scalar-tensor coupling — solutions with positive cosmological constant do not exist in our model.

Keywords: Black Holes, Classical Theories of Gravity

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Contents

1 Introduction

Einstein's theory of General Relativity (GR) is up to the present day the best theory we have to describe the gravitational interaction. In recent detections of gravitational waves from colliding black hole binaries by the LIGO collaboration [1–5] no deviations from GR have been found. Next to the direct observation of gravitational waves (one of the predictions of the theory), these observations have also conclusively shown that black holes do, indeed, exist in the universe. Accepting this, however, leads to a problem: black hole solutions of standard GR are plagued with physical, i.e. space-time singularities at their gravitational center. These are, in general, not observable for an outside observer due to the existence of an event horizon, but it shows that this classical theory of gravity possesses limits. In particular, up to today it seems impossible to reconcile Quantum physics, which is the fundamental basis of the Standard Model of Particle physics, with GR. One of the best candidates is String Theory, which contains the assumed mediator of the gravitational interaction, the graviton, naturally. Since the energy scale at which Quantum Gravity (QG) theories should become relevant are out of reach for present day accelerators, there are different ways to test whether extensions of GR are necessary.

One such possibility is to observe strong gravity events such as the collision of two black holes or the recently observed collision of two neutron stars using both gravitational waves as well as Gamma-rays [6–8]. This allows to test gravity theories to a very high precision with the help of multi-messenger gravitational wave (GW) observations.

Another way to test gravity theories is to study theoretical predictions that are also detectable at low(er) energies. One such prediction of String Theory is the Anti-de Sitter/Conformal Field Theory (AdS/CFT) correspondence [9–12] that states that a gravity theory in a d-dimensional AdS space-time possesses exactly the same number of degrees of freedom as the $(d-1)$-dimensional CFT on the conformal boundary of AdS. This duality is a weak-strong coupling correspondence and has mainly been used in the context of classical d-dimensional GR in AdS describing a strongly coupled CFT on the $(d-1)$-dimensional boundary with the extra dimension giving the renormalization group (RG) flow. One such application is the description of high-temperature superconductors in terms of holographic duals [13–18], another one the application to heavy ion collisions and the quark-gluon plasma (see [19] and references therein).

In recent years, Horndeski gravity models [20] have gained a lot of attention. These models include, in general, higher curvature invariants and non-linear couplings between scalar, vector and tensor fields. One such example are scalar-tensor gravity models in which a scalar field, the so-called galileon, is non-minimally coupled to the tensor part [21–28]. These models contain a shift symmetry of the scalar field that leads to the existence of a conserved Noether current. Consequently, black hole solutions have been studied in these models [29, 30], which, however, have a diverging norm of the Noether current on the horizon. In [31] solutions with vanishing Noether current have been constructed. Interestingly, in these same models it has been conjectured that globally regular, star-like solutions in asymptotically flat space-time should not be able to carry Galilean "hair", while it has been suggested that the no-go theorem for astrophysically relevant solutions could be avoided in asymptotically de Sitter (dS) space-time [32].

In this paper, we discuss solutions of a Galileon scalar-tensor gravity model with a linear coupling between the scalar field and the Gauss-Bonnet term including a cosmological constant. Recent multi-messenger GW astronomy that puts strong bounds on the GW speed, disfavoures these models [33] in their original version without cosmological constant. In fact, originally these models had been studied in order to solve the dark energy problem through the inclusion of a dynamical scalar field which renders the cosmological solutions self-accelerating. In this paper, we will show that the explicit presence of a positive cosmological constant does neither allow black hole nor star-like, solitonic solutions for relevant, small values of the scalar-tensor coupling. On the other hand, we will demonstrate that the presence of a negative cosmological constant in the model allows for solitonic solutions as well as new branches of black hole solutions as compared to the case with vanishing cosmological constant. Hence, while the motivation to study these solutions is no longer given in an astrophysical and/or cosmological context, we suggest that the dual description of strongly coupled phenomena within the AdS/CFT correspondence could lead to interesting new models in the future.

Our paper is organized as follows: in section 2, we give the model and equations, while in section 3 and 4 we discuss soliton and black hole solutions, respectively. Section 5 contains our summary and conclusions.

2 The model

The model we are studying in this paper is a Horndeski scalar-tensor model which possesses a shift symmetry in the scalar field $\phi \to \phi + a_\mu x^\mu + c$, where a_μ is a constant co-vector and c is a constant. Its action reads:

$$S = \int \mathrm{d}^4x \sqrt{-g}\left[R - 2\Lambda + \frac{\gamma}{2}\phi\mathcal{G} - \partial_\mu\phi\partial^\mu\phi\right], \tag{2.1}$$

where the Gauss-Bonnet term $\mathcal{G}$ is given by

$$\mathcal{G} = R^{\mu\nu\rho\sigma}R_{\mu\nu\rho\sigma} - 4R^{\mu\nu}R_{\mu\nu} + R^2 \ . \tag{2.2}$$

γ is the scalar-tensor coupling and $\Lambda \neq 0$ is the cosmological constant.[1] Units are chosen such that $16\pi G \equiv 1$ and the scalar-tensor coupling γ is related to the α used in [30] by $\gamma = 4\alpha$. For $\Lambda = 0$, this model and its black hole solutions have been studied in [29–31], while it was recently pointed out that solitonic, i.e. star-like solutions that are globally regular, do not exist in this model [32]. In the following, we will demonstrate that this does not hold true for the model with $\Lambda < 0$.

Varying the action (2.1) with respect to the metric and the scalar field gives the following gravity equations:

$$G_{\mu\nu} + \Lambda g_{\mu\nu} - \partial_\mu\phi\partial_\nu\phi + \frac{1}{2}g_{\mu\nu}\partial_\alpha\phi\partial^\alpha\phi + \frac{\gamma}{2}\mathcal{K}_{\mu\nu} = 0\,, \tag{2.3}$$

where

$$\mathcal{K}_{\mu\nu} = (g_{\rho\mu}g_{\sigma\nu} + g_{\rho\nu}g_{\sigma\mu})\nabla_\lambda(\partial_\gamma\phi\epsilon^{\gamma\sigma\alpha\beta}\epsilon^{\delta\eta}R_{\delta\eta\alpha\beta})\,, \tag{2.4}$$

as well as the scalar field equation

$$\Box\phi = -\frac{\gamma}{2}\mathcal{G} \ . \tag{2.5}$$

We are interested in spherically symmetric and static solutions and hence choose for the Ansatz:

$$ds^2 = -N(r)\sigma(r)^2dt^2 + \frac{1}{N(r)}dr^2 + r^2\left(d\theta^2 + \sin^2\theta d\varphi^2\right)\ , \ \phi = \phi(r)\ . \tag{2.6}$$

Inserting this Ansatz into the equations of motion (2.3) and (2.5) results in a coupled system of non-linear ordinary differential equations that has to be solved subject to the appropriate boundary conditions. The explicit form of the gravity equations (2.3) reads:

$$4\gamma(1-N)\phi'' - 2\gamma(3N-1)\phi'\frac{N'}{N} + r^2\phi'^2 + 2r\frac{N'}{N} - \frac{2}{N} + 2 + 2\Lambda\frac{r^2}{N} = 0\,, \tag{2.7}$$

$$\frac{1}{N^2} + 3\gamma\phi'\left(2\frac{\sigma'}{\sigma} + \frac{N'}{N}\right) - \left[1 + (r+\gamma\phi')\left(2\frac{\sigma'}{\sigma} + \frac{N'}{N}\right) - \frac{1}{2}r^2\phi'^2\right]\frac{1}{N} - \Lambda\frac{r^2}{N} = 0\,, \tag{2.8}$$

$$\mathcal{F}'\left(2\gamma\phi' - \frac{r}{N}\right) + \mathcal{F}\left[2\gamma\phi'' + 2\gamma\phi'\left(2\frac{N'}{N} + \frac{\sigma'}{\sigma}\right) - \frac{r}{N}\left(\frac{N'}{N} + \frac{\sigma'}{\sigma}\right) - \frac{1}{N}\right] - \frac{r}{N}\phi'^2 - \frac{N'}{N^2} - 2r\frac{\Lambda}{N^2} = 0\,, \tag{2.9}$$

where $\mathcal{F} = \frac{N'}{N} + 2\frac{\sigma'}{\sigma}$ and the prime now and in the following denotes the derivative with respect to r. Finally, the scalar field equation (2.5) is:

$$\gamma\left[(N-1)\left(4\frac{\sigma''}{\sigma} + 2\frac{N''}{N} + 6\frac{\sigma'N'}{\sigma N} + \frac{N'}{N}\right) + 4\frac{\sigma'N}{\sigma N} + 2\frac{N'^2}{N}\right] + 2r^2\left[\phi'' + \left(\frac{\sigma'}{\sigma} + \frac{N'}{N} + \frac{2}{r}\right)\phi'\right] = 0\ . \tag{2.10}$$

Note that the system of equations does not depend on $\phi(r)$ explicitly, but only on $\phi'(r)$.

[1]Note that the action can be supplemented by suitable boundary and counter terms in order to render the action finite. For the discussion in the following these are, however, not necessary.

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The model contains a shift symmetry $\phi \to \phi + a_\mu x^\mu + c$, which leads to the existence of a locally conserved Noether current J^r with norm $(J_r J^r)^{1/2}$ that results from the invariance of the kinetic term under this transformation and the fact that the Gauss-Bonnet term is a total divergence in 4-dimensional space-time, respectively. In our choice of coordinates the norm reads [31]:

$$(J_r J^r)^{1/2} = \frac{\gamma(N-1)}{2r^2}\left(\frac{N'}{N} + \frac{2\sigma'}{\sigma}\right) - \phi' \ . \tag{2.11}$$

For $\gamma = 0$ the system of equations corresponds to Einstein gravity including a cosmological constant. For $\gamma > 0$, the asymptotically non-vanishing curvature of the space-time sources the scalar field — even at $r \to \infty$. It is, thus, not surprising that the solution to the equations of motion does not behave like a "pure" (Anti-) de Sitter ((A)dS) space-time asymptotically. For $r \to \infty$, we find the following behaviour for the metric functions and the scalar field derivative:

$$N(r) = C_1 - \frac{\lambda}{3}r^2 - \frac{M}{r} + \mathrm{O}(r^{-2}) \ , \ \sigma(r) = 1 + \mathrm{O}(r^{-2}) \ , \ \phi(r) = C_0 - C_2 \ln(r) + \mathrm{O}(r^{-2}) \tag{2.12}$$

where C_0 is a constant. λ, C_1 and C_2 fulfil

$$\lambda\left(3 + \frac{10}{9}\gamma^2\lambda^2\right) = 3\Lambda \quad , \quad C_1 = \frac{16\gamma^4\left(\frac{\lambda}{3}\right)^4 + 12\gamma^2\left(\frac{\lambda}{3}\right)^2 + 1}{40\gamma^4\left(\frac{\lambda}{3}\right)^4 + 14\gamma^2\left(\frac{\lambda}{3}\right)^2 + 1} \quad , \quad C_2 = -\frac{2}{3}\gamma\lambda \tag{2.13}$$

and the parameter M determines the gravitational mass of the solution (see [34] for a summary and references therein):

$$M_{\mathrm{grav}} = \frac{8\pi}{16\pi G}M = 8\pi M \, , \tag{2.14}$$

where in the last equality we have used our convention $16\pi G = 1$. While the field $\phi(r)$ diverges for $r \to \infty$, note that only $\phi'(r)$ appears in the equations of motion and falls of like $\phi'(r) = -C_2/r + \mathrm{O}(r^{-2})$.

The relation between λ and Λ implies that the signature of these constants is equal, i.e. negative (positive) Λ implies negative (positive) λ, and that $|\lambda| < |\Lambda|$ for $\gamma \neq 0$ with $|\lambda|$ a decreasing function with Λ fixed and γ increasing. To state it differently, for a fixed value of Λ, the asymptotic space-time deviates increasingly from (A)dS for increasing γ.

However, for relevant, small values of γ our solutions are — but only to linear order in γ — global (A)dS with a scalar field $\psi(r) := \phi'(r)$ that falls of linearly. However, our system is very different to the Gauss-Bonnet-scalar field system used in the description of holographic superconductors (see e.g [35–39]). While the condensation of the scalar field sets in at sufficiently low temperature of the black hole in the latter model, the scalar field is directly sourced by the Gauss-Bonnet term and is hence non-trivial whenever $\gamma \neq 0$ in our case. This leads to the observation that while for holographic superconductors, the scalar field fall-off on the conformal boundary is determined by the dimension of the space-time and the mass of the scalar field, the scalar field $\phi(r)$ in our model has to be constant on the AdS boundary or diverge logarithmically. Now, since the scalar field possesses a shift symmetry, we can always scale the constant to zero, such that only one possible behaviour remains. This is reflected in the fact, that only the derivative $\phi'(r)$ appears in the system of equations.

The detailed study of the applications in the context of the AdS/CFT correspondence is left as a future work, in particular since the coupling to a U(1) gauge field seems important in order to be able to construct phase diagrams. However, in order to get an idea if and how the application e.g. in the context of holographic superconductors would work, we can evaluate the action given in (2.1) on the solutions to the equations of motion and check whether the scalar field has a normalizable or non-normalizable fall-off on the AdS boundary. For that first note that we can integrate the term $\partial_\mu\phi\partial^\mu\phi$ in (2.1) by parts which gives

$$S = \int \mathrm{d}^4x\sqrt{-g}\left[R - 2\Lambda + \frac{\gamma}{2}\phi\mathcal{G} + \phi\Box\phi\right] + \int \mathrm{d}^3x\sqrt{-g_\infty}\phi\partial^r\phi|_{r\to\infty}\,, \tag{2.15}$$

where the last term is to be evaluated on the boundary $r\to\infty$. The last two terms in the 4-dimensional integral vanish when evaluated on the solutions to the equation of motions. The remaining terms in the 4-dimensional integral read (after suitable substraction of boundary terms):

$$\sqrt{-g}\,[R - 2\Lambda] = 2\sigma\left(-rN' - N + 1 - r^2\Lambda\right) \tag{2.16}$$

and using the expansion of the fields for $r\to\infty$ (see (2.12)) we find:

$$\sqrt{-g}\,[R - 2\Lambda]\,|_{r\to\infty} = (\lambda - \Lambda)r^2 + 1 - C_1\,. \tag{2.17}$$

This demonstrates that we have to think about suitable counterterms different to the ones used in standard holographic applications to remove the UV divergence. This is currently under investigation. Note, however, that taking only terms linear in γ into account leads to $\lambda = \Lambda$, $C_1 = 1$ and the 4-dimensional part of (2.15) is regulated. The boundary term is formally equal to the one that appears in standard holographic superconductors, where the value of the scalar field on the AdS boundary is interpreted as the source of the dual CFT if the divergence in the boundary action can be regulated by suitable counterterms. However, in the case of minimal coupling, the scalar field normally has power-law behaviour, while here it has logarithmic form. Hence, the boundary term in (2.15) has to be regulated by non-standard terms. This is currently under investigation and will be reported elsewhere. Again, let us state that we believe that the coupling to the electromagnetic field will be crucial in this respect.

Let us finally remark that the solution with $\Lambda > 0$ possesses a horizon at $r_c = \sqrt{3/\lambda}$ which — to linear order in γ — is equal to the cosmological horizon of dS space-time. However, as we will show below, neither soliton nor black hole solutions with (approximately) dS asymptotics exist, though the expansion (2.12) is valid for both signs of Λ.

3 Solitonic solutions

As pointed out in [32], star-like, globally regular and asymptotically flat solutions with non-trivial scalar field do not exist in the model we are studying here. However, when $\Lambda < 0$, these solutions exist, as we will show in the following. Let us start with the behaviour of the functions close to $r = 0$. We find:

$$N(r) = 1 + \frac{n_2}{2} r^2 + \mathrm{O}(r^3) \ , \quad \sigma(r) = \sigma_0 \left(1 + \frac{\sigma_2}{2} r^2 + \mathrm{O}(r^3)\right) \ , \quad \phi(r) = \frac{\phi_2}{2} r^2 + \mathrm{O}(r^3) \ , \tag{3.1}$$

where n_2, σ_2 and ϕ_2 fulfil the following relations:

$$n_2 = \frac{2\Lambda}{6\gamma\phi_2 - 3} \ , \quad \sigma_2 = \frac{\gamma\Lambda\phi_2}{3(1 - 2\gamma\phi_2)^2} \tag{3.2}$$

and

$$8\gamma^3\phi_2^4 - 12\gamma^2\phi_2^4 + 6\gamma\phi_2^2 + \left(\frac{2}{3}\gamma^2\Lambda^2 - 1\right)\phi_2 - \frac{2}{9}\gamma\Lambda^2 = 0 \ . \tag{3.3}$$

$\sigma_0 \equiv \sigma(0)$ is a constant to be determined numerically (see below for numerical results). This expansion already implies that both $\gamma \neq 0$ and $\Lambda \neq 0$ are necessary to obtain solitonic solutions. In order to make this more evident, let us look at the expansion in γ — we would expect γ in any realistic model to be small and hence the expansion in γ can give hints on how the existence of the soliton manifests itself. We find:

$$N(r) = 1 - \frac{\Lambda}{3} r^2 + \gamma^2 N_2(r) + \mathrm{O}(\gamma^4) \, , \ \sigma(r) = 1 + \gamma^2 \Sigma_2(r) + \mathrm{O}(\gamma^4) \, , \ \ \phi'(r) = \gamma\varphi_1(r) + \mathrm{O}(\gamma^3) \, , \tag{3.4}$$

where the function $N_2(r)$, $\Sigma_2(r)$ and $\varphi_1(r)$ are given by:

$$N_2(r) = -\frac{2}{9}\Lambda^2 + \frac{10}{81}\Lambda^3 r^2 + \frac{2}{r}\left(-\frac{\Lambda^3}{27}\right)^{1/2} \arctan\left(r\left(-\frac{\Lambda}{3}\right)^{1/2}\right) \ , \tag{3.5}$$

and

$$\Sigma_2(r) = \frac{\Lambda^2}{27(\Lambda r^2 - 3)} \ , \quad \varphi_1(r) = \frac{2r\Lambda^2}{3(\Lambda r^2 - 3)} \ . \tag{3.6}$$

We conclude that only for $\gamma \neq 0$ and $\Lambda < 0$ we can have globally regular solutions. For $\Lambda > 0$, the function φ_1 tends to infinity at the cosmological horizon $r_c = \sqrt{3/\Lambda}$. Hence, *globally regular dS solutions do not exist in our model*, at least not for physically relevant, small values of γ.

It is also important to note that the norm of the Noether current (2.11) fulfils $\sqrt{J_r J^r}(r = 0) = 0$ and that it is finite in the full interval of the radial coordinate r for $\Lambda < 0$ (see numerical results). Moreover, we can only define the gravitational mass of the soliton at quadratic order in γ. The mass parameter M (see (2.12)) can be found by considering the γ expansion for $r \to \infty$ and is given by

$$M = -\pi\gamma^2 \left(-\frac{\Lambda^3}{27}\right)^{1/2} + \mathrm{O}(\gamma^4) \ . \tag{3.7}$$

For $\gamma = 0$, $\Lambda < 0$ the solution corresponds to global AdS and consequently has mass equal to zero, while the solitonic solutions for $\gamma > 0$ possesses negative mass for $\Lambda < 0$.

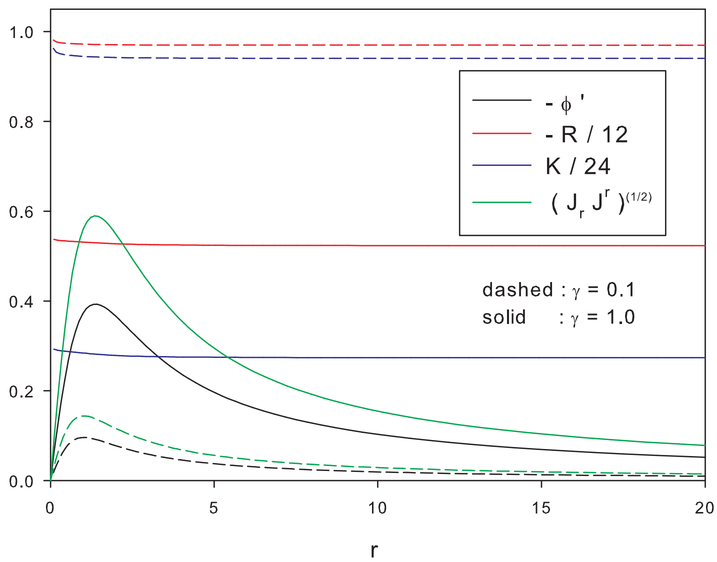

Figure 1. We show the dependence of $-\phi'$, the Ricci scalar R, the Kretschmann scalar $K = R_{\nu\mu\rho\sigma}R^{\nu\mu\rho\sigma}$ and the norm of the Noether current $(J_r J^r)^{1/2}$ on the radial coordinate r for soliton solutions with $\Lambda = -3$ and $\gamma = 0.1$ (dashed) and $\gamma = 1.0$ (solid), respectively.

3.1 Numerical construction

In the following, we will choose Λ negative since our analysis above has shown that globally regular dS solutions do not exist. In order to solve the equations (2.7) – (2.10) numerically, suitable combinations can be made and the system reduces to two equations of first order (the equations for N and σ) as well as one equation of second order (the equation for ϕ). We will hence fix 4 boundary conditions in the following.[2] Using the results of the expansion close to $r = 0$, we choose the following conditions for globally regular solutions:

$$N(0) = 1 \ , \ \phi(0) = 0 \ , \quad \phi'(0) = 0 \quad , \quad a(r \to \infty) \to 1 \ . \tag{3.8}$$

In figure 1 we show $\phi'(r)$ for $\Lambda = -3$ and two values of γ together with the Kretschmann scalar $K = R_{\nu\mu\rho\sigma}R^{\nu\mu\rho\sigma}$, the Ricci scalar R and the norm of the Noether current. As can be seen, the space-time is perfectly regular everywhere, contains no physical singularities and possesses a finite norm of the Noether current. Moreover, ϕ' tends asymptotically to zero and increases in value over the whole range of r when increasing γ. The norm of the Noether current is zero at $r = 0$, increases to a maximal value close to the center of the soliton and then falls off to zero asymptotically. With the increase of γ, the norm increases, but stays qualitatively the same. The Kretschmann scalar K and the Ricci scalar R deviate increasingly from their "pure" AdS values $K_{\rm AdS} = -8\Lambda = 24$ and $R_{\rm AdS} = 4\Lambda = -12$ for increasing γ. This is nothing else but the statement made above that the space-time is

[2] Since the equations of motion depend only on ϕ', the system is effectively a system of three 1st order equations and $\phi(0)$ is a free parameter, which can be chosen to be equal to zero due to the shift symmetry in the model.

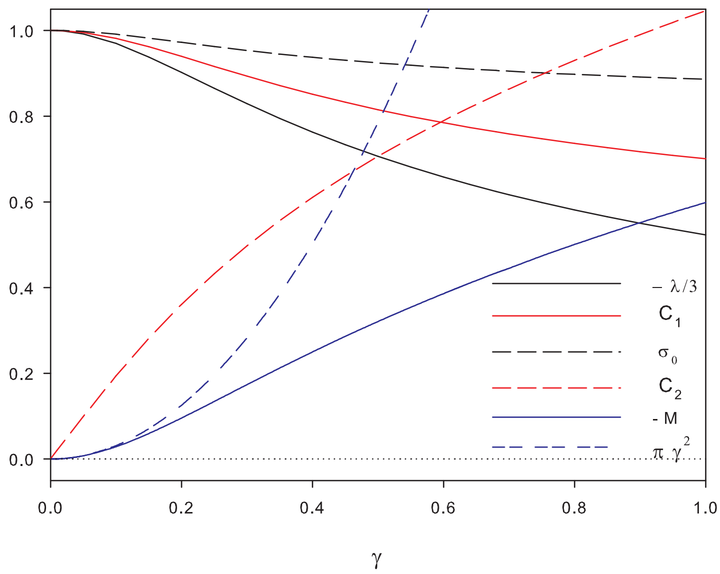

Figure 2. We show the dependence of the coefficients appearing in the asymptotic expansion (see (2.12)) as well as σ_0 (see (3.1)) on γ for soliton solutions with $\Lambda = -3$. Note that the negative value of the mass parameter $-M$, which for our choice of $\Lambda = -3$ reads $-M = \pi\gamma^2 + O(\gamma^4)$, fits the curve $\pi\gamma^2$ very well for small values of γ.

asymptotically AdS only to linear order in γ. This becomes also clear by virtue of figure 2, in which we give the dependence of the coefficients appearing in the asymptotic expansion (see (2.12)) as well as the value of σ_0 (see (3.1)) on γ for $\Lambda = -3$. Again, for the allowed range of the parameter γ, the value of σ_0 stays perfectly finite. The mass parameter M is zero for $\gamma = 0$, which corresponds to global AdS, and becomes increasingly negative when increasing γ for Λ fixed. We have hence found a continuous branch of solitonic solutions that is directly connected to global AdS.

4 Black hole solutions

For $\gamma = 0$, our model has a black hole solution, namely the Schwarzschild-AdS solution (SAdS) (see e.g. [34] and references therein) with

$$N(r) = 1 - \frac{\Lambda}{3}r^2 - \frac{M}{r}\,, \quad \sigma(r) \equiv 1 \;, \quad \phi(r) \equiv 0\,, \tag{4.1}$$

and the mass parameter M is related to the horizon r_h by $M = r_h - \frac{\Lambda}{3}r_h^3$.

In order to understand the deformation of this black hole solution in the presence of the scalar field, we have first studied the expansion of the solution in powers of γ. We find:

$$N(r) = 1 - \frac{\Lambda}{3}r^2 - \frac{M}{r} + \gamma^2\tilde{N}_2(r) + \mathrm{O}(\gamma^4) \;, \quad \sigma(r) = 1 + \gamma^2\tilde{\Sigma}_2(r) + \mathrm{O}(\gamma^4) \;,$$
$$\phi'(r) = \gamma\Phi_1(r) + \mathrm{O}(\gamma^3)\,, \tag{4.2}$$

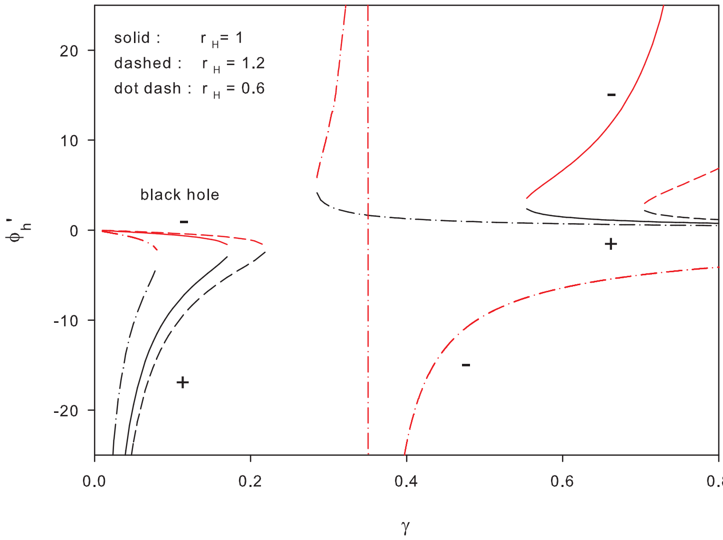

Figure 3. We show the domain of existence of black hole solutions with scalar hair and $\Lambda = -3$ in the ϕ'_h–γ– plane. The curves denote the critical value of γ (see (4.9)) for the four different branches (see also (4.7)) for $r_h = 1.2$ (dashed), $r_h = 1.0$ (solid) and $r_h = 0.6$ (dotted-dashed). Note that the $+$ and $-$ indicate the two different branches as given by (4.7). Moreover, the vertical dotted-dashed line is the value of γ at which the denominator of (4.7) becomes zero for $r_h = 0.6$.

and function $\Phi_1(r)$ reads:

$$\Phi_1(r) = \frac{\left(r^2 + r r_h + r_h^2\right)\left(\frac{2}{9}r^3 r_h \Lambda^2 + \frac{1}{9}r_h^4\Lambda^2 - \frac{2}{3}r_h^2\Lambda + 1\right)}{r^4 r_h\left(\frac{\Lambda}{3}r^2 + \frac{\Lambda}{3}r r_h + \frac{\Lambda}{3}r_h^2 - 1\right)} \ . \tag{4.3}$$

In particular, we find from this expression the dominant asymptotic term $\Phi_1(r \to \infty) \sim \frac{2}{3}\frac{\Lambda}{r}$, which is in excellent agreement with our numerical construction (see below). The expressions for $\tilde{N}_2(r)$ and $\tilde{\Sigma}_2(r)$ are very lengthy, that is why we do not present them here. Let us just note that, asymptotically, the expansion in γ is equivalent to the γ expansion of the soliton solution, see (3.4).

The temperature of a static black hole is given in terms of its surface gravity κ evaluated at the horizon $r = r_h$:

$$T_H = \frac{\kappa}{2\pi}|_{r=r_h} \quad , \quad \kappa^2 = -\frac{1}{4}g^{tt}g^{ij}\partial_i g_{tt}\partial_j g_{tt}|_{r=r_h} \ , \quad i,j = 1,2,3 \ . \tag{4.4}$$

For our Ansatz (2.6) and using (2.7) we find:

$$4\pi T_H = \left(N'\sigma\right)_{r=r_h} = \frac{1 - \Lambda r_h^2}{r_h + \gamma\phi'|_{r=r_h}}\sigma(r_h) \, , \tag{4.5}$$

where $\phi'|_{r=r_h}$ can be expressed in terms of Λ, r_h and γ (see (4.7) below).

4.1 Numerical construction

In order to construct black hole solutions numerically, we choose — like in the soliton case — $a(r \to \infty) \to 1$, while we now have to impose boundary conditions on the regular

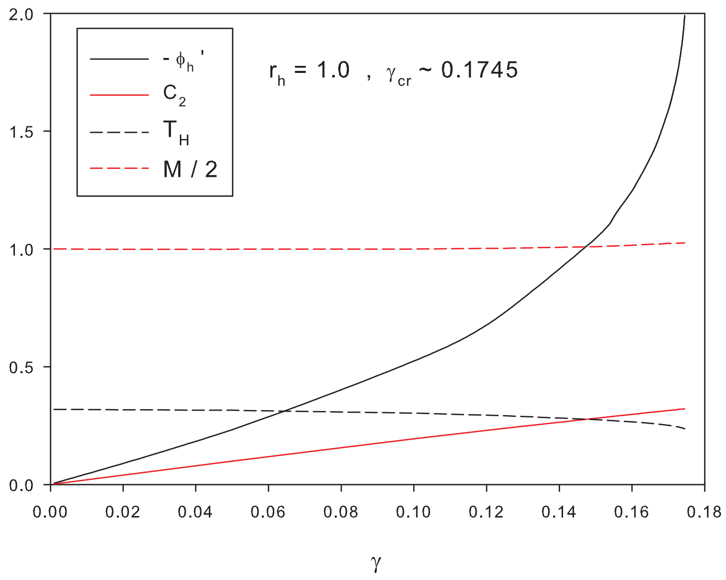

Figure 4. We show the temperature $T_{\rm H}$, the constant C_2 (see (2.12)), $-\phi'_h$ (see (4.7)) as well as the mass parameter M in dependence of γ for $\Lambda = -3$ and $r_h = 1$. Note that in this case $\gamma_{\rm cr} \approx 0.1745$.

(non-extremal) horizon $r = r_h$. These conditions read:

$$N(r_h) = 0 \ , \quad \phi(r_h) = 0 \tag{4.6}$$

as well as the following condition for the scalar field derivative $\phi'(r)$:

$$\phi'|_{r=r_h} = \frac{\pm\sqrt{\Delta}|\Lambda r_h^2 - 1| + 2\Lambda\gamma^2 r_h^3 - 6\Lambda\gamma^2 r_h + \Lambda r_h^5 - r_h^3}{2\gamma(2\Lambda\gamma^2 - \Lambda r_h^4 + r_h^2)} \tag{4.7}$$

with

$$\Delta = 4\gamma^4\Lambda^2 r_h^2 - 24\gamma^4\Lambda + 8\gamma^2\Lambda r_h^4 - 12\gamma^2 r_h^2 + r_h^6 \ . \tag{4.8}$$

The requirement $\Delta \geq 0$ gives the intervals in γ for which black holes with regular horizon and non-trivial scalar hair exist. We find: (assuming that $\gamma \geq 0$):

$$\gamma_{\rm cr}(\Lambda)^{(\pm)} = \left(\frac{\pm\sqrt{3}r_h^2\sqrt{\Lambda^2 r_h^4 - 2\Lambda r_h^2 + 3} - 2\Lambda r_h^4 + 3r_h^2}{2\Lambda(\Lambda r_h^2 - 6)}\right)^{1/2} \ . \tag{4.9}$$

Solutions with regular horizon then exist in the intervals $\gamma \in [0 : \gamma_{\rm cr}^{(-)}]$ and for $\gamma \geq \gamma_{\rm cr}^{(+)}$. Moreover, the denominator in (4.7) can become zero leading to the divergence of ϕ'_h. Hence depending on the choices of Λ and r_h there might be one value of γ, where solutions with regular horizon do not exist.

Equations (4.7) and (4.9) indicate that we should expect to have four branches for $\Lambda \neq 0$. This is shown in figure 3 for $\Lambda = -3$ and different values of r_h: the curves in this plot show the value of $\phi'|_{r=r_h} \equiv \phi'_h$ in dependence of γ_{cr}. The plus and minus signs indicate the different branches as given by (4.7), while the different intervals in γ are obvious from this plot. Note that for $\Lambda = 0$ only the branches close to $\gamma = 0$ exist. Hence, the non-vanishing cosmological constant leads to the existence of new branches that are disconnected from the $\gamma = 0$, $\Lambda = 0$ limit. For decreasing r_h the value of γ_{cr} decreases such that for $r_h \to 0$, the branches disappear. On the other hand, the extend in γ of the two disconnected branches increases with decreasing r_h. One could then speculate that the limit $r_h \to 0$ exists, corresponding to a soliton solution. Although we have shown above that soliton solutions actually exist in our model for $\Lambda < 0$, we find, however, that the branches never reach the corresponding soliton solution which has $\phi'_h = 0$. To state it differently: the soliton solutions presented above do not arise in a smooth limit taking $r_h \to 0$ for the black hole solutions.

We have also constructed the solutions on the branches that exist for large enough values of γ and find that these have — in general — $\phi' \to \infty$ for some intermediate $\tilde{r}$ with $\tilde{r} \in [r_h : \infty[$. This is why we do not discuss them further here.

In figure 4 we show the dependence of the black hole temperature, the constant C_2 (see (2.12)), the value $-\phi'_h$ (see (4.7)) as well as the mass parameter M in dependence of γ for $\Lambda = -3$ and $r_h = 1$. In this case, the interval in which solutions exist is $\gamma \in [0 : \gamma_{\rm cr}^{(-)}]$ with $\gamma_{\rm cr}^{(-)} \approx 0.174$ and for $\gamma \gtrsim 0.550$ with the denominator of (4.7) diverging at $\gamma = 2/3 \approx 0.667$. The temperature of the black hole $T_{\rm H}$ decreases from its "pure" AdS value $T_{\rm H,AdS} = (-\Lambda r_h + r_h^{-1})/(4\pi) \approx 0.3183$ at $\gamma = 0$ when increasing γ, while the coefficient C_2 increases. If we interpret C_2 as on order parameter, we find that C_2 increases with decreasing temperature $T_{\rm H}$, a phenomenon typically observed in superconductors. For $\gamma_{\rm cr}^{(-)} \approx 0.174$ the derivative of the scalar function at the horizon diverges, which makes the black hole temperature $T_{\rm H}$ by virtue of (4.4) tend to zero as $\gamma \to \gamma_{\rm cr}^{(-)}$, while C_2 stays finite. For $\gamma \in [0.174 : 0.550]$ no globally regular black hole solutions exist, while for $\gamma \geq 0.550$ and $\gamma \neq 0.667$ black holes with non-trivial scalar field and regular horizon at $r = r_h$ exist, but as mentioned above, these solutions become singular at a finite value of $r > r_h$. Finally, the mass parameter M does not depend strongly on γ and stays close to its SAdS value $M = r_h - \frac{\Lambda}{3} r_h^3 \equiv 2$ for our choice of parameters $r_h = 1$, $\Lambda = -3$.

5 Conclusions

In this paper, we have presented evidence that the non-existence theorem for globally regular solutions of shift-symmetric scalar-tensor gravity models does not extend to the case with negative cosmological constant. We have constructed globally regular, solitonic solutions that have AdS asymptotics to linear order in γ. The corresponding black hole solutions possess a regular horizon at $r = r_h$, but do not tend to the soliton solutions in the limit $r_h \to 0$. We also observe that the presence of the negative cosmological constant allows new branches of black hole solutions, which, however, possess a diverging scalar field derivative at finite distance outside the horizon. As mentioned above, it will be interesting to understand the application of our solutions in the context of the AdS/CFT correspondence, e.g. in the holographic description of high-temperature superconductors. The parameter γ triggers the existence of a non-trivial scalar field in the space-time and

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for black holes we find a typical superconductor behaviour, namely the order parameter increases with decreasing temperature. In the standard approach to holographic superconductors [13–18], the scalar field is minimally coupled to an electromagnetic field (and a tensor gravitational field if backreaction is taken into account) and this coupling triggers the spontaneous formation of scalar hair below a critical temperature of the planar AdS black hole. For our model it will be interesting to see how the existence of a non-trivial scalar field on the conformal boundary can be interpreted, especially in the context of the existence of "gaps" in γ, where regular black holes with scalar hair do not exist. This is currently under investigation.

Our results also indicate that neither solitonic nor black hole solutions in a space-time with positive cosmological constant exist, and, we have, in fact, not been able to construct the solutions with the appropriate boundary conditions. We believe that this is related to the fact that the system of ordinary differential equations of the model is effectively a system of three 1st order equations and, consequently, only 3 boundary conditions are not trivial. Since, however, for the construction of black hole solutions, we would need to fix the metric function $N(r)$ to be equal to zero on both the regular horizon r_h as well as the cosmological horizon r_c and require in addition the scalar field to be regular at these two points, the number of boundary conditions needed appears too large for the system of equations.

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