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Quantum Cosmology of the Big Rip: Within GR and in a Modified Theory of Gravity

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Abstract: Quantum gravity is the theory that is expected to successfully describe systems that are under strong gravitational effects while at the same time being of an extreme quantum nature. When this principle is applied to the universe as a whole, we use what is commonly named "quantum cosmology". So far we do not have a definite quantum theory of gravity or cosmology, but we have several promising approaches. Here we will review the application of the Wheeler–DeWitt formalism to the late-time universe, where it might face a Big Rip future singularity. The Big Rip singularity is the most virulent future dark energy singularity which can happen not only in general relativity but also in some modified theories of gravity. Our goal in this paper is to review two simple setups of the quantisation of the Big Rip in a Friedmann–Lemaître–Robertson–Walker universe within general relativity and in a modified theory of gravity.

Keywords: quantum cosmology; future singularities; modified theories of gravity; holographic dark energy; Eddington-inspired-Born-Infeld theory

1. Introduction

Cosmology has made giant steps in the last years given the impressive amount of observations and theoretical advancements. However, cosmology is facing many challenging questions, such as the fundamental cause of the recent acceleration of the Universe as predicted by SNeIa observations almost twenty years ago [1,2] and later on confirmed by several types of cosmological and astrophysical data (see [3] for a recent account on this issue). The straightforward approach is to assume a cosmological constant that kicked in recently and is currently dominating the late-time energy density budget of the universe. This approach is in agreement with the current observations [4]; however, it faces many challenging theoretical issues: *why is it so tiny? why has this cosmological constant begun to be important only right now?* (e.g., [5–8]). These questions lead to other natural questions: *what happens if the cosmological constant is not quite constant?* A question as simple as this one led to a great interest in exploring other possible scenarios to explain the late-time acceleration of the universe by invoking either an additional matter component in the Universe, which we name dark energy (DE) [9–11], or by appropriately modifying the laws of gravity (for a very recent account on this issue, see, for example, [12,13] and the extensive list of references provided therein).

Among the plethora of dark energy models studied in the last years, we will focus on what is known as the holographic Ricci dark energy scenario (HRDE). This model is characterised by its simplicity, yet it has strong physical roots. In fact, this model is rooted in the holographic principle and therefore has a quantum gravity inspirational origin. The idea behind the holographic dark energy model is that the energy density of a given system is bounded by a magnitude proportional to the inverse square of a length characterising that system [14,15]. When this principle is applied to the universe as a whole, we get the holographic dark energy [16,17]. It turns out that there are many different ways of characterising the size of the universe. One possibility is to characterise the square of such a length as the inverse of the Ricci curvature of the universe. When the size of the universe is characterised in such a way, we end up with the HRDE model [18]. It turns out that the observationally preferred HRDE model induces a Big Rip (BR) singularity [19–21] in the future [22,23]. We remind that a BR singularity is characterised by a blow of the curvature of space-time at a finite cosmic time in such a way that geodesic cannot be extended beyond that point [24]. In fact, at the singularity, the size of the Universe, its expansion rate, and acceleration rate diverge. All these phenomena are accompanied by the laceration of space-time itself.

On the other hand, among the vast zoo of modified theories of gravity, we will concentrate on what is known as the Eddington-inspired-Born-Infeld theory as an example. First of all, let us say a few words about why we should care about modified theories of gravity; that is, why we should think about theories beyond general relativity (GR). Certainly, Einstein's theory of GR has been an extremely successful theory for more than a century. However, the theory is expected to break down at some points at very high energies where quantum effects are expected to become crucial, such as in the past expansion of the Universe where GR predicts a big bang singularity or in its future evolution where some DE singularities can show up. In addition, GR on its own cannot explain the current acceleration of the universe unless an exotic new component is evoked on the matter content of the universe [25]. We will focus on one particular alternative theory of gravity, dubbed Eddington-inspired-Born-Infeld (EiBI) theory. It was recently proposed in [26], pioneered in [27], and has attracted much attention [28–34]. The EiBI theory has been shown to be able to remove the big bang singularity for a radiation-dominated universe through a loitering effect or a bounce in the past [26]. The ability of the theory to smooth other cosmological singularities in a phantom-dominated universe has also been studied in [35–38]. Unfortunately, it was found that even though the EiBI theory can lead to the avoidance of the big bang and the alleviation of some smoother singularities, the BR singularity is still unavoidable.

In these two scenarios (HRDE and EiBI), a BR singularity can unavoidably be the final stage of the universe—at least from a classical point of view. Given that the BR is characterised by being an event where energies of Planck scale can be reached, we expect quantum effects to be important. This is a crucial point, as it highlights that quantum physics can be extremely important, even for systems of a macroscopic scale. We will next describe a quantum framework based on the Wheeler–DeWitt equation for the BR in an HRDE and in an EiBI theory. Furthermore, we will regard the DeWitt condition (i.e., the vanishing of the wavefunction near the singularity) as a potential hint to the avoidance of the classical singularity, even though the probability interpretation is still not clear due to the lack of a complete quantum theory of gravity. In addition, our work is motivated by the fact that some DE singularities might be favoured by observations [39].

The paper is organised as follows: In Section 2, we briefly review the definition of a BR singularity and its physical consequences. In Section 3, we deal with the quantum cosmology of the HRDE model. Then, in Section 4, we carry out the quantum analysis of a BR within the EiBI theory. Finally, we conclude in Section 5.

2. The Classical Big Rip Singularity

The Universe on its largest scale can be described by a Friedmann–Lemaître–Robertson–Walker (FLRW) universe whose metric reads

$$ds^2 = -dt^2 + a^2(t)\left(\frac{dr^2}{1-kr^2} + r^2 d\theta^2 + r^2 \sin^2\theta d\varphi\right), \tag{1}$$

where $k = -1, 0, 1$ for open, flat, and closed spatial geometries, respectively. For simplicity and given that our results are independent from the value k, we will set it to zero from now on. The simplest cosmological model that induces a BR corresponds to a FLRW universe filled with a perfect fluid with a constant equation of state such that $w < -1$. In that case, regardless of whether other (standard) matter components are filling the universe, the asymptotic behaviour of its scale factor will be of the sort

$$a(t) \propto (t_{BR} - t)^{\frac{2}{3(1+w)}}, \tag{2}$$

where t_{BR} corresponds to the moment where the BR takes place. It can be shown that the Hubble rate aand its cosmic derivative blow up at $t = t_{BR}$, and most importantly, the curvature is ill-defined at that space-time point. In addition, as was shown in [24], the geodesics cannot be extended beyond that point. Schematically, the BR can be explained as shown in Figure 1.

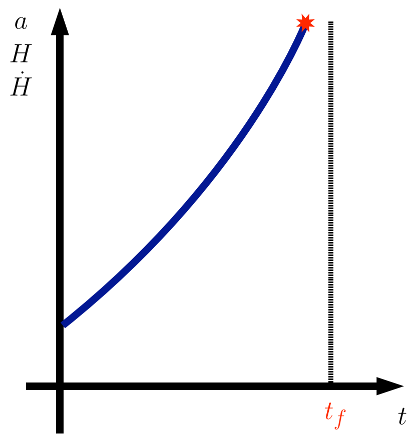

Figure 1. Schematic plot of the Big Rip (BR) singularity. In this plot, t_f corresponds to the time of the occurrence of BR.

In addition, it can be shown that any bounded structure in this kind of model will be destroyed [40,41]. Indeed, this can be easily proven by considering the motion of a spherical Newtonian object with mass M and a test particle rotating around the mass M with a physical radius r. The radial motion can be described through an effective potential, $\dot{r}^2 = -2V_{eff}$, which reads [40,41]

$$V_{eff} = -\frac{1}{2}\frac{\ddot{a}}{a}r^2 - \frac{GM}{r} + \frac{1}{2}\frac{L^2}{r^2}, \tag{3}$$

where the dot stands for the derivative with respect to the cosmic time, L stands for the constant angular momentum per unit mass of the test particle, and G is the gravitational constant. We remind that the existence of a bound structure with a circular orbit around the massive body M corresponds to the existence of a minimum of the potential V_{eff}. Given that the term $\frac{\ddot{a}}{a}$ blows up close to the BR, that minimum disappears and the bounded structures are destroyed (cf. Figure 2).

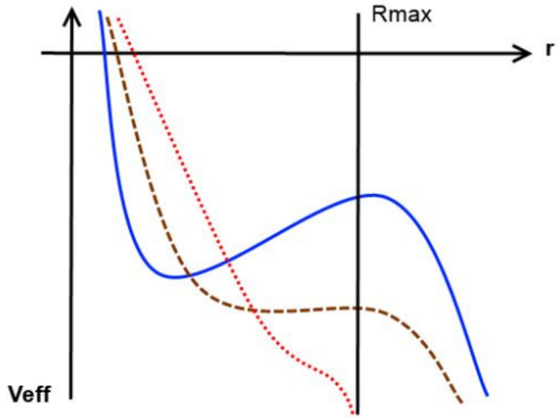

Figure 2. Plot that schematically shows the destruction of bounded structures when approaching the BR.

3. Quantum Cosmology of the Big Rip within the Holographic Ricci Dark Energy

When DE is described by an HRDE, then the asymptotic behaviour of the universe will be characterised by the energy density [18]:

$$\rho_{\mathrm{H}} \approx \frac{3\beta}{8\pi G}\left(\frac{1}{2}\frac{dH^2}{dx} + 2H^2\right), \tag{4}$$

where $x = \ln(a)$, and β is a positive dimensionless parameter that measures the strength of the holographic component. Consequently, the energy density of the HRDE scales asymptotically as

$$\rho_{\mathrm{H}} = \frac{3H_0^2}{8\pi G}\Omega_{p0}\left(\frac{a}{a_0}\right)^{-2\left(2-\frac{1}{\beta}\right)}, \tag{5}$$

where H_0 is the current Hubble rate and Ω_{p0} is an integration constant which quantifies the effective amount of DE in the HRDE model [18]. Therefore, whenever $0 < \beta < 1/2$, the Universe not only enters an accelerated state, but also super accelerates ($\dot{H} > 0$) in the future, hitting a BR at a finite cosmic time, where a quantum analysis is required.

It can be shown that the Wheeler–DeWitt equation for this model reads [42]

$$\left[\partial_x^2 + \gamma g\left(x\right)\right]\Psi\left(x\right) = 0, \tag{6}$$

where

$$\gamma \equiv \left(\pi H_0 a_0^3/G\hbar\right)^2, \quad g\left(x\right) = \Omega_{p0} x^{-\frac{2}{3}\left(1-\frac{2}{\beta}\right)}. \tag{7}$$

Therefore, the first-order WKB approximation gives the solution

$$\Psi\left(x\right) \approx \left(g\left(x\right)\right)^{-\frac{1}{4}}\left[A_1 e^{ih(x)} + A_2 e^{-ih(x)}\right], \tag{8}$$

where A_1 and A_2 are constants of integration and $h(x) = \int\sqrt{\gamma g\left(x\right)}dx$. It can be shown that the first-order WKB approximation is valid in the region that complies (large scale factor or x) because the following inequality holds [42]

$$q\left(x\right) \equiv \frac{1}{\gamma}\left|\frac{5\left[g\prime\left(x\right)\right]^2 - 4\left[g\prime\prime\left(x\right)\right]\left[g\left(x\right)\right]}{16\left[g\left(x\right)\right]^3}\right| \ll 1, \tag{9}$$

where a prime stands for derivatives with respect to x. As can be seen, the wave function is oscillating and decaying because $g(x)$ diverges and the prefactor of Equation (8) approaches zero when $x \to \infty$.

We remind at this regard that $0 < \beta < 1/2$, and therefore the exponent in the expression of $g(x)$ is always positive; i.e., $g(x)$ is proportional to a positive power of x. Therefore, it can be concluded that the wave function indeed vanishes at the BR. Therefore, the DeWitt condition is fulfilled and we can interpret our result as a singularity avoidance. It can be equally shown that our result is not changed by also considering the other matter component of our universe. This is not surprising, as, for example, dark matter (DM) decays faster that DE and it is subdominant by the time the BR is reached.

4. Quantum Cosmology of the Big Rip Model within the Eddington-Inspired Born-Infeld Model

The gravitational action of the EiBI theory proposed in [26] reads ($8\pi G = c = 1$)

$$\mathcal{S}_{EiBI} = \frac{2}{\kappa} \int d^4x \left[\sqrt{|g_{\mu\nu} + \kappa R_{\mu\nu}(\Gamma)|} - \lambda \sqrt{-g} \right] + S_m(g), \tag{10}$$

where $|g_{\mu\nu} + \kappa R_{\mu\nu}|$ is the determinant of the tensor $g_{\mu\nu} + \kappa R_{\mu\nu}$. The parameter κ characterizes the theory, and λ corresponds to the effective cosmological constant at the low curvature regime. S_m is the matter Lagrangian. The theory is constructed on the Palatini formalism, and the field equations are obtained by varying (10) with respect to $g_{\mu\nu}$ and the connection Γ. In an FLRW universe filled with a perfect fluid whose energy density and pressure are ρ and p, respectively, the modified Friedmann equation is [36]

$$\kappa H^2 = \frac{8}{3} \left[\bar{\rho} + 3\bar{p} - 2 + 2\sqrt{(1+\bar{\rho})(1-\bar{p})^3} \right] \frac{(1+\bar{\rho})(1-\bar{p})^2}{\left[(1-\bar{p})(4+\bar{\rho}-3\bar{p}) + 3\frac{d\bar{p}}{d\bar{\rho}}(1+\bar{\rho})(\bar{\rho}+\bar{p}) \right]^2}, \tag{11}$$

where $\bar{\rho} \equiv \kappa\rho$ and $\bar{p} \equiv \kappa p$. If the Universe is filled with a phantom energy whose equation of state is a constant $w < -1$, the universe will end up with a BR where the energy density and the scale factor diverge. In fact, the asymptotic Friedmann equation and its cosmic time derivative become

$$H^2 \approx \frac{4\sqrt{|w|^3}}{3\left(3w+1\right)^2} \rho \to \infty, \qquad \dot{H} \approx \frac{2\sqrt{|w|^3}}{\left(3w+1\right)^2} |1+w| \rho \to \infty, \qquad t \to t_{BR}. \tag{12}$$

The derivation of the Wheeler–DeWitt equation of the EiBI model relies on an appropriate and legitimate Hamiltonian. As shown in [29], this can be reached by considering an alternative action which is dynamically equivalent to the action (10):

$$\mathcal{S}_a = \lambda \int d^4x \left[R(q) - \frac{2\lambda}{\kappa} + \frac{1}{\kappa} \left(q^{\alpha\beta} g_{\alpha\beta} - 2\sqrt{\frac{g}{q}} \right) \right] + S_m(g), \tag{13}$$

where $q_{\mu\nu}$ is the auxiliary metric, $q^{\mu\nu}$ and q are its inverse and determinant, respectively. $R(q)$ is the scalar curvature constructed solely from the auxiliary metric $q_{\mu\nu}$. In Reference [28], it was shown that the field equations obtained by varying the action (13) with respect to $g_{\mu\nu}$ and the auxiliary metric $q_{\mu\nu}$ are the same as those deduced from the action (10). Classically, these actions are dynamically equivalent. Starting from the action (13) and inserting the FLRW ansatz, we can obtain the Lagrangian of this model in which the matter field is described by a perfect fluid (see Reference [29])

$$\mathcal{L} = \lambda M b^3 \left[-\frac{6\dot{b}^2}{M^2 b^2} - \frac{2\lambda}{\kappa} + \frac{1}{\kappa} \left(\frac{N^2}{M^2} + 3\frac{a^2}{b^2} - 2\frac{Na^3}{Mb^3} \right) \right] - 2\rho N a^3,$$

where N and M are the lapse functions of $g_{\mu\nu}$ and $q_{\mu\nu}$, respectively, and b is the scale factor of $q_{\mu\nu}$.

After choosing a gauge to fix the lapse function N, we obtain the Hamiltonian to construct the Wheeler–DeWitt equation. The Wheeler–DeWitt equation of this model reads [29]

$$\left[\frac{\partial^2}{\partial x^2} + V_1\Big(a,x\Big)\right]\Psi(a,x) = 0, \tag{14}$$

where we rewrite the potential $V_1(a,x)$ as

$$V_1(a,x) = \frac{24}{\kappa\hbar^2}e^{6x}\left[2 - 3\delta + \Big(\lambda + \kappa\rho(a)\Big)^2\delta^3\right], \tag{15}$$

where $\delta \equiv a^2e^{-2x}$ and $x = \ln(\sqrt{\lambda}b)$. Near the classical singularity where $a \to \infty$, the behavior of the potential can be classified as follows:

- If a^2 diverges slower than e^{2x} (i.e., $\delta \to 0$), the second term in the bracket in (15) is negligible compared with the first term. However, whether the first term dominates over the third term depends on the exact form of $\rho(a)$ and δ. In either case, the potential reaches positive infinite values when both a and x go to infinity.
- If a^2 diverges faster than e^{2x} (i.e., $\delta \to \infty$), the potential can be approximated as

$$V_1\Big(a,x\Big) \approx \frac{24}{\kappa\hbar^2}\Big(\lambda + \kappa\rho\Big(a\Big)\Big)^2 a^6 \tag{16}$$

  when a goes to infinity.
- If a^2 diverges comparably with e^{2x}, the potential can also be approximated as in Equation (16), because the phantom energy density blows up when $a \to \infty$.

Therefore, we find that the potential $V_1(a,x)$ goes to positive infinity when $a \to \infty$ for all values of x. With the help of the first-order WKB approximation, we find that the wave function of Equation (14) is a decaying and oscillating function, and it vanishes when $a \to \infty$ for all values of x.

We can also obtain a second Wheeler–DeWitt equation [29]

$$\left[\frac{\partial^2}{\partial y^2} + V_2\Big(a,y\Big)\right]\Psi(a,y) = 0\,, \qquad V_2(a,y) = \frac{32}{3\kappa\hbar^2}y^2\left[2 - 3\eta + \Big(\lambda + \kappa\rho(a)\Big)^2\eta^3\right], \tag{17}$$

by choosing another factor ordering and introducing a new variable $y \equiv (\sqrt{\lambda}b)^{3/2}$ and $\eta \equiv a^2y^{-4/3}$. Using a similar method as used in the first Wheeler–DeWitt equation to classify the behavior of the potential $V_2(a,y)$ near the classical singularity, we find that the potential goes to positive infinity when $a \to \infty$ for all values of y. Qualitatively, using the WKB approximation, we can claim that the wave function $\Psi(a,x)$ and $\Psi(a,y)$ vanish when $a \to \infty$. Thus, the DeWitt criterion is satisfied and the BR is expected to be avoided.

5. Conclusions and Further Discussions

We have focused on the quantum cosmology of the BR singularity within (i) GR for the HRDE model and (ii) the EiBI theory with a perfect fluid with a constant and sufficiently negative equation of state. For an alternative quantum approach within GR, please see [43–45] (cf. also [46,47]).

We have shown that the DeWitt condition is fulfilled in both models, and therefore the BR is avoided. In fact, if the wave function vanishes close to the singularity, then we can somehow conclude that the probability of the universe of reaching that stage is almost zero. Of course, the DeWitt condition can only be regarded as a guidance in the currently incomplete theory of quantum cosmology. In fact, from a more fundamental point of view, the initial condition for our universe as well as an almost vanishing probability distribution at singularities should arise within the theory in a natural way as a consequence of some other fundamental requirements (e.g., the normalizability of the wave function), and should not be imposed by hand. Because we lack a complete and consistent quantum gravity theory, we will stick to the DeWitt condition as our guiding principle for singularity avoidance.

It is equally important to highlight that the probability amplitudes for wave packets that should vanish close to the region of configuration space corresponding to the classical singularity require an appropriate Hilbert space associated with its inner product which defines a correct measure factor. Unfortunately, it is not obvious that this can always be done in a straightforward way in quantum cosmology [48].

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Abbreviations

The following abbreviations are used in this manuscript:

BR	Big Rip
DE	dark energy
DM	dark matter
FLRW	Friedmann–Lemaître–Robertson–Walker
HRDE	holographic Ricci dark energy
EiBI	Eddington-Inspired-Born-Infeld

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