Marcus and Substitutivity *

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Received: 12.11.2012
Final version: 01.03.2013

BIBLID [0495-4548 (2013) 28: 78; pp. 359-374]
DOI: 10.1387/theoria.6938

ABSTRACT: The paper discusses Marcus's formulation of the principle of substitutivity. She relied on a notion of logical form in which certain problematic kinds of context are analyzed away. I defend a variant formulation of the principle in which the problematic contexts are accommodated in their own right.

Keywords: opacity; anaphora; indexicality; explanation.

RESUMEN: El artículo discute la formulación de Marcus del principio de sustituibilidad. Se apoyó en una noción de forma lógica en la que el análisis elimina algunos tipos problemáticos de contexto. Defiendo una formulación variante del principio en la cual los contextos problemáticos se acomodan por derecho propio.

Palabras clave: opacidad; anáfora; indexicalidad; explicación.

1. Introduction

A major thread that runs through the work of Ruth Barcan Marcus[1] is her debate with Quine over the question of the intelligibility of *de re* modal language (see Marcus 1990a for a summation). In (Quine 1955), the main symptom of the unintelligibility of *de re* modal language is said to be the failure of coreferential singular terms to interchange *salva veritate* within the scope of modal operators. From this it is supposed to follow that the notion of objectual satisfaction is inapplicable to *de re* formulae, hence quantifying-in makes no sense. A response that was once favored by Marcus is to reconstrue the semantics of quantification substitutionally, as in (Marcus 1961). But as she had already pointed out (Marcus 1948), Smullyan (1948) had demonstrated that Quine's premise, that the rule of Identity Elimination (=E) fails in modal languages, is incorrect. Quine's later 'animadversions' about *de re* modality (e.g., in Quine 1966), accuse it of commitment to an invidious Aristotelian essentialism, but work by Marcus and Parsons (Marcus 1967; Parsons 1969) shows that this charge has little force.

And there matters rest so far as *de re* modality is concerned. But of course, this is not the whole story. For the argument that where =E fails, quantifying-in makes no sense, is still of relevance to other cases in which =E *does* seem to fail. First, a Quinean paradigm of failure of substitutivity (Quine 1961, 22)

(1) Giorgione is so-called because of his size; Giorgione is Barbarelli; therefore, Barbarelli is so-called because of his size.

* An earlier version of this paper was given at an APA session on the philosophy of Ruth Marcus in 2000. I thank Marcus and members of the audience for their comments on that occasion. I also thank a referee for THEORIA for criticisms which improved my penultimate draft.

[1] All page references to Marcus's writings in this paper are to (Marcus 1993).

Here, a use of =E leads us astray. In addition, quantifying-in produces something *uninterpretable*; that is, no complete proposition can be assigned to the quantified sentence (even though it is obtained by Existential Introduction (∃I) from a complete proposition):

(2) Someone is such that he is so-called because of his size.

He cannot be a pronoun bound by the initial *someone* and *also* an indexical or a name. More carefully, a standard understanding of the English excludes this (we could devise some conventions that would make (2) artificially interpretable, as we could for any nonsense string). But *he* would have to be a name to be a suitable antecedent for *so-called*. (2) is true if interpreted substitutionally, but it is a bug, not a feature, of the substitutional interpretation of *someone is such that*… that it assigns a truth-value to (2).

In another example (Fine 1989, 222-3; see also Linsky 1967, 104),

(3) The man behind Fred saw him leave; the man behind Fred = the man in front of Bill; therefore, the man in front of Bill saw him leave

we again find failure of =E, and, relatedly, uninterpretability as the outcome of quantifying-in:

(4) Someone is such that he saw him leave.

Syntax excludes interpreting *him* as coindexed with *he* and *someone*, which means that, barring a surreptitious conversion of its semantics from anaphoric to deictic, *him* can only be a variable that is free in (4).

However, there is at least one case where it looks as if we have failure of =E combined with the acceptability of quantifying-in, the case of attitude ascriptions. For although

(5) Lex fears Superman; Superman = Clark; therefore Lex fears Clark

seems wrong, we would not object to

(6) Lex fears Superman; therefore, someone is such that Lex fears him

on the grounds that the conclusion is intuitively *uninterpretable*.

One response to these data is to accept that the illustrated uses of =E are all as incorrect as they appear, while of the three uses of ∃I displayed in (2), (4) and (6), the one in (6) involves a special maneuver that explains its validity. I will develop this response in the rest of the paper against the backdrop of an attempt by Marcus to formulate =E with the right restrictions to block incorrect uses.

2. Marcus and Cartwright on =E

As discussed in (Marcus 1975), Cartwright (1971) accepts that (1) is a counterexample to =E, but distinguishes this rule of inference from Leibniz's Law, which may be formulated as *if a = b then every property of a is a property of b or as if a = b then whatever is true of a is true of b*. Cartwright holds that the Law is not threatened by (1), because the expression (presumably expression-*type*) *is so-called because of his size* does not express a property, or a condition that can be true or false of things. Marcus is sceptical that the fate of the inference-rule can come apart from that of the Law so easily (1975, 103), but it seems to me that we can make a stronger objection, namely, that the Law does no better with (1) than does the inference rule. For it is perfectly fine to attribute to Giorgione the property of being so-called because of his size (that token of the predicate is not defective *qua* expressing a property); equally, it is unproblematically true of Giorgione that he is so-called because of his size.[2] Moreover, Giorgione is Barbarelli. Yet Barbarelli does *not* have the property of being so-called because of his size; equally, it is *not* true of Barbarelli that he is so-called because of his size.

Marcus's own proposal is that the rule of inference is only applicable to logical forms, in which various kinds of ambiguity to which ordinary discourse is subject have been eliminated (1975, 105). She points out that

(7) Giorgione is so-called because of his size

could mean that Giorgione is called 'Shorty' because of his size if it immediately follows an utterance of *Shorty is so-called because of his size* (I will incorporate this point in §3). She then proposes the following formulation of =E, improving on Cartwright's version (*op. cit.*, 108):

(8) For all proper names *a* and *b* (indexed to preserve univocality), $a = b$ expresses a true proposition just in case, for all sentences *P*, *S* and *S**, if *S* is a restatement of *P* in logical form and if *S** is like *S* save for containing an occurrence of *b* where *S* contains an occurrence of *a*, then *S* expresses a true proposition only if *S** does.

However, while the requirement that the rule be applied only to logical forms can deal with problem cases in which substitution affects syntactic structure, (8) does not seem to me to make much headway with (1) and (3).[3] The minor premise of (3) may be written

(9) The man behind Fred_1 saw him_1 leave

using indices to mark anaphoric relations. If we wish to replace *the man behind Fred* with the coreferential *the man in front of Bill*, we either have to use the same index on *Bill* or a different one. If we use the same one, we get a falsehood if the man in the middle did not see Bill leave; if we use another index *i*, we get something uninterpretable ('him_1' would be a free variable); and if we use no index, *him* becomes deictic. (8) fails to help us here.

Does (8) do better with (7)? Though Marcus is not explicit, her point about the context-sensitivity of *so* and the context-insensitivity of logical forms indicates that at logical form, she would eliminate *so-called* in favor of *called NN*, where *NN* is the expression the context provides by which the relevant entity is called. This allows free use of =E; for example, we can substitute the sole occurrence of *Giorgione* used as a name of big Giorgio in *Giorgione is called* Giorgione *because of his size* (Barbarelli is called that for the same reason), and the worry about *Shorty* no longer arises.

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[2] I intend this judgement to be intuitively plausible, but the theory developed in this paper offers further support. 'So' is treated as a demonstrative, and there doesn't seem to be any reason why a demonstrative in a complex predicate should render tokens defective *qua* property-expression.

[3] For structure-affecting substitutions, see (Fine 1989, III). I assume substitution replaces all and only the contents of some node in the minor premise's parse tree.

But this proposal about the logical form of (7) seems unacceptable to me, as does its analogue for (9), which requires *him*$_1$ to be replaced at logical form by *Fred*. It belongs to the semantics of an anaphoric pronoun that its semantic value is recovered in the course of interpretation from an autonomously referring expression that anchors it, a process that simply disappears under the replacement proposal. The semantics of *so* also disappears. *So* is a demonstrative: (7) means *Giorgione is called by* ***that*** *name because of his size*. We should therefore expect a formal semantics for a language with the substitution-blocking *so* device to use apparatus of a kind standardly deployed in the semantics of demonstratives, e.g., the LD framework of (Kaplan 1989b).[4]

Despite these objections to Marcus's proposed version of =E, I agree that the rule is correct if formulated in a way that is appropriate for the expressive resources of the language in question. Our two examples of misapplication of =E indicate that what we need is a restriction to the effect that a substitution may be made in $S(a)$ using the major premise $a = b$ so long as the substitution has no *truth-condition-altering side-effects*. The side-effects in our fallacious substitutions are, in (1), that the referent of *so* is altered, and in (3), that the antecedent of *him* is changed, so these truth-condition-altering substitutions are to be forbidden. It is this *semantic* criterion, of not altering truth-condition, that blocks fallacious uses of =E. But semantic criteria are in principle unavoidable, since in natural language, the ways of affecting truth-condition as a result of a substitution are open-ended. A purely syntactic criterion is only available for a formal language, in which ways of making a truth-condition-altering substitution are precisely circumscribed. I turn now to some examples.

3. Demonstrating words

In sketching a treatment of *so-called*,[5] I will work round the problematic *because of* by supposing we are concerned with a formal language L_F containing a three-place predicate $F(x, t, z)$, to be read *x is called t for z*, as in *Giorgione is called* Giorgione *for his size* and *Giorgione is called so for his size* (I assume *so-called* is derived from *called so*, and take possessive descriptions such as *his size* to be singular terms).[6] I propose to ignore any extensionality problems this may give rise to. *t* is either *so*, or for some syntactically sim-

[4] Montalbetti (2003, 133) points to pronoun-like behavior in *so* which threatens my classification of it as a demonstrative. For example, in *April is so-called for when she was born, and June is too*, the sloppy reading, on which the second conjunct means *June is called* June *for when she was born*, dominates. But in *April is called that for when she was born, and June is too*, the strict reading (*June is called* April *for when she was born*) dominates the sloppy one, if the latter even exists. Here *so* contrasts with *that* as *his* does: the sloppy reading of *John loves his wife and Bill does too* is more favored, absent priming, while *John loves that woman and Bill does too* doesn't have a sloppy reading. But this contrast between *so* and *that* is consistent with *so* being a demonstrative. When the elided *called that for when she was born* is restored for interpretation, the context of evaluation doesn't change, and the newly explicit *that* defaults to the *demonstrandum* the context already provides (if it is indexed, as in Kaplan's system, the index will just be copied). But when the elided *so-called for when she was born* is restored the context changes, since the context is the linguistic environment of *so*. The second *so* therefore has the option of referring to *June*, and *ceteris paribus* prefers the more salient (because closer) name.

[5] Ch. 8.2 of (Forbes 2006) is a compressed version of some of the material in this section.

[6] In a more realistic account, the third argument of F would be a proposition or state of affairs.

ple constant c among the constants of L_F, t is a quotation name $\underline{c}$ of c. L_F is otherwise a standard first-order language, except that the interpretation of F, $V_A(F)$, is constrained by the principle that $<x, c, z>$ is in $V_A(F)$ only if $V_A(c) = x$ (Giorgione can be called *Giorgione* for some reason only if *Giorgione* refers to Giorgione).

An L_F-*discourse* is a sentence or set of sentences of L_F in which occurrences of *so* are distinctively numbered, and for each individual constant c, occurrences of c are distinctively numbered.

A *context* μ is a function from a set of positive integers (perhaps empty) to a set of pairs each of the form $<j, c>$, where j is a positive integer and c is an individual constant of L_F. $\mu(i) = <j, c>$ means that the occurrence of *so* numbered i in the sentence or discourse being evaluated refers to the occurrence of the name c numbered j. If $\mu(i) = <j, c>$ we define $\mu_1(i) = j$ and $\mu_2(i) = c$. Technically, we also require stipulations ensuring that each context is *suitable* for any structure with which it is paired in the evaluation of a discourse that has occurrences of *so* (this means a *so* always refers to a name in the structure's domain), and that such a context is *defined for* the discourse being evaluated (each occurrence of a *so* refers to some name actually in the discourse).

A context μ for an L_F-discourse may be displayed using arrows that link occurrences of *so* to the expressions they demonstrate in the discourse. For example, for the conditional

(10.1) If Caravaggio was called so for his birthplace and Guercino was called so for his squint, then Guercino was called so for his squint and Caravaggio was called so for his birthplace

or in L_F,

(10.2) $F(C^{(1)}, so^{(1)}, his_C\ birthplace)\ \&\ F(G^{(1)}, so^{(2)}, his_G\ squint) \rightarrow$
$F(G^{(2)}, so^{(3)}, his_G\ squint)\ \&\ F(C^{(2)}, so^{(4)}, his_C\ birthplace)$

there are various contexts available. The most likely is the context μ^* defined by

(11) $\mu^*(1) = <1, C>; \mu^*(2) = <1, G>; \mu^*(3) = <2, G>; \mu^*(4) = <2, C>$.

and displayed, for (10.2), as follows:

(12.1) $F(C, so, his_C\ birthplace)\ \&\ F(G, so, his_G\ squint) \rightarrow$
$F(G, so, his_G\ squint)\ \&\ F(C, so, his_C\ birthplace)$.

With contexts displayed explicitly, there is no need to write in superscripts on the occurrences of *so* or the individual constants.

The semantics of L_F is given by a standard recursive definition of the concept $(A, \mu) \vDash_h f$, read as "ϕ is true in the structure A and context μ relative to the assignment h". Here ϕ is an L_F -wff, and h is a partial function into D_A defined for all the variables free in ϕ; A is suitable for μ; and μ is defined for $\Delta = \{\phi\}$. If ϕ is a formula in which a specific context is displayed, as in (12.1), we may speak of its truth-value (relative to h) in A *simpliciter*.

The clauses of the semantics are as would be expected, μ being appealed to only to give the reference of an occurrence of *so*. Following Kaplan (1989b, 544-6), we define a general notion of *content* in a structure A and context μ relative to an assignment h, which appeals to D_A for individual constants, to h for free variables, and to μ for *so* (I postpone the details of the treatment of descriptions):

(13.1) $[\![c^{(k)}]\!]_{A,\mu,h} = V_A(c)$, c a syntactically simple constant with optional superscript; $[\![\underline{c}]\!]_{A,\mu,h} = V_A(\underline{c}) = c$;

(13.2) $[\![so^{(i)}]\!]_{A,\mu,h} = \mu_2(i)$;

(13.3) $[\![v]\!]_{A,\mu,h} = h(v)$, v an individual variable for which h is defined.

The clauses for atomic and complex formulae based on (13.1)–(13.3) are completely standard. For instance, for atomic formula, negations and quantifiers, we have

(14.1) if $\phi(t_1,\ldots,t_n)$ is an atomic wff of L_F, where each t_i is either a variable or a constant or an occurrence of *so*, then $(A,\mu) \vDash_h \phi(t_1,\ldots,t_n)$ iff: $<[\![t_1]\!]_{A,\mu,h},\ldots,[\![t_1]\!]_{A,\mu,h}>$ is in $V_A(\phi)$;

(14.2) $(A,\mu) \vDash_h {\sim}\phi$ iff $(A,\mu) \nvDash_h \phi$;

(14.3) $(A,\mu) \vDash_h$ (*some/every v:* ψv)[ϕv] iff for some/every x in D such that $(A,\mu) \vDash_{h^\frown(v \Rightarrow x)} \psi v$, we also have $(A,\mu) \vDash_{h^\frown(v \Rightarrow x)} \phi v$.[7]

This leaves us to define validity and consequence. Sentences (10.1) and (10.2) have the appearance of logical truths, since, ignoring superscripts, they seem to be instances of $(p \,\&\, q) \rightarrow (q \,\&\, p)$. But with the *demonstranda* of their occurrences of *so* unspecified, they are like $\varphi(that) \rightarrow \varphi(that)$ in Kaplan's system, logically true only if the homonymous demonstratives are coreferential (Kaplan 1989a, 587). However, with the context μ^* defined in (11) provided to fix the reference of the *so*'s, it is clear that changes in the interpretation of the remaining non-logical vocabulary will not affect truth-value; that is, sentence-type (10.2) expresses a logically true proposition in the context μ^* (there are other contexts in which it does not even express a truth). So one notion of validity is that of a sentence which is *valid in a context:* if μ is defined for $\{\sigma\}$ and A is suitable for μ, $(A,\mu) \vDash \sigma$. But for *so*-free sentences, which are not context-sensitive, we have the standard concept of *universal* validity. A *so*-free sentence is universally valid iff it is a logical truth of conventional first-order logic.

We can define semantic consequence in the predictable way:

(15) $\Sigma \vDash_\mu \sigma$ iff μ is defined for $\Delta = \Sigma \cup \{\sigma\}$ and for every A suitable for μ, if $(A,\mu) \vDash \tau$ for every τ in Σ, then $(A,\mu) \vDash \sigma$.

To assess (1) in these terms, we treat its premises and conclusion as a single discourse for which μ is defined, with *so* having two occurrences. Displaying the context $\mu^\dagger$ defined by $\mu^\dagger(1) = <1,G>$, $\mu^\dagger(2) = <2, B>$, we will then have

[7] Closed sentences are evaluated with respect to the null variable-assignment; when a determiner binding a variable is encountered, the current assignment h is extended by assigning an object a to the variable v, resulting in $h^\frown(v \Rightarrow a)$ (the syntax excludes double-binding).

(16) $F(G, so, his\ size), G = B \nvDash F(B, so, his\ size)$

(without the arrows, replace $\nvDash$ with $\nvDash_{\mu\dagger}$). A context-relative valid argument is obtained by replacing $\mu\dagger$ with μ^* defined as $\mu^*(1) = <1, G>$, $\mu^*(2) = <1, G>$ (the second *so* also refers to the initial *Giorgione*).

It is clear that the failure of quantifier introduction on G in the minor premise of (16) has nothing to do with G or its position being "not purely referential". The semantics of individual constants in the framework we have just set out is entirely standard. Nor is the problem just that $V(so)$ is stipulated to be a subset of $D \times D_{L_F} \times D$, since this is consistent with

(17) $(some\ x: person(x))[\ F(x, so, his\ size)]$

being logically false. (17) should rather come out uninterpretable. One way of achieving this would be to write the syntax of L_F so that $F(x, so, his\ size)$ is not well-formed; but it is perfectly well-formed—trouble only arises when a *so* tries to link to the *x*. The best option seems to be to modify (13.2), which defines the content of $so^{(i)}$ in a context μ and structure A. Instead of setting $[\![so^{(i)}]\!]_{A,\mu,h}$ equal to $\mu_2(i)$, we make the assignment of a content conditional:

(18) $[\![so^{(i)}]\!]_{A,\mu,h} = \mu_2(i)$ if $\mu_2(i)$ is an individual constant of L_F; $[\![so^{(i)}]\!]_{A,\mu,h}$ is undefined otherwise.

We redefine a context to be a function μ whose range is a set of pairs $<j, \alpha>$, where j is a positive integer and α is *any well-formed expression* of L_F. Then in conjunction with (18), leaving everything else as it is, the derivation of a truth-condition for (17) "crashes" when it tries to call the value of *so* and finds it undefined.

Explaining why (17) cannot be inferred from the minor premise of (16) is part of the more general project of providing an adequate system of inference for which we will have such results as:

(19.1) $F\ (G, so, his\ size), G = B \nvdash F(B, so, his\ size)$

(19.2) $F(G, so, his\ size), G = B \vdash F(B, so, his\ size)$

(19.3) $F(G, so, his\ size) \nvdash (\exists y)\ F(x, so, his\ size)$.

(19.4) $F(G, so, his\ size) \vdash (\exists x)\ F(x, so, his\ size)$.

The most natural notion of proof for this consequence relation is one in which a proof is a list of sequents of the form $\Gamma \vdash \sigma$, in which σ and every member of Γ is a sentence-in-context, as illustrated in (19.1)–(19.4). Intro and Elim rules apply to the sentence-in-context on the right of $\vdash$, with appropriate adjustments to the premise-assumption set on the left. The Elimination rule for *so* is that from $\Gamma \vdash \ldots F(t_1, so^{(i)}, t_2)\ldots$ we may infer $\Gamma \vdash \ldots F(t_1, \underline{c}, t_2)\ldots$, where c is the constant $\mu_2(i)$ (i.e., the one the i'th occurrence of *so* refers to in the context in which the inference is being carried out). The Introduction rule is that from $\Gamma \vdash \ldots F(t_1, \underline{c}, t_2)\ldots$, we may infer $\Gamma \vdash \ldots F(t_1, so^{(i)}, t_2)\ldots$, where the context μ is extended to a new context μ' defined for i, and $\mu'(i)$ is some occurrence of c on the current line (in terms of constructing a proof, this involves drawing a new arrow).[8]

=E and quantifier introduction rules are the only primitive rules that replace individual constants. Assuming that such replacement is the *only* way that quantifier introduction works, then we must require that a name can be substituted in =E or replaced by a variable in a use of a QI rule only if it is not targeted by an arrow. Thus the connection between substitution-resistance and the uninterpret- ability of quantifying-in is quite simple in this case: the condition that prevents substitution is the very same one that blocks quantifier introduction.

To end this part of our discussion, here is a proof of (19.2):

(20.1)	$F(G, so, his\ size) \vdash F(G, so, his\ size)$	Premise
(20.2)	$G = B \vdash G = B$	Premise
(20.3)	$F(G, so, his\ size) \vdash F(G, \text{'}G\text{'}, his\ size)$	1 *so*-E
(20.4)	$F(G, so, his\ size), G = B \vdash F(B, \text{'}G\text{'}, his\ size)$	2,3 =E
(20.5)	$F(G, so, his\ size), G = B \vdash F(B, so, his\ size)$	4 *so*-I

As this proof illustrates, the rules for *so* are context-manipulation rules, so that context can change from line to line in a proof. So the appropriate notion of soundness for these rules is not that of preserving validity relative to a fixed context, but rather, relative to a variable one. More precisely, where μ is defined for $\Sigma \cup \{\sigma\}$ and μ' is defined for $\Sigma' \cup \{\sigma'\}$, we require of each rule R that if $\Sigma' \vdash_{\mu'} \sigma'$ may be inferred from $\Sigma \vdash_{\mu} \sigma$ using R, then if $\Sigma \vDash_{\mu} \sigma$, then $\Sigma' \vDash_{\mu'} \sigma'$. The rules for connectives are all sound, since in their use, $\mu = \mu'$. And the soundness of the *so*-rules is easy to demonstrate. So the system is sound, since every proof begins with a sequent of the form $\sigma \vdash \sigma$, like (20.1).

[8] If a term in a formula ϕ is targeted by an arrow from outside ϕ, ϕ cannot be removed by assumption-discharge with the rules $\vee$E and $\exists$E, while in a use of $\rightarrow$I to infer $\Gamma \vdash p \rightarrow q$ from $\Gamma, p \vdash q$ any arrow from a *so* in q to a constant in p must contract to maintain its target as p moves across $\vdash$. We also require that $\rightarrow$E and &E cannot be applied if the antecedent or eliminated conjunct is linked to by the consequent or retained conjunct, and that in ~E, neither major nor minor premise contains *so* (we can often use *so*-E and *so*-I to work round these restrictions). Multi-premise rules require a notion of identity of formula; e.g., to apply $\rightarrow$E to $\Sigma \vdash p \rightarrow q$ we need the very same p in $\Gamma \vdash p$ on another line. To reason with formulae containing *so*, we must extend our notion of same formula to include the condition that corresponding *so's* target tokens of the same type.

4. *Argument anaphora*

In our other example of a misuse of =E,

(3) The man behind Fred saw him leave; the man behind Fred = the man in front of Bill; therefore, the man in front of Bill saw him leave,

the anaphoric pronoun *him* in the minor premise or conclusion of (3) is assigned content by inheritance from an argument expression functioning as its antecedent. Standardly, indices are used to capture this:

(21) The man behind Fred_1 saw him_1 leave; the man behind Fred = the man in front of Bill; therefore, the man in front of Bill_2 saw him_2 leave.

The subscripts reflect how the English is understood: once the substitution is made, the last *him* inherits its reference from *Bill.*

A pronoun may be referentially dependent upon a proper name or a definite description (these are the *terms*), so it is time to stop being coy about the treatment of definite descriptions. For the purposes of this paper, we employ a Fregean account which takes them to be singular terms which give rise to presupposition-failures if they are improper. Let us say that a presupposition-failure generates a truth-value gap in atomic formulae and that ϕ is undefined iff ϕ has an undefined atomic subformula.[9] We stipulate

(22) $[\![\textit{the } v{:}\phi]\!]_{A,\mu,h}$ = the unique $a \in \mathrm{D}_A$ such that $(A, \mu) \vDash_{h^\frown(v \Rightarrow a)} \phi$ if there is such an a; $[\![\textit{the } v{:}\phi]\!]_{A,\mu,h}$ is undefined otherwise.[10]

If a sentence contains indexed terms and pronouns with the same indices, we restrict evaluation to functions h defined for those indices which assign them the same domain elements as the terms to which they are attached, and we don't allow pronouns with unanchored indices. So we are guaranteed, for any indexed pronoun ρ_i, that $[\![\rho_i]\!]_{A,\mu,h} = h(i)$.

Co-indexing does not block substitution of a term t with a coreferential t', since the content of the pronoun is the *content* of the term, not, as in the previous case, the term itself. So if Fred = Jed, then the man behind Jed_1 saw him_1 leave (the index attaches to the *position*). The problem cases, as (21) illustrates, arise when terms are embedded in complex terms. The same goes for applying ∃I (the only quantifier rule in whose rule-scheme the term t may be a definite description): we cannot quantify away the entire description in *the man behind Fred saw him leave*, since this produces

(23) (*some x: person*(*x*))[*x saw* $\textit{him}_1$ *leave*].

Him's antecedent *Fred* has simply disappeared. Like (17), then, (23) should be undefined.

We can make inferences with the likes of *the man behind* $\textit{Fred}_1$ *saw* $\textit{him}_1$ *leave*. This entails that the man behind Fred saw someone leave, and given that the man behind Fred is the man in front of Bill, that the man in front of Bill saw Fred leave. We as-

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[9] This is controversial. See (Soames 1989, 560) for discussion.

[10] The context μ plays no role in (21), but will have a role when we mix pronouns and *so*.

sume a rule of *depronominalization*, (D), which stipulates that pronouns anaphoric on some term may be replaced by that term; and that the rule of Alphabetic Variant be extended to apply to indices as well as variables. We then require of =E and ∃I that they not apply to any term that properly contains anaphoric pronouns. We can illustrate how this works with an example that combines both the devices we have considered so far, the demonstrative *so* and pronouns: *Giorgione*$_1$ *knows he*$_1$ *is so-called because of his*$_1$ *size, therefore Giorgione*$_1$ *is so-called because of his*$_1$ *size*.[11] Deriving this requires a meaning-postulate (FK) embodying the factivity of *knows*, which allows us to arrive at the conclusion in four steps:[12]

(24.1)	$\mathrm{K}_{Giorgione_1}(F(he_1, so, his_1\ size))$	Premise
(24.2)	$\mathrm{K}_{Giorgione_1}(F(he_1, \text{‘Giorgione’}, his_1\ size))$	1, so-E
(24.3)	$\mathrm{K}_{Giorgione}(F(Giorgione_1, \text{‘Giorgione’}, his_1\ size))$	2, D
(24.4)	$F(Giorgione_1, \text{‘Giorgione’}, his_1\ size)$	3, FK
(24.5)	$F(Giorgione_1, so, his_1\ size)$	4, so-I

Speakers can carry anaphoric reference some distance into unambiguous discourses, but at any point, perhaps prompted by stylistic or processing constraints, a pronoun can be replaced by a name it depends on, and subsequent coindexed pronouns come to depend on the new occurrence of the name, the original occurrence losing its index. This is how we prepare in (24.3) for the detachment in (24.4). Separate constraints (like those in note 8) prevent detachment rules from leaving us with free indices.

Where does this leave Marcus's idea, embodied in (8), that =E (and presumably ∃I) are correct rules so long as they are applied only to logical forms? (8) is in the service of conventional formulations of =E and ∃I and requires that amenable logical forms be imposed on sentences of the problematic types. What we have shown here is that when the language of such sentences is provided with a plausible system of inference, we can derive unproblematic sentences from the problematic ones by such rules as *so*-

[11] Predelli (2010) distinguishes between regular and 'obstinate' demonstratives, and regards the *so* of *Giorgione is so-called because of his size* as obstinate: there is no alternative to treating it as referring to 'Giorgione'. Similarly, in (i) *Guercino said that Giorgione was so-called because of his size*, 'Guercino' is "unreachable" by *so*, 'presumably due to syntactic constraints of this or that kind' (Predelli 2010, 7). But I am unpersuaded that there is a special 'obstinate' use of *so*. The premise of our argument, *Giorgione*$_1$ *knows he*$_1$ *is so-called because of his*$_1$ *size*, shows that *so* can easily reach outside its clause. The reason that it does not do so in (i) is that the speaker is reporting a speech-act of Guercino's in which *so-called* is used to specify part of the content of what Guercino said. But we naturally assume a context in which *Guercino* is not used to specify the speech-act content, while *Giorgione* is. This makes it seem that 'Guercino' is unreachable by *so*. However, if Guercino is discussing people who are nicknamed *Guercino* because they squint, then an informed audience can read the report *Guercino said that Giorgione was so-nicknamed because of his squint* in the way Predelli suggests grammar rules out.

[12] Each line in this proof begins with a suppressed '$\mathrm{K}_G(F(he_1, so, his_1\ size)) \vdash$' with a link from *so* back to *Giorgione*.

E and depronominalization and apply =E and ∃I in the conventional way to the derived sentences. We then need only add restrictions to the standard versions of these rules to prevent attempted application of them before the admissible premises for their application are obtained. But such restrictions fall under a general type of restriction that is always implicit in natural deduction, that application of a rule should have no truth-condition-altering side-effect.

5. Attitude ascriptions

In the two cases we have considered, that of *so* and of pronouns, the failure of =E to preserve truth and of ∃I to preserve interpretability have gone hand in hand. An example of substitution failure in which the substituted terms are purely referential, the device explaining substitution-failure is essentially the same as one of the two above, and yet quantifier introduction is not blocked, would break the connection. We end by considering a case which *appears* to satisfy this description but will argue that properly understood, the appearance is mere appearance. This is a happy outcome for Quine's view that quantification into substituting-resisting position is incoherent.

What rule of =E is appropriate for languages in which attitude ascriptions can be expressed? If we are in agreement that

(5) Lex fears Superman; Superman = Clark; therefore Lex fears Clark

has a reading on which it is a fallacy, then we have the option of saying that the position of *Superman* in the minor premise on such a reading is not purely referential. But Marcus would be among the last to assent to this proposal, and besides, we have seen no motivation for it in our previous cases, where the problem arose from =E's affecting the semantic value of some other, dependent, term, *so* or a pronoun, and wasn't related to the semantics of the substituted term. Such a proposal would, moreover, make the preservation of interpretability in

(6) Lex fears Superman; therefore, someone is such that Lex fears him

puzzling, since it is not obvious how a device of pure reference, such as a variable, could function in a position that is not purely referential—Quine's point.

Despite her interest in the *Giorgione* case, Marcus does not seem to have considered the possibility that substitution-failure in attitude ascriptions is to be accounted for by the covert presence of something analogous to *so-called*. Perhaps such a proposal seemed to her to threaten (6), but we shall see that this need not be so. The idea would be that (5) fails for essentially the same reason that (1) fails: replacing *Superman* has the side-effect of changing another expression's content. This in turn would be because the minor premise of (5), (25.1) below, is underlyingly (25.2) for the reading on which (5) is invalid:

(25.1) Lex fears Superman
(25.2) Lex fears Superman *as such*.

In logical form the sentential adverb phrase *as such* is prefixed as a complex sentential operator, consisting in the preposition *as* and the demonstrative *such*. The claim is that

(25.1) optionally contains a phonetically null operator *O* with a semantics like *as such*; when the operator is present, =E fails. For (25.2) we would write more formally

(26) *as such*(*fears*(*Lex*, *Superman*))

which expresses a proposition relative to a Kaplan-style context that fixes a reference for the *such*. We display contexts in the arrow notation, so that the intended meaning for (26) is given by

(27) *as such*(*fears*(*Lex*, *Superman*))

(27) exhibits the context μ such that $\mu(1)$ = <1, *Superman*>. In this context, (26) expresses the same proposition as the context-independent *Lex fears Superman as Superman*.

The main idea behind the semantics is that intensional transitives like *fear* are assigned a *set* of alternative extensions instead of a single extension. *As* is assigned a function which looks at the content of *such* in a given context μ and produces another function, one which selects an extension from the available alternatives for each intensional transitive in the scope of the *as such* in question. Which extension-selector is produced by *as* can vary with the content of *such*. So whether or not (26) comes out true in a context μ will depend on the action of the extension-selector produced by *as* on *fear*. When *Superman* is the content of *such*, we get an extension-selector that picks an extension for *fear* containing <Lex, the Man of Steel>. When *Clark* is the content of *such*, we get an extension-selector that picks an extension for *fear* not containing <Lex, the Man of Steel>.[13]

On the proof-theoretic side, we require the L$_F$-restriction on the rule of =E, that targeted constants cannot be substituted. There is also an Elimination rule for *as such*, namely, that from *as such*$^{(i)}(\phi)$ we can infer ϕ. The semantic justification for this is that if *as such*$^{(i)}(\phi)$ holds, this means that each relevant pair of objects belongs to at least one of the extensions in the semantic value of each intensional verb ψ in ϕ. Then if ϕ is evaluated independently, it will come out true, so long as we arrange for the absence of *as such* to trigger a union operation on $[\![\psi]\!]$ resulting in a single extension that contains each pair of objects in at least one extension in $[\![\psi]\!]$.

The soundness of *as such*-Elimination is important for solving the puzzle of why quantifier introduction, as illustrated in

(6) Lex fears Superman; therefore, someone is such that Lex fears him

produces an interpretable conclusion. Assuming that only individual constants can be targeted, the derivation of a truth-condition for a sentence containing a *such* that targets a variable will crash. But it follows that the conclusion of (6) is uninterpretable only if we suppose it to have been inferred by essentially one step of ∃I. The reason (6) strikes us as acceptable is that when we interpret *Lex fears Superman* as (26), *Lex*

[13] For technical details in the framework of neo-Davidsonian event-semantics, see further (Forbes 2000, 154–64); at p.156 I thank David Kaplan for pointing out an extensionality problem for the multi-extension 'atomic' semantics I am now describing, which I resolved in that paper by quantifying over events. But it would be too tangential to Marcus's work to pursue this here.

fears Superman as such, we take (6) to be an enthymeme in which a step of *as such* Elimination is implicitly performed before ∃I is applied.

This means that the case of attitude ascriptions is not really different from the others we have investigated: the mechanism explaining substitution-resistance also blocks quantifier introduction, *if* it is present. The superficial difference is that in ordinary English, the mechanism's presence or absence is not indicated on the surface, whereas the difference between, say, *Giorgione is so-called* and *Giorgione is called 'Giorgione'*, is easy to see.

Objectual attitude ascriptions are the easier case. Ideally, we would like an absolutely uniform treatment of objectual and propositional ascriptions; but it is unclear how this is to be accomplished. One reasonable philosophical account of the phenomenon of substitution-resistance in objectual attitude ascriptions is that we can stand in attitude relations to objects under *modes of presentation* of those objects. Furthermore, we may stand in such a relation to a certain object under one mode but not under another. The differing extensions for an intensional transitive like *fear* show which experiencer-theme pairs enter or drop out of the verb's extension as modes of presentation vary. We may say that objectual attitude relations are relations *to* Russellian entities that hold *under* Fregean ones. Names are modally but not cognitively equivalent because only cognitive operators like *as such* invoke the Fregean entities.

A uniform account of propositional and objectual ascriptions would require that propositional attitude relations are also relations to Russellian entities that hold under Fregean ones. But here we run into difficulties, since it seems that the only candidate for the second term of a propositional relation is a proposition. Fregeans and Russellians characteristically disagree over the nature of propositions (there is no comparable disagreement about the objects in objectual ascriptions). So it is unclear how a theory of propositional ascriptions could combine Frege and Russell.

In her later work, Marcus (1990b, 240-1) made the interesting proposal that Russellians abandon the notion of proposition, which she argues is ineliminably 'language-centered', and exchange it for the 'object-centered' notion of a *state of affairs* (SOA). So-called "propositional" attitude relations are really relations to states of affairs, which are complexes of objects and properties. The idea that *modal* properties are fundamentally features of SOA's is one to which I am highly sympathetic (see Forbes 1989, ch. 5), but Marcus's suggestion is more radical. And she argues that substituting SOA's for propositions helps Russellians with issues about the rationality of those who apparently take "assentive" attitudes towards contradictory Russellian propositions (1990b, 248-51), as in the example

(28) Lex fears that Superman is nearby and Clark is not.[14]

The advantages for Russellians of replacing a fear with a contradictory proposition as its content with fearing an impossible SOA are not so clear to me.[15] But once we have

[14] Suppose Lex {hopes/plans} to use Clark as a hostage to thwart Superman.

[15] Marcus says (*loc. cit.*) that the language-centred theorist is "baffled" by the question, does Lex fear Clark is nearby or does he not? She then argues that if we take the object-centred view that a belief is a dispositional relation to a SOA, namely, to act as if that SOA a obtained, "a puzzle has been solved". But I

distinguished SOA's and propositions in Marcus's way, we can think of propositions as *modes of presentation of SOA's*. We are then in a position to distinguish believing an impossible SOA from believing a *manifestly* impossible SOA, a distinction reflected in the following two formulations:

(29.1) *as such*(*fears*(Lex, *that Superman is nearby and Clark is not*))

(29.2) *as such*(*fears*(Lex, *that Superman is nearby and Superman is not*)).

Here the *that*-clauses refer to SOA's, and the indicated subsentences invoke modes of presentation of them. We should therefore extend our semantics to propositional attitude verbs and accommodate the introduction of SOA's: an extension for *fear* may include experiencer-SOA pairs as well as experiencer-object pairs, *fear* is assigned multiple extensions, and *as such* chooses between them as before. Substitution is again forbidden for targeted expressions; we simply expand the range of expressions that can be targeted. More carefully, substitution of expressions that are either targeted themselves, or that contain a targeted proper constituent, is forbidden. So there is no interchanging *that Superman is nearby and Clark is not* and *that Superman is nearby and Superman is not* even though they both refer to the same SOA. Similarly, there is no quantifying into any position within the extent of the delimiter of a targeted expression.

The treatments of objectual and propositional ascriptions are now exactly parallel. But have we paid a high price for this uniformity? There is a difficulty in the area of action-explanation. We will accept that

(30) Anyone who fears Superman and believes Superman is nearby will, *ceteris paribus*, try to avoid encountering Superman on the occasion of acquiring the belief.[16]

This is a correct principle if *fears*, *believes* and *try* are in the scope of (the unvoiced counterpart *O* of) *as such*. But (30) seems to be an instance of the more general

(31) Anyone who fears a certain person and believes that that person is nearby will, *ceteris paribus*, try to avoid encountering that person on the occasion of acquiring the belief.

However, (31) contains only variables, and variables cannot be targeted, so there is no *as such* in (31) if (31) expresses a complete proposition. But in that case, a step of ∀E on (31),[17] followed by =E, will produce the apparently false

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find myself baffled by the question what it would be to act as if the SOA *Clark is nearby and Clark is not nearby* obtains, assuming univocal *Clark*.

[16] Strictly, we ought to say something more cautious along the lines of *try to do something that he or she thinks within his or her powers and thinks will significantly reduce the chances of encountering that person on that occasion, if he or she believes the chances of an encounter are too high without such steps being taken*.

[17] It's not really ∀E that is applied to (31) unless we are thinking of a regimentation in which the unbound anaphora in (31) is removed by replacing *someone* with a wide-scope universal. But we can certainly *instantiate* (31) and then use =E to infer (32) below, however exactly the instantiation is to be explained.

(32) Anyone who fears Superman and believes Clark is nearby will, *ceteris paribus*, try to avoid encountering Superman on the occasion of acquiring the belief.

How should we deal with this difficulty? The problem is that (31) apparently instantiates two *referentially transparent* ascriptions, that is, in our framework, ones without occurrences of *as such* to block substitutions such as the substitution of *Clark* that results in (32). The substitution carries us from an apparently predictive principle to one that isn't. Of course, logic only promises to preserve truth, not explanatoriness or predictiveness. Nevertheless, it is puzzling that (31) seems apt for deploying in explanation of a subject's furtive behavior, yet its instances fail us.

How the subject thinks of the object or SOA is relevant to explanatoriness, and I see only two ways consonant with my approach of accounting for principles like (31).[18] One proposal is that when we understand (31) to be true, we are implicitly inserting quantifiers over modes of presentation: if you fear someone under a certain m.p. and believe under that m.p. that that person is nearby, then… Of course, this interpretation allows us to instantiate the objectual variable with a name and the m.p. variable with a name of a coreferential name, so that we get the likes of (27) only as a special case.

Alternatively, it may be that principles like (31) are not understood as exceptionless: there is a suppressed "typically", and the, or one, atypical case is precisely when the subject has two different ways of thinking of the same individual. For subjects who have only one way of thinking of the object in question, the generalizations are correct. But for subjects with more than one way of thinking of the object, instantiation has to be followed by *as such* Introduction in the scope of intensional verbs. The introduced occurrences of *as such* then follow the anaphoric dependencies for their interpretation. From (31) this process would produce, informally,

(33) Anyone who fears Superman$_1$ as such and believes that person$_1$, as such, is nearby will, *c.p.*, try to avoid encountering that person$_1$ as such

and there is no route from here to the unwanted (32). (33) is also correct in the typical case, but is overkill if we are assuming that the subject only has one way of thinking of the Man of Steel.[19]

Clearly, then, the problem of how commonsense psychological explanation works is of very great relevance to the semantic issues we have been discussing here. Exploring their interrelationships would surely be an appropriate way of extending Marcus's legacy.

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[18] A very different approach, based on making liberal use of *ceteris paribus* qualifiers, is developed in (Braun 2000).

[19] A principle that lacks intensional operators in its consequent may appear to be explanatory of brute behavior, e.g., 'anyone who dislikes Superman and thinks he can get away with it will hit him'. Nothing I have said blocks substitution of 'Clark' for 'him', since the latter is not within the scope of an intensional verb. But I am inclined to think that if the quoted principle is correct, so is 'anyone who dislikes Superman and thinks he can get away with it will hit Clark', certainly so if we prefix 'since Superman is Clark'.

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