DOI: 10.1016/j.mechmachtheory.2017.05.023

Multioperation capacity of parallel manipulators basing on generic kinematic chain approach

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Abstract

The idea of designing multioperation mechanisms capable of performing different tasks has gained prominence in the last years. These mechanisms, commonly called reconfigurable mechanisms, have the ability to change their configuration. At present, this type of mechanisms is capturing the attention of design engineers because of their great potential in many industrial applications. In this paper, the basis for the development of a methodology intended for the analysis and design of multioperational parallel manipulators is presented. First, the structural synthesis of 6 degree-of-freedom (dof) kinematic chains that can form a 6 dof manipulator is established. Next, a general purpose approach for non-redundant parallel manipulators (PM) will be presented. This procedure enables obtaining the Jacobian matrices of any 6 dof or low-mobility PM whose kinematic chains belong to the library of chains derived from the structural synthesis. To demonstrate the versatility of the procedure, it will be applied to three PM: the first one, a 6 dof PM, the second one, a reconfigurable 6 dof PM, and finally, a low-mobility PM.

Keywords: Multioperation, Parallel manipulator, Generic kinematic chain, Reconfigurable

1. Introduction

Traditionally, in mechanism and machine design, one of the main targets was to create a robust and reliable mechanical system intended for efficiently carrying out a certain particular task. Later on, the need of more adaptableness of the system was materialized into adjustable mechanisms, in which a greater flexibility was demanded. This requirement of higher compliance capacity was achieved by adjusting one or more dimensions using fixing or liberating elements such as screws and extensible bars. In the last 20 years, this initial idea has progressed into a new design concept based on mechanisms capable of performing different operation modes so that they can execute different tasks [1]. These mechanisms with the ability to change their configuration are typically called reconfigurable mechanisms [2-5]. Nowadays, the study of these mechanisms is becoming a relevant field of investigation in specific forums such as [6, 7] because they are demonstrating their potential in a great variety of industrial applications such as packing and folding, structures and deployable antennas, biomedical applications, and so on [8].

At the present time, there exist various studies related to parallel manipulators (PMs) with six degree-of-freedom (dof) kinematic chains in which it is possible to obtain low-mobility PMs by constraining some of the inputs of the manipulator. Subsequently, the investigations that share a common research line with the present paper will be cited. Y. Jin et al. [9, 10] propose several 6 dof PMs with three kinematic chains in which by means of a selective actuation different motions are achieved: spherical (3R), translational (3T), mixed motions (2T1R and 1T2R) or the complete motion of 6 dof. For that, a 2 dof actuator, having one rotation and a translation in the same axis, is installed in each kinematic chain. The same authors present a methodology in [11] for the synthesis of PMs with three kinematic chains having partially decoupled 6 dof.

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P. Fanghella et al. [12] describe a methodology to synthetize PMs by using kinematotropic chains that allow changing the number of dof, their nature or both effects at the same time. Even maintaining the motion pattern of the platform, their geometric characteristics can change. As an example, they show a PM with a Schönflies motion in the end-effector which can change the direction of the rotation axis between two established orientations. X. Kong et al. [13] propose an approach for the synthesis of PMs with multiple operation modes. In particular, they develop several 3 dof PMs that can perform both spherical motion and spatial translational motion.

A. D. Finistauri et al. define a procedure in [14, 15] to generate reconfigurable PMs. The proposed PMs consist of two modules, each of them having three 6 dof kinematic chains, both connected to the fixed and moving platform. Whereas the first module has its three chains permanently linked to the moving platform, the kinematic chains of the second module can be disconnected from the moving platform. While some dof of the first module are being restricted (by blocking one rotation or one translation of the kinematic chains), the chains belonging to the second module are being accordingly disconnected.

Certainly, using a 6 dof manipulator it is possible, theoretically, to perform any type of spatial motion. Nevertheless, for that, the required motion has to be programmed according to the kinematic structure of the chains of the manipulator, and not all the kinematic chains adapt with the same facility to the tracing of a defined motion. For example, a simple translational motion can be easily achieved with a manipulator formed by a certain cartesian structure in its kinematic chains. In fact, a motion as simple and so typical as this one, using partially cartesian kinematic machines, very common in machine-tool applications, is achieved by blocking the unnecessary actuators. However, if we try to get this same motion with a manipulator in which its chains yield a more coupled kinematics, as for example the Gough platform, it will not be that simple. To get that purpose the controlling of all of the degrees of freedom is required, resulting in a much more complex task. As a consequence, to get a similar precision, the number of interpolation points in the trajectory has to be increased. On the other hand, some other aspects that have a greater impact in this case have to be considered, such as calibration, workspace singularities or the interferences among the legs of the manipulator. All of this results in the necessity of studying the functional operation of specific designs of the manipulator.

To do so, the concept of displacements of group structure type will be considered. The set of displacements of a solid in space can be arranged in 12 algebraic entities having group structure [16, 17]. Each of these groups has a specific pure motion pattern. A manipulator in which its moving platform owns a motion pattern of group structure type, has the great advantage that this pattern remains invariable for any posture of the workspace. Under these circumstances, the approach of any type of path planning is more simple. Therefore, the hypothesis of this work is that the greater the quantity of group structure type displacements the moving platform can achieved, the higher the practical effectiveness of the corresponding PM will be. The way of obtaining the set of displacement groups of the platform consists in fixing conveniently the different actuators of the mechanism. Then, the kinematics of the resulting PM is simplified thanks to two causes: the blocking of some dof and the capability to generate permanent motion patterns along the workspace. Consequently, the control of the machine will be faster and more efficient.

To get the set of all possible 12 motion patterns a manipulator with at least 6 dof is required. Hence, in the next section the structural synthesis of 6 dof kinematic chains that can form the manipulator is established. Next, a general purpose approach for non-redundant PM will be developed, subjected to some geometric conditions that will be later on explained, but that they do not imply any limitation to the multioperation capacity (variety of motions) of the platform. This procedure enables obtaining the Jacobian matrices of any 6 dof or low-mobility PM whose kinematic chains belong to the library of chains derived from the previously cited structural synthesis.

2. Structural synthesis of 6 dof kinematic chains

Prior to initiating the structural synthesis, the characteristics of the PM will be defined. The legs of the PM will be non-redundant kinematic chains. Thus, the maximum number of first class kinematic pairs in each chain will be six, knowing that by conveniently orienting them the required 6 dof can be achieved. Similarly, the maximum number of translational joints in each chain will be three, so that positioning them along the three directions in space the 3 translational dof can be obtained.

Initially, only first class kinematic pairs will be considered: prismatic joints (P) and revolute joints (R). Any pair of higher class, such as cylindrical joint (C), universal joint (U) or spherical joint (S) can be obtained by combining P and R pairs.

Afterwards, a methodology for kinematic analysis will be developed, thus some geometrical constraints will be now established. These constraints have the only purpose of simplifying the obtained designs and also restricting

DOI: 10.1016/j.mechmachtheory.2017.05.023

the many possible cases that can arise, knowing that this does not imply a loss of generality at the time of obtaining a certain motion pattern:

1. The directions of two consecutive joints of the chain form 0° or 90°. The same occurs with the directions of the pairs that connect the chains with the moving platform.
2. In the case of a P joint and a R joint being parallel and consecutive (or a R and a P), the distance between the axis of the revolute joint and the slider guide will be considered null. Then, this PR (or RP) results in a cylindrical joint.
3. If the axes of two R joints are consecutive and non-parallel, then the distance must be null. For example, the sequence RR forms a universal joint and the sequence RRR a spherical joint. Particular exceptions such as the SS kinematic chain, with 5 dof in the end-effector and one idle dof, are solved by including a non-null distance resulting in chains SUR, URS, RUS, …, instead of SS chain.
4. The following notation will be used to define the relative position of the different joints forming a chain. Those set of R joints, in which their axes are parallel, will be written with an underline, in italics or in bold. Let us use an example: the chain RRR*RR*R which generates the displacements 3T3R. The first three RRR pairs are parallel between them (they generate 2 translational dof and 1 rotation); the next two *RR* (in italics) are parallel between them and form 90° with the previous set (they generate 1 translational dof and 1 rotation); the last R pair forms 90° with respect to the previous set and defines the third rotational dof. In the case of having three groups of revolute joints (with parallel axes in each group), the third group will be written in bold, as for example the RR*RR***RR** chain. Finally, the actuated joint will be underlined (in case of coinciding with the underline of the first group, then a double underline will be established in the corresponding pair, as for example RR*RR***RR**).

Taking into consideration the previous premises, there exist four assemblages of prismatic and rotational joints that enable obtaining all the possible 6 dof kinematic chains: PPPRRR, PPRRRR, PRRRRR y RRRRRR. In subsequent sections, these four chains and their permutations will be examined.

2.1. PPPRRR chains and their permutations

By permuting the relative positions of the prismatic and revolute pairs of the chain PPPRRR, the following 20 chains are derived: PPPRRR, PPRPRR, PPRRPR, PPRRRP, PRPPRR, PRPRPR, PRPRRP, PRRPPR, PRRPRP, PRRRPP, RPPPRR, RPPRPR, RPPRRP, RPRPPR, RPRPRP, RPRRPP, RRPPPR, RRPPRP, RRPRPP y RRRPPP.

To get three rotational dof in the chain, the axes of the three R pairs will form 90°. Taking into account the defined geometric constraints, additional combinations can be obtained. For example, the chain PPPRRR can result into a PPPS chain or a PPCU chain, according to the disposition of the PR central pairs (parallel or perpendicular). In Table 1, the possible 63 chains that can be obtained from the 20 permutations of PPPRRR are displayed. The resulting chains that have been previously obtained appear crossed out. This table, and subsequent ones, are included in the Appendix for the shake of clarity.

2.2. PPRRRR chains and their permutations

By permuting the relative positions of the prismatic and revolute pairs of the chain PPRRRR, the next 15 chains are obtained: PPRRRR, PRPRRR, PRRPRR, PRRRPR, PRRRRP, RPPRRR, RPRPRR, RPRRPR, RPRRRP, RRPPRR, RRPRPR, RRPRRP, RRRPPR, RRRPRP, RRRRPP.

Considering the adopted geometric criteria, the following cases can arise:

- Three consecutive R pairs, two of them being parallel (RRR): this combination results in one rotational pair and one universal joint (RU), where the axis of the R joint and the first axis of the U joint are parallel, which is indicated with the underline.

- The sequence PRR, where the direction of the translational joint P is parallel to the direction of the two R joints, results in a cylindrical pair and a rotational pair with parallel axes, denoted as CR.

- The sequence PRRP yields two possibilities: CRP or PRC, depending on existence of parallelism with any of the prismatic guides. Nonetheless, the chain CC cannot be obtained as the guides of the two prismatics have to be perpendicular among them.

- The sequence RRRR yields two universal joints, UU or UU, as both cases have the same motion pattern 1T3R. Note that the notation UU ≡ RRRR symbolizes that the parallel axes are the ones associated with the last R joint and the first R joint of the resulting universal joints.

DOI: 10.1016/j.mechmachtheory.2017.05.023

In Table 2, the 151 possible chains obtained from the 15 permutations of PPRRRR are collected. Again, the chains that have been previously obtained are crossed out.

2.3. PRRRRR chains and their permutations

By permuting the relative positions of the pairs in the chain PRRRRR, the following 6 chains are obtained: PRRRRR, RPRRRR, RRPRRR, RRRPRR, RRRRPR y RRRRRP.

Given the structure of these chains, 2 translational dof have to be achieved by combining the five rotational pairs. The two only options with a maximum number of parallel rotational axes are the following:

- Three parallel revolute joints and the remaining two forming 90°. This is the case of the sequence PRRRRR. The combination of any of the parallel R pairs with the prismatic will always yield a cylindrical joint. Otherwise, the three translations would not be achieved. For example, the sequence PRRR will result in CRR. Because of this same reason, the combination of the prismatic with any of the remaining R cannot yield a cylindrical joint.

- Two groups formed by two R joints: the joints in each group having parallel axes and those axes being perpendicular to the ones of the other group; the remaining R perpendicular to the previous group. This yields the sequence PRR*RR*R.

Any other combination with less number of R joints with parallel axes is obtained by carrying out the permutations of the two cited sequences: PRRRRR y PRR*RR*R. Indeed, let us analyse the case RRR: it is obvious that the R joints will not maintain their parallelism when the intermediate R starts to rotate, thus RRR =RRR. In this way, realizing all the possible permutations of the group of rotational pairs RRRRR 8 possible permutations are achieved: RRRRR, RRRRR, RRRRR, RRRRR, RRRRR, RRRRR, RRRRR y RRRRR. By combining those permutations with a P joint, all of the possible variants of the six chains derived from the generic chain PRRRRR will be obtained. The sequence RR*RR*R will be analysed in the same way.

The possible permutations of the chain PRRRRR are collected in Table 3 of the Appendix. In that table, the 69 possible combinations of joints P, R, C, U and S are also displayed. Similarly, in the appendix, Table 4 shows all the possible permutations of the chain PRR*RR*R and the 40 possible combinations of joints P, R, C, U and S.

2.4. RRRRRR chains

In this case, only one possible permutation is possible. The 6 revolute joints must be combined in such a way that they generate 3 translational and 3 rotational dof. Bearing in mind the defined geometric criteria, different variants can be obtained. In particular, two options with maximum number of parallel axes arise:

- Three groups of parallel joints, each group being connected to the previous one by axes forming 90° (for example: **RR**RR*RR*)

- One group having three R joints with parallel axes, another group with two parallel R and perpendicular to the first group, and finally, the remaining R joint perpendicular to the previous group (for example: RRR*RR*R).

Coming from these two chains and creating all possible permutations, the remaining variants of the 6R chain are obtained. In Table 5 all the variants of the chain **RR**RR*RR* are shown, which are obtained by permuting the groups of parallel R. The repeated cases are crossed out. A total of 10 variants are obtained. In Table 6, all the variants of RRR*RR*R chain are displayed. The left column shows the 3 possible combinations of the two groups of pairs *RR*R. Those combinations are at the same time combined with the group RRR. Finally, after eliminating the repeated cases, a total of 16 different combinations are derived.

In Tables 7 and 8, the permutations of the **RR**RR*RR* and RRR*RR*R chains previously displayed in Tables 5 and 6 are collected, obtaining 24 possible combinations of joints P, R, C, U and S. The chains SS, SRRR and RRRS have been directly eliminated as they only define 3 rotational dof and 2 translations.

In total, the number of obtained kinematic chains is 347. All these chains could be automatically analyzed (by software programming) according to the procedure that will be explained in the next section.

3. Modelization of a generic kinematic chain

In this section a general-purpose approach based on the characterisation and analysis of a generic kinematic chain will be developed. This generic chain can be afterwards configured or particularized so as to obtain the parallel manipulators commonly used in practice. It also enables to easily implement the reconfigurability option by

DOI: 10.1016/j.mechmachtheory.2017.05.023

blocking some inputs and deriving the corresponding constraint equations. It probably exists more than one alternative to get this target, but the one proposed here has demonstrated to have a great potential to cover a broad range of possible chains.

Basing on this generic chain, PMs with a maximum of six chains will be generated, as additional chains would only constitute passive legs that would not incorporate any practical utility. Besides, the minimum number of chains will be three, being distributed in different planes in such a way that the rigidity is ensured in the three directions. A common case would be a 6 dof PM formed by three chains, with the actuators located on the fixed platform so as to avoid floating weights. In a manipulator as this one the multioperation capability can be achieved by adequately blocking the actuators of each leg depending on the desired motion pattern the moving platform has to generate.

This general-purpose methodology deals with those robots with a parallel kinematic structure whose kinematic chains (the ones linking the fixed and moving platform) are of serial type. These chains consist of several links connected among them, as well as joined with both platforms, by kinematic joints. The most habitual type of joints are: revolute joint (R), prismatic joint (P), cylindrical joint (C), spherical (S) and universal joint (U).

The generic kinematic chain is defined in the simplest way because afterwards, to analyse the real legs of the PM, it is necessary to work with the equations derived from this chain. Bearing this in mind, some hypotheses have been assumed:

- The generic chain is formed only by prismatic and revolute joints. In the case of the R joint, the associated unit vector defined the direction of the rotation axis, while for the P joint the corresponding unit vector defines the sliding direction.
- These two types of kinematic joints are alternatively located along the serial chain.
- The relative position between two consecutive joints is restricted to the two simplest options: either they have parallel axes or perpendicular axes.
- To achieve the required generality, it is necessary to form the generic chain with six R and six P joints.
- By restricting some of the position parameters of this generic chain, different configurations of 6 dof kinematic chains are obtained.

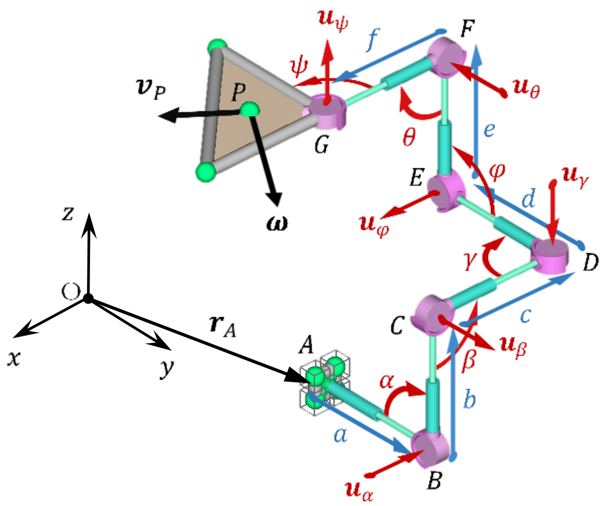

Figure 1: Generic kinematic chain. Joints and parameters

In Fig. 1, the generic serial kinematic chain, together with its variables and corresponding vectors, is shown. The first link is connected to the fixed base, while the last one is joined to the moving platform. A reference point P, considered as the terminal point, will be used to set the velocity equation of the leg.

Initially, the loop equation must be defined. The vectors $\boldsymbol{r}_P$ and $\boldsymbol{r}_A$ being the position vectors of the two extreme points of the loop, then:

$$\boldsymbol{r}_P = \boldsymbol{r}_A + \boldsymbol{a} + \boldsymbol{b} + \boldsymbol{c} + \boldsymbol{d} + \boldsymbol{e} + \boldsymbol{f} + \boldsymbol{GP} \tag{1}$$

DOI: 10.1016/j.mechmachtheory.2017.05.023

Next, each vector associated with a variable-length link is expressed as the product of its module (length) times the unit vector of the corresponding P joints:

$$\boldsymbol{r}_P = \boldsymbol{r}_A + a \cdot \boldsymbol{u}_a + b \cdot \boldsymbol{u}_b + c \cdot \boldsymbol{u}_c + d \cdot \boldsymbol{u}_d + e \cdot \boldsymbol{u}_e + f \cdot \boldsymbol{u}_f + \boldsymbol{GP} \tag{2}$$

In Eq. (2), $\boldsymbol{r}_A$ is the only magnitude that remains constant, thus by making the derivative with respect to time it yields:

$$\begin{gathered} \boldsymbol{v}_P = \dot{a} \cdot \boldsymbol{u}_a + \dot{b} \cdot \boldsymbol{u}_b + \dot{c} \cdot \boldsymbol{u}_c + \dot{d} \cdot \boldsymbol{u}_d + \dot{e} \cdot \boldsymbol{u}_e + \dot{f} \cdot \boldsymbol{u}_f + \\ + \boldsymbol{\omega}_b \times \boldsymbol{b} + \boldsymbol{\omega}_c \times \boldsymbol{c} + \boldsymbol{\omega}_d \times \boldsymbol{d} + \boldsymbol{\omega}_e \times \boldsymbol{e} + \boldsymbol{\omega}_f \times \boldsymbol{f} + \boldsymbol{\omega} \times \boldsymbol{GP} \end{gathered} \tag{3}$$

where $\boldsymbol{v}_P$ is the linear velocity of the coupler point, and the angular velocities of the elements as well as that of the moving platform, are given by:

$$\begin{aligned} \boldsymbol{\omega}_b &= \dot{\alpha} \cdot \boldsymbol{u}_\alpha \\ \boldsymbol{\omega}_c &= \boldsymbol{\omega}_b + \dot{\beta} \cdot \boldsymbol{u}_\beta \\ \boldsymbol{\omega}_d &= \boldsymbol{\omega}_c + \dot{\gamma} \cdot \boldsymbol{u}_\gamma \\ \boldsymbol{\omega}_e &= \boldsymbol{\omega}_d + \dot{\varphi} \cdot \boldsymbol{u}_\varphi \\ \boldsymbol{\omega}_f &= \boldsymbol{\omega}_e + \dot{\theta} \cdot \boldsymbol{u}_\theta \\ \boldsymbol{\omega} &= \boldsymbol{\omega}_f + \dot{\psi} \cdot \boldsymbol{u}_\psi \end{aligned} \tag{4}$$

As it was pointed out at the initial part of Section 2, from this general configuration it is feasible to obtain several kinematic chains commonly used in practice which are formed by other typical types of joints, such as the U, S or C joints. The process is indeed very simple (see Fig. 2):

- U joint: two consecutive R joints having perpendicular axes and with a null-length link between them.
- S joint: three consecutive R joints with mutually perpendicular axes and having null distance.
- C joint: formed by a P joint followed by a R joint whose axis is parallel to the sliding direction of the P.

Taking into account these considerations, the process to get other more complex kinematic joints, such as the PR or the CS, is a straightforward matter. On the other hand, evidently, all the constant-length links of the chain (bars) are modelized by simply restricting the distance associated with the corresponding P joint to remain constant.

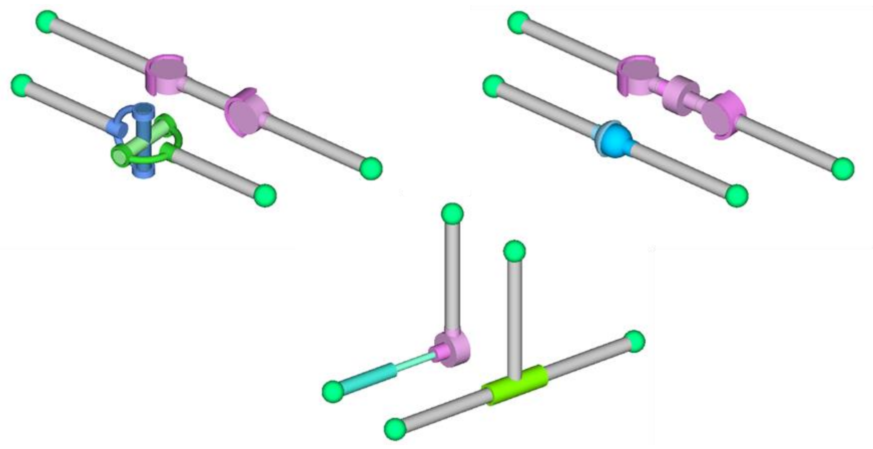

Figure 2: Combinations of P and R joints.

Nevertheless, note that not all the combinations are feasible. Additional constraints have to considered as not all the combinations make sense. As an example, all the possible combinations of three consecutive joints (bearing in mind that consecutive joints are parallel or perpendicular) are shown in Fig. 3. In Fig. 3 (a), the cases in which any of the unit vectors is parallel to another one are displayed. In Fig. 3 (b), two of the three unit vectors are parallel. All the cases in Figs. 3 (a) and 3 (b) have practical sense and, besides, the mathematical relations among the unit vectors can be easily devised. The most obvious cases without any practical sense are those in which the three consecutive unit vectors are parallel, as the ones shown in Fig. 3 (c). These cases have idle rotations or displacements.

DOI: 10.1016/j.mechmachtheory.2017.05.023

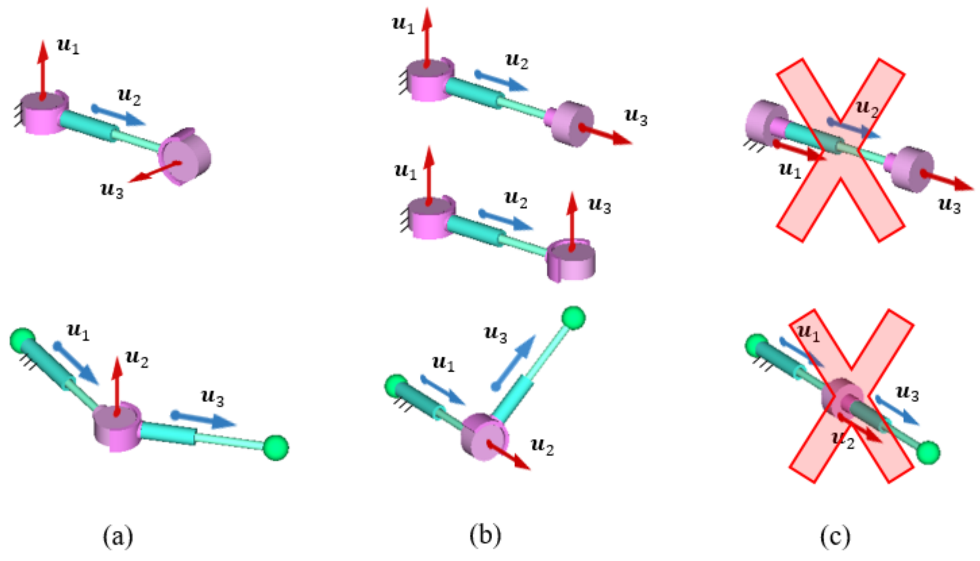

Figure 3: Combinations of three consecutive joints.

Subsequently, some illustrative examples will be shown to demonstrate how this generic chain can be particularized to obtain those commonly used kinematic chains. Additionally, it will be exposed the way the proposed approach enables incorporating the reconfiguration ability.

4. Kinematic chain particularized to the 6-UPS parallel manipulator

The first example will be one of the most well-known PMs, the Gough-Stewart platform. In this case, all the legs of the manipulator are of the same typology, UPS, as represented in Fig. 4.

The UPS chain shown in Fig. 4 is easily derived from the proposed generic kinematic chain (Fig. 1) by particularizing the following parameters:

$$a = b = d = e = f = 0 \qquad \psi = 0 \tag{5}$$

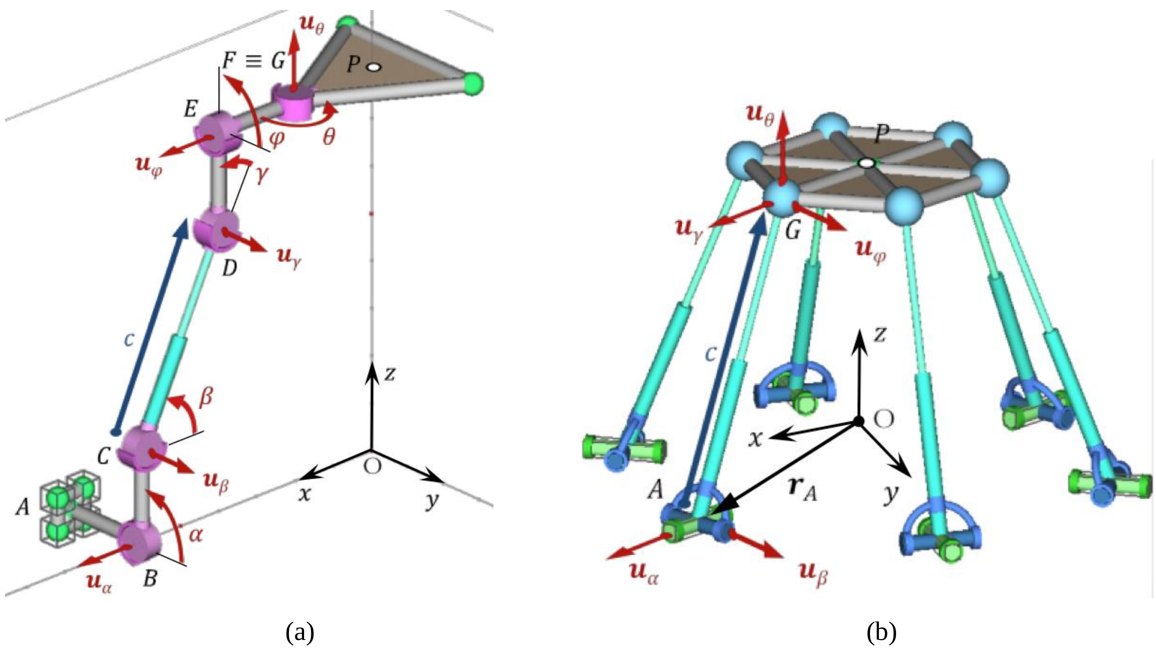

Figure 4: Particularization of the generic kinematic chain to the 6-UPS manipulator.

DOI: 10.1016/j.mechmachtheory.2017.05.023

Then, from the generic chain we get the UPS chain displayed in Fig. 4 (a), which can be modelized as the chains of the 6-UPS PM shown in Fig. 4 (b), by taking into account that the two first R joints form a U joint and, similarly, the last three joints form a spherical joint.

To obtain the velocity equation, we establish the loop equation of the chain of Fig. 4 (a):

$$\boldsymbol{r}_P = \boldsymbol{r}_A + \boldsymbol{c} + \boldsymbol{GP} \tag{6}$$

And differentiating with respect to time, the velocity equation yields:

$$\boldsymbol{v}_P = \dot{c} \cdot \boldsymbol{u}_c + \boldsymbol{\omega}_c \times \boldsymbol{c} + \boldsymbol{\omega} \times \boldsymbol{GP} \tag{7}$$

Where:

$$\begin{aligned} &\boldsymbol{c} = c \cdot \boldsymbol{u}_c \\ &\boldsymbol{\omega}_c = \dot{\alpha} \cdot \boldsymbol{u}_\alpha + \dot{\beta} \cdot \boldsymbol{u}_\beta \\ &\boldsymbol{\omega} = \boldsymbol{\omega}_c + \dot{\gamma} \cdot \boldsymbol{u}_\gamma + \dot{\varphi} \cdot \boldsymbol{u}_\varphi + \dot{\theta} \cdot \boldsymbol{u}_\theta \end{aligned} \tag{8}$$

This mechanism has 6 dof. The output variables of the velocity problem are the components of $\boldsymbol{v}_P$ and $\boldsymbol{\omega}$. The input variables correspond to the $\dot{c}_i$ velocities of each leg i. To get a system of equations that establishes a direct relation between input and output variables, the passive variables $\dot{\alpha}_i$ and $\dot{\beta}_i$ have to be eliminated from the final formulation of the velocity problem. The process to achieve this is indeed very systematic: multiplying the whole velocity equation by a unit vector in such a way that the passive terms are eliminated and only input and output velocities remain. In this case, the term $\boldsymbol{\omega}_c \times \boldsymbol{c}$ has to be eliminated, as it includes the passive velocity $\boldsymbol{\omega}_c$, thus by multiplying Eq. (7) by the unit vector $\boldsymbol{u}_c$ it yields:

$$\boldsymbol{u}_c \cdot \boldsymbol{v}_P = \dot{c} \cdot \boldsymbol{u}_c \cdot \boldsymbol{u}_c + \boldsymbol{u}_c \cdot (\boldsymbol{\omega}_c \times \boldsymbol{c}) + \boldsymbol{u}_c \cdot (\boldsymbol{\omega} \times \boldsymbol{GP}) \tag{9}$$

After carrying out the corresponding operations, we get:

$$\boldsymbol{u}_c \cdot \boldsymbol{v}_P + (\boldsymbol{u}_c \times \boldsymbol{GP}) \cdot \boldsymbol{\omega} = \dot{c} \tag{10}$$

By posing the resulting scalar equation for each of the six legs, the final system of equations of the velocity problem is obtained. As this system is linear in the input and output velocities, it can be expressed in matrix form (Eq. (11)), obtaining thereby the well-known Jacobian matrices:

$$\begin{bmatrix} (\boldsymbol{u}_{c_1})^T & (\boldsymbol{u}_{c_1} \times \boldsymbol{G}_1\boldsymbol{P})^T \\ (\boldsymbol{u}_{c_2})^T & (\boldsymbol{u}_{c_2} \times \boldsymbol{G}_2\boldsymbol{P})^T \\ (\boldsymbol{u}_{c_3})^T & (\boldsymbol{u}_{c_3} \times \boldsymbol{G}_3\boldsymbol{P})^T \\ (\boldsymbol{u}_{c_4})^T & (\boldsymbol{u}_{c_4} \times \boldsymbol{G}_4\boldsymbol{P})^T \\ (\boldsymbol{u}_{c_5})^T & (\boldsymbol{u}_{c_5} \times \boldsymbol{G}_5\boldsymbol{P})^T \\ (\boldsymbol{u}_{c_6})^T & (\boldsymbol{u}_{c_6} \times \boldsymbol{G}_6\boldsymbol{P})^T \end{bmatrix} \begin{bmatrix} \boldsymbol{v}_P \\ \boldsymbol{\omega} \end{bmatrix} = [I] \begin{bmatrix} \dot{c}_1 \\ \dot{c}_2 \\ \dot{c}_3 \\ \dot{c}_4 \\ \dot{c}_5 \\ \dot{c}_6 \end{bmatrix} \tag{11}$$

5. Reconfigurable and low-mobility parallel manipulators

Taking advantage of the generic kinematic chain approach reconfigurable parallel manipulators can be obtained. Additionally, by blocking some of the actuators, low-mobility parallel manipulators can be achieved. As it will be demonstrated, the proposed methodology enables analysing this type of robots as well as obtaining the corresponding constraint equations.

We illustrate the process making use of two manipulators: on the one hand, the 3-UPS parallel manipulator, which can be reconfigured to obtain manipulators of lower mobility. On the other hand, the low-mobility 3-RPS robot in which the moving platform yields only 3 dof [18]. As it will be exposed, once we get the velocity equations of the initial reconfigurable manipulator, by following a very simple process as the same one in previous Section 4, the velocity equations corresponding to the low-mobility robot are directly obtained together with the necessary constraint equations.

The 3-UPS is a 6 dof parallel manipulator formed by three kinematic chains, as represented in Fig. 5 (a). Each kinematic chain incorporates two actuators in order to get the total dof. The first input corresponds to the rotational dof of the first axis of the universal joint (α angle) and the second actuation is applied in the prismatic joint of the leg (c length). Coming from this 6 dof robot, we can apply the reconfiguration option so as to get the low-mobility 3-RPS robot, which is displayed in Fig. 5 (b). This is done by blocking one of the inputs of the initial robot. In this

DOI: 10.1016/j.mechmachtheory.2017.05.023

case, blocking the first rotational joint so that α angle remains constant, the manipulator becomes the 3-RPS represented in Fig. 5 (b), in which only one actuator remains in each leg (the prismatic joint) and, accordingly, the robot has only three dof in its moving platform.

The velocity problem of each robot is established in subsequent sections, making use of the generic kinematic chain approach. It is important to emphasize that if the aim would be analysing only the low-mobility manipulator (as for example, the aforementioned 3-RPS), this can be perfectly done by directly particularizating the generic chain into the RPS chain of the robot of interest.

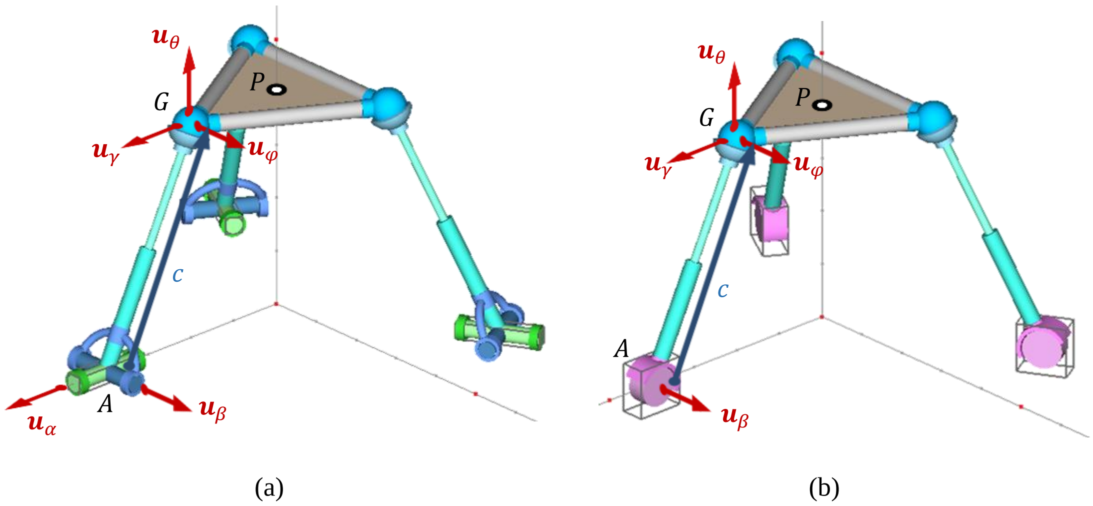

Figure 5: Manipulators (a)3-UPS and (b) 3-RPS

5.1. Reconfigurable 3-UPS manipulator

For the 3-UPS PM, the two inputs of each chain are: the prismatic input (defined by variable c) and the first revolute joint of the U joint (defined by the angular variable α). This manipulator can be reconfigured into a low-mobility manipulator by blocking one of the inputs. As previously explained, to get later on the 3-RPS we will restrict the angular variable of the first R joint. Mathematically, this is done by simply including the condition $\dot{\alpha} = 0$ in the velocity equations.

The chain UPS has been previously analysed in Section 4, obtaining the velocity expression given by Eq. (7). To eliminate the passive variables and get a direct relation between inputs ($\dot{c}_i$ and $\dot{\alpha}_i$) and outputs (components of $\boldsymbol{v}_P$ and $\boldsymbol{\omega}$), we proceed as follows:

First, as it was done in Section 4, apply the dot product to Eq. (7) multiplying by the unit vector $\boldsymbol{u}_c$, obtaining for each leg the velocity equation (10), which in our case of three legs yields:

$$\begin{bmatrix} (\boldsymbol{u}_{c_1})^T & (\boldsymbol{u}_{c_1} \times \boldsymbol{G}_1\boldsymbol{P})^T \\ (\boldsymbol{u}_{c_2})^T & (\boldsymbol{u}_{c_2} \times \boldsymbol{G}_2\boldsymbol{P})^T \\ (\boldsymbol{u}_{c_3})^T & (\boldsymbol{u}_{c_3} \times \boldsymbol{G}_3\boldsymbol{P})^T \end{bmatrix} \begin{bmatrix} \boldsymbol{v}_P \\ \boldsymbol{\omega} \end{bmatrix} = \begin{bmatrix} \dot{c}_1 \\ \dot{c}_2 \\ \dot{c}_3 \end{bmatrix} \tag{12}$$

Second, multiplying by unit vector $\boldsymbol{u}_\beta$:

$$\boldsymbol{v}_P \cdot \boldsymbol{u}_\beta = \dot{c} \cdot \boldsymbol{u}_c \cdot \boldsymbol{u}_\beta + (\boldsymbol{\omega}_c \times \boldsymbol{c}) \cdot \boldsymbol{u}_\beta + (\boldsymbol{\omega} \times \boldsymbol{GP}) \cdot \boldsymbol{u}_\beta \tag{13}$$

And operating as follows:

$$(\boldsymbol{\omega}_c \times \boldsymbol{c}) \cdot \boldsymbol{u}_\beta = \left(\boldsymbol{u}_\beta \times (\dot{\alpha} \cdot \boldsymbol{u}_\alpha + \dot{\beta} \cdot \boldsymbol{u}_\beta)\right) \cdot \boldsymbol{c} = \dot{\alpha} \cdot (\boldsymbol{u}_\beta \times \boldsymbol{u}_\alpha) \cdot \boldsymbol{c} + \dot{\beta} \cdot (\boldsymbol{u}_\beta \times \boldsymbol{u}_\beta) \cdot \boldsymbol{c} \tag{14}$$

$$\boldsymbol{u}_\beta \cdot \boldsymbol{v}_P = \dot{c} \cdot 0 + \dot{\alpha} \cdot (\boldsymbol{u}_\beta \times \boldsymbol{u}_\alpha) \cdot c \cdot \boldsymbol{u}_c + \dot{\beta} \cdot 0 - (\boldsymbol{u}_\beta \times \boldsymbol{GP}) \cdot \boldsymbol{\omega} \tag{15}$$

Then, the three remaining equations of velocities are achieved:

DOI: 10.1016/j.mechmachtheory.2017.05.023

$$\boldsymbol{u}_\beta \cdot \boldsymbol{v}_P + (\boldsymbol{u}_\beta \times \boldsymbol{GP}) \cdot \boldsymbol{\omega} = \dot{\alpha} \cdot c \cdot (\boldsymbol{u}_\beta \times \boldsymbol{u}_\alpha) \cdot \boldsymbol{u}_c \tag{16}$$

Again, expressed in a matrix form it yields:

$$\begin{bmatrix} (\boldsymbol{u}_{\beta_1})^T & (\boldsymbol{u}_{\beta_1} \times \boldsymbol{G}_1\boldsymbol{P})^T \\ (\boldsymbol{u}_{\beta_2})^T & (\boldsymbol{u}_{\beta_2} \times \boldsymbol{G}_2\boldsymbol{P})^T \\ (\boldsymbol{u}_{\beta_3})^T & (\boldsymbol{u}_{\beta_3} \times \boldsymbol{G}_3\boldsymbol{P})^T \end{bmatrix} \begin{bmatrix} \boldsymbol{v}_P \\ \boldsymbol{\omega} \end{bmatrix} = \begin{bmatrix} \dot{\alpha}_1 \cdot c_1 \cdot (\boldsymbol{u}_{\beta_1} \times \boldsymbol{u}_{\alpha_1}) \cdot \boldsymbol{u}_{c_1} \\ \dot{\alpha}_2 \cdot c_2 \cdot (\boldsymbol{u}_{\beta_2} \times \boldsymbol{u}_{\alpha_2}) \cdot \boldsymbol{u}_{c_2} \\ \dot{\alpha}_3 \cdot c_3 \cdot (\boldsymbol{u}_{\beta_3} \times \boldsymbol{u}_{\alpha_3}) \cdot \boldsymbol{u}_{c_3} \end{bmatrix} \tag{17}$$

From equations (12) and (17), the Jacobians of the direct and inverse problems are clearly:

$$[J_D] \begin{bmatrix} \boldsymbol{v}_P \\ \boldsymbol{\omega} \end{bmatrix} = [J_I] [\dot{c}_1 \quad \dot{c}_2 \quad \dot{c}_3 \quad \dot{\alpha}_1 \quad \dot{\alpha}_2 \quad \dot{\alpha}_3]^T \tag{18}$$

The inverse Jacobian being:

$$[J_I] = \begin{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} & \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \\ \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} & \begin{bmatrix} c_1 \cdot (\boldsymbol{u}_{\beta_1} \times \boldsymbol{u}_{\alpha_1}) \cdot \boldsymbol{u}_{c_1} & 0 & 0 \\ 0 & c_2 \cdot (\boldsymbol{u}_{\beta_2} \times \boldsymbol{u}_{\alpha_2}) \cdot \boldsymbol{u}_{c_2} & 0 \\ 0 & 0 & c_3 \cdot (\boldsymbol{u}_{\beta_3} \times \boldsymbol{u}_{\alpha_3}) \cdot \boldsymbol{u}_{c_3} \end{bmatrix} \end{bmatrix} \tag{19}$$

5.2. 3-R<u>P</u>S low-mobility manipulator

As it has been explained, the way of obtaining the equations associated to the 3-RPS is almost direct. We include the constraint associated with the blocking of the rotational actuator, which implies $\dot{\alpha}_i = 0$. Making these velocities null results in:

$$[J_D] \begin{bmatrix} \boldsymbol{v}_P \\ \boldsymbol{\omega} \end{bmatrix} = [J_I] \begin{bmatrix} \dot{c}_1 \\ \dot{c}_2 \\ \dot{c}_3 \end{bmatrix} \tag{20}$$

Where the Jacobians are now:

$$[J_D] = \begin{bmatrix} (\boldsymbol{u}_{c_1})^T & (\boldsymbol{u}_{c_1} \times \boldsymbol{G}_1\boldsymbol{P})^T \\ (\boldsymbol{u}_{c_2})^T & (\boldsymbol{u}_{c_2} \times \boldsymbol{G}_2\boldsymbol{P})^T \\ (\boldsymbol{u}_{c_3})^T & (\boldsymbol{u}_{c_3} \times \boldsymbol{G}_3\boldsymbol{P})^T \\ (\boldsymbol{u}_{\beta_1})^T & (\boldsymbol{u}_{\beta_1} \times \boldsymbol{G}_1\boldsymbol{P})^T \\ (\boldsymbol{u}_{\beta_2})^T & (\boldsymbol{u}_{\beta_2} \times \boldsymbol{G}_2\boldsymbol{P})^T \\ (\boldsymbol{u}_{\beta_3})^T & (\boldsymbol{u}_{\beta_3} \times \boldsymbol{G}_3\boldsymbol{P})^T \end{bmatrix} \qquad [J_I] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{21}$$

As it can be observed, the direct Jacobian is the same as that of the 3-UPS, while in the inverse Jacobian the last three rows are cancelled. Precisely, the nullity of these rows, due to the imposed constraint $\dot{\alpha}_i = 0$, establishes the constraint equations that relate the output velocities of this low-mobility manipulator. Therefore, these constraint equations are:

$$\boldsymbol{u}_{\beta_i} \cdot \boldsymbol{v}_P + (\boldsymbol{u}_{\beta_i} \times \boldsymbol{G}_i\boldsymbol{P}) \cdot \boldsymbol{\omega} = 0 \qquad i = 1, 2, 3 \tag{22}$$

DOI: 10.1016/j.mechmachtheory.2017.05.023

6. Conclusions

In this work, the basis of a methodology for the analysis and design of parallel manipulators capable of achieving multiple operation modes has been presented. To get the set of all possible 12 motion patterns in the end-effector, the structural synthesis of 6 dof kinematic chains that can form a 6 dof manipulator has been done. Then, a general-purpose approach based on the characterisation and analysis of a generic kinematic chain is presented. This generic chain can be afterwards configured or particularized so as to obtain many parallel manipulators commonly used in practice. It also enables to easily implement the reconfigurability option by blocking some inputs and deriving the corresponding constraint equations. Basing on this generic chain, we illustrate the procedure for a 6 dof PM, a reconfigurable PM and a low-mobility PM. The approach proposed in this paper has demonstrated to have a great potential to cover the analysis of a broad range of possible chains to be implemented in reconfigurable parallel manipulators.

Funding sources:

This work was supoorted by the Spanish Government through the *Ministerio de Economía y Competitividad* (Project DPI2015-67626-P (MINECO/FEDER, UE)), the financial support from the University of the Basque Country (UPV/EHU) under the program UFI 11/29 and the support to the research group, through the project with ref. IT949-16, given by the *Departamento de Educación, Política Lingüística y Cultura* of the Regional Government of the Basque Country.

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DOI: 10.1016/j.mechmachtheory.2017.05.023

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DOI: 10.1016/j.mechmachtheory.2017.05.023

Appendix

Permutation	Kinematic chain	Possible chains
1	PPPRRR	PPPS, PPCU
2	PPRPRR	PPRPU, PCPU, PCCR, ~~PPCU~~, PPRCR
3	PPRRPR	PPUPR, PCRPR, ~~PCCR~~, PCRC, ~~PPRCR~~, PPUC
4	PPRRRP	PPSP, PCUP, ~~PCRC~~, ~~PPUC~~
5	PRPPRR	PRPPU, CPPU, CPCR, ~~PCPU, PCCR~~, PRPCR
6	PRPRPR	PRPRPR, CPRPR, ~~CPCR~~, CCPR, CPRC, CCC, ~~PCRPR~~, ~~PCCR, PCRC~~, PRCPR,PRCC, ~~PRPCR~~, PRPRC
7	PRPRRP	PRPUP, CPUP, CCRP, ~~CCC, CPRC, PCUP~~, ~~PCRC~~, PRCRP, ~~PRCC, PRPRC~~
8	RPPPRR	RPPPU, ~~CPPU, CPCR~~, RPPCR
9	RPPRPR	RPPRPR, ~~CPRPR, CPCR, CPRC, CCPR, CCC~~, RPCPR, RPCC, RPPRC
10	RPPRRP	RPPUP, ~~CPUP, CCRP, CCC, CPRC~~, RPCRP, ~~RPCC, RPPRC~~
11	PRRPPR	PUPPR, CRPPR, CRPC, PUPC, ~~PRCPR, PRCC, CCC, CCPR~~
12	PRRPRP	PUPRP, CRPRP, ~~PRCRP, PUPC, CCRP, CRPC~~, CRCP, ~~PRCC, CCC~~
13	PRRRPP	PSPP, CUPP, PUCP, ~~CRCP~~
14	RPRPPR	RPRPPR, ~~CRPPR, RPCPR~~, RPRPC, ~~CCPR, CRPC, RPCC, CCC~~, RCPPR, RCPC
15	RPRPRP	RPRPRP, ~~CRPRP, RPCRP~~, RCPRP, RPRCP, ~~RPRPC, CCRP~~, ~~CRCP, CRPC, CCC, RPCC~~, RCCP, ~~RCPC~~
16	RPRRPP	RPUPP, ~~CUPP~~, RCRPP, ~~RPRCP, CRCP, RCCP~~
17	RRPPPR	UPPPR, ~~RCPPR~~, UPPC, ~~RCPC~~
18	RRPPRP	UPPRP, ~~RCPRP~~, UPCP, ~~UPPC, RCCP, RCPC~~
19	RRPRPP	UPRPP, ~~RCRPP~~, UCPP, ~~UPCP, RCCP~~
20	RRRPPP	SPPP, ~~UCPP~~

Table 1: Possible chains obtained from the permutations of PPPRRR

DOI: 10.1016/j.mechmachtheory.2017.05.023

Permutation	**Kinematic chain**	**Variants with pairs RR**	**Possible chains**
1	PPRRRR	PPRRRR	PPRS,PCS
		PPRRRR	PPUU,PCRU
		PPRRRR	PPSR,PCUR
		PPRRRR	~~PPRS,PPSR,PCS~~,PPUU
2	PRPRRR	PRPRRR	CPS,~~PCS~~,PRCU
		PRPRRR	PRPRU,CPRU,PRCU,CCU,~~PCRU~~
		PRPRRR	PRPUR,CPUR,PRCRR,CCRR,~~PCUR~~
		PRPRRR	~~CPS,PCS~~,PRPS,PRCU,CCU
3	RPPRRR	RPPRRR	RPPS,~~CPS~~,RPCU
		RPPRRR	RPPRU,~~CPRU~~,RPCU,~~CCU~~
		RPPRRR	RPPUR,~~CPUR~~,RPCRR,~~CCRR~~
		RPPRRR	~~CPS~~,RPCU,~~RPPS,CCU~~
4	PRRPRR	PRRPRR	PRRPU,CRPU,~~PRCU~~,PRRCR,CRCR
		PRRPRR	PUPU,CRPU,~~PRCU~~,PUCR,~~CCU~~,CRCR
		PRRPRR	PUPRR,CRPRR,~~PRCRR~~,PUCR,CRCR,~~CCRR~~
		PRRPRR	PUPU,PUCR,~~PRCU~~,CRPU,~~CCU~~,CRCR
5	PRRRPR	PRRRPR	PRUPR,~~PRRCR,CRCR~~,CUPR,CUC,PRUC
		PRRRPR	PURPR,~~PUCR~~,PURC,CRRPR,~~CRCR~~,CRRC
		PRRRPR	PSPR,PSC,CUPR,CUC,~~CRCR,PUCR~~
		PRRRPR	~~PSPR,PSC~~,CUPR,CUC,~~CRCR,PUCR~~
6	PRRRRP	PRRRRP	PRSP,CSP,~~PRUC,CUC~~
		PRRRRP	PUUP,~~PURC~~,CRUP,~~CRRC~~
		PRRRRP	PSRP,~~PSC~~,CURP,~~CUC~~
		PRRRRP	~~PSRP,PRSP,PSC,CSP,CUC~~,PUUP
7	RPRRPR	RPRRPR	RPUPR,RPUC,~~CUPR,CUC~~,RCRPR,RCCR,RCRC
		RPRRPR	RPRRPR,RPRRC,RPRCR,~~CRRPR~~,RCRPR,~~CRRC~~, ~~CRCR~~,RCRC
		RPRRPR	RPUPR,RPUC,RPRCR,~~CUPR,CUC,CRCR~~,RCCR, RCRC
		RPRRPR	RPUPR,~~CUPR~~,RPUC,~~CUC~~,RCRPR,RCRC,RCCR, RPRCR
8	RRPPRR	RRPPRR	RRPPU,RRPCR,RCPU,~~RCCR~~
		RRPPRR	UPPU,UPCR,RCPRR,RCPU,RCCR
		RRPPRR	UPPRR,UPCR,RCPRR,~~RCCR~~
		RRPPRR	UPPU,UPCR,RCPU,~~RCCR~~

DOI: 10.1016/j.mechmachtheory.2017.05.023

9	RPRPRR	RPRPRR	RPRPU,RPRCR,RCPU,RCCR,RPCU,CRPU,CRCR
		RPRPRR	RPRPU,RPRCR,RPCU,RCPU,RCCR,CCU,CRPU,CRCR
		RPRPRR	RPRPRR,RPRCR,RCPRR,RCCR,CRPRR,CRCR,CCRR,RPCRR
		RPRPRR	RPRPU,RPRCR,RPCU,RCPU,RCCR,CCU,CRCR,CRPU
10	RRRRPP	RRRRPP	RSPP,RUCP
		RRRRPP	UUPP,URCP
		RRRRPP	SRPP,SCP
		RRRRPP	SRPP,SCP,RSPP,UUPP
11	RRRPRP	RRRPRP	RRCRP,RRCC,RUPRP,RUPC,RUCP
		RRRPRP	URPRP,URPC,URCP,UCRP,UCC
		RRRPRP	SPC,SCP,UCRP
		RRRPRP	SPRP,SPC,SCP,UCRP,UCC
12	RRRPPR	RRRPPR	RUPPR,RUPC,RRCPR,RRCC
		RRRPPR	URPPR,URPC,UCPR,UCC
		RRRPPR	SPPR,SPC,UCPR
		RRRPPR	SPPR,SPC,UCPR,UCC
13	RRPRRP	RRPRRP	RRPUP,RRPRC,RRCRP,RRCC,RCUP, RCRC
		RRPRRP	UPUP,UPRC,RCUP,RCRC,UCC,UCRP
		RRPRRP	UPRRP,UPRC,UCRP,RCRRP,RCRC
		RRPRRP	UPUP,UPRC,RCUP,RCRC,UCC,UCRP
14	RPRRRP	RPRRRP	RPUC,RPSP,RCRC,RCUP,CUC,CSP
		RPRRRP	RPRUP,RPRRC,CRUP,CRRC,RCUP,RCRC
		RPRRRP	RCRC,RPUC,RPURP,CUC,RCRRP,CURP
		RPRRRP	RPSP,CSP,RCRC,RCUP,CUC,RPUC
15	RRPRPR	RRPRPR	RRPRPR,RCRPR,RRCPR,RRPRC,RRCC,RCCR,RRPCR,RCRC
		RRPRPR	UPRPR,UPRC,UPCR,UCC,RCRPR,RCCR,RCRC,UCPR
		RRPRPR	RCRC,RCRPR,UPRPR,UPRC,UCPR,RCCR,UPCR
		RRPRPR	UPRPR,UCPR,UPRC,UPCR,UCC,RCRPR,RCRC,RCCR

Table 2: Possible chains resulting from the permutations of PPRRRR

DOI: 10.1016/j.mechmachtheory.2017.05.023

Permutation	Chain	Variants with RRR	Possible chains
1	PRRRRR	PRRRRR	CRS
		PRRRRR	PURU,CRRU
		PRRRRR	PSRR,CURR
		PRRRRR	CUU,CSR,~~CRS~~,PRSR,PRRS,PRUU
		PRRRRR	PUS,CRS
		PRRRRR	PSU,CUU
		PRRRRR	~~PSRR~~,~~CSR~~,PUUR,CURR,~~PRSR~~
		PRRRRR	~~PSU~~,CUU,~~CSR~~,~~PUS~~,~~CRS~~
2	RPRRRR	RPRRRR	~~CRS~~
		RPRRRR	RCRU
		RPRRRR	RPURR,~~CURR~~,RCRRR
		RPRRRR	~~CRS~~,RCS,~~CUU~~,~~CSR~~
		RPRRRR	RCS,~~CRS~~
		RPRRRR	~~CUU~~,RCRU
		RPRRRR	~~CSR~~,~~CURR~~,RCUR,RPSR
		RPRRRR	~~CSR~~,~~RCS~~,~~CUU~~,~~CRS~~
3	RRPRRR	RRPRRR	~~RCS~~,RRCU
		RRPRRR	UCU,~~RCRU~~
		RRPRRR	UCRR
		RRPRRR	~~RCS~~,RRCU
		RRPRRR	~~UPS~~,UCU,~~RCS~~
		RRPRRR	UCU,~~RCRU~~
		RRPRRR	UPUR,~~RCUR~~,UCRR
		RRPRRR	UPS,UCU,RCS
4	RRRPRR	RRRPRR	~~RRCU~~
		RRRPRR	~~UCU~~,URCR
		RRRPRR	SCR,~~UCRR~~
		RRRPRR	RUPU,~~RRCU~~,RUCR
		RRRPRR	~~UCU~~,URPU,URCR
		RRRPRR	SPU,~~UCU~~,SCR
		RRRPRR	~~SCR~~,~~UCRR~~
		RRRPRR	~~SPU~~,~~UCU~~,~~SCR~~
5	RRRRPR	RRRRPR	RRUPR,RRRCR,RRUC
		RRRRPR	~~URCR~~
		RRRRPR	~~SCR~~,SRC
		RRRRPR	RSC,RSPR,~~RUCR~~
		RRRRPR	UUC,~~URCR~~
		RRRRPR	~~SCR~~,SRC,SRPR
		RRRRPR	UUC,~~SRC~~,~~SCR~~,UUPR,SRPR,~~RSC~~,~~RSPR~~
		RRRRPR	~~SRC~~,UUC,~~RSC~~,~~SRPR~~,UUPR,~~RSPR~~
6	RRRRRP	RRRRRP	RRSP,~~RRUC~~
		RRRRRP	URUP,URRC
		RRRRRP	~~SRC~~
		RRRRRP	~~RSC~~,~~RRSP~~,RUUP,RRUC,RSRP
		RRRRRP	~~UUC~~,USP
		RRRRRP	SUP,RUUP,~~SRC~~
		RRRRRP	~~SRC~~,~~UUC~~,~~RSC~~,SRRP,UURP,~~RSRP~~
		RRRRRP	~~SUP~~,~~USP~~,~~UUC~~,~~SRC~~,~~RSC~~,~~RSRP~~

Table 3: Possible chains obtained from permutations of PRRRRR

DOI: 10.1016/j.mechmachtheory.2017.05.023

Permutation	**Kinematic chain**	**Variants with joints RRRR**	**Possible chains**
1	PRRRRR	PRRRRR	PRUU,CUU
		PRRRRR	PRSR,CSR
		PRRRRR	PUUR,CRUR
		PRRRRR	PRRS,CRS,PRSR,CSR,PRUU,CUU
		PRRRRR	PUUR,PUS,CRS
		PRRRRR	PSU,CUU
		PRRRRR	PSRR,PRSR,CSR
		PRRRRR	PUS,PSU,CUU,CRS,CSR
2	RPRRRR	RPRRRR	RPUU,CUU,RCRU
		RPRRRR	RPSR,CSR,RCUR
		RPRRRR	RPRUR,CRUR,RCUR
		RPRRRR	RPRS,CRS,RCS,RPSR,CSR,RPUU,CUU
		RPRRRR	RPRS,RCS,CRS
		RPRRRR	RPUU,CUU,RCRU
		RPRRRR	RPSR,CSR,RPSR,RCUR
		RPRRRR	RPUU,RPRS,RPSR,CUU,CRS,CSR,RCS
3	RRPRRR	RRPRRR	RRPRU,RCRU,RRCU
		RRPRRR	RRPUR,RCUR,RRCRR
		RRPRRR	UPUR,RCUR,UCRR
		RRPRRR	RRPS,RRCU,RCS
		RRPRRR	UPS,RCS,UCU
		RRPRRR	UPRU,UCU,RCRU
		RRPRRR	UPUR,UCRR,RCUR
		RRPRRR	UPS,RCS,UCU
4	RRRPRR	RRRPRR	RUPU,RUCR,RRCU
		RRRPRR	RUPRR,RUCR,RRCRR
		RRRPRR	URPRR,URCR,UCRR
		RRRPRR	RUPU,RUCR,RRCU
		RRRPRR	URPU,URCR,UCU
		RRRPRR	SPU,SCR,UCU
		RRRPRR	SPRR,SCR,UCRR
		RRRPRR	SPU,SCR,UCU

DOI: 10.1016/j.mechmachtheory.2017.05.023

5	RRRRPR	RRRRPR	RURPR,RURC,RUCR
		RRRRPR	RSPR,RSC,RUCR
		RRRRPR	UUPR,UUC,URCR
		RRRRPR	RSPR,RSC,RUCR
		RRRRPR	UUPR,UUC,URCR
		RRRRPR	SRPR,SRC,SCR
		RRRRPR	SRPR,RSPR,SRC,RSC,UUPR,UUC
		RRRRPR	RSC,SRC,RSPR,SRPR,UUC,UUPR,SCR
6	RRRRRP	RRRRRP	RUUP,RURC
		RRRRRP	RURRP,RRURP,RURC,RRUC
		RRRRRP	UURP,UUC
		RRRRRP	RRSP,RSRP,RSC,RUUP
		RRRRRP	USP,UUC,UURP
		RRRRRP	SUP,SRC
		RRRRRP	SRRP,SRC,RSRP,RSC
		RRRRRP	USP,SUP,RSC,SRC,UUC

Table 4: Possible chains resulting from the permutations of PRRRRR

DOI: 10.1016/j.mechmachtheory.2017.05.023

	Permutations of chain RRRRRR and equivalences				
RRRR=RRRR	RRRRRR	RRRRRR=RRRRRR =RRRRRR	RRRRRR=RRRRRR	RRRRRR= RRRRRR	RRRRRR= RRRRRR
	RRRRRR =RRRRR R	RRRRRR=RRRRRR	RRRRRR=RRRRRR =RRRRRR	RRRRRR= RRRRRR	———
RRRR=RRRR =RRRR	RRRRRR =RRRRR R	———	———	———	———
RRRR=RRRR =RRRR	———	———	———	———	———

Tabla 5: Variants of chain **RR**RRRR

	Permutations of chain RRRRRR and equivalences				
RRR	RRRRRR	RRRRRR=RRRRRR	RRRRRR=RRRRRR= RRRRRR	RRRRRR= RRRRRR	———
	RRRRRR=RRRRRR	RRRRRR	RRRRRR	———	———
RRR=RRR	RRRRRR	RRRRRR	RRRRRR	———	———
RRR	RRRRRR	RRRRRR=RRRRRR	RRRRRR=RRRRRR	RRRRRR= RRRRRR	RRRRRR
	RRRRRR	———	———	———	———

Table 6: Variants of chain RRRRRR

DOI: 10.1016/j.mechmachtheory.2017.05.023

Permutation of RRRRRR	**Possible chains**
RRRRRR	**RU**UR
RRRRRR	RSU
RRRRRR	USR
RRRRRR	UUU
RRRRRR	RSU,RUS,RRSR
RRRRRR	UUU,URS,USR
RRRRRR	SRU,UUU,RSU
RRRRRR	SUR,USR,RSRR
RRRRRR	UUU,SUR,SRU,RUS,URS,USR,RSU
RRRRRR	SUR,SRU,RUS,URS

Table 7: Possible chains obtained from permutations of **RR**RRRR

Permutations of RRRRRR	**Possible chains**
RRRRRR	RRUU
RRRRRR	RURU
RRRRRR	RSRR
RRRRRR	URS
RRRRRR	RUUR
RRRRRR	UUU
RRRRRR	USR
RRRRRR	RRUU,RRSR
RRRRRR	SRU
RRRRRR	UURR,RSRR
RRRRRR	RRSR
RRRRRR	URUR
RRRRRR	UURR
RRRRRR	RSRR,RRSR
RRRRRR	RSU
RRRRRR	SUR

Table 8: Possible chains obtained from permutations of RRRRRR