DOV M. GABBAY AND JOHN WOODS: *The Rise of Modern Logic: From Leibniz to Frege.* [*Handbook of the History of Logic*, vol. 3]. Elsevier North Holland, Amsterdam, 2004, 770pp.

This volume contains essays on the most representative logicians of the period covered by the book (1685-1900) such as Leibniz, Bolzano, Schröder, Peirce or Frege, but also on philosophers such as Kant, Hegel or Husserl, whose reflections on logic have had a considerable influence on the development of this discipline. We begin with some general remarks on the structure and aims of the book and leave the commentaries about some of the papers contained in it for the end of this review.

As it is said in the Preface, "what is striking about this period is the earliness and persistence of what could be called "the mathematical turn in logic"" (p. vii). Even though, the Editors remind us, this mathematical turn took different forms (at least in the second half of the nineteenth century and the beginning of the twentieth century): algebraic presentation of the laws and inferences of logic (*algebraist approach*) and foundation of mathematics from the concepts and rules of inference of logic (*logicist approach).* But there was indeed a third way of understanding the link between logic and mathematics: that which understood logic as an aid for the formalization of mathematical theories and proceeded later to the mathematical study of logic as a branch of mathematics (*formalist approach*). This was the way in which Peano, Hilbert and their respective schools understood the relationship between logic and mathematics. This approach is, from our standpoint, "the precursor of the modern view that […] logic is indeed a branch of pure mathematics" (p. vii), which the Editors endorse to the algebraist approach. Unfortunately, there is no chapter in the book dedicated to the contributions of Peano and his school to the rise of modern logic (Hilbert's contributions are dealt with in volume 5 of the Handbook, untitled *Logic from Russell to Church*, which has just come out). This is quite misleading not only because Peano constitutes in many respects the link between the algebraic and logicist traditions, but also due to the fact that one of the most repeated thesis of the book is that "at the beginning of the twenty-century the algebra of logic was superseded by the mathematical logic of G. Frege (1848-1925) and G. Peano (1858-1932)" (p. 389). Indeed, without a close study of these two authors and of how Russell's logic incorporated and transformed the heritage of Peano (symbolic logic) and Frege (logical analysis), no one can grasp the transition from nineteenth to twentieth century and the rise of modern mathematical logic.

In accordance with the views expressed by the Editors, the main bulk of the pages concerning the logicians of the second half of the nineteenth century deals with the algebraic and mathematical logicians (to use Grattan-Guinness' expression), but with a considerable disproportion: 334 pages dedicated to the algebraic logicians versus 91 pages to the mathematical logicians. One of the reasons for this disproportion is obviously that Frege is the only author considered on the side of mathematical logicians. But another reason is that some of the papers devoted to the algebraic logicians are concerned more or less with the same subjects. This is the case, for example, of Th. Hailperin's paper and V. Sánchez Valencia's one, which deal both with almost the same authors (Boole, British logicians and algebraists, De Morgan, Peirce, Schröder, etc.) and topics (logic of classes, logic of relations and propositional logic). It would surely be better to have just one introduction to the development of the algebraic tradition and then essays for each of the most representative authors of this tradition: Boole and his successors (Jevons and Venn), Peirce and Schröder.

Despite all that has been said in the last two paragraphs, the Editors have succeeded in assembling an extraordinary collection of papers and the book itself is a *must-have* for all graduate students and researchers interested in the history of logic. We proceed now to briefly review some of the papers included in the book, concentrating ourselves on the articles about the algebraic logicians cited before.

Th. Hailperin's article *Algebraical Logic 1685-1900* is a comprehensive study on the development of algebraic logic from Leibniz to Peirce and Schröder. Obviously, the sections on Boole are excellent (the author is one of the most authoritative of Boole's scholars) and, generally speaking, the expositions are very clear and vigorous, even though they are often too schematic and sometimes incomplete. In section 7, for example, the author explains "How the logic of relations began". He focuses on De Morgan's well known contributions to the topic and on Peirce's first attempt to build an "algebra for the logic of relatives" in his "Notation for the Logic of Relatives" (Peirce, 1870). But there is not a section for explaining "the development of the logic of relations" in the hands of Peirce and Schröder (even though we can find it in the next paper by V. Sánchez Valencia). Nor can we find any mention of Schröder in section 9, dedicated to "Propositional Logic" (although the author devoted the second volume of his monumental *Vorlesungen über die Algebra der Logik* (Schröder, 1891) to the topic), but we find an unexpected subsection on Frege's propositional logic (recall this is a chapter on *Algebraical Logic* and we have a later chapter on Frege). Moreover, the subsection on Peirce's propositional logic focuses only on Peirce's article "On the Algebra of Logic" (Peirce, 1880), but there is no mention of the paper "On the Algebra of Logic: A Contribution to the Philosophy of Notation" (Peirce, 1885), surely the most outstanding work (with Frege's *Begriffschrift*) of the nineteenth century for his contributions to propositional and quantificational logic (the author refers only to this article when talking about truth values). Fortunately, we have an adequate review of it in Risto Hilpinen's essay (to be reviewed later).

The essay *The Algebra of Logic* by V. Sánchez Valencia is a lengthy study on the development of the algebraist tradition in logic. As the author remarks the four main actors in this development are Boole, De Morgan, Peirce and Schröder, even the focus of his concern is the work of Boole and De Morgan. In Part 1, the author analyzes the mathematical background that influenced De Morgan and Boole (for example: lagrangian algebras, symbolical algebra or Hamilton's logic), but it must be said that very little is said about the way in which this influence took course. In Parts 2-5, the author explains the "logic of monadic predicates" (or "absolute terms") of Boole, Jevons, Peirce and Schröder respectively. Finally, in Parts 6-8, the author's concern is the "logic of relations" of De Morgan, Peirce and Schröder. The analysis of Boole's logic is quite interesting, but the methodological perspective adopted is quite disputable. Basically, what the author tries to do is to present a *Reconstructed Boole's Logic* (his own words) in terms of modern logic. For example, some of the entries in the sections on Boole's logic are *The vocabulary*, *Construction rules*, *Derivations*, *Derived rules* (where he mentions *Boole's Cut Rule*), etc. Obviously, Boole neither used this terminology, nor conceived his logic in terms of these notions, simply because they were not at his disposal. So there is a serious danger of misunderstanding the way in which Boole understood his calculus of logic using all this conceptual machinery.

On the contrary, the analysis of Peirce's contributions to the algebra of logic is basically chronological and it goes directly to the heart of the matter: the development of quantificational logic from the algebra of logic. Nevertheless, there are some points to dispute. For example, the author says (p. 462) that Peirce's "treatment of absolute terms in (Peirce, 1870) is essentially quantificational" and that despite the fact that "in his initial papers Peirce lacks a notation for quantifiers [...] he had very early the notion of quantification". It is true that in (Peirce, 1870) the author tried to express quantified statements (particularly, the existential ones), as Sanchez Valencia correctly argues in pages 469-470, but this doesn't imply necessarily that by this time Peirce had the notion (even implicit) of quantification. In the last part of his essay, the author pays attention to "the way in which Schröder developed Peirce's theory of quantification and to his formalization of the algebra of relations" (p. 531). The conclusion of the author is that "against current wisdom, illustrated for instance in (Goldfarb, 1979, 252), we have to conclude that to arrive at a calculus of relations [...] was not the only aim of Schröder's third volume. He engaged as well in the development of the theory of quantification as the proof theoretical framework in which to carry proofs, as the semantic framework in which meaning could be captured" (p. 536). This is very remarkable because, as we will explain immediately, Goldfarb's thesis is part and parcel of a much extended misunderstanding about Schröder's theory of quantification which makes a correct account of his contribution to the rise and development of modern logic impossible.

V. Peckhaus' paper on *Schröder's Logic* is an excellent introduction to Schröder's life and work in logic: its sources, the aim of this work, the problems handled, etc. In the first section, the author makes an introduction to the significance of Schröder approach to logic and a re-evaluation of the place deserved to Schröder in the historiography of logic (and this is, from our standpoint, completely necessary). In the next four sections, the author makes an overview to Schröder's life and writings in logic, his algebraic programme and the sources of his work. The rest of the sections are devoted to Schröder's algebra of logic: Schröder's calculus of domains and classes and his calculus of propositions, Schröder's method of resolution of logical problems, the quasi-axiomatic presentation of his logic and, finally, Schröder's logic of relatives and its relationship with his pasigraphic project (with a comparison with Leibniz' and Frege's one). Like Sánchez Valencia, the author recognizes that Schröder developed a "full- fledged theory of quantification" (p. 557), but he remarks later that "for Schröder the use of $\sum$ and $\prod$ in logic is perfectly analogous to arithmetic. The existential quantifier and the universal quantifier are therefore interpreted as possible indefinite logical addition or disjunction, and logical multiplication or conjunction, respectively" (p. 576).

Peckhaus' emphasis on this interpretation belongs also to the "current wisdom" mentioned before, which includes not only (Goldfarb, 1979, 354), but also (Moore, 1980, 98), (Grattan-Guinness, 1997, 20), (Brady, 2000, 149), etc. Nonetheless, this current wisdom ought to be seriously qualified: In explaining the meaning of his fifteenth *postulate* [*Festsetzung*] for the algebra of relatives, Schröder defines semantically the quantifiers' symbols. For example, he says about the universal quantified statement $\prod_u A_u$ that it "means that the statement A_u is true for each one of these objects *u* […] Therefore, the statement $\prod_u A_u$ will have the truth value 1 when and only when, for any *u* considered, $A_u = 1$" (Schröder, 1895a, 36-37). This definition of the quantifiers is completely analogous to the one he has given previously in (Schröder, 1891) and to the one given by Peirce in (Peirce, 1885). From this definition, Schröder offers a justification of the schemes [*Schemata*] or principles [*Prinzipien*] which regulate the use of the universal and existential quantifiers in the algebra of relatives. Only later, when he introduces a new list of auxiliary schemas, which express some rules relative to what nowadays is called *multiple quantification*, Schröder proposes for its justification "taking in consideration mainly (but this is not indispensable and talking strictly ought to be avoided), *for didactical* reasons, the usual expression of $\prod$ and $\sum$ as [if they were] a "explicit" product or aggregate of a multiplicity of terms" (*Ibid.*, 111). We have to conclude then that the interpretation of the existential quantifier and the universal quantifier in terms of possible indefinite logical conjunction or disjunction is not the only one (and surely not the main one) and that the emphasis on this interpretation, "current wisdom" in the historiography on Schröder's logic, is a serious obstacle for interpreting and evaluating correctly Schröder's contributions to modern quantificational logic. (It is not unthinkable to assume that Peirce was aware of the use of the quantifiers in the semantical sense explained before and that for this reason he claims in his review of (Schröder, 1895a) that Schröder "often" uses his "general algebra of logic" (the term by which Peirce refers to his own quantificational logic) in the development of the algebra of relatives (Peirce, 1933, 282)).

R. Hilpinen's *Peirce's Logic* is just the essay which anyone who would like to introduce himself to the study of Peirce's logic must read, for it deals with all the topics relevant to the correct understanding of the multiple-faced logic of Ch. S. Peirce. In the introductory first section, the author states explicitly that "the emergence of formal or mathematical logic in the 19th and the early 20th century was the outcome of two parallel and partly independent lines of development, whose key figures were Charles S. Peirce and Gottlob Frege" (p. 611). The prominence of Peirce and Frege is due to the fact that they "were the first logicians who construed quantifiers as variable binding operators. They invented quantification theory independently of each other, at approximately the same time" (p. 612). Peirce's development of quantification theory from his logic of relatives is precisely the topic of section 2, which also deals with Peirce's axiomatization and truth value analysis of propositional logic. The following sections are on Peirce's logic in the context of his semiotics and pragmatism (where the author points out Peirce's interpretation of quantifiers in terms of games with perfect information), Peirce's existential graphs, Peirce on modalities and possible words (where the author explains his modal interpretation of conditional statements), Peirce's contributions to three valued logic and, finally, Peirce's theory of reasoning (with emphasis on his analysis of non-deductive argumentation, *i.e.*, induction and abduction). Indeed, Hilpinen's essay helps to clarify or complete some important aspects relative to the development of Peirce's logic touched on in previous essays, even though his explanations are sometimes insufficient. For example, the author correctly observes that "the main purpose of his [Peirce's] logical work is the analysis of logical inference rather than practical facilitation of reasoning" (p. 612). This, we must remark, is the reason he replaced the sign of identity by the sign $-\!\!<$ to represent the *copula*, but nor Hilpinen's neither Hailperin's or Sánchez Valencia's essays make any reference to this important question. Analogously, in explaining the development of quantification theory from his algebra of relatives, the author just says that "the definition of relative product and sum requires a quantifier" (p. 614). This is obviously true, but it doesn't explain for itself the transition from the algebra of relatives to the *general algebra of logic* (Peirce's quantificational logic). The reasons are by far more complex and it must be said again that not one of the authors reviewed tries to explain them.

I. Grattan-Guinness' *The Mathematical Turns in Logic* is a short paper on the interplay between, on one side, logic and mathematics and, on the other, algebraic and mathematical logic. He mentions four main differences between the two traditions of symbolic logic: part-whole theory versus set theory (Russell), laws and properties versus axioms, mathematics applied to logic versus logic applied to mathematics and, finally, attention upon adjectives and names versus attention upon the quantitative expressions "*all*, *every*, *any*, *a*, *some* and *the*" (Russell again). Obviously, each of these heads would deserve a long commentary which probably would exceed the aim of this review. It is not clear if the author thinks of these differences between the two traditions as the reasons the algebraic tradition was superseded by the mathematical one. Nevertheless they were probably only different aspects of the main reason that algebraic logic was partially abandoned at the beginning of the twentieth century, *i.e.*, "the fact that the algebra of logic became to be regarded as not suitable for the new research goal, the logical foundations of the whole of mathematics" (p. 390)[1].

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[1] G. H. Moore's and I. Grattan-Guinness' papers cited above are respectively: "Beyond First-Order Logic: The Historical Interplay between Mathematical Logic and Axiomatic Set Theory", *History and Philosophy of Logic* (1), pp. 95-137 and "Vida en común, vidas separadas. Sobre las interacciones entre matemáticas y lógicas desde la Revolución Francesa hasta la Primera Guerra Mundial", *Theoria* (12), pp. 13-37. For the rest of the references see the bibliography available in V. Peckhaus' paper in the book reviewed.