INCOMMENSURABILITY AND MEASUREMENT

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ABSTRACT: Does incommensurability threaten the realist's claim that physical magnitudes express properties of natural kinds? Some clarification comes from measurement theory and scientific practice. The standard (empiricist) theory of measurement is metaphysically neutral. But its representational, operational and axiomatic aspects give rise to several kinds of a one-sided metaphysics. In scientific practice, the scales of physical quantities (e.g. the mass or lenght scale) are indeed constructed from measuring methods which have incompatible axiomatic foundations. They cover concepts which belong to incommensurable theories. I argue, however, that the construction of such scales commits us to a modest version of scientific realism.

Keywords: Archimedean axiom, incommensurability, measurement theory, physical magnitudes, physical quantities, scientific realism.

1. Incommensurability

Thomas S. Kuhn's famous book *The Structure of Scientific Revolutions* claims that competing theories are *incommensurable*. Incommensurability has three aspects: change of the problems which the scientists attempt to solve, change in the meaning of crucial theoretical concepts, and change of the world within which science is practised. In the 1969 postscript, Kuhn focuses on change of meaning and the associated problems of translatability:[1] Competing theories are associated with different classification systems as regards phenomena. It is only partially possible to communicate about observational or experimental evidence in a theoretically neutral language. Thus it is impossible to express the crucial concepts of one theory in the language of the other theory, and vice versa. Following Quine's work on ontological relativity, Kuhn argues that for these reasons the usual (Tarskian) concept of truth as correspondence to what is 'really there' is no longer tenable:

> There is, I think, no theory-independent way to reconstruct phrases like 'really there'; the notion of a match between the ontology of a theory and its 'real' counterpart in nature now seems to me illusive in principle. (Kuhn 1970, p. 206)

Indeed any substantial change in the conceptual foundations of a discipline raises most serious problems for scientific realism. Scientific realists claim that science aims at true explanations of the observable phenomena (Leplin 1984). In order to cope with scientific revolutions, they have to assume that either some referential truths about natural kinds, or some approximately true laws of nature, or both, can be expressed in a universal scientific language. From a realist's point of view, the language of modern science derives from mathematical physics since Galileo. Incommensurability challenges scientific realism by denying the possibility of a unique scientific language. If we have no language to bridge the conceptual gap between competing theories, Kuhn is right in claiming that it is neither possible to explain whether theories refer nor why one of them should come closer to truth than the other one.

2. The language of physics

At least for modern physics, incommensurability seems to have much less dramatic consequences for the uniqueness of scientific language than Kuhn's arguments suggest. Most physicists would agree that competing theories are in a certain sense incommensurable, and that a change in the conceptual foundations of science gives rise to a dramatic change of our world view. But most of them would never agree that changes in the meaning of theoretical concepts preclude the existence of terms in which such conceptual changes can be expressed, and in which the superiority of one theory over another one can be validated. Richard P. Feynman (whom no one could suspect of having been affected too much by professional philosophy of science) wrote about the change in the meaning of mass in the transition to relativistic mechanics:

> Philosophically we are completely wrong with the approximate law. Our entire picture of the world has to be altered even though the mass changes only a little bit. (...) Even a small effect sometimes requires profound changes in our ideas. (Feynman 1963, p. 4)

These words confirm Kuhn's view of incommensurability only partially. We should note that Feynman makes a distinction between the *philosophical import* of a law and the *language of physics* in which it is expressed. Like Kuhn, he emphasizes that the world views associated with non-relativistic mechanics and special relativity are completely different, even though approximately equal laws derive from both theories. On the other hand, he speaks about *the mass* which is subject to a small quantitative change when we pass from the non-relativistic to the relativistic description of a motion. Kuhn's account of scientific revolutions tells us that these descriptions rely on incommensurable concepts. A non-relativistic mass is constant but a relativistic mass depends on its velocity in a given inertial frame. Kuhn would argue that in speaking about *the mass*, Feynman uses the concept ambiguously.

If we look at scientific practice, however, we see *both* concepts at work in pre-established harmony. Physicists have been using the non-relativistic and relativistic concepts of mass for decades, without running into ambiguity or confusion. The conceptual choice depends on the problems they have to solve and the theoretical tools they need for solving them. Quite often they use even both concepts simultaneously. Many models combining a non-relativistic description of a process with a relativistic correction have been developed in subatomic physics. Such models work perfectly, and they can be independently tested (Falkenburg 1995, 1996). Thus it seems that in scientific practice, the linguistic problems posed by incommensurability can be overcome. Physicists construct a mass scale that covers all mass values we may assign to any object in the subatomic, macroscopic or cosmological domain. They suppose that the scale represents a class of physical properties of material things and their parts. They assign mass values to electrons, to billard balls, and to black holes. In so doing, they seem to suppose that these numbers represent commensurable physical magnitudes.

Kuhn would not interpret such scientific practice as a case against his view of incommensurability. He would rather cite his 1961 paper *The Function of Measurement in Modern Physical Science* where he showed that in advanced disciplines, theory consolidation and comparison rely on *measurement*. Advanced science develops experiments for generating observable phenomena, and precise measuring methods for giving a quantitative account of the phenomena. Once the phenomena can be measured, competing theories *have* to have almost identical empirical content. That is, their quantitative predictions must agree approximately for most of the observable phenomena. Empirically, they compete only in being capable of coping with the quantitative anomalies of the rival theory.

Thus in Kuhn's view, measurements put very strong constraints on theory development. Measuring results force incommensurable theories to come as close to each other as possible in their quantitative predictions. Surely Kuhn would never say that they have to come close to *truth*. My point is that he is right on incommensurability, and most probably on the illusiveness of the truth of full-fledged theories, but that he underestimates the unifying power and the referential import of the theoretical language in which measurements are expressed. Let me develop my argument beginning with a sketch of formal measurement theory and its empiricist background.

3. Measurement theory

Modern measurement theory has an empiricist origin. Helmholtz (1887) founded it. Campbell (1920) elaborated it in more detail. Nagel (1931), Hempel (1952), and Carnap (1966) put it in the framework of logical empiricism. The Suppes school reformulated it in model-theoretic terms and developed it as an abstract theory which is expressed in the formal language of modern, structuralist mathematics. Due to the monumental work *Foundations of measurement* (Krantz 1971) abstract measurement theory has become such a perfect axiomatic theory that modern textbooks present it as applied mathematics (Narens 1985).

Philosophers of science agree about the technical aspects but not about the metaphysics of measurement. Within the debate on scientific realism, they discuss whether physical quantities such as length, mass, charge, or temperature represent real properties of physical phenomena or not. Some undoubted technical features of measurement theory may serve as my starting point for metaphysical clarification.

Formally, measurement theory is based on the axiomatization of relational structures. From an empiricist point of view, these relational structures are approximate models of empirical phenomena. According to Carnap or Suppes, measurement is based on ordering the phenomena of a given class into relational structures. Ordering gives rise to non-numerical relational structures which are models of the phenomena, that is, to empirical relational structures. An empirical relational structure is established in two steps. First, we have to choose two kinds of *empirical operations* acting on phenomena or entities: an operation of concatenation (such as combining rods along a straight line), and an operation of comparison (such as setting two rods parallel to each other, and observing which one is longer). Secondly, we have to choose certain *axioms* to fix a formal relational structure in correspondence to the empirical ordering obtained from concatenation and comparison (such as the axioms of extensive measurement). Measurement is the numerical comparison of an element of an empirical structure with an arbitrary unit. Thus to measure a phenomenon, two further systematic steps are required. Namely, we have to choose a *unit* (such as the standard meter) under the elements of the empirical structure, and then assign to a phenomenon a *number* which expresses its relative magnitude in comparison to the unit. A *quantity* is usually identified with the function which assigns real numbers to the elements of an empirical structure, in relation to the unit. The *scale* of a quantity is the range of the function, that is, the numerical representation of all magnitudes which are admitted by the axioms.

Abstract measurement theory can be separated from its empiricist background and elaborated as applied mathematics. It starts from given axioms for formal relational structures, and deals primarily with the formal problems of mapping them into the real numbers. The choice of the axioms is constrained by the goal of measuring. A formal relational structure is only good for measurement if it is uniquely representable by numbers, that is, if it has numerical models which are unique up to isomorphism. Therefore, it is required that two theorems derive from the axioms: a *representation theorem* which guarantees the existence of a homomorphism from a given relational structure to real numbers; and a *uniqueness theorem* which determines the homomorphism uniquely up to isomorphic transformations, for example up to the linear transformations preserving the structure of a temperature scale.

Abstract measurement theory is primarily tailored for ideal measurements, that is for the mapping of simple relational structures into the real numbers. For more realistic applications, the theory becomes complicated. For example, measurement errors, or the fuzziness of empirical structures, may be taken into account by implementing probabilistic assumptions into the axioms.[2] Another interesting way of modifying the theory is to weaken its axiomatic basis. For example, without the Archimedean axiom it no longer holds that all elements of a relational structure are representable by real numbers.[3] This means that there are *mathematically* incommensurable elements of a relational structure.

The mathematical concept of incommensurability was Kuhn's analogue for introducing his distinct notion of incommensurability (Mühlhölzer 1989, p. 14). Indeed both concepts turn out to be closely related via the Archimedean axiom of measurement theory. In empirical science, the Archimedean axiom guarantees that we may extend the familiar scales of physical quantities such as mass or length to subatomic or cosmological domains. Hilbert emphasized that the validity of the axiom can be empirically tested. According to his celebrated article *Axiomatisches Denken*, the empirical possibility to express inneratomic as well as celestial distances in terms of terrestrial measurements proves that the axiom is valid (Hilbert 1918, p. 149). Inneratomic, terrestrial, and cosmological distances, however, are subject to quantum theory, classical physics, and the special and general theories of relativity. Since these theories are incommensurable in Kuhn's sense, the Archimedean axiom has direct import for the question of whether incommensurable theories necessarily give rise to the incommensurability of physical concepts that express measurable magnitudes such as mass or lenght. If the axiom is empirically valid the *measured* quantities in the subatomic, terrestrial, and celestial domains are *not* incommensurable in Kuhn's sense, in the context of axiomatic measurement theory.[4]

From an empiricist point of view, measurement has three aspects. 1. It has an *operational basis* that gives rise to a well-defined empirical ordering of the phenomena. 2. It depends on *axioms* that are strong enough to determine a numerical representation. 3. It implies the *referential claim* that the axioms, and their numerical representation, express the operational basis in an empirically adequate way. Abstract measurement theory expresses these aspects in such a way that they can no longer be separated. The operational and axiomatic aspects are interwoven in the axiomatic definition of a non-numerical (algebraic) relational structure. The referential aspect is formally expressed by theorems which derive from the axioms of the relational structure plus number theory. These theorems state that the corresponding algebraic structure can be uniquely represented by real numbers. Abstract measurement theory is based on the axiomatic method of modern mathematics. It says nothing about the empirical meaning of the formal operations of concatenation and comparison on which it is built. And it makes no claims about the existence of concrete models of abstract relational structures, or their numerical representations.

4. Metaphysical disagreement

Philosophers of science start to disagree about measurement when they consider abstract measurement together with its concrete background, that is, when they study its links to empirical applications. Looking at the applications of a theory, however, does not yet force metaphysical debates upon us. In my view, the empiricist theory of measurement presented by the Suppes school is metaphysically neutral. It is just a mathematical theory about the structure of the use of numerical methods in empirical science. In this regard, it is equal to *any* mathematical theory of empirical science itself.

A closer look at experimental physics shows indeed that measurement theory accounts correctly for the way in which numerical models represent the theoretical axioms underlying a measurement method, and the structure of the phenomena arising from certain experimental operations. The distinction of the operational, axiomatic, and referential (or representational) aspects of measurement, and the investigation of their formal relationships, is neither restricted to an empiricist view of science nor to the empiricist standard examples of measuring lengths by means of rigid rods and so on. Even the sophisticated experiments of subatomic physics may be completely explained in terms of measurement theory as sketched above. They are based on *empirical operations* which result first in the construction and calibration of an electron or proton beam and a particle detector, later in a period of data taking, and finally in the comparison of the observed properties of the particle tracks which had been taken by the detector. The design and performance of such an experiment and the quantitative data analysis that makes the measurement complete are firmly based on theoretical laws functioning as *axioms of measurement*. These measuring laws depend crucially on classical and quantum models of what goes on in the experiment. In addition, they depend crucially on the theoretical construction of numerical scales of physical quantities such as mass and charge, and on the assumption that the observed particle tracks are *uniquely representable* by numbers that denote the corresponding physical magnitudes of certain kinds of particles.

Only when a single aspect of measurement is overemphasized do we make a decisive step towards diverging metaphysical views about physical quantities. If we consider exclusively the operational aspects of measurement, we are led to *operationalism*. Bridgman proposed the extreme view that each measuring method defines another quantity (Bridgman 1927). Ellis' account of operational definitions is more sophisticated. Ellis (1966, p. 34-36) suggests that physical quantities are cluster concepts which derive from all measuring methods giving approximately the same quantitative results. Nevertheless, in *Basic Concepts of Measurement* he still rejects the way in which scientific realism explains why certain measuring results should agree, namely the existence of physical properties which correspond to such cluster concepts.

Focusing on the axiomatic aspects of measurement leads to *holism*. Any measurement depends on a theory about the structure of the phenomena which are measured. Any theory, even if it is made up of several heterogeneous models, can be restated in terms of the axiomatic method. Sneed and his followers emphasized mainly the theoretizity of dynamic quantities such as mass or force (Sneed 1971, Balzer/Moulines/Sneed 1987). But the familiar spatio-temporal quantities depend no less crucially on theory. Modern physics knows alternative space-time theories which give rise to distinct axiomatizations of measurement. One has the choice between Euclidean or non-Euclidean measurements of length, non-relativistic or relativistic measurements of velocity and mass, and so on. The measurement of a quantity never stands on its own. It is always embedded in a complicated framework of assumptions about the laws of nature, ceteris paribus clauses, etc. Such reasoning results in the Duhem-Quine-Thesis, and in Kuhn's view that the crucial concepts of rival theories have incommensurable meaning even though they may give rise to approximately equal numerical predictions. Kuhn's incommensurability thesis is not only a case against empiricism or scientific realism, but also against Ellis' cluster concepts of physical quantities. In modern physics, the scales of length, mass, charge etc. rely on a cluster of measuring methods stemming from incommensurable theories. That is, they are conceptual clusters made up of incompatible concepts.[5]

A related anti-representationalist view of measurement is *conventionalism*. It results from focusing on the arbitrary assumptions which are necessarily built in any measurement. The unit of the scale of a quantity is always arbitrarily chosen. In addition, many measuring methods depend on theoretical concepts without any operational content. Typical examples are: the basic assumption of ordinary length measurement that rods are rigid; or the famous Einstein convention in the relativistic definition of simultaneity (Einstein 1905, § 1).

Many philosophers of science emphasize the representational (or referential) aspects of measurement against operationalism, holism, and conventionalism. They tend either to empiricism or to scientific realism. Empiricists claim that the axiomatic representation of measurement is constrained by the structure of the phenomena: the axioms of a measurement cannot be arbitrarily chosen since they have to be empirically adequate. Thus even though any measuring method contains arbitrary elements, the choice of the axioms is much more than a matter of convention. It is anchored in the phenomena. But in explaining the domain of the axioms, empiricists defend ontological parsimony. They argue that there is no epistemic justification to our extending the relational structures underlying a measurement from a finite empirical domain to an infinite domain of unobservables.

Scientific realists, on the other hand, defend the view that physical quantities such as length, mass, or charge express the real properties of natural kinds such as electrons. For them, quanties are classes of magnitudes, and magnitudes are properties that come in degrees. Typically they are platonists, that is, they reify properties and magnitudes (Armstrong 1987, 1988). Swoyer (1987) and Ellis (1987) emphasize that physical properties are first order universals, and their quantitative relations second order universals. Kyburg (1984, p.17) has a related position. He defends the views that physical magnitudes are abstract objects, and that quantities should not be interpreted as functions from empirical structures into the real numbers but as functions from empirical structures into classes of magnitudes.

The formal structure of measurement theory explains obviously at least which diversity of metaphysical views could arise. Is there any hope of bringing the diverging lines of reasoning together? How are they related? Is one of them of predominant importance? For clarification purposes, let me recall some traditional philosophical views about physical quantities which were developed together with modern physics.

5. Representationalism

Empiricists and scientific realists share a representational view of theoretical concepts. According to the empiricist approach, the numerical values of physical magnitudes represent classes of empirical phenomena. These classes arise from certain empirical operations. In the view of scientific realism, physical magnitudes represent properties of physical objects. To reduce physical quantities to their representational aspects results obviously in a *referential* or *extensional view* of the meaning of physical concepts.

Modern physics started from such a referential interpretation of theoretical concepts. Newton conceives of space and time as real entities. His concepts of absolute space and time have no operational meaning at all. His absolute space is an immaterial substance with the only physical effect of causing the pseudo-forces that are observed in non-inertial frames. The associated absolute time-flow is an immaterial process of perfect uniformity. (Both concepts are closely related to 17th century theism. In the last analysis, Newton supposed them to be grounded in God.)

Newton's concept of mass according to which mass is the product of volume and density is also referential. It depends on atomism. It is only meaningful if density is unterstood in terms of atoms per volume. Newton does not mention atomism in the definitions at the beginning of the *Principia*. But in the comment to the third rule of reasoning in book III, he tells us that the primary qualities of all bodies, "extension, hardness, impenetrability, mobility, and inertia", result from the corresponding properties of all parts of the bodies; and he continues:

> and thus we conclude the least parts of all bodies to be also all extended, and hard, and impenetrable, and movable, and endowed with their proper inertia. And this is the foundation of all philosophy. (Newton 1729, p. 399)

Thus in Newton's view, natural philosophy, or physics, is founded in theoretical concepts which refer to atoms. Atoms are extended, hard, impenetrable, movable, and have inertial mass. The corresponding observable properties of mechanical bodies derive from the same properties of unobservables. The mass of a body is simply the sum of the masses of its atoms. Therefore, Newton's famous definition of mass, or "quantity of matter", should be interpreted as follows. Mass is the product of volume and density whereby the density of a substance is the number of atoms in a given unit volume of that substance. In this way, the concept of *inertial* mass is defined. (Newton has no concept of gravitational mass, only a concept of weight. In his view, weight is not a primary quality of matter but a relational property which depends on gravity, that is, on the force another body exerts on a body according to the law of gravitation).

Understood in this way, Newton's definition of mass aims at specifying a dynamic quantity in terms of atomic units. This is analogous to the modern way of specifying atomic weight in multiples of the nucleon mass. Newton defines mass as a quantity which is a multiple of a given unit instead of explaining it as a physical property or quality of bodies. He gives only a definition for the numerical concept of the dimensionless ratio of the mass values of a body and its least parts. I think this is typical of an exclusively referential, or extensional, view of the meaning of physical quantities. A magnitude is not defined as a kind of property (or *intensional* entity) but only as a number that denotes the multiple of a natural unit. Newton's definition of the "quantity of matter" should be taken as literal: it defines only the *quantity* of matter, measured in atomic units, but not the *quality* of inertia as a dynamic property of matter.

Such an extensional definition of mass as a quantity has the enormous advantage that mathematical physics can dispense with the metaphysical assumption of dynamic dispositions or internal powers of bodies. But it has also two disadvantages. First, the definition depends *necessarily* on atomism. It has no mathematical meaning if there is no *finite* natural unit of mass, that is, if matter is infinitely divisible. Second, the definition does not discriminate between *coextensive* physical predicates such as inertial and gravitational mass. Since both kinds of mass are identified in general relativity we may object that it depends on our theories whether coextensive physical properties should be distinguished or not. In Newton's view, however, *all* primary qualities of bodies are coextensive. Therefore, his definition of mass does not even admit to make the distinction between spatio-temporal properties such as extension and movability, and dynamic properties such as hardness, impenetrability, and inertia. In giving an *exclusively* extensional definition of matter, Descartes was obviously more logical.

The problem arises since Newton's extensional concept of mass is operationally void. His definition of mass refers to unobservable atomic units, and is thus essentially non-operational. It is extensional, or referential, *because* it does not give rise to any measuring method. Newton needs to refer to the number of atoms since he has no measurement for mass as an intrinsic property of bodies. In Newton's day, however, atoms were empirically unaccessible. To measure the number of atomic units corresponding to the mass of a body was beyond the experimental method. Surely Newton was well aware that a theoretical concept without any operational content is of no use in empirical science. Therefore he tied his referential concept of mass to the empirical concept of weight. In doing so, he relied on the empirical proportionality of mass and weight (Newton 1729, p. 1).

Mach's famous criticism is in the same thrust. His objections against Newton's concept of mass point at the missing clarity of a referential definition without any empirical or operational content. On the basis of rejecting atomism, Mach is justified in arguing that the definition is circular.[6] In addition, he argues that even if atomism is granted the definition generally does not work. In his view, determining a mass by counting atoms works only for bodies made up of the same stuff whereas for chemically different bodies, the remaining theoretical presuppositions are merely multiplied (Mach 1883, p. 210 f.).[7]

6. Operationalism

Mach suggests the following way to avoid the unclear reduction of mass to the number of unobservable atoms. Mass should not be defined as a monadic predicate but as a relational concept. According to Newton's second and third law, the negative ratio of the relative accelerations that two moving bodies exert upon each other is the ratio of their masses. The mass of a single body cannot be defined, only the mass ratio of two bodies that accelerate each other. Their mass ratio is the negative inverse proportion of their mutual accelerations (Mach 1883, p. 211). This is an explicit theoretical definition for a dimensionless quantity that replaces Newton's full-fledged concept of mass. The resulting concept of a mass ratio has an obvious operational content since the relative acceleration of two bodies can always be measured. The concept is not operational in a *strict* sense since the definition gives rise to the ways of measuring a mass ratio, and not vice versa. We may take it as operational, however, in the weaker sense of a theoretical concept which is sufficiently determined by its operational basis.

Mach's definition makes it possible to define a relational concept of mass. The relative mass of a body is given as a number in units of the mass of another body. The advantage of this concept is that in contradistinction to Newton's atomic units, the units are now empirically given. The disadvantage is that it does not allow the distinction of the dimensions of mass or of other dynamic quantities such as momentum, force, energy, or charge. Newton's definition of mass in terms of atomic units *aims* at least at the dimension 'mass', even though missing it. From an empiricist point of view, however, the apparent loss is indeed a gain. Mach defines the mass ratio explicitly by an acceleration ratio since he wants to *reduce* dynamic properties such as mass or force to spatio-temporal quantities such as acceleration. For him, all dynamic distinctions are reducible to distinct phenomenological features. It is not by accident that he eliminates from physics the dimension of mass as a class of properties of their own. The dimensional concept of mass belongs to the world view of scientific realism. It expresses the assumption of a class of magnitudes which are identified with properties of natural kinds. In Mach's view its elimination is possible since operationally, the numerical value of a mass ratio can be reduced to a proportion of the observed accelerations of two empirical bodies. (This idea has later been realised by means of stating the Ramsey sentence of a theory.)

Mach's operational reduction of mass ratios and relative masses to mutual accelerations, however, is deficient from an axiomatic point of view. His theoretical definition of the mass ratio relies on a crucial idealisation. It deals with two isolated bodies. Considering only the mutual acceleration of two bodies neglects their interactions with the rest of the world. For many practical purposes, the idealisation may give rise to a very good approximate description of the relative empirical motion arising from the interaction of two bodies on their own. But for the *definition* of a theoretical concept, empirical approximations are obviously not good enough. Due to the law of universal gravitation, all bodies in the world are accelerating each other. Thus from an axiomatic point of view, it does not suffice to define a mass ratio solely from the mutual acceleration of *two* bodies. Therefore Mach's concept of a mass ratio does not satisfy even the (weak) necessary condition of an operational concept to be *determined* by its operational basis. The correct operational definition of mass ratios requires a sophisticated Lagrangian axiomatization of mechanics that admits the description of the mutual accelerations of *many* bodies simultaneously (Schmidt 1993). To make the operational definition of the mass ratios in a system of given bodies complete, we would have to take into account in the last analysis all bodies in the universe.

In considering *strict* operationalism, it becomes only more obvious that operational definitions are bound to an axiomatic approach. According to Bridgman (1927), a quantity is defined by the associated measuring method. Let me ignore the problem of multiplying quantities which Ellis (1968, p. 34-36) wants to overcome with his cluster concepts. Even the paradigm of operationalism, Einstein's *definition of simultaneity*, derives from theoretical principles. In the famous 1905 paper on special relativity, Einstein defines the concept 'simultaneous' referring to the synchronisation of clocks by the transfer of light signals between inertial frames. We all know that the definition implies a non-operational element, the assumption that light travels with the same speed back and forth which has been called the Einstein convention (Einstein 1905, § 1). But in addition, it depends on the *principle of relativity* that is non-operational as well.

The principle of relativity says that the laws of physics are identical in all inertial frames. In the introduction of the 1905 paper, Einstein states it as a principle which does not only hold for the laws of mechanics but also for the laws of electrodynamics and optics. His starting point are some phenomena of electrodynamics which give support to the expectation that the principle is universally valid. His empirical example is that the magnitude of the current in a conductor moving in a magnetic field depends on the relative motion alone. In addition, he mentions the null results of the Michelson-Morley type experiments. He emphasizes that two presuppositions are sufficient for deducing a simple and consistent electrodynamics of moving bodies: the principle of relativity, and the principle that the vacuum speed of light does not depend on the motion of the light source. Logically, the second principle is related to the first one as follows. By applying the principle of relativity to the transmission of light signals, we obtain the same speed of light in all inertial frames.

A closer look at Einstein's operational definition of simultaneity shows that it is indeed based on the principle of relativity. Einstein adhered to Mach in rejecting theoretical definitions that are operationally void. But in contradistinction to Mach, he did not want to criticise the conceptual foundations of Newton's mechanics but to develop a new theory. Thus he could not presuppose a full-fledged theory for defining operational concepts. He had to work the other way round. This did not at all mean that he started from a discussion of empirical operations. His starting point was the principle of relativity. It told him *which* kinds of empirical operation were admitted for measurements that agree with the laws of electrodynamics in all inertial frames: above all, the transmission of light signals. According to Newtonian mechanics alone, it is admitted to synchronise two clocks in a common rest frame, and to accelerate one of them innocuously for time measurements in another inertial frame. According to the principle of relativity it is *not* admitted, as the twin paradox strikingly shows.[8] Einstein's operational definitions of simultaneity, and of the lenght of a rigid rod, are powerful enough for deriving the laws of special relativity only because they are chosen in agreement with the principle of relativity. It can indeed be shown that the Lorentz transformation derives from the principle of relativity, the assumption of a finite speed of light, plus a few other principles of physics (Mittelstaedt 1995).

7. The axiomatic method

Bridgman (1927) interpreted Einstein's definitions from an empiricist point of view. According to him, physical quantities reduce to expressions for empirical operations. He cut operational definitions off their axiomatic background. Einstein had a completely different methodology. His main principles were the *universality* of theoretical principles and measuring methods, and the *agreement* of both. In realising these methodological principles, he started always from theoretical considerations. When quantum mechanics and the Copenhagen interpretation arose he did not hesitate to accuse Heisenberg and Bohr of operationalism. Against their alleged operational view of the observables of quantum mechanics, he claimed that the theory determines what are the data, and not vice versa (Heisenberg 1969, p. 92; Scheibe 1992, p. 125).

Einstein was obviously a proponent of the axiomatic method. His view of the relation between theories and their data brings us back to theoretical holism, and to the problem of incommensurability. How can we judge on the truth of a theory on the basis of measuring methods which depend on the conceptual foundations of the very theory under debate? To clarify this question, let us have a look at Hilbert's axiomatic method and its use in physics.

According to Hilbert, theoretical terms such as the concepts of Euclidean geometry have no formal or informal meaning on their own. They are only determined by a system of axioms. That is, their sense can not be pinned down by explicit definitions. It is only implicitly given by axioms instead of definitions. In addition, their reference is not fixed. They may apply to *many* kinds of entities inside or outside pure mathematics. It has come into use to say that axioms give *implicit definitions* of concepts. This is due to Bernays (1922, p. 95) who emphasized that in Hilbert's view, geometrical concepts such as 'point', 'straight line', or 'lying between', are only "implicitly characterised" by the axioms in which they occur. A system of axioms does not determine isolated concepts but a whole theory, and a class of formal or empirical structures as its models.

The axioms of a theory must satisfy several conditions. They have to be *independent* and *consistent* (Hilbert 1918, p. 148). Their applications in physics should in addition be *empirically adequate*. Hilbert used to say that there is a pre-established harmony between physical theory, and experience (Hilbert 1930). In his view, Einstein claims correctly that there *are* empirical data that may give external support to a theory, even though the theory comes first. From an axiomatic point of view, the adequacy requirement may be expressed as follows. Internal consistency is not the only consistency constraint of an empirical theory. In addition, the empirical structures and measurements which result from applying a theory to the phenomena should be approximately consistent with the axioms. The external consistency requirement is obviously non-logical. It demands that an empirical or phenomenological structure be an approximate model of a theory.

A logically correct theory yields consistent theoretical predictions that *may* give rise to discrepancies with the measuring results. Such discrepancies being anomalies in Kuhn's sense; they *may* provoque scientific revolutions. A good example is the anomalous perihelion of Mercury. It was found on the basis of Newtonian mechanics alone, but has only been explained from Einstein's general theory of relativity. The measuring methods for the test of a theory may well depend on the theory under debate. But to perform a measurement means that the numerical measuring *results* are *not* determined by the theory under debate.

In a certain sense, the axiomatic method is grist on Kuhn's mill. By a set of axioms, the sense of theoretical concepts is given implicitly. The reference of an axiomatic theory is left completely open. A system of axioms that is taken to be universally valid defines a world view. Since the concepts occurring in the axioms cannot be defined explicitly, it is impossible to translate them term-by-term into the concepts of a rival theory which has another axiomatic basis.

Kuhn's claims concerning holism, incommensurability and the illusiveness of reference depend crucially on the assumption that a theory expresses a *world view*, or is considered to be universally valid. In scientific practice, however, no one would claim that in a preliminary stage of research the axioms underlying a theory or a corresponding measuring method should hold forever, and for all parts of physics. Hilbert was well aware that the physical theories of his day were of limited scope. The axiomatic method does not aim at generating a holistic world view all of a sudden. It establishes rather the process of integrating increasingly more fragments of theoretical knowledge into decreasingly many axiomatic systems. Two distinct theories can either be unified by combining their axioms (if they are compatible), or by reducing their axioms approximately to those of a third theory (if they are not). Hilbert's standard expression for the unification of axiomatic theories was *Tieferlegung der Fundamente* (Hilbert 1918, p. 148). A favourite example of axiomatic unification is Einstein's reconciliation of mechanics and electrodynamics by means of the principle of relativity, respectively the special theory of relativity.

To incorporate a physical theory into a more embracing axiomatic framework is only possible if it fits in with more of physics. Thus in addition to independence, consistency, and adequacy, the axioms of a physical theory should satisfy a condition concerning the extendability of their scope. They must not contradict well-established theorems from neighbouring domains of knowledge (Hilbert 1918, p. 150 f.). Hilbert gave two examples for this condition being satisfied. Kinetic theory is in agreement with thermodyamics; and some laws of geometrical optics derive from his own axiomatization of radiation theory.

We see that the axiomatic method does not only make Kuhn's holistic views more precise. It shows also how the problems posed by incommensurability might be overcome. Theories are no longer incommensurable as soon as their laws are embedded into a common axiomatic framework, by means of exact or approximate reductions. The rational response to Kuhn's challenge has indeed been to initiate detailed formal investigations of inter-theoretical relations (Balzer/Moulines/Sneed 1987; Mühlhölzer 1989; Bartels 1994; Scheibe 1997). But this is not the whole story. In scientific practice, several semantic problems arising from incommensurable theories can *not* be resolved by means of the axiomatic method. For two reasons, looking for inter-theoretic relations that may be expressed in axiomatic terms is quite often of no help in closing the conceptual gaps between two given theories.

First, many times axiomatization comes long *after* some crucial laws of a new theory have been developed, tested by well-established measuring methods, and fitted into the rest of physics. The scientists have to cope with incommensurability long before a new theory is embedded into the existing architectonics of physics. They are only able to embed a new theory into well-established physics and to develop independent measuring methods for it when they have already mastered the conceptual gaps between the new theory and the old concepts. Imagine the development of quantum mechanics after two decades of the old quantum theory. The old quantum theory was heterogeneous. It was not good for axiomatization even though Hilbert attempted to put it in quasi-axiomatic shape around 1922/23. But it was the link between the classical concepts and later quantum mechanics (Darrigol 1992; Falkenburg 1997, 1998). It anticipated the crucial quantum principles while maintaining the classical model of orbiting electrons. When the latter had to be finally abandoned, substantial parts of the new theory were already expressed in the familiar language of physical quantities and anchored in the phenomena. They were interpreted in terms of classical physics and of Bohr's correspondence principle. After quantum mechanics had become subject of the axiomatic method, Heisenberg (1930, p. 78 ff.) suggested to derive its basic laws from classical mechanics by means of quantisation rules, and to interpret them in terms of correspondence.

Second, we can not be sure whether all theories of physics may indeed be unified. Nancy Cartwright argues that the idea of a unified physics is fictitious. Actually any reductionism in physics encounters the challenge of quantum theory. The incorporation of irreducible quantum effects into an otherwise classical world is still a hard nut to crack for the axiomatic method, as it was eighty years ago.[9] The classical and the quantum theories are best examples of incommensurable theories in Kuhn's sense. They are associated with diverging world views. The quantum description of a physical process cannot be reduced to a classical description, or vice versa, at a non-probabilistic level and within a theory that admits unification with relativity. But we can neither dispense with the classical nor with the quantum theories. At the present stage of knowledge, quantum physics commits us to theoretical pluralism.

8. The scales of physical quantities

Neither representationalism, nor operationalism, nor the axiomatic method on its own can sufficiently explain what physical quantities are. Any reasonable metaphysics of measurement should not rely exclusively on the representational, or operational, or axiomatic, features of measurement but consider them all together. Indeed abstract measurement theory is tailored to do so. It is metaphysically neutral in that it combines the axiomatic method with an operational and representational approach, instead of favouring one of these approaches at the expense of the others. Measurement theory on its own neither commits us to empiricism or scientific realism, nor is it sufficient to reject Kuhn's anti-referential conclusions drawn from incommensurability. It simply does not support a decisive position concerning the truth of physical theories, or the reference of their concepts.

In another regard, measurement theory is *not* metaphysically neutral. It does not suffice to support a metaphysics of measurement that agrees with scientific practice. It is only good for describing such measurement methods and defining such operational concepts that have a unique axiomatic basis. But it is too strong to cope with the actual demands of theoretical pluralism. It does not open any way for constructing a common domain of all modern physical theories in terms of scales of physical quantities.

The scale of a quantity can be operationally defined from a chain of measurements. For example, the length scale is established by a chain of measurements performed with measuring devices such as: star parallaxis, geodesic instruments, ruler, micrometric screw, microscope, electron microscope, particle scattering in high energy physics. The scale of a quantity may be considered to represent a cluster concept in Ellis' sense. It stems from all such empirical and experimental operations that give rise to measurements which are uniquely representable by real numbers and which have overlapping numerical ranges. In the overlap of the ranges, any two measurements of the same phenomenon must give approximately the same results. According to Ellis (1968, p. 41 f.), this condition can be made precise in such a way that is necessary and sufficient for defining a scale.

However, the neat definition is illogical. Each measuring method comes with its own axioms. In general, the scale of a quantity is based on measurements stemming from several incommensurable theories. The axiomatic foundations of the corresponding operational definitions may be incompatible. Measurement theory simply inherits the crucial limitations of the axiomatic method. Concerning the striking axiomatic gaps between classical and quantum concepts, Ellis' cluster concepts of quantities turn out to rest on inconsistent foundations. For a scale covering such conceptual gaps, the formal continuity established by the Archimedean axiom of measurement theory seems to be illusive.

Scientific practice is much sloppier than abstract measurement theory. The sloppy constructions of scales of length, time, and mass measurement are empirically most successful. Many modern technologies rely on them. For example, they gave rise to the world-wide standardization of precision measurements to atomic units. (Think of the precision clockwork of your quartz watch.) I have already mentioned (in section 3) that in Hilbert's view this matter of fact indicates that the Archimedean axiom is empirically valid. This means, strikingly enough, that an empirically adequate axiom of measurement theory is at odds with the pluralistic structure of modern physics.

How is it possible that the construction of the length or mass scales from the subatomic to the cosmological domain is empirically successful? The lenght scale covers the size of the universe, the size of this sheet of paper and the distance of quarks within a proton or neutron. The size of the universe is obtained from models of general relativity (above all the *big bang*-model) plus many kinds of astrophysical data. The size of this sheet of paper is measured with a ruler. The distance of quarks within the nucleon has been measured from lepton-nucleon scattering in high energy physics, or predicted from models of quantum chromodynamics. Similarly, the mass scale embraces the mass values for electrons, billard balls, or black holes as I mentioned in the beginning (section 2). Electrons are subject to quantum electrodynamics, the motions of billard balls obey classical mechanics, black holes belong to the domain of general relativity. According to Kuhn, each theory generates a world view. If we adopt his holistic philosophy in face of theoretical pluralism, we have to conclude that the scales of physical quantities span a fragmented world.

In constructing such scales *without* despairing of a fragmented world, the physicists trust in general principles. The axiomatic method has indeed a grip on many of them. Above all, the symmetries and invariances of non-relativistic or relativistic physics are powerful axiomatic tools (Wigner 1939, 1979). They establish a general framework of theory construction, relations of approximate reduction between alternative axioms, and other pleasant features of a unified physics. The associated conservation laws such as energy-momentum conservation, charge conservation, parity conservation (or non-conservation) give rise to axiomatic definitions of general concepts. Theoretical concepts defined in terms of symmetries and conservation laws do not depend on specific theories. As far as they give rise to smooth transitions between non-relativistic and relativistic laws, they may hold for the whole scale of a quantity. They are keystones in the construction of the language of physics.

The dimensional algebra of physical quantities is another keystone. The dimension of a quantity expresses the algebraic property of belonging to the class of magnitudes making up the corresponding scale. (The class of magnitudes making up a scale can be operationally defined from a chain of measurements. We should not forget, however, that some members of the chain may have incompatible axiomatic foundations.) Some authors identify the dimension of a quantity with its scale. More strictly, the scale is the numerical representation of a class of magnitudes of a certain dimension. The dimension depicts the algebraic or combinatoric properties of a magnitude. It disregards the numerical representation of a magnitude. In this way, the numerical relations between magnitudes, or the quantitative content of physical laws, are also not taken into consideration.

Correspondingly, the dimensional algebra of physics deals with the non-numerical features of physical quantities and laws. The dimensional algebra of quantities has also been axiomatized within abstract measurement theory. (Krantz 1971, Chapter 10.) The axioms imply that the dimensions on both sides of an equation expressing a physical law must combine to the same kind of quantity, e.g. [mass] x [length] x $[\text{time}]^{-2}$ in the law of force. Such general algebraic or combinatorial properties of quantities enter into all physical theories. They give rise to the well-known Π-Theorem of dimensional analysis (Bridgman 1949) as well as to the consideration that an ultimate theory of physics has to be expressed in terms of dimensionless quantities (Whyte 1954). The dimensional analysis of physical problems is a useful heuristic tool for constructing the models of a known or unknown theory. Indeed it is a very strong constraint of theory construction. It relies on the assumption that some basic algebraic features of the classical concepts of lenght, mass, etc. apply also to structures in the subatomic and cosmological domains. The assumption is strong enough to construct scales which satisfy the Archimedean axiom. On the other hand, it is weak enough to admit that in the quantum domain, the resulting algebra of observables has non-Boolean sectors.

The remaining conceptual gaps in the scales are closed by means of bridge principles. Bridge principles are used to tie a theory to more accessible aspects of reality (Cartwright 1983, p. 132). Quite often, they link some crucial terms of the vocabulary of two incommensurable theories, for example the concepts of mass in non-relativistic mechanics and in special relativity, or the concepts of position or of momentum in classical and quantum mechanics. Such conceptual links are usually justified by relations of approximate reduction that hold between some specific laws or models of both theories. Some famous bridge principles have a formal shape that derives from an axiomatic theory, others do not. Ehrenfest's theorem, a useful principle to bridge the conceptual gap between the classical and the quantum domain, is formal. It derives from quantum mechanics. Bohr's famous correspondence principle is informal. It gave rise to the development of quantum mechanics. It established most important links between classical and quantum concepts in the days of the old quantum theory, as it does today (Darrigol 1992; Falkenburg 1997, 1998).

9. The inherent metaphysics of measurement

Nancy Cartwright (1983) has emphasized that the actual indispensibility of bridge principles is a case for theoretical pluralism. She argues against the axiomatic method and the related search for a unified physical theory. Probably she would say that the pragmatic construction of scales by means of bridge principles commits us to instrumentalism. I would not agree. Sometimes, empirical success in constructing a scale can not be explained from any known bridge principle. A striking case is found in particle physics (Falkenburg 1996). Momentum measurements from particle tracks have been performed for decades with enormous empirical success. In order to measure particle momenta from macroscopic particle tracks, classical laws which apply to individual tracks are combined with probabilistic quantum laws which do *not* apply to individual tracks. The resulting quasi-classical measurement theory is incoherent. It can be shown that if energy dissipates along the tracks *none* of the known bridge principles of physics legitimates the measuring method. The empirical success of the method is a case for the Archimedean axiom. The measurements give coherent empirical results only because the incommensurable classical and quantum concepts that enter the measuring laws express related physical magnitudes.

The well-known miracle argument in favour of scientific realism should not be brought up here again. The scope of my paper was to show that Kuhn's view of incommensurability and its consequences neglect the unifying power and the referential import of the language in which measurements are expressed (see end of section 2). The language of measurement is expressed in terms of physical quantities such as lenght, time, or mass. Many measurements of subatomic physics or astrophysics presuppose that the measured quantities range from a subatomic to a cosmological scale, in accordance with the Archimedean axiom and despite theoretical pluralism. To complete my argument I have indeed merely to insist on the ontological commitment of the Archimedean axiom. The scale of a physical quantity is constructed in such a way that the axiom is satisfied. The Archimedean axiom, however, commits us to believe in the existence of a continuum of entities which span the scale. Obviously, to construct a scale and to use it for measurements is *not* metaphysically neutral. The construction of a scale makes no sense if we do not believe that it ranges over related physical predicates. The use of such predicates in a measurement makes no sense if we do not believe that they express what is measured, that is, physical magnitudes. To believe in such magnitudes, however, means to believe in physical properties.

The Archimedean axiom does *not* commit us to identify physical properties with abstract entities of their own. We may adhere empiricism or naturalism, and prefer an extensional account of magnitudes. From a logical point of view, physical magnitudes stand for classes of concrete objects, and the scale of a quantity is a class of such classes which expresses relations between concrete systems or processes. We may conceive of the physical properties behind these relations cautiously in terms of dispositions. This results in the following modest version of scientific realism. Physical magnitudes express dispositions of processes or systems to behave in well-ordered ways under certain kinds of measurement.

Whatever metaphysical view of magnitudes or properties we are willing to defend, the Archimedean axiom commits us to continua of physical magnitudes. Whether we conceive of magnitudes as entities of their own or not, we have to conceive of them as coming in degrees. To construct the length, time, and mass scales from zero to the infinite means to believe that each of these scales expresses an ordering of related magnitudes of a certain kind. To construct them in spite of theoretical pluralism from a subatomic to a cosmological scale means to maintain that our actual scientific knowledge of natural kinds is fragmented, but that the kinds of magnitudes we measure in nature are not.

Notes

1 Cf. Kuhn 1970, pp. 198 ff., compared to pp. 148-150. See also Hoyningen-Huene 1993, pp. 206-222.

2 See Kyborg 1984, chapter 9, p. 183 ff., and the literature quoted there.

3 See Krantz 1971, p. 25: "In addition to the types of axioms just described, a rather odd axiom is usually stated as part of each system. It is called *Archimedean* because it corresponds to the Archimedean property of real numbers: for any positive number x, no matter how small, and for any number y, no matter how large, there exists an integer such that $nx \geq y$. This simply means that any two numbers are comparable, i.e., their ratio is not infinite." See also Narens 1985, part II.

4 I shall come back to this crucial point in sections 8 and 9.

5 I shall come back to Ellis' cluster concepts in section 8.

6 To criticize atomism, Mach refers to Newton's *rules of reasoning* (Mach 1883, p. 466 f). In so doing, he takes advantage of Newton's authority for supporting an anti-atomism which the author of rule III evidently did *not* have in mind.

7 Modern subatomic physics shows that the criticism is substantial. The problem is only shifted from the mass of chemically different bodies to the mass of the subatomic constituents of the chemical elements, that is in the last analysis, to the masses and charges of different quarks. Contrarily to Mach and Newton, however, modern physics resolves the problem by assuming intrinsic dynamic properties in which the subatomic constituents of matter differ.

8 In the 1905 paper, §4, Einstein mentions the strange consequence ("eigentümliche Konsequenz") of the Lorentz transformation that a moving clock should be delayed, compared to a clock at rest. The so-called twin paradox stems from applying the principle of relativity to the moving clock and the clock at rest, and asking how there can be an *absolute* delay of one of both clocks.

9 Hilbert (1918, p. 151) emphasizes that quantum theory contradicts Maxwell's electrodynamics. This year we celebrate 70 years of Heisenberg's uncertainty relations, without having a definite quantum mechanics of measurement.

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