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Existence of Solutions for a System of Integral Equations Using a Generalization of Darbo's Fixed Point Theorem

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Abstract: In this paper, an extension of Darbo's fixed point theorem via θ-F-contractions in a Banach space has been presented. Measure of noncompactness approach is the main tool in the presentation of our proofs. As an application, we study the existence of solutions for a system of integral equations. Finally, we present a concrete example to support the effectiveness of our results.

Keywords: fixed point; measure of noncompactness; coupled fixed point; integral equations

1. Introduction and Preliminaries

Integral equations are equations in which an unknown function emerges under an integral sign. Integral equations are handled naturally in applied sciences, such as physics and engineering. Furthermore, especially integral equations have been connected with many applications in actuarial science (ruin theory), inverse problems, Marchenko equation (inverse scattering transform), radiative transfers and Viscoelasticity. (see, for example [1].)

One of the strong tools in solving integral equations is fixed point theory. Fixed point theory is one of highly active fields for research in nonlinear analysis. Some new and interesting results in this direction can be found in [2,3].

The existence of solutions for nonlinear integral equations have been perused in many papers via applying the measures of noncompactness approach which was initiated by Kuratowski [4]. The Kuratowski measure of noncompactness has absorbed many researchers studying the fields of functional equations, ordinary and partial differential equations and many other branches. In fact, since measures of noncompactness are functions which are suitable for measuring the degree of noncompactness of a given set, they are very useful instrumentations in functional analysis such as the metric fixed point theory and the operator equation theory in Banach spaces (see [5,6]). Recently, in [7] the concepts of α-ψ and β-ψ condensing operators have been defined and using them some new fixed point results via the technique of measure of noncompactness have been presented.

For more details on the theory of measure of noncompactness, its applications and its relations with nonlinear analysis we refer the reader to [8–13].

In this paper, first we collect some indispensable concepts and results that will be applied throughout this text. Then, we obtain some new fixed point theorems utilizing the measure of noncompactness. In the second section, we apply our results to obtain coupled fixed points. Finally, in order to demonstrate the applicability of our results, we investigate the existence of solutions for a system of integral equations.

Throughout this paper, let $\mathfrak{E}$ be a real Banach space with norm $||.||$ and Λ be a nonempty subset of $\mathfrak{E}$. We mark by $\overline{\Lambda}$ and $Convc(\Lambda)$ the closure and the closed convex hull of Λ, respectively. In addition, let $\mathfrak{M}(\mathfrak{E})$ denotes the family of all nonempty and bounded subsets of $\mathfrak{E}$ and let $\mathfrak{N}(\mathfrak{E})$ be the collection of all relatively compact subsets of $\mathfrak{E}$. Let $\mathfrak{R}$ denotes the set of all real numbers and $\mathfrak{R}_+ = [0, +\infty)$. Moreover, let $\overline{B}(\iota, r)$ be the closed ball with center ι and radius r. Furthermore, let $\overline{B}_r$ indicates the ball $\overline{B}(0, r)$.

The following Definition of a measure of noncompactness is adapted from [14].

Definition 1. *We say that a mapping* $\mathfrak{m} : \mathfrak{M}(\mathfrak{E}) \longrightarrow \mathfrak{R}_+$ *is a measure of noncompactness in the Banach space* $\mathfrak{E}$ *if:*

1° *The family* $ker\mathfrak{m} = \{\Lambda \in \mathfrak{M}(\mathfrak{E}) : \mathfrak{m}(\Lambda) = 0\}$ *is nonempty and* $ker\mathfrak{m} \subset \mathfrak{N}(\mathfrak{E})$*;*
2° $\Lambda \subset \Sigma \Longrightarrow \mathfrak{m}(\Lambda) \leq \mathfrak{m}(\Sigma)$*;*
3° $\mathfrak{m}(\overline{\Lambda}) = \mathfrak{m}(\Lambda)$*;*
4° $\mathfrak{m}(Conv\Lambda) = \mathfrak{m}(\Lambda)$*;*
5° $\mathfrak{m}(\lambda\Lambda + (1-\lambda)\Sigma) \leq \lambda\mathfrak{m}(\Lambda) + (1-\lambda)\mathfrak{m}(\Sigma)$ *for all* $\lambda \in [0, 1]$*;*
6° *If* $\{\Lambda_n\}$ *is a sequence of closed sets from* $\mathfrak{M}(\mathfrak{E})$ *such that* $\Lambda_{n+1} \subset \Lambda_n$ *for* $n = 1, 2, \cdots$, *and if* $\lim_{n\to\infty} \mathfrak{m}(\Lambda_n) = 0$*, then* $\Lambda_\infty = \cap_{n=1}^{\infty} \Lambda_n \neq \emptyset$*.*

In 2012, Wardowski [15] presented a significant generalization of the Banach contraction principle. He introduced a new class of control functions $\mathcal{F}$ which provide a large number of contractions.

Let Γ indicates the set of all functions $W : (0, \infty) \to \mathfrak{R}$ such that:

$(W1)$ W is strictly increasing, i.e., for all $\rho, \varrho \in (0, \infty)$ such that $\rho < \varrho$, one has $W(\rho) < W(\varrho)$,
$(W2)$ $\lim_{n\to\infty} \rho_n = 0$ if and only if $\lim_{n\to\infty} W(\rho_n) = -\infty$, for all sequence $\{\rho_n\}$ of positive values,
$(W3)$ $\lim_{\rho\to 0^+} \rho^{v} W(\rho) = 0$, for some $v \in (0, 1)$.

Let Δ be the following subfamily of Γ consists of all functions $\mathcal{W} : \mathfrak{R}^+ \to \mathfrak{R}$ so that

$(\mathcal{W}_1)$ $\mathcal{W}$ is a continuous and strictly increasing mapping;
$(\mathcal{W}_2)$ $\lim_{n\to\infty} t_n = 0$ iff $\lim_{n\to\infty} \mathcal{W}(t_n) = -\infty$, for each sequence $\{t_n\} \subseteq R^+$.

Example 1. *If* $\mathcal{W}_1(t) = \ln(t)$*, or* $\mathcal{W}_2(t) = 1 - \frac{1}{t^p}$*, where* $p > 0$*, or* $\mathcal{W}_3(t) = 1 - \frac{1}{e^t - 1}$*, or* $\mathcal{W}_4(t) = \frac{1}{e^{-t} - e^t}$*, then* $\mathcal{W}_i \in \Delta,\ i = 1, 2, 3, 4$*.*

Consider $\mathcal{U}(t) = -\frac{1}{t} + t$ for $t > 0$. Note that $\lim_{\rho\to 0^+} \rho^{v} \mathcal{U}(\rho) = -\infty$ $(0 < v < 1)$, that is, $\mathcal{U} \in \Delta$, but it is not a Wardowski mapping.

As in [16], let Θ indicates the family of all functions $\theta : \mathfrak{R} \to \mathfrak{R}$ such that:

(θ_1) $\lim_{n\to\infty} \theta^n(t) = -\infty$ for all $t > 0$;
(θ_2) $\theta(t) < t$ for all $t \geq 0$;
(θ_3) θ is an increasing continuous mapping.

Example 2. *Take* $\theta_1(t) = t - \tau$ $(\tau > 0)$*,* $\theta_2(t) = t^3 - 1$ $(t \leq 1)$ *and* $\theta_2(t) = \sqrt[3]{t} - 1$ $(t \geq 1)$*. Then* $\theta_i \in \Theta$ *for* $i = 1, 2$*.*

Now we remind two significant theorems playing a main designation in the fixed point theory. These theorems is extracted from [17] and [18] respectively.

Theorem 1. *Let Ω be a nonempty, bounded, closed and convex subset of a Banach space $\mathfrak{E}$. Then each continuous and compact mapping $W : \Omega \to \Omega$ possesses at least one fixed point in the set Ω.*

The above formulated theorem organizes the well known Schauder fixed point principle.

The Darbo fixed point theorem (the generalization of Schauder fixed point principle), is regulated as below.

Theorem 2. *Let Ω be a nonempty, bounded, closed and convexsubset of a Banach space $\mathfrak{E}$ and let $\Upsilon : \Omega \to \Omega$ be a continuous mapping. Assume that there exists a constant $\eta \in [0, 1)$ such that $\mathfrak{m}(\Upsilon\Lambda) \leq \eta\mathfrak{m}(\Lambda)$ for any nonempty subset Λ of Ω, where $\mathfrak{m}$ is a MNC defined in $\mathfrak{E}$. Then Υ admits at least a fixed point in Ω.*

2. Main Results

The Darbo contraction principle [18] is an applicable instrumentation for solving problems in nonlinear analysis. In this section, we want to extend it using the concept of θ-$\mathfrak{W}$-contractions.

For simplicity, a nonempty, bounded, closed and convex subset Ω of a Banach space $\mathfrak{E}$ is indicated by NBCC, shortly.

Theorem 3. *Let Ω be an NBCC subset of a Banach space $\mathfrak{E}$ and let $\Upsilon : \Omega \to \Omega$ be a continuous operator such that*

$$\mathcal{W}\big(\mathfrak{m}(\Upsilon\Lambda)\big) \leq \theta\big(\mathcal{W}(\mathfrak{m}(\Lambda))\big), \tag{1}$$

for all $\Lambda \subseteq \Omega$, where $\mathcal{W} \in \Delta$, $\theta \in \Theta$ and $\mathfrak{m}$ is an arbitrary MNC. Then Υ has at least one fixed point in Ω.

Proof. Define a sequence $\{\Omega_n\}$ such that $\Omega_0 = \Omega$ and $\Omega_{n+1} = \overline{Conv}(\Upsilon(\Omega_n))$ for all $n \in \mathbb{N}$.

Let there exists an $N \in \mathbb{N}$ such that $\mathfrak{m}(\Omega_N) = 0$. So, Ω_N is relatively compact and Theorem 1 yields that Υ possesses a fixed point. So, we can suppose that $\mathfrak{m}(\Omega_n) > 0$ for each $n \in \mathbb{N}$.

It is clear that $\{\Omega_n\}_{n\in\mathbb{N}}$ is a sequence of NBCC sets such that

$$\Omega_0 \supseteq \Omega_1 \supseteq \cdots \supseteq \Omega_n \supseteq \Omega_{n+1}.$$

On the other hand,

$$\begin{aligned} \mathcal{W}(\mathfrak{m}(\Omega_{n+1})) &= \mathcal{W}(\mathfrak{m}(\Upsilon\Omega_n)) \\ &\leq \theta(\mathcal{W}(\mathfrak{m}(\Omega_n))) \\ &\leq \theta^2(\mathcal{W}(\mathfrak{m}(\Omega_{n-1}))) \\ &\leq \theta^{n+1}(\mathcal{W}(\mathfrak{m}(\Omega_0))). \end{aligned} \tag{2}$$

Tending $n \to \infty$ in (3) and applying (θ_1), we have $\lim_{n\to\infty} \mathcal{W}\big(\mathfrak{m}(\Omega_{n+1})\big) = -\infty$. According to the fact that $\mathcal{W} \in \Delta$, we obtain that

$$\lim_{n\to\infty} \mathfrak{m}(\Omega_{n+1}) = 0.$$

According to principle (6°) of Definition 1 we evolve that the set $\Omega_\infty = \bigcap_{n=1}^{\infty} \Omega_n$ is a nonempty, closed and convex set and it is stable under the operator Υ and belongs to $Ker\mathfrak{m}$. Then in view of the Schauder theorem, Υ has a fixed point. □

Taking $\theta(t) = t - \tau$, for all $t \in \mathfrak{R}$, we conclude that:

Corollary 1. *Let Ω be an NBCC subset of a Banach space $\mathfrak{E}$ and let $\Upsilon : \Omega \to \Omega$ be a continuous operator such that*

$$\tau + \mathcal{W}\big(\mathfrak{m}(\Upsilon\Lambda)\big) \leq \mathcal{W}\big(\mathfrak{m}(\Lambda)\big), \tag{3}$$

for all $\Lambda \subseteq \Omega$, where $\mathcal{W} \in \Delta$, τ is an arbitrary positive amount and $\mathfrak{m}$ is an arbitrary MNC. Then Υ admits at least one fixed point in Ω.

Remark 1. *We can get the Darbo's fixed point theorem in the above corollary if we take $\mathcal{W}(t) = \ln t$, for all $t > 0$.*

3. Coupled Fixed Point

The notion of coupled fixed point has been introduced by Bhaskar and Lakshmikantham [19].

Definition 2. *We say that $(\iota, \kappa) \in \mathfrak{E}^2$ is a coupled fixed point of a mapping $\Upsilon : \mathfrak{E} \times \mathfrak{E} \to \mathfrak{E}$ if $\Upsilon(\iota, \kappa) = \iota$ and $\Upsilon(\kappa, \iota) = \kappa$.*

The following Theorem which is adapted from [13] helps to construct new measures from arbitrary measures.

Theorem 4. *Suppose that$\mathfrak{m}_1, \mathfrak{m}_2, \ldots, \mathfrak{m}_n$ are measures of noncompactness in Banach spaces $\mathfrak{E}_1, \mathfrak{E}_2, \ldots, \mathfrak{E}_n$, respectively, the function $f : [0,\infty)^n \longrightarrow [0,\infty)$ is a convex function and $f(\iota_1, \cdots, \iota_n) = 0$ if and only if $\iota_i = 0$ for all $i = 1, 2, \cdots, n$. Then*

$$\tilde{\mathfrak{m}}(\Lambda) = f(\mathfrak{m}_1(\Lambda_1), \mathfrak{m}_2(\Lambda_2), \ldots, \mathfrak{m}_n(\Lambda_n)),$$

is a measure of noncompactness in $\mathfrak{E}_1 \times \mathfrak{E}_2 \times \ldots \times \mathfrak{E}_n$, where Λ_i denotes the natural projection of Λ into $\mathfrak{E}_i$, for all $i = 1, 2, \ldots, n$.

From now on, we assume that $\mathcal{W}$ is a sub-additive mapping unless otherwise stated. For instance, any concave function $f : [0,\infty) \to [0,\infty)$ with the reservation that $f(0) \geq 0$, is a sub-additive function.

Theorem 5. *Let Ω be an NBCC subset of a Banach space $\mathfrak{E}$ and let $\Upsilon : \Omega \times \Omega \to \Omega$ be a continuous function such that*

$$\mathcal{W}\big(\mathfrak{m}(\Upsilon(\Lambda_1 \times \Lambda_2))\big) \leq \frac{1}{2}\big[\theta\big(\mathcal{W}\big(\mathfrak{m}(\Lambda_1) + \mathfrak{m}(\Lambda_2)\big)\big)\big] \tag{4}$$

for all subsets Λ_1, Λ_2 of Ω, where $\mathfrak{m}$ is an arbitrary MNC and θ and $\mathcal{W}$ are as in Theorem 3. Then Υ embraces at least a coupled fixed point.

Proof. Consider $\widetilde{\Upsilon} : \Omega^2 \to \Omega^2$ by

$$\widetilde{\Upsilon}(\iota, \kappa) = (\Upsilon(\iota, \kappa), \Upsilon(\kappa, \iota)).$$

Clearly, $\widetilde{\Upsilon}$ is continuous. We show that $\widetilde{\Upsilon}$ satisfies all the conditions of Theorem 3. Let $\Lambda \subset \Omega^2$ be a nonempty subset. We know that $\tilde{\mathfrak{m}}(\Lambda) = \mathfrak{m}(\Lambda_1) + \mathfrak{m}(\Lambda_2)$ is a (MNC) [14] , where Λ_1 and Λ_2 denote the natural projections of Λ into $\mathfrak{E}$. From (4) we have

$$\begin{aligned}
\mathcal{W}(\widetilde{\mathfrak{m}}(\widetilde{\Upsilon}(\Lambda))) &\leq \mathcal{W}(\widetilde{\mathfrak{m}}(\Upsilon(\Lambda_1 \times \Lambda_2) \times \Upsilon(\Lambda_2 \times \Lambda_1))) \\
&= \mathcal{W}(\mathfrak{m}(\Upsilon(\Lambda_1 \times \Lambda_2)) + \mathfrak{m}(\Upsilon(\Lambda_2 \times \Lambda_1))) \\
&\leq \mathcal{W}(\mathfrak{m}(\Upsilon(\Lambda_1 \times \Lambda_2))) + \mathcal{W}(\mathfrak{m}(\Upsilon(\Lambda_2 \times \Lambda_1))) \\
&\leq \frac{1}{2}\left[\theta(\mathcal{W}(\mathfrak{m}(\Lambda_1) + \mathfrak{m}(\Lambda_2)))\right] \\
&+ \frac{1}{2}\left[\theta(\mathcal{W}(\mathfrak{m}(\Lambda_2) + \mathfrak{m}(\Lambda_1)))\right] \\
&\leq \theta(\mathcal{W}(\mathfrak{m}(\Lambda_1) + \mathfrak{m}(\Lambda_2))) \\
&= \theta(\mathcal{W}(\widetilde{\mathfrak{m}}(\Lambda))).
\end{aligned}$$

Now, from Theorem 3 we deduce that $\widetilde{\Upsilon}$ has at least a fixed point which implies that Υ has at least a coupled fixed point. □

Taking $\theta(t) = t - 2\tau$ $(\tau > 0)$ in Theorem 5 we have:

Corollary 2. *Let Ω be an NBCC subset of a Banach space $\mathfrak{E}$ and $\Upsilon : \Omega \times \Omega \to \Omega$ be a continuous function such that*

$$\tau + \mathcal{W}[\mathfrak{m}(\Upsilon(\Lambda_1 \times \Lambda_2))] \leq \frac{1}{2}\mathcal{W}[\mathfrak{m}(\Lambda_1) + \mathfrak{m}(\Lambda_2)] \tag{5}$$

for any subsets Λ_1, Λ_2 of Ω, where $\mathfrak{m}$ is an arbitrary (MNC), and $\mathcal{W}$ is as in Theorem 3. Then Υ has at least a coupled fixed point.

The subadditivity assumption of $\mathcal{W}$ has been omitted in the following theorem.

Theorem 6. *Let Ω be an NBCC subset of a Banach space $\mathfrak{E}$ and let $\Upsilon : \Omega \times \Omega \to \Omega$ be a continuous function such that*

$$\mathcal{W}(\mathfrak{m}(\Upsilon(\Lambda_1 \times \Lambda_2))) \leq \theta(\mathcal{W}(\max\{\mathfrak{m}(\Lambda_1), \mathfrak{m}(\Lambda_2)\})) \tag{6}$$

for all subsets Λ_1, Λ_2 of Ω, where $\mathfrak{m}$ is an arbitrary MNC and θ and $\mathcal{W}$ are as in Theorem 3. Then Υ possesses at least a coupled fixed point.

Proof. Take $\widetilde{\Upsilon} : \Omega^2 \to \Omega^2$ by

$$\widetilde{\Upsilon}(\iota, \kappa) = (\Upsilon(\iota, \kappa), \Upsilon(\kappa, \iota)).$$

It is clear that $\widetilde{\Upsilon}$ is continuous. We show that $\widetilde{\Upsilon}$ satisfies all the conditions of Theorem 3. We know that $\widetilde{\mathfrak{m}}(\Lambda) = \max\{\mathfrak{m}(\Lambda_1), \mathfrak{m}(\Lambda_2)\}$ is a (MNC) [14], where Λ_1 and Λ_2 denote the natural projections of Λ into $\mathfrak{E}$. Let $\Lambda \subset \Omega^2$ be a nonempty subset. From (6) we have

$$\begin{aligned}
\mathcal{W}(\widetilde{\mathfrak{m}}(\widetilde{\Upsilon}(\Lambda))) &\leq \mathcal{W}(\widetilde{\mathfrak{m}}(\Upsilon(\Lambda_1 \times \Lambda_2) \times \Upsilon(\Lambda_2 \times \Lambda_1))) \\
&= \mathcal{W}(\max\{\mathfrak{m}(\Upsilon(\Lambda_1 \times \Lambda_2)), \mathfrak{m}(\Upsilon(\Lambda_2 \times \Lambda_1))\}) \\
&= \max\{\mathcal{W}(\mathfrak{m}(\Upsilon(\Lambda_1 \times \Lambda_2))), \mathcal{W}(\mathfrak{m}(\Upsilon(\Lambda_2 \times \Lambda_1)))\} \\
&\leq \max\left\{\theta(\mathcal{W}(\max\{\mathfrak{m}(\Lambda_1), \mathfrak{m}(\Lambda_2)\})), \theta(\mathcal{W}(\max\{\mathfrak{m}(\Lambda_2), \mathfrak{m}(\Lambda_1)\}))\right\} \\
&= \theta(\mathcal{W}(\max\{\mathfrak{m}(\Lambda_1), \mathfrak{m}(\Lambda_2)\})) \\
&= \theta(\mathcal{W}(\widetilde{\mathfrak{m}}(\Lambda))).
\end{aligned}$$

Now, in view of Theorem 3 we deduce that $\widetilde{\Upsilon}$ possesses at least a fixed point, that is, Υ has at least a coupled fixed point. □

Corollary 3. *Let Ω be an NBCC subset of a Banach space $\mathfrak{E}$ and let $\Upsilon : \Omega \times \Omega \to \Omega$ be a continuous function such that*

$$\tau + \mathcal{W}\big(\mathfrak{m}(\Upsilon(\Lambda_1 \times \Lambda_2))\big) \leq \mathcal{W}\big(\max\{\mathfrak{m}(\Lambda_1), \mathfrak{m}(\Lambda_2)\}\big) \tag{7}$$

for all subsets Λ_1, Λ_2 of Ω, where $\mathfrak{m}$ is an arbitrary (MNC), $\tau > 0$ and $\mathcal{W}$ is as in Theorem 3. Then Υ has at least a coupled fixed point.

4. Application

This section of the article is dedicated to discussing the existence of solutions for the following system of equations:

$$\begin{cases} \mu_1(\iota) = f\left(\iota, \mu_1(\rho(\iota)), \mu_2(\rho(\iota)), \int_0^{\varrho(\iota)} g\left(\iota, \kappa, \mu_1(\rho(\kappa)), \mu_2(\rho(\kappa))\right) d\kappa\right) \\ \mu_2(\iota) = f\left(\iota, \mu_2(\rho(\iota)), \mu_1(\rho(\iota)), \int_0^{\varrho(\iota)} g\left(\iota, \kappa, \mu_2(\rho(\kappa)), \mu_1(\rho(\kappa))\right) d\kappa\right) \end{cases} \tag{8}$$

where $\iota \in [0, T]$.

Let $C[0, T]$ be the space of all real functions which are bounded and continuous on the interval $[0, T]$ with the usual norm

$$\|\iota\| = \sup\{|\iota(t)| : t \in [0, T]\}.$$

The modulus of continuity of a function $\iota \in C[0, T]$ is as

$$\omega(\iota\epsilon) = \sup\{|\iota(t) - \iota(s)| : t, s \in [0, T], |t - s| \leq \epsilon\}.$$

Uniform continuity of ι on $[0, T]$ yields that $\omega(\iota\epsilon) \to 0$ as $\epsilon \to 0$.

Now, let $\omega(\Lambda, \varepsilon) = \sup\{\omega(\iota\varepsilon) : \iota \in \Lambda\}$. The Hausdorff measure of noncompactness for all bounded sets Λ of $C[0, T]$ is as follows:

$$\omega(\Lambda) = \lim_{\epsilon \to 0} \left\{ \sup_{\iota \in \Lambda} \omega(\iota\epsilon) \right\}.$$

(See more detail in [13].)

Theorem 7. *Suppose that:*

(i) $\rho, \varrho : [0, T] \longrightarrow [0, T]$ *are continuous functions,*
(ii) *The function $f : [0, T] \times \mathbb{R}^3 \longrightarrow \mathbb{R}$ is continuous and there exists a function $\mathcal{W} \in \Delta$ so that*

$$\mathcal{W}\Big(\Big|f(\iota, \mu_1, \mu_2, \kappa) - f(\iota\nu_1, \nu_2, z)\Big|\Big) \leq \theta\Big(\mathcal{W}\Big(\max\Big\{\Big|\mu_1 - \nu_1\Big|, \Big|\mu_2 - \nu_2\Big|\Big\} + \Big|\kappa - z\Big|\Big)\Big), \tag{9}$$

(iii) $g : [0, T] \times [0, T] \times \mathbb{R}^2 \longrightarrow \mathbb{R}$ *is continuous,*
(iv) *The inequality*

$$\mathcal{W}^{-1}\Big(\theta(\mathcal{W}(r + G_r))\Big) + M \leq r$$

has a positive solution r_0, where $M = \max\{f(\iota, 0, 0, 0) : \iota \in [0, T]\}$, and $G_r = \sup\left\{\left|\int_0^{\varrho(\iota)} g(\iota, \kappa, \mu_1, \mu_2) d\kappa\right| : \iota \in [0, T], ||\mu_1||, ||\mu_2|| \leq r\right\}$.

Then the system of integral Equations (8) possesses at least one solution in the space $(C[0, T])^2$.

Proof. Let $\Upsilon : C[0,T] \times C[0,T] \longrightarrow C[0,T]$ be defined by

$$\Upsilon(\mu_1,\mu_2)(\iota) = f\big(\iota,\mu_1(\rho(\iota)),\mu_2(\rho(\iota)),\int_0^{\varrho(\iota)} g(\iota,\kappa,\mu_1(\rho(\kappa)),\mu_2(\rho(\kappa)))d\kappa\big). \tag{10}$$

According to given assumptions, we conclude that the function $\Upsilon(\mu_1,\mu_2)$ is continuous for arbitrarily $\mu_1,\mu_2 \in C[0,T]$. Furthermore, from our assumptions, we obtain that

$$\begin{aligned}|\Upsilon(\mu_1,\mu_2)(\iota)| &= \Big|f\big(\iota,\mu_1(\rho(\iota)),\mu_2(\rho(\iota)),\int_0^{\varrho(\iota)} g(\iota,\kappa,\mu_1(\rho(\kappa)),\mu_2(\rho(\kappa)))d\kappa\big)\Big| \\ &\le \Big|f\big(\iota,\mu_1(\rho(\iota)),\mu_2(\rho(\iota)),\int_0^{\varrho(\iota)} g(\iota,\kappa,\mu_1(\rho(\kappa)),\mu_2(\rho(\kappa)))d\kappa\big) - f(\iota,0,0,0)\Big| \\ &+ \Big|f(\iota,0,0,0)\Big|.\end{aligned}$$

Furthermore, we have

$$\begin{aligned}&\mathcal{W}\Big(\Big|f\big(\iota,\mu_1(\rho(\iota)),\mu_2(\rho(\iota)),\int_0^{\varrho(\iota)} g(\iota,\kappa,\mu_1(\rho(\kappa)),\mu_2(\rho(\kappa)))d\kappa\big) - f(\iota,0,0,0)\Big|\Big) \\ &\le \theta\Big(\mathcal{W}\Big(\max\{\Big|\mu_1(\rho(\iota))\Big|,\Big|\mu_2(\rho(\iota))\Big|\} + \Big|\int_0^{\varrho(\iota)} g(\iota,\kappa,\mu_1(\rho(\kappa)),\mu_2(\rho(\kappa)))d\kappa\Big|\Big)\Big) \\ &\le \theta\Big(\mathcal{W}\Big(\max\{\|\mu_1\|,\|\mu_2\|\} + \Big|\int_0^{\varrho(\iota)} g(\iota,\kappa,\mu_1(\rho(\kappa)),\mu_2(\rho(\kappa)))d\kappa\Big|\Big)\Big) \\ &\le \theta\big(\mathcal{W}\big(\max\{\|\mu_1\|,\|\mu_2\|\} + G_{\max\{\|\mu_1\|,\|\mu_2\|\}}\big)\big).\end{aligned}$$

Thus,

$$\begin{aligned}&\Big|f\big(\iota,\mu_1(\rho(\iota)),\mu_2(\rho(\iota)),\int_0^{\varrho(\iota)} g(\iota,\kappa,\mu_1(\rho(\kappa)),\mu_2(\rho(\kappa)))d\kappa\big) - f(\iota,0,0,0)\Big| \\ &\qquad\le \mathcal{W}^{-1}\Big(\theta\big(\mathcal{W}\big(\max\{\|\mu_1\|,\|\mu_2\|\} + G_{\max\{\|\mu_1\|,\|\mu_2\|\}}\big)\big)\Big).\end{aligned}$$

From the above calculations, we have

$$\|\Upsilon(\mu_1,\mu_2)(\iota)\| \le \mathcal{W}^{-1}\Big(\theta\big(\mathcal{W}\big(\max\{\|\mu_1\|,\|\mu_2\|\} + G_{\max\{\|\mu_1\|,\|\mu_2\|\}}\big)\big)\Big) + M. \tag{11}$$

Along of inequality (11) and applying (iv), the function Υ maps $(\bar{B}_{r_0})^2$ into $\bar{B}_{r_0}$.

Now, we shall prove the continuity of function Υ on $(\bar{B}_{r_0})^2$. So, fix $\varepsilon > 0$ and take $\mu_1,\mu_2,\nu_1,\nu_2 \in \bar{B}_{r_0}$ arbitrarily such that $\|\mu_i - \nu_i\| \le \varepsilon$ for all $i = 1,2$. Then, for all $\iota \in [0,T]$, we obtain that

$$
\begin{aligned}
&\mathcal{W}\Big(\Big|\Upsilon(\mu_1,\mu_2)(\iota)-\Upsilon(\nu_1,\nu_2)(\iota)\Big|\Big)\\
&\le \mathcal{W}\Big(\Big|f\big(\iota,\mu_1(\rho(\iota)),\mu_2(\rho(\iota)),\int_0^{\varrho(\iota)} g(\iota,\kappa,\mu_1(\rho(\kappa)),\mu_2(\rho(\kappa)))d\kappa\big)\\
&-f\big(\iota\nu_1(\rho(\iota)),\nu_2(\rho(\iota)),\int_0^{\varrho(\iota)} g(\iota,\kappa,\nu_1(\rho(\kappa)),\nu_2(\rho(\kappa)))d\kappa\big)\Big|\Big)\\
&\le \theta\Big(\mathcal{W}\Big(\max_{i=1,2}\{|\mu_i(\iota)-\nu_i(\iota)|\}+\Big|\int_0^{\varrho(\iota)} g(\iota,\kappa,\mu_1(\rho(\kappa)),\mu_2(\rho(\kappa)))d\kappa\\
&-\int_0^{\varrho(\iota)} g(\iota,\kappa,\nu_1(\rho(\kappa)),\nu_2(\rho(\kappa)))d\kappa\Big|\Big)\Big)\\
&\le \theta\Big(\mathcal{W}\Big(\max_{i=1,2}\{\|\mu_i-\nu_i\|\}\\
&+\Big|\int_0^{\varrho(\iota)} g(\iota,\kappa,\mu_1(\rho(\kappa)),\mu_2(\rho(\kappa)))d\kappa-g(\iota,\kappa,\nu_1(\rho(\kappa)),\nu_2(\rho(\kappa)))d\kappa\Big|\Big)\Big)\\
&\le \theta\Big(\mathcal{W}\Big(\max_{i=1,2}\{\|\mu_i-\nu_i\|\}+TQ^{\varepsilon}_{r_0}\Big)\Big)<\mathcal{W}(\max_{i=1,2}\{\|\mu_i-\nu_i\|\}+TQ^{\varepsilon}_{r_0}),
\end{aligned}
$$

where

$$
Q^{\varepsilon}_{r_0}=\sup\{|g(\iota,\kappa,\mu_1,\mu_2)-g(\iota,\kappa,\nu_1,\nu_2)|:\iota,\kappa\in[0,T],||\mu_i||,||\nu_i||\le r_0,||\mu_i-\nu_i||\le\varepsilon\}.
$$

The continuity of g on the compact set $[0,T]^2\times[-r_0,r_0]^2$ yields that $Q^{\varepsilon}_{r_0}\longrightarrow 0$ as $\varepsilon\longrightarrow 0$. Thus, $\Big|\Upsilon(\mu_1,\mu_2)(\iota)-\Upsilon(\nu_1,\nu_2)(\iota)\Big|\Rightarrow 0$ as $\varepsilon\longrightarrow 0$. That is, Υ is a continuous function on $(\bar{B}_{r_0})^2$. Now, we show that Υ satisfies all the conditions of Theorem 6. Let Λ_1,Λ_2 be nonempty and bounded subsets of $\bar{B}_{r_0}$. Assume that $\varepsilon>0$ is an arbitrary constant. Also, we take $\iota_1,\iota_2\in[0,T]$, with $|\iota_2-\iota_1|\le\varepsilon$, $|\rho(\iota_2)-\rho(\iota_1)|\le\varepsilon$ and $\mu_j\in\Lambda_j$ for all $j=1,2$. Then we have

$$
\begin{aligned}
&\Big|\Upsilon(\mu_1,\mu_2)(\iota_1)-\Upsilon(\mu_1,\mu_2)(\iota_2)\Big| \qquad (12)\\
&\le\Big|f\big(\iota_1,\mu_1(\rho(\iota_1)),\mu_2(\rho(\iota_1)),\int_0^{\varrho(\iota_1)} g(\iota_1,\kappa,\mu_1(\rho(\kappa)),\mu_2(\rho(\kappa)))d\kappa\big)\\
&-f\big(\iota_2,\mu_1(\rho(\iota_2)),\mu_2(\rho(\iota_2)),\int_0^{\varrho(\iota_2)} g(\iota_2,\kappa,\mu_1(\rho(\kappa)),\mu_2(\rho(\kappa)))d\kappa\big)\Big|\\
&\le\Big|f\big(\iota_1,\mu_1(\rho(\iota_1)),\mu_2(\rho(\iota_1)),\int_0^{\varrho(\iota_1)} g(\iota_1,\kappa,\mu_1(\rho(\kappa)),\mu_2(\rho(\kappa)))d\kappa\big)\\
&-f\big(\iota_2,\mu_1(\rho(\iota_1)),\mu_2(\rho(\iota_1)),\int_0^{\varrho(\iota_1)} g(\iota_1,\kappa,\mu_1(\rho(\kappa)),\mu_2(\rho(\kappa)))d\kappa\big)\Big|\\
&+\Big|f\big(\iota_2,\mu_1(\rho(\iota_1)),\mu_2(\rho(\iota_1)),\int_0^{\varrho(\iota_1)} g(\iota_1,\kappa,\mu_1(\rho(\kappa)),\mu_2(\rho(\kappa)))d\kappa\big)\\
&-f\big(\iota_2,\mu_1(\rho(\iota_2)),\mu_2(\rho(\iota_2)),\int_0^{\varrho(\iota_1)} g(\iota_1,\kappa,\mu_1(\rho(\kappa)),\mu_2(\rho(\kappa)))d\kappa\big)\Big|\\
&+\Big|f\big(\iota_2,\mu_1(\rho(\iota_2)),\mu_2(\rho(\iota_2)),\int_0^{\varrho(\iota_1)} g(\iota_1,\kappa,\mu_1(\rho(\kappa)),\mu_2(\rho(\kappa)))d\kappa\big)\\
&-f\big(\iota_2,\mu_1(\rho(\iota_2)),\mu_2(\rho(\iota_2)),\int_0^{\varrho(\iota_2)} g(\iota_1,\kappa,\mu_1(\rho(\kappa)),\mu_2(\rho(\kappa)))d\kappa\big)\Big|\\
&+\Big|f\big(\iota_2,\mu_1(\rho(\iota_2)),\mu_2(\rho(\iota_2)),\int_0^{\varrho(\iota_2)} g(\iota_1,\kappa,\mu_1(\rho(\kappa)),\mu_2(\rho(\kappa)))d\kappa\big)\\
&-f\big(\iota_2,\mu_1(\rho(\iota_2)),\mu_2(\rho(\iota_2)),\int_0^{\varrho(\iota_2)} g(\iota_2,\kappa,\mu_1(\rho(\kappa)),\mu_2(\rho(\kappa)))d\kappa\big)\Big|.
\end{aligned}
$$

Using condition (9) we have

$$
\begin{aligned}
&\Big|\Upsilon(\mu_1,\mu_2)(\iota)-\Upsilon(\nu_1,\nu_2)(\iota)\Big|\\
&\le \omega_{r_0}(f,\varepsilon)+\mathcal{W}^{-1}\Big(\theta\Big(\mathcal{W}\big(\max\{|\mu_1(\rho(\iota_1))-\mu_1(\rho(\iota_2))|,|\mu_2(\rho(\iota_1))-\mu_2(\rho(\iota_2))|\}\big)\Big)\Big)\\
&+\mathcal{W}^{-1}\Big(\theta\Big(\mathcal{W}\big(\Big|\int_0^{\varrho(\iota_1)} g(\iota_1,\kappa,\mu_1(\rho(\kappa)),\mu_2(\rho(\kappa)))d\kappa-\int_0^{\varrho(\iota_2)} g(\iota_1,\kappa,\mu_1(\rho(\kappa)),\mu_2(\rho(\kappa)))d\kappa\Big|\big)\Big)\Big)\\
&+\mathcal{W}^{-1}\Big(\theta\Big(\mathcal{W}\big(\Big|\int_0^{\varrho(\iota_2)} g(\iota_1,\kappa,\mu_1(\rho(\kappa)),\mu_2(\rho(\kappa)))d\kappa-\int_0^{\varrho(\iota_2)} g(\iota_2,\kappa,\mu_1(\rho(\kappa)),\mu_2(\rho(\kappa)))d\kappa\Big|\big)\Big)\Big)\\
&\le \omega_{r_0}(f,\varepsilon)+\mathcal{W}^{-1}\Big(\theta\Big(\mathcal{W}\big(\max\{\omega(\mu_1,\varepsilon),\omega(\mu_2,\varepsilon)\}\big)\Big)\Big) \qquad (13)\\
&+\mathcal{W}^{-1}\Big(\theta\Big(\mathcal{W}\big(\Big|\int_{\varrho(\iota_1)}^{\varrho(\iota_2)} g(\iota_1,\kappa,\mu_1(\rho(\kappa)),\mu_2(\rho(\kappa)))d\kappa\Big|\big)\Big)\Big)\\
&+\mathcal{W}^{-1}\Big(\theta\Big(\mathcal{W}\big(\int_0^{\varrho(\iota_2)} |g(\iota_1,\kappa,\mu_1(\rho(\kappa)),\mu_2(\rho(\kappa)))-g(\iota_2,\kappa,\mu_1(\rho(\kappa)),\mu_2(\rho(\kappa)))d\kappa|\big)\Big)\Big)\\
&\le \omega_{r_0}(f,\varepsilon)+\mathcal{W}^{-1}\Big(\theta\Big(\mathcal{W}\big(\max\{\omega(\mu_1,\varepsilon),\omega(\mu_2,\varepsilon)\}\big)\Big)\Big)+\mathcal{W}^{-1}\Big(\theta\Big(\mathcal{W}(\omega(\varrho,\varepsilon)U_{r_0})\Big)\Big)\\
&+\mathcal{W}^{-1}\Big(\theta\Big(\mathcal{W}(T\omega_{r_0}(g,\varepsilon))\Big)\Big)
\end{aligned}
$$

where

$$
\begin{aligned}
&\omega_{r_0}(f,\varepsilon)\\
&=\sup\{|f(\iota_1,u,v,z)-f(\iota_2,u,v,z)|:\iota_1,\iota_2\in[0,T],|\iota_2-\iota_1|\le\varepsilon,||u||,||v||\le r_0,|z|\le G_{r_0}\},\omega_{r_0}(g,\varepsilon)\\
&=\sup\{|g(\iota_1,\kappa,u,v)-g(\iota_2,\kappa,u,v)|:\iota_1,\iota_2,\kappa\in[0,T],|\iota_2-\iota_1|\le\varepsilon,||u||,||v||\le r_0\},\\
&U_{r_0}=\sup\{|g(\iota,\kappa,u,v)|:\iota,\kappa\in[0,T],u,v\in[-r_0,r_0]\}.
\end{aligned}
$$

Since in (13), μ_i was an arbitrary element of Λ_i for $i=1,2$, we obtain that

$$
\begin{aligned}
\omega(\Upsilon(\Lambda_1\times\Lambda_2),\varepsilon)&\le \omega_{r_0}(f,\varepsilon)+\mathcal{W}^{-1}\Big(\theta\Big(\mathcal{W}\big(\max\{\omega(\Lambda_1,\varepsilon),\omega(\Lambda_2,\varepsilon)\}\big)\Big)\Big)\\
&+\mathcal{W}^{-1}\Big(\theta\Big(\mathcal{W}(\omega(\varrho,\varepsilon)U_{r_0})\Big)\Big)+\mathcal{W}^{-1}\Big(\theta\Big(\mathcal{W}(T\omega_{r_0}(g,\varepsilon))\Big)\Big).
\end{aligned}
$$

The uniform continuity of f, ϱ and g on the compact sets $[0,T]\times[-r_0,r_0]^2\times[-G_{r_0},G_{r_0}]$, $[0,T]$ and $[0,T]^2\times[-r_0,r_0]^2$, respectively, yields that $\omega_{r_0}(f,\varepsilon)\longrightarrow 0$, $\omega(\varrho,\varepsilon)\longrightarrow 0$ and $\omega_{r_0}(g,\varepsilon)\longrightarrow 0$ as $\varepsilon\longrightarrow 0$. Therefore,

$$
\omega(\Upsilon(\Lambda_1\times\Lambda_2))\le\mathcal{W}^{-1}\Big(\theta\Big(\mathcal{W}\big(\max\{\omega(\Lambda_1),\omega(\Lambda_2)\}\big)\Big)\Big).
$$

Thus, we obtain that

$$
\mathcal{W}(\omega(\Upsilon(\Lambda_1\times\Lambda_2)))\le\theta\Big(\mathcal{W}\big(\max\{\omega(\Lambda_1),\omega(\Lambda_2)\}\big)\Big) \qquad (14)
$$

Therefore, Theorem 6 concludes that the operator Υ admits a coupled fixed point. That is, the system of functional integral Equation (8) has at least one solution in $(C[0,T])^2$. □

5. Example

Example 3. *Suppose that the following system of integral equations be given:*

$$\begin{cases} \iota(t) = \frac{1}{2}e^{-t^2} + \frac{\arctan\iota(t)+\sinh^{-1}\kappa(t)}{8\pi+t^8} \\ +\frac{1}{8}\int_0^t \frac{s(|sin\iota(s)| + \sqrt{(1+\iota^2(s))(1+sin^2\kappa(s))})}{e^t(1+\iota^2(s))(1+sin^2\kappa(s))}ds \\ \kappa(t) = \frac{1}{2}e^{-t^2} + \frac{\arctan\kappa(t)+\sinh^{-1}\iota(t)}{8\pi+t^8} \\ +\frac{1}{8}\int_0^t \frac{s(|sin\kappa(s)| + \sqrt{(1+\kappa^2(s))(1+sin^2\iota(s))})}{e^t(1+\kappa^2(s))(1+sin^2\iota(s))}ds. \end{cases} \tag{15}$$

We observe that this system of integral Equation (15) is a special case of the system (8) with

$$\rho(t) = \varrho(t) = t, \qquad t \in [0,1],$$

$$f(t,\iota,\kappa,p) = \frac{1}{2}e^{-t^2} + \frac{\arctan\iota + \sinh^{-1}\kappa}{8\pi + t^8} + \frac{p}{8},$$

and

$$g(t,s,\iota,\kappa) = \frac{s(|sin\iota| + \sqrt{(1+\iota^2)(1+sin^2\kappa)})}{e^t(1+\iota^2)(1+sin^2\kappa)}.$$

We need to verify the conditions (i)–(iv) *of Theorem 7 to show that the above system has a solution.*

Condition (i) is clearly evident. We define $\mathcal{W}(t) = \ln t$ *and* $\theta(t) = t - \ln 8$*. Now, we have*

$$\begin{aligned} &\mathcal{W}\Big(\Big|f(t,\iota,\kappa,m) - f(t,u,v,n)\Big|\Big) \\ &\leq \ln\Big(\frac{|\arctan\iota - \arctan u| + |\sinh^{-1}\kappa - \sinh^{-1}v|}{8\pi + t^8} + \frac{|m-n|}{8}\Big) \\ &\leq \ln\Big(\frac{\arctan|\iota - u|}{8\pi} + \frac{|\kappa - v|}{8\pi} + \frac{|m-n|}{8}\Big) \\ &\leq \ln\big(\max\{|\iota-u|,|\kappa-v|\} + |m-n|\big) - \ln 8 \\ &= \theta\big(\mathcal{W}\big(\max\{|\iota-u|,|\kappa-v|\} + |m-n|\big)\big). \end{aligned}$$

So, we observe that f *satisfies condition (ii) of Theorem 7. Furthermore,*

$$M = \sup\{|f(t,0,0,0)| : t \in [0,1]\} = \sup\{\frac{1}{2}e^{-t^2} : t \in [0,1]\} \leq 0.5$$

Obviously, condition (iii) of Theorem 7 is valid, that is, g *is continuous on* $[0,T] \times [0,T] \times \mathbb{R}^2$*, and*

$$\begin{aligned} G_r &= \sup\left\{\Big|\int_0^t \frac{s(|sin\iota(s)| + \sqrt{(1+\iota^2(s))(1+sin^2\kappa(s))})}{e^t(1+\iota^2(s))(1+sin^2\kappa(s))}ds\Big| : t,s \in [0,1], \iota,\kappa \in [-r,r]\right\} \\ &\leq \sup\frac{t^2}{e^t} \leq 1. \end{aligned}$$

Furthermore, clearly every $r \geq 0.15$ *satisfies the inequality appears in condition (iv), i.e.,*

$$\mathcal{W}^{-1}\Big(\theta\Big(\mathcal{W}\Big(r + G_r\Big)\Big)\Big) + M < \mathcal{W}^{-1}\Big(\theta\Big(\mathcal{W}\Big(r+1\Big)\Big)\Big) + 0.5 = \frac{r+1}{8} \leq r.$$

Consequently, the conditions of Theorem 7 are fulfilled and so, the above system of integral equations admits at least one solution in $\{C[0,T]\}^2$.

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References

1. Sachs, E.W.; Strauss, A.K. Efficient solution of a partial integro-differential equation in finance. *Appl. Numer. Math.* **2008**, *58*, 1687–1703. [CrossRef]
2. Petruşel, A.; Petruşel, G. On Reich's strict fixed point theorem for multi-valued operators in complete metric spaces. *J. Nonlinear Var. Anal.* **2018**, *2*, 103–112.
3. Petruşel, A. Local fixed point results for graphic contractions. *J. Nonlinear Var. Anal.* **2019**, *3*, 141–148.
4. Kuratowski, C. Sur les espaces complets. *Fund. Math.* **1930**, *15*, 301–309. [CrossRef]
5. Muskhelishvilli, N. *Singular Integral Equations: Boundary Problems of function Theory and Their Applications to Mathematical Physics*; Springer: Amsterdam, The Netherlands, 1958. 9994-7. [CrossRef]
6. Akhmerov, R.R.; Kamenskii, M.I.; Potapov, A.S.; Rodkina, A.E.; Sadovskii, B.N. *Measures of Noncompactness and Condensing Operators*; Birkh Auser: Basel, Switzerland, 1992. 0348-5727-7. [CrossRef]
7. ur Rehman, H.; Gopal, D.; Kumam, P. Generalizations of Darbo s fixed point theorem for new condensing operators with application to a functional integral equation. *Demonstr. Math.* **2019**, *52*, 166–182. [CrossRef]
8. Aghajani, A.; Mursaleen, M.; Shole Haghighi, A. A generalization of Darbo's theorem with application to the solvability of systems of integral equations. *J. Comput. Appl. Math.* **2014**, *260*, 68–77. [CrossRef]
9. Banaei, S.; Ghaemi, M.B. A generalization of the Meir-Keeler condensing operators and its application to solvability of a system of nonlinear functional integral equations of Volterra type. *Sahand Commun. Math. Anal.* **2019**, *15*, 19–35. [CrossRef]
10. Banaei, S. An extension of Darbo's theorem and its application to existence of solution for a system of integral equations. *Cogent Math. Stat.* **2019**, *6*. [CrossRef]
11. Banaei, S. Solvability of a system of integral equations of Volterra type in the Fréchet space $L^p_{loc}(\mathbb{R}_+)$ via measure of noncompactness. *Filomat* **2018**, *32*, 5255–5263. [CrossRef]
12. Banaei, S.; Ghaemi, M.B.; Saadati, R. An extension of Darbo's theorem and its application to system of neutral diferential equations with deviating argument. *Miskolc Math. Notes* **2017**, *18*, 83–94. [CrossRef]
13. Banás, J.; Jleli, M.; Mursaleen, M.; Samet, B. *Advances in Nonlinear Analysis via the Concept of Measure of Noncompactness*; Springer: Singapore, 2017.
14. Banás, J.; Goebel, K. *Measures of Noncompactness in Banach Spaces*; Lecture Notes in Pure and Applied Mathematics; Springer: New York, NY, USA, 1980.
15. Wardowski, D. Fixed points of a new type of contractive mappings in complete metric spaces. *Fixed Point Theory Appl.* **2012**, *94*, 2012. [CrossRef]
16. Parvaneh, V.; Hussain, N.; Khorshidi, M.; Mlaiki, N.; Aydi, H. Fixed point results for generalized F-contractions in modular metric and fuzzy metric spaces. *Fixed Point Theory Appl.* **2019**, *7*, 887, doi:10.3390/math7100887. [CrossRef]
17. Agarwal, R.P.; Meehan, M.; O'Regan, D. *Fixed Point Theory and Applications*; Cambridge University Press: Cambridge, UK, 2004.
18. Darbo, G. Punti uniti in trasformazioni a codominio non compatto. *Rendiconti del Seminario Matematico della Universita di Padova* **1955**, *24*, 84–92.
19. Gnana Bhaskar, T.; Lakshmikantham, V. Fixed point theorems in partially ordered metric spaces and applications. *Nonlinear Anal.* **2006**, *65*, 1379–1393. [CrossRef]