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Translational parallel manipulator with Pa^2 kinematic joints

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Abstract.

The kinematic pair Pa^2 is composed of two interlinked parallelograms. It has two degrees of freedom that generate a translational plane variable with position. It has a structure different from the PaPa pair, which is composed also by two parallelograms but generates a constant translational plane. Currently, the Pa^2 pair is used at conceptual level but it is not used in almost any practical application. There are advantages and drawbacks in using it. The main drawback is the high number of redundant constraints that this pair possesses. However, substituting carefully the revolute joints by spherical joints can eliminate these redundant constraints. Also, this pair constitutes a more rigid structure that replaces adequately the problematic passive prismatic joints.
In this paper, will be presented a preliminary study of a translational parallel manipulator (PM) based on the use of the Pa^2 pair: the 3-PPa^2 that contains redundant constraints in its global structure. To study the potentiality of the PM presented in this paper, the following analyses will be done: position and velocity (direct and inverse kinematics), workspace and singularity analysis. Also the potentiality to be optimised will be studied.

Key words: Pa^2 joint, translational parallel manipulator, kinematic analysis.

1 Introduction

In pick-and-place operations, three translations are commonly required to move the object from one position to another. In this case, a 6-DOF fully-parallel ma-

nipulator is too expensive to be used because six actuators are required to control the posture of end-effector. Therefore, it is more economical to employ lower mobility parallel robots since less actuators are used. Additionally, lower mobility parallel robots have simpler architecture and simpler forward and inverse kinematics.
In the field of parallel manipulators generating three translations, many architectures [1,2] have been proposed by academic researchers in past years. Between them, Delta [2] robot could be highlighted as a successful application of a lower mobility architecture, which is famous for its high acceleration capabilities. The end-effector is triangular and is connected to the legs by revolute (R) joints. Also, each leg contains a parallelogram whose coupler link generates a circumferential translation with respect to its opposite bar.
In this paper, to explore new three-translation generators, the Pa^2 pair, which contains two interlinked parallelograms, is used to build a translational PM.

2 PMs with Pa and Pa^2 pairs

A Pa kinematic pair (also called Π pair) is composed of one parallelogram, which can generate a circumferential translation with respect to the fixed bar. An early application of Pa pairs in the structural synthesis of parallel robots can be found in the manipulator developed at Maryland University (or in its equivalent without redundant restrictions, the Delta robot). Its moving platform can perform three translations in 3D space, and its legs are RRPaR kinematic chains. In the field of academic research, the Pa pair was firstly introduced by Wohlhart [3]. After then, different architectures with Pa pairs have been proposed [4].
In comparison with the Pa pair, Pa^2 pairs have received little attention by researchers. As far as we know, there are no PMs with Pa^2 pairs in the scientific bibliography. However, it is worth noticing that some structures (called Pa^2 equivalents) with similar functions as Pa^2 have been involved in some research developments.
A more similar case is proposed by Angeles [5]. He constructed a Π^2 pair with two parallelograms sharing the same moving platform and the same fixed one, while the joints in Π^2 are replaced by universal joints. This Π^2 pair is characterized by a 2-DOF displacement generator being its displacements elements of the T^3 subgroup. In addition, the points of the moving platform have a spherical motion with respect to the base plate. These kinematic characteristics are similar to those of the Pa^2 pair.
Another architecture with a function equivalent to a Pa^2 joint is the use of two Pa pairs in the Micro Finger [6]. In this case, the two Pa pairs are connected in serial instead of in parallel. The two Pa pairs are interlinked with an angle, while the first Pa's moving platform is the fixed plate of the second Pa. Similar to the Micro Finger, in [7] two Pa pairs connected in serial are also used as a Schönflies Motion Generator.

In the design proposed in this paper, the Pa pairs are located as shown in figure 1. Using Malishev's formula, the mobility analysis of the Pa^2 pair has been done obtaining two translational degrees of freedom (2T). Also, there are eight geometrical redundant restrictions introduced by the R joints. Most of them can be eliminated replacing carefully some R joints by S joints.

3 Pa^2 kinematic joint study

As shown in the figure 1, a Pa^2 pair has three parts: two Pa pairs parallel to each other and a third Pa pair cross-linked with the other Pa pairs. The two parallel Pa pairs form two planes: upper and lower, being the upper plane always parallel to the lower one. In this Pa^2 pair, only R joints are used to connect the two adjacent links. The axes of the R joints in the two parallel Pa pairs have the same direction, while the axes in the cross-linked Pa are perpendicular to them.

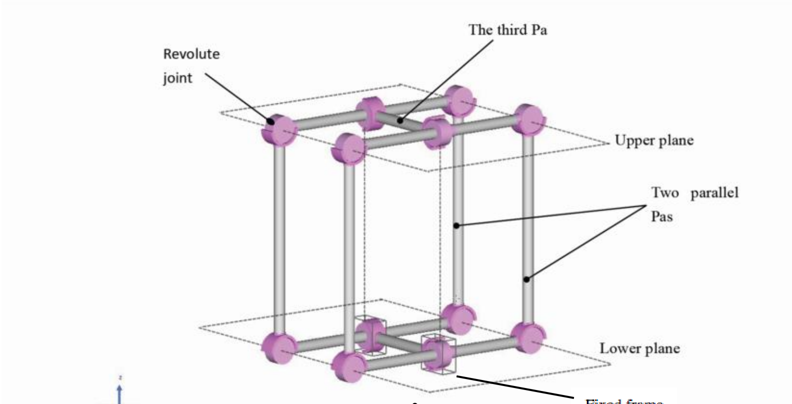

Fig.1 Pa^2 joint

In order to study the potentiality of the Pa^2 pair, a fixed reference frame O-$X_FY_FZ_F$ is defined as shown in the figure 2. The origin of the reference frame O is located at the center of the lower link, X_F axis is perpendicular to the cross-linked Pa^2 plane and goes through the center of the lower link, Y_F axis is defined along the lower link, while Z_F axis is obtained applying the right hand rule. In addition, point P is defined at the center of the upper link.

Without considering the third parallelogram, the two parallel parallelograms would generate 1-DOF circumferential translation along X_F axis if they move simultaneously with respect to the lower plane with an angle β. Similarly, the third Pa could just generate a circumferential translation along Y_F axis with an angle α without the constraints of the two parallel parallelograms. Thus, α and β are rotational parameters of Pa^2. Once interlinking the two parallel Pa pairs with the third Pa, the new structure would generate 2-DOF translations with respect to the plane X_FY_F. Besides, any point located at the upper plane has a spherical trajectory with respect to the fixed one with a radius of OP (the common length of Pa^2). As a consequence, this pair is a translational generator of dimension 2 included in $\{T_3\}$. However, this bond does not constitute a subgroup because the plane of translations is not constant.

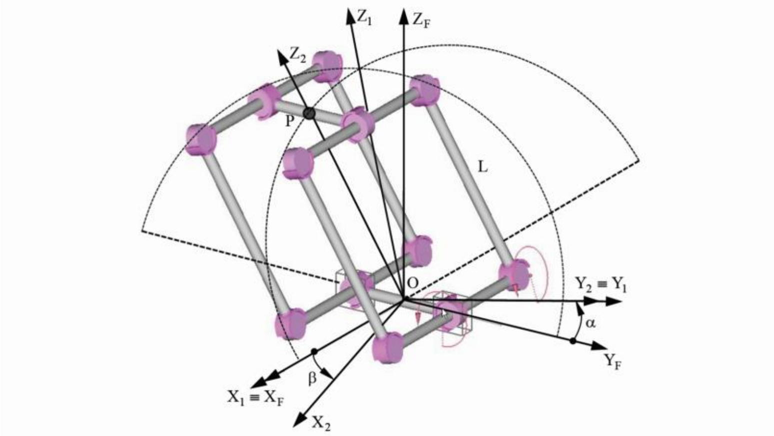

Fig.2 Pa^2 joint frames

Although, a Pa^2 pair is more complicated than a Pa joint in terms of motions involved, the position of any point of the upper plane onto the trajectory surface can be obtained by two sequential rotations.

In a first step, the cross-linked Pa pair rotates an angle α around the axes of its revolute joints. In a second step, the two parallel Pa pairs rotate an angle β around the axes of their revolute joints. Two local reference frames corresponding to the first step and the second one are defined as shown in the figure 2. When the values of α and β are 0º, the Pa^2 pair is in the posture shown in figure 1.

Thus, the position of point P expressed in the fixed frame is,

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \mathrm{L}\, sin\beta \\ -\mathrm{L}\, cos\beta\, sin\alpha \\ \mathrm{L}\, cos\beta\, cos\alpha \end{bmatrix} \tag{1}$$

Thus, an position equation of point P expressed the fixed frame can be obtained,

$$x^2 + y^2 + z^2 = \mathrm{L}^2 \tag{2}$$

In general, any point onto the upper plane will draw a spherical trajectory. Compared with other kinematic joints, Pa^2 has greater rigidity.

4 Position problems of the 3-PPa² PM

In this section, the loop closure equations of the 3-PPa^2 parallel manipulator are obtained.

As shown in figure 3, a fixed reference frame is defined along the three linear actuators located at points A_1, A_2 and A_3 respectively. Also, three local reference frames are defined by rotating the fixed frame around the actuator's axes corresponding to three legs respectively, which are depicted in figure 3.
Vectors **i**, **j**, and **k** denote the unit vectors of X_0, Y_0 and Z_0 axes respectively. Vectors $\mathbf{u}_i$, $\mathbf{v}_i$ and $\mathbf{w}_i$ are unit vectors corresponding to the local frame of i^{th} leg. In addition, OA_i denotes the stroke of the i^{th} leg, defined as s_1, s_2, and s_3 while L and ℓ are geometrical dimensions of the manipulator.

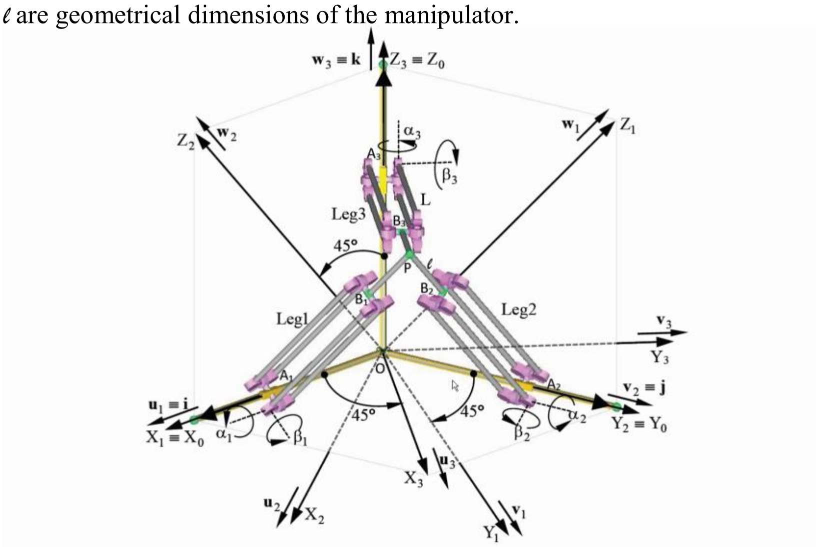

Fig.3 3-PPa[2]

4.1 Direct position problem of 3-PPa² PM

The purpose of the direct position problem is to obtain the position of the end-effector, defined by the position vector $\mathbf{x}_p = \begin{bmatrix} x_p & y_p & z_p \end{bmatrix}^T$ of the reference point P, as a function of input parameters, i.e., the strokes of actuators s_1, s_2, and s_3.
The absolute position of point P can be expressed for leg 1,

$$\mathbf{x}_P = \mathbf{x}_{A_1} + \mathbf{x}_{A_1B_1} + \mathbf{x}_{B_1P} \tag{3}$$

$$\mathbf{x}_{A_1} = s_1 i \tag{4}$$

$$\mathbf{x}_{A_1B_1} = {}^0_1\mathbf{R}\left(\mathrm{L}\,sin\beta_1\mathbf{u}_1 - \mathrm{L}\,cos\beta_1\,sin\alpha_1\mathbf{v}_1 + \mathrm{L}\,cos\beta_1\,cos\alpha_1\mathbf{w}_1\right) \tag{5}$$

$$\mathbf{x}_{B_1P} = {}^0_1\mathbf{R}\left(0\mathbf{u}_1 + 0\mathbf{v}_1 + \ell\mathbf{w}_1\right) \tag{6}$$

where $\mathbf{u}_1$, $\mathbf{v}_1$ and $\mathbf{w}_1$ denote the unit vectors corresponding to the leg 1 local frame and ${}^0_1\mathbf{R}$ denotes the matrix that rotates leg 1 local frame (O- $X_1Y_1Z_1$) to the fixed frame (O- $X_0Y_0Z_0$).

Equivalently, the position vectors of legs 2 and 3 are obtained by rotating leg 1. Imposing that the modules of vectors $\mathbf{x}_{\mathrm{AiB}i}$ must be equal to L, the nonlinear equations for 3-PPa^2 are obtained,

$$\left(\mathrm{x}_p - s_1\right)^2 + \left(\mathrm{y}_p - n\right)^2 + \left(\mathrm{z}_p - n\right)^2 = L^2 \tag{7}$$

$$\left(\mathrm{x}_p - n\right)^2 + \left(\mathrm{y}_p - s_2\right)^2 + \left(\mathrm{z}_p - n\right)^2 = L^2 \tag{8}$$

$$\left(\mathrm{x}_p - n\right)^2 + \left(\mathrm{y}_p - n\right)^2 + \left(\mathrm{z}_p - s_3\right)^2 = L^2 \tag{9}$$

where $n = \frac{\sqrt{2}}{2} l$. These equations lead to a quadratic univariate polynomial that must be solved numerically.

4.2 Inverse Position Problem of 3-PPa^2 PM

The aim of inverse position problem is to obtain the values of the input parameters s_1, s_2 and s_3 for a given posture of the end-effector defined by the position vector $\mathbf{x}_\mathrm{p}$.

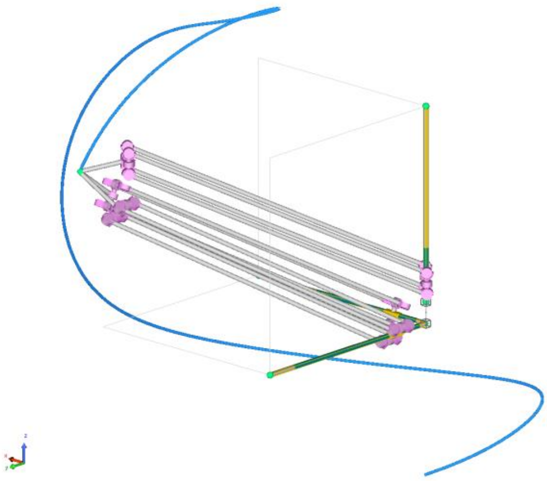

Fig. 4 3-PPa^2 path.

From equations (7)-(9), all inputs can be obtained in a straightforward way:

$$\mathrm{s}_1 = x_\mathrm{p} \pm \sqrt{\mathrm{L}^2 - \left(y_\mathrm{p} - \mathrm{n}\right)^2 - \left(z_\mathrm{p} - \mathrm{n}\right)^2} \tag{10}$$

$$\mathrm{s}_2 = y_\mathrm{p} \pm \sqrt{\mathrm{L}^2 - \left(x_\mathrm{p} - \mathrm{n}\right)^2 - \left(y_\mathrm{p} - \mathrm{n}\right)^2} \tag{11}$$

$$s_1 = z_p \pm \sqrt{L^2 - (x_p - n)^2 - (y_p - n)^2} \tag{12}$$

where $n = \frac{\sqrt{2}}{2}\ell$. In figure 4 is shown a path of the 3-PPa^2 obtained using GIM software (http://www.ehu.eus/compmech/software/).

4.3 Workspace

For the manipulator under study, the workspace is defined by the intersection of three cylinders,

$$(y_p - n)^2 + (z_p - n)^2 \le L^2 \tag{13}$$

where $n = \frac{\sqrt{2}}{2}\ell$. In figure 5 is shown a 3D view of the workspace.

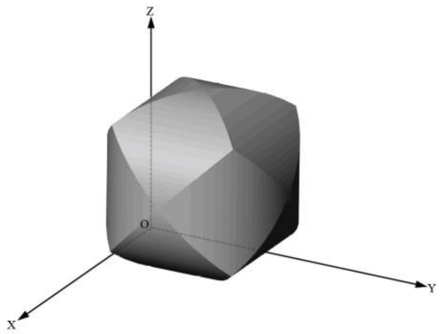

Fig. 5 Workspace.

5 Singularities

In order to analyze the singularities, Jacobian matrices are obtained. Differentiating equation (3) for each leg and dot-multiplying both sides by $\mathbf{x}_{A_iB_i}$, the velocity equation in matrix form is obtained,

$$\begin{bmatrix} \mathbf{x}^{T}_{A_1B_1} \\ \mathbf{x}^{T}_{A_2B_2} \\ \mathbf{x}^{T}_{A_3B_3} \end{bmatrix} \begin{bmatrix} \dot{x}_p \\ \dot{y}_p \\ \dot{z}_p \end{bmatrix} = \begin{bmatrix} L\,sin\beta_1 & 0 & 0 \\ 0 & -L\,sin\beta_2 & 0 \\ 0 & 0 & -L\,sin\beta_3 \end{bmatrix} \begin{bmatrix} \dot{s}_1 \\ \dot{s}_2 \\ \dot{s}_3 \end{bmatrix} \tag{14}$$

When the manipulator is in the inverse kinematic singularity, the end-effector loses one or more degrees in one direction. For this manipulator, this singularity occurs when the value of any β_i (i =1, 2, 3) is 0º or 180º (Figure 6).

A direct kinematic singularity happens when the rank of the direct Jacobian reduces. For the manipulator under study, this singularity appears when any two of legs are parallel to each other, or the three legs are on the same plane.

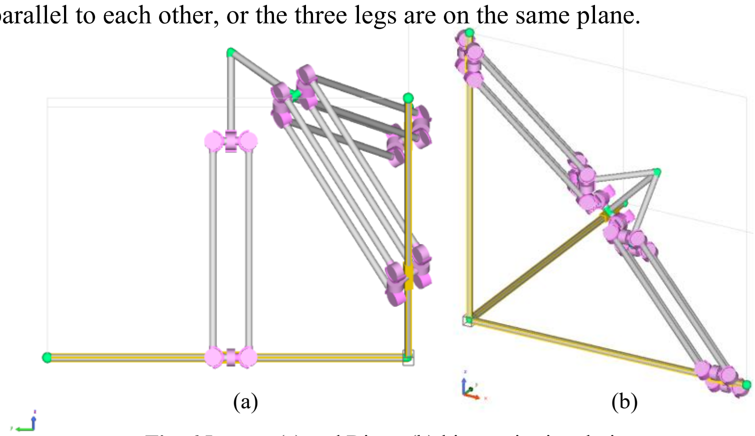

Fig. 6 Inverse (a) and Direct (b) kinematic singularity.

6 Conclusions

In this paper, the potential use of the Pa^2 pair as a part of a PM's kinematic chain is studied. From a theoretical approach, this kinematic structure has been proposed in some designs. However, the authors have not found an analyzed PM including this type of pairs. The manipulator presented in this paper, 3-PPa^2, includes redundant restrictions and constitutes a preliminary study of the use of this type of Pa^2 pairs.

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