THE WAY OF LOGIC INTO MATHEMATICS

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ABSTRACT: Using a contextual method the specific development of logic between c. 1830 and 1930 is explained. A characteristic mark of this period is the decomposition of the complex traditional philosophical omnibus discipline logic into new philosophical subdisciplines and separate disciplines such as psychology, epistemology, philosophy of science, and formal (symbolic, mathematical) logic. In the 19th century a growing foundational need in mathematics provoked the emergence of a structural view on mathematics and the reformulation of logic for mathematical means. As a result formal logic was taken over by mathematics in the beginning of the 20th century as is shown by sketching the German example.

Keywords: Revolution in logic; philosophical and mathematical contexts of the emergence of mathematical logic, "logical question", symbolic algebra, absolute algebra, calculus of operations, algebra of logic, combinatorics, general doctrine of forms, institutionalization of mathematical logic in Germany.

1. Introduction

There is no doubt that the modern shape of logic can hardly be compared with the state of logic some 100 or 150 years ago. The changes are so radical that it is easy to understand that historians of logic are inclined to describe them in terms of Thomas S. Kuhn's *Structure of Scientific Revolutions* (Kuhn *1962*). Donald Gillies is an example. In 1992 he edited a volume *Revolutions in Mathematics* (Gillies (ed.) *1992*) collecting papers which discuss the possibility of applying Kuhn's theory of scientific revolutions and his ideas concerning changes of paradigms to formal sciences. In his own contribution Gillies discussed the history of logic as an example of paradigmatic changes isolating a "Fregean Revolution in Logic" (Gillies *1992*).

In the following presentation I will proceed in three steps. First I will discuss Gillies's method to diagnose a revolutionary character of the changes in logic. I will conclude that his method is incapable to explain the specific development of logic between c. 1830 and 1930. Alternatively I will offer a contextual method which will be used in a second step to present the reasons for these changes. These reasons can be seen in the decomposition of the complex traditional philosophical omnibus discipline logic into new philosophical subdisciplines and separate disciplines such as psychology, epistemology, philosophy of science, and formal (symbolic, mathematical) logic. At the same time a growing foundational need in mathematics (calculus, algebra, geometry) could be observed, provoking the emergence of a structural view on mathematics, the formulation of universal theories (universal algebra, pangeometry etc.), and the reformulation of logic for mathematical means. In a final third step I will describe the ways in which formal logic was taken over by mathematics in the beginning of the 20th century by sketching the German example.

2. Is there a Fregean Revolution in Logic?

Readers may wonder that a Fregean revolution in logic could be questioned at all. Modern first-order logic started with Gottlob Frege's (1848-1925) *Begriffsschrift* of *1879*, they say, and thus, no doubt, Frege is the father of modern logic. But, what does "father of modern logic" mean? It may be regarded as synonymous with "founder of modern logic" as W.V. Quine did. He wrote: "(...) I have long hailed Frege as the founder of modern logic, and viewed Boole, De Morgan and Jevons as forerunners" (Quine *1985*). This confession was published in a review of Desmond MacHale's biography (MacHale *1985*) of the British algebraist of logic George Boole (1815-1864), and it was part of an argument from a "truer perspective" against MacHale's falling in "with the tradition (...) of representing Boole as the father of modern logic."[1] Quine had to face the fact that modern logic has at least two sources. Beside the logicistic approach connected to the names of Gottlob Frege, Giuseppe Peano and Bertrand Russell there was the earlier algebraic approach by logicians like the British George Boole and William Stanley Jevons, the American Charles S. Peirce and the German Ernst Schröder. Quine therefore admits:

> The avenue from Boole through Peirce to the present is one of continuous development, and this, if anything, is the justification for dating modern logic from Boole. (...) But logic became a substantial branch of mathematics only with the emergence of general quantification theory at the hands of Frege and Peirce. I date modern logic from there.
>
> Frege got there first. His initial and succeeding contributions were richer and more refined, moreover, than any other logical work until the present century, when the current from Boole through Peirce and the current from Frege converged.

This quotation shows that there are not only two *sources* of modern logic, but also at least two *perspectives* on innovative developments in science. The first is the "perspective of priority" centred around the search for first formulations and first occurrences of seminal ideas. The second is the "genetic perspective" which complements the former by taking the reception of new ideas into account. In my opinion only the second perspective allows a suitable presentation of the *development* of a scientific discipline. Nobody adhering to this second perspective would deny that Frege was the first who formulated a quantification theory which includes first-order logic. He would furthermore agree that the invention of quantification theory was a decisive step towards modern logic. He will, however, ask whether the new idea became effective at all, or whether is was only retrospectively recognized as a new idea. According to this approach it makes no sense to start modern logic with Leibniz as Wolfgang Lenzen suggested (cf. Lenzen *1984*). Leibniz's logical calculi -which are in some respects more powerful than the Boolean algebra of logic- came to the fore only after the new logic had already been invented (cf. Peckhaus *1994a*). And even Frege might be a doubtful case as expressed by Geraldine Brady in her paper on "From the Algebra of Relations to the Logic of Quantifiers" (*forthcoming*):

> The restriction to first-order logic (i.e. Peirce's restriction to "the first-intentional logic of relatives in his paper of 1885) has been enormously fruitful in modern mathematical logic, and, although modern foundation literature ascribes first-order logic primarily to Frege's seminal paper (which is, of course, a book) (1879), we support the case that it was Peirce's work, as systematized and extended by Schröder (1895), that was a primary influence on Löwenheim (1915) and Skolem (1920, 1922 (1923)), as reflected by their notations and methods.

The discussion above should have shown how difficult it can be to identify a proper starting-point of a certain development. Such difficulties restrict the power of explanations using the notion of revolution in order to qualify developments. This scheme of explanations is restricted to the comparison of theories at certain stages of development. It fails to explain the reasons for these developments.

Although Kuhn's term "paradigm" had been criticized as too vague and ambiguous, Gillies thinks that "it has (...) just the right degree of precision for the subject-matter in hand, the analysis of revolutions in science and mathematics" (Gillies *1992*, 270). Gillies analyses the Fregean revolution in logic as an abandonment of the Aristotelian paradigm "whose core was the theory of syllogism" in favour of the Fregean paradigm "whose core was propositional and first-order predicate calculus" (271). In order to prove his claim that this change of cores (whether they are really the cores or not is not discussed in the paper) are in fact changes in paradigms, Gillies uses what he calls "the textbook criterion for paradigms." He justifies it by the following suggestion (ibid.):

> If a historian wishes to identify the paradigm of a group of scientists at a certain time and place, he or she should examine the textbooks that were used to teach novices the knowledge they needed to become fully recognized members of a group. The contents of these textbooks will then (more or less) define the paradigm accepted by the group.

Applied to the development of logic one should find

> that pre-revolutionary textbooks expound Aristotelian logic with particular emphasis on the theory of the syllogism, while these topics should disappear from the post-revolutionary textbooks to be replaced by an account of propositional and first-order predicate calculus.

Gillies presents John Neville Keynes's *Studies and Exercises in Formal Logic* as an example of a pre-revolutionary textbook. It was first published in *1884*, a fourth edition appeared in 1906. The books by Elliott Mendelson *Introduction to Mathematical Logic* of *1964* and *A Course in*

Mathematical Logic by John L. Bell and Moshe Machover (*1977*) are taken as post-revolutionary textbooks.

For Gillies the revolution was evoked by Frege's *Begriffsschrift* (*1879*) which appeared five years *before* the pre-revolutionary book by Keynes was published. One should concede that the later date of publication does not disqualify his choice because "many years were to elapse before the new paradigm was properly formed, and more years still before it succeeded in ousting the old paradigm" (Gillies *1992*, 272).

Gillies tries to prove his thesis by a simple quantitative evaluation of his examples. He states that the complete third part of Keynes's book is devoted to the syllogism, i.e., 29 % of its size. In contrast to this, the words "Aristotle" or "syllogism" are not even mentioned in the post-revolutionary books. Frege's remarks in his *Begriffsschrift* sum up to 1 1/2 pages of 101 pages in Bynum's translation (Frege *1972*), i.e., 1.5 % of its size. Therefore Gillies takes it for granted that the syllogism disappeared from logic textbooks and that Aristotelian logic lost its former importance. In comparing the Fregean revolution in logic with the Einsteinian revolution in physics he goes even further (Gillies *1992*, 283-284):

> Aristotelian logic is very often omitted altogether from textbooks, and rarely, if ever, would someone nowadays consider formulating a logical argument in syllogistic form. So, while Newtonian mechanics is still taught and used, Aristotelian logic is, to a large extent, neither taught nor used.

Gillies' analysis is based on a mixture of presuppositions and prejudices leading to unprovable consequences. He obviously maintains

1. that the syllogism was an undisputed core element of 19th century research on logic;
2. that the logic book of Keynes (which is much more than a textbook, containing, e.g., Keynes's theory of existential import) was a typical (paradigmatic) logic book;
3. that today the syllogism is no more a topic of logic books, being a completed domain of knowledge which cannot and need not be revised or enlarged, and that it has no applications.

All these assertions can be doubted.

1. At least since Descartes' times the syllogism and with it all of formal logic was heavily debated. Above all the syllogism's significance for solving philosophical problems and for finding truths was disputed. Nevertheless glancing through several 19th century logic books shows that the syllogism was no longer a central topic. This can be shown by playing Gillies' game of counting pages. Georg Friedrich Wilhelm Hegel's *Wissenschaft der Logik*, e.g., comprises two volumes. The first volume, *Die objective Logik*, was published in *1812*, the second, *Wissenschaft der subjectiven Logik oder die Lehre vom Begriff*, in *1816*. Under the heading "The Inference of the

Existence" ("Der Schluß des Daseyns") only 30 of these 1029 pages, i.e., 2.9 % of the total size, deal with mediated inferences, including seven pages of polemics against the formal view on logic. The neo-Aristotelian and anti-Hegelian Friedrich Adolf Trendelenburg deals with the syllogism on 19 of the 691 pages of his *Logische Untersuchungen* (2.75 %). Herman Ulrici, one of the spokesmen of a revival of formal logic against Hegel's abandonment, discusses the syllogism on 9 of the 218 pages of his *Compendium der Logik* (4.13 %). As a last example Ernst Schröder's *Vorlesungen über die Algebra der Logik* should be mentioned, in which the "syllogisms of the ancient" ("Syllogismen der Alten") are discussed on 38 pages (*1891*, §§ 42-44). But his monumental three volumes have a total of 1985 pages, so these are only 1.9 %.

2. Keynes's *Formal Logic* is far from being a typical British 19th century logic book. Its main purpose was to give an overview on the doctrines of Aristotelian logic[2] in a country where the philosophy of inductive sciences and inductive logic as their methodology were predominant. Formal logic was brought to a broader audience only in *1826* when Richard Whately's successful *Elements of Logic* appeared. The state of logical knowledge in England before this publication was characterized by Thomas M. Lindsay (Lindsay *1871*, 557):

> Before the appearance of this work, the study of the science had fallen into universal neglect. It was scarcely taught in the universities, and there was hardly a text-book of any value whatever to be put into the hands of the student.

And still in *1894* John Venn deplores the poorness of logical knowledge in England at least in comparison with Germany (Venn *1894*, 533). Far more influential than Keynes's book was William Stanley Jevons's *Elementary Lessons in Logic* (*1870*) which was published, according to Risse's bibliography (Risse *1973*), in seven editions and 31 further printings between 1870 and 1968. Jevons presents the syllogism on 43 of 352 pages (12.22 %). Already in *1864* Jevons had published the booklet *Pure Logic* containing his elaboration of George Boole's algebra of logic and its utilization for questions of the traditional philosophical logic. There he discusses the relation of his system to the syllogism on 8 of 74 pages in order "to show the power of its method" (52). This is a quota of 10.81 %. But only on 1 page a syllogistical mode is discussed (FELAPTON), i.e., 1.35 %.

3. Gillies ignores the fact that the syllogistic theory is still today (maybe more than in the 19th century) a vivid topic of research. "Aristotelian logic" differs, however, significantly from the logic of Aristotle. Several modifications, extensions and applications can be observed. The discussion on the quantification of the predicate between William Hamilton and Augustus De Morgan in the mid of the 19th century is an early example. It concerned a modification of the standard forms of categorical judgements (cf. Heath *1966*). Other variants of modifications of the syllogism include,

e.g., the syllogism of proportional quantifiers (Thompson *1992*) or the syllogism with uncertain propositions (Spiess *1989*). The Aristotelian syllogistic has been reconstructed and axiomatized (cf. Łukasiewicz *1951*, Ebbinghaus *1964*, Bird *1964*), and applied to the philosophy of language, metaphysics, moral and political philosophy (cf. Sommers *1982*, Englebretsen *1987*).

Gillies' textbook criterion is obviously inappropriate to prove what it was to prove. Admittedly, in 19th century syllogism was an accepted part of logic (even critics of formal logic as Hegel, Schleiermacher and others did not question the validity of, e.g., the mode BARBARA) but it stood not in the core of the logical discussion, and its value for philosophy was heavily questioned. It was only taken to the fore in order to test new methods and results. And Frege's new logic did not block its further development.

I don't want to deny that the emergence of first-order logic was a decisive step in the development of logic. On the contrary! I wanted to show that the changes in logic cannot be explained and not even described in an appropriate way in terms of the theory of scientific revolutions (at least in Gillies's version). Obviously the development of logic since the second half of the 19th century went in two directions. Whereas the syllogism plays a significant rôle in theories of representating vague knowledge, mathematical logic is applied in metamathematics, theoretical computer science and mathematical domains. A historical methodology should be able to explain the emergence of such split.

As a method to deal with such developments I would like to propose a "contextual historiography of scientific disciplines," a methodology elaborated in a research project under the direction of Christian Thiel entitled "Case Studies towards a Social History of Formal Logic" (1985-1990).[3] The leading idea of this methodology is to investigate the emergence and development of new ideas and methods within their historical context. It aims at combining the traditional history of ideas with the social history of science in order to explain motives, initial reasons, and the background for scientific developments. One of the lessons which can be drawn from this method is that the history of mathematical logic cannot be written by simply listing first formulations and inventions. Such procedure would at best help to establish the existence of a new development leading to the present state of affairs. But it wouldn't be able to give any explanation of the emergence of this development, and it seduces to forget all of its circumstances.

3. The Emergence of the New Logic

In the following sections I will give a short overview on the philosophical and mathematical contexts in which the new logic emerged.[4] My aim is to answer the question why the new logic was created by mathematicians although logic undoubtedly belongs to the domain of philosophy.

3.1. The Philosophical Context in Great Britain

The development of the new logic starts in 1847, completely independent from earlier anticipations, e.g. from Leibniz (cf. Peckhaus *1994a*), when the British mathematician George Boole publishes his pamphlet *The Mathematical Analysis of Logic* (Boole *1847*). Boole mentions that it was the priority struggle concerning the quantification of the predicate between the Edinburgh philosopher William Hamilton (1788-1856) and the London mathematician Augustus De Morgan (1806-1871) which stimulated this study. He thus refers to a broad philosophical discussion indicating a vivid interest in formal logic in Great Britain. This interest was, however, a new interest, not even 20 years old. On the contrary, neglecting formal logic can be regarded as a mark of British philosophy up to *1826* when Richard Whately published his *Elements of Logic*.[5] In his preface Whately adds an extensive report on the languishing research and education in formal logic in England. He complaines that only very few students of the University of Oxford became good logicians and that (*1826*, xv)

> by far the greater part pass through the University without knowing any thing of all of it; I do not mean that they have not learned by rote a string of technical terms; but that they understand absolutely nothing whatever of the principles of the Science.

Thomas Lindsay, the translator of Friedrich Ueberweg's eminent *System der Logik* (*1857*, translation *1871*), is very critical of the scientific qualities of Whately's book, but he, nevertheless, emphasizes its outstanding rôle for the rebirth of formal logic in Great Britain (Lindsay *1871*, 557):

> Before the appearance of this work, the study of the science had fallen into universal neglect. It was scarcely taught in the universities, and there was hardly a text-book of any value whatever to be put into the hands of the students.

One year after the publication of Whately's book George Bentham's *An Outline of a New System of Logic* appeared (*1827*) which came along as a commentary to Whately. Bentham's book was critically discussed by William Hamilton in a review article published in the *Edinburgh Review* (*1833*). With this review Hamilton founded his reputation as the "first logical name in Britain, it may be in the world."[6] Hamilton propagated a revival of the Aristotelian scholastic formal logic without, however, one-sidedly preferring the syllogism. His logical conception was focused on a revision of the standard forms by quantifying the predicates of judgements.[7] The controversy about the priority arose, when De Morgan, in a lecture "On the Structure of the Syllogism" (De Morgan *1846*) given to the Cambridge Philosophical Society on 9 November 1846, also proposed to quantify predicates. The dispute indicates a new interest in *research* on formal logic, but it represents only one side of the effect released by Whately's book. Another direction of research stood in the direct tradition of Humean empiricism and the philosophy of inductive sciences: the inductive logic of John Stuart Mill (1806-1873), Alexander Bain (1818-1903) and others. Boole's logic was in clear opposition to inductive logic, an opposition which was only made explicit, however, by his successor William Stanley Jevons (cf. Jevons *1877-1878*). Boole refers to Hamilton, but his influence should not be overemphasized. In his main work on the *Laws of Thought* (1854) Boole goes back to the logic of Aristotle quoting from the Greek original. This can be interpreted as a sign that the influence of the contemporary philosophical discussion was not as important as his own words might suggest. In writing a logic book he is doing philosophy, and thus it is a matter of course that his results should be related to the philosophical discussion of his time. This does not mean, of course, that the thoughts expressed in it are really influenced by this discussion.

3.2. The Philosophical Context in Germany

It seems clear that, as regards the 18th century dichotomy between German and British philosophy as represented by the philosophies of Kant and Hume, Hamilton and Boole stood on the Kantian side. And in Germany as well, the philosophical discussion on logic after Hegel's death was determined by the Kantian influence. In the preface to the second edition of his *Kritik der reinen Vernunft* of *1787* Immanuel Kant (1723-1804) wrote (*KrV* B VIII) that logic has followed the safe course of a science since the earliest times. For Kant this can be shown by the fact that logic was not allowed to take any step backwards since Aristotle. But he regards it as curious that it took no step forwards either. Thus, logic seems to be closed and complete. Formal logic, in Kant's terminology the analytical part of the general logic, does not play a prominent rôle in Kant's system of transcendental philosophy. In any case it is a negative touchstone of truth (*KrV* B 84). Georg Wilhelm Friedrich Hegel (1770-1831) goes further in denying any relevance of formal logic for philosophy (Hegel, *Wissenschaft der Logik*, *1812/13*, I, Introduction, XV-XVII). Referring to Kant he maintains that from the fact that logic did not change since Aristotle one should infer that it needs a complete recast (ibid., XV). Hegel created logic as the foundational science of his philosophical system, defining it as "the science of the *pure idea*, i.e., the idea in the *abstract element of reasoning*" (*Encyclopädie der philosophischen Wissenschaften*, *1830*, 27). Logic thus coincides with metaphysics (ibid., 34).

This was the state of the art when the philosophical discussion on logic in Germany started after Hegel's death. This logic reform discussion stood under the label "the logical question", a term coined by the Neo-Aristotelian Adolf Trendelenburg. In *1842* he published a paper entitled "Zur Geschichte von Hegel's Logik und dialektischer Methode" with the subtitle "Die logische Frage in Hegel's Systeme". But what is the logical question according to Trendelenburg? He formulates this question explicitly towards the end of his article: "Is Hegel's dialectical method of pure reasoning a scientific procedure?" (*1842*, 414). In answering this question in the negative, he gives the occasion to rethink the status of formal logic within a theory of human knowledge without, however, proposing to return to the old (scholastic) formal logic. In consequence the term "the logical question" was subsequently used in a less specific way. Georg Leonard Rabus, the early chronicler of the logic reform discussion, writes that the logical question emerged from doubts concerning the justification of formal logic (*1880*, 1). Although this discussion is clearly *connected* to formal logic the claimed reform does not *concern* a reform of formal logic. The reason is given by the Neo-Kantian Wilhelm Windelband who writes in a brilliant survey on the 19th century logic (*1904*, 164):

> It is in the nature of things that in this enterprise (i.e. the reform of logic) the lower degree of fruitfulness and developability power was on the side of formal logic. Since the reflection on the rules of the correct progress of thinking, the technique of correct thinking, had indeed been brought to perfection by former philosophy, presupposing a naive world view. What Aristotle had created in a stroke of genius, was decorated with finest filigree work in Antiquity and the Middle Ages: an art of proving and disproving which culminates in a theory of reasoning, and then constructing from this the doctrines of judgements and concepts. If one has once accepted the foundations, the safely assembled building is not to be shaken: it can only be refined here and there and perhaps adapted to new scientific requirements.

Concerning the English mathematical logic Windelband is very critical. Its quantification of the predicate allows the correct presentation of extensions in judgements, but it "drops hopelessly" the vivid sense of all judgements, which are inclined to claim or deny a material relation between subject or predicate. It is "a logic of the conference table", which cannot be used in the vivid life of science, a "logical sport" which has, however, its merits in exercising the final acumen (ibid., 166-167).

The reform efforts concerned above all two domains: (1) the problem of a foundation of logic itself which was approached by psychological and physiological means leading to a new discussion on the question of priority between logic and psychology, and to various forms of psychologism and anti-psychologism (cf. Rath *1994*, Kusch *1995*); (2) the problem of logical applications focussing the interest on the methodological part of traditional logic. The reform of applied logic attempted to bring philosophy up to the stormy development of mathematics and sciences in that time.

Both reform procedures had a destructive effect on the shapes of logic and philosophy. The struggle on psychologism led to a removal of psychology (especially in its new, experimental form) from the body of philosophy in the beginning of the 20th century. Psychology became a new, autonomous scientific discipline. The debate on methodology emerged in the creation of the philosophy of science which was separated from the body of logic. The philosopher's ignorance of the development of formal logic caused a third process of removal: a part of formal logic was taken from the domain of competence of philosophy and incorporated into mathematics where it was instrumentalized for foundational tasks.

3.3. The Mathematical Context in Great Britain

As mentioned earlier the influence of the philosophical discussion on logic in Great Britain on the emergence of the new logic should not be overemphasized. Of greater importance were mathematical influences. Most of the new logicians can be related to the so-called "Cambridge Network" (Cannon *1978*, 29-71), i.e. the movement aiming at a reform of British science and mathematics which started at Cambridge. One of the roots of this movement was the foundation of the Analytical Society in 1812 (cf. Enros *1983*) by Charles Babbage (1791-1871), George Peacock (1791-1858) and John Herschel (1792-1871). In respect to mathematics Joan L. Richards called this act a "convenient starting date for the nineteenth-century chapter of British mathematical development" (Richards *1988*, 13). One of the first achievements of the Analytical Society was a revision of the Cambridge Tripos by adopting the Leibnitian notation for the calculus and abandoning the customary Newtonian theory of fluxions: "the principles of pure D-ism in opposition to the Dot-age of the University," as Babbage wrote in his memories (Babbage *1864*, 29). It may be assumed that this successful movement released by a change in notation might have stimulated an interest for operating with symbols. This lead step by step to a change in the concept of mathematics towards abstract mathematics. The new research on the calculus had parallels in innovative approaches to algebra. In the first place the development of symbolical algebra has to be mentioned which was codified by George Peacock in his *Treatise on Algebra* (*1830*) and further propagated in his famous report for the British Association for the Advancement of Science (Peacock *1834*, especially 198-207). The main feature of symbolical algebra is the "principle of the permanence of equivalent forms": "Whatever form is algebraically equivalent to another when expressed in general forms, must continue to be equivalent, whatever these symbols denote" (Peacock *1834*, 198). Peacock starts with a distinction between arithmetical and symbolical algebra, which is based on the common restrictive understanding of arithmetic as the doctrine of quantity. A generalization of Peacock's concept can be seen in Duncan F. Gregory's (1813-1844) "calculus of operations". Gregory is most interested in *operations* with symbols. He defines symbolical algebra as "the science which treats of the combination of operations defined not by their nature, that is by what they are or what they do, but by the laws of combinations to which they are subject" (*1840*, 208). In his much praised paper "On a General Method in Analysis" (*1844*) Boole makes the calculus of operations the basic methodological tool for analysis. But, following Gregory, he goes further, claiming more applications. He cites Gregory who writes in *1842* that a symbol is defined algebraically "when its laws of combination are given;

and that a symbol represents a given operation when the laws of combination of the latter are the same as those of the former" (ibid., 153-154). It is possible that a symbol for an arbitrary operation can be applied to the same operation (ibid., 154). It is thus necessary to distinguish between the arithmetical algebra and a symbolical algebra which has to regard symbolical, but non-arithmetical fields of application. As an example Gregory mentions the symbols a und $+a$. They are isomorphic in arithmetic, but in geometry they have to be interpreted differently. a can refer to a point marked by a line whereas the combination of signs $+a$ additionally expresses the direction of the line. Therefore a symbolical algebra has to distinguish between both symbols. He deplores that the unequivocalness of notation prevailed due to the persistence of mathematical practice. Clear notation has nothing but advantages, and Gregory thinks that our minds would be "more free from prejudice, if we never used in the general science symbols to which definite meanings had been appropriated in the particular science" (ibid., 158).

Boole adopts this criticism almost word for word. In his *Mathematical Analysis of Logic* Boole claims that the reception of symbolic algebra and its principles was delayed by the fact that in most interpretations of mathematical symbols the idea of quantity was involved. These connotations of quantitative relationships are for Boole the result of the context of the emergence of mathematical symbolism, but not a universal principle of mathematics (Boole *1847*, 3-4). Boole reads the principle of the permanence of equivalent forms as a principle of independence from interpretations in an "algebra of symbols". By applying this algebra of symbols to another field, the field of logic, he tries to free the principle from the idea of quantity in order to get further affirmation of this principle. For logic this implies that only the principles of a "true Calculus" should be presupposed. This calculus is characterized as "method resting upon the employment of Symbols, whose laws of combination are known and general, and whose results admit of a consistent interpretation" (ibid., 4). He stresses (ibid.):

> It is upon the foundation of this general principle, that I purpose to establish the Calculus of Logic, and that I claim for it a place among the acknowledged forms of Mathematical Analysis, regardless that in its objects and in its instruments it must at present stand alone.

Boole expresses logical propositions in symbols whose laws of combination are based on the mental acts represented by them. Boole thus attempts a psychological foundation of logic, mediated, however, by language. The central mental act in Boole's early logic is the act of election used for building classes. Man is able to separate objects from an arbitrary collection which belong to given classes, and to distinguish them from others. The symbolic representation of these mental operations follows certain laws of combination which are similar to those of symbolic algebra. Logical theorems can thus be proved like mathematical theorems. Boole's opinion has of course consequences for the place of logic in philosophy: "On the principle of a true classification, we ought no longer to associate Logic and Metaphysics, but Logic and Mathematics" (ibid., 13).

Although Boole's logical considerations became more and more philosophical with time, aiming at the psychological and epistemological foundations of logic itself, his initial interest was not to reform logic but to reform mathematics. He wanted to establish an abstract view on mathematical operations without regarding the objects of these operations. When claiming for the calculus of logic "a place among the acknowledged forms of Mathematical Analysis" (*1847*, 4) he doesn't simply want to include logic into traditional mathematics. The superordinate discipline was a new mathematics. This is expressed in Boole's writing: "It is not the essence of mathematics to be conversant with the ideas of number and quantity" (*1854*, 12).

3.4. The Mathematical Context in Germany

The results of this examination of the British situation at the time when the emergence of the new logic started -reform of mathematics, but initially no interest in a reform of logic, establishing an abstract view on mathematics focussing not on mathematical objects but on symbolic operations with arbitrary objects- can be transposed to the situation in Germany without problems.

The most important representative of the German algebra of logic was the mathematician Ernst Schröder (1841-1902) who was regarded as the completer of the Boolean period in logic (cf. Bocheński *1956*, 314). In his first logical pamphlet *Der Operationskreis des Logikkalkuls* (*1877*) he presented a critical revision of Boole's logic of classes, stressing the idea of the duality between logical addition and logical multiplication introduced by William Stanley Jevons in *1864*. In 1890 Schröder started the big project to publish the monumental *Vorlesungen über die Algebra der Logik* (*1890*, *1891*, *1895*, *1905*) which remained unfinished although it grew up to three volumes with four parts of which one appeared only posthumously. Contemporaries regarded the first volume alone as completing the algebra of logic (cf. Wernicke *1891*, 196).

Schröder's opinion concerning the question to which end logic has to be studied (cf. Peckhaus *1991*, *1994b*) can be drawn from an autobiographical note, published in *1901*, the year before his death. It contains Schröder's own survey on his scientific aims and results. Schröder divides his scientific production into three fields:

(1) A number of papers dealing with some current problems of his science.

(2) Studies to create an "absolute algebra," i.e., a general theory of connections. Schröder underlines that such studies represent his "very own object of research" of which only little was published at that time.

(3) Work on the reform and development of logic.

Schröder writes (*1901*) that his aim was

> to design logic as a calculating discipline, especially to give access to an exact handling of relative concepts, and, from now on, by emancipation from the routine claims of spoken language, to withdraw any fertile soil from "cliché" in the field of philosophy as well. This should prepare the ground for a scientific universal language that, widely differing from linguistic efforts like Volapük (a universal language like Esperanto being very popular at that time in Germany), looks more like a sign language than like a sound language.

Schröder's own division of his fields of research shows that he consider himself a logician: His "very own object of research" is the "absolute algebra," an object that is in respect to its basic problems and fundamental assumptions similar to the modern abstract or universal algebra. What is the connection between logic and algebra in Schröder's research? From the quoted passages one could assume that they belong to two separate fields of research, but this is not the case. They are intertwined in the framework of his heuristic idea of a general science. He writes in his autobiographical note (*1901*):

> The disposition for schematizing, and the aspiration to condense practice to theory advised Schröder to prepare physics by perfecting mathematics. This required deepening -such as of mechanics and geometry- above all of arithmetic, and subsequent to it he became by the time aware of the necessity, to reform the source of all these disciplines, logic.

Schröder's universal claim becomes obvious. His scientific efforts serve for providing the requirements to found physics as the science of material nature by "deepening the foundations," to quote a famous metaphor later used by David Hilbert (*1918*, 407) in order to illustrate the objectives of his axiomatic programme.

Schröder regards the formal part of logic that can be formed as a "calculating logic," using a symbolical notation, as a *model* of formal algebra that is called "absolute" in its last state of development. He formulates this algebraic programme in his *Lehrbuch der Arithmetik und Algebra* published in *1873*, and a first step to its development is taken in the school programme pamphlet *Über die formalen Elemente der absoluten Algebra* (*1874*). Other results and applications occur throughout his further publications. In the first chapter of his *Lehrbuch* Schröder defines (pure) mathematics as the "science of number" (*1873*, 2). This definition differs from the traditional doctrine of mathematics as the science of quantity. But what is Schröder's concept of number? Schröder leaves this open, because the concept of number goes through "a progressive and not yet ended expansion or development." He hints at the discovery of hypercomplex number systems, and remarks that a lot of other types of numbers could be imagined (ibid.). In any case, he says, the number is a sign that is created arbitrarily to reach most different aims. In the *Programmschrift* Schröder uses these ideas for a very general definition of a "number field" that is not restricted to mathematics (*1874*, 3). The base of formal or absolute algebra is the assumption that there is an

> unlimited manifold of objects (of any kind) that are conceptually distinguished from one another -by a feature or a boundary. Any element of this assumed manifold is designated with letters *a*, *b*, *c* ... (...) The given manifold can be called a *number field* in the widest sense of the word.

The kind of objects that are comprised in such a "number field" is not determined. Examples of possible objects are "proper names, concepts, propositions, algorithms, numbers (of pure mathematics), symbols for dimensions and operations, points and systems of points, or any geometrical objects, quantities of substances, etc.".

But what is "formal algebra"? The theory of formal algebra "in the narrowest sense of the word" includes "those investigations on the laws of algebraic operations (...) that refer to nothing but general numbers in an unlimited number field without making any presuppositions concerning its nature" (*1873*, 233). Formal algebra therefore prepares "studies on the most different number systems and calculating operations that might be invented for particular purposes" (ibid.). In the *Lehrbuch* Schröder formulates a four-step programme of formal algebra (ibid., 293-294):

(1) Formal algebra compiles all assumptions that can serve for defining connectives for numbers of a number field.
(2) Formal algebra compiles for every premise or combination of premises the complete set of inferences, a task that Schröder calls "separation."
(3) Formal algebra investigates in which particular number fields the defined operations hold.
(4) Formal algebra has finally to decide "what geometrical, physical, or generally reasonable meaning these numbers and operations can have, what real substratum they can be given."

Steps (1) and (2) show that Schröder proceeds in a combinatorial way. Only after having finished the semantical steps (3) and (4) formal algebra becomes an "absolute algebra."

What is the rôle of logic in this programme? Schröder's logical considerations in his *Lehrbuch* are due to his naive definition of a set being the base of a constructive introduction of natural numbers via computability. According to Schröder a set is a mental collection of several units. Every object that can be counted is a unit. From this notion of a set restricted to collections of units he distinguishes collections of, e.g., possible solutions of an equation, or of possible values of ambiguous expressions. Schröder discusses, e.g., the problem of symbolizing the relation between $\sqrt{9}$ and its values +3 and -3 (*1873*, 27). He interprets the relation logically as the superordination of a "general value" over its "particular values". To express possible relations between extensions of concepts Schröder introduces five logical relations including equality and the subsumption relations which became so important in his later logical writings. Schröder stresses that, if negation is symbolized additionally, a complete terminology has been created to represent relations between the extensions of concepts with the help of formulas. These formulas "join closely the scheme of the mathematical sign language, and insert themselves harmonically into the whole sign apparatus" (ibid., 29). It should also be stressed that these signs are nevertheless newly created, and not borrowed from mathematics. Later in his book he additionally introduces the logical connectives conjunction and disjunction as interpreted algebraic connectives (ibid., 145-147). The fundamentals laid down in these early writings govern all of Schröder's philosophy of logic up to its peak, the logic of relatives, published in its first (algebraic) part in the third volume of his *Vorlesungen* (*1895*). The notion of "relatives" is for Schröder the main device for applying structural results to most different domains of objects.

It has to be stressed that Schröder writes his early considerations on formal algebra and logic without any knowledge of the results of his British predecessors. But what are his sources? In his *Lehrbuch* Schröder mentions Martin Ohm's *Versuch eines vollkommen consequenten Systems der Mathematik* (*1822*, ^{2}1828/1829), Hermann Günther Graßmann's *Lehrbuch der Arithmetik* (*1861*), Hermann Hankel's *Theorie der complexen Zahlensysteme* (*1867*) and Robert Graßmann's *Formenlehre oder Mathematik* (*1872a-f*). These sources show that Schröder stands in the tradition of German combinatorial algebra and algebraic analysis.

Schröder's combinatorial approach can be traced back to Carl Friedrich Hindenburg (1741-1808) and his Combinatorial School (cf. Jahnke *1990a,b*). Hindenburg followed the Leibnitian idea of an *ars combinatoria* within the heuristic *ars inveniendi*. He regarded algebra as a special form of combinatorics and focused on methods to form combinations. In Hindenburg's conception combinatorics appeared to be a general structure theory of formulas.

Martin Ohm (1789-1854) was one of the disciples of Heinrich August Rothe (1773-1842) who was a member of the Hindenburg School. In his main work *Versuch eines vollkommen consequenten Systems der Mathematik*, published in 9 parts from 1822 on, Ohm wants to transfer the rigour of Euclid's foundations of geometry to all of mathematics. He distinguishes between number (or "not-named number") and quantity (or "named number"). Ohm questions the then dominating quantitative mathematics by subordinating quantitative mathematics under non-quantitative mathematics. Like Boole after him he regards operations with non-named numbers, i.e. symbols, as representatives for mental operations (Ohm *1853*, VI-VII).

Most important for Schröder were the writings of Hermann Günther Graßmann (1809-1877). Graßmann's *Lehrbuch der Arithmetik* (1861) is one of the basic sources for Schröder's own considerations on arithmetic and algebra. Furthermore, Schröder's conception of algebra as general theory of operations resembles Graßmann's philosophical convictions conveyed in his pioneering *Lineale Ausdehnungslehre* (*1844*). There Graßmann distinguishes between real and formal sciences. Formal sciences are divided into those treating the general laws of reasoning and those dealing with the particular which is posited in reasoning. The former is dialectics (logic), the latter pure mathematics (*1844*, X). In the second edition (*1878*, XXII) he adds that logic itself has a purely mathematical side, which can be called "formal logic". He defines pure mathematics as the doctrine of forms (*1844*, XX), and opens the *Lineale Ausdehnungslehre* with an "Overview on the general doctrine of forms" (*1844*, 1-14) which comprises a general theory of connecting operations forming the base for later applications in the domain of extensive quantities, e.g., to directed lines, i.e. vectors.

Graßmann's considerations did not gain the reception they deserved. Only when Hermann Hankel (1839-1873) hinted at the fundamental importance of Graßmann's work in his *Theorie der complexen Zahlensysteme* (*1867*) did mathematicians become interested in Graßmann's book. Like Graßmann, Hankel opens his book with a general doctrine of forms in which general connections of objects are discussed. This general doctrine is applied to the arithmetic of natural and complex numbers.

From there it is only a small step to the logical interpretation of general connective operations. This step is done by Robert Graßmann (1815-1901), Hermann Günther Graßmann's younger brother. He develops an algebraic logical system, similar to the systems of Boole and Jevons, but without any knowledge of his British predecessors and of the contemporary philosophical discussion. He publishs *1872(a-f)* a *Formenlehre oder Mathematik* comprising in a general doctrine of quantities (*1872b*), a doctrine of concepts or logic (*1872c*), a doctrine of connections or combinatorics (*1872d*), a doctrine of number or arithmetic (*1872e*) and a doctrine of the exterior or of extensions (*1872f*). These parts of mathematics are distinguished from one another according to the different results of the additive and multiplicative connections of a unit with itself. It is through Robert Graßmann's book that Schröder recognizes the possibility to express conjunction and disjunction of concepts using algebraical signs. One could even say that Schröder learns his logic not from Boole, but from Robert Graßmann (cf. Peckhaus *1996*).

Like the British tradition and independent of it the German algebra of logic was connected to new trends in algebra. It differed from the British counterpart in its combinatorial approach. In both traditions algebra of logic was invented within the enterprise to reform basic notions of mathematics which lead to the emergence of structural abstract mathematics. The algebraists wanted to design algebra as "pan-mathematics", i.e. as a general discipline which embraces all mathematical disciplines as special cases. The independent attempts in Great Britain and Germany were combined when Schröder learned about the existence of Boole's logic in late 1873, early 1874. Schröder finally enriched the Boolean class logic by adopting Charles S. Peirce's theory of quantification and adding a logic of relatives according to the model of Peirce and De Morgan.

The first interest of the new logicians was to utilize logic for mathematical and scientific purposes, and only in a second step, yet as an indispensable consequence of the attempted applications, the reform of logic came into the view. What has been said of the representatives of the algebra of logic also holds for proponents of competing logical systems such as Gottlob Frege and Giuseppe Peano. Both wanted to use logic in their quest for mathematical rigour which was questioned by the stormy development of mathematics.

4. Institutionalization of Mathematical Logic in Germany

The description given above should have made plausible, why the innovative impulses in the development of formal logic in the end of the 19th century grew from mathematicians and not from philosophers. The interest in a further elaboration of formal logic was caused by mathematical needs. This is, however, no sufficient reason for the way of logic into mathematics as maintained in the title of this presentation which refers to processes of academic institutionalization. The German representatives of mathematical logic started very early to lecture on their topic at universities. Already in the summer term of 1876 Ernst Schröder gave a lecture course on "Logik auf mathematischer Grundlage" ("Logic on a mathematical base") at the Polytechnical School at Darmstadt, and Gottlob Frege started his lectures on "Begriffsschrift" at Jena in the winter term of 1879 (cf. Peckhaus *1992*). These academic activities preceded by far those in other countries -Bertrand Russell, e.g., gave the first English course on mathematical logic at Trinity College Cambridge at the turn of the century (cf. Grattan-Guinness *1992*, xii), and Kazimierz Twardowski gave a course "On Reforming Tendencies in Formal Logic" in the academic year 1899/1900 in the then Polish Lvov (cf. Woleński *1995*)- but both failed to become really effective, and mathematical logic did not get a footing in academic life. The situation changed when in David Hilbert's successful axiomatic programme the need for research on logic became obvious (cf. Peckhaus *1990a*, *1992*, *1995*). When Hilbert first formulated his axiomatic programme in his *Grundlagen der Geometrie* (*1899*) he did not care about logic. He believed that the means of traditional logic would be sufficient to prove the consistency of axiomatic systems. Only when the logical and set-theoretical antinomies were discovered which proved Frege's system of the *Grundgesetze der Arithmetik* (Frege *1893/1903*), the most elaborated then existing logical system, inconsistent, Hilbert felt inclined to insert logic into his axiomatic programme and to found his axiomatic systems on axiomatized logic and set-theory. With this he formulated the mathematicians' claim on logic. In Germany Hilbert was responsible for the emergence of symbolic logic as a mathematical subdiscipline, but the old logic was not replaced. The new logic was rather taken out of its domain. The institutionalization of symbolic logic in Germany proceeded even more slowly. The first official lectureship for "Mathematical Logic and

Related Fields" was granted to Ernst Zermelo in 1908 (cf. Peckhaus *1990b*), and only in 1943 the first German chair for Mathematical Logic and Foundations was created in Münster and filled with Heinrich Scholz.

Notes

1 MacHale writes more exactly: "Boole has been called the 'Father of Symbolic Logic' and the 'Founder of Pure Mathematics', but he is just as deserving of the title, 'Father of Computer Science,'" MacHale *1985*, 72.

2 It had a similar effect as Friedrich Ueberweg's *System der Logik und Geschichte der logischen Lehren* in Germany. Ueberweg's book was first published in *1857* and translated into English in *1871*.

3 Cf. Padilla Gálvez *1991*, Peckhaus *1986*, forthcoming, Thiel *1996*.

4 A detailed account of these processes can be found in my forthcoming book *Logik, mathesis universalis und allgemeine Wissenschaft*.

5 Risse (*1973*) lists 9 editions up to 1848 and 28 further printings until 1908. Van Evra (*1984*, 2) mentions 64 printings in USA until 1913.

6 This opinion can be found in a letter of De Morgan's to Spalding of 26 June 1857 (quoted in Heath *1966*, xii) which was, however, not sent. For George Boole, Hamilton is one of the "two greatest authorities in logic, modern and ancient" (*1847*, 81). The other authority is Aristotle. This reverence to Hamilton might not be without irony because of Hamilton's disregard of mathematics.

7 Cf. Hamilton *1859–1866*, vol. 4 (1866), 287.

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