

# Superstring at the boundary of open supermembrane interacting with D=4 supergravity and matter supermultiplets

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**ABSTRACT:** We present the complete supersymmetric and  $\kappa$ -symmetric action for the 4-dimensional interacting system of open supermembrane, dynamical supergravity and 3-form matter multiplets. The cases of a single 3-form matter multiplet and a quite generic model with a number of nonlinear interacting double 3-form multiplets are considered. In all cases the fermionic parameter of the  $\kappa$ -symmetry is subject to two apparently different projection conditions which suggests that the ground state of the system, in particular a domain junction, might preserve at most 1/4 of the spacetime supersymmetry.

The boundary term of the open supermembrane action, needed to preserve the  $\kappa$ -symmetry, has the meaning of the action of a superstring. The Wess-Zumino term of this superstring action is expressed in terms of real linear superfield playing the role of Stückelberg field for the 3-form gauge symmetry. This indicates that this symmetry is broken spontaneously by the superstring at the boundary of supermembrane.

**KEYWORDS:** p-branes, Supergravity Models, Superspaces, Supersymmetric Effective Theories

ARXIV EPRINT: [1906.09872](https://arxiv.org/abs/1906.09872)

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## Contents

<b>1</b>	<b>Introduction</b>	<b>1</b>
<b>2</b>	<b>Open supermembrane interacting with single 3-forms matter and supergravity</b>	<b>3</b>
2.1	Supermembrane action in the background of 3-form supergravity and 3-form matter	3
2.2	Supergravity interacting with closed supermembrane	6
2.3	Interaction with supergravity and single 3-form matter multiplet	7
2.4	Superstring at the boundary of open supermembrane coupled to supergravity and single 3-form matter multiplet	8
<b>3</b>	<b>Interacting system of double three form multiplets, supergravity and open supermembrane</b>	<b>11</b>
<b>4</b>	<b>Open supermembrane, nonlinearly self-interacting double 3-form matter multiplets and supergravity</b>	<b>13</b>
<b>5</b>	<b>Conclusion</b>	<b>16</b>
<b>A</b>	<b>Torsion constraints of minimal supergravity</b>	<b>18</b>
<b>B</b>	<b>Super-Weyl symmetry of the minimal supergravity constraints</b>	<b>19</b>
<b>C</b>	<b>Useful equations on supermembrane worldvolume</b>	<b>19</b>

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## 1 Introduction

Eleven dimensional supermembrane [1, 2], presently also known under the name of M2-brane, is one of the most important fundamental objects of the hypothetical underlying M-theory. Its consistency in curved superspace background subject this to the equations of motion of the 11-dimensional supergravity [3–5], which is believed to provide the low energy limit of the M-theory.

The simpler 4D cousin of M2-brane also attracted an interest already in late 80-th [6]. Different aspects of its interaction with  $\mathcal{N} = 1$   $D = 4$  supergravity and matter multiplets were the subject of study in [7–15]. The selfconsistency of nontrivial interaction with supermembrane requires matter and supergravity supermultiplets to include three form fields, thus leading naturally to the so-called variant superfield representations [7, 9–13, 16–22].

In particular the interaction of closed supermembrane with supergravity and matter in the models of the type appearing in string theory compactifications was studied in [14] while [15] considered supermembrane interaction with supersymmetric  $SU(N)$  Yang-Mills

(SU( $N$ ) SYM) theory and its effective description by Veneziano-Yankelovich (VY) action [23]. In the latter case the accounting for the presence of supermembrane allowed to solve a long-standing problem [24] of the missing contribution to the tension of BPS saturated domain-wall configurations, for which the membrane serves as a core. This also allowed us to find in [15] the explicit BPS domain wall solutions in this theory.

In [15] also the interacting action for open supermembrane carrying string on its boundary and rigid supersymmetric theories: generalized Wess-Zumino models including VY/SYM model, was briefly discussed. To our best knowledge the generic case of D=4 interacting system of open supermembrane, superstring on its end, supergravity and p-form matter has not been studied yet. The aim of this paper is to create a basis to feel this gap.<sup>1</sup> We present the complete superfield action for such an interacting system and prove its  $\kappa$ -symmetry which is an important property indicating that the ground state of this dynamical system preserves a part of supersymmetry (and hence is a stable BPS state). The complete interacting action can be split on the sum of the terms describing supergravity plus matter system ( $S_{\text{sugra+matter}}$ ), supermembrane ( $S_{p=2}$ ) and superstring at the end of open supermembrane ( $S_{p=1}$ )

$$S = S_{\text{sugra+matter}} + S_{p=2} + S_{p=1} . \tag{1.1}$$

We will describe these ingredients step by step for the case of coupling to different formulations of supergravity and different types of matter system. Clearly,  $S_{p=2}$  and  $S_{p=1}$  can be treated as actions of supermembrane and superstring at the end of supermembrane in the background of supergravity and matter multiplets.

Our action can be used to describe an effective field theories of string compactifications with open branes and branes at the boundary of open branes<sup>2</sup> and to study the role of open and intersecting supermembranes in supersymmetric generalizations and/or deformations of the constructions from [35–39]. It will be interesting to search for the supersymmetric domain wall junction solutions (see [40, 41]) of the equations of motion with open membrane sources which follow from our interacting actions.

In string theory the system of 4D open supermembranes with superstrings at the boundary of their worldvolume can be obtained from the network of higher p-branes on flux vacua of the type considered in [42] and [43]. In particular, the systems of connected domain walls and strings which appear from networks of D7-, D5- and D3-branes in compactifications on wrapped Calabi-Yau manifolds are described in the approach of calibrations in [42] were the explicit examples in the case of toroidal orientifold vacua and the Klebanov-Strassler geometry [44] have been discussed in more detail. In [43] the authors classify the particles, strings and membranes arising from wrapped  $p$ -branes which

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<sup>1</sup>The actions for open M2 brane (D=11 supermembrane) ending on M9-brane (the Horava-Witten end-of-world nine-brane) and M5-brane were found in [25, 26] and [27]. However, as the off-shell superfield description of the 11D supergravity and 10D matter was not known (and is still unknown), the 11D supergravity and 10D matter ( $E_8$  SYM) at the boundary of 11D spacetime were considered as a background obeying the ‘free’ equations of motion without superbrane sources. Probably a bypass to the complete but gauge fixed Lagrangian description of the dynamical system including, besides open 11D supermembrane, also 11D supergravity and 10D matter, can be reached on the line of [28–30].

<sup>2</sup>See for instance [31–33] and [34] and refs. therein for the description of a (not always direct) way from 10D branes and supergravity to 4D effective theories.

have a charges conserved modulo some integer number  $q$ , and discuss the catalyses of their annihilation by fluxes and  $\mathbb{Z}_q$  gauge symmetry associated with those. The actions of the type considered in this paper can be used to describe the effective field theory of such compactifications with networks of Dp-branes.

The rest of this paper is structured as follows. In section 2 we present the interacting action of the dynamical system of open supermembrane, supergravity and a single three form matter multiplet which is the master system used later as a basis to construct the actions for more complicated interacting systems. The closed supermembrane action and its  $\kappa$ -symmetry is described in section 2.1, the interaction of dynamical supergravity with closed supermembrane is the subject of section 2.2.

In section 2.3 we discuss the breaking of the  $\kappa$ -symmetry in the case of open supermembrane and show that it can be partially restored by adding to the open supermembrane action a certain boundary term which can be interpreted as an action for closed superstring at the end of open supermembrane. The additional projection conditions on the  $\kappa$ -symmetry parameter at the worldsheet presented there suggest that the open supermembrane (and supermembrane junctions) can preserve not more than one quarter of the spacetime supersymmetry. The spontaneous breaking of the three form gauge symmetry by superstring at the boundary of supermembrane and gauge fixed form of the boundary superstring action is also discussed in section 2.3. This section is finished by describing the most general form of the interacting action for the dynamical system under consideration, which includes, in particular, the mass term for the single 3-form multiplet.

The interacting system of open supermembrane, double three form supermultiplet and supergravity is described in section 3. In section 4 we present an action for quite generic interacting system including, besides open supermembrane with superstring at its ends and double 3-form supergravity, a nonlinear interacting system of  $n$  double-three form multiplets. This system is constructed with the use of special geometry and possesses symplectic  $\text{Sp}(2n + 2|n)$  invariance provided the  $2(n + 1)$  charges carried by the open supermembrane transform as symplectic vector. This interacting action, generalizing the action for the system with closed supermembrane studied in [14], can be used to investigate the role of open branes and branes at the boundary of open branes in the effective actions of the models originating in string compactifications. We conclude in section 5. Some useful equations are collected in the appendices.

## 2 Open supermembrane interacting with single 3-forms matter and supergravity

### 2.1 Supermembrane action in the background of 3-form supergravity and 3-form matter

The action for a supermembrane in a supergravity background and also in the background of supergravity and 3-form matter multiplet(s) can be written in the following form

$$S_{p=2} = \int_{W^3} d^3\xi \sqrt{|h|} |\mathcal{Z}| + \int_{W^3} C_3 . \tag{2.1}$$

In the first, Dirac-Nambu-Goto term of this action  $\xi^m = (\xi^0, \xi^1, \xi^2)$  are local coordinates on the worldvolume  $W^3$  of the supermembrane, which is defined as a surface in superspace  $\Sigma^{(4|4)}$  with coordinates  $z^M = (x^\mu, \theta^{\dot{\alpha}}, \bar{\theta}^{\dot{\alpha}})$  with the use of coordinate functions  $z^M(\xi) = (x^\mu(\xi), \theta^{\dot{\alpha}}(\xi), \bar{\theta}^{\dot{\alpha}}(\xi))$ ,

$$W^3 \in \Sigma^{(4|4)} : \quad z^M = z^M(\xi) . \quad (2.2)$$

$h = \det h_{mn}$  is the determinant of the induced metric

$$h_{mn} = E_m^a \eta_{ab} E_n^b , \quad E_m^a = \partial_m z^M(\xi) E_M^a(z(\xi)) \quad (2.3)$$

which is constructed from the pull-back  $E^a(z(\xi)) = d\xi^m E_m^a$  of the bosonic supervielbein of the supergravity superspace,

$$E^A(z) = (E^a, E^\alpha, \bar{E}^{\dot{\alpha}}) = dz^M E_M^A(z) . \quad (2.4)$$

Finally,  $\mathcal{Z}$  denotes the pull-back  $\mathcal{Z}(z(\xi))$  of a covariantly chiral superfield  $\mathcal{Z}(z)$  of a special type which we describe below. Now we just notice that, as any covariantly chiral superfield,  $\mathcal{Z}(z)$  obeys the constraint

$$\bar{\mathcal{D}}_{\dot{\alpha}} \mathcal{Z} = 0 , \quad (2.5)$$

where  $\bar{\mathcal{D}}_{\dot{\alpha}} = -(\mathcal{D}_\alpha)^*$  is the spinor covariant derivative defined by decomposition of the covariant differential on supervielbein,

$$\mathcal{D} = E^A \mathcal{D}_A = E^a \mathcal{D}_a + E^\alpha \mathcal{D}_\alpha + \bar{E}^{\dot{\alpha}} \bar{\mathcal{D}}_{\dot{\alpha}} . \quad (2.6)$$

Supervielbein (2.4) is restricted by minimal supergravity constraints which we present in the appendix A (see also [45] and refs therein).

In the second, Wess-Zumino term of the supermembrane action (2.1),  $C_3$  is the pull-back of a 3-form potential defined in curved superspace and having the field strength 4-form expressed in terms of the above chiral superfield  $\mathcal{Z}$  by<sup>3</sup>

$$\begin{aligned} H_4 = dC_3 = & \frac{1}{2} E^b \wedge E^a \wedge E^\alpha \wedge E^\beta \sigma_{ab}{}_{\alpha\beta} \bar{\mathcal{Z}} + \frac{1}{2} E^b \wedge E^a \wedge E^{\dot{\alpha}} \wedge E^{\dot{\beta}} \bar{\sigma}_{ab}{}^{\dot{\alpha}\dot{\beta}} \mathcal{Z} + \\ & + \frac{1}{12} E^c \wedge E^b \wedge E^a \wedge \epsilon_{abcd} E^\alpha \sigma_{\alpha\dot{\beta}}^d \bar{\mathcal{D}}^{\dot{\beta}} \bar{\mathcal{Z}} + \frac{1}{12} E^c \wedge E^b \wedge E^a \wedge \epsilon_{abcd} E^{\dot{\beta}} \sigma_{\alpha\dot{\beta}}^d \mathcal{D}^\alpha \mathcal{Z} + \\ & + \frac{i}{192} E^d \wedge E^c \wedge E^b \wedge E^a \epsilon_{abcd} ((\bar{\mathcal{D}}\bar{\mathcal{D}} - 3R)\bar{\mathcal{Z}} - (\mathcal{D}\mathcal{D} - 3\bar{R})\mathcal{Z}) . \end{aligned} \quad (2.7)$$

Here  $R = (\bar{R})^*$  and  $G_a = (G_a)^*$  are main superfields of minimal (and variant) off-shell supergravity (see appendix A).

The form  $H_4$  is closed,  $dH_4 = 0$ , when the supervielbein obeys the minimal supergravity constraints. However, the requirement that it is exact, i.e. that there exists a 3-form  $C_3$

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<sup>3</sup>To our best knowledge, the closed 4-form (2.7) in supergravity superspace was first presented in [18] and its super-Weyl invariance was noticed in [13].

such that  $H_4 = dC_3$ , requires the chiral superfield  $\mathcal{Z}$  to be special, namely to be constructed in terms of real superfield prepotential  $\mathcal{P} = \mathcal{P}^*$ ,

$$\mathcal{Z} = -\frac{1}{4} (\bar{\mathcal{D}}_{\dot{\alpha}} \bar{\mathcal{D}}^{\dot{\alpha}} - R) \mathcal{P}. \quad (2.8)$$

As a result, the component content of  $\mathcal{Z}$  is different from that of the usual chiral superfield  $\Phi = -\frac{1}{4} (\bar{\mathcal{D}}_{\dot{\alpha}} \bar{\mathcal{D}}^{\dot{\alpha}} - R) \mathbb{K}$  constructed from the complex potential  $\mathbb{K} \neq \mathbb{K}^*$ : the  $F$ -component of that superfield is given by a complex linear combination of real scalar and a divergence of a real vector instead of two real scalars (scalar and pseudoscalar) in the case of  $\Phi$  (see e.g. [15] and refs. therein for more details). Hence the name of single three form supermultiplet for the field content of the special chiral superfield (2.8) with an arbitrary superfield  $\mathcal{P}$ .

Now, the real 3-form potential  $C_3$ , the pull-back of which enters the second term in (2.1), is expressed in terms of the same real superfield  $\mathcal{P}$  by

$$\begin{aligned} C_3 = & -iE^a \wedge E^\alpha \wedge \bar{E}^{\dot{\alpha}} \sigma_{a\alpha\dot{\alpha}} \mathcal{P} - \frac{1}{4} E^b \wedge E^a \wedge E^\alpha \sigma_{ab\alpha}{}^\beta \mathcal{D}_\beta \mathcal{P} + \\ & + \frac{1}{4} E^b \wedge E^a \wedge \bar{E}^{\dot{\alpha}} \tilde{\sigma}_{ab}{}^{\dot{\beta}}{}_{\dot{\alpha}} \bar{\mathcal{D}}_{\dot{\beta}} \mathcal{P} + \frac{1}{48} E^c \wedge E^b \wedge E^a \epsilon_{abcd} \left( \tilde{\sigma}^{d\dot{\alpha}\alpha} [\mathcal{D}_\alpha, \bar{\mathcal{D}}_{\dot{\alpha}}] \mathcal{P} + 2G^d \mathcal{P} \right). \end{aligned} \quad (2.9)$$

Of course, (2.9) is the gauge fixed form of the potential corresponding to the field strength (2.7). However, there exists a residual gauge invariance with respect to additive transformations of real prepotential superfield  $\mathcal{P}$  with real linear superfield  $\mathbb{L}$ ,

$$\delta \mathcal{P} = \mathbb{L}, \quad (2.10)$$

$$(\bar{\mathcal{D}}_{\dot{\alpha}} \bar{\mathcal{D}}^{\dot{\alpha}} - R) \mathbb{L} = 0, \quad (\mathcal{D}^\alpha \mathcal{D}_\alpha - \bar{R}) \mathbb{L} = 0. \quad (2.11)$$

Such transformation of the prepotential results in the gauge transformation of the super-space 3-form (2.9)

$$\delta C_3 = d\alpha_2 \quad (2.12)$$

by closed 3-form  $d\alpha_2$  constructed from the real linear superfield  $\mathbb{L}$  (2.11) as follows

$$\begin{aligned} d\alpha_2 = & -iE^a \wedge E^\alpha \wedge \bar{E}^{\dot{\alpha}} \sigma_{a\alpha\dot{\alpha}} \mathbb{L} - \frac{1}{4} E^b \wedge E^a \wedge E^\alpha \sigma_{ab\alpha}{}^\beta \mathcal{D}_\beta \mathbb{L} + \\ & + \frac{1}{4} E^b \wedge E^a \wedge \bar{E}^{\dot{\alpha}} \tilde{\sigma}_{ab}{}^{\dot{\beta}}{}_{\dot{\alpha}} \bar{\mathcal{D}}_{\dot{\beta}} \mathbb{L} + \frac{1}{48} E^c \wedge E^b \wedge E^a \epsilon_{abcd} \left( \tilde{\sigma}^{d\dot{\alpha}\alpha} [\mathcal{D}_\alpha, \bar{\mathcal{D}}_{\dot{\alpha}}] \mathbb{L} + 2G^d \mathbb{L} \right). \end{aligned} \quad (2.13)$$

Clearly,  $d\alpha_2 = C_3|_{\mathcal{P} \rightarrow \mathbb{L}}$ .

The *closed* supermembrane action is invariant under local fermionic  $\kappa$ -symmetry transformations of the coordinate functions

$$i_\kappa E^a := \delta_\kappa z^M E_M^a = 0, \quad i_\kappa E^\alpha := \delta_\kappa z^M E_M^\alpha = \kappa^\alpha, \quad i_\kappa E^{\dot{\alpha}} := \delta_\kappa z^M E_M^{\dot{\alpha}} = \bar{\kappa}^{\dot{\alpha}} \quad (2.14)$$

the fermionic parameters of which obey the conditions

$$\kappa_\alpha = -i \frac{\mathcal{Z}}{|\mathcal{Z}|} \Gamma_{\alpha\dot{\alpha}} \bar{\kappa}^{\dot{\alpha}}, \quad \bar{\kappa}_{\dot{\alpha}} = -i \frac{\bar{\mathcal{Z}}}{|\mathcal{Z}|} \kappa^\alpha \Gamma_{\alpha\dot{\alpha}}, \quad (2.15)$$

where

$$\Gamma_{\alpha\dot{\alpha}} = \frac{i}{3!\sqrt{h}} \sigma_{\alpha\dot{\alpha}}^a \epsilon_{abcd} \epsilon^{mnk} E_m^b E_n^c E_k^d, \quad (2.16)$$

is imaginary,  $(\Gamma_{\alpha\dot{\alpha}})^* = -\Gamma_{\alpha\dot{\alpha}}$ , and obeys  $\Gamma_{\alpha\dot{\alpha}}\Gamma^{\dot{\alpha}\beta} = \delta_{\alpha}^{\beta}$ .

Finally let us comment on the dimension of special chiral superfield  $\mathcal{Z}$ . If we read eq. (2.1) literally, we should conclude that the dimension of  $\mathcal{Z}$  is 3 in the mass units,  $[\mathcal{Z}] = M^3$ . This is because the tension of the supermembrane,  $T_2$ , is included in  $\mathcal{Z}$  as a multiplier. To make its presence explicit we should redefine  $\mathcal{Z} \mapsto T_2\mathcal{Z}$  and consider  $\mathcal{Z}$  to be dimensionless,  $[\mathcal{Z}] = M^0$ . We can also consider (2.1) as an action with  $T_2$  formally set to be 1 and containing a dimensionless  $\mathcal{Z}$ . We will prefer such interpretation of our action.

## 2.2 Supergravity interacting with closed supermembrane

As we have already stated, the action (2.1) can describe the supermembrane moving in the background of a three-form supergravity as well as in the background of supergravity and 3-form matter multiplet(s). In the first case the above special chiral superfield  $\mathcal{Z}$  should be treated as conformal compensator of a 3-form supergravity.

The name of 3-form supergravity is attributed to two variant formulations of minimal supergravity [16, 17], presently referred to as single three form supergravity and double three form supergravity [22]. In the (super-)Weyl invariant formulation of these versions of  $\mathcal{N} = 1$  supergravity the conformal compensator of minimal supergravity  $\mathcal{Z}$  has a special form: it is expressed in terms of real prepotential superfield  $\mathcal{P}$  as in (2.8). In the case when this prepotential is an independent ('fundamental') superfield, we arrive at single three form supergravity [7, 10, 12, 17, 20, 21, 47] in its Weyl invariant formulation of [20]. If the chiral compensator is expressed in terms of composite real prepotential given by real or imaginary part of a complex linear superfield  $\Sigma$ ,

$$\mathcal{P} = \Im m \Sigma := \frac{i}{2}(\bar{\Sigma} - \Sigma), \quad (\mathcal{D}^\alpha \mathcal{D}_\alpha - \bar{R})\Sigma = 0, \quad (\bar{\mathcal{D}}_{\dot{\alpha}} \bar{\mathcal{D}}^{\dot{\alpha}} - R)\bar{\Sigma} = 0, \quad (2.17)$$

we are dealing with double three form supergravity, which was actually described already in [49] and [50]. Its coupling to matter and application to the effective theory of string compactifications was the subject of recent [22].

The action for the interacting system of double three form supergravity and closed supermembrane was presented in [13]. The dynamical system including also a set of nonlinearly self interacting double 3-form matter multiplets was described and studied in [14]. The single three form supergravity interacting with supermembrane was the subject of [7, 10, 12]. Here we will consider the case of supergravity interacting with open supermembrane and superstring at the boundary of the open supermembrane.

But first let us write the action for the interacting system of supergravity and closed membrane in super-Weyl invariant formulation of supergravity. It reads

$$S = S_{\text{sugra}} + S_{p=2}, \quad (2.18)$$

where  $S_{p=2}$  has the form of (2.1), with (2.8) and (2.9), and<sup>4</sup>

$$S_{\text{sugra}} = -\frac{3}{4\kappa^2} \int d^8z E (\mathcal{Z}\bar{\mathcal{Z}})^{\frac{1}{3}} - \frac{m}{2\kappa^2} \left( \int d^6\zeta_L \mathcal{E} \mathcal{Z} + \text{c.c.} \right). \quad (2.19)$$

Here  $E = \text{sdet}(E_M^A(z))$  is the superdeterminant (Berezinian) of the supervielbein,  $m$  is the gravitino mass, proportional to the cosmological constant ( $m = 0$  for the case of Poincaré supergravity) and  $d^6\zeta_L \mathcal{E}$  is the chiral measure (see [51] and refs. therein). This is related to the complete superspace measure  $d^8z E$  by

$$\int d^8z E \mathbb{Y} = -\frac{1}{2} \int d^6\zeta_L \mathcal{E} (\bar{\mathcal{D}}\bar{\mathcal{D}} - R) \mathbb{Y}, \quad (2.20)$$

where  $\mathbb{Y}$  is an arbitrary superfield.

The super-Weyl transformations leaving invariant the action (2.18), (2.19), (2.1) are described in appendix B (see eqs. (B.2)–(B.5) and (B.1)). This can be used to set the chiral superfield  $\mathcal{Z}$  equal to unity. The super-Weyl symmetry of the action (2.19) is thus realized by Stückelberg mechanism with a pure gauge superfield  $\mathcal{Z}$ . Hence the name of conformal compensator used for  $\mathcal{Z}$  superfield in the action (2.19).

### 2.3 Interaction with supergravity and single 3-form matter multiplet

The case of simplest interacting system of supergravity, supermembrane and a 3-form matter multiplet is described by the action

$$S = S_{\text{sugra+matter}} + S_{p=2}, \quad (2.21)$$

where  $S_{p=2}$  has the form of (2.1) and

$$S_{\text{sugra+matter}} = -\frac{3}{4\kappa^2} \int d^8z E \Omega(\mathcal{Z}, \bar{\mathcal{Z}}) - \frac{1}{2\kappa^2} \left( \int d^6\zeta_L \mathcal{E} \mathcal{W}(\mathcal{Z}) + \text{c.c.} \right). \quad (2.22)$$

Here  $\mathcal{W}(\mathcal{Z})$  is superpotential,

$$\Omega(\mathcal{Z}, \bar{\mathcal{Z}}) = e^{-\frac{\kappa^2}{3} K(\mathcal{Z}, \bar{\mathcal{Z}})}, \quad (2.23)$$

and  $K(\mathcal{Z}, \bar{\mathcal{Z}})$  is the Kähler potential. The action (2.22) is invariant under the super-Weyl transformations (B.2), (B.3), (B.4) with  $\Upsilon = \Upsilon(\mathcal{Z})$ ,  $\tilde{\Upsilon} = \tilde{\Upsilon}(\bar{\mathcal{Z}})$ , supplemented by the Kähler transformations of the Kähler potential,

$$K(\mathcal{Z}, \bar{\mathcal{Z}}) \mapsto K(\mathcal{Z}, \bar{\mathcal{Z}}) + 6\Upsilon(\mathcal{Z}) + 6\tilde{\Upsilon}(\bar{\mathcal{Z}})$$

and  $\mathcal{W}(\mathcal{Z}) \mapsto \mathcal{W}(\mathcal{Z}) e^{-6\Upsilon(\mathcal{Z})}$ . In the case of nonvanishing superpotential, these transformations can be used to gauge this to a constant  $m$ ,

$$K(\mathcal{Z}, \bar{\mathcal{Z}}) \mapsto \mathcal{K}(\mathcal{Z}, \bar{\mathcal{Z}}) = K(\mathcal{Z}, \bar{\mathcal{Z}}) + \frac{2}{\kappa^2} \ln |\mathcal{W}(\mathcal{Z})| - \frac{2}{\kappa^2} \ln |m|, \quad \mathcal{W}(\mathcal{Z}) \mapsto m. \quad (2.24)$$

In the case of  $\mathcal{W}(\mathcal{Z}) = m\mathcal{Z}$  and  $K(\mathcal{Z}, \bar{\mathcal{Z}}) = -\frac{2}{\kappa^2} \ln |\mathcal{Z}|$ , in which (2.22) reduces to (2.19), the transformation (2.24) removes the chiral superfield  $\mathcal{Z}$  from the action. This indicates that, as stated, the action (2.19) describes the (3-form) supergravity only.

<sup>4</sup>As super-Weyl invariant action for three-form supergravity eq. (2.19) was discussed in [20]. See [46] for the description of new minimal and nonminimal off-shell formulations of supergravity as super-Weyl-invariant couplings of the old minimal supergravity to a compensating supermultiplet.



## 2.4 Superstring at the boundary of open supermembrane coupled to supergravity and single 3-form matter multiplet

When the supermembrane worldvolume is not closed,  $\partial W^3 = W^2 \neq \emptyset$ , its action (2.1) is not invariant under the above described  $\kappa$ -symmetry,

$$\delta_\kappa S_{p=2} = \int_{W^2=\partial W^3} i_\kappa C_3, \quad (2.25)$$

$$i_\kappa C_3 = -iE^a \wedge E^\alpha \sigma_{a\alpha\dot{\alpha}} \bar{\kappa}^{\dot{\alpha}} \mathcal{P} - iE^a \wedge \bar{E}^{\dot{\alpha}} \kappa^\alpha \sigma_{a\alpha\dot{\alpha}} \mathcal{P} - \frac{1}{4} E^b \wedge E^a \left( \kappa^\alpha \sigma_{ab\alpha}{}^\beta \mathcal{D}_\beta \mathcal{P} - \tilde{\sigma}_{ab}{}^{\dot{\beta}}{}_{\dot{\alpha}} \bar{\kappa}^{\dot{\alpha}} \bar{\mathcal{D}}_{\dot{\beta}} \mathcal{P} \right). \quad (2.26)$$

Neither the open supermembrane action (2.1) is invariant under the gauge transformations (2.10): we find (2.12) and

$$\delta_{\text{gauge}} S_{p=2} = \int_{W^2=\partial W^3} \alpha_2, \quad (2.27)$$

where  $\alpha_2$  is defined by (2.13).

To compensate these nonvanishing variations, it is necessary to put at the boundary of supermembrane a superstring. For the gauge symmetry the mechanism of compensation refers to the Wess-Zumino term of the superstring action which is given by integral over the worldsheet of a 2-form potential  $B_2$ ,

$$- \int_{W^2} B_2 = - \int_{W^3} dB_2 \equiv - \int_{W^3} H_3. \quad (2.28)$$

The sum of the Wess-Zumino terms of string and membrane

$$\int_{W^3} C_3 - \int_{W^2} B_2 = \int_{W^3} (C_3 - dB_2) \quad (2.29)$$

will be invariant under 3-form gauge transformations (2.12), (2.13) if 2-form potential transforms under these as a Stückelberg field,

$$\delta C_3 = d\alpha_2, \quad \delta B_2 = \alpha_2. \quad (2.30)$$

This is possible if  $B_2$  in (2.28) is the pull-back of the superspace 2-form with the field strength expressed by

$$H_3 = dB_2 = -iE^a \wedge E^\alpha \wedge \bar{E}^{\dot{\alpha}} \sigma_{a\alpha\dot{\alpha}} L - \frac{1}{4} E^b \wedge E^a \wedge E^\alpha \sigma_{ab\alpha}{}^\beta \mathcal{D}_\beta L + \frac{1}{4} E^b \wedge E^a \wedge \bar{E}^{\dot{\alpha}} \tilde{\sigma}_{ab}{}^{\dot{\beta}}{}_{\dot{\alpha}} \bar{\mathcal{D}}_{\dot{\beta}} L + \frac{1}{48} E^c \wedge E^b \wedge E^a \epsilon_{abcd} \left( \tilde{\sigma}^{d\dot{\alpha}\alpha} [\mathcal{D}_\alpha, \bar{\mathcal{D}}_{\dot{\alpha}}] L + 2G^d L \right) \quad (2.31)$$

( $H_3 = C_3|_{\mathcal{P} \rightarrow L}$ ) in terms of the real tensor multiplet  $L$

$$(\bar{\mathcal{D}}_{\dot{\alpha}} \bar{\mathcal{D}}^{\dot{\alpha}} - R) L = 0, \quad (\mathcal{D}^\alpha \mathcal{D}_\alpha - \bar{R}) L = 0 \quad (2.32)$$

which is transformed as Stückelberg superfield under (2.10),

$$\delta\mathcal{P} = \mathbb{L}, \quad \delta L = \mathbb{L}. \quad (2.33)$$

In the absence of open supermembrane, the action of closed superstring with Wess-Zumino term (2.28) contains also a Nambu-Goto term including the pull-back to worldline of the real linear superfield  $L$ ,

$$\frac{1}{2} \int_{W^2} d^2\sigma \sqrt{-\gamma} |L|. \quad (2.34)$$

Here  $d^2\sigma = d\sigma^0 \wedge d\sigma^1$ ,  $\sigma^i = (\sigma^0, \sigma^1)$  are local worldsheet coordinates and  $\gamma = \det \gamma_{ij}$  is the determinant of the metric induced on the worldsheet  $W^2$ ,

$$\gamma_{ij} = E_i^a \eta_{ab} E_j^b, \quad E_i^a = \partial_i z^M(\sigma) E_M^a(z(\sigma)). \quad (2.35)$$

When the string is situated at the end of membrane, the term (2.34) should be modified to  $\frac{1}{2} \int_{W^2} d^2\sigma \sqrt{-\gamma} |\mathcal{P} - L|$  as, after such a modification, the Nambu-Goto term will respect the gauge symmetry (2.33) which also leaves invariant the sum of the Wess-Zumino terms of the superstring and the supermembrane as well as the Dirac-Nambu-Goto term of the supermembrane action. Thus we arrive at the following action for superstring at the boundary of supermembrane

$$S_{p=1} = \frac{1}{2} \int_{W^2} d^2\sigma \sqrt{-\gamma} |\mathcal{P} - L| - \int_{W^2} B_2, \quad (2.36)$$

where the field strength of  $B_2$  has the form of (2.31),  $L$  is the pull-back of a real linear superfield obeying (2.32). Finally  $\mathcal{P}$  in (2.36) is the pull-back to the worldsheet of the real prepotential superfield defining the special chiral superfield through eq. (2.8) and the three form potential through eq. (2.9).

The actions given by the sum of (2.36) and (2.1) is also invariant under the local fermionic  $\kappa$ -symmetry (2.14) with parameters restricted, besides (2.15), by the projection conditions

$$\kappa_\alpha = \frac{\mathcal{P} - \mathbb{L}}{|\mathcal{P} - \mathbb{L}|} P_\alpha^\beta \kappa_\beta, \quad \bar{\kappa}_{\dot{\alpha}} = \frac{\mathcal{P} - \mathbb{L}}{|\mathcal{P} - \mathbb{L}|} \bar{P}_{\dot{\alpha}}^{\dot{\beta}} \bar{\kappa}_{\dot{\beta}} \quad (2.37)$$

where

$$P_\beta^\alpha = \frac{1}{2\sqrt{-\gamma}} \epsilon^{ij} E_i^a E_j^b \sigma_{ab\beta}^\alpha, \quad \bar{P}_{\dot{\alpha}}^{\dot{\beta}} = (P_\alpha^\beta)^* = -\frac{1}{2\sqrt{-\gamma}} \epsilon^{ij} E_i^a E_j^b \tilde{\sigma}_{ab\dot{\alpha}}^{\dot{\beta}} \quad (2.38)$$

obey

$$P^2 = \mathbb{I}, \quad \bar{P}^2 = \mathbb{I}. \quad (2.39)$$

For a particular case of superstring at the boundary of open supermembrane interacting with Veneziano-Yankelovich effective description of the SYM theory the (flat super-space version of the) action (2.36) was found in [15] where the above  $\kappa$ -symmetry was also presented.

The new property of the interacting system including supergravity, which follows from its the diffeomorphism gauge invariance, is that the superstring and supermembrane Goldstone fields, this is to say bosonic and fermionic coordinate functions, become Stückelberg (pure gauge) fields which do not carry degrees of freedom. This allows to fix their values by imposing, e.g.

$$x^i(\sigma) = \sigma^i, \quad x^2(\sigma) = 0, \quad x^3(\sigma) = 0, \quad \theta^{\dot{\alpha}}(\sigma) = 0, \quad \bar{\theta}^{\dot{\alpha}}(\sigma) = 0 \quad (2.40)$$

in the case of superstring and

$$x^m(\xi) = \xi^m, \quad x^3(\xi) = 0, \quad \theta^{\dot{\alpha}}(\xi) = 0, \quad \bar{\theta}^{\dot{\alpha}}(\xi) = 0 \quad (2.41)$$

in the case of supermembrane.

Let us stress that, when dynamical supergravity described by the superfields which are varied in the action, is not present, like in flat superspace system discussed in [15], eqs. (2.41) and (2.40) describe a particular configuration of open supermembrane and superstring at the boundary of this supermembrane. In contrast, when supergravity is dynamical the diffeomorphism invariance is the gauge symmetry of the system and (2.41) and (2.40) describe just gauge fixed conditions for such a symmetry spontaneously broken by open supermembrane and superstring at its boundary.

The superstring at the boundary of supermembrane also breaks the gauge symmetry (2.10), (2.12), (2.13), characteristic for the three form potential. When action is written with the use of the Stückelberg real linear superfield  $L$ , as in (2.36), this symmetry is formally maintained (realized dynamically) as  $\mathcal{P} - L$  is invariant under (2.33). However, we can fix the gauge under this symmetry by setting  $L=0$  and in this gauge (2.36) reduces to

$$S_{p=1}|_{L=0} = \frac{1}{2} \int_{W^2} d^2\sigma \sqrt{-\gamma} |\mathcal{P}|. \quad (2.42)$$

Notice that the Wess-Zumino term of the superstring vanishes in this gauge. The remaining Nambu-Goto type term (2.42) is sufficient to compensate (2.27) with (2.26) provided the  $\kappa$ -symmetry parameter is restricted, besides (2.15), also by the condition (cf. (2.37))

$$\kappa_{\alpha} = \frac{\mathcal{P}}{|\mathcal{P}|} P_{\alpha}^{\beta} \kappa_{\beta}, \quad \bar{\kappa}_{\dot{\alpha}} = \frac{\mathcal{P}}{|\mathcal{P}|} \bar{P}_{\dot{\alpha}}^{\dot{\beta}} \bar{\kappa}_{\dot{\beta}}, \quad (2.43)$$

where  $P_{\alpha}^{\beta} = (\bar{P}_{\dot{\alpha}}^{\dot{\beta}})^*$  is defined in (2.39).

When restoring the membrane tension in the gauge fixed action (2.42), it takes the form  $S_{p=1}|_{L=0} = \frac{T_2}{2} \int_{W^2} d^2\sigma \sqrt{-\gamma} |\mathcal{P}|$  which makes manifest that the effective tension of the string at the end of supermembrane is defined by the supermembrane tension:  $T_1(\sigma) = T_2 |\mathcal{P}(z(\sigma))|$ . To stress this, it is instructive to write once the action of open supermembrane and the superstring at its boundary with the membrane tension written explicitly:

$$S_{p=2} + S_{p=1} = T_2 \int_{W^3} d^3\xi \sqrt{|h|} |\mathcal{Z}| + T_2 \int_{W^3} (C_3 - H_3) + \frac{T_2}{2} \int_{W^2=\partial W^3} d^2\sigma \sqrt{-\gamma} |\mathcal{P} - L|. \quad (2.44)$$

Notice that in the  $L = 0$  gauge the form of the action looks like a counterpart of the Fayet-Iliopoulos term, but with the real prepotential superfield  $V$  of the U(1) SYM model replaced by the real prepotential superfield  $\mathcal{P}$  of the three form multiplet, and with superspace integration replaced by the integration over the worldsheet. This later results in the explicit breaking of the three-form gauge symmetry (2.10) in the (gauge fixed) action including (2.42), while when the action contains (2.36), the three form gauge symmetry is maintained but realized with Stückelberg mechanism as in (2.33).

As far as the breaking of the three form gauge symmetry is allowed, we can add one more term to the supergravity plus matter part (2.22) of the action (1.1). These is the mass term for the 3-form matter multiplet,  $\int d^8z E \frac{\mathcal{P}^2}{(\mathcal{Z}\bar{\mathcal{Z}})^{1/3}}$  [19]. As we have already introduced the Stückelberg real linear superfield  $L$  in the bulk, thus stressing the spontaneous character of the breaking of the 3-form gauge symmetry by superstring at the boundary of supermembrane, we can write this term with maintaining formal gauge invariance as  $\int d^8z E \frac{(\mathcal{P}-L)^2}{(\mathcal{Z}\bar{\mathcal{Z}})^{1/3}}$ . Then the most general (up to inclusion of higher derivative terms)  $S_{\text{sugra+matter}}$  part of the action (1.1) reads

$$S_{\text{sugra+matter}} = -\frac{3}{4\kappa^2} \int d^8z E \Omega(\mathcal{Z}, \bar{\mathcal{Z}}) - \frac{1}{2\kappa^2} \left( \int d^6\zeta_L \mathcal{E} \mathcal{W}(\mathcal{Z}) + \text{c.c.} \right) - \mathbf{m}^4 \int d^8z E \frac{(\mathcal{P} - L)^2}{(\mathcal{Z}\bar{\mathcal{Z}})^{1/3}}. \tag{2.45}$$

Here  $\mathbf{m}$  is a constant of dimension of mass. The remaining parts of the interacting action (1.1) are given in eq. (2.44). (This latter clearly indicates that the mass dimension of  $\mathcal{Z}$  and  $\mathcal{P}$  in (2.45) is 0 and -1, respectively).

One might observe the possibility to add to the action a true counterpart of the Fayet-Iliopoulos term constructed from the real prepotential superfield:  $\int d^8z E \mathcal{P}$ . However, taking into account the relation of the chiral and full superspace integration measure (2.20) it is easy to observe that actually this is an equivalent form of the F-term with linear superpotential for the special chiral superfield  $\mathcal{Z}$  (2.8),  $\int d^8z E \mathcal{P} = \int d^6\zeta_L \mathcal{E} \mathcal{Z} + \text{c.c.}$

Notice that the form of the mass term in (2.45) is fixed by the requirement of the super-Weyl invariance under (B.2)–(B.5) with (B.1). For the case of Veneziano-Yankelovich effective theory of SYM such a term was considered in [19] and [15].

### 3 Interacting system of double three form multiplets, supergravity and open supermembrane

The open supermembrane part of the action for the interacting systems including a number of different 3-form matter multiplets and supergravity can be easily obtained from the above action for the case of supergravity interacting with a single three form multiplet. The key point is to define the composite special chiral superfield  $\mathcal{Z}$  and its real prepotential superfield  $\mathcal{P}$  in terms of several ‘fundamental’ superfields. No need to stress that such a redefinition generically would result in a possible changes of the superfield matter part of the action and in any case would produce a different set of the equations of motion.

Below we would like to discuss the actions and symmetry of such interacting systems beginning from the case of open supermembrane coupled to the double three form matter and supergravity.

A particular case of composite special chiral superfield  $\mathcal{Z}$  is reached when the prepotential in (2.8) is constructed as

$$\mathcal{P} = \Im \Sigma := \frac{i}{2}(\bar{\Sigma} - \Sigma) \quad (3.1)$$

from the complex linear superfield  $\Sigma$  obeying

$$(\mathcal{D}^\alpha \mathcal{D}_\alpha - \bar{R})\Sigma = 0. \quad (3.2)$$

Eq. (3.2) is solved by

$$\Sigma = \mathcal{D}_\alpha \Xi^\alpha \quad (3.3)$$

with an independent spinor superfield  $\Xi^\alpha$ .

A special chiral superfield  $\mathcal{Z}$  defined in (2.8) and (3.1), which we denote below by  $\frac{i}{2}\mathcal{S}$ ,

$$\mathcal{S} = \frac{1}{4}(\bar{\mathcal{D}}_{\dot{\alpha}}\bar{\mathcal{D}}^{\dot{\alpha}} - R)\Sigma, \quad \bar{\mathcal{S}} = \frac{1}{4}(\mathcal{D}^\alpha \mathcal{D}_\alpha - \bar{R})\bar{\Sigma}, \quad (3.4)$$

has as its F-component a linear combination of two divergences of real vectors (instead of two scalars in the case of usual chiral superfield  $\Phi$ , see [22] and refs. therein for details). Hence the name of double three form multiplet for the component content of the special chiral superfield  $\mathcal{S}$ .

There is also a related superspace reason for such a name. With  $\mathcal{Z} = \frac{i}{2}\mathcal{S}$  defined in (2.8) and (3.1), the real exact form (2.7) is equal to doubled real part of the complex exact form

$$\begin{aligned} H_4 &= dC_3 = 2\Re\bar{\mathcal{H}}_4, \quad (3.5) \\ \bar{\mathcal{H}}_4 &= d\bar{\mathcal{A}}_3 = -\frac{i}{4}E^b \wedge E^a \wedge E^\alpha \wedge E^\beta \sigma_{ab\alpha\beta} \bar{\mathcal{S}} - \\ &\quad -\frac{i}{4!}E^c \wedge E^b \wedge E^a \wedge \epsilon_{abcd} E^\alpha \sigma_{\alpha\dot{\beta}}^d \bar{\mathcal{D}}^{\dot{\beta}} \bar{\mathcal{S}} + \\ &\quad +\frac{1}{384}E^d \wedge E^c \wedge E^b \wedge E^a \epsilon_{abcd} (\bar{\mathcal{D}}\bar{\mathcal{D}} - 3R)\bar{\mathcal{S}}. \end{aligned} \quad (3.6)$$

The complex three form potential for (3.6) can be chosen to be

$$\begin{aligned} \bar{\mathcal{A}}_3 &= \frac{1}{2}E^a \wedge E^\alpha \wedge \bar{E}^{\dot{\alpha}} \sigma_{a\alpha\dot{\alpha}} \bar{\Sigma} - \frac{i}{8}E^b \wedge E^a \wedge E^\alpha \sigma_{ab\alpha}{}^\beta \mathcal{D}_\beta \bar{\Sigma} + \\ &\quad +\frac{i}{8}E^b \wedge E^a \wedge \bar{E}^{\dot{\alpha}} \tilde{\sigma}_{ab}{}^{\dot{\beta}}{}_{\dot{\alpha}} \bar{\mathcal{D}}_{\dot{\beta}} \bar{\Sigma} + \frac{1}{3!}E^c \wedge E^b \wedge E^a \bar{\mathcal{A}}_{abc} \end{aligned} \quad (3.7)$$

with

$$\begin{aligned} \bar{\mathcal{A}}_{abc} &= \frac{i}{16}\epsilon_{abcd} \left( \tilde{\sigma}^{d\dot{\alpha}\alpha} \mathcal{D}_\alpha \bar{\mathcal{D}}_{\dot{\alpha}} \bar{\Sigma} - 2i\mathcal{D}^d \bar{\Sigma} + G^d \bar{\Sigma} \right) \\ &= \frac{i}{32}\epsilon_{abcd} \left( \tilde{\sigma}^{d\dot{\alpha}\alpha} [\mathcal{D}_\alpha, \bar{\mathcal{D}}_{\dot{\alpha}}] \bar{\Sigma} + 2G^d \bar{\Sigma} \right). \end{aligned} \quad (3.8)$$

The real 3-form potential for (2.7) is now given by (twice the) real part of the complex potential,

$$C_3 = 2\Re\bar{\mathcal{A}}_3 = \mathcal{A}_3 + \bar{\mathcal{A}}_3, \quad (3.9)$$

and we actually have two gauge invariances of the type (2.12), a one-parametric combination of which should act on our Stückelberg two-form,

$$\delta\mathcal{A}_3 = d\beta_2, \quad \delta\bar{\mathcal{A}}_3 = d\bar{\beta}_2, \quad \delta B_2 = 2\Re\beta_2 = \beta_2 + \bar{\beta}_2. \quad (3.10)$$

The expression (3.7) is clearly gauge fixed and the residual gauge symmetry preserving this form of the complex 3-form potential is generated by the following transformations of complex linear and real linear superfields

$$\delta\Sigma = \tilde{\mathbb{L}} + i\mathbb{L}, \quad \delta\bar{\Sigma} = \tilde{\mathbb{L}} - i\mathbb{L}, \quad \delta L = \mathbb{L}. \quad (3.11)$$

The transformations ‘parametrized’ by the second real linear multiplet,  $\tilde{\mathbb{L}}$ , leave invariant the real 3-form  $C_3$  which enters the supermembrane action. However, it is convenient to introduce the Stückelberg real linear multiplet superfield  $\tilde{L}$  also for these transformations,

$$\delta\tilde{L} = \tilde{\mathbb{L}}, \quad (3.12)$$

so that  $(\Sigma - \tilde{L} - iL)$  and its c.c. are gauge invariant.

The simplest action for the supergravity and double three form matter supermultiplet(s) can be obtained by substituting (3.1) for  $\mathcal{P}$  and  $(i/2)\mathcal{S}$  for  $\mathcal{Z}$  into (2.45). However, with the above described Stückelberg realization of the two three form gauge symmetries we can write the action with a more general mass term, thus arriving at

$$S_{\text{sugra+matter}} = -\frac{3}{4\kappa^2} \int d^8z E \Omega(\mathcal{S}, \bar{\mathcal{S}}) - \frac{1}{2\kappa^2} \left( \int d^6\zeta_L \mathcal{E} \mathcal{W}(\mathcal{S}) + \text{c.c.} \right) - m^4 \int d^8z E \frac{(\Sigma - \tilde{L} - iL)(\bar{\Sigma} - \tilde{L} + iL)}{(\mathcal{S}\bar{\mathcal{S}})^{1/3}}. \quad (3.13)$$

Thus the coupling of open supermembrane to the simplest double three form matter and supergravity is described by the action (1.1) with (3.13), and

$$S_{p=2} + S_{p=1} = \frac{T_2}{2} \int d^3\xi \sqrt{|h|} |\mathcal{S}| + T_2 \int_{W^3} (\mathcal{A}_3 + \bar{\mathcal{A}}_3 - H_3) + \frac{T_2}{4} \int_{W^2=\partial W^3} d^2\sigma \sqrt{-\gamma} |\Sigma - \bar{\Sigma} - 2iL|. \quad (3.14)$$

#### 4 Open supermembrane, nonlinearly self-interacting double 3-form matter multiplets and supergravity

In this section we would like to consider a coupling of open supermembrane and superstring at the boundary of supermembrane to the dynamical system of  $n$  self-interacting double three form multiplets and supergravity [22] which is of the type appearing in string compactifications. The interaction of closed supermembrane with such a system was studied in [14].

Let us consider a set of  $(n + 1)$  special chiral superfields  $\mathcal{S}^I$ ,  $I = 0, 1, \dots, n$  which are defined by a nonlinear interacting generalization of the above discussed (3.4). We describe them below in eq. (4.4) and now just state that each of them carry two three form fields among their components.

Following [22] and [14], let us consider  $\mathcal{S}^I$  as coordinates of a special Kähler manifold with holomorphic prepotential  $\mathcal{G}(\mathcal{S})$  homogeneous of order two,

$$\mathcal{G}(w\mathcal{S}) = w^2\mathcal{G}(\mathcal{S}). \tag{4.1}$$

Then

$$\mathcal{G}_I(\mathcal{S}) = \partial_I\mathcal{G}(\mathcal{S}) = \mathcal{G}_{IJ}(\mathcal{S})\mathcal{S}^J \tag{4.2}$$

and

$$\mathcal{G}_{IJ}(\mathcal{S}) := \partial_I\partial_J\mathcal{G}(\mathcal{S}) \tag{4.3}$$

are homogeneous of degrees one and zero, respectively. We define our special chiral superfields by [14, 22]

$$\begin{aligned} \mathcal{S}^I &= \frac{1}{4} (\bar{\mathcal{D}}_{\dot{\alpha}}\bar{\mathcal{D}}^{\dot{\alpha}} - R) \mathcal{M}^{IJ}(\Sigma_J - \bar{\Sigma}_J) \\ &= \frac{i}{2} (\bar{\mathcal{D}}_{\dot{\alpha}}\bar{\mathcal{D}}^{\dot{\alpha}} - R) \mathcal{M}^{IJ} \Im\mathfrak{m}\Sigma_J, \end{aligned} \tag{4.4}$$

where  $\Sigma_J = (\bar{\Sigma}_J)^*$  are complex linear superfields,

$$(\mathcal{D}^\alpha\mathcal{D}_\alpha - \bar{R})\Sigma_J = 0, \quad (\bar{\mathcal{D}}_{\dot{\alpha}}\bar{\mathcal{D}}^{\dot{\alpha}} - R)\bar{\Sigma}_J = 0, \tag{4.5}$$

and the real symmetric matrix  $\mathcal{M}^{IJ}$  is the inverse of the imaginary part of (4.3),

$$\mathcal{M}^{IJ}\mathcal{M}_{JK} = \delta^I_K, \quad \mathcal{M}_{IJ} := \Im\mathfrak{m}\mathcal{G}_{IJ}. \tag{4.6}$$

Generically, the relation (4.4) is nonlinear, while in a particular case of  $\mathcal{G}_{IJ}(\mathcal{S}) = i\delta_{IJ}$ ,  $\mathcal{M}^{IJ} = \delta_{IJ}$  and it reduces to the set of  $n + 1$  independent relations (3.4). Notice that the homogeneity of the holomorphic prepotential implies that

$$\begin{aligned} \mathcal{G}_I &= \mathcal{G}_{IJ}(\mathcal{S})\mathcal{S}^J = \frac{1}{4} (\bar{\mathcal{D}}_{\dot{\alpha}}\bar{\mathcal{D}}^{\dot{\alpha}} - R) \mathcal{G}_{IK}\mathcal{M}^{KJ}(\Sigma_J - \bar{\Sigma}_J) \\ &= \frac{i}{2} (\bar{\mathcal{D}}_{\dot{\alpha}}\bar{\mathcal{D}}^{\dot{\alpha}} - R) \Im\mathfrak{m}(\mathcal{G}_{IK}\mathcal{M}^{KJ}\Sigma_J), \end{aligned} \tag{4.7}$$

so that the composite chiral superfield  $\mathcal{G}_I(\mathcal{S})$  in (4.2) is also special, of the type define in eq. (3.4).

This observation makes manifest that the composite chiral superfield

$$\mathcal{S} = q_I\mathcal{S}^I - p^I\mathcal{G}_I(\mathcal{S}) \tag{4.8}$$

with constants  $q_I$  and  $p^I$  is also special. Namely, it can be defined by equation (2.8) with  $\mathcal{Z} = \frac{i}{2}\mathcal{S}$  and real prepotential given by a linear combination

$$\mathcal{P} = q_I\mathcal{P}^I - p^I\tilde{\mathcal{P}}_I \tag{4.9}$$

of the composite real superfields

$$\mathcal{P}^I \equiv \Im(\mathcal{M}^{IJ}\Sigma_J), \quad \tilde{\mathcal{P}}_I \equiv \Im(\mathcal{G}_{IJ}\mathcal{M}^{JK}\Sigma_K) \quad (4.10)$$

which serve as prepotentials for  $\mathcal{S}^I$  and  $\mathcal{G}_I(\mathcal{S})$  (see (4.4) and (4.7)),

$$\mathcal{S}^I = \frac{i}{2}(\bar{\mathcal{D}}^2 - R)\mathcal{P}^I, \quad \mathcal{G}_I(\mathcal{S}) = \frac{i}{2}(\bar{\mathcal{D}}^2 - R)\tilde{\mathcal{P}}_I. \quad (4.11)$$

The composite superfield (4.8) has been used in [14] to couple the closed supermembrane to double three form matter and supergravity. Here we will be using it in the open supermembrane action (2.1) setting  $-2iZ \mapsto \mathcal{S} = q_I\mathcal{S}^I - p^I\mathcal{G}_I(\mathcal{S})$  (4.8) and  $\mathcal{P} \mapsto q_I\mathcal{P}^I - p^I\tilde{\mathcal{P}}_I$  (4.9).

The supermembrane action in the background of nonlinearly self-interacting three form matter and supergravity reads

$$S_{p=2}(q_I, p^I) = \frac{1}{2} \int d^3\xi \sqrt{|\hat{g}|} |q_I\mathcal{S}^I - p^I\mathcal{G}_I(\mathcal{S})| + q_I \int_{W^3} \mathcal{C}_3^I - p^I \int_{W^3} \tilde{\mathcal{C}}_{3I} \quad (4.12)$$

where  $\mathcal{C}_3^I$  and  $\tilde{\mathcal{C}}_{3I}$  have the form of (2.9) with  $\mathcal{P}^I$  and  $\tilde{\mathcal{P}}_I$  from eqs. (4.10),  $\mathcal{S}^I$  is given by (4.4) and  $\mathcal{G}_I(\mathcal{S})$  is defined in (4.2) and has the form of (4.7) due to (4.1).

It is not difficult to check that the special chiral superfield (4.4) does not change under the transformations

$$\delta\Sigma_I = \tilde{\mathbb{L}}_I + \mathcal{G}_{IJ}\mathbb{L}^J, \quad \delta\bar{\Sigma}_I = \tilde{\mathbb{L}}_I + \bar{\mathcal{G}}_{IJ}\mathbb{L}^J, \quad (4.13)$$

with real linear superfields  $\mathbb{L}^I$  and  $\tilde{\mathbb{L}}_I$ ,

$$(\bar{\mathcal{D}}_{\dot{\alpha}}\bar{\mathcal{D}}^{\dot{\alpha}} - R)\mathbb{L}^I = 0 = (\mathcal{D}^{\alpha}\mathcal{D}_{\alpha} - \bar{R})\mathbb{L}^I, \quad (\bar{\mathcal{D}}_{\dot{\alpha}}\bar{\mathcal{D}}^{\dot{\alpha}} - R)\tilde{\mathbb{L}}_I = 0 = (\mathcal{D}^{\alpha}\mathcal{D}_{\alpha} - \bar{R})\tilde{\mathbb{L}}_I. \quad (4.14)$$

As in a simpler cases discussed in previous section, (4.13) generate a particular case of the gauge transformations of super-3-forms,

$$\delta\mathcal{C}_3^I = d\beta_2^I, \quad \delta\tilde{\mathcal{C}}_{3I} = d\tilde{\beta}_{2I} \quad (4.15)$$

with  $d\beta_2^I$  and  $d\tilde{\beta}_{2I}$  expressed in terms of  $\mathbb{L}^I$  and  $\tilde{\mathbb{L}}_I$  as in (2.13).

Thus for closed supermembrane the above action (4.12) is invariant under (4.13). To reach the same for an open supermembrane we should add to (4.12) with  $\partial W^3 \neq \emptyset$  the action of superstring the worldsheet of which is the boundary of the worldvolume,  $W^2 = \partial W^3$ ,

$$S_{p=1}(q_I, p^I) = \frac{1}{2} \int_{W^2} d^2\sigma \sqrt{-\gamma} |q_I(\mathcal{P}^I - L^I) - p^I(\mathcal{P}_I - \tilde{L}_I)| - q_I \int_{W^2} B_2^I + p^I \int_{W^2} \tilde{B}_{2I}. \quad (4.16)$$

In this boundary term of the supermembrane action  $\mathcal{P}^I$  and  $\mathcal{P}_I$  are ‘electric’ and ‘magnetic’ parts of the composite real prepotentials (4.9) defined by eqs. (4.10), and  $H_3^I = dB_2^I$  and  $\tilde{H}_{3I} = d\tilde{B}_{2I}$  are expressed in terms of the real linear superfields  $L^I$  and  $\tilde{L}_I$  as in (2.31).

The sum of the open supermembrane and superstring actions (4.12) and (4.16) is invariant under the  $\kappa$ -symmetry (2.14) provided the parameters obey

$$\kappa_{\alpha} = -i \frac{q_I\mathcal{S}^I - p^I\mathcal{G}_I(\mathcal{S})}{|q_I\mathcal{S}^I - p^I\mathcal{G}_I(\mathcal{S})|} \Gamma_{\alpha\dot{\alpha}}\bar{\kappa}^{\dot{\alpha}}, \quad \bar{\kappa}_{\dot{\alpha}} = -i \frac{q_I\bar{\mathcal{S}}^I - p^I\bar{\mathcal{G}}_I(\bar{\mathcal{S}})}{|q_I\bar{\mathcal{S}}^I - p^I\bar{\mathcal{G}}_I(\bar{\mathcal{S}})|} \kappa^{\alpha}\Gamma_{\alpha\dot{\alpha}}, \quad (4.17)$$



and

$$\begin{aligned}\kappa_\alpha &= \frac{q_I(\mathcal{P}^I - L^I) - p^I(\mathcal{P}_I - \tilde{L}_I)}{|q_I(\mathcal{P}^I - L^I) - p^I(\mathcal{P}_I - \tilde{L}_I)|} P_\alpha{}^\beta \kappa_{\beta}, \\ \bar{\kappa}_{\dot{\alpha}} &= \frac{q_I(\mathcal{P}^I - L^I) - p^I(\mathcal{P}_I - \tilde{L}_I)}{|q_I(\mathcal{P}^I - L^I) - p^I(\mathcal{P}_I - \tilde{L}_I)|} \bar{P}_{\dot{\alpha}}{}^{\dot{\beta}} \bar{\kappa}_{\dot{\beta}},\end{aligned}\tag{4.18}$$

with the projectors defined in (2.16) and (2.39).

The sum of the open supermembrane and superstring actions is also invariant under the gauge symmetry (4.13) supplemented by the Stückelberg-type transformations of the real linear superfields:

$$\delta\Sigma_I = \tilde{\mathbb{L}}_I + \mathcal{G}_{IJ}\mathbb{L}^J, \quad \delta L^I = \mathbb{L}^I, \quad \delta\tilde{L}_I = \tilde{\mathbb{L}}_I.\tag{4.19}$$

These leave invariant the chiral superfields (4.4) as well as the combinations

$$\Sigma_I - \tilde{L}_I - \mathcal{G}_{IJ}L^J.\tag{4.20}$$

A quite general action for nonlinearly self interacting system of the 3-form multiplets and supergravity, of a kind which appear in string compactifications, with a spontaneously broken (realized by Stückelberg mechanism) 3-form gauge symmetry reads

$$\begin{aligned}S_{\text{sugra+matter}} &= -\frac{3}{4\kappa^2} \int d^8z E \Omega(\mathcal{S}^I, \bar{\mathcal{S}}^I) - \frac{1}{2\kappa^2} \left( \int d^6\zeta_L \mathcal{E} \mathcal{W}(\mathcal{S}^I) + \text{c.c.} \right) - \\ &\quad - c_2 \int d^8z E \frac{\mathcal{M}^{IJ} \left( \Sigma_I - \tilde{L}_I - \mathcal{G}_{IK}L^K \right) \left( \bar{\Sigma}_J - \tilde{L}_J - \mathcal{G}_{JL}L^L \right)}{(\mathcal{S}^P \mathcal{M}_{PQ} \bar{\mathcal{S}}^Q)^{\frac{1}{3}}}.\end{aligned}\tag{4.21}$$

This can be included as matter plus supergravity part in the interacting action (1.1) together with the open supermembrane and superstring at the boundary of supermembrane actions (4.12) and (4.16). Such an action will be invariant under the  $\text{Sp}(2n+2|\mathbb{Z})$  symmetry, characteristic for string compactifications, provided  $\Omega(\mathcal{S}^I, \bar{\mathcal{S}}^I)$  and  $\mathcal{W}(\mathcal{S}^I)$  are invariant and the supermembrane charges  $(p^I, q_I)$  in (4.12) and (4.16) are transformed as symplectic vector. Actually the quantization of these charges breaks the possible  $\text{Sp}(2n+2|\mathbb{R})$  symmetry of the supergravity plus matter (super)field system to its discrete subgroup  $\text{Sp}(2n+2|\mathbb{Z})$  (see [14] for a discussion of quantization of  $(p^I, q_I)$  without a reference to the higher dimensional origin of the D=4 domain wall system).

## 5 Conclusion

In this paper we present the actions describing the interacting dynamical system of open supermembrane, quite generic 3-form matter and supergravity. The similar interacting system containing closed supermembrane was studied in [14]. The action of open supermembrane in Veneziano-Yankelovich effective theory of  $\mathcal{N} = 1$  SYM was discussed in [15].

We have begun by writing the action for the most general interacting system of open supermembrane, single three form matter and supergravity, which serves as a master case

for the derivation of the actions for more complicated systems. A particular case of this action describes the interacting system of open supermembrane and single three form supergravity.

Then we describe the interacting actions for the open supermembrane interacting with double 3-form matter multiplet and supergravity. Finally, we present the action for a quite generic system of open supermembrane, a number of nonlinearly interacting double three form multiplets and supergravity which has a special Kähler structure and possesses an invariance under symplectic transformations; the dynamical field theoretical systems of such a type appear in string theory compactifications.

To preserve the  $\kappa$ -symmetry, the open supermembrane action should include a boundary term which have a natural interpretation of the action for superstring at the boundary of supermembrane. In contrast with the  $\kappa$ -symmetry, the three form gauge symmetry is broken when the supermembrane is open. This breaking can be interpreted as spontaneous and the 3-form gauge symmetry can be formally maintained with the use of Stückelberg mechanism. To this end we have to introduce the pure gauge real linear superfield(s)  $L$  ( $L^\Lambda = (L^I, \tilde{L}_I)$ ) and the Wess-Zumino term of the action of superstring at the end of supermembrane is constructed with the use of this (these) supermultiplet(s). The gauge fixed version of this superstring action contains the Nambu-Goto-type term only and the tension of superstring at the end of supermembrane is expressed in terms of real prepotential superfield(s)  $\mathcal{P}$  ( $\mathcal{P}^\Lambda$ ), either fundamental or composite, of the special chiral superfield(s) describing 3-form matter supermultiplet(s) and/or conformal compensator of the 3-form supergravity interacting with the open supermembrane.

As 3-form gauge symmetry is broken due to the presence of open supermembrane, or realized by Stückelberg mechanism, we find natural to consider also the terms breaking the 3-form gauge symmetry in the supergravity plus matter part of the action: the generalized mass terms.

The inclusion of the actions of open supermembranes with closed strings at the boundary of supermembranes into effective field theories (EFT) of string theory flux compactifications is inline with the completeness conjecture [52].<sup>5</sup> The EFT action of such a type can be obtained e.g. from type IIB compactifications on wrapped Calabi-Yau manifolds of the dynamical systems including the networks of D7-, D5- and D3-branes discussed in [42] and [43]. In recent [53], appearing on the net slightly after the first version of the present paper, the actions for network of open supermembranes and strings were obtained independently and completed by an interesting actions for spacetime filling 3-branes which are given by a Wess-Zumino-type terms only and do not break any supersymmetry.

To resume, we believe that the actions presented in this paper as well as its generalizations discussed in [53] will be useful to construct the effective actions for phenomenologically interesting models of string theory compactifications with open branes and branes at the

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<sup>5</sup>Actually in [52] Polchinski proposed two completeness principles, the second of which, most relevant for our discussion, states that “in any fully unified theory, for every gauge field there will exist electric and magnetic sources” obeying the Dirac quantization conditions  $eg = 2\pi n$  “with the minimum relative Dirac quantum  $n = 1$  (more precisely, the lattice of electric and magnetic charges is maximal)” [52]. The first of the completeness principles relate any charge quantization to the existence of magnetic monopoles.

boundary of open branes. The natural next step in development of our formalism is to obtain equations of motion for 3-form matter and supergravity from our action and to search for their solution describing open supermembrane systems and supermembrane junctions.

## Acknowledgments

The author is thankful to Sergei Kuzenko and especially to Dima Sorokin for useful conversations and for collaboration at early stages of this work which was supported in part by the Spanish MINECO/FEDER (ERDF EU) grant PGC2018-095205-B-I00, by the Basque Government Grant IT-979-16, and by the Basque Country University program UFI 11/55.

## A Torsion constraints of minimal supergravity

Our notations are those of [11, 12, 45]; they are close but not identical to that of [51]. In particular we use the mostly minus metric conventions,  $\eta_{ab} = \text{diag}(+, -, -, -)$  and the set of our superspace constraints contains  $R_{\alpha\dot{\beta}}{}^{cd} = 0$  instead of  $T_{ab}{}^c = 0$  in [51].

The superspace constraints of minimal supergravity and their consequences can be collected in the following expressions for superspace torsion and curvature 2-forms

$$T^a = \mathcal{D}E^a = -2i\sigma_{\alpha\dot{\alpha}}^a E^\alpha \wedge \bar{E}^{\dot{\alpha}} - \frac{1}{8}E^b \wedge E^c \varepsilon^a{}_{bcd} G^d, \quad (\text{A.1})$$

$$T^\alpha = \mathcal{D}E^\alpha = \frac{i}{8}E^c \wedge E^\beta (\sigma_c \tilde{\sigma}_d)_\beta{}^\alpha G^d - \frac{i}{8}E^c \wedge \bar{E}^{\dot{\beta}} \epsilon^{\alpha\beta} \sigma_{c\beta\dot{\beta}} R + \frac{1}{2}E^c \wedge E^b T_{bc}{}^\alpha, \quad (\text{A.2})$$

$$T^{\dot{\alpha}} = \mathcal{D}\bar{E}^{\dot{\alpha}} = \frac{i}{8}E^c \wedge E^\beta \epsilon^{\dot{\alpha}\dot{\beta}} \sigma_{c\beta\dot{\beta}} \bar{R} - \frac{i}{8}E^c \wedge \bar{E}^{\dot{\beta}} (\tilde{\sigma}_d \sigma_c)^{\dot{\alpha}}{}_{\dot{\beta}} G^d + \frac{1}{2}E^c \wedge E^b T_{bc}{}^{\dot{\alpha}}, \quad (\text{A.3})$$

$$R^{ab} = \frac{1}{2}R^{\alpha\beta} (\sigma^a \tilde{\sigma}^b)_{\alpha\beta} - \frac{1}{2}R^{\dot{\alpha}\dot{\beta}} (\tilde{\sigma}^a \sigma^b)_{\dot{\alpha}\dot{\beta}}, \quad (\text{A.4})$$

$$\begin{aligned} R^{\alpha\beta} \equiv \frac{1}{4}R^{ab} \sigma_{ab}{}^{\alpha\beta} &= -\frac{1}{2}E^\alpha \wedge E^\beta \bar{R} - \frac{i}{8}E^c \wedge E^{(\alpha} \tilde{\sigma}_c{}^{\dot{\gamma}\beta)} \bar{\mathcal{D}}_{\dot{\gamma}} \bar{R} + \\ &+ \frac{i}{8}E^c \wedge E^\gamma (\sigma_c \tilde{\sigma}_d)_\gamma{}^{(\beta} \mathcal{D}^\alpha) G^d - \frac{i}{8}E^c \wedge \bar{E}^{\dot{\beta}} \sigma_{c\dot{\gamma}\dot{\beta}} W^{\alpha\beta\dot{\gamma}} + \frac{1}{2}E^d \wedge E^c R_{cd}{}^{\alpha\beta}, \end{aligned} \quad (\text{A.5})$$

$R^{\dot{\alpha}\dot{\beta}} = (R^{\alpha\beta})^*$ , and in the following equations for main superfields

$$\mathcal{D}_\alpha \bar{R} = 0, \quad \bar{\mathcal{D}}_{\dot{\alpha}} R = 0, \quad (\text{A.6})$$

$$\bar{\mathcal{D}}^{\dot{\alpha}} G_{\alpha\dot{\alpha}} = -\mathcal{D}_\alpha R, \quad \mathcal{D}^\alpha G_{\alpha\dot{\alpha}} = -\bar{\mathcal{D}}_{\dot{\alpha}} \bar{R}, \quad (\text{A.7})$$

$$\bar{\mathcal{D}}_{\dot{\alpha}} W^{\alpha\beta\dot{\gamma}} = 0, \quad \mathcal{D}_\alpha \bar{W}^{\dot{\alpha}\dot{\beta}\dot{\gamma}} = 0, \quad (\text{A.8})$$

$$\mathcal{D}_\gamma W^{\alpha\beta\dot{\gamma}} = \bar{\mathcal{D}}_{\dot{\gamma}} \mathcal{D}^{(\alpha} G^{\beta)\dot{\gamma}}, \quad \bar{\mathcal{D}}_{\dot{\gamma}} \bar{W}^{\dot{\alpha}\dot{\beta}\dot{\gamma}} = \mathcal{D}_\gamma \bar{\mathcal{D}}^{(\dot{\alpha}} G^{\dot{\beta})\dot{\gamma}}. \quad (\text{A.9})$$

As a consequence, the superalgebra of superspace covariant derivatives  $\mathcal{D}_A$  (2.6) in the case of minimal supergravity contains the following anticommutators of fermionic covariant derivatives

$$\{\mathcal{D}_\alpha, \bar{\mathcal{D}}_{\dot{\alpha}}\} = 2i\sigma_{\alpha\dot{\alpha}}^a \mathcal{D}_a, \quad (\text{A.10})$$

$$\{\mathcal{D}_\alpha, \mathcal{D}_\beta\} V_\gamma = -\bar{R} \epsilon_{\gamma(\alpha} V_{\beta)}, \quad (\text{A.11})$$

$$\{\bar{\mathcal{D}}_{\dot{\alpha}}, \bar{\mathcal{D}}_{\dot{\beta}}\} V_{\dot{\gamma}} = R \epsilon_{\dot{\gamma}(\dot{\alpha}} V_{\dot{\beta})}. \quad (\text{A.12})$$

## B Super-Weyl symmetry of the minimal supergravity constraints

The super-Weyl transformations with covariantly chiral superfield parameter  $\Upsilon$ ,

$$\bar{\mathcal{D}}_{\dot{\alpha}}\Upsilon = 0, \quad (\text{B.1})$$

which leave invariant minimal (and 3-form) supergravity constraints, as well as, for instance, the action (2.18) with (2.19) and (2.1), are defined by [54]

$$E^a \mapsto \tilde{E}^a = e^{\Upsilon + \bar{\Upsilon}} E^a, \quad (\text{B.2})$$

$$E^\alpha \mapsto \tilde{E}^\alpha = e^{2\bar{\Upsilon} - \Upsilon} \left( E^\alpha - \frac{i}{2} E^a \bar{\mathcal{D}}_{\dot{\alpha}} \bar{\Upsilon} \tilde{\sigma}_a^{\dot{\alpha}\alpha} \right), \quad (\text{B.3})$$

$$\bar{E}^{\dot{\alpha}} \mapsto \tilde{\bar{E}}^{\dot{\alpha}} = e^{2\Upsilon - \bar{\Upsilon}} \left( \bar{E}^{\dot{\alpha}} - \frac{i}{2} E^a \tilde{\sigma}_a^{\dot{\alpha}\alpha} \mathcal{D}_\alpha \Upsilon \right), \quad (\text{B.4})$$

$$\mathcal{Z} \mapsto e^{-6\Upsilon} \mathcal{Z}. \quad (\text{B.5})$$

It is useful to notice that under these transformations

$$(\bar{\mathcal{D}}\bar{\mathcal{D}} - R) \dots \mapsto e^{-4\Upsilon} (\bar{\mathcal{D}}\bar{\mathcal{D}} - R) e^{2\bar{\Upsilon}} \dots, \quad (\text{B.6})$$

$$\mathcal{P} \mapsto e^{-2\Upsilon - 2\bar{\Upsilon}} \mathcal{P}, \quad (\text{B.7})$$

$$E \mapsto e^{2\Upsilon + 2\bar{\Upsilon}} E, \quad (\text{B.8})$$

$$\mathcal{E} \mapsto e^{6\Upsilon} \mathcal{E}, \quad (\text{B.9})$$

$$\mathcal{W} \mapsto e^{-6\Upsilon} \mathcal{W}. \quad (\text{B.10})$$

## C Useful equations on supermembrane worldvolume

The orientation of the volume element of  $W^3$  is defined by

$$d\xi^m \wedge d\xi^n \wedge d\xi^k = d^3\xi \epsilon^{mnk}, \quad \epsilon^{012} = 1, \quad (\text{C.1})$$

The invariant measure with induced metric on  $W^3$  can be written in the following equivalent form

$$d^3\xi \sqrt{h} = -\frac{1}{3} * E_a \wedge E^a, \quad d^3\xi \delta \sqrt{h} = - * E_a \wedge \delta E^a \quad (\text{C.2})$$

with the Hodge star operation defined by

$$*E^a = \frac{1}{2} d\xi^n \wedge d\xi^m \sqrt{h} \epsilon_{mnk} h^{kl} E_l^a. \quad (\text{C.3})$$

The definition of the  $\kappa$ -symmetry projector in (2.16) implies

$$d^3\xi \sqrt{h} \Gamma_{\alpha\dot{\alpha}} = \frac{i}{3!} \sigma_{\alpha\dot{\alpha}}^a \epsilon_{abcd} E^b \wedge E^c \wedge E^d, \quad (\text{C.4})$$

$$*_3 E_a \wedge E^\beta (\sigma^a \tilde{\Gamma})_{\beta}{}^\alpha = -\frac{1}{2} E^b \wedge E^c \wedge E^\beta (\sigma_{bc})_{\beta}{}^\alpha. \quad (\text{C.5})$$

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