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Review article

Critical integration in neural and cognitive systems: Beyond power-law scaling as the hallmark of soft assembly

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ABSTRACT

Inspired by models of self-organized criticality, a family of measures quantifies long-range correlations in neural and behavioral activity in the form of self-similar (e.g., power-law scaled) patterns across a range of scales. Long-range correlations are often taken as evidence that a system is near a critical transition, suggesting interaction-dominant, softly assembled relations between its parts. Psychologists and neuroscientists frequently use power-law scaling as evidence of critical regimes and soft assembly in neural and cognitive activity. Critics, however, argue that this methodology operates at most at the level of an analogy between cognitive and other natural phenomena. This is because power-laws do not provide information about a particular system's organization or what makes it specifically cognitive. We respond to this criticism using recent work in Integrated Information Theory. We propose a more principled understanding of criticality as a system's susceptibility to changes in its own integration, a property cognitive agents are expected to manifest. We contrast critical integration with power-law measures and find the former more informative about the underlying processes.

1. Should we expect cognitive agents to behave like critical systems?

Cognitive performance is the result of many interacting processes comprising brain, body, and environment (e.g., Beer, 2000, 2008; Byrge et al., 2014; Chiel and Beer, 1997). These interactions can be complex and nonlinear, and often require explanations in terms of emergent properties and global parameters (e.g., Kelso, 1995; Turkheimer et al., 2019). In general, however, even when not explicitly advocated, a framework of modular thinking prevails when seeking functionalist explanations of cognitive phenomena. Such explanations rely on the assumption of near-decomposability (Simon, 1996). Cognitive function is assumed to be the result of dedicated information-processing mechanisms typically associated with a particular kind of cognitive capability. Empirical observations repeatedly challenge assumptions of modularity by showing that cognitive performance is embedded in a series of correlations occurring at various scales, both in terms of the processes involved (neural, musculoskeletal, hormonal, interactive, ecological, social) as well as timescales (coordination in neural population, microgenesis of movement and perception, streams of behavior, learning, habits, development, and so on; Anderson et al., 2012; Kello et al., 2008, 2010). Such cross-scale correlations are not expected in nearly-decomposable systems (Simon, 1996). This fact leads to alternative proposals concerning the organization of environmentally situated brains and bodies. A cognitive organization that fits empirical observations is one where multiple interacting components can be recombined from moment to moment on the basis of context, task, and developmental history. Such systems are sometimes referred to as displaying multiplicative interactions, interaction-dominant dynamics (Van Orden et al., 2003) or soft assembly (Kello and Van Orden, 2009), in contradistinction to 'near-decomposable' component-dominant dynamics or hard-assembled modular organizations (Simon, 1977).

Several hypotheses have been proposed to account for interaction-dominant dynamics in cognitive systems: e.g., that neural systems operate near a critical point in an order-disorder phase transition (Beggs, 2008; Chialvo, 2010), that the brain is able to integrate functionally differentiated parts by finding states of metastable equilibrium (Kelso, 1995), or that neural networks operate as a 'dynamic core' of regions

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active across multiple tasks and integrating more specialized regions depending on context (Varela, 1995; Bell and Shine, 2016; Shine et al., 2019). An influential reformulation of some of these ideas is provided by Integrated Information Theory (Tononi et al., 2016). Formulated as an answer to the problem of measuring conscious states, Integrated Information Theory proposes that a system is conscious to the extent that it generates integrated information, that is, intrinsic information that is irreducible to the sum of its parts, considered independently. Integrated information, φ, is measured as the distance between the information circulation of the system as a whole and the information circulation under a minimal partition (i.e., the partition that minimizes this distance). Interestingly, different studies show that some systems present maximum integrated information precisely at their critical points (Aguilera and Di Paolo, 2019; Aguilera, 2019; Kim and Lee, 2019; Khajehabdollahi et al., 2019; Arsiwalla et al., 2017). This suggests there is a link between integrated information and criticality.

An empirical test for the presence of interaction-dominant dynamics in cognitive systems is the observation of organized patterns of long-term correlations in performance variables during the execution of repeated tasks. Evidence of self-similarity (power-law scaling) in these correlations has driven some researchers to draw analogies with the physics of phase transitions in aggregate material processes, such as in the case of self-organized criticality (Bak et al., 1987; Beggs, 2008). By this analogy, operating near critical points would be a way for neural and cognitive systems to generate complex spatiotemporal patterns spanning many different correlated scales, implying system-wide coherence and very high sensitivity to perturbations (Muñoz, 2018). The implications of this analogy are profound; it suggests that cognitive systems are softl assembled and mostly characterized by interaction-dominant modes of engagement with the environment in the way many complex natural systems are. Multiple scales, accordingly, would be involved in any cognitive phenomenon (such is the proposal) and the underlying processes cannot be easily compartmentalized into neural activity, bodily dynamics, and agent-environment coupling as (causally and conceptually) distinct explanatory domains. This is in opposition with traditional assumptions which map functions to neural structures/processes in that both the mapping itself is put into question as well as the concept of neatly defined functions that pre-exist cognitive performance in contrast to functions that emerge dynamically *with* the performance (see Anderson et al., 2013; Pessoa, 2014).

Evidence and arguments in favour of this dynamical perspective have been criticized for different reasons. These include (1) empirical concerns relating to measures of power-law scaling, (2) the unwarranted implication that these measures necessarily always entail all or any of the associated properties, such as self-organized criticality, soft assembly, interaction-dominant dynamics, and context sensitivity, and (3) the lack of models or principled arguments whereby we should expect human cognition to behave in analogous ways to other complex, self-organized natural processes. The first and second criticisms are, in our opinion, satisfactorily addressed in Van Orden et al. (2005). There, aside from technical consideration in measuring fractal scaling, the authors propose that the appropriate point of view is to consider fractal scaling as a consequence of criticality and not the opposite. It follows that the relevant level of study should be that of emergent phenomena and fractal scaling themselves.

1.1. Criticality, models, and cognitive processes

The third line of criticism (Farrell et al., 2006; Wagenmakers et al., 2012) asserts that (a) criticality is only well understood in simple and abstract models (e.g., the Ising model, rice piles, the branching process model) which are generally highly homogeneous and operate under very specific boundary conditions, and (b) advocates for power-law scaling approaches simply assume that cognitive systems displaying power-law scaling share properties with these abstract models, and this is not necessarily true. The conclusions drawn, therefore, would seem to operate at best at the level of an analogy or metaphor rather than as principled models that can yield falsifiable predictions (Wagenmakers et al., 2012; Mora and Bialek, 2011; Muñoz, 2018). Note that this does not put into question the role of metaphor in theory and model-making. Valid models may be framed as analogies (as opposed to simulacra) as proposed by Hesse (1966), (see also Morgan and Morrison, 1999). However, some ideas in science play only or mostly the role of metaphors or analogues (a role we do not discount), while others can act as metaphors as well as lead to generative models (which one could consider formalizable metaphors). The criticism expresses the worry that, without the support of specifically cognitive models, power-law scaling approaches fail to suitably explain certain types of criticality as a consequence of relevant properties of cognition.

To put this worry in other words, the mere observation of typical markers of criticality in cognitive systems does not guarantee enough of a distinction between such systems and other physical systems. There is a concern about underspecification and insufficiently justified analogies. This is partially considered in Van Orden et al. (2005), where the authors argue that "[a] fractal account does not necessarily require details of internal mechanisms to answer the theoretical questions that are posed. In the case of opaque complex systems, it is their behavior that motivates the hypothesis of self-organization, not the details of interacting primitives" (p. 122). However, even if microscopic details could be disregarded (and some may contest this), our position is that in cognitive processes there are meso-scale factors that matter and are specifically cognitive (e.g., agent-environment asymmetries, processes of learning and adaptation), and that adequate scaling measures can capture some of these properties better than flat one-dimensional exponents. To fully answer this worry, we should provide an argument concerning the organization of cognitive agents and the reasons that this organization leads us to expect that criticality should be (in the appropriate circumstances) an observed phenomenon in cognitive systems.

We offer an argument to address this last line of criticism. We argue that the main problem with power-law scaling approaches is that they are based on measures that have been developed in models in which the underlying organization is completely 'flat' or homogeneous, and therefore cannot capture specific organizational aspects of cognitive systems. A fundamental feature of cognitive organization, according to various theories of embodied cognition, concerns the ongoing, precarious, and self-sustaining integration of the agent's own identity (Di Paolo et al., 2017; Kauffman, 2000; Juarrero, 2000; Silberstein and Chemero, 2012; Varela, 1997). For instance, Bailly and Longo (2008) propose that living matter behaves in an 'extended critical situation', sustained far from equilibrium by 'the dynamic integration and the regulation of its components' We should expect that this central feature of all extant cognitive and living systems constrains the kinds of viable cognition in general. If we can derive from the basic requirement of active, far from equilibrium integration an expectation that such systems will show properties of criticality, soft assembly, etc. then we will have shown that the empirical observations verify an expected consequence of how cognitive agents are put together. The arrow would be inverted from an inference about mechanisms following observed critical behavior, to a verification of a (particular kind of) critical behavior that should be expected in self-integrating systems. In addition, cognitive systems would be located within but also distinguished from the rest of the family of physical systems that exhibit critical behavior (since not all of them exhibit the property of dynamic integration).

1.2. Alternative indices of fractal scaling

Before expanding this argument, we shall admit that picture portrayed above about flat exponents of criticality is, to some extent, a simplification. Although much work relies on uni-dimensional scaling exponents, a variety of more complex methods exist (Kello et al., 2010).

Multifractal scaling indices, for instance, extend critical exponents into a continuous dimension of exponents, offering a richer characterization of the different temporal scales of a system. It has been argued that they better characterize the multiplicative interactions in a cognitive systems (Ihlen and Vereijken, 2010). Similarly, recurrence quantification analyses offer better characterizations of the nonlinear dynamics of a system (Wallot et al., 2015). These more elaborated methods, however, are susceptible to the same criticisms we have discussed: it is in general not obvious how to connect the observed indices with the underlying, specifically cognitive processes or how these underlying processes relate to the analytical solutions of simpler models of criticality.

Closer to our perspective, some techniques are also more precise in depicting the relationships between different systems or subsystems. For instance, complexity matching analysis shows that interacting systems tend to attune their behavior at different scales and so enhance their coordination and maximize information transfer (Abney et al., 2014). It has been noted however that in some cases the matching of long-term fluctuations could be the result of short-term coupling processes (Delignières and Marmelat, 2014), and that apparently similar matching phenomena could in principle be divided between simple statistical matching and genuine complexity matching, without fractal exponents giving us information about one situation or the other. Still, some methods have been proposed that could overcome these difficulties (Almurad et al., 2018). For example, detrended cross-correlation analysis (a generalization of detrended fluctuation analysis) allows the investigation of power-law cross-correlations between time series in the presence of nonstationarity (Podobnik and Stanley, 2008). It has been proposed that such cross-correlation analysis, in combination with different time lags, can be used to describe the type of interaction in complexity matching processes (Almurad et al., 2018). Multifractal signatures in complexity matching have also been proposed to provide a more detailed view of the interaction between processes (Delignières et al., 2016). We believe that these advances move in the right direction, from flat fractal exponents to more detailed depictions of the non-uniform interactions in complex processes. Still, the problem lies in that it is not obvious how to interpret these measures and, as their proponents recognise, caution is needed (Delignières et al., 2016). One of the advantages of fractal exponents, is that at least in toy models they have a direct relation with the analytical exponents (e.g., the behavior of order parameters or the susceptibility of a system) derived from the analysis of critical phase transitions. These exponents can even be linked to families of phenomena like the Ising or percolation universality classes (Fisher, 1974). Although these different measures advance in solving some of the shortcomings of fractal exponents for understanding cognitive systems, they move further away from our analytical understanding of critical phase transitions and the insights found in models of criticality.

The criticism that we wish to respond to, and that is not entirely misguided in our view, is the one that says that ideas such as soft assembly and criticality are only metaphorical and that the often-invoked analogy with physical systems that show similar observable characteristics lacks cognitive specificity. We answer this criticism by providing a conceptual intermediary justification–in the form of models that can capture cognitive relevant properties, such as agent-environment asymmetries, and the requirement of sustained integration–that addresses the question of why we should expect certain behavior from specifically cognitive systems, and this goes beyond (but does not negate) the analogical links to physical systems.

We propose that the development of indices of criticality that are connected with the organizational structure of a cognitive system can address this gap between the observable properties and the specific underlying processes that generate them. We support our claim by reviewing recent analytical work of this type of measures in kinetic Ising models of infinite size (Aguilera and Di Paolo, 2019), connected with work in Integrated Information Theory. We model the existence of differentiated functional regions and show how integrated information φ at different parts of the system is related to the specific underlying organization. In this way, the proposed model, while remaining abstract, captures aspects of the level of autonomy and agency of different parts of a system that have been proposed in theories of embodied cognition (Di Paolo et al., 2017). Finally, we comment on how this kind of work can help integrate different views concerning the properties of neural organization.

2. Integrated information as susceptibility to organizational changes near criticality

Systems operating in a critical regime generate power-law scaling only for specific dependent variables and under specific conditions. Different methods can detect the presence of power-law scaling in the system's variables, but they do not provide a clear picture of how these variables interact or what role they play (if any) in the emergence of the critical phenomenon. For this reason, claims about soft assembly or interaction-dominant dynamics after detecting power-law scaling can be quite vague. Moreover, such claims make it difficult to address questions about what is and what is not part of the interaction-dominant system: Can an observed variable display power-law scaling without being part of an interaction-dominant ensemble? And the other way around? The problem with uniform power-law scaling analyses is that they measure a one-dimensional index, a critical exponent, that describes how close the fluctuation of a signal is to scale invariance. To capture the specificity of the organization of a system, a measure needs to have some capacity to reveal relational properties between its components. This should not be mistaken with a reductionist ambition of disassembling an emergent process into independent components. But if the emergent phenomena of fractal scaling in cognitive phenomena is the object of study, we should at least be able to identify the boundaries of the irreducible interactive system producing it. By themselves, power-law measures are inadequate for this task. And even if heuristic methods can be developed, as in cases like complexity matching experiments, the interpretation of the results is far from obvious (Delignières et al., 2016).

Instead of power-law scaling and long-range correlations, critical behavior can be more precisely described by a divergence of the system's susceptibility or Fisher information of the system, depicting a second order phase transition (Binder, 1987; Salinas, 2001; Prokopenko et al., 2011). The susceptibility of a system is a measure of how sensitive a quantity of the system is to changes in the underlying parameters (e.g., how much a material will become magnetized if we change a magnetic field). We can think of classical models of ferromagnetic phase transitions (e.g., Ising models). The simplest Ising model showing a critical phase transition is the Curie–Weiss magnet (also known as the infinite-range Ising model, Kochmański et al., 2013). In this model, units are influenced by an external magnetization H and all of them are coupled to other units with a coupling parameter J. If we consider the average activity of units for $H = 0$ for a model of infinite size (Fig. 1A), we observe that there is a critical point at $J = 1$, in which there is a symmetry-breaking transition that goes from a disordered state ($m = 0$) to an ordered state (in which m is polarized to either a positive or negative value). This critical point is characterized by power-law distributions of different variables of the model (Kochmański et al., 2013). For example, if we think of the duration of 'avalanches' in the system, the equivalent kinetic Curie–Weiss model of infinite size is equivalent to a branching process (Slade, 2008) where the size of an avalanche has the form $P(S) \sim \frac{1}{\sqrt{\pi}} S^{-1.5}$ (Fig. 1B). It is well known that the magnetic susceptibility – the derivative of the magnetization with respect to an applied magnetic field – has a power-law singularity diverging at the critical point (Fig. 1C, dashed). Similarly, the susceptibility with respect to the coupling constant J (Fig. 1C, solid) also diverges at the critical point. A high susceptibility in these examples implies that the behavior of the system (i.e. its average magnetization) is going to change more significantly with changes in the parameters than in cases of lower susceptibility. This critical transition is essentially a one-dimensional phenomenon in a system with a homogeneous distribution, in which

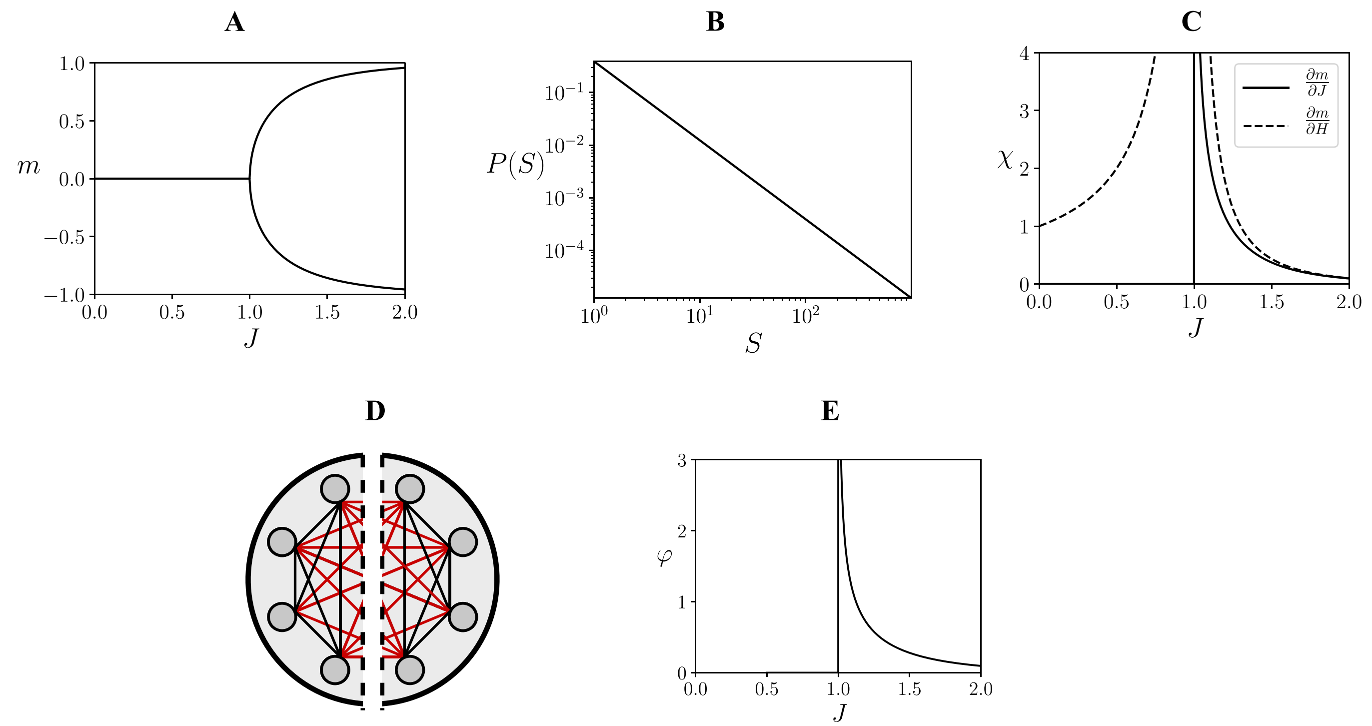

Fig. 1. Behavior of an infinite-range Ising model with a critical point. (A) Values of average activity m respect to couplings J, showing a critical point at $J = 1$. (B) Power law distribution of avalanche size S at the critical point $J = 1$, where probability avalanche size is defined by $P(S) \sim \frac{1}{\sqrt{\pi}} S^{-1.5}$. (C) Susceptibility respect to the magnetic field $\frac{\partial m}{\partial H}$ (dashed line) and respect to the coupling value $\frac{\partial m}{\partial J}$, both diverging at the critical point. (D) Example of a partition removing part of the connections in the system. Red lines represent couplings affected by the partition and black lines those not affected. (E) Integrated information φ for the minimum partition, diverging at the critical point similarly to $\frac{\partial m}{\partial J}$.

changes in one parameter (e.g., external field, temperature, or coupling strength) define a second order phase transition (Salinas, 2001).

We can apply this more principled characterization of criticality to neural systems. Recent work on Integrated Information Theory (Aguilera and Di Paolo, 2019; Aguilera, 2019) has shown that (in some models and under specific assumptions) integrated information can be described by a type of susceptibility that is no longer one-dimensional but that takes into account the different ways in which a system can be decomposed. The conclusion is that a system is more integrated when it is more susceptible to parametric changes caused by partitions in the system. A partition consists in breaking the coupling between two parts of the system (and replacing their mutual influence by uncorrelated noise, see Fig. 1D). Specifically, integrated information depends on the susceptibility of the system in the direction of the parameter space determined by the minimum information partition (i.e., the partition that has the least susceptibility, Fig. 1E). Susceptibility can be measured in different directions of the parameter space, and the minimum information partition indicates the partition that defines the least susceptible direction for all possible partitions. In other words, if we think of all the possible interventions we could make in the internal processes that compose a system, then the system would be highly integrated if it is very sensitive to any of the interventions that challenge its integrity. This is equivalent to saying that a system showing high integrated information is one that is very susceptible to events that affect the coherence of its internal structure.

Following this, we can offer a different reading of what it means for a cognitive system to be in a critical state. Although a system can be in a critical point with respect to an external parameter (e.g., temperature), a system can also be critical with respect to the parameters of its internal organization (e.g., internal couplings). This is a sense of criticality that connects more directly to questions about a cognitive system's organization such as whether it is softly assembled or interaction-dominant. We believe that the extended characterization of critical transitions as susceptibilities in multiple directions, in particular the ones that correspond to different possible partitions of the system, can bridge the gap between macroscopic descriptions of scale-invariant dynamics and the underlying processes. This would help address the worry that conclusions about a system's organization from observed power laws are too general to distinguish cognitive systems from other physical systems. Integrated information can give a better characterization of the interaction dominant-structure of a specifically cognitive system. We illustrate this idea with an example.

3. Power-law scaling vs integrated information in an agent-environment system

To clarify the difference between power-law scaling and an integrated information approach to criticality, we use an example we present elsewhere (Aguilera and Di Paolo, 2019). The model shows how susceptibility at critical points describes the degree of individual and joint integration of two interacting systems composed of a large number of interacting units. There is a system A (an 'agent') in interaction with an 'environment' E (Fig. 2A). Units in A, representing different internal variables, have a recurrent coupling value J_R. Variables (units) in E are only connected to units in A, with bidirectional coupling values J_C. We have shown analytically that the system is at a critical point when $J_R = 2 - J_C^2$ (Fig. 2). At this critical point, both the agent A and the joint system AE (composed of both A and E) reach the point of maximum information integration φ, i.e., maximally interaction-dominant dynamics (Fig. 2B and C shows φ for $J_C = 0.8$ and $J_C = 1.2$).

How easy is it to capture the underlying organization of the system from measuring data series using power-law scaling approaches and using integrated information? In other words, how easy is it to delineate the boundary that divides an agent and the environment? For infinite systems, φ can be computed analytically. We show that φ_A and φ_{AE}

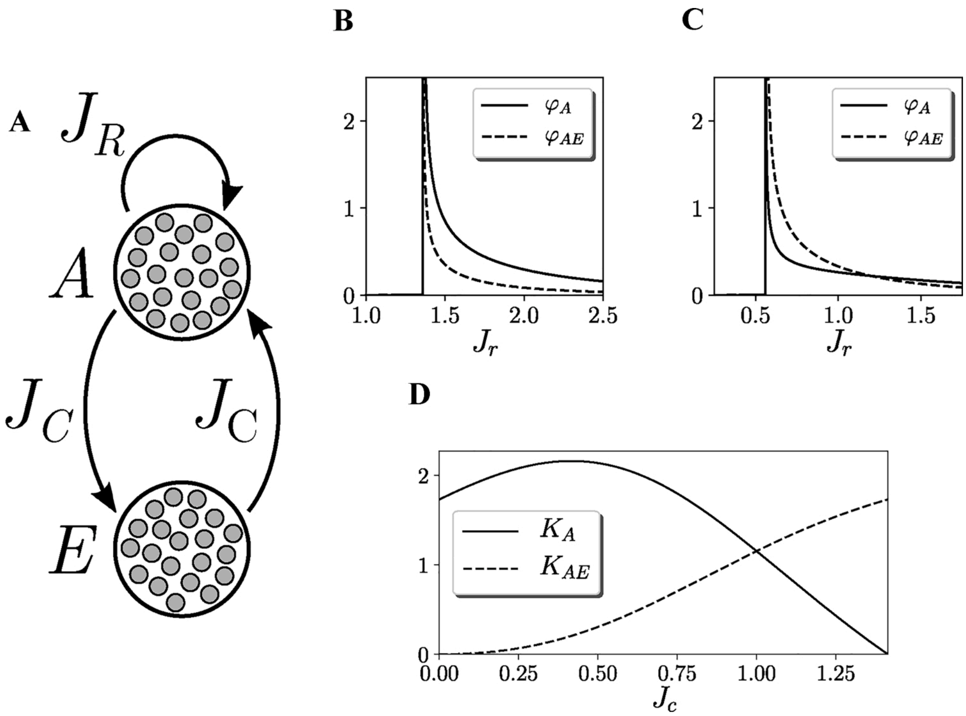

Fig. 2. Asymmetric interaction in a kinetic Ising model. (A) Basic agent A with self coupling J_R, connected to an environment E with bidirectional couplings J_C. (B, C) Values of integrated information for the agent and environment nodes of the model at stability for $J_C = 0.8$ (left) and $J_C = 1.2$ (right) and different values of J. (D) Constants K multiplying the diverging integrated information value near the critical point $J_R = 2 - {J_C}^2$, showing the level of irreducibility of system A and system AE. Figure adapted from Aguilera and Di Paolo (2019), were a detailed analysis can be found.

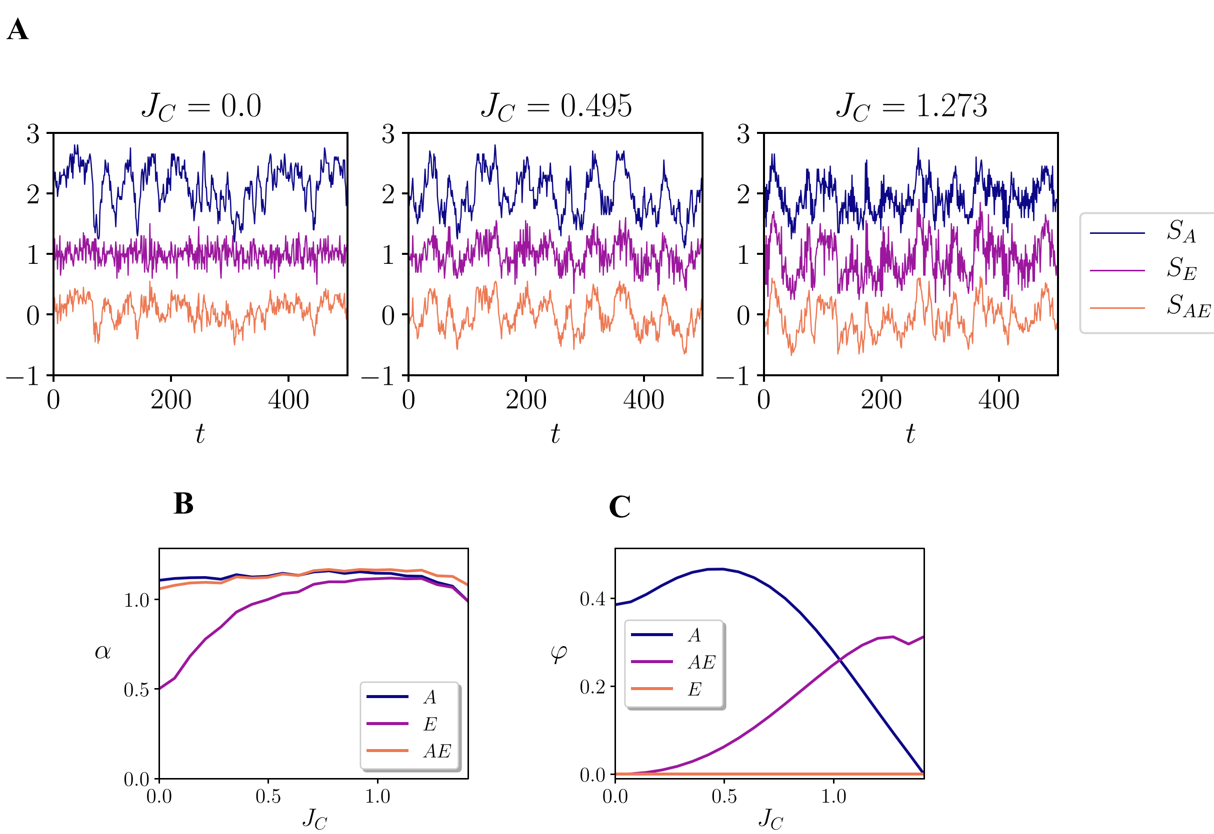

Fig. 3. Power-law scaling and integrated information analyses. (A) Time series representing simulations of $S_A(t)$, $S_E(t)$ and $S_{AE}(t)$ for different values of J_C (an offset value has been added for visiblity). (B) Power law exponents of the behavior of the systems in Fig. 2, measured using Detrended Fluctuation Analysis (DFA). Exponents $\alpha = 1$ represent scale-free dynamics, while $\beta = 0.5$ represent independent random noise. (C) Integrated information of different clusters of the system, showing a transition from integration in $A$to integration in AE.

diverge at the critical point and tend to infinity. However, their value is multiplied by a constant that depends on the coupling strength J_C (Fig. 2D), showing that for $J_C < 1$, A is more integrated than the joint system AE. For the other conditions ($J_C > 1$) the opposite occurs and A and E do not operate as differentiated systems. This shows – in the idealized case of an infinite system with homogeneous regions – how we can determine the boundary of an interaction-dominant ensemble. Interaction-dominance, therefore, does not imply the dissolution of a cognitive system into its environmental background.

Can the same be done for a finite system using either power-law scaling measures or integrated information? We illustrate each case in a finite version of the model at the critical point $J_R = 2 - J_C{}^2$, where regions A and E have $N = 1000$ units each. We run a simulation for 10^5 steps to reach a stationary regime, and run for 10^6 further updates computing the activation values for the different regions $S_A(t) = \frac{1}{N}\sum_{i\in A} s_i(t)$, $S_E(t) = \frac{1}{N}\sum_{i\in E} s_i(t)$ and $S_{AE}(t) = \frac{1}{2N}\sum_{i\in AE} s_i(t)$ (Fig. 3A) to apply a power-law scaling analysis. We also calculate the distributions of $P(S_A(t))$, $P(S_E(t))$, $P(S_AE(t))$ using the methods described in Aguilera (2019) to compute φ.

To measure the presence of power-law scaling we apply Detrended Fluctuation Analysis (DFA), a standard tool for characterizing the exponent of the scaling. As an example, we apply DFA to the activation time series $S_A(t)$, $S_E(t)$ and $S_{AE}(t)$. The DFA algorithm extracts an index α, which indicates the presence of long-range correlations when $0.5 < \alpha < 1.5$, being $\alpha = 0.5$ the case with no temporal correlations. With this measure, we can inspect the behavior of the long-term correlations of the activity of units in region A, region E, and the combination of both. In this way, we test what power-law scaling can tell us about the internal assembly of the systems components.

Results are shown in Fig. 3B. First, we observe that for the subsystem E, α goes to 0.5 when J_C is small, indicating the absence of correlations in the time series. When J_C is large, the system always presents long-range correlations in A, E and AE. Note that even when $J_C = \sqrt{2}$ and $J_R = 0$ (A has no self-coupling and its activity is only correlated through E) A and E still present long-range correlations although they are not, individually, interaction-dominant. The DFA analysis captures the long-range correlations generated by the presence of a critical point but, as these long-range correlations are propagated throughout the system, the measure cannot identify which parts of the system are organized in an interaction-dominant ensemble. This means that, even if we observe through power-law scaling measures that units in A behave with interaction-dominant dynamics, it is unclear whether this behavior is due to the self-coupling of units within A, the interaction with E, or a combination of both. There is a disconnection between the dynamical effects that we can measure using scaling measures, and the mechanisms that generate the observed patterns. For instance, critical dynamical patterns in the brain might be caused by neural mechanisms, but they might as well be generated by external power-law patterns (in the body or the environment) driving their dynamics. The point is that power-law scaling analysis of data series does not provide an explanation of where and how interaction-dominant dynamics are generated.

Let us now analyze the integrated information of the system. We numerically compute the values of integration for networks of size $N = 1000$ for regions A and E (Fig. 3C). The integration of unit E alone is zero (since it has no self-connections that can be partitioned). As the coupling J_C changes, we observe that integration φ can distinguish which region of the system is more integrated. For low values of J_C, self-couplings J_R dominate at the critical point and unit A is the most integrated part of the system, whereas when agent-environment couplings J_C dominate at the critical point and the joint system $AE$is the most integrated. Thus, a measure that is related with a critical susceptibility of the system *in response to its possible partitions* is more effective for characterizing its organization. One-dimensional indices of criticality are suited for completely homogeneous systems. In systems comprising regions with different mechanisms, measures of criticality using integrated information, reflecting its susceptibility to a multiplicity of parameters (i.e., possible partitions), can better capture aspects of the organization that are missed by traditional measures.

Here we have measured critical integration in a model, but how can such measures be performed using experimental data in the laboratory? Several measures of integration are discussed in the extensive literature on the topic (see e.g., Mediano et al., 2019), although the relation of the different versions of φ with criticality and susceptibility indices should be carefully reviewed as indicated by Aguilera (2019), Aguilera and Di Paolo (2019).

4. Extended critical integration: adaptivity, robustness, autonomy

We have proposed that integrated information and susceptibility to internal partitions are better suited for characterizing the processes of a system showing interaction-dominant dynamics. How should we interpret this proposal in terms of the properties of critical systems? Intuitively, high integration suggests a high level of robustness, while paradoxically high susceptibility indicates a degree of fragility in the system. Traditionally, criticality is understood as a state maintaining a compromise between robustness (resilience to external perturbations, which is a property of ordered phases) and flexibility (responsiveness to environmental stimuli, which is a property of disordered phases) (Muñoz, 2018). Moreover, some critical systems have been shown to display dynamical properties that are robust to changes in their structure (Goodarzinick et al., 2018). High susceptibility is also linked with a large dynamic range of responses of a system to diverse intensities of stimuli, where variations in input correspond robustly to variations in response (Kinouchi and Copelli, 2006). Moreover, many properties at critical points are quite robust and largely independent of small-scale details, giving rise to *universality* in large-scale behavior (Fisher, 1974).

Integrated information has also been associated with the autonomy of a system (Marshall et al., 2017). From ecological and enactive perspectives, interaction-dominant dynamics accompany the emergence of intentionality (Van Orden et al., 2003) and adaptive autonomy (Aguilera et al., 2015) in an agent. This suggests that a high level of integration/susceptibility to internal partitions corresponds to the constitution of the robust identity of an autonomous system. Still, this claim must be taken with caution. In many models, criticality is the result of a fine-tuning of the parameters, and in a model like the one mentioned in the previous section critical states can rapidly disappear with a slight variation of the coupling strength. In contrast with the idea in physics of criticality as a singular point, it has been proposed that living organisms are 'extended' critical systems (Bailly and Longo, 2008). Recent models shows that extended regions of criticality can be generated by the presence of structural heterogeneity in a network (Moretti, 2013). Similarly, another model presented by Aguilera and Di Paolo (2019) examines a network with three infinite homogeneous regions, (two of them composing a self-regulating agent, A and B, and the other its environment E, Fig. 4A). The agent in this model, under particular combinations of parameters, has a region (not just a point) of high integration robust to changes in the environmental dynamics (Fig. 4).

In general, we suggest that integration/susceptibility to internal partitions is a property that allows us to identify a system as interaction-dominant (i.e., as a system susceptible to changes in the interactions that constitute it). However, not all systems will maintain integration in a robust manner, and the emergence of self-sustaining cognitive identities (as proposed in enactive perspectives, Di Paolo et al., 2017) should be linked with the ability to maintain regions of extended critical integration in the face of changing interactions with the environment.

5. Discussion

Ideas derived from the study of complex systems can be useful for measuring the organizational properties of neural and cognitive

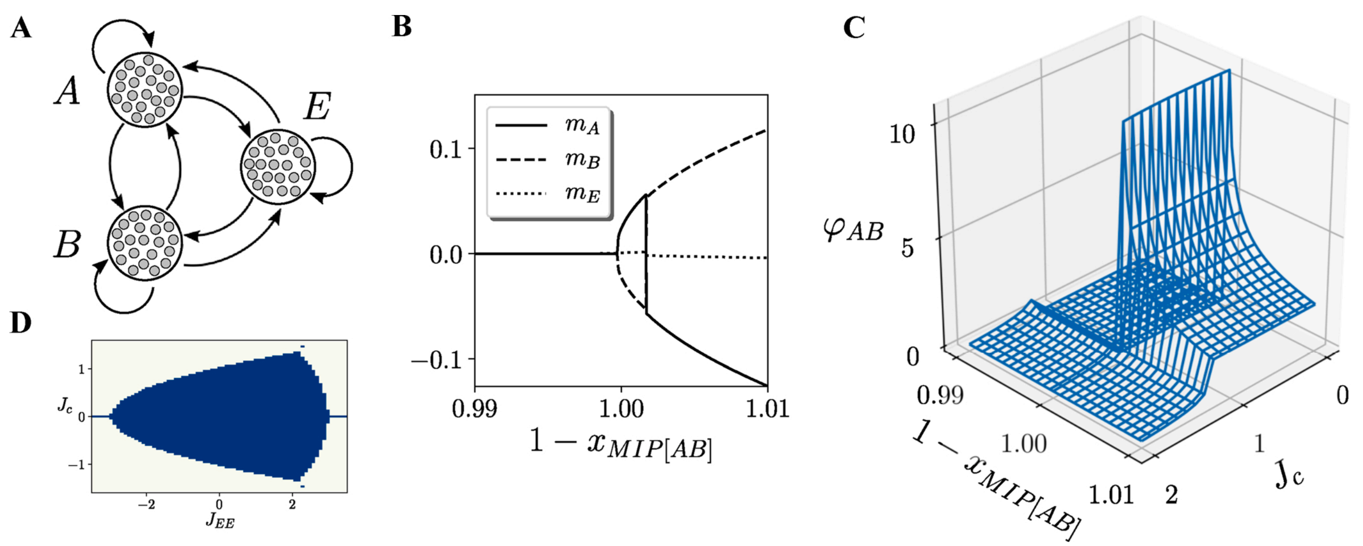

Fig. 4. Adaptive integration in a kinetic Ising model. (A) Adaptive sensorimotor system connected to an environment. (B) Values of the mean fields of the stable solution for agent-environment couplings $J_c = 1$. (C) Values of $\varphi_{AB}(\tau \to \infty)$ for different values of J_c. (F) The blue area represents the surface in J_c and environmental self-couplings J_{EE} where $\varphi_{AB}(\tau \to \infty)$ diverges, i.e., the area of 'extended criticality'. Figure reproduced from Aguilera and Di Paolo (2019), were a detailed analysis can be found.

systems. Power-law scaling measures are some of the most widely used. However, these measures have been developed for homogeneous simple models. As general emergent parameters, they are too disconnected from the specificity of a system's organization. As an alternative, we have considered here an approach to soft assembly based on critical integration that can answer the criticisms levelled against inferences based only on observed power laws.

Explicating the relations between criticality and properties of the organization of cognitive systems can be achieved by interpreting Integrated Information in terms of the susceptibility of critical systems along specific directions in their internal parameter space. These directions are connected to the effects of different partitions affecting the interaction between a system's components. We suggest that research in power-law scaling in neuroscience and cognitive science can be further developed by taking into account measures that are more directly informative about the organization of underlying processes, thus helping clarify how these processes relate to macroscopic measures of cognition and other levels of explanation.

We should note that one drawback of the proposal is that it is based on information-theoretical measures that are hard to compute from time-series data. Some versions of IIT (Tononi et al., 2016) are known for being impracticable in networks larger than 10–15 binary units. However, alternatives exist to simplify the calculations and they can be applied to time-series data, potentially to those found in behavior science (Barrett and Seth, 2011; Oizumi et al., 2016; Mediano et al., 2019). Furthermore, we have proposed that, at least for large networks, integrated information can be well captured by susceptibility measures. In information theory susceptibility in different directions of the parameter space is captured by Fisher information. Although computing Fisher information is also more complicated than computing fractal exponents, several methods exist for computing susceptibilities from time-series data (Klein and Mélard, 1995; Dobos and Abonyi, 2013; Wang and Shang, 2018). Moreover, simpler measures in the form of sensitivity to perturbations might capture aspects of a system critical integration. A simplified way to explore some of the ideas developed here could be to quantify how fractal exponents respond in front of different kinds of perturbations that challenge the integrity of a system. Studies of sensitivity to context variations can be found in the literature about fractal scaling, e.g., experiments in reaction time tasks introducing variability in stimulus presentation (Kello et al., 2007), introducing white noise in inter-trial intervals (Holden et al., 2011), or a variety oscillatory response intervals (Amon et al., 2018). In another study, body-tool interactions where studied by analysing the changes in fractal exponents with the introduction of noise in motor behavior (Dotov et al., 2010).

Finally, we believe that the presented framework could motivate experiments aimed at characterizing critical exponents related to the integration of a subject's activity or of the whole coupled system of sensorimotor interaction. Susceptibility to changes in the task environment can be compared to susceptibility to perturbations of the subject's action (e.g., using virtual reality setups or controlled environments like in Dotov et al., 2010), potentially reproducing a scenario like the one in Fig. 2. All combined, these research avenues can help delineate research directions merging experimental tools and guidelines for the design of new studies, with models that ground measures and intuitions on stronger theoretical foundations.

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