Science of the Total Environment 646 (2019) 670–684

Contents lists available at ScienceDirect

Science of the Total Environment

journal homepage: www.elsevier.com/locate/scitotenv

# Stochastic diffusion models to describe the evolution of annual heatwave statistics: A three-factor model with risk calculations

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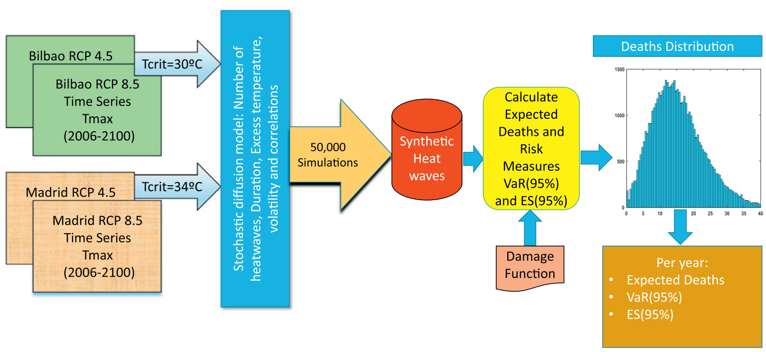
## HIGHLIGHTS

- Probabilistic assessment of evolution of annual characteristics of heatwaves
- Stochastic diffusion models to represent annual statistics of heatwaves
- Long-term high-resolution time series contain information on stochasticity.
- Assessment of human health impacts from heatwaves in the context of climate change
- Risk metrics such as Value at Risk and Expected Shortfall are proposed.

## GRAPHICAL ABSTRACT




## ARTICLE INFO

*Article history:*
Received 11 June 2018
Received in revised form 12 July 2018
Accepted 12 July 2018
Available online xxxx

Editor: SCOTT SHERIDAN

*Keywords:*
Heatwaves
Climate change
Stochastic diffusion modelling
Risk
Uncertainty

## ABSTRACT

In view of risk assessments this paper proposes a stochastic diffusion model to characterise statistics of extreme events when climate- or environmental variables surpass critical thresholds. The proposed three-factor model captures trend and volatility of such statistics and could prove valuable for climate and environmental impact analysis in many systems such as human health, agriculture or ecology. The model supports decisions in view of lowering risks to acceptable levels.

We illustrate the development of the model for heatwave impacts on human health in the context of climate change. We propose a generic model composed of three random processes characterising annual statistics of heatwaves: a Poisson process characterising the number of heatwaves, a Gamma process characterising mean duration and a truncated Gaussian process capturing mean excess temperature of heatwave days. Additionally, potential correlations between the three processes are taken into account.

The model is calibrated with data obtained from a regional climate model for two cities in Spain. The suitability of the model for probabilistic analysis is tested with Monte Carlo simulations. We assess the time-dependent probability distributions of heatwave-related mortality and demonstrate how to obtain relevant risk metrics such as the 95th percentile and the average of the 5% of worst cases (ES (95%)).



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https://doi.org/10.1016/j.scitotenv.2018.07.158

# 1. Introduction

## 1.1. Heatwaves

According to the World Health Organisation and the World Meteorological Organisation (WMO, 2015), heatwaves are among the most hazardous meteorological events, although in the past they have received far less attention than other apparently more spectacular and violent events such as floods, cyclones and hurricanes. Heatwaves can pose significant threats to human health, ecosystems and to energy-, water- and transport systems. They represent an important socio-economic problem. The Intergovernmental Panel on Climate Change, IPCC, warns of an increase in frequency, duration and magnitude of heatwaves in future decades (IPCC, 2012, 2013). The 2003 heatwave, which affected all of Europe, was presumably responsible for up to 70,000 deaths in 16 countries (Robine et al., 2008), and according to Stott et al. (2004), such severe heatwaves could become unexceptional events by 2040. Christidis et al. (2015) project a clear trend of increase in the frequency of heatwaves and a sharp reduction in the return period for the more extreme cases.

Mueller et al. (2016) found that the probability of hot summers is currently ten times higher compared to a scenario without climate change. Within the next two decades regions such as the Mediterranean, Western US, Canada, the Sahara and Southern Asia will start to be particularly affected by hot summers. The PESETA project estimated an increase in mortality between 1% and 4% for each degree Celsius increase in temperature in Europe, which would result in 30,000 additional deaths by 2030, and between 50,000 and 110,000 by 2080 (Paci, 2014).

Heatwaves are generally more serious in urban areas, due to the heat island effect, though rural areas are also susceptible to suffer severe impacts. Cities are especially vulnerable due to the urbanisation process leading to a gradual increase of the proportion of population living in urban areas.

Heatwaves typically occur when temperatures exceed thresholds according to climatologic or epidemiological criteria. Epidemiological thresholds depend on local climatic conditions and may be modified by other variables such as pollution or humidity and wind. The definition of a heatwave event depends on local climate and geographical conditions.

Perkins and Alexander (2013) argue that definitions for heatwaves are ambiguous and inconsistent and that in some cases it is high daytime temperatures that co-occur with high nocturnal temperatures or with high humidity that are critical. Windy conditions can also modify heat stress (WMO, 2015).

Areas that have not been at risk or at lower risk of extreme heat so far might become vulnerable in the future. In this context it becomes crucial to assess future hazards and the occurrence of high-impact events, in terms of intensity, duration and frequency. In view of proof of concept we consider here a single time series of maximum daytime temperature and look for a stochastic diffusion model that can characterise annual statistics of daily exceedances above a critical threshold temperature. Such a model could then be widely applied for assessing health impacts dependent on climatic- or epidemiological thresholds.

## 1.2. Stochastic diffusion models

For climate change impact- and adaptation assessments a good representation of the future evolution of relevant extreme events such as heatwaves is crucial. Such a representation should capture both evolution of trends and variability of its defining characteristics, as well as potential correlations.

Climatic variables, such as temperature, are typically available in the form of time series. These time series can stem either from observed historical observations or as outputs from climate models. In this context, stochastic models provide a means to capture generic information such as trends or variability of the number of annual heatwaves as well as related indicators such as duration and intensity. They allow summarising an entire time series through a model with calibrated parameters for deterministic and stochastic components. Such models can then be used to more easily compare time series from different origins (e.g. from different climate models). The calibrated model can be used for risk assessments, e.g. to compute risk metrics from probability distributions.

Stochastic diffusion models could be especially suited to estimate probability distributions of potential variables characterising heatwaves in terms of number of events, duration and intensity. A diffusion process is defined as a solution to a stochastic differential equation which generates a probabilistic distribution for each time $t$. A general introduction to such models can be found in Kloeden and Platen (1999) and in Dixit and Pindyck (1994). We can distinguish between discrete stochastic processes, where the variable takes on discrete values such as number of heatwaves per year and continuous processes in which the variable takes on values in a continuous range such as heatwave intensity.

Some authors have previously modelled heat wave characteristics using a Poisson process approach. Furrer et al. (2010) develop a model that considers the frequency modelled by a Poisson distribution, the duration as a geometric distribution and the intensity of heatwaves as a conditional generalized Pareto distribution. They calibrated the model with historical series of daily maximum temperature at three different stations. Wang et al. (2015) used the model of Furrer et al. (2010) to study heatwaves in China with information from the 30 Coupled Model Intercomparison Project Phase 5 (CIMP5) General Circulation Models (GCMs). Keellings and Waylen (2014) studied the maximum and minimum daily temperatures in Florida, considering the frequency, intensity, duration of heat waves. They used historical data from 1949 to 2000 to study the variability of heatwaves characteristics. They examined the changes in heat wave characteristics between two equal time periods. Abaurrea et al. (2007) analyse the summer maximum daily temperature in the Ebro river basing during the period 1951–2004; they calibrated a statistical model using a nonhomogeneous Poisson process and used it to obtain medium-term predictions of extreme heat events.

Our approach considers frequency of heatwaves, as well as their annual mean duration and the temperature exceedance on heatwave days. It allows for time-dependent changes in these characteristics caused by projected climate change of the 21st century. The model we propose with the three variables is a “mixed” model containing information on the evolution of annual statistics of heatwaves. It consists of two discrete processes (number of heatwaves per year and mean duration of heatwaves) and one continuous process (mean excess temperature of heatwave days). Previous applications with up to three variables have been conducted for continuous stochastic diffusion processes (Abadie et al., 2014). The approach allows for possible correlations among the three stochastic variables. Once calibrated, we demonstrate how to use the model for computing risk metrics of extreme events.

## 1.3. Risk metrics

After the model calibration, Monte Carlo simulations can be run in order to obtain annual distributions of the measures of interest: e.g. number of heatwaves as well as statistical properties such as the expected values and risk measures. We use risk measures with roots in financial engineering: Value at Risk (VaR) and Expected Shortfall (ES). They both characterise the risk properties of the probability density functions at a given percentile, e.g. the 95% level:

- Value at Risk — VaR (95%) is the value of a variable that is only surpassed in 5% of cases, i.e. the 95th percentile.
- Expected Shortfall — ES (95%) is the mean of the 5% worst cases, i.e. the mean of cases when the 95th percentile is exceeded.

The Expected Shortfall is more sensitive to the shape of the distribution tails than is the 95th percentile. Although Value at Risk is a widely used risk measure, the Expected Shortfall has been shown to have better statistical properties (Artzner et al., 1999).

These measures of risk provide valuable information for climate change impact assessments and adaptation. They allow to calculate the effects of high-impact low-probability events and can be an important input to decision-making. In this way the adaptation decisions can be made using the risk measures that reflect the impacts of worst cases and not only the expected damages.

The remainder of the paper is organized as follows: Section 2 describes the materials and methods used. Section 3 shows the results for heatwaves and associated mortality risk and discusses the generalization and transferability of such an approach. Section 4 presents the main conclusions of the study.

## 2. Materials and methods

### 2.1. Heatwave characterisation and temperature time series

For this study, a "heatwave" is defined in terms of maximum daytime temperature $T_{max}$ exceeding a certain threshold value $T_{crit}$ during one or more consecutive days.

For daily maximum temperature $T_{max}$ we assume two climatic projections between 2006 and 2100 (95 years) based on Representative Concentration Pathway (RCP4.5 and RCP8.5) scenarios presented in IPCC's Fifth Assessment Report AR5 (van Vuuren et al., 2014). RCP8.5 is a high $CO_2$ emissions scenario, driven by strong reliance on fossil fuels, and high population growth. RCP4.5, instead, is an intermediate emissions scenario characterised by emissions reductions obtained with climate policies.

Projections of $T_{max}$ for each scenario are taken from Scoccimarro and Gualdi (2014) who downscaled simulations from the Coupled Models Intercomparison Project (CMIP5) using the Rossby Center Atmosphere (RCA4) regional model.

We demonstrate the development of the model in view of heatwave related mortality impacts for two cities in Spain: Madrid and Bilbao. Based on epidemiological studies from Díaz et al. (2015), the critical thresholds $T_{crit}$ above which mortality increases, are 34 °C for Madrid and 30 °C for Bilbao. For consistency with the epidemiological model, the analysis is conducted for the summer period (from 1 June to 30 September) and heatwave days outside that period are disregarded. Table 1 shows that this approach captures the majority of heatwave days (93%–99.6%).

**Table 1**
Heatwave days occurring within summer months (1 June to 30 September) and outside of summer months for the period of 2006–2100 (number of days).

| City | Madrid | | Bilbao | |
|---|---|---|---|---|
| Scenario | RCP 8.5 | RCP 4.5 | RCP 8.5 | RCP 4.5 |
| Summer | 3549 | 2761 | 960 | 900 |
| Other months | 54 | 11 | 67 | 34 |
| Total | 3603 | 2772 | 1027 | 934 |

### 2.2. Stochastic model requirements

The objective is to identify a stochastic model that is able to reproduce annual heatwave characteristics of the original $T_{max}$ time series. The characteristics we are interested in are the number of heatwaves in a given year, their mean duration in days and the mean exceedance above $T_{crit}$ in °C on heatwave days. The model allows to characterise annual degree-days. Degree-days refers to number of days in heatwave multiplied with their mean excess temperature and is widely used for impact models, such as in epidemiology or in the energy sector.

For each city and RCP scenario the model is calibrated with a time series of maximum daytime temperatures $T_{max}$.

Potential acclimatisation processes that imply an increasing threshold temperature $T_{crit}$ over time are not considered here. However, the model framework is general enough to allow the introduction of dynamic thresholds.

The calculations refer only to two climatic scenarios RCP 8.5 and RCP 4.5 for Madrid and Bilbao, but the model framework is general enough to be transferable to other climate scenarios and other cities.

### 2.3. Hypothesising a model

We consider the following three variables for which we want to reproduce time-dependent deterministic and stochastic components on an annual basis:

i) number of heatwaves,
ii) mean duration of heatwaves in days
iii) mean excess temperature (in °C) on heatwave days

For each variable a model is proposed based on first principles and on parsimony and then calibrated with the available time series. As heatwaves (i) occur as discrete random events, it is reasonable to represent their annual occurrence with a Poisson process. As the heatwave duration (ii) is a positive number of days we have chosen a Gamma distribution that allows to calibrate both mean and volatility of duration. Finally, given that the mean excess temperature (iii) must be characterised by a continuous distribution that cannot have negative values, a truncated Gaussian process is hypothesised for this distribution. We include a sensitivity analysis using a Gamma distribution for excess temperature. In addition, we consider that the expected values of the three processes can change over time. For all three processes we consider the possibility of an exponential increase. This choice is due to considerations of parsimony and a preliminary analysis of the time series obtained with the climate model data (Appendix A).

In accordance with the foregoing, the deterministic time-dependent components of the three processes are considered in the following way:

i) The number of annual heatwaves is estimated with Eq. (1) allowing for exponential increase over time:

$$\lambda(t) = \lambda(0)e^{\alpha t} \tag{1}$$

where $\lambda(0)$ is the expected number of heatwaves in year $t = 0$ and $\lambda(t)$ is the expected number of heatwaves in year $t$.

ii) The expected value of the annual mean duration of heatwaves is estimated in accordance with Eq. (2):

$$dur(t) = dur(0)e^{\gamma t} \tag{2}$$

where $dur(0)$ is the expected mean duration of heatwaves in the initial year $t = 0$ and $dur(t)$ is the expected mean duration in year $t$. The process allows for an exponential increase it time.

iii) The expected value of the mean excess temperature is estimated with Eq. (3):

$$temp(t) = temp(0)e^{\beta t} \tag{3}$$

where $temp(0)$ is the mean excess temperature expected during heatwave days in the initial year $t = 0$ and $temp(t)$ is the mean excess temperature expected in year $t$. This process also allows for an exponential increase in time.

In summary, the following processes are hypothesised:

i) A Poisson process that generates an annual number of heatwaves: $X_{1,t}$.
ii) A Gamma process that determines the mean annual duration of heatwaves: $X_{2,t}$.
iii) A truncated Gaussian process characterising mean annual excess temperature of heatwave days: $X_{3,t}$.

Potential correlations linking the three processes are considered as follows: ($\rho_{1,2}$) captures the potential correlation between the number of heatwaves per year (i) and mean duration (ii) while ($\rho_{2,3}$) captures the potential correlation between mean duration (ii) and mean excess temperature (iii).

Note that by definition, in the case of the Poisson processes hypothesised for i) the mean and variability are captured by the same parameter, whereas for the Gamma and Gaussian processes the volatilities $\sigma_G$ and $\sigma_N$ needs to be calibrated.

In the next step the model is calibrated with a daily $T_{max}$ time series.

### 2.4. Model calibration

The calibration of the model is done using nonlinear least squares. Detailed information on goodness-of-fit statistics and confidence intervals is provided in Appendix A and a summary of the results is shown in Table 2.

For most cases the coefficients are highly significant and the R-squared values are very high indicating the adequacy of models (see Appendix A).

The number of heatwaves per year, mean duration and mean excess temperature tend to increase exponentially over time, except for Bilbao under RCP4.5, where the exponential parameters α, β and γ are not significantly different from 0 (see Appendix A).

Note the negative correlation $\rho_{1,2}$ between the number of heatwaves and mean duration as well as the positive correlation $\rho_{2,3}$ between mean duration and mean excess temperature. $\rho_{1,2}$ can be expected to be negative due to the finite length of the heat wave season: The limits would be a single heat wave with duration of the entire heat wave season, whereas the other extreme would be every second day being a heatwave. $\rho_{2,3}$ is believed to be positive as heat wave intensity can be expected to increase with the number of consecutive heat wave days: we would not expect a one-day heatwave to be at extremely high temperatures whereas in a

**Table 2**
Calibrated model parameters.

| Parameter | Units | Madrid RCP 8.5 | Madrid RCP 4.5 | Bilbao RCP 8.5 | Bilbao RCP 4.5 |
|---|---|---|---|---|---|
| $\lambda(0)$ | (−) | 6.2894 | 6.5990 | 4.3355 | 5.8720 |
| $\alpha$ | $(y^{-1})$ | 0.0055 | 0.0028 | 0.0058 | −0.0005 |
| $dur(0)$ | (days) | 3.0640 | 3.2557 | 1.4925 | 1.8373 |
| $\gamma$ | $(y^{-1})$ | 0.0088 | 0.0042 | 0.0039 | −0.0021 |
| $temp(0)$ | (°C) | 1.5922 | 1.8662 | 2.5698 | 2.6408 |
| $\beta$ | $(y^{-1})$ | 0.0096 | 0.0036 | 0.0051 | 0.0024 |
| $\sigma_G$ | (days) | 2.1653 | 1.6383 | 1.3981 | 0.6738 |
| $\sigma_N$ | (°C) | 0.6159 | 0.6570 | 1.1477 | 1.1539 |
| $\rho_{1,2}$ | (−) | −0.2742 | −0.2676 | −0.0704 | −0.0412 |
| $\rho_{2,3}$ | (−) | 0.7001 | 0.5807 | 0.1945 | 0.2809 |

**Table 3**
Expected values of number of heatwaves (−), mean duration (days) and mean exceedance (°C) for selected years.

| Parameter | Year | Madrid RCP 8.5 | Madrid RCP 4.5 | Bilbao RCP 8.5 | Bilbao RCP 4.5 |
|---|---|---|---|---|---|
| $\lambda(t)$ | 2025 | 6.98 | 6.96 | 4.84 | 5.82 |
| | 2050 | 8.01 | 7.46 | 5.60 | 5.74 |
| | 2075 | 9.19 | 8.01 | 6.47 | 5.67 |
| | 2100 | 10.55 | 8.59 | 7.48 | 5.60 |
| $dur(t)$ | 2025 | 3.62 | 3.53 | 1.61 | 1.77 |
| | 2050 | 4.51 | 3.92 | 1.77 | 1.68 |
| | 2075 | 5.62 | 4.35 | 1.95 | 1.59 |
| | 2100 | 7.01 | 4.83 | 2.15 | 1.51 |
| $temp(t)$ | 2025 | 1.91 | 2.00 | 2.83 | 2.76 |
| | 2050 | 2.43 | 2.19 | 3.22 | 2.93 |
| | 2075 | 3.09 | 2.39 | 3.65 | 3.12 |
| | 2100 | 3.93 | 2.62 | 4.15 | 3.31 |

**Table 4**
Results of the Monte Carlo simulation expressed with means for number of heatwaves per year (−), duration (days) and excess temperature (°C), standard deviation of duration (days), standard deviation of excess temperature (°C) and correlation coefficients (−).

| Parameter | Year | Madrid RCP 8.5 | Madrid RCP 4.5 | Bilbao RCP 8.5 | Bilbao RCP 4.5 |
|---|---|---|---|---|---|
| $\lambda(t)$ | 2025 | 6.59 | 6.97 | 4.86 | 5.82 |
| | 2050 | 8.03 | 7.47 | 5.61 | 5.75 |
| | 2075 | 9.22 | 8.02 | 6.48 | 5.68 |
| | 2100 | 10.54 | 8.61 | 7.49 | 5.61 |
| $dur(t)$ | 2025 | 3.62 | 3.52 | 1.60 | 1.77 |
| | 2050 | 4.50 | 3.92 | 1.78 | 1.68 |
| | 2075 | 5.61 | 4.35 | 1.96 | 1.59 |
| | 2100 | 7.01 | 4.83 | 2.16 | 1.51 |
| $temp(t)$ | 2025 | 1.91 | 2.00 | 2.80 | 2.74 |
| | 2050 | 2.43 | 2.19 | 3.21 | 2.92 |
| | 2075 | 3.09 | 2.39 | 3.65 | 3.11 |
| | 2100 | 3.93 | 2.62 | 4.15 | 3.30 |
| $\sigma_G$ | 2025 | 2.18 | 1.64 | 1.39 | 0.67 |
| | 2050 | 2.17 | 1.64 | 1.40 | 0.67 |
| | 2075 | 2.17 | 1.63 | 1.40 | 0.67 |
| | 2100 | 2.17 | 1.64 | 1.40 | 0.67 |
| $\sigma_N$ | 2025 | 0.62 | 0.66 | 1.18 | 1.19 |
| | 2050 | 0.61 | 0.66 | 1.16 | 1.18 |
| | 2075 | 0.62 | 0.65 | 1.15 | 1.17 |
| | 2100 | 0.62 | 0.66 | 1.15 | 1.16 |
| $\rho_{1,2}$ | 2025 | −0.28 | −0.27 | −0.07 | −0.04 |
| | 2050 | −0.28 | −0.27 | −0.07 | −0.04 |
| | 2075 | −0.28 | −0.27 | −0.07 | −0.04 |
| | 2100 | −0.28 | −0.27 | −0.07 | −0.04 |
| $\rho_{2,3}$ | 2025 | 0.68 | 0.56 | 0.19 | 0.28 |
| | 2050 | 0.67 | 0.56 | 0.20 | 0.28 |
| | 2075 | 0.67 | 0.56 | 0.21 | 0.29 |
| | 2100 | 0.67 | 0.56 | 0.20 | 0.28 |

**Table 5**
Mean excess temperature of heatwave days (°C). Mean, 95th percentile and mean of the 5% worst cases (ES (95%)) for selected years.

| Year | Madrid RCP 8.5 (°C) | | | Madrid RCP 4.5 (°C) | | |
|---|---|---|---|---|---|---|
| | Mean | 95th percentile | ES (95%) | Mean | 95th percentile | ES (95%) |
| 2025 | 1.91 | 2.99 | 3.36 | 2.00 | 3.12 | 3.45 |
| 2050 | 2.43 | 3.47 | 3.82 | 2.19 | 3.30 | 3.62 |
| 2075 | 3.09 | 4.14 | 4.45 | 2.39 | 3.49 | 3.80 |
| 2100 | 3.93 | 4.97 | 5.26 | 2.62 | 3.73 | 4.04 |
| Year | Bilbao RCP 8.5 (°C) | | | Bilbao RCP 4.5 (°C) | | |
| | Mean | 95th percentile | ES (95%) | Mean | 95th percentile | ES (95%) |
| 2025 | 2.80 | 4.78 | 5.26 | 2.74 | 4.74 | 5.25 |
| 2050 | 3.21 | 5.14 | 5.63 | 2.92 | 4.89 | 5.39 |
| 2075 | 3.65 | 5.56 | 6.05 | 3.11 | 5.06 | 5.56 |
| 2100 | 4.15 | 6.04 | 6.53 | 3.30 | 5.24 | 5.73 |

heatwave of long duration we may expect very high temperatures during some of the days.

Based on the parameter values in Table 2, the expected values for the three processes are shown in Table 3 for selected years. For example, in the case of Madrid, 10.55 heatwaves are expected in the year 2100 under scenario RCP 8.5, with an expected mean duration of 7.01 days and an expected mean excess temperature of 3.93 °C on heatwave days, equivalent to a mean $T_{max}$ of 37.93 °C during heatwave days. The expected values have to be interpreted consistently with the results presented in Table 2 and Appendix Tables A.1–A.4. For example, the reported change over time among expected values for Bilbao RCP4.5 reflects coefficients not statistically significantly different from 0, so that no trend in number, duration and intensity of heatwaves can be realistically expected in this case. In all other cases, expected values show an increase in frequency, mean duration and mean excess temperature over time, which tends to be more pronounced for RCP8.5 compared to RCP4.5.

### 2.5. Monte Carlo simulation

Because of the complex relationships between the three stochastic processes with their variability and correlations and the very low computational cost of a single simulation, we decided to use the Monte Carlo methodology to compute the risk measures with a high accuracy. For a general introduction to Monte Carlo simulations in the context of stochastic diffusion models we refer the reader to Abadie and Chamorro (2013).

50,000 Monte Carlo simulations were performed for selected years (2025, 2050, 2075, 2100). In a first step, each simulation for each year generates the number of heatwaves.

With the Poisson distribution, the probability of observing $k$ events in an interval is given by Eq. (4).

$$e^{-\lambda}\frac{\lambda^k}{k!} \tag{4}$$

There is a slight probability that there may not be any extreme events ($k=0$) in a given year. This is expressed by Eq. (5):

$$f(0,\lambda(t)) = e^{-\lambda(t)} \tag{5}$$

Therefore, in the case of Bilbao in the year 2100, under scenario RCP 8.5 there is a 0.058% probability that there will be no heatwaves.

The random values of annual number of heatwaves have been generated from a Poisson distribution using the parameters from Eq. (1) for each year. Similarly the values generated by Eqs. (2) and (3) together with volatilities and correlations are used to generate random samples for duration and excess temperature. Truncation and correlations are accounted for as described in Section 2.5.1.

#### 2.5.1. Truncation and correlation

The excess temperature must always be greater or equal to zero.

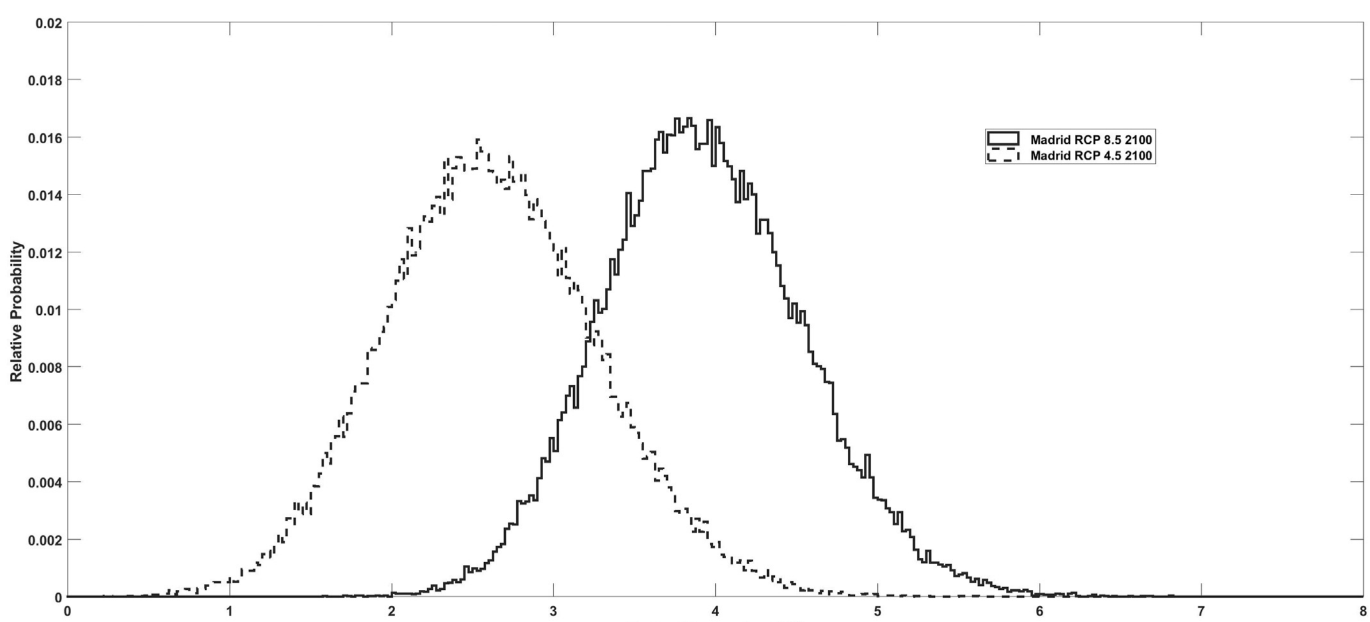


**Fig. 1.** Distribution of mean excess temperature in Madrid in 2100 over 34 °C under scenarios RCP 8.5 and RCP 4.5.

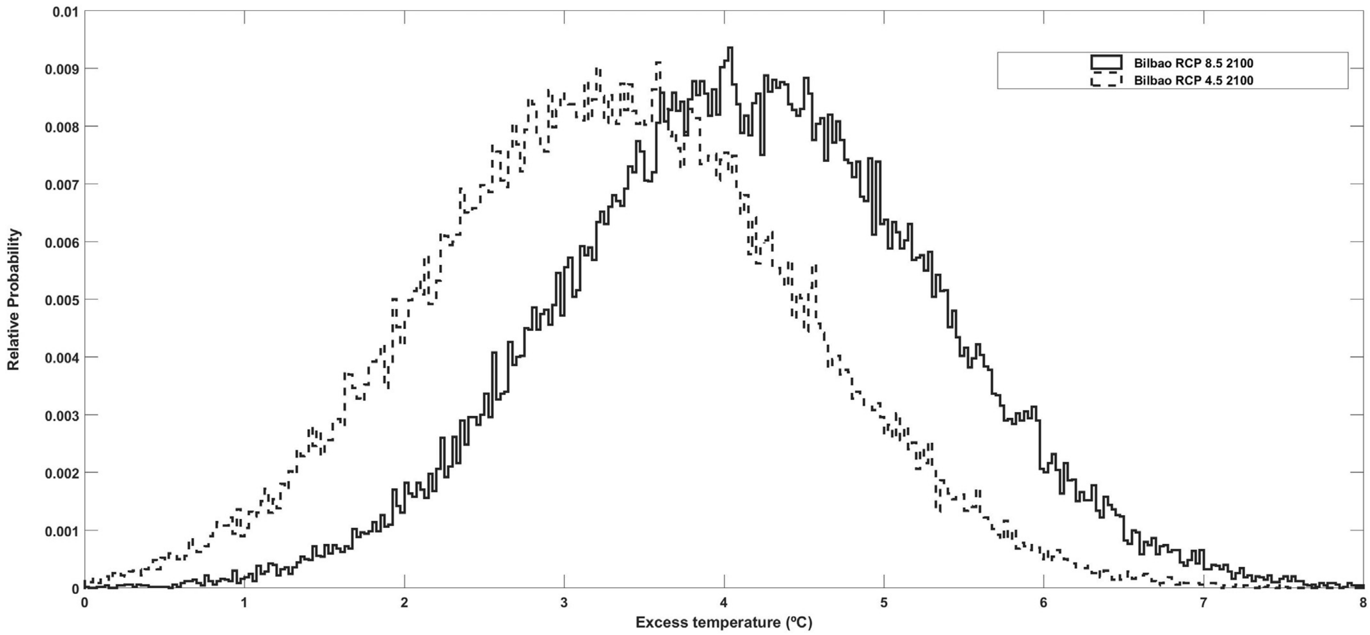


**Fig. 2.** Distribution of mean excess temperature in Bilbao in 2100 over 30 °C under scenarios RCP 8.5 and RCP 4.5.

Mean excess temperature is obtained from a zero-truncated normal distribution correlated with mean duration. See Johnson et al. (2005) for the corresponding equations to obtain the parameters for the non-truncated distribution.

Correlation is simulated by obtaining samples $v_2$ (normalised mean duration) and $v_3$ (normalised mean excess temperature) using Eq. (6):

$$v_2 = x_2; v_3 = \rho_{2,3}x_2 + x_3\sqrt{1-(\rho_{2,3})^2} \tag{6}$$

where $x_2$ and $x_3$ are two independent samples of duration and excess temperature which are normalised in advance by subtracting the means of their distribution and dividing by the standard deviation of that distribution. Following the operation with Eq. (6) the samples $v_2$ and $v_3$ are transformed back to the original probability space to obtain variables with the original mean and standard deviation, now including the appropriate correlation.

In a similar way, we generate random samples to account for the correlation between the number of heatwaves and duration.

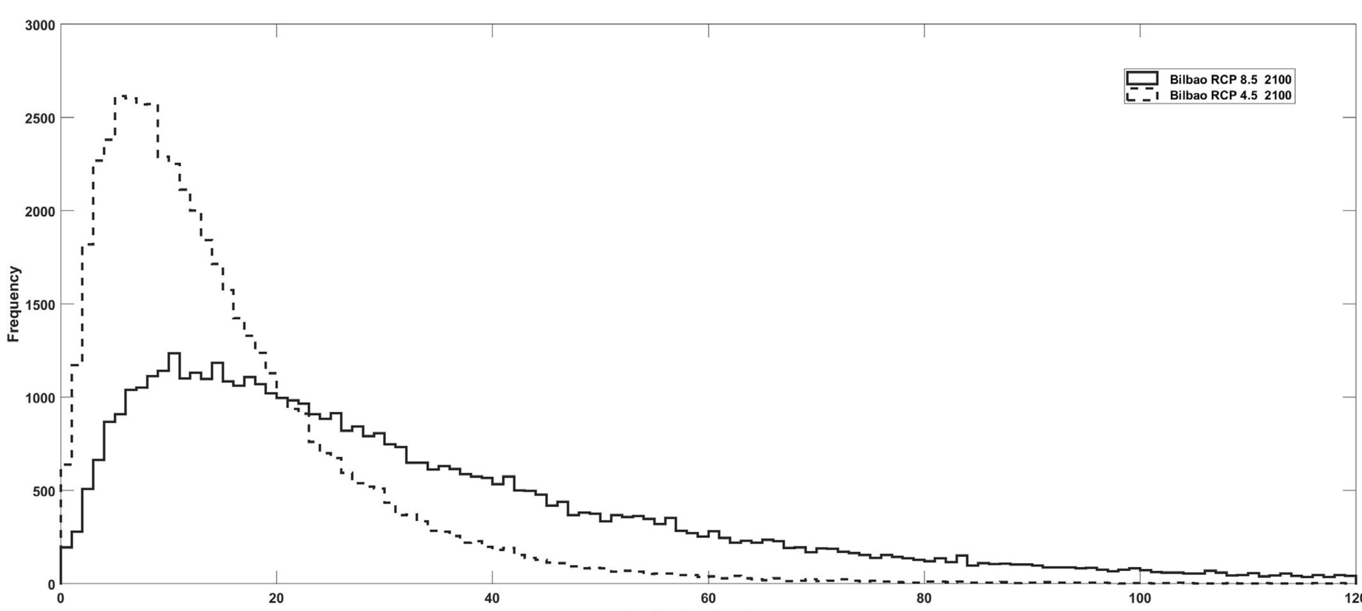


**Fig. 3.** Distribution of number of deaths from heatwaves in Bilbao in 2100 under scenarios RCP 8.5 and RCP 4.5.

### 2.5.2. Convergence

Table 4 shows the sample means of the Monte Carlo simulation for numbers of events, mean duration, mean excess temperature, volatility and correlations. As can be seen by comparison with Table 3, convergence has been achieved: the differences between the theoretical values of the parameters and those obtained via the Monte Carlo simulations are minimal.

## 3. Results and discussion

In this section we analyse the resulting distributions of the annual statistics of heatwaves and calculate risk metrics for mean excess temperature and for mortality.

### 3.1. Excess temperature risk

The results for expected mean excess temperature during heatwave days, the 95th percentile (Value at Risk VaR (95%)) and the mean for the 5% of worst cases (Expected Shortfall ES (95%)) are shown in Table 5.

According to the model, in Madrid in 2100, under scenario RCP8.5 there is a 5% probability that the mean annual excess temperature of heatwave days will exceed 38.97 °C (4.97 °C + 34.00 °C). For the same scenario, the mean for the 5% worst cases would be 39.26 °C (5.26 °C + 34.00 °C). Excess temperature risk tends to increase over time for all scenarios, although the magnitude of increase is more severe under scenario RCP8.5 than under scenario RCP4.5. Results for Bilbao show higher values for expected excess temperature risks for all selected years. For RCP 8.5 and RCP 4.5 we visualise the distributions of excess temperature in 2100 for the two cities as obtained with the Monte Carlo simulations (Figs. 1–2).

### 3.2. Climate change impact assessment: mortality risk

In this section we illustrate the use of the model for mortality risk projections using an epidemiological equation (Eq. (7)) where annual heatwave-related mortality is proportional to the degree-days (Díaz et al., 2015). We obtain the annual number of heatwave days by multiplying the number of heatwave events ($X_{1,t}$) with the mean duration of heatwaves ($X_{2,t}$). The heatwave days are then multiplied by the mean excess temperature ($X_{1,t}$) to obtain the degree days: ($X_{1,t} * X_{2,t} * X_{3,t}$).

**Table 6**
Heatwave-related mortality risk projections for Madrid and Bilbao under RCP 4.5 and RCP 8.5 (number of deaths per year). Mean, 95th percentile and mean of the 5% worst cases (ES (95%)) for selected years.

| Year | Madrid RCP 8.5 | | | Madrid RCP 4.5 | | |
|---|---|---|---|---|---|---|
| | Mean | 95th percentile | ES (95%) | Mean | 95th percentile | ES (95%) |
| 2025 | 194.66 | 561.93 | 821.42 | 191.53 | 494.35 | 677.31 |
| 2050 | 341.48 | 854.09 | 1159.62 | 248.25 | 602.02 | 803.58 |
| 2075 | 610.62 | 1346.04 | 1730.07 | 320.07 | 732.90 | 956.37 |
| 2100 | 1096.86 | 2157.52 | 2631.64 | 416.18 | 905.77 | 1154.18 |
| **Year** | **Bilbao RCP 8.5** | | | **Bilbao RCP 4.5** | | |
| | Mean | 95th percentile | ES (95%) | Mean | 95th percentile | ES (95%) |
| 2025 | 11.87 | 40.46 | 62.01 | 15.38 | 39.77 | 52.91 |
| 2050 | 17.30 | 54.52 | 81.62 | 15.35 | 39.72 | 52.79 |
| 2075 | 25.01 | 73.45 | 105.47 | 15.27 | 39.47 | 52.93 |
| 2100 | 35.88 | 98.05 | 135.77 | 15.21 | 39.33 | 52.94 |

Note: We intentionally do not correct for population growth in order to only account for mortality changes due to changes in heatwave dynamics (The corresponding population numbers are Madrid: 2.88 million, and Bilbao: 0.354 million).

**Table 7**
Number of annual heatwave-related deaths per 10,000 inhabitants. Mean, 95th percentile and mean of the 5% worst cases (ES (95%)) for selected years.

| Year | Madrid RCP 8.5 | | | Madrid RCP 4.5 | | |
|---|---|---|---|---|---|---|
| | Mean | 95th percentile | ES (95%) | Mean | 95th percentile | ES (95%) |
| 2025 | 0.68 | 1.95 | 2.85 | 0.67 | 1.72 | 2.35 |
| 2050 | 1.19 | 2.97 | 4.03 | 0.86 | 2.09 | 2.79 |
| 2075 | 2.12 | 4.67 | 6.01 | 1.11 | 2.54 | 3.32 |
| 2100 | 3.81 | 7.49 | 9.14 | 1.45 | 3.15 | 4.01 |
| **Year** | **Bilbao RCP 8.5** | | | **Bilbao RCP 4.5** | | |
| | Mean | 95th percentile | ES (95%) | Mean | 95th percentile | ES (95%) |
| 2025 | 0.34 | 1.14 | 1.75 | 0.43 | 1.12 | 1.49 |
| 2050 | 0.49 | 1.54 | 2.31 | 0.43 | 1.12 | 1.49 |
| 2075 | 0.71 | 2.07 | 2.98 | 0.43 | 1.12 | 1.50 |
| 2100 | 1.01 | 2.77 | 3.84 | 0.43 | 1.11 | 1.50 |

We use the epidemiological parameters from the site-specific studies (Linares et al., 2014; Díaz et al., 2015).

$$M_t = RM \times B \times \{X_{1,t} \times X_{2,t} \times X_{3,t}\} \quad (7)$$

where:

$M_t$ is the heatwave-related mortality in the year $t$ (deaths).
$RM$ is the heatwave mortality risk (°C$^{-1}$) expressed as mortality increase per 1 °C temperature increase above the threshold temperature $T_{crit}$. (Madrid: $T_{crit}$ = 34 °C with RM = 6.54% = 0.0654; Bilbao: $T_{crit}$ = 30 °C with RM = 5.66% = 0.0566).
$B$ is the background daily mortality rate (deaths per day) for death from natural causes during heatwave days (June–September) established for the period 2000–2008 (Madrid: B = 57.5, Bilbao: B = 9.26). We intentionally do not correct for population growth in order to only account for mortality changes due to changes in heatwave dynamics. The corresponding population numbers are for the year 2000: Madrid (2.88 million) and Bilbao (0.354 million).
$X_{1,\,t}$ is the number of heatwaves in year $t$ (−).
$X_{2,\,t}$ is the mean duration of heatwaves in year $t$ (days).
$X_{3,\,t}$ is the mean excess temperature on heatwave days in year $t$ (°C).

Applying this calculation (Eq. (7)) for each Monte Carlo simulation we obtain a distribution of annual mortality $M_t$. We display the distributions for Bilbao for RCP4.5 and RCP8.5 (Fig. 3). We then calculate the annual means and the risk measures: 95th percentile and Expected Shortfall ES (95%) (see Table 6).

The simulations indicate that in Bilbao in 2100, under scenario RCP 8.5, a mean of 36 deaths is expected, and there is a 5% probability that the number of deaths could exceed 98.05. The average of the 5% worst case simulations would be 135.77. Depending on the scenario and the city we find the Expected Shortfall ES (95%) to be around two to four times the mean value. This information on worst case scenarios can help in setting up appropriate adaptation plans. For Bilbao, for the year 2100, the distributions are visualised for RCP 4.5 and 8.5 in Fig. 3.

To improve comparability between the two cities we include information on the annual heatwave related deaths per 10,000 inhabitants using the population numbers above (Table 7).

We additionally include a summary of the results of the sensitivity analysis using a Gamma distribution for the mean excess temperature with the same mean and volatility as the original data. The Gamma distribution is only slightly different to the one obtained with the truncated normal distribution (Appendix B, Fig. B.1). When predicting mortality this small difference is further diluted as mortality is a function of all of the three processes involved. The final difference in projected mortality turns out to be negligible (Appendix B, Table B.1).

### 3.3. Generalization and transferability of the approach

The stochastic diffusion models allow capturing increasing trends and volatility in a very versatile manner, including correlations between the involved processes. This could open up a much wider set of uses for analysing climatic time series.

The methodology developed for heatwaves can be transferred to other situations, such as other geographic locations and other temperature thresholds (e.g. climatic thresholds instead of epidemiological thresholds). It can be directly applied to study heatwaves characterised by high wet-bulb temperatures (Im et al., 2017) or to study cold-waves. The method is extendable to include time-changing thresholds to account for acclimatisation processes. Instead of heatwave impacts on health, the approach could be adapted to characterise low-probability high-impact events in other sectors such as energy or agriculture.

## 4. Conclusions

A single long-term (>90 years) high resolution time series (e.g. maximum daily temperature) can be used to hypothesise a generic stochastic model able to reproduce evolution of trends and volatility of annual heatwave characteristics.

The model couples three stochastic processes characterising the annual number of heatwaves (Poisson process), their mean duration (Gamma process) as well as the mean excess temperature on heatwave days (truncated Gaussian process). The calibration reveals a positive correlation between mean annual heatwave duration and excess temperature and a negative correlation between mean duration and annual number of heatwaves.

We illustrate the versatility of such a model in view of risk assessments of heatwave-related mortality projections using epidemiological models. We show how calculating the associated risk metrics such as Value at Risk and Expected Shortfall can provide valuable information for decision making in climate change adaptation.


## Acknowledgements

Luis M. Abadie is grateful for financial support received from the Basque Government via project GIC12/177-IT-399-13 and also from the Science and Innovation Ministry of Spain (ECO2015-68023).

Marc B. Neumann acknowledges financial support from the Ramón y Cajal Research Fellowship of the Ministry of Economy and Competitiveness of Spain (no. RYC-2013-13628).


## Appendix A. Calibration results

**Table A.1**
Madrid RCP 8.5 calibration.

| Number of heatwaves per year | | | | | | |
|---|---|---|---|---|---|---|
| Number of obs | | | | | | 95 |
| R-squared | | | | | | 0.9174 |
| Adj R-squared | | | | | | 0.9157 |
| Root MSE | | | | | | 2.524696 |
| Res. dev. | | | | | | 443.5399 |
| Parameter | Coefficient | Std. Err. | t | P > \|t\| | [95% Conf. Interval] | |
| λ(0) | 6.289359 | 0.448781 | 14.01 | 0 | 5.398168 | 7.180549 |
| α | 0.005493 | 0.001165 | 4.71 | 0 | 0.003179 | 0.007807 |
| Duration of heatwaves (days) | | | | | | |
| Number of obs | | | | | | 95 |
| R-squared | | | | | | 0.8383 |
| Adj R-squared | | | | | | 0.8348 |
| Root MSE | | | | | | 2.176978 |
| Res. dev. | | | | | | 415.3851 |
| Parameter | Coefficient | Std. Err. | t | P > \|t\| | [95% Conf. Interval] | |
| *dur*(0) | 3.064066 | 0.353374 | 8.67 | 0 | 2.362335 | 3.765796 |
| γ | 0.008795 | 0.001776 | 4.95 | 0 | 0.005268 | 0.012322 |
| Temperature excess over 34 °C | | | | | | |
| Number of obs | | | | | | 95 |
| R-squared | | | | | | 0.9498 |
| Adj R-squared | | | | | | 0.9488 |
| Root MSE | | | | | | 0.6192655 |
| Res. dev. | | | | | | 176.5249 |
| Parameter | Coefficient | Std. Err. | t | P > \|t\| | [95% Conf. Interval] | |
| temp(0) | 1.592232 | 0.098329 | 16.19 | 0 | 1.39697 | 1.787493 |
| β | 0.009556 | 0.00094 | 10.17 | 0 | 0.007691 | 0.011422 |

Volatility of excess temperature = 0.6159.
Volatility of duration heatwaves = 2.1653.
Correlation of duration and excess temperature = 0.7001.
Correlation of number of heatwaves and duration = −0.2742.
Note: In this case all the parameters values are significantly different from zero.

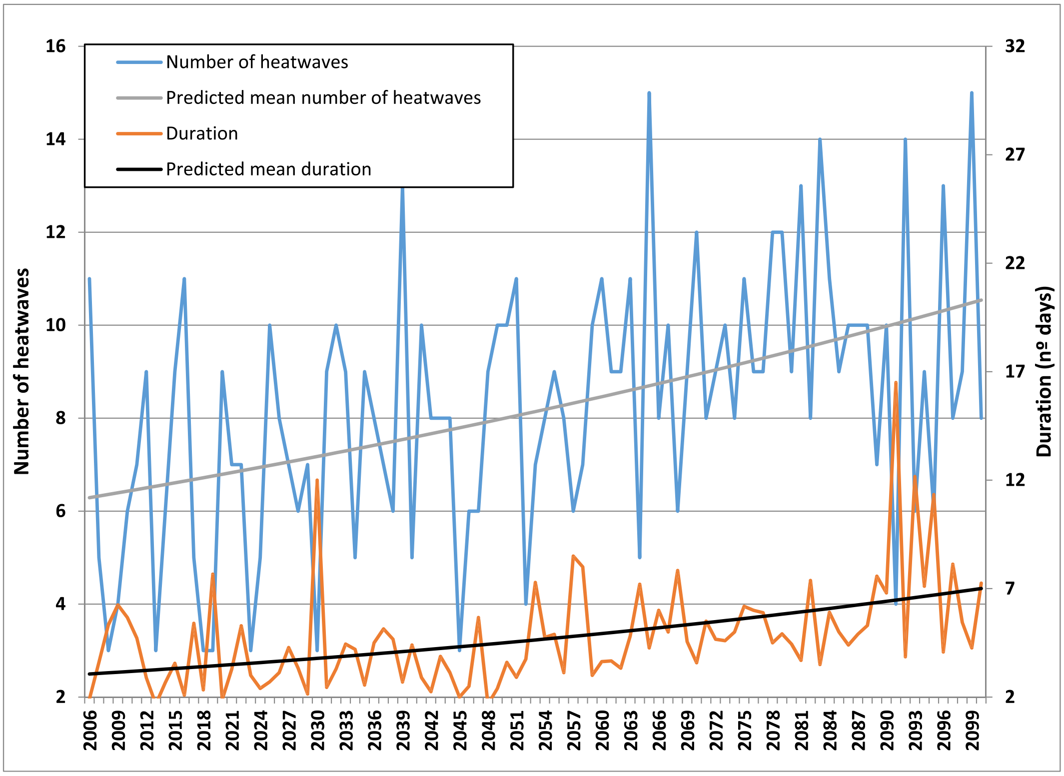


**Fig. A.1.** Number of heatwaves and duration in Madrid under scenario RCP 8.5.

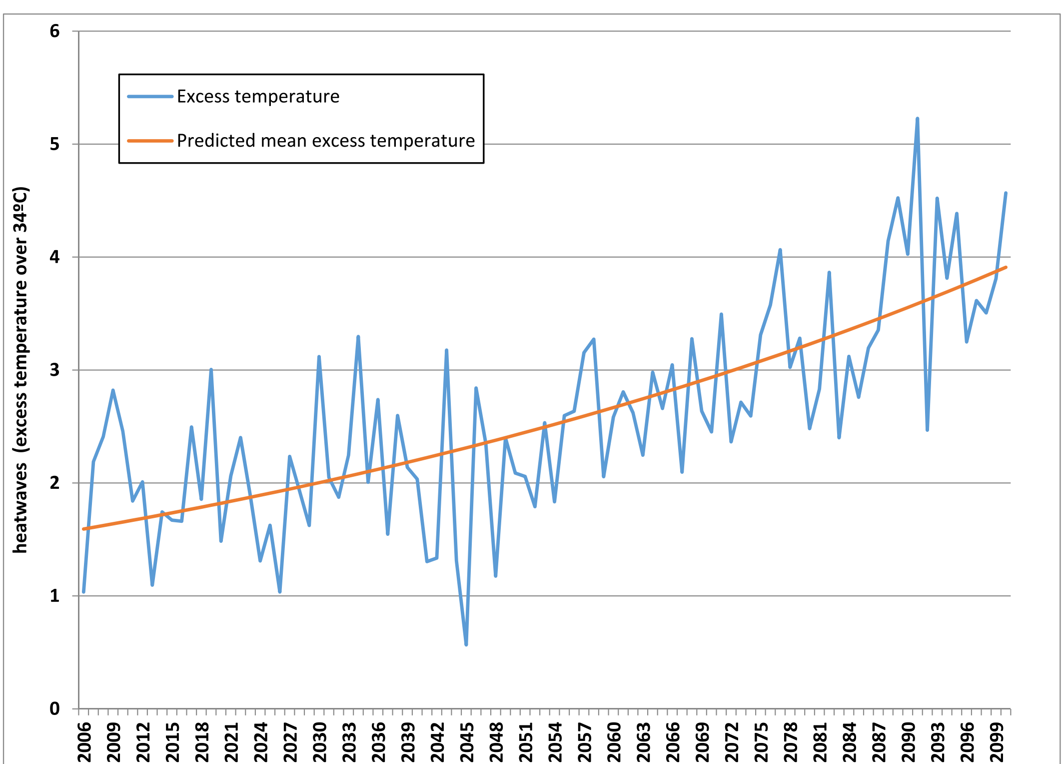


**Fig. A.2.** Excess temperature in Madrid over 34° C under scenario RCP 8.5.

**Table A.2**
Madrid RCP 4.5 Calibration.

| Number of heatwaves per year | | | | | | |
|---|---|---|---|---|---|---|
| Number of obs | | | | | | 95 |
| R-squared | | | | | | 0.9125 |
| Adj R-squared | | | | | | 0.9106 |
| Root MSE | | | | | | 2.372549 |
| Res. dev. | | | | | | 431.7303 |
| Parameter | Coefficient | Std. Err. | t | P > \|t\| | [95% Conf. Interval] | |
| $\lambda(0)$ | 6.598955 | 0.451464 | 14.62 | 0 | 5.702437 | 7.495473 |
| $\alpha$ | 0.002826 | 0.001179 | 2.4 | 0.019 | 0.000484 | 0.005168 |
| Duration of heatwaves (days) | | | | | | |
| Number of obs | | | | | | 95 |
| R-squared | | | | | | 0.8588 |
| Adj R-squared | | | | | | 0.8558 |
| Root MSE | | | | | | 1.647124 |
| Res. dev. | | | | | | 362.3928 |
| Parameter | Coefficient | Std. Err. | t | P > \|t\| | [95% Conf. Interval] | |
| $dur(0)$ | 3.255747 | 0.30275 | 10.75 | 0 | 2.654546 | 3.856948 |
| $\gamma$ | 0.004204 | 0.001557 | 2.7 | 0.008 | 0.001112 | 0.007297 |
| Temperature excess over 34 °C | | | | | | |
| Number of obs | | | | | | 95 |
| R-squared | | | | | | 0.9211 |
| Adj R-squared | | | | | | 0.9194 |
| Root MSE | | | | | | 0.6605721 |
| Res. dev. | | | | | | 188.7937 |
| Parameter | Coefficient | Std. Err. | t | P > \|t\| | [95% Conf. Interval] | |
| temp(0) | 1.866188 | 0.123258 | 15.14 | 0 | 1.621422 | 2.110954 |
| $\beta$ | 0.003611 | 0.00112 | 3.22 | 0.002 | 0.001388 | 0.005835 |

Volatility of excess temperature = 0.6570.
Volatility of duration heatwaves = 1.6383.
Correlation of duration and excess temperature = 0.5807.
Correlation of number of heatwaves and duration = −0.2676.
Note: In this case all the parameters values are significantly different from zero.

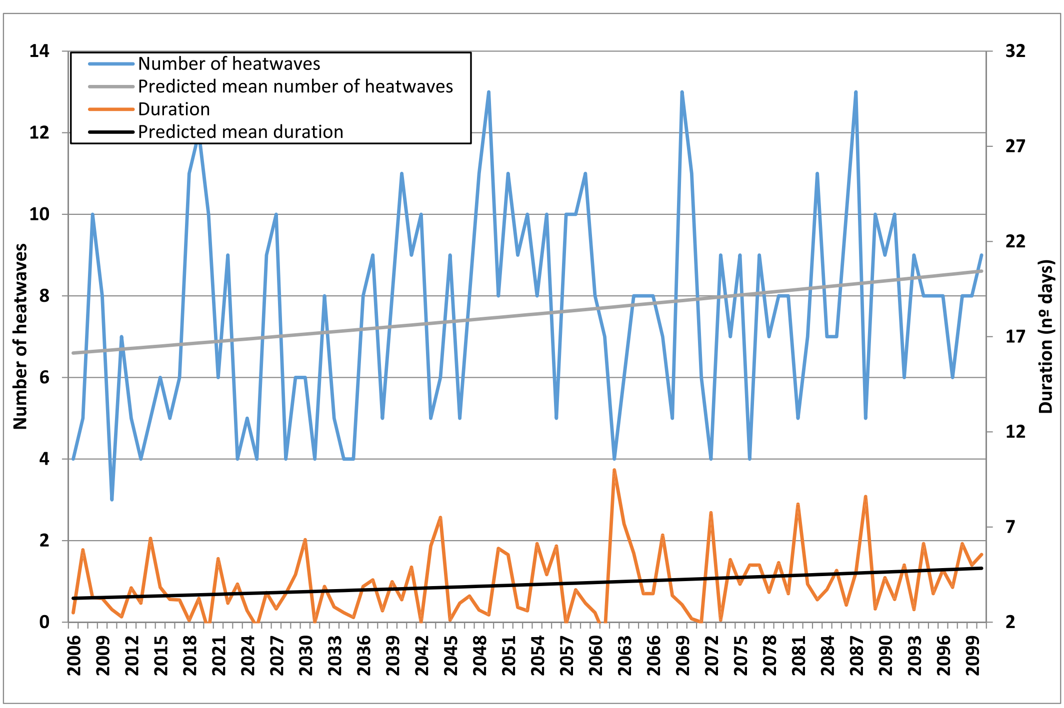


**Fig. A.3.** Number of heatwaves and duration in Madrid under scenario RCP 4.5.

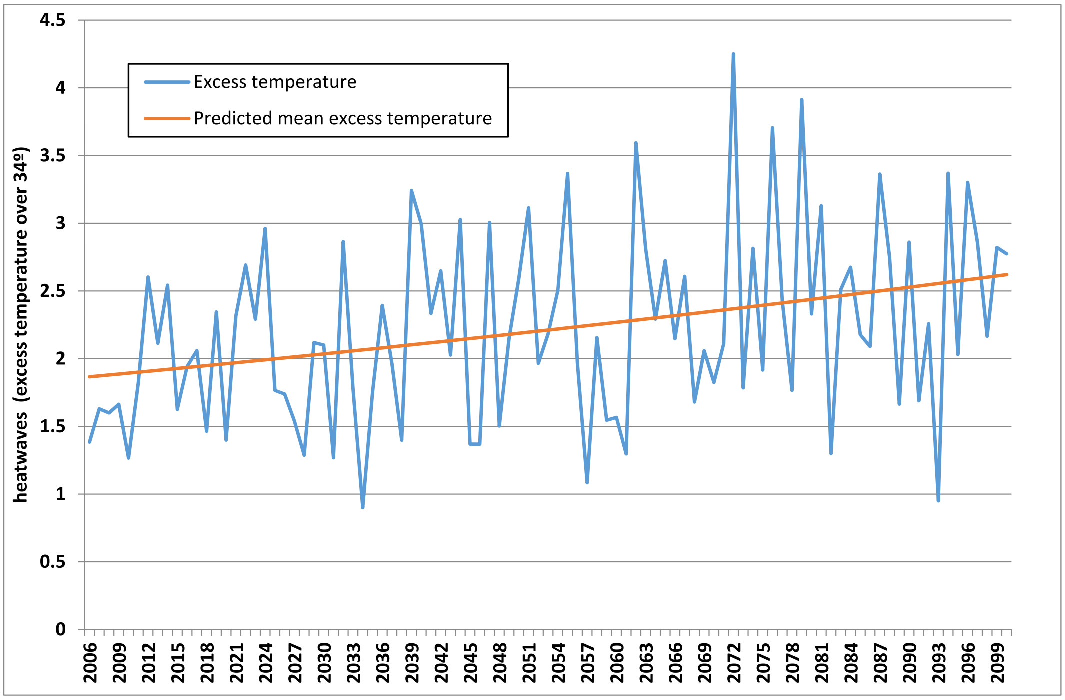


**Fig. A.4.** Excess temperature in Madrid over 34° C under scenario RCP 4.5.

**Table A.3**
Bilbao RCP 8.5 calibration.

| Number of heatwaves per year | | | | | | |
|---|---|---|---|---|---|---|
| Number of obs | | | | | | 95 |
| R-squared | | | | | | 0.8382 |
| Adj R-squared | | | | | | 0.8347 |
| Root MSE | | | | | | 2.587336 |
| Res. dev. | | | | | | 448.1964 |
| Parameter | Coefficient | Std. Err. | t | P > \|t\| | [95% Conf. Interval] | |
| λ(0) | 4.335485 | 0.456661 | 9.49 | 0 | 3.428647 | 5.242324 |
| α | 0.005761 | 0.001711 | 3.37 | 0.001 | 0.002363 | 0.009159 |
| Duration of heatwaves (days) | | | | | | |
| Number of obs | | | | | | 95 |
| R-squared | | | | | | 0.6294 |
| Adj R-squared | | | | | | 0.6215 |
| Root MSE | | | | | | 1.405602 |
| Res. dev. | | | | | | 332.2655 |
| Parameter | Coefficient | Std. Err. | t | P > \|t\| | [95% Conf. Interval] | |
| *dur*(0) | 1.492538 | 0.260421 | 5.73 | 0 | 0.975394 | 2.009683 |
| γ | 0.003892 | 0.002941 | 1.32 | 0.189 | −0.00195 | 0.009732 |
| Temperature excess over 30 °C | | | | | | |
| Number of obs | | | | | | 95 |
| R-squared | | | | | | 0.8943 |
| Adj R-squared | | | | | | 0.8923 |
| Root MSE | | | | | | 1.153854 |
| Res. dev. | | | | | | 294.7674 |
| Parameter | Coefficient | Std. Err. | t | P > \|t\| | [95% Conf. Interval] | |
| temp(0) | 2.569776 | 0.207341 | 12.39 | 0 | 2.158037 | 2.981515 |
| β | 0.00508 | 0.001328 | 3.82 | 0 | 0.002442 | 0.007717 |

Volatility of excess temperature = 1.1477.
Volatility of duration heatwaves = 1.3981.
Correlation of duration and excess temperature = 0.1945.
Correlation of number of heatwaves and duration = −0.0704.
Note: In this case all the parameters values with the exception of γ are significantly different from zero.

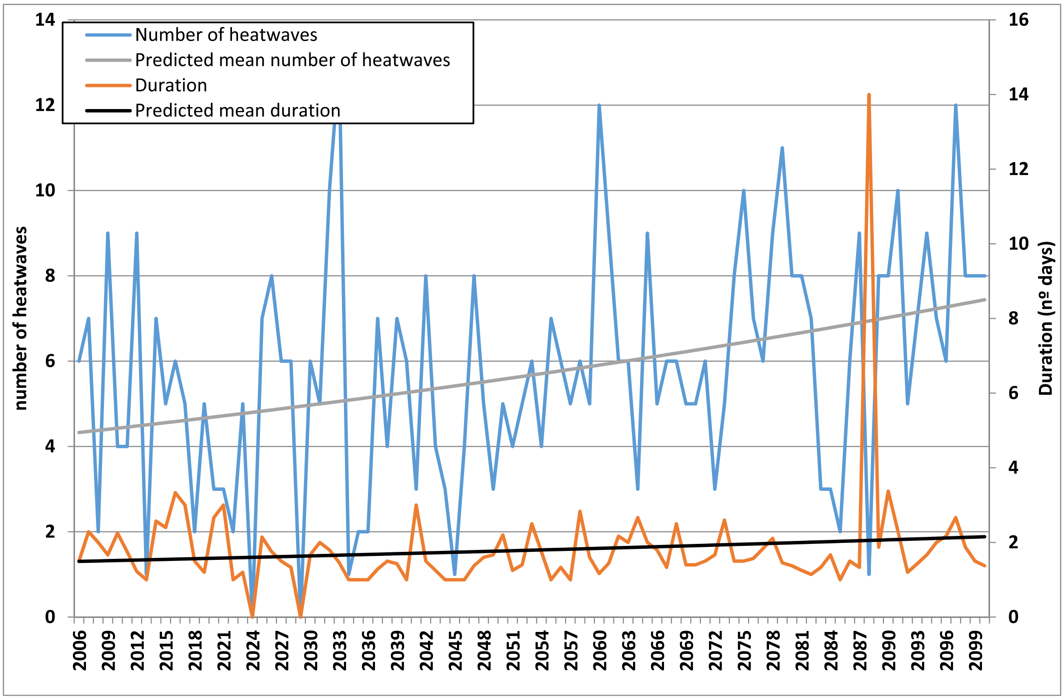


**Fig. A.5.** Number of heatwaves and duration in Bilbao under scenario RCP 8.5.

The spike in Fig. A.5 corresponds to the original data from Scoccimarro and Gualdi (2014) and has been generated by their climate model. It represents a year where just one heatwave episode occurs that lasts 14 days.

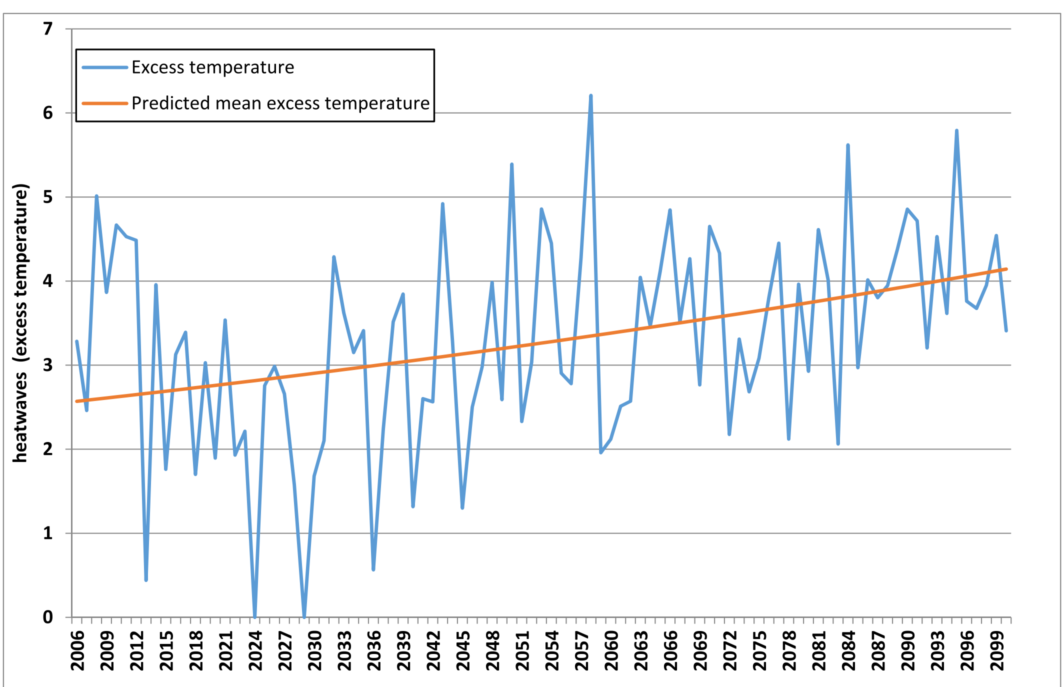


**Fig. A.6.** Excess temperature in Bilbao over 30 °C under scenario RCP 8.5.

**Table A.4**
Bilbao RCP 4.5 Calibration.

| Number of heatwaves per year | | | | | | |
|---|---|---|---|---|---|---|
| Number of obs | | | | | | 95 |
| R-squared | | | | | | 0.8203 |
| Adj R-squared | | | | | | 0.8165 |
| Root MSE | | | | | | 2.718816 |
| Res. dev. | | | | | | 457.6143 |
| Parameter | Coefficient | Std. Err. | t | P > \|t\| | [95% Conf. Interval] | |
| λ(0) | 5.872042 | 0.559381 | 10.5 | 0 | 4.761223 | 6.982861 |
| α | −0.00046 | 0.00177 | −0.26 | 0.796 | −0.00397 | 0.003057 |
| Duration of heatwaves (days) | | | | | | |
| Number of obs | | | | | | 95 |
| R-squared | | | | | | 0.8615 |
| Adj R-squared | | | | | | 0.8585 |
| Root MSE | | | | | | 0.6773774 |
| Res. dev. | | | | | | 193.5669 |
| Parameter | Coefficient | Std. Err. | t | P > \|t\| | [95% Conf. Interval] | |
| *dur*(0) | 1.837252 | 0.144545 | 12.71 | 0 | 1.550215 | 2.12429 |
| γ | −0.00208 | 0.001522 | −1.37 | 0.175 | −0.0051 | 0.000941 |
| Temperature excess over 30 °C | | | | | | |
| Number of obs | | | | | | 95 |
| R-squared | | | | | | 0.8697 |
| Adj R-squared | | | | | | 0.8669 |
| Root MSE | | | | | | 1.160055 |
| Res. dev. | | | | | | 295.7858 |
| Parameter | Coefficient | Std. Err. | t | P > \|t\| | [95% Conf. Interval] | |
| temp(0) | 2.640783 | 0.223204 | 11.83 | 0 | 2.197544 | 3.084022 |
| β | 0.002375 | 0.001471 | 1.61 | 0.11 | −0.00055 | 0.005296 |

Volatility of excess temperature = 1.1539.
Volatility of duration heatwaves = 0.6738.
Correlation of duration and excess temperature = 0.2809.
Correlation of number of heatwaves and duration = −0.0412.
Note: In this case all the exponential parameters α, γ and β are not significantly different from zero.

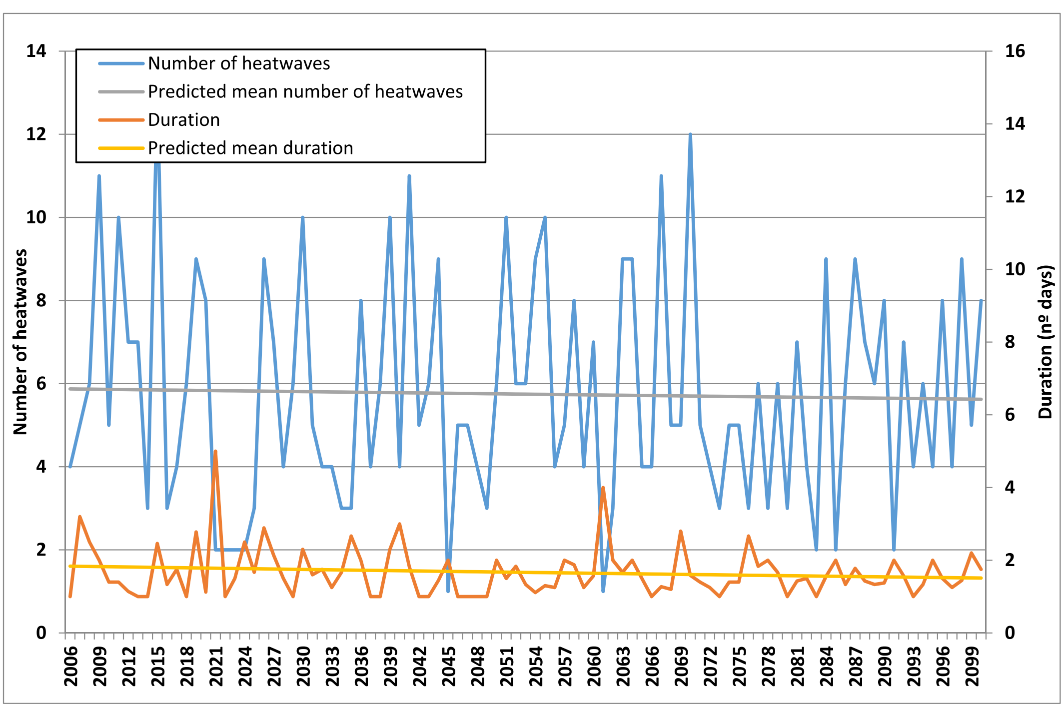


**Fig. A.7.** Number of heatwaves and duration in Bilbao under scenario RCP 4.5.

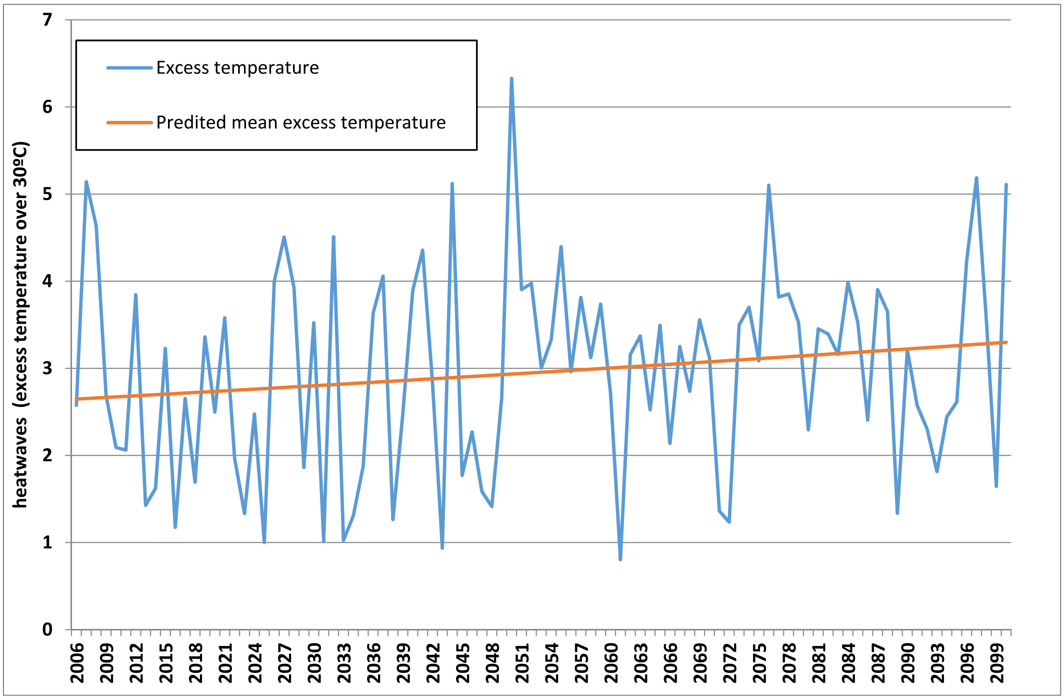


**Fig. A.8.** Excess temperature in Bilbao over 30 °C under scenario RCP 4.5.

## Appendix B. Sensitivity analysis with Gamma distribution for excess temperature

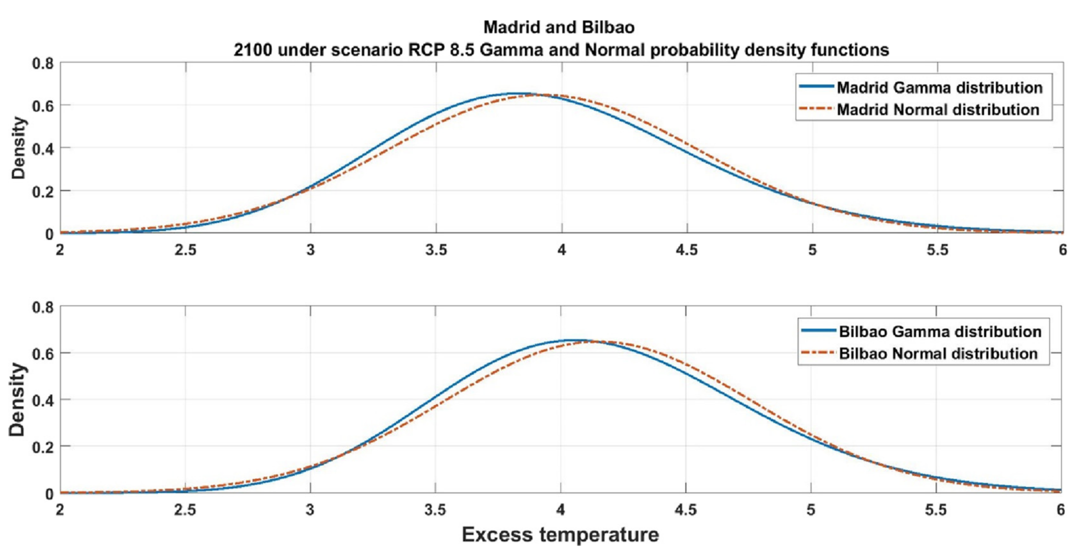


**Fig. B.1.** Excess temperature distribution in 2100 for Madrid and Bilbao using normal truncated and Gamma distributions.

**Table B.1**
Absolute difference in number of deaths when using the Gamma distribution instead of the truncated normal distribution for excess temperature (number of deaths).

| Year | Madrid RCP 8.5 (no. deaths) | | | Madrid RCP 4.5 (no. deaths) | | |
|---|---|---|---|---|---|---|
| | Mean | 95th | ES (95%) | Mean | 95th | ES (95%) |
| 2025 | −0.71 | −3.90 | −7.65 | −0.34 | 0.50 | −5.77 |
| 2050 | 0.03 | −3.60 | 7.15 | 0.37 | −5.68 | −2.53 |
| 2075 | 0.67 | −9.48 | 8.97 | −0.49 | −5.84 | 2.05 |
| 2100 | −3.29 | 1.61 | −11.55 | 0.61 | 1.62 | 0.72 |
| Year | Bilbao RCP 8.5 (no. deaths) | | | Bilbao RCP 4.5 (no. deaths) | | |
| | Mean | 95th | ES (95%) | Mean | 95th | ES (95%) |
| 2025 | −0.23 | −0.95 | −1.50 | −0.14 | −0.22 | −0.33 |
| 2050 | −0.03 | −0.03 | 0.32 | −0.12 | 0.01 | −0.56 |
| 2075 | 0.09 | −0.39 | −0.45 | −0.10 | 0.02 | −0.52 |
| 2100 | 0.13 | −0.31 | −0.30 | −0.04 | 0.01 | −0.60 |

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