Guest Editor's Introduction

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The so-called 'Quine-Putnam Indispensability Argument', as characterized e.g. by Mark Colyvan (2001), is an argument for mathematical Platonism, the claim that at least some of our mathematical theories (specifically, those that are indispensable to empirical science) are true of a realm of abstract mathematical objects. But Hilary Putnam (2012) complains that *his* indispensability argument was never intended as an argument for *Platonism*, but only for a non-ontological form of mathematical *realism*, where what is defended is mathematical objectivity, but not the existence of mathematical objects.

Precisely what does Putnam's mathematical 'realism without ontology' amount to, and is Putnam's right that his indispensability considerations establish his realism without establishing Platonism?

Putnam (2012) suggests that the reason for resisting Platonism despite the indispensable presence of mathematics in empirical science is that the apparently ontologically committing mathematical claims used can be replaced by modal-structural translations (in the manner of Hellman (1989), which develops the proposal of Putnam (1967)). On the 'mathematics as modal logic' picture, the mathematical claims used in science are objectively true, but their truth doesn't require the existence of mathematical objects, Platonistically construed, but only the objectivity of their modal translations.

In the contemporary debate between Platonism and anti-Platonism, then, while most take it that acceptance of the indispensability of mathematics supports *Platonism*, Putnam falls squarely on the *anti-Platonist* side. But Putnam (2012) is highly critical of contemporary *fictionalism* as an anti-Platonist account of mathematics. Putnam argues that Field's *dispensabilist* fictionalism, if it could be made to work, would adequately respond to *Putnam's* indispensability argument, but that alternative *instrumentalist* versions of fictionalism (such as is offered by Rosen 2001) would fail as a response. Putnam's concerns about instrumentalist fictionalism include the complaint that, in order to resist the indispensability argument fictionalists presuppose an abstract/concrete divide, but they cannot characterize what they take to be the truths about 'the concrete world' except by reference to mathematics, and the complaint that, in refusing to believe that the explanations offered by our ordinary scientific theories are *true*, holding instead that they are *nominalistically adequate*, fictionalists abandon the scientific project of explanation.

As **Concha Martinez Vidal**'s discussion makes clear, Putnam's objections to contemporary instrumentalist fictionalism are rather puzzling. In particular, she argues, both

Field's *dispensabilist* fictionalism, and Putnam's modal structural realism, also require that we can make sense some kind of brute notion of 'concreteness'. For Field's fictionalism, in order to make sure that his nominalistically-stated theories (expressed in a nominalistic vocabulary) don't place restrictions on the abstract realm (and thereby contradict conservativeness), he has to relativize their universally quantified claims to all *non-mathematical* things. And to the extent that Putnam's 'objectivity without objects' view depends on the success of Hellman's modal structural interpretation of applied mathematics, this account also needs some way of pinning down the actual concrete world to modalize on (via a 'non-interference proviso'). It would seem that Putnam's modal structural account can avoid ontological realism if and only if instrumental fictionalism can.

But can *either* account really avoid commitment to mathematical objects? One concern in the attempt to avoid ontology by showing that existence claims in mathematics can be replaced by modal claims is that a commitment to objective modal claims concerning consistency and logical consequence is itself a commitment to mathematical objects —particularly, to sets— given that to claim that a theory is logically consistent (for example) *just is* to claim that it has a model. Indeed, as **José Miguel Sagüillo** points out, in his 1971 book, *Philosophy of Logic*, Putnam argues explicitly for an account of logical validity in terms of sets. If we wish to maintain the view that the modal structural interpretation of mathematics avoids commitment to mathematical objects, then it looks as though we must reject the reduction of modal claims to claims about the existence of set theoretic models, and instead accept primitive modality.

Otávio Bueno, in his contribution, is unsatisfied with the proposal to accept modal facts as primitive, given that it provides no answer to the question of "what grounds the possibility of structures satisfying the axioms of ZFC" (Bueno, p. 210). Perhaps, Bueno suggests, Putnam's insight that the use of mathematics in science requires only its objectivity and not the existence of mathematical objects can be secured in another way. Rather than offering modal structural translations of mathematics, Bueno proposes to leave mathematics as it is, and accept the indispensable quantification over mathematical objects in empirical science. Nevertheless, Bueno claims, such acceptance does not force a commitment to an ontology of abstract mathematical objects, given the possibility of recognising an ontologically neutral reading of our quantifiers. On Bueno's proposal, mere quantification over a domain of objects does not require acceptance of the existence of the objects quantified over. Rather, existence can be indicated in other ways (e.g. via the ascription of an existence predicate).

If quantifier commitment involves no ontological commitment, then we are owed an account of when a theory *is* committed to the real existence of an object it quantifies over. One option would be to offer a *causal* criteria: we are committed to the existence of objects our theory posits as causes. **Sorin Bangu** offers reasons for thinking that a focus on *causal role* as making the difference between theoretical posits whose existence we should take as confirmed by their indispensable presence in our scientific theories and those whose existence is not confirmed would be in conflict with a form of methodological naturalism that many scientific realists would wish to accept. If, for example, mathematical posits play an indispensable explanatory role in our scientific theories, then perhaps this suffices to confirm the existence of the mathematical objects posited even if mathematical objects play no causal role (though see **Susan Vineberg's** contribution for some reasons to be suspicious of the claim that the use of mathematics in explanation confirms the existence of mathematical objects).

An alternative route to distinguishing between those objects we quantify over in empirical science whose existence is confirmed by their role in our scientific theories, and those that are merely instrumental posits, is to argue that mathematical objects differ from, say, the physical objects quantified over in our theories in virtue of what **Matteo Plebani** calls their 'preconceived' nature, that being the fact that their properties appear to be fixed by the way we characterize them in a way that is not true of the physical objects to which we take our theories to be genuinely ontologically committed. If mathematical posits are distinguished from physical posits by virtue of being preconceived, then this would provide support for the idea that the question of "**how the concrete world is** and **whether there are abstract objects** are orthogonal" (Plebani, p. 250). It is this intuition that seems ultimately to stand behind Putnam's own skepticism about the ontological implications of his indispensability argument when he asks, "if any entities do not interact with us or with the empirical world *at all*, then doesn't it follow that *everything would be the same if they didn't exist?*" (Putnam 2000, 33) So perhaps a defence of mathematical objects as preconceived would provide the key to seeing the indispensability considerations as supporting the objectivity of mathematics but not the existence of mathematical objects.

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