Diffusion and Games in Social and Economic Networks

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Resumen

El ser humano es un ser social: las elecciones de las personas y su bienestar dependen en buena medida de las acciones de sus contactos. Las redes sociales son una importante fuente de recursos para sus integrantes: a través de amigos, familiares y conocidos, los individuos obtienen información, ayuda y otro tipo de recursos. La gran incidencia que las redes sociales tienen en numerosos ámbitos de la vida hace necesario comprender: (i) cómo la estructura de las redes sociales de las personas afecta a los resultados que éstas obtienen (en términos laborales, de salud, de influencia, etc.) y (ii) cómo el comportamiento de las mismas se ve afectado por la estructura que presenta la red de la que forman parte.

Esta tesis contribuye al conocimiento de estas cuestiones. El primer capítulo muestra cómo los resultados laborales de las personas pueden explicarse parcialmente por las características de las red en la que están inmersos, en la medida en que éstas obtienen información laboral de sus redes sociales. En este sentido, analizamos el papel que las redes juegan como *difusoras* de información laboral y sus consecuencias en el empleo. El segundo y tercer capítulo, en cambio, analizan la influencia que las redes sociales tienen en el *comportamiento* de las personas en contextos en los que los resultados que los individuos obtienen dependen no sólo de sus propias acciones sino de las acciones de sus contactos. Dada la naturaleza estratégica de este tipo de situaciones, estos dos capítulos se analizan desde el enfoque de la Teoría de Juegos.

Capítulo 1. *Are close-knit networks good for employment?*

Uno de los ámbitos en los que las redes sociales han mostrado tener una mayor relevancia es el del mercado de trabajo (Goyal, 2007, Jackson y Rogers, 2007, Vega-Redondo, 2007, Jackson, 2010). Uno de los primeros estudios en documentar la importancia de las redes sociales en el mercado laboral fue el trabajo de Myers y Schutz (1951), que constató que el 62 % de todos los trabajadores entrevistados en el sector textil en Nueva Inglaterra habían encontrado su empleo a través de algún contacto, frente a un 23 % de trabajadores que había solicitado directamente el empleo. El 15 % restante obtuvo su puesto de trabajo a través de medios diseñados exclusivamente para tal fin: agencias de empleo, anuncios de ofertas laborales, etc. Ese descubrimiento fue seguido de más investigaciones con resultados similares (véase Rees, 1966; Sheppard y Beliitsky, 1966; Granovetter; 1974; Staiger, 1990; Montgomery; 1991; Corcoran et al. 1980, Campbell y Marsden, 1990, entre otros muchos). El estudio de Rees y Schutz (1970), por ejemplo, mostró la relevancia de las redes de contactos a la hora de obtener un empleo independientemente del ámbito profesional considerado. Sorprendentemente −y a pesar de la gran diversidad de medios que han surgido a lo largo de los años para la búsqueda de empleo− el uso de las redes de contactos para la obtención de trabajo se ha incrementado en las últimas décadas (Ioannides y Loury, 2004).

A la vista de estos resultados han surgido ciertos interrogantes. Por ejemplo, ¿cómo afecta la estructura de la red al flujo de información laboral entre individuos? ¿Qué características específicas de la red afectan a la probabilidad de recibir empleo a través de contactos y cómo? ¿Puede una distinta posición en la red explicar los patrones de desigualdad que se observan entre individuos en el mercado laboral? La literatura económica ha tratado de dar respuestas a estas cuestiones a lo largo de los años. En este sentido, se ha identificado cómo el número de contactos que los individuos tienen (su *degree*) tiene una influencia positiva en sus resultados laborales: un número mayor de contactos se traduce en una mayor probabilidad de obtener empleo (Calvó-Armengol, 2004) y en un salario esperado más elevado (Cappellari y Tatsiramos, 2015). Además, existen correlaciones a lo largo del tiempo en el estado laboral de individuos conectados entre sí, lo cual puede explicar parcialmente los patrones de segregación observados en el mercado laboral −ya sea por grupo étnico, género o situación geográfica− o las decisiones de los individuos de permanecer o abandonar dicho mercado (Calvó-Armengol y Jackson, 2004, 2007). Por otro lado, la probabilidad de que una persona obtenga información laboral de un contacto varía dependiendo de la fuerza del vínculo que le une a dicho contacto: si el vínculo es débil −el contacto es un mero conocido− la probabilidad de obtener información laboral del mismo es mayor que si el vínculo es fuerte −es un amigo íntimo o un familiar. En efecto, Granovetter (1973) constató que, mientras que un 16.7 % de los individuos entrevistados reportaron haber encontrado su empleo a través de un enlace fuerte −alguien a quien veían regularmente− un 83.4 % de los individuos encontraron su trabajo a través de un enlace débil −alguien a que apenas frecuentaban−. Su explicación a tal descubrimiento fue que los lazos débiles tienden a proporcionar a las personas información diferente de la que obtienen de otros contactos, pues estos individuos actúan como «puentes» entre grupos de personas que de otra forma estarían aislados. Los vínculos fuertes, por el contrario, tienden a proporcionar información redundante y poco novedosa.

Existen, sin embargo, otras cuestiones abiertas en relación al impacto que ciertas propiedades de las redes de las personas tienen en sus resultados laborales. Una de las características más prominentes de las redes sociales reales es su alto grado de agrupación: los amigos de un individuo son a menudo amigos entre sí o tienen amigos en común, sus respectivos amigos en muchas ocasiones se conocen, y así sucesivamente (Jackson y Rogers, 2007). En terminología de redes, las redes agrupadas se caracterizan por un gran número de relaciones transitivas, lo cual se traduce en un considerable número de triángulos, cuadrados y otros ciclos de enlaces en la red (Holland y Leinhardt, 1971; Watts y Strogatz, 1998, Vega-Redondo, 2007; Jackson, 2010). Aunque la importancia de la agrupación de la red ha sido ampliamente discutida en la literatura sociológica, no existe un acuerdo respecto a si las redes altamente agrupadas son beneficiosas o perjudiciales y por qué (Burt, 2001, 2009; Jackson et al. 2017). Las relaciones en las redes agrupadas son normalmente más fuertes, lo cual favorece la confianza y la cooperación entre individuos (Granovetter, 1973; Burt, 1992; Coleman, 1988a, 1988b; Bloch et al. 2008; Lippert y Spagnolo, 2011; Jackson et al. 2012). Sin embargo, las redes agrupadas pueden inhibir el flujo de información novedosa, ya que el contenido de la información recibida en este tipo de redes puede ser redundante (Burt, 2001, Blau, 2017). Este último punto es el argumento de Granovetter (1973) en *La fuerza de los lazos débiles.* En relación con esta teoría surge una pregunta adicional: ¿reside la ventaja de los vínculos débiles exclusivamente en el *contenido* de la información que estos vínculos aportan o en otro tipo de propiedades de red asociados a ellos, tales como su menor presencia en ciclos de enlaces?

El presente capítulo trata de esclarecer estas cuestiones. Analizamos cómo la agrupación de la red −reflejada en la presencia de ciclos de enlaces− influye en los resultados laborales de las personas: su probabilidad de empleo y su salario esperado. De manera similar a Calvó-Armengol (2004) y Calvó-Armengol y Jackson (2004; 2007) consideramos un modelo en el que los agentes están conectados a través de una red social y los individuos empleados con información laboral transmiten esta información a sus amigos directos en situación de desempleo. Calvó-Armengol (2004) analiza cómo el número de contactos directos e indirectos afecta a la probabilidad de empleo e identifica un efecto positivo del número de contactos directos en dicha probabilidad (esto es, un efecto positivo del *degree*), así como un efecto negativo del número de vecinos indirectos (un efecto negativo del *second-order degree*). Por otro lado, muestra cómo un cambio en la distribución del *degree* en redes regulares y sin ciclos tiene efectos no-monótonos en el empleo.[1] Finalmente, propone un modelo de formación de redes en el cual la decisión de los individuos de abandonar el mercado laboral es endógena: depende de los recursos que esperan obtener de la red. Calvó-Armengol y Jackson (2004; 2007) analizan el papel de los salarios en un modelo similar: estudian los patrones de correlación desde una perspectiva dinámica. Este enfoque permite explicar la desigualdad en los salarios de los agentes y en sus decisiones de abandono del mercado laboral en función de su estado inicial.[2]

El objetivo de este capítulo es estudiar el papel que el agrupamiento de la red, reflejado en la presencia de ciclos de enlaces, tiene en los resultados laborales de sus integrantes. En nuestro análisis, aislamos el impacto de los ciclos de aquel que podría venir inducido por otras propiedades de la red, como la distribución del *degree* y del *second-order degree.* Probamos cómo la probabilidad de empleo de los individuos es diferente dependiendo de los ciclos cortos (triángulos y cuadrados) que forman con sus vecinos, aun cuando sólo se tiene en consideración un periodo. Esto es, mostramos cómo los flujos de información hacia un individuo no sólo se ven afectados por su *degree* y su *second-order degree*, sino también por la geometría que crean sus enlaces y los enlaces de sus respectivos vecinos. Dejando constante el *degree* y el *second-order degree* de un individuo, la presencia de triángulos y cuadrados en su vecindario reduce su probabilidad de empleo. De este modo, el desempleo en una red g puede ser mayor que en otra g' si el grado de agrupamiento en g es mayor que en g', incluso si la distribución conjunta del *degree* y el *second-order degree* es la misma en ambas

[1] Dado que el número de contactos directos e indirectos determina la probabilidad de empleo de los agentes, la tasa de desempleo en dos redes con el mismo número de enlaces puede ser diferente si la geometría de estas redes es distinta, sencillamente por el hecho de que el *degree* y el *second-order degree* de sus respectivos integrantes puede no ser el mismo. Calvó-Armengol (2004) ilustra este punto con un ejemplo.

[2] Los individuos desempleados obtienen más (menos) información cuando sus contactos están empleados (desempleados), dando lugar a formas robustas de correlación en los salarios y en el estado laboral de agentes conectados entre sí. Calvó-Armengol y Jackson (2004) muestran con un ejemplo cómo dos agentes con el mismo número de vecinos pero un distinto *average path length* pueden presentar una diferente probabilidad de empleo en el largo plazo, lo cual sugiere un efecto negativo de la agrupación de la red en el empleo. Sin embargo, su anális no explora la incidencia específica que los ciclos de enlaces tienen en los flujos de información, *ceteris paribus.*

redes. Este efecto viene causado por el hecho de que estos ciclos generan afiliación estocástica en los flujos de información procedente de distintos contactos. A diferencia de lo que ocurre en una red sin ciclos −donde los flujos de información entre contactos ocurren de manera independiente− la probabilidad de que un individuo A reciba información de un vecino B depende positivamente de la probabilidad de que A reciba información de otro vecino C si ambos vecinos están unidos (es decir, si A, B y C forman un triángulo) o si B y C tienen un amigo en común D (en cuyo caso A,B, C, y D forman un cuadrado). Esta falta de independencia en los flujos de información en redes con ciclos hace que sus integrantes sean más propensos a recibir varias ofertas de empleo de diferentes contactos de forma simultánea, pero también más propensos a no recibir ninguna oferta de éstos. Dado que recibir varias ofertas de empleo no supone ninguna ventaja cuando se analiza la probabilidad de empleo (los individuos sólo pueden aceptar un trabajo a la vez), el efecto resultante de los ciclos en la probabilidad de empleo es negativo.

El agrupamiento perjudica a la probabilidad de empleo, pero también al salario esperado. Mostramos cómo los ciclos cortos tienen una incidencia negativa en el salario esperado, dado que la probabilidad de estar desempleado es mayor cuando los individuos están inmersos en ciclos y los agentes desempleados tienen un salario esperado igual a cero. Condicionado a estar empleado, por el contrario, el salario esperado de los individuos es mayor cuando éstos forman ciclos con sus contactos. Este resultado se explica por el hecho de que los individuos inmersos en ciclos son más propensos a recibir varias ofertas laborales a la vez y por tanto pueden escoger entre distintas opciones. El hecho de recibir distintas ofertas simultáneamente no presenta ninguna ventaja cuando se analiza la probabilidad de empleo, ya que los individuos sólo pueden aceptar un empleo. Sin embargo, recibir varias ofertas laborales a la vez constituye una ventaja cuando se analiza el salario esperado ya que los individuos pueden escoger entre trabajos con salarios distintos.

Una vez identificados los *efectos estáticos* de los ciclos sobre el empleo, analizamos sus *efectos dinámicos* (esto es, en el largo plazo). Consideramos distintas redes que difieren en el número de ciclos que los individuos forman con sus vecinos, y analizamos el estado estacionario de la cadena de Markov inducida por nuestro modelo. Mostramos cómo los ciclos de enlaces organizan la probabilidad de empleo en el estado estacionario en el sentido de la dominancia estocástica de primer orden. Las personas en vecindarios altamente agrupados presentan una menor probabilidad de empleo a largo plazo que aquellas en vecindarios menos agrupados. Este resultado pone de manifiesto el efecto de los ciclos cortos de enlaces en los patrones de desigualdad observados dentro de una red, así como entre redes distintas. La ciclos de longitud mayor (pentágonos, hexágonos, etc.) −que son irrelevantes en el modelo estático− tienen una relevancia marginal en el modelo dinámico.

En la misma línea, mostramos cómo los ciclos cortos refuerzan los patrones de correlación en el empleo a largo plazo identificados en Calvó-Armengol y Jackson, (2004). De acuerdo con estos patrones, los individuos con amigos empleados (desempleados) presentan mayor (menor) tendencia a estar empleados porque su entorno (no) les proporciona información laboral. Esto se traduce en patrones de desigualdad y segregación laboral a lo largo del tiempo. La afiliación estocástica en los flujos de información inducida por los ciclos cortos refuerza estos patrones: dado que la transición del desempleo al empleo es más difícil cuando los individuos forman parte de ciclos, los individuos en vecindarios agrupados tienen una menor tendencia a estar empleados. Si a esto añadimos la correlación en el estado laboral que existe entre vecinos en la red, el resultado son fluctuaciones laborales más prolongadas y persistentes en redes con un mayor grado de agrupamiento.

En resumen, la contribuciones de este capítulo primero son varias:

- Sacamos a la luz un mecanismo detrás de *La fuerza de los lazos débiles* (Granovetter, 1973). Tal y como argumenta Granovetter (1973), existen distintas formas de medir la fuerza de un lazo social. Una manera se basa en el número de contactos que dos personas tienen en común, esto es, en los ciclos que ambas forman, «*cuanto más fuerte sea la unión entre A y B, mayor será el número de individuos del grupo S con los que ambos estarán relacionados mediante lazos fuertes o débiles*». Si de acuerdo con esta concepción consideramos «débiles» a aquellos enlaces que no cierran ciclos de red cortos (triángulos o cuadrados) y «fuertes» a aquellos que cierran algún ciclo corto, nuestro análisis muestra cómo los lazos débiles son efectivamente más propensos a proporcionar información laboral que los lazos fuertes. Sin embargo, la ventaja de los lazos débiles no reside en el contenido de la información proporcionada por estos, sino en el hecho de que el flujo de información a través de ellos es independiente de los flujos de información entre otros agentes en la red.

- Contribuimos al debate clásico en sociología respecto a si el agrupamiento en las redes sociales es una ventaja o una desventaja para los individuos inmersos en ellas. En nuestro contexto, los ciclos constituyen una desventaja a la hora de obtener empleo, ya que los individuos son menos propensos a recibir al menos una oferta laboral si forman ciclos con sus contactos. Sin embargo, en otros contextos en los que el proceso de difusión en la red es semejante al que tiene lugar en nuestro modelo, la afiliación estocástica inducida por los ciclos podría beneficiar a los individuos inmersos en ellos.[3]

- Tal y como argumentan Jackson et al. (2017), «*las medidas estadísticas que han sido diseñadas para capturar el agrupamiento han tendido a focalizarse exclusivamente en tríos −grupos de exactamente tres nodos−. Aunque esto es útil, parece razonable conjeturar que las propiedades asociadas a la agrupación de un grafo no pueden ser capturadas imponiendo las limitaciones implicadas por estas medidas*». El capítulo ilustra cómo la agrupación de la red y sus efectos van más allá de lo capturado por el coeficiente de agrupamiento clásico (que sólo tiene en cuenta la presencia de triángulos en la red). Mostramos cómo los cuadrados son igualmente relevantes para la difusión incluso en el modelo estático y cómo el coeficiente de agrupamiento clásico puede dar una medida poco precisa del grado de agrupamiento de la red, al no tener en cuenta la presencia de este tipo de ciclos (cuadrados).

- Por último, la literatura económica ha mostrado cómo las fricciones en la búsqueda de trabajo conducen a una lenta dinámica de empleo (Pissarides, 1985; Bentolilla y Bertola, 1999; Mortensen y Pissarides, 1999; Burgess et al., 2001). Además, las tasas de entrada y salida del desempleo se ven afectadas por las características personales de los individuos, como su nivel de educación (Maarten y Wolbers, 2000), o su número de contactos directos e indirectos (Calvó-Armengol, 2004). El presente capítulo identifica otro factor que incide en estos procesos: la geometría creada por los enlaces de la red. Dado que las fluctuaciones en el empleo son un determinante importante de la economía, nuestros resultados pueden contribuir a la comprensión de algunos hechos estilizados de tipo macroeconómico, como son los patrones de desigualdad que se observan en las sociedades interconectadas.

Capítulo 2. *Clustering in Network Games*

En las sociedades actuales, el comportamiento y bienestar de los individuos no puede entenderse sin considerar la estructura social en la que están inmersos. Por su condición gregaria, el ser humano rara vez toma decisiones sin considerar a los demás, pues el rendimiento esperado de su comportamiento a menudo se ve afectado por las decisiones de otras personas. En tiempos de coronavirus, por ejemplo, una persona puede decidir acudir a un determinado lugar sólo si espera que otros individuos no van a hacerlo. En otros contextos, la misma persona puede querer llevar a cabo una acción (como acudir a una huelga o reciclar su basura) únicamente si anticipa que otras personas van a actuar del mismo modo. El elevado impacto que el entorno social ejerce en el comportamiento de los individuos ha generado una inmensa literatura sobre juegos en redes que ha permitido entender mejor fenómenos de distinta naturaleza (véase Calvó-Armengol y Jackson, 2004, 2007; Calvó-Armengol et al. 2009; Calvó-Armengol y Zenou, 2004; Ballester et al. 2010; Bramoullé y Kranton, 2007; Jackson et al. 2012, entre otros).

Existe una variedad de aspectos de la estructura social que pueden tener una incidencia en el comportamiento humano. Una característica intrínseca de las redes sociales reales es su elevado grado de agrupamiento, manifestado en una amplia presencia de relaciones transitivas y de ciclos de enlaces en la red (Holland y Leinhardt, 1971; Watts y Strogatz, 1998, Vega-Redondo, 2007; Jackson, 2010). La literatura sociológica ha analizado ampliamente cómo el comportamiento de las personas puede verse afectado por el grado de agrupamiento de la red que integran señalando distintos mecanismos de incidencia. Uno de ellos se basa en la información: en una red agrupada y densa es más probable que una mala conducta sea detectada y divulgada, pues la existencia de un elevado número de relaciones transitivas hace que cualquier comportamiento inapropiado se detecte más rápidamente en este tipo de redes (Coleman, 1990; Greif, 1993). Como

[3] El proceso de difusión considerado en este capítulo presenta dos importantes rasgos: (i) la capacidad de cada agente para poder ser transmisor o receptor de la información transmitida depende de su estado (empleado o desempleado) y (ii) el bien transmitido es rival. Es razonable pensar que los ciclos de red pueden generar afiliación estocástica en otros procesos de difusión que presenten estos rasgos, como podría ser la transmisión de una enfermedad. En este caso (i) se cumple, ya la capacidad para transmitir la enfermedad o contagiarse depende del estado de cada agente (depende de si tiene la enfermedad o no). Por otro lado, si los individuos tienen un tiempo limitado y sólo pueden interactuar con un cierto número de personas, la enfermedad podría considerarse como un «bien» rival. En este caso formar parte de ciclos podría ser beneficioso para un individuo, pues podría presentar una menor probabilidad de contagiarse como consecuencia de la afiliación generada por los ciclos en el proceso de difusión.

consecuencia, los incentivos de los individuos a comportarse de acuerdo a las reglas establecidas son más grandes cuando la red presenta un grado de agrupamiento mayor. Otro mecanismo relacionado vendría dado por la capacidad que tiene un grupo social para imponer sanciones colectivas en función de su grado de agrupamiento: los individuos pueden coordinarse más fácilmente para imponer sanciones colectivas cuando integran un grupo altamente cohesionado (Coleman, 1988a). Con base a ambos argumentos, las redes sociales agrupadas fomentarían la confianza entre agentes e inhibirían la aparición de polizones (*free riders*).

Desde el punto de vista de la Teoría de Juegos, algunos estudios han mostrado cómo los individuos pueden tener distintos incentivos a ayudar a sus contactos dependiendo del grado de agrupamiento de la red (Jackson et al. 2012): la existencia de amistades comunes entre individuos les induce a prestar ayuda a sus contactos ante la posibilidad de perder las amistades que tienen en común si la ayuda no es prestada. Sin embargo, son pocos los estudios que analicen el impacto que el agrupamiento de la red tiene en el comportamiento de los individuos desde una perspectiva de juegos, lo cual deja abiertas varias cuestiones. Por ejemplo, ¿cómo afecta el agrupamiento de la red al comportamiento de los individuos en otro tipo de contextos? ¿es la amenaza de sanciones colectivas el único mecanismo por el cual el agrupamiento puede afectar al comportamiento de las personas? ¿cómo depende la respuesta del tipo de juego en cuestión?

El propósito de este segundo capítulo es estudiar cómo el comportamiento de los individuos se ve afectado por el grado de agrupamiento de las redes sociales que éstos integran. Concentramos nuestra atención en dos formas de interacción estratégica: juegos de sustitutivos estratégicos y de complementarios estratégicos. En juegos de complementarios estratégicos el rendimiento que los individuos obtienen de sus acciones son mayores cuanto mayor es el número de individuos en su vecindario (directo o indirecto) que juegan la misma acción. Este tipo de interacción estratégica refleja decisiones como la adopción de una nueva tecnología: por cuestiones de compatibilidad, por ejemplo, los individuos obtienen más (menos) beneficios al adoptar la tecnología si sus contactos (no) lo hacen. Los juegos de sustitutivos estratégicos reflejan situaciones donde el rendimiento que los individuos obtienen de llevar a cabo una determinada acción son más altos (bajos) cuanto mayor (menor) es el número de vecinos que escoge la acción contraria. Este tipo de juegos se aplica, por ejemplo, a la decisión de proveer un bien público: el beneficio que un individuo obtiene al no contribuir a la financiación de un bien público es mayor si algún otro vecino contribuye al mismo, pues en este caso puede disfrutar del bien de forma gratuita actuando como polizón.

Cuando se analizan las interacciones estratégicas que tienen lugar en redes surgen dos problemas fundamentales. En primer lugar, dada la interdependencia existente entre distintas propiedades de la red es problemático −y en ocasiones imposible− aislar el impacto que cada característica de la red tiene en el comportamiento de los jugadores. La razón estriba en que el cambio de una propiedad de la red modifica simultáneamente otras características de la misma y, por tanto, es aventurado atribuir los efectos del cambio a una propiedad de la red en concreto. En segundo lugar, aun cuando se tienen en cuenta las interacciones estratégicas que tienen lugar en una red específica, los individuos a menudo pueden condicionar su comportamiento a la observación de una gran variedad de aspectos de la red y coordinarse alcanzando un equilibrio (véase, por ejemplo, Bramoullé y Kranton, 2007). Para solventar estos problemas, este capítulo asume que los individuos tienen información incompleta sobre la red en la que están inmersos, lo cual permite realizar predicciones más concretas sobre la incidencia que ciertas propiedades de la red tienen en su comportamiento y en su bienestar. Este enfoque es además realista, puesto que en la vida real las personas rara vez tienen información completa sobre la red social que integran e, incluso cuando la tienen, su capacidad para procesar toda esta información es limitada (Jannick y Larrick, 2005; Dessi et al. 2014).

El marco teórico del presente capítulo es semejante al de Galeotti et al. (2010). Estos autores consideran un escenario en el que los jugadores sólo conocen su tendencia a interactuar con otros −su *degree*− y la *distribución del degree* de la población. Los jugadores no conocen ningún otro aspecto de la red (ni siquiera la identidad de sus contactos). El *degree* de cada agente se interpreta como su *tipo*; los jugadores conocen su tipo pero no el tipo de otros jugadores. Bajo este marco de información, los individuos tienen que decidir qué acción jugar (por ejemplo, contribuir o no contribuir a la provisión de un bien público) teniendo en cuenta que el beneficio que obtendrán de la misma dependerá de lo que los otros jugadores escojan (por ejemplo, si un jugador decide no contribuir y nadie contribuye no podrá disfrutar del bien mientras que si decide no contribuir y alguien lo hace podrá usar el bien de forma gratuita). La principal aportación de Galeotti et al. (2010) es mostrar que las acciones de los jugadores son no crecientes (no decrecientes) en su *degree* en el equilibrio Bayesiano bajo sustitutivos estratégicos (complementarios estratégicos). Este resultado indica que las relaciones sociales crean ventajas personales para los jugadores, ya que los pagos esperados de los agentes con mayor *degree* son mayores que las de aquellos con menor *degree* en el equilibrio Bayesiano de ambos juegos.

Nuestro marco teórico se construye a partir del modelo de Galeotti et al. (2010), pero lo extiende al dotar a los jugadores de cierta información sobre el grado de agrupamiento de la red. Asumimos que cada jugador conoce su *degree* (que define su tipo), la *distribución del degree y* máximo número de enlaces que pueden existir entre sus vecinos, información a la que denominamos *agrupamiento percibido.* Este escenario de información se da en ciertas situaciones de la vida real en las que las personas conocen su tendencia a interactuar con otros y pueden anticipar el grado en el que sus futuros contactos pueden conocerse entre ellos. Por ejemplo, una persona que es presentada en un nuevo círculo de amigos puede anticipar que la mayoría de los individuos con los que interactuará se conocerán entre sí. Por el contrario, un estudiante de primer año puede esperar que la mayor parte de los individuos con los que irá a clase no habrán tenido contacto previo. En la mayor parte de las situaciones de la vida real, las expectativas de los individuos en relación al agrupamiento en sus vecindarios se sitúa entre estos dos extremos.

Las contribuciones de este capítulo son la siguientes:

- Mostramos que el comportamiento de los individuos puede ser diferente dependiendo de la información que éstos tienen del grado de agrupamiento de sus vecindarios, *ceteris paribus.* Bajo sustitutivos estratégicos (complementarios estratégicos), las acciones de los jugadores en el equilibrio Bayesiano no decrecen (no crecen) cuando el *agrupamiento percibido* aumenta. Si tomamos la provisión de un bien público como ejemplo de juego de sustitutivos estratégicos y la adopción de una tecnología como ejemplo de complementarios estratégicos, nuestros resultados indican que un mayor agrupamiento percibido no reduce la contribución de los individuos al bien público y no incrementa la adopción de la nueva tecnología. La explicación de este resultado es la siguiente: al igual que en Galeotti et al. (2010), la popularidad de cada jugador (su *degree*) determina su comportamiento y, por tanto, las expectativas de los jugadores sobre las acciones de sus vecinos corresponden a sus expectativas sobre los *degrees* de sus vecinos. La diversidad de tipos de vecinos que un individuo espera tener (en términos de popularidad) disminuye a medida que su agrupamiento percibido es mayor, pues los enlaces entre los vecinos de un individuo generan correlaciones en los *degrees* de dichos vecinos. Es decir, un jugador espera tener una mayor diversidad de vecinos cuando sabe que puede formar un número bajo de triángulos con ellos y una menor diversidad de vecinos cuando sabe que puede formar un número alto de triángulos con éstos. Como consecuencia, bajo sustitutivos estratégicos (complementarios estratégicos) los jugadores esperan tener *al menos un tipo* de vecino que contribuya al bien (adopte la tecnología) con mayor probabilidad cuando el agrupamiento percibido es bajo y por consiguiente sus acciones no decrecen (no crecen) cuando el agrupamiento percibido aumenta.[4]

- Ponemos de manifiesto un mecanismo alternativo por el cual el agrupamiento percibido de la red podría modificar el comportamiento de los individuos. Nuestros resultados se encuentran en línea con los argumentos de Coleman (1988a, 1988b), en el sentido de que un mayor agrupamiento percibido desincentiva a los individuos a actuar como polizones y les induce a contribuir al bien común. Sin embargo, este resultado no se debe a la amenaza de sanciones colectivas por parte de otros agentes, sino meramente a las creencias que los individuos tienen sobre la diversidad de sus contactos en términos de popularidad. Los individuos esperan tener amigos más diversos en términos de popularidad cuando su agrupamiento percibido es bajo, y estas expectativas son las que les inducen a adoptar la acción «más arriesgada»: no contribuir al bien público bajo sustitutivos estratégicos (a riesgo de quedarse sin el mismo si ningún agente lo provee) y adoptar la tecnología bajo complementarios estratégicos (aún a riesgo de no poder disfrutar de la misma si ninguno de sus contactos la adopta).

Capítulo 3. *Network Perception in Network Games*

En las interacciones estratégicas que tienen lugar entre agentes conectados a través de una red social, las acciones de los individuos dependen de las acciones de sus vecinos, que a su vez dependen de las de sus vecinos

[4] En los juegos considerados en este capítulo, los individuos tienen que escoger entre dos acciones: jugar 1 −contribuir al bien público en el caso de sustitutivos estratégicos y adoptar la tecnología en el caso de complementarios estratégicos− o jugar 0. Bajo sustitutivos estratégicos (complementarios estratégicos), los agentes juegan 0(1) en equilibrio si esperan que al menos un vecino jugará 1.

y así sucesivamente. Por tanto, la información que los jugadores tienen sobre la red en la que están inmersos tiene una influencia fundamental en su comportamiento y en su bienestar, que pueden verse afectados si dicha información cambia.

La literatura económica ha analizado tradicionalmente los juegos en redes sociales bajo dos enfoques. El primero asume que los individuos tienen información completa sobre la red de la que forman parte. Este supuesto −considerado por ejemplo en Bramoullé y Kranton (2007), Goyal y Moraga-González (2001) o Ballester et al. (2006) entre otros− presenta un inconveniente: cuando los jugadores tienen un conocimiento completo de la estructura social subyacente puede existir una desconcertante variedad de equilibrios, lo que impide sacar conclusiones sobre la incidencia específica que distintas características de la red tienen en el comportamiento de las personas. Un segundo bloque de trabajos −entre los que se encuentra, por ejemplo, Jackson y Yariv (2005; 2007) o Sundararajan (2008)− asume que la información que los individuos tienen sobre su red se limita a algunas características concretas de la misma. Galeotti et al. (2010), por ejemplo, consideran un escenario en el que los jugadores conocen su *degree* (que se interpreta como su tipo) y la *distribución del degree* de la red, pero ningún otro aspecto de ésta. Estos supuestos permiten realizar predicciones más concretas sobre el comportamiento de equilibrio. En particular, Galeotti et al. (2010) muestran cómo, bajo estas condiciones, las acciones de equilibrio de los jugadores son no crecientes (no decrecientes) en su *degree* en juegos de sustitutivos estratégicos (complementarios estratégicos), lo que se traduce en mayores pagos esperados para los individuos más populares.

Asumir que los agentes tienen tal nivel de desconocimiento de la red es realista en ciertos contextos. Un individuo puede decidir acudir a un congreso, preparar una oposición, votar a un partido político, etc. basándose exclusivamente en el *número* de personas que espera que hagan lo mismo y puede tener información a este respecto (puede conocer el número de personas que acudió a una edición anterior del congreso, tener acceso a encuestas de intención de voto, etc.). Sin embargo, este agente puede no ser capaz de deducir otros aspectos de la red en cuestión a partir de esta información, como, por ejemplo, si las personas que van a acudir al congreso se conocen previamente, si tienen amigos en común, la distancia en términos de enlaces que separa a cada una de ellas, etc. En este tipo de situaciones −en las que el conocimiento que los agentes tienen de la estructura social se reduce a algunos aspectos concretos de la misma− el marco teórico de Galeotti et al. (2010) es razonable, pues la percepción que los agentes tienen de la red se limita exclusivamente a la información que poseen sobre la misma.

En muchos contextos de la vida real, la información que los individuos tienen sobre la red en la que están inmersos no es completa. Sin embargo, suelen conocer la identidad de las personas con las que interactúan y algunos aspectos de la posición que éstas ocupan en la red. Por ejemplo, pueden saber el grado de popularidad de sus contactos, si sus respectivos amigos pertenecen a círculos distintos o se conocen entre sí, etc. A partir de esta información − e incluso cuando es más reducida− los individuos pueden deducir otros aspectos de la red social que integran y formarse una representación mental de la misma. Pensemos, por ejemplo, en una persona que se incorpora a una organización. Esta persona puede tener cierta información sobre la red integrada por los miembros de la organización a la que se incorpora: puede conocer el tamaño de la misma, el número de trabajadores en cada departamento, su perfil, etc. A medida que interactúa con los individuos, puede obtener más información sobre la estructura de la red: puede ver quién interactúa con quién, qué personas coinciden en el mismo horario, quiénes tienen contactos en común, etc. A partir de esta información, este individuo puede formarse una idea más precisa del entorno social en el que está integrada. En efecto, la investigación en psicología social muestra cómo esto verdaderamente ocurre: las personas tienden a formarse una un sociograma mental que captura las relaciones existentes entre los distintos individuos con los que tratan (Hecker, 1993; Kilduff and Tsai, 2003). En este tipo de situaciones −en las que los individuos pueden deducir más aspectos de su red a partir de la información que tienen− el modelo de Galeotti et al. (2010) no parece un reflejo ajustado del conocimiento que los agentes tienen sobre su red social. Surgen entonces las siguientes cuestiones:

- ¿Cómo podemos modelar la percepción que los agentes tienen de su red social en este tipo de situaciones? ¿Qué características de la red afectan a la percepción que los individuos tienen de ésta? ¿Qué aspectos de la red pueden deducir a partir del conocimiento de otros elementos de la misma?

- ¿Son las predicciones de Galeotti et al. (2010) robustas a cambios sutiles en sus supuestos de información? ¿existe un efecto monótono de incrementar la información que los jugadores tienen de la red en el número y/o estructura de los equilibrios que se pueden alcanzar?

El presente capítulo trata de dar respuesta a estas cuestiones. Está organizado en dos bloques:

1. **Percepción de la red.** Analizamos cómo los individuos perciben la red social de la que forman parte cuando la información que poseen de la misma es limitada. Con tal propósito, consideramos distintos escenarios de información y exploramos qué aspectos de la estructura social subyacente pueden deducir a partir de la información que tienen de la red en cada uno de estos escenarios. Comenzamos por un escenario en el que los jugadores tienen información muy limitada sobre la red que integran −conocen exclusivamente su *degree* y la *distribución del degree* de la red− y a partir de ahí vamos incrementando la cantidad de información que los agentes poseen. Mostramos cómo distintas redes pueden ser factibles para los agentes dada la información que poseen en todos estos escenarios y analizamos la distribución de probabilidad de las mismas. De la misma manera, ilustramos cómo los agentes calculan la probabilidad de ocupar una determinada posición en la red teniendo en consideración todas las redes que consideran factibles y su distribución de probabilidad.

   Descubrimos una relación entre ciertas nociones de simetría entre nodos, tales como la *equivalencia automórfica* (Hanneman y Riddle, 2005) y la *equivalencia estructural* (Lorrain y White, 1971), y la percepción de los agentes de su entorno social. Tal relación implica que, bajo todos los escenarios de información considerados, las personas presentan un sesgo cognitivo hacia estructuras sociales más asimétricas. Esto es, cuando varias geometrías de red son factibles dada la información de los individuos, éstos asignan un peso probabilístico mayor a aquellas con un *grupo automórfico* de menor orden. Este resultado tiene un efecto en el comportamiento y bienestar de los agentes, como mostramos en la segunda parte del capítulo.

2. **Juegos en redes.** Estudiamos las interacciones estratégicas que tienen lugar en la red considerando el marco teórico desarrollado en la primera parte. Comenzamos por mostrar cómo las predicciones de Galeotti et al. (2010) se rompen cuando relajamos sutilmente sus supuestos. Cuando los agentes tienen información sobre su *degree* y la *distribución del degree de la red* −y a partir de esta información deducen la probabilidad de que sus amigos tengan *degrees* particulares− los equilibrios simétricos pueden ser múltiples y exhibir diversos patrones. Bajo este escenario de información, los equilibrios simétricos son iguales en todas las redes con el mismo tamaño y distribución del *degree*. Sin embargo, dado que el bienestar alcanzado en estos equilibrios varía dependiendo de la geometría de la red, establecemos un condición suficiente para que una determinada estructura de red sea eficiente (el bienestar de sus integrantes sea máximo). Finalmente, ilustramos cómo el hecho de incrementar la información de la que disponen los jugadores tiene efectos no monótonos en el número y la estructura de los equilibrios.

Las aportaciones de este capítulo son varias:

- Caracterizamos formalmente los mapas cognitivos de los individuos sobre la red social en la que están inmersos a partir de la información que poseen. Nuestro marco contribuye a la literatura existente en psicología social, sociología (Krackhardt; 1987, Carley; 1986; Michaelson y Contractor, 1992; Freeman, 1992; Kumbasar et al. 1994; Casciaro, 1998; Johnson y Orbach; 2002; Janicik y Larrick, 2005) y economía (Dessi et al. 2016) en relación a la percepción que los individuos tienen de su entorno social. En particular, mostramos que algunas nociones clásicas de equivalencia entre nodos son relevantes a la hora de explicar la percepción que los individuos tienen de sus redes sociales. Probamos que todos los jugadores tienen creencias simétricas sobre la estructura de la red que componen cuando sólo tienen información sobre su *degree* y a la *distribución del degree de la red*. Sin embargo, esta simetría en la percepción de la red desaparece a medida que el radio de información de los individuos se incrementa ligeramente.

- El marco teórico propuesto permite analizar las interacciones estratégicas entre jugadores cuando éstos tienen creencias racionales sobre la posición que sus contrincantes ocupan en la red. En nuestro modelo, los jugadores creen que sus vecinos presentan atributos particulares (por ejemplo, tienen un cierto *degree*) con probabilidades que calculan teniendo en cuenta el conjunto de redes factibles y su distribución de probabilidad. Ilustramos cómo esta diferencia entre nuestro modelo y el de Galeotti et al. (2010) hace que el equilibrio alcanzado no sea único ni necesariamente monótono no creciente (no decreciente) en el *degree* de los jugadores bajo sustitutivos estratégicos (complementarios estratégicos).[5]

[5]Galeotti et al. (2010) realizan algunos supuestos respecto a las creencias que los jugadores tienen sobre el degree de otros

- Ponemos de manifiesto los efectos que tiene manipular la información sobre la red de la que disponen los jugadores en las interacciones estratégicas que tienen lugar en la misma. La introducción de información incompleta como solución al problema de multiplicidad de equilibrios se ha encontrado con una crítica fundamental: el resultado alcanzado por los jugadores depende en gran medida de cómo se introduce esta falta de información −qué aspectos de la red se asume que conocen y cuáles desconocen (Weinstein y Yildiz, 2007)−. Si bien está crítica podría aplicarse a cualquier juego Bayesiano, el presente capítulo muestra que es particularmente relevante en los juegos que tienen lugar en redes: dada la interdependencia que existe entre las distintas medidas de la red, un ligerísimo cambio en la información que los agentes tienen sobre algún elemento de la misma puede cambiar sustancialmente la percepción que tienen sobre ésta y por ende, en los equilibrios y el bienestar alcanzados.

- Por último, señalamos el papel que el tamaño del *grupo automórfico* de la red −medida de su grado de simetría− tiene a la hora de analizar el comportamiento y el bienestar de los individuos cuando estos interactúan en la misma. Si bien esta propiedad de la red ha tendido a ser ignorada en la literatura de juegos, este capítulo pone de manifiesto su relevancia a la hora de explicar la percepción que los individuos tienen de su entorno social, así como las decisiones que estos toman en contextos de interacción estratégica.

agentes. En particular, prueban que bajo sustitutivos estratégicos (complementarios estratégicos) el equilibrio es monótono no creciente (no decreciente) cuando todos los jugadores creen que la probabilidad de tener un vecino con un degree alto depende negativamente (positivamente) de su propio degree, o es independiente del mismo. En este capítulo, analizamos los juegos relajando estos supuestos.

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Capítulo 1

Are close-knit networks good for employment?

1.1. Introduction

The key role of social networks in shaping socio-economic phenomena is well documented across a variety of contexts.[1] The importance of social contacts and the architecture of relationships has been particularly recognized in labor economics as a key source of employment information (Ionamides and Datcher, 2005; Beaman, 2016). Granovetter (2018) concludes that people rely primarily on contacts when finding a job, independently of the occupation, skill, location, or socio-economic background.[2] Cappellari and Tatsiramos (2015) estimate that an additional employed friend increases the probability of finding a job by 3.7 %.[3] Topa (2001) and Conley and Topa (2002) document geographic, ethnic, and race correlations in employment, suggesting a network-based flow of information about jobs. Other stream of literature analyzes referral systems in hiring, reporting a better performance of referred employees (Burks et al., 2015), a higher probability of being hired (Brown et al., 2016; Pallais and Sands, 2016), and a longer tenure in the firm (Dustmann et al., 2016). Thus, employers may benefit from the employees network to increase the search efficiency in the hiring process (Barr et al., 2019).[4]

Most of the above mentioned literature has focused on the effects of network size, whereas finer details of network architecture have received less attention. Granovetter (1973) documents that, while 16.7 % of interviewed individuals report having found their jobs through a *strong tie* (i.e. someone they saw regularly), 83.4 % found their job through a *weak tie* (a contact they meet «occasionally»).[5] He argues that the strength of the tie between two individuals is intimately related to the *«overlap in their friendship circles»*, in such a way that individuals linked through strong ties are expected to have a higher proportion of common contacts, integrating densely-connected groups deprived of information from other parts of the social system. In the terminology of this chapter, they are more likely to belong to short network cycles. Weak ties, on the contrary, may act as bridges between such densely connected neighborhoods, enjoying an advantage for getting novel information with respect to strong ties. Many studies have tested Granovetter's argument (see e.g. Killworth and Bernard, 1974; Friedkin, 1980; Montgomery, 1992) or applied his idea to different frameworks, ranging from the diffusion of knowledge among organizations (e.g. Hansen, 1999) to innovation or creativity (e.g. Ruef, 2002 and Baer, 2010). Recently, Gee et al. (2017) have analyzed the role of weak and strong ties in

[1]See Goyal (2007), Vega-Redondo (2007), Jackson (2010), and Jackson et al. (2017).

[2]The weight given to the network-driven information differs across these dimensions though. See also Myers and Schultz, 1951; Corcoran, et al. 1980; Marsden and Campbell, 1990; Montgomery, 1991; Marmaros and Sacerdote, 2002; Munshi, 2003; Franzen and Hangarther, 2006; Pellizari, 2010; and Bentolila et al., 2010, among many others.

[3]Some studies directly explore the variation of employment status of some network members on other network integrants (see e.g. Topa, 2001; Bayer et al., 2008; Beaman, 2011; Beaman and Magruder, 2012).

[4]Beaman et al. (2018) identify a potential cause of the different labor outcomes of males and females by documenting a significant tendency of men to refer few women, compared to men.

[5]Granovetter (1973) defines the strength of a tie as the «*amount of time, the emotional intensity, the intimacy (mutual confiding), and the reciprocal services which characterize the tie*».

job search. Using data on six million Facebook users and creating a proxy for job help by identifying people who eventually worked for the same employer as a pre-existing friend, they report that people are more likely to find a job through a weak tie because of their relatively large number, but strong ties are individually more beneficial at the margin. Note that they measure tie strength between two agents as the number of friends they have in common, or−alternatively−the number of triangles, in which both participate.[6] This evidence notwithstanding, little is known about what drives this *strength of weak ties* phenomenon. Is this effect driven solely by the differences in the content of the information, or is it partly due to the different network embeddedness of weak vs. strong ties?

This chapter analyzes theoretically whether close-knittedness influences employment prospects and wages of individuals and groups. Close-knit neighborhoods, one of the most prevalent features of real-world social networks (Jackson and Rogers, 2007; Jackson, 2010), are characterized by high transitivity of relationship: friends of friends are typically friends; and so are friends of friends of friends, and so on. In network terminology, highly clustered networks contain a large number of triangles, squares, and other short network cycles.

To study the role of short cycles in the flow of job market information, we build on a model developed by Calvó-Armengol (2004) and Calvó-Armengol and Jackson (2004, 2007).[7] People are distributed on a network and employed individuals who hear about vacancies pass the information on to their unemployed network neighbors.[8] Calvó-Armengol (2004) shows that higher degree and lower second-order degree increase the individual probability of employment and detects a non-monotonic effect of shifts of the degree distribution on employment in cycle-free regular networks. Calvó-Armengol and Jackson (2004) study patterns of correlations from a dynamic perspective. They show that such a network model generates unemployment correlations across time and path-connected individuals, exhibits duration dependence and persistence, and allows to understand inequalities and drop-out decisions.[9] Calvó-Armengol and Jackson (2004) provide several examples suggesting that other network features might play a role, but they do not isolate their effects formally. Last, Calvó-Armengol and Jackson (2007) analyze wages in a similar model. Our contribution to this literature is to model explicitly the role of local network density of links, measured by the presence of short cycles, isolating their effect from that of the first- and second-order degree.

We report two sets of results. The first result has implications that go beyond the labor-market application of this study: we show that short network cycles generate *stochastic affiliation* in diffusion processes.[10] More precisely, if two friends of individual i are friends or have another common friend, the information flows from these two friends to i are schochastically affiliated even in a one-period model, whereas these flows are independent if they are connected neither directly nor indirectly in any other way.[11] This implies that the diffusion does not only depend on the number of first- and second-order connections but also on the geometry created by these links. Thus, two neighborhoods or two networks where the players have the same number of first- and second-order neighbors (i.e. the same *joint* distribution of degree and second-order degree) but differing clustering patterns induce different diffusion dynamics and thus different employment and wage distributions. Since this result is fairly general and does not rely on many of the model assumptions, our work provides a microfoundation for *why* short networks cycles, namely triangles and squares, shape diffusion on networks.

Our second set of results concerns the micro- and macroeconomic implications of short network cycles. We prove that, *ceteris paribus*, people involved in three- or four-cycles have worse employment prospects. The intuition stems from information affiliation: stochastic affiliation increases the probability of mismatch between vacancies and job candidates, leading to diffusion inefficiencies. That is, network cycles are a source of labor-market frictions when employers rely on referrals. Our numerical simulations show that this result persists in the steady state of a dynamic Markov chain. In the long run, network cycles organize the employment

[6] Bian et al. (2015) show empirically that weak ties are better able to provide job information than strong ones, but strong ties are better at mobilizing diverse forms of favoritism that is particularly relevant for high-earning positions.

[7] Calvó-Armengol (2004) and Calvó-Armengol and Jackson (2004, 2007) have in turn been partially inspired by the early contributions of Boorman (1975) and Diamond (1981).}

[8] See also Montgomery (1991). Galeotti and Merlino (2014) model endogenous formation of networks.

[9] Unemployed agents obtain more (less) information when their contacts are employed (unemployed), giving rise to robust forms of correlations in wages and employment status of path connected agents.

[10] Stochastic affiliation is a strong form of correlation; see Section 1.3.1 for a formal definition.

[11] Some results in Calvó-Armengol (2004) and Calvó-Armengol and Jackson (2004, 2007) rely on the assumption that the inflow of information from two neighbors of i to i is independent. Our contribution is to show that this assumption fails even in the static model if the two neighbors are connected or if they share a friend $j \neq i$. We further explore the consequences.

probabilities in the sense of the first-order stochastic dominance. People in densely-knit neighborhoods have lower employment prospects than individuals in comparable positions and close-knit networks exhibit higher unemployment rates than more loosely-knit societies with the same joint distributions of first- and second-order degrees. Hence, short network cycles have important short- and long-run consequences for inequality within and across networks. We further show that economically relevant and statistically strong effects are exclusively limited to short cycles in our model. Importantly, none of these results relies on spatial segregation of low- and high-clustering people. Although spatial correlations in the employment of path-connected people, characterized in Calvó-Armengol and Jackson (2004, 2007), weaken the negative impact of short cycles in integrated societies, we show that their detrimental role can persistent in the steady state. Hence, policies aiming at the integration of dense and loosely-knit agents may have a limited effect.

These findings extend for wages. Leaving constant other network features, three- and four- cycles reduce the expected wage of agents involved in these cycles. However, this negative effect is driven by the unemployment probability. Conditional on having a job, the expected wages of individuals involved is short cycles are actually higher. The affiliation of information flows increases the probability of multiple job offers in dense neighborhoods but such multiplicity is not longer redundant if we analyze wages: multiple job offers allow agents to select among offers with different wages if unemployed or accept better-paid jobs if employed. Hence, the direction of the effect of short network cycles may vary in function of one's socio-economic environment.

Short network cycles further affect other features of the employment dynamics. Most importantly, they increase serial correlations of employment. This has two implications. Firstly, since the variability of the steady-state employment is virtually unaffected by network cycles but serial correlation increases considerably, the fluctuations of employment within denser neighborhoods or in closely-knit societies exhibit larger amplitude. This effect is generated by a combination of factors. First, network diffusion generates employment time correlations, as well as correlations between connected nodes. Employed friends maintain their contacts employed due to the diffusion channel, while unemployed agents make their contacts more vulnerable (Calvó-Armengol and Jackson, 2004). However, the affiliation in job-market information flows caused by network short cycles reinforces these effects and extends them to network neighborhoods. On the one hand, it slows down the transition between different employment states: it maintains employment within cycled groups of employed individuals while it also perpetuates unemployment in unemployed cycled neighborhoods. On the other hand, the spatial correlation makes the transition from employment to unemployment in a neighborhood more difficult. However, if several members of an employed neighborhood lose their job, they make their circles more vulnerable and drag the whole neighborhood towards unemployment; in contrast, if a positive shock hits an unemployed neighborhood the network externality spills over more easily in cycled neighborhoods. The combination of these factors results in longer, more persistent employment fluctuations, with more pronounced booms and troughs in closely-knit neighborhoods and networks. Secondly, the combination of higher time persistence of employment and labor market transitions with lower unemployment causes that, even though the probability of maintaining a job is unaffected, the likelihood of remaining unemployed across periods is enhanced by close-knittedness.

The findings of this chapter have important implications for two fields, labor economics and network theory. As for the former, we deepen our understanding of the micro- and macroeconomic effects of social networks in labor markets and the economy as a whole. First, we uncover one possible mechanism behind the strength of weak ties (Granovetter, 1973): since weak connections are less likely to be embedded in short cycles,[12] the strength of weak ties lies not only on the informational content but also on the lack of correlation in the information they provide.[13] Granovetter's (1973) argues that the benefits of weak ties derive from their ability to transmit information to larger audiences and to provide novel information. We show that individuals immersed in loose-knit communities may enjoy information advantages even when they are in contact with the same number of people and when the kind of information they receive is of the same nature as in highly cohesive neighborhoods. Thus, weak ties might be relevant not solely because of their bridging role, but due to their capacity to provide independent information.

[12]Granovetter (1973) argues that *«transitivity, (...), is claimed to be a function of the strength of ties, rather than a general feature of social structure»*. According to Marsden and Campbell (1984), the overlap of social circles is a particularly good measure of the strength of a tie. Louch (2002) indeed observes that when two contacts of an individual speak to each other frequently or know each other for a long time, i.e. they can be thought of as *strong ties* according to Granovetter (1973) definition, the probability that they are mutually linked increases by 178 % and 278 %, respectively.

[13]Conversely, this mechanism provides a rationale behind the *structural holes* argument of Burt (2005): one of the advantages of bridging holes is the diversification of information flows from otherwise disconnected parts of the network.

Second, taking into account that the empirical evidence consistently corroborates that real-life labor markets heavily rely on employee referrals, we show that short network cycles−and consequently, the clustering coefficient, social cohesion, and network close-knittedness−are important determinants of individual and aggregate labor-market performance. Short cycles affect inequality and they may be a relevant source of labor-market frictions. They reinforce the propagation of idiosyncratic shocks, shaping labor fluctuations over time and space in our model. The detected impact is particularly in line with the literature that shows that search frictions lead to sluggish aggregate employment dynamics and labor market churning (e.g. Pissarides, 1985; Bentolilla and Bertola, 1999; Mortensen and Pissarides, 1999; Burgess et al., 2001). There is a literature on how unemployment entry and exit rates are affected by personal characteristics, such as education (Maarten and Wolbers, 2000); Calvó-Armengol and Jackson (2004) position networks among the determinants. We identify another source of sluggishness: the geometry of one's social environment. This channel deserves a deeper empirical investigation. More generally, since employment fluctuations and employment transitions are strongly correlated with business cycles and aggregate macroeconomic volatility cannot be explained without consideration of labor-market fluctuations (Kydland, 1995; Mortensen and Pissardies, 1999), our results may contribute to the understanding of key stylized facts of macroeconomic dynamics.

Last, we add a new potentially negative aspect to the list of economic consequences of recruitment process that rely on employee referrals. Calvó-Armengol (2004) shows that popular friends and too much connectivity in a network may be detrimental and Calvó-Armengol and Jackson (2004, 2007) implies that, under employee referral programs, initially disadvantaged groups are persistently worse off. Munshi and Rosenzweig (2006) report that networks may prevent people from exploiting new opportunities. We show that, apart from connectivity, network cohesion contributes to inequality and lead to labor-market search inefficiencies.

As for network theory, we provide three novel insights. First, we show that network close-knittedness goes beyond the clustering coefficient as both the number and the organization of cycles of different lengths determine to what extent one benefits from the social capital embedded in social structures. Triadic closure and clustering (i.e. cycles of length three) have received a great deal of attention in the literature across fields. Our focus on cycles links naturally two concepts based on triangles, the clustering coefficient and the concept of network support proposed recently by Jackson et al. (2012). However, both concepts ignore the role of longer cycles in shaping socio-economic phenomena. We show formally that cycles of length larger than three also play a role and why.

Second, numerous studies across fields suggest that local clustering may affect diffusion on networks (e.g. Calvó-Armengol and Jackson, 2004; Centola, 2010; Acemoglu et al., 2011; Campbell, 2013). However, it is difficult to disentangle the effect of clustering from that of other network features, which is a stumbling block for any causal claims regarding the effect of short cycles and for a microfoundation of their role in network processes. Since the transmission of information in this chapter resembles the network diffusion of many other phenomena, our results may have implications beyond the labor market literature. We show formally that clustering matters keeping constant the first- and second-order degree distributions−the two features typically considered in the diffusion literature−and that short network cycles induce correlation in diffusion. Since cycles already impact diffusion in our static, one-period model, they will likely play a relevant role in the dynamics of other network phenomena and abstracting from them may thus provide an incomplete picture of the mechanisms behind network diffusion.

Last, we contribute to the long-standing debate in economic sociology regarding whether dense neighborhoods are beneficial or detrimental (Burt, 2001, 2009; Jackson et al. 2017).[14] We report that, even in the same context, the answer depends on the particular question: close-knittedness may be both beneficial and detrimental and both directions coexist even in the same network and application. More precisely, network cohesion may be beneficial in well-off neighborhoods that are successful maintaining high employment and in times of high employment whereas it may hurt the same people in times of economic unease.

The chapter is organized as follows. Section 1.2 presents the model. Section 1.3 analyzes the static version of the model while the dynamic analysis can be found in Section 1.4. Section 1.5 introduces wages into the model. Finally, Section 1.6 concludes.

[14]Relationships in close-knit networks are typically stronger, enabling trust and cooperation (Granovetter, 1973; Burt, 1992; Coleman, 1988a, 1988b; Bloch et al., 2008; Lippert and Spagnolo, 2011; Jackson et al., 2012). However, close-knit networks inhibit the flow of novel information and individuals in tight neighborhoods may receive redundant information (Granovetter, 1973, 2005; Burt, 2001, Blau, 2017).

1.2. Model

To study the role of short cycles in labor markets, we build on the model of Calvó-Armengol (2004) and Calvó-Armengol and Jackson (2004, 2007). First, we present and analyze the static, one-period version of the model where we introduce and consider the impact of short cycles. See Section 1.4 for the analysis of the dynamics and long-run consequences.

1.2.1. The network

People are distributed on an undirected fixed network that is used to disseminate information about job openings. The network $g = (N, E)$ is characterized by a set of nodes $N = \{1, .., n\}$ and a set of edges or links E between them. We write $g_{ij} = 1$ if individuals i and j are directly linked in g and $g_{ij} = 0$ otherwise. Let $A = (g_{ij})_{i,j\in N}$ be the $n \times n$ symmetric adjacency matrix of the network, with $g_{ii} = 0$. To simplify notation, we sometimes denote a link between i and j by ij. G is the set of all feasible networks.

The set of i's direct contacts in g is defined as $N_i(g) = \{j \in N : g_{ij} = 1\}$; let $n_i(g) = |N_i(g)|$ be the (first-order) *degree* of individual i. Analogously, denote the set of i's second-order or indirect neighbors (neighbors of i's neighbors) as $N_i^2(g) = \{k \in N : g_{ij}g_{jk} = 1 \text{ for some } j \in N,\, i \neq k\}$ and $n_i^2(g) = \left|N_i^2(g)\right|$. Observe from the previous definitions that $N_i(g)$ and $N_i^2(g)$ may have a non-empty intersection; that is, some contacts of i may simultaneously be i's indirect contacts. For a pair of nodes i and j, define $N_{ij}(g) = \{k \in N : k \in N_i(g) \cap N_j(g)\}$ as the set of common contacts of both i and j, with $n_{ij}(g) = |N_{ij}(g)|$. The set of contacts of i that are not shared with j is $N_{i-j}(g) = N_i(g) \backslash N_{ij}(g)$, with $n_{i-j}(g) = |N_{i-j}(g)|$. Distance between nodes i and j in network g is the length of the shortest path between them, denoted as $d_{ij}(g)$. Naturally, $d_{ij}(g) = 1$ if $ij \in E$, $d_{ij}(g) = 2$ if there is $k \in N$ such that $ik, kj \in E$ but $ij \notin E$, and so on.

This work focuses on the effect of cycles on the probability of receiving information about job vacancies through network contacts.

Definition 1 (Cycle). *A $K-$cycle $Z_K(g)$ is a sequence of distinct nodes $i_1, i_2, \ldots, i_{K-1}, i_K$, such that $K > 2$, $g_{i_k i_{k+1}} = 1$ for each $k \in (1, \ldots, K-1)$, and $g_{i_1 i_K} = 1$.*

In words, a $K-$cycle in a network g is a sequence of K linked nodes starting and ending in the same node. A cycle may be equivalently defined as the set of edges. For example, a *three-cycle* or a *triangle* is a path passing through three edges: ij, jk, ki. Equivalently, we may refer to a triangle as the sequence $\{i, j, k\}$.[15] Analogously, a *four-cycle* or a *square* is a path through four edges *ij,jk,kl,li* or a sequence $\{i, j, k, l\}$. We denote $S_S(g)$ the set of all three- and four-cycles and $S_L(g)$ the set of all K-cycles for $K > 4$ in g; $S(g) = S_S(g) \cup S_L(g)$ thus corresponds to the set of all K-cycles in g. Last, $S_S^i(g)$, $S_L^i(g)$, and $S^i(g)$ are the sets of the corresponding cycles, in which i is involved.

The clustering coefficient of individual i is the fraction of$i$'s direct contacts who are neighbors themselves. In the above terminology, the coefficient measures the number of triangles in i's neighborhood divided by the number of all possible triangles among all i's contacts. Formally,[16]

$$C_i(g) = \frac{\sum_{j\neq i;k\neq j;k\neq i} g_{ij}g_{ik}g_{jk}}{\sum_{j\neq i;k\neq j;k\neq i} g_{ik}g_{ij}}$$

Clustering coefficient reflects the density or closeknittedness within a node's neighborhood; when the coefficient is high, the neighborhood is densely interconnected because most of i's contacts are linked. The average clustering coefficient of network g is simply $C(g) = \frac{1}{n}\sum_{i=1}^{n} C_i(g)$ and measures the overall level of local density within the network.

Note that the clustering coefficient counts the triangles but abstracts from longer cycles. Lind et al. (2005) propose a coefficient that keeps track of the fraction of four-cycles as follows:

$$C_i^4(g) = \frac{\sum g_{ij}g_{jm}g_{mk}g_{ik}}{\sum g_{ij}g_{ik}g_{jm}}$$

[15] In graph theory, the term n-cycle is sometimes used as a description of a circle network of n nodes. In Figure 1.2, g_b and g_c would be examples of three- and four-cycles, respectively, under that terminology. In this chapter, cycles may also represent a part of more complex architectures, rather than the whole network.

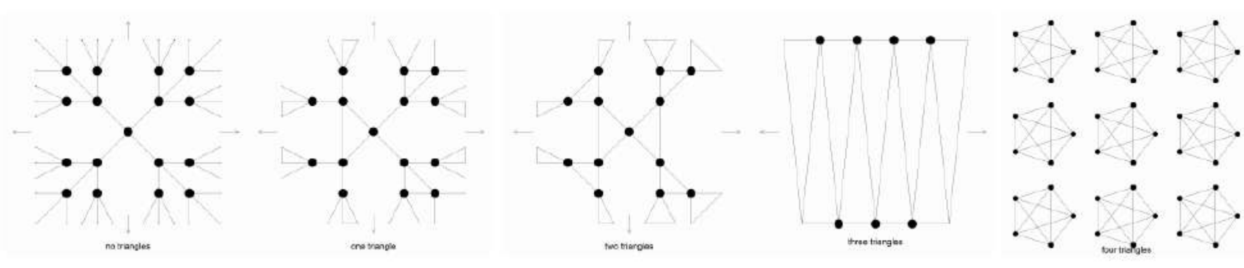
[16] The coefficient is not defined for $n_i(g) < 2$. In such a case, some authors consider clustering to be equal to zero (e.g. Vega-Redondo, 2007), while others leave it undefined.

where i, j, k and m are nodes of the network. For node i, the number of squares is given by the number of common neighbors among i's contacts (i.e. that is sequences $\{i, j, k, m\}$ such that $g_{ij}g_{jm}g_{mk}g_{ik} = 1$). Again, $C^4(g) = \frac{1}{n}\sum_{i=1}^{n} C_i^4(g)$.

A distinct but related concept is *support*, proposed by Jackson et al. (2012): A link jk is supported if $\exists\, i \in N : i \in N_{jk}(g)$; the link is unsupported if $N_{jk}(g) = \oslash$. A link jk is supported by i if $i \in N_{jk}(g)$; the link is unsupported by i if $i \notin N_{jk}(g)$. The more links are supported by i, the higher is i's embeddedness. Hence, network support is another measure of close-knittedness, but it differs from the clustering coefficient (see Jackson et al., 2012).

Certain network architectures play a particular role in our analysis. First, a *tree* is a network with no cycles of any length; that is, $S(g) = \emptyset$ and $C_i(g) = C_i^4(g) = 0$ for each $i \in N$. A *cycle network* is the only graph of n nodes and n links containing one n-cycle. In Figure 1.2, networks $g_b - g_d$ depict a triangle, square, and pentagon, respectively; network g_g represents a hexagon. A *regular network* is a network in which $|n_i(g)|$ is equal for all $i \in N$; a regular network is symmetric in the number of direct and indirect contacts of each individual but not necessarily in other network features.[17] A special case of regular network is a *vertex-transitive network*, where no node can be distinguished from any other based on its neighborhood since they all have a structurally identical neighborhood, second-order neighborhood, and so on. There are several formal definitions of these networks (see e.g. Weisstein, 2016), but an important property of these graphs is that each node occupies a structurally equivalent position in the network.[18] Hence, every vertex-transitive graph is regular, and each node has the same degree, second-order degree, and the clustering coefficient. However, the converse is not true; not all regular networks are vertex-transitive, and not all networks in which all nodes have the same clustering coefficient are vertex-transitive. For instance, every node in a network may have the same number of links but they can still differ in other characteristics, such as the clustering coefficient or global centrality. Examples of vertex-transitive networks include empty and complete networks, circles, cubes, many lattice networks, and Caley graphs. Figure 1.1 displays examples of vertex-transitive networks; all nodes in the five networks occupy an identical position. In case of the first three networks, all vertices have four connections and twelve second-order neighbors, but the number of triangles involving a given node increases from zero in case of the leftmost network to two in case of the middle network. The fourth and fifth networks are also vertex-transitive with $n_i(g) = 4$ for all i, but each node is involved in several three- and four-cycles. To see the difference between clustering and support, note that the clustering coefficient of each node increases from zero to one as we move from the left to the right. Nevertheless, the support is already maximal in the third and fourth networks where every single edge is supported, while the clustering is lower than one in these networks because each node has contacts who are not connected.

Figure 1.1 Vertex-transitive networks with degree four but increasing clustering from the left to the right.



1.2.2. Information flows

In our model, initially each worker is employed.[19] Then, every agent looses her job with probability $b \in (0, 1)$. Afterwards, each node may hear about a vacancy with probability $a \in (0, 1)$, regardless of her

[17] Some authors call regular networks symmetric (e.g. Calvó-Armengol, 2004). However, the term symmetric network exists in graph theory and it is more restrictive than regularity.

[18] In fact, vertex-transitive graphs are also called node-symmetric (Chiang and Chen, 1995), a name that reflects better the main feature of these networks.

[19] This assumption is inconsequential. All the results hold if all people start with the same probability of being employed.

employment status. We assume that all jobs are identical.[20] Loosing the job and hearing about a vacancy are independently distributed and independent across individuals.

At this point, each worker can be in one out of four possible situations or status:

1. **Status 1**. Each person is employed and has heard about a new job with probability $\alpha = a(1-b)$.
2. **Status 2**. She is unemployed and has not heard about any offer with probability $\beta = b(1-a)$.
3. **Status 3**. She is unemployed but has heard about an offer with probability $\delta = ab$
4. **Status 4.** She is employed with no offer to pass to contacts with probability $\gamma = (1-a)(1-b)$.

We label these different status as 1 to 4 respectively, and define the random variables $X_1^i(g)$, $X_2^i(g)$, $X_3^i(g)$, $X_4^i(g)$ as the number of i's contacts who are in each status. Particular realizations of these variables are denoted as $x_1^i(g)$, $x_2^i(g)$, $x_3^i(g)$ and $x_4^i(g)$, with $x_1^i(g)+x_2^i(g)+x_3^i(g)+x_4^i(g) = n_i(g)$. Similarly, the variables $X_m^{jk}(g)$ and $X_m^{j-k}(g)$ measure respectively the number of agents in $N_{jk}(g)$ and $N_{j-k}(g)$ who are in status $m \in \{1,2,3,4\}$; $x_m^{jk}(g)$ and $x_m^{j-k}(g)$ denote again their realizations. Define $Y_i^s(g)$ a random variable, such that $y_i^s(g) = 1$ if i is in state $s = \{1,2,3,4\}$; $y_k^s(g) = 0$ otherwise. Unemployed individuals who hear about a job (those in status 3) immediately accept the offer. Workers who are employed and have heard about a vacancy (in status 1) pass the offer to one of their unemployed neighbors (in status 2) uniformly at random. As a result, status 1 and 2 play a key role in our analysis.

The model assumes that all agents at any point in time have perfect information about the labor status of their direct contacts.[21] Observe that only individuals in state 1 can pass an offer to one of their direct contacts in status 2 (i.e. those who lost their job but have not heard about any offer) who immediately accepts the offer. Therefore, an individual's contacts who are in status 1 will be called *potential providers* and her second-order contacts (the contacts of her contacts) who happen to be in status 2 will be named *competitors*. By construction, it is possible that an individual in status 2 simultaneously receives several offers from her different contacts. In such a case, she accepts one of them at random and the others remain unfilled. These redundant offers may generate search frictions in the labor market.

Assume that node i is unemployed. We represent the information flow from $j \in N_i(g)$ to i through a random variable $I_j^i(g)$ taking value 1 when i receives information from j and 0 otherwise. $I_j^i(g)$ is a function of the state of j and j's network position. First, it depends on whether j is in state 1 (a provider), an event with probability α. If so, she passes an offer to one of her unemployed neighbors with equal probability. Thus, i receives an offer from j with probability $\frac{1}{X_2^{j\setminus i}(g)+1}$, where $X_2^{j\setminus i}(g)$ denotes the number of j's contacts other than i who are in state 2. Then,

$$I_j^i(g) = \begin{cases} 1 & \text{with probability} \quad \left(\frac{\alpha}{X_2^{j\setminus i}(g)+1}\right) \\ 0 & \text{with probability} \quad \left(1 - \frac{\alpha}{X_2^{j\setminus i}(g)+1}\right) \end{cases} \tag{1.2.1}$$

1.2.3. The probability of receiving an offer through the network and the unemployment rate

Denote $P^i(g)$ the probability that node i receives a job offer from at least one neighbor in network g when she is is status 2 (i.e. unemployed). With this notation, we can write the employment probability of node i as $E_i(g) = (1-b) + ba + b(1-a)P^i(g)$, which can be interpreted as the individual employment prospect in network g. The employment rate of the network is $E(g) = \frac{1}{n}\sum_{i\in N} E_i(g)$; the unemployment rate is thus $U(g) = 1 - E(g)$.

[20] In Section 1.5, we relax this assumption and allow for wages to differ across jobs.

[21] In Calvó-Armengol (2004), employed individuals with a job offer pass the vacancy to one of their contacts who lost their job. Unemployed individuals may have later received an offer but this is not observed by other agents. Though this is not explicitly stated, it can be seen in the proof of his Proposition 1, where the decision to pass information depends on b instead of β. We change slightly this assumption and assume that the decision depends on β. If we maintained the assumption of Calvó-Armengol (2004), our results would in fact be reinforced.

Since a and b are exogenously given and the same for all individuals, the only difference in (un)employment rates across nodes and networks arises from $P^i(g)$ so we analyze how this probability depends on network cycles.

$P^i(g)$ depends on the employment status of i's direct and indirect (second-order) contacts. The status of i's direct neighbors determines the number of i's potential providers, $X_1^i(g)$, while the status of the contacts of the potential providers determines the number of competitors $(X_2^{j\setminus i}(g))$ and thus the probability with which any potential provider j passes an offer to i (see (1)). As we show below, both three- and four-cycles affect the probability with which i receives at least one offer through her contacts.

Assume that $y_i^2(g) = 1$ and $y_j^1(g) = 1$, $j \in N_i(g)$. Since each node $k \in N_j(g) \setminus i$ may be unemployed (that is, in status 2) with probability β, the probability that i does not receive any offer from j is[22]

$$q_j(n_j(g)) = \sum_{h=0}^{n_j(g)-1} \binom{n_j(g)-1}{h} \beta^h (1-\beta)^{n_j(g)-1-h} \frac{h}{h+1} \tag{1.2.2}$$

In a similar vein, the probability that i does not receive any offer from j, conditionall on knowing the status of $k \in N_j(g) \setminus i$, is

$$q_j(n_j(g) \mid y_k^2(g) = x) = \sum_{h=0}^{n_j(g)-2} \binom{n_j(g)-2}{h} \beta^h (1-\beta)^{n_j(g)-2-h} \ \frac{h+x}{h+x+1} \tag{1.2.3}$$

Expressions (1.2.2) and (1.2.3) directly lead to the following claim:

Claim 1. (Individual Probability). *Assume that $y_i^2(g) = 1$ (unemployed) and let $y_j^1(g) = 1$ for a node $j \in N_i(g)$ (potential provider of i). Then, $q_j(n_j(g))$ is increasing in $n_j(g)$ and independent of network cycles, and $q_j(n_j(g) \mid y_k^2(g) = 1) > q_j(n_j(g) \mid y_k^2(g) = 0) = q_j(n_j(g) - 1)$.*

Claim 1 illustrates first that the probability with which i does not receive any offer from a particular provider j (or, conversely, the probability that she does, i.e. $1 - q_j(n_j(g))$ only depends on the number of (potential) competitors in j's neighborhood and this probability is not affected by the presence of cycles within i's or j's neighborhoods. Hence, network cycles do not affect the *individua*l decision to pass information about jobs. However, Example 1 below illustrates that cycles do affect the probability of getting an offer from *at least one* contact, altering one's employment prospects. Second, Claim 1 shows how knowing the status of a neighbor of a neighbor affects this probability and that knowing that one neighbor does not need a job is equivalent to having one competitor less. These latter results play a key role if a neighbor of a neighbor can simultaneously be a direct neighbor.

Example 1 (Effect of short cycles). Consider networks g_b, g_c, g_e, and g_f in Figure 1.2 and assume that $y_i^2(g) = 1$ in each network. The probability that node 1 receives a job offer from at least one neighbor in network g, $P^1(g)$, satisfies the following:[23]

$$P^1(g_e) > P^1(g_c) > P^1(g_b) \tag{1.2.4}$$

Note that node 1's degree is the same in the three networks, while her second-order degree is the same in networks g_b and g_e but lower in g_c. However, individual 1 is more likely to be employed in the tree network g_e, followed by the square network g_c, as compared to the triangle network g_b even though the number of competitors is the same in the first two networks and even though node 1 has one competitor less in g_c than in g_e. Similarly, $P^1(g_f)$ may be larger or lower than $P^1(g_b)$, depending on the parameters. If, say, $\alpha = 0.9$ and $\beta = 0.01$, $P^1(g_f) = 0.9886 > P^1(g_b) = 0.9810$, despite the fact that the number of 1's potential competitors is greater in g_f than in g_b. ■

Claim 1 and Example 1 jointly deliver several messages. First, the geometry of network neighborhoods affects the probability of receiving a job offer, beyond the number of first- and second-order neighbors. We

[22] Since $q_j(n_j(g))$ is identical for any $k \in N_j(g)$ including i, we do not index this probability by i throughout the chapter to simplify the notation.

[23] Detailed calculations can be found in Appendix.

particularly point to the role of short cycles. Since Claim 1 shows that their presence does not affect the probability of receiving an offer from one particular individual, their role in Example 1 stems from the lack of independence of information flows coming from different contacts j and k if they belong to a cycle with i (operationalized by a multivariate random variable $\left(I_j^i(g), I_k^i(g)\right)$. The example particularly suggests that network cycles affect individuals negatively and their length may matter. Last but not least, the impact of cycles is economically relevant: their effect in Example 1 rivals with that of two-link-away connections. We formalize these observations in the next section.

1.3. Results

1.3.1. Cycles and affiliation of information flows

In cycle-free networks (e.g. trees), $N_i(g) \cap N_i^2(g) = \emptyset$ and $N_{jk}(g) = \{i\}$ if $j, k \in N_i(g)$, implying that information flows from neighbors j and k to i are independent. The probability that neighbor j passes an offer to i depends on j's status and on $X_2^{j \setminus i}$, but it depends neither on the status of any other $k \in N_i(g)$ nor on the status of any second-order contact $s \in N_k(g) \setminus i$, $k \neq j$.

In contrast, if $j, k \in N_i(g)$ and $jk \in E$ (i.e. j and k form a three-cycle with i), the information flow from j to i depends on both j's and k's status and, similarly, that from k depends on k's and j's status. More precisely, the probability that j transmits information about a job opening to i depends on whether j is employed and possesses such information and on whether k also needs a job. If k does, k will provide no information to i and this is independent of cycles. However, since j and k are friends, k now competes with i for information from j, decreasing i's probability of receiving information from j. Such *indirect* effect of the status of k on $P_j^i(g)$−and thus on $P^i(g)$−is a direct consequence of the cycle $\{i, j, k\}$. This indirect effect is missing when j and k do not belong to any short cycle with i.

Similar dependence appears when j and k belong to a four-cycle together with i. If a four-cycle $\{i, j, s, k\}$ is present, the lack of independence in information flows from j and k to i comes from the fact that the probabilities of receiving a job offer from each of them depend on a common variable, the status of individual s.

We formalize these statements using the concept of stochastic affiliation:[24][25]

Proposition 1 (Affiliation). *Assume $y_i^2(g) = 1$ and consider $j, k \in N_i(g)$.*
(a) $I_j^i(g)$ and $I_k^i(g)$ are strictly affiliated if either $S_S^i(g) = \{i, j, k\}$ or $S_S^i(g) = \{i, j, s, k\}$ for one $s \neq i, j, k$.
(b) Conditional on the status of j's and k's neighbors who form three or four-cycles with i, $I_j^i(g)$ and $I_k^i(g)$ are conditionally independent.

Strict affiliation in information flows from node i's potential providers implies that, conditional on receiving an offer from a provider j with high probability, the probability that i receives another offer from a provider k increases if the two providers are connected directly or through another node $s \neq i$.

We illustrate the intuition behind the proof using an example. Consider a triangle network g_b in Figure 1.2 composed of individuals i, j, and k and assume that node i is unemployed. Denote $f(I_j^i(g_b), I_k^i(g_b))$ the joint density function of $I_j^i(g_b)$ and $I_k^i(g_b)$. First, consider that $j \in N_i(g)$ is employed and hears about an open position; this happens with probability α. Only j will pass information to i, but not k, when k is employed but does not have information about other jobs, an event that has probability $1 - \alpha - \beta$. Node j transmits a job offer to i with probability 1 in such a case. In contrast, i only receives a job offer from j with probability $\frac{1}{2}$ if k needs a job and thus competes with i for j's information, an event that has probability β. As a result and due to the network symmetry,

$$f(1,0) = f(0,1) = \alpha\beta\frac{1}{2} + \alpha(1 - \alpha - \beta).$$

[24] This concept was introduced to economics by Milgrom and Weber (1982). Two random variables are affiliated if, conditional on observing a low (high) value of one variable, the probability of observing a low (high) value of the other variable increases. Formally, two random variables X and Y are affiliated if for all $x < x'$ and for all $y < y'$:

$$f(x', y) * f(x.y') \leq f(x', y') * f(x, y)$$

where *f(x,y)* is the joint density function of variables X and Y. Since two independent random variables are affiliated according to the above expression, we say that two variables are *strictly* affiliated if the condition holds with strict inequality.

[25] All proofs are relegated to Appendix.

Second, i receives no offer from any of her neighbors if either none of them has any information to pass or one of them does but passes it along to a competitor, leading to

$$f(0,0) = (1-\alpha)^2 + 2\alpha\beta\frac{1}{2}$$

Last, a node does not lose the job and hears about a vacancy with probability α. Therefore, i might receive two offers from both j and k with probability $f(1,1) = \alpha^2$.

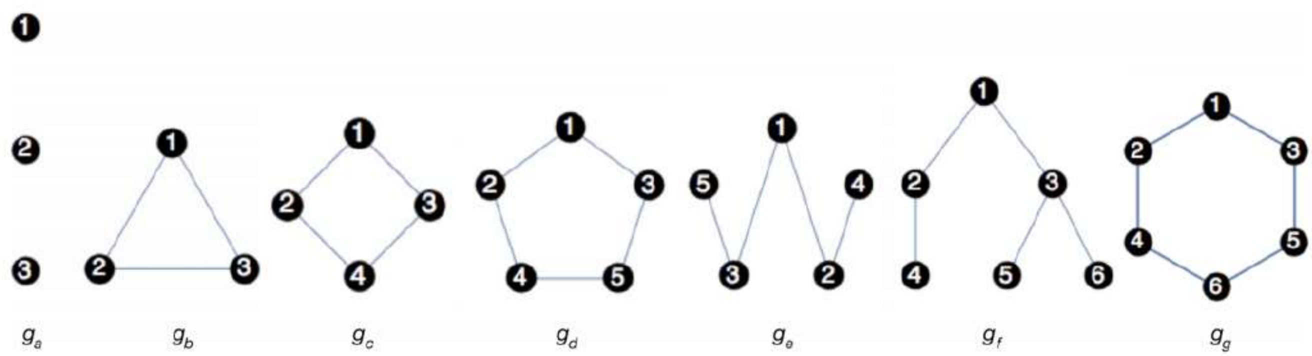
Note that

$$f(1,1) * f(0,0) = \alpha^2\Big((1-\alpha)^2 + \alpha\beta\Big) > \alpha^2\Big((1-\alpha-\beta) + \beta\frac{1}{2}\Big)^2 = f(0,1) * f(1,0)$$
$$(1-\alpha)^2 + \beta\alpha > (1-\alpha)^2 - \beta(1-\alpha-\frac{1}{4}\beta),$$

which implies that $I_j^i(g_b)$ and $I_k^i(g_b)$ are strictly affiliated.

To illustrate the case of a four-cycle, consider the four-node network g_c in Figure 1.2 and take the perspective of a node i with $N_i(g_c) = \{j,k\}$ and $N_i^2(g_c) = \{s\}$. In this example, it cannot happen that i receives an offer with probability 1 from j and with probability $\frac{1}{2}$ from k, or viceversa. If $y_s^2(g_c) = 0$ (i.e. s does not need a job), both potential providers j and k can only pass a job along to i with probability 1. Conversely, if $y_s^2(g_c) = 1$, both potential providers j and k will pass information to i with a lower probability $\frac{1}{2}$. Hence, the probabilities that each of the two neighbors of i pass information to her are not independent.

Figure 1.2 Networks with different local connectivity and clustering patterns



Part (b) of Proposition 1 shows that the affiliation in the information flows is the only reason for such dependence. Once we condition on the status in the corresponding neighborhoods, the information flows become independent.

1.3.2. Effects of cycles on employment

Proposition 1 shows that network cycles generate affiliation in information flows but provides no prediction concerning the direction of the effect. Calvó-Armengol (2004) shows that, within cycle-free networks, direct contacts are beneficial whereas second-order contacts are detrimental for the individual probability of employment. However, example 1 indicates that this does not necessarily hold in networks that contain short cycles. Let us denote $\Big(n_i(g); \{n_j(g)\}_{\forall j \in N_i(g)}\Big)$ as the *joint degree distribution* of i. The following proposition characterizes the effects of short cycles, keeping constant the joint degree distribution (i.e. the number of direct contacts and the number of contacts of her contacts for each i):

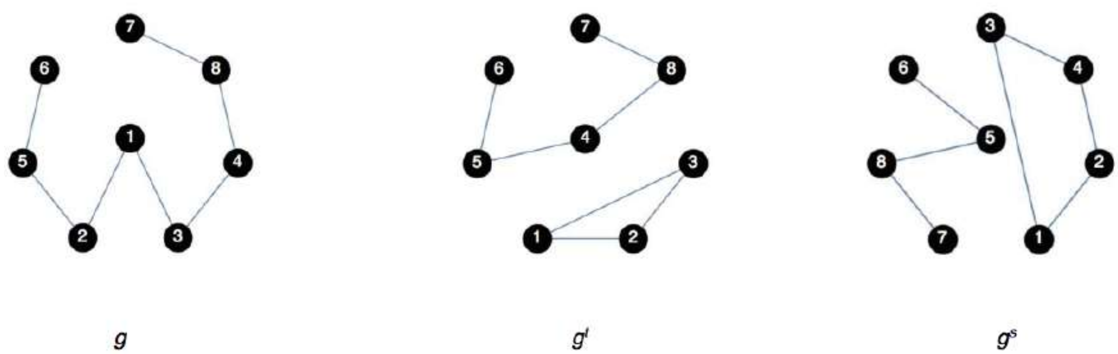
Proposition 2. (Effects of cycles). *Let* $g = (N,E)$ *and* $g^x = (N^x, E^x)$, $x \in \{t,s\}$, *be three networks, such that* $N \subseteq N^x$, $\Big(n_i(g); \{n_j(g)\}_{\forall j \in N_i(g)}\Big) = \Big(n_i(g^x); \{n_j(g^x)\}_{\forall j \in N_i(g^x)}\Big)$ *for* $\forall i \in N$ *and* $x \in \{t,s\}$, $S_S(g^t) = S_S(g) \cup \{i,j,k\}$ *and* $S_S(g^s) = S_S(g) \cup \{i,j,z,k\}$ *for some* $i,j,k,z \in N$. *Then,*

(i) $P^h(g) > P^h(g^t)$ *for* $h \in \{i,j,k\}$ *and* $P^f(g) = P^f(g^t)$ *for all* $f \in N \setminus \{i,j,k\}$.
(ii) $P^h(g) > P^h(g^s)$ *for* $h \in \{i,j,k,z\}$ *and* $P^f(g) = P^f(g^s)$ *for all* $f \in N \setminus \{i,j,k,z\}$.
(iii) $P^h(g^s) > P^h(g^t)$ *for* $h \in \{i,j,k\}$ *and* $P^h(g^s) = P^h(g^t)$ *for* $f \in N \setminus \{i,j,k,z\}$.

Figure 1.3 provides an example of the networks in Proposition 1. Networks g, g^t and g^s have the same *joint* distribution of degree and second-order degree and the set of cycles is the same with one exception: g^t (g^s) has one additional triangle (square) compared to g.[26]

Part (i) indicates that if an individual has the same degree and her neighbors also have the same degrees in both networks but she is involved in one triangle more in g^t than in g (e.g. the case of nodes 1, 2, and 3 in Figure 1.3), she is less likely to get a job offer through her network contacts in g^t. In contrast, the remaining nodes (i.e. nodes $4-7$ in Figure 1.3) are unaffected by the difference between g and g^t. Part (ii) shows that the same holds for a square but, as Part (iii) indicates, the impact of a triangle is larger that that of a square.

Figure 1.3 Example of g, g^t and g^s from Proposition 2.



A direct consequence of Proposition 2 for labor-market outcomes is that under the conditions of the proposition, the rate of employment of i is lower if i belongs to a triangle or square in g^t or g^s, respectively: $E_i(g) > E_i(g^s) > E_i(g^t)$. That is, i is more likely to be unemployed in $g^x, x \in \{s,t\}$ than in g and more so if the additional cycle is a triangle rather than a square, while $E_i(g) = E_i(g^s) = E_i(g^t)$ for the nodes that are not involved in any of the additional cycles.

To illustrate the intuition behind the proposition, let us compare a tree network g, in which $S_S(g) = \emptyset$, with a network g^t such that $S_S(g^t) = \{i,j,k\}$. Let $N_i(g) = N_i(g^t) = \{j,k,v,\ldots,z\}$. Assume $ik, ij \in E \cap E^t$, and $jk \in E^t$ but $jk \notin E$. If $i \in N$ is in status 2 (i.e. i needs a job), the probability that she does not receive any offer from $j \in N_i(g)$ is $R^i_j(g) = \alpha q_j(n_j) + (1-\alpha)$. In the following illustration, to simplify notation we omit the superscript i and the dependence on g to write R_j.

Since $S_S(g) = \emptyset$, the probability that i does not receive any offer from any neighbor in g is simply the product of the individual probabilities over all i's neighbors. Therefore, the probability that i receives at least one offer from her contacts is (see Proposition 1 in Calvó-Armengol, 2004):

$$P^i(g) = 1 - R_j R_k R_v \ldots R_z = 1 - \prod_{h \in N_i(g)} R_h. \tag{1.3.1}$$

Since the only difference between g and g^t relevant for i is that $jk \in E$, Proposition 2 shows $I^i_j(g^t)$ and $I^i_k(g^t)$ are affiliated. As a result, $R_{jk} \neq R_j R_k$, where R_{jk} is the probability that i does not receive any offer neither from j nor from k. Consequently,

$$P^i(g^t) = 1 - R_{jk} R_v \ldots R_n = 1 - R_{jk} \prod_{h \in N_i(g) \setminus \{j,k\}} R_h. \tag{1.3.2}$$

Note from (1.3.1) and (1.3.2) that $P^i(g) > P^i(g^t)$ if $R_{jk} > R_j R_k$.

[26]Note that keeping the second order degree constant implies that the number of neighbors of i's contacts is constant; thus, whenever the distribution of degree and second order degree is the same in the different networks, the conditions of Proposition 2 hold.}

Table 1.1 presents the probabilities R_{jk} conditional on the status of j and k for networks g and g^t under the conditions of Proposition 1.[27] The first column of the table lists the four possible combinations of the status of j and k: both j and k are in status 1 and thus they are potential providers of i (row 1), two cases in which one of them is a provider while the other one is not (rows 2 and 3), and the fourth situation where none of them is a provider of i (row 4). The second and third columns, denoted respectively as g and g^t, contain the probabilities that i does not receive any offer neither from j nor k under each scenario in any of the two networks. The probabilities R_{jk} are obtained by simply adding up the four expressions in the corresponding columns.[28]

Table 1.1 Probability that i does not receive any offer from $j, k \in N_i(g)$ in g^t and g.

R_{jk}/status	g^t	g	$g^t - g$
j, k providers	$\alpha^2 q_j(n_j-1)q_k(n_k-1)$	$\alpha^2 q_j(n_j)q_k(n_k)$	$\alpha^2 \left[q_j(n_j-1)q_k(n_k-1) - q_j(n_j)q_k(n_k)\right]$
j is provider, k not	$\alpha q_j(n_j) - \alpha^2 q_j(n_j-1)$	$\alpha(1-\alpha)q_j(n_j)$	$\alpha^2 \left[q_j(n_j) - q_j(n_j-1)\right]$
k is provider, j not	$\alpha q_k(n_k) - \alpha^2 q_k(n_k-1)$	$\alpha(1-\alpha)q_k(n_k)$	$\alpha^2 \left[q_k(n_k) - q_k(n_k-1)\right]$
j, k not providers	$(1-\alpha)^2$	$(1-\alpha)^2$	0

The last column (labeled $g^t - g$) in Table 1.1 is the difference between these probabilities across the two networks. If we add up the four rows of this last column, we obtain

$$R_{jk} - R_j R_k = \alpha^2 \Big[\Big(q_j(n_j) - q_j(n_j-1)\Big)\Big(1 - q_k(n_k-1)\Big) + \Big(q_k(n_k) - q_k(n_k-1)\Big)\Big(1 - q_j(n_j)\Big)\Big] > 0. \quad (1.3.3)$$

by Claim 1. Therefore, $P^i(g) > P^i(g^t)$.

Table 1.1 further illustrates how the affiliation affects the information flows from j and k to i. In g^t, the affiliation increases the likelihood that i receives two offers from both j and k or none, while decreasing the probability of receiving just one offer. When j and k are providers (row 1 in Table 1.1), $R_{jk} - R_j R_k < 0$ by Claim 1. In contrast, two events in Table 1.1 (one neighbor can pass information to i while the other cannot, corresponding to cases in rows 2 and 3) yield a lower probability of not getting at least one offer in g^t than in g, $R_{jk} - R_j R_k > 0$. Expression (1.3.3) shows that the opposing effects are non-neutral and the aggregate effect on i is negative (higher probability of not getting at least one offer). In g^t, i receives two offers simultaneously from j and k more often but she can accept only one of the jobs while the other one remains unfilled. Therefore, short network cycles, by inducing affiliation, increase the likelihood of mismatch between candidates and jobs; affiliation decreases the employment prospects of people in closely-knit neighborhoods. In economic terms, transitivity in relationships and overlapping in the neighborhoods of different individuals, through the effect of affiliation of information flows, prevent an efficient diffusion and generate labor market frictions.[29]

The above argument extends naturally when the starting network g is not a tree, and to cycles of length four. Regarding the former, $P^i(g)$ and $P^i(g^t)$ would be more complex than a product of R_h's if $S(g) \neq \emptyset$, but the comparison between the corresponding expressions (1.3.1) and (1.3.2) would again reduce to the comparison of the probabilities $R_{jk}(g)$ and $R_j(g)R_k(g)$. As for four-cycles, the argument is the same, except that the affiliation, and the fact that $R_{jk}(g^s) \neq R_j(g^s)R_k(g^s)$, stems from the status of contacts of neighbors such as $s \in N_{jk}(g^s)$. If s is employed, her status makes it more likely that *both* j and k share a job information with i and when s is unemployed it is more likely that i does not receive any offer from any of them.

Proposition 2 has several implications and raises a few issues that are worth stressing here. First, note that the above comparison holds both within and across networks. That is, it does not matter whether we compare two individuals across two networks as in Proposition 2 or two individuals within the same network.

[27] Appendix A.2. contains the detailed computation of the probabilities in Table 1.1

[28] Note that $R_{jk}(g) = R_j(g)R_k(g)$ but $R_{jk}(g^t) \neq R_j(g^t)R_k(g^t)$.

[29] Note that this effect is stronger that a simple diversification argument. A higher probability of receiving two offers or none, and a lower probability of having just one offer could be equivalent if the individual could benefit more from receiving two offers than from receiving just one. However, two offers are equivalent to just one offer because the individual can only have one job. Thus, the negative effect is stronger, because people cannot fully enjoy the advantage of receiving multiple job offers. In other words, even a slightly risk loving individual would, *ceteris paribus*, prefer not to be involved in cycles.

The reason is that, in the static, one-period version of the model, only the local neighborhood matters. More precisely, the employment probability of an individual is determined only by the status of agents in her first- and second-order neighborhood and by the geometry of these local neighborhoods. Therefore, any two nodes with the same degree, second-order degree, and, say, the same numbers of squares but different number of triangles could also be compared. The following two remarks clarify this point:

Remark 1. *In the one-period model, m-cycles do not affect employment prospects for any $m > 4$.*

Remark 2. *In the one-period model, $I^i_g(g)$ is unaffected by cycles that do not contain i.*

It is important to note that none of the above two remarks would hold in a dynamic model (see Section 1.4, where we analyze the dynamics).

Second, what can we say about the unemployment at the network level? Proposition 2 can be interpreted as an individual as well as a network-level result. Since the employment prospects of agents only differ in the probability $P^i(g)$, the next statement directly follows from Proposition 2:

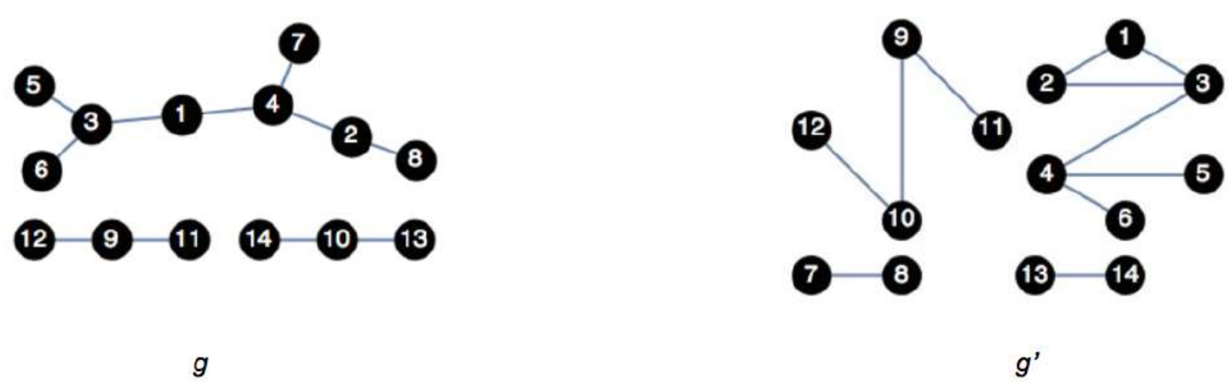
Corolary 1 (Network level). Consider networks g, g^t, and g^s from Proposition 2 with $N = N^t = N^s$. Then, the unemployment rate in each network is such that:

$$U(g^t) > U(g^s) > U(g) \tag{1.3.4}$$

In words, our results allow us to compare two networked societies in terms of unemployment as long as the assumptions in Proposition 2 hold: when two networks have the same *joint degree distribution* and they only differ in one short cycle, the network that contain the cycle has higher unemployment rate and the effect is stronger for a triangle than for a square.

The above discussion raises a natural question: can we compare two societies if we relax some of the assumptions of Proposition 2? The answer is no. The following example shows that having the same *joint degree distribution*, $\Big(n_i(g); \{n_j(g)\}_{\forall j \in N_i(g)}\Big)$, in both networks is a necessary condition. If we relax this assumption, for example by only maintaining constant the degree distribution, Proposition 2 may fail to hold.

Figure 1.4. Networks in Example 2



Example 2 (Equal joint degree distribution as a necessary condition). *Consider networks g and g' in Figure 1.4. Both networks have the same degree distribution, but the joint distribution differs across them. At the individual level, being involved in more cycles does not necessarily translate in lower unemployment if the number of direct and indirect contacts is not held constant. For instance, node 1 belongs to the cycle $\{1, 2, 3\}$ in network g' while she is not involved in any cycle in network g. Nevertheless, $P^1(g') = 0.1334 > P^1(g) = 0.1285$ for $a = 0.1$ and $b = 0.2$.*[30] *The reason is that node 1 competes with a smaller number*

[30]See Section A.2 for the derivation of all the probabilities.

of agents in g' than in g for information and this benefits her more than the harm that comes from the affiliation induced by the triangle. From the perspective of the entire network, although $S(g') \setminus S(g) = \{1, 2, 3\}$, $E(g') = 0.84013 > E(g) = 0.84012$ for $a = 0.1$ and $b = 0.2$. Hence, having less cycles does not guarantee lower unemployment if the joint distribution of the direct and two-links-away friends is not held fixed. ■

Example 2 illustrates that, if two networks have the same distributions of degrees and neighbors' degrees but a distinct joint distribution of both variables, the (un)employment rates cannot be generally ranked according to Corollary 1. This is an important observation: Example 2 shows that Proposition 1 and Corollary 1 cannot be generalized by relaxing the assumption of holding fixed the joint distribution of connectivity and second-order connectivity.

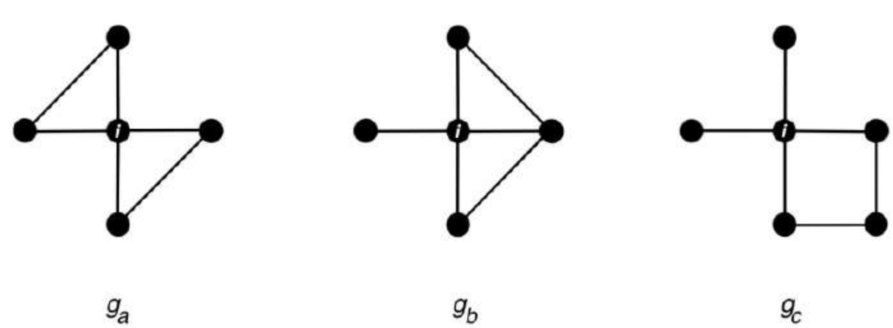
Next we compare specific networks with different clustering coefficient.

Corolary 2 (Vertex-transitive network). *Consider two vertex-transitive networks g and g', with $\Big(n_i(g); \{n_j(g)\}_{\forall j \in N_i(g)}\Big) = \Big(n_i(g'); \{n_j(g')\}_{\forall j \in N_i(g')}\Big)$ for $\forall i \in N$. If $C_i(g') \geq C_i(g)$, $C_i^4(g') \geq C_i^4(g)$ for all i, and at least one of them is satisfied with strict inequality, then $U(g') > U(g)$.*

The corollary is a direct consequence of Proposition 2 and the definition of vertex-transitive networks (see Section 1.2). Remember that a network is vertex-transitive if all the nodes occupy identical positions. Hence, they have the same degree, second-order degree, clustering coefficient, global centrality etc. As a consequence, if we compare two such networks that have the same number of neighbors and number of neighbors' contacts, we can compare their unemployment rates in line with both Proposition 2 and Corollary 1. To provide an illustration, Corollary 2 predicts that unemployment increases as we move from the first to the third network in Figure 1.1

According to Proposition 2 what matters for labor prospects is the number of short cycles, while we refer to the clustering coefficients in Corollary 2. Due to the popularity of the clustering coefficient, one may ask why we do not link unemployment directly to the coefficient in Proposition 2. The main reason is that the relationship between the number of cycles and the clustering coefficients is not one-to-one. As an illustration, consider node i in networks g_a - g_c in Figure 1.5. Although $C_i(g_a) = C_i(g_b) = \frac{1}{3}$, $P^i(g_a) \neq P^i(g_b)$ because the two triangles in g_b additionally form a four-cycle that affects i's the information flows and thus the employment prospects of those involved in the four-cycle. Similarly, $C_i^4(g_b) = C_i^4(g_c)$ but $P^i(g_b) \neq P^i(g_c)$. In vertex-transitive networks to which we refer in Corollary 2, differences in the number of cycles in which agents are involved are controlled for, since all agents occupy identical positions. The short cycles in their neighborhoods are then distributed equally. However, more complex architectures contain more complex interactions between the two clustering coefficients and cycles of different lengths, depending on who is involved in which cycle. Therefore, we express our main results in terms of additional network cycles to illustrate their *ceteris paribus* effect. A result a là Proposition 2 that would link employment to the clustering coefficient should account for both the number of cycles and their distribution across agents.

Figure 1.5. Relation between short cycles and the clustering coefficients



1.4. Dynamic analysis

In this section, we examine the long-run consequences of network cycles in labor markets. Calvó-Armengol and Jackson (2004) showed in a similar model that the unemployment rate is positively correlated across time periods and path-connected agents, and that there is duration dependence. They further provide numerous examples illustrating that the network topology shapes the long-run labor-market performance. In this section, we separate the impact of short cycles on long-run employment and inequality patterns from that of other network features.

In the proposed setup, time evolves in discrete steps, denoted by t. At the beginning of each period t, each worker may be employed or unemployed, depending on her employment status in period $t-1$. If she starts employed, she has a probability $b \in (0,1)$ of losing the job. Afterwards, all workers (employed and unemployed) hear about a vacancy with probability $a \in (0,1)$. As in the static model, losing a job and hearing about a vacancy are independently distributed and independent across individuals and periods. Afterwards, unemployed individuals who have heard about a job accept it immediately, while those employed pass it along to one of their unemployed connections at random.

We again assume that, at $t = 1$, all nodes start being employed.[31] The only difference between the first and the subsequent periods is the initial employment status; in subsequent periods, not necessarily all individuals are employed at the beginning of the period, but they may bring unemployment from the previous period.

Let the *state* at the beginning of period t be the vector $s_t = (E_{1t}, \ldots, E_{nt})$, where $E_{it} = 0$ if node i is unemployed when period t starts and $E_{it} = 1$ if she is employed. Each state s_t has an associated employment rate $E_t = \frac{\sum E_{it}}{n}$ and unemployment rate $U_t = 1 - \frac{\sum E_{it}}{n}$. From $t = 2$ on, the state s_t will be determined by the parameters a and b, the network architecture g that channels the flow of information about job openings and the employment state of the previous period s_{t-1}; the rest of the history is irrelevant. This constitutes a Markov chain.

Since the state space is finite, we can represent the transition distribution probability through the transition matrix P with element (i, j) given by $p_{ij} = \Pr(S_{t+1} = j \mid S_t = i)$. Each row of P contains the probabilities of each possible state s_{t+1}, i.e. all possible combinations of employment and unemployment for all nodes given that they started the period in state s_t.

In our setup, the Markov chain is time-homogeneous, so that the transition matrix P is the same in each period, and the $t-$period transition probability can be computed as the $t-$th power of the transition matrix,P^t. As a result, the dynamics can be modeled as a finite-state irreducible and aperiodic Markov chain and the process converges to a unique limit distribution (Young, 1993). Note that the initial state becomes irrelevant after a certain number of periods and the limit probabilities over states only depend on the parameters of the model, a and b, as well as on the network architecture g. We are interested in the steady-state probability distribution over all possible employment states: $lim_{t\to\infty} P^t = \Pi$, where Π is a matrix in which each row is the limit distribution of the process and yields the probability of each possible state s. From this limit probability distribution over states, we first ask whether the effects of short cycles identified in Section 1.3 persist in the long run. In addition, we study other moments of the distribution, the space correlations and the persistence of employment, and we particularly focus how they change as we systematically manipulate the close-knittedness of network neighborhoods.

Methodologically speaking, we proceed as follows. In Section 1.4.1, we first compute the exact limit distribution of the proposed Markov chain for a few simple networks and all values of $a, b \in (0,1)$. However, certain statistics do not have a general closed-form solutions even in case of very simple networks and computing the limit distributions for larger and more complex network structures is infeasible. As a consequence, the remaining analysis relies on Monte Carlo methods studying how network cycles shape (un)employment patterns in a set of carefully selected networks after convergence to the steady state, given the values of a and b. In this chapter, we present the results of the simulations for $a = b = 0.1$. The conclusions are qualitatively robust to alternative parameter constellations.[32] To facilitate the comparison of the long-run dynamics across networks, we simulate a large number of identical networked *economies* for each network architecture. Each economy is composed of multiple identical networks, while the number of people in each economy is held constant. This way, we might have different number of networks within each economy but the same number of

[31] The model converges to a unique steady-state distribution, independently of the initial state of the system. The initial employment state is thus irrelevant after convergence.

[32] The analysis for different values is available from the authors upon request.

random events across compared scenarios. This enables us to compare not only the averages of the variables of interest but also their variability across economies.[33]

In the numerical experiments, we simulate 100 economies for 10,000 periods for each parameter constellation and report the unemployment patterns in the last 1,000 periods to analyze the steady state distribution.[34]

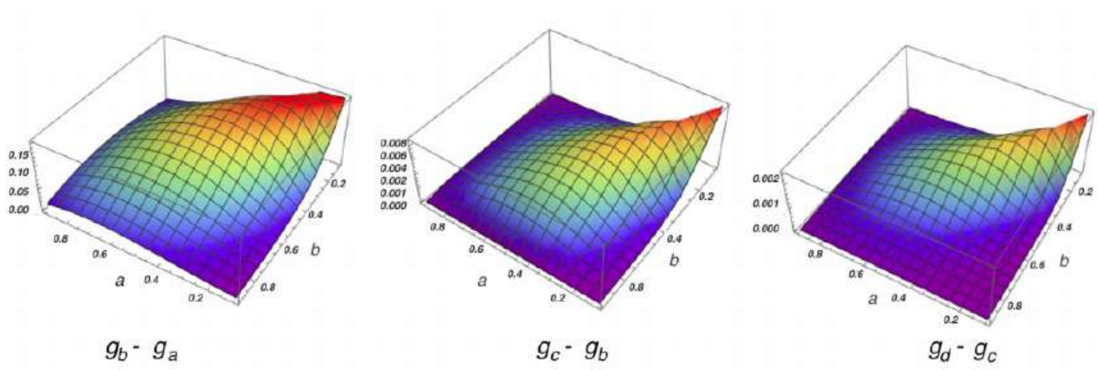
We present three subsections. Section 1.4.1 analyzes the extent to which the length of network cycles matters in the long-run employment. Section 1.4.2 studies how network cycles shape long-run inequality within networks, and Section 1.4.3 compares across homogeneous networks.

1.4.1. Long-run unemployment in cycle networks

In this section, we characterize the steady state distributions of the Markov chain for a few cycle networks (see Section 1.2 for a defintion). Consider the networks g_a through g_d in Figure 1.2 (an empty network with $n = 3$, a triangle, a square, and a pentagon, respectively). The one-period analysis shows that three- and four-cycles matter and the former have a larger impact on employment than the latter. Here, we use the cycle networks to show that this result persists in the long run. The dynamic model further allows to test whether the effect of cycles extends to cycles of longer lengths and, since the effect seems to diminish with the cycle length, whether the affect vanishes as we increase the cycle length.

Since the networks g_a - g_d are simple and the positions of all nodes symmetric in each of them, we computed the transition matrices, the limit state distributions in function of a and b, and the associated average steady-state employment rates for each node and network.[35] Figure 1.6 plots the differences in employment rates between a few pairs of networks in a function of the parameters (a, b). The figure reveals that the steady-state employment rate in networks g_a to g_d can be ranked as $E[g_a] < E[g_b] < E[g_c] < E[g_d]$ for any $a, b \in (0, 1)$. That is, as long as the values of a and b differ from zero and one, the steady state employment rates scale up with the length of the cycle (from three to five) in each network. This is the first evidence corroborating that persistence of the negative role of short network cycles, characterized in the one-period model, in the long run. Below, we corroborate this using alternative manipulations of network close-knittedness. Moreover, the above ranking shows that the impact extends for five-cycles and scales down with the cycle length.

Figure 1.6. Differences in steady-state employment rates in networks g_a, g_b, g_c, and g_d



Although we are able to obtain explicitly the limit distributions as a function of a and b for the above networks, certain statistics cannot be computed explicitly and the complexity of the limit distributions increases dramatically with the network size. In the following, we thus provide numerical experiments for $a = b = 0{,}1$ in cycle networks. To that aim, we complement the networks g_a - g_d, analyzed above, with the hexagon network g_g in Figure 1.2. Table 1.2 reports the long-run labor-market information from the simulation exercise. Each cell reports the average steady-state statistic listed in the first column for the

[33] The higher the number of nodes the lower the variation of the employment rate around the mean, but a decrease in variation due to the number of nodes is not what we try to capture.

[34] We performed robustness checks to see how fast the model converges and it actually converges to the steady state relatively fast. We are thus confident that running the model for 10,000 periods and selecting the last 1,000 provides a good approximation of the steady-state distributions.

[35] We include the empty network for comparison.

network specified on top of the corresponding column in the last 1,000 periods (out of the simulated 10,000) from 100 economies composed of 60 individuals, each economy containing a certain number of independent networks (see the row labeled as *Number of networks*).

We report four types of information in Table 1.2 (as well as Tables 1.3, A1.10, and A1.11):

A. *Labor-market statistics* report the size of each economy, the number of economies, the total number of the corresponding networks,[36] the average employment rate, and its standard deviation and coefficient of variation.

B. *Time and spatial correlations* report the serial correlations in average employment and the simple-matching coefficients in employment of connected and two-links-away individuals.[37]

C. *Transition rates* provide information about the changes and persistence of the labor-market status of agents.[38]

D. *Kolmogorov-Smirnov, Wilcoxon, and Fligner-Killeen tests* report the statistics (and p-values) of non-parametric tests of equality of distributions, means, and variances of employment, respectively, across all pairs of networks after convergence.

Table 1.2. Long-run labor market statistics in cycle networks

	Networks				
	Empty ($n = 3$)	Triangle	Square	Pentagon	Hexagon
A. Employment statistics					
Population size:	60	60	60	60	60
Number of economies:	100	100	100	100	100
Number of networks:	2000	2000	1500	1200	1000
Employment rate (E):	0.5275008	0.681182	0.6891573	0.6922612	0.6917226
St.Dev. of E:	0.06481294	0.06961622	0.06978304	0.0694386	0.06949965
Coef.variation. of E:	0.1228679	0.1021991	0.1012585	0.1003069	0.1004733
B. Time and spatial correlations					
Correlation(E_{t-1}, E_t):	0.8118573	0.8038626	0.8043265	0.799333	0.8001368
Correlation(E_{t-2}, E_t):	0.6596495	0.6496083	0.6493387	0.6424061	0.6433073
Average SM (1st neighbors):	NA	0.6379833	0.6263433	0.6232033	0.62293
Average SM (2nd neighbors):	NA	NA	0.61296	0.5965467	0.5964967
C. Transition rates					
Fraction keeping employed:	0.4801037	0.6267303	0.633977	0.6367135	0.6361628
Fraction keeping unemployed:	0.4251008	0.2643752	0.2556523	0.2521913	0.2527253
Fraction lost job:	0.0474005	0.05444417	0.05518517	0.0555535	0.05555967
Fraction found job:	0.047395	0.05445033	0.05518517	0.05554167	0.05555217
Conditional: EE/(EU+EE)	0.9101419	0.9200731	0.9199243	0.9197513	0.9196792
Conditional: UU/(UE+UU)	0.8996922	0.8292159	0.8224629	0.8195134	0.8197982

D. Kolmogorov-Smirnov, Wilcoxon, and Fligner-Killeen tests	D (p)	W (p)	χ^2 (p)
Empty vs. triangle	0.74598 (0)	549147871 (0)	52.86039 (0.043962)
Empty vs. square	0.76736 (0)	467889490 (0)	37.82941 (0.341365)
Empty vs. pentagon	0.77816 (0)	432566331 (0)	33.90109 (0.472507)
Empty vs. hexagon	0.77788 (0)	440002393 (0)	38.55862 (0.311763)
Triangle vs. square	0.0477 (0)	4676213322 (0)	47.92431 (0.071471)
Triangle vs. pentagon	0.06376 (0)	4553235062 (0)	43.51337 (0.127136)
Triangle vs. hexagon	0.05963 (0)	4570659747 (0)	42.71902 (0.173410)
Square vs. pentagon	0.01658 (0)	4877763199 (0)	33.32336 (0.500604)
Square vs. hexagon	0.01586 (0)	4895893172 (0)	54.78864 (0.017759)
Pentagon vs. hexagon	0.00422 (0.3354)	5018302682 (0.1552)	32.32549 (0.597889)

[36] Remember that a network does not necessarily corresponds to an economy as an economy can contain one or more networks; see above.

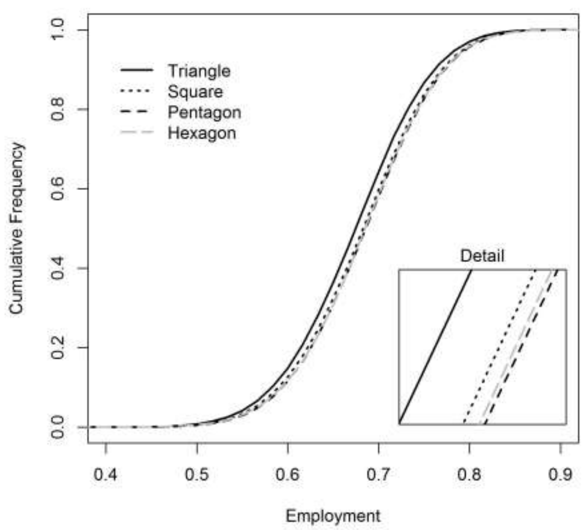
[37] Simple-matching coefficient reports the fraction of network links, in which both nodes at the end of the link share the employment status. For instance, the value of 0.6 means that 60 % of connected members of all networks in all simulated economies in the last 1,000 periods were either both employed or both unemployed.

[38] We label EE the fraction of people who preserved a job from one period to the next one, UU is the corresponding fraction of those who remained unemployed, EU is the fraction of the population that ended employed in one period but finished unemployed in the next, while UE is the reverse case. For example, a row labeled as EE reporting 0.80 refers to that 80 % of people across all the simulated network and economies in the last 1,000 periods preserved their jobs. The other cases are interpreted accordingly.

Table 1.2 corroborates that $E[g_a] < E[g_b] < E[g_c] < E[g_d]$. Both the mean and the whole distributions are all statistically different from each other ($p < 0{,}00001$; Kolmogorov-Smirnov and Wilcoxon rank-sum tests); the variances of the average employment do not differ systematically though.[39] The differences are relatively small between the triangle, square, and pentagon economies, and the marginal increases in the employment rates across these three networks decrease with the length of the shortest cycle. The increase is over 15 % from the empty network to the triangle, 0,79 % from the triangle to the square, and 0,31 % from the square to the pentagon. Quantitatively speaking, in an economy composed of $1,000,000$ workers, there would be roughly $7,975$ and $11,079$ unemployed individuals more in the triangle economy in the long-run, as compared to the square and pentagon economies, respectively. Figure 1.7 reveals that the different economies can be ranked in the sense of the first-order stochastic dominance: the limit distribution of employment in the pentagon economy first-order stochastically dominates (FOSD, hereafter) the square economy, which FOSD the triangle.[40]

This notwithstanding, although the long-run employment distributions of the pentagon and hexagon economies FOSD all the remaining ones, these two economies generate very similar employment distributions. In fact, we can reject the equality of neither the distributions nor their moments ($p > 0{,}15$ in all cases). This suggests that economically relevant and statistically strong long-run impact of network cycles is limited to relatively short network cycles.

Figure 1.7. Cumulative density function of the employment rate in cycle networks



The simulated data enable to analyze other long-run effects of network cycles that go beyond the employment rates. For instance, one important macroeconomic question concerns the employment fluctuations. We thus ask how network cycles affect the volatility and structure of employment. As mentioned above, we find no systematic effects of network cycles on employment volatility across our comparison networks. Even though we confirm the time and network correlations in employment and that the correlations diminish with time and network distance, we detect virtually no systematic impact of network-cycle length on the values of all these correlations in this exercise.

In sum, our first numerical experiment complements our theoretical results in that network cycles decrease employment prospects even in the long run and their impact diminishes with the length of the cycle. Furthermore, although relatively longer cycles should theoretically induce associations in information flows once the model is repeated, statistically strong and economically relevant long-run effects of network cycles are limited to short cycles, such as triangles and squares and to a lesser extent five-cycles. The effects do not

[39] The non-parametric Fligner-Killeen tests of equality of variances only allow to reject the equality in two cases at 5 % and one at 10 %, but the ranking is not systematic.

[40] To focus on cycles, the empty network is omitted in Figure 1.7 but the ranking in the sense of the first-order stochastic dominance extends for the empty network.

seem to go beyond five-cycles after convergence.[41] In the following sections, we limit our attention to the role triangles.[42]

1.4.2. Network cycles and long run inequality

In the previous subsection, all nodes occupy symmetric positions, which precludes within-network comparisons. However, an important question is whether network cycles can be a source of long-run inequality. In this section, we provide two experiments. First, we investigate whether triangles affect the inequality persistently even if the nodes with low and high clustering are not segregated in the network. Then, we analyze the impact of the clustering coefficient in an economy simulated on a real-life friendship network.

Different clustering in an integrated network. We explore a carefully designed network, in which some individuals have higher clustering than others. Specifically, we analyze the long-run effects of short cycles in the network depicted in Figure 1.8. The network is composed of 28 individuals with equally sized first- and second-order neighborhoods ($n_i(g) = 3$ and $n_i^2(g) = 6$ for each i) but differing clustering patterns. In particular, 7 nodes have clustering equal to zero, whereas 21 agents belong to one three-cycle (i.e. have clustering equal to 1/3).

More importantly for our purpose, the low- and high-clustering individuals are not segregated in the network: each low-clustering individual is only connected with high-clustering individuals and each high-clustering node is connected to one low-clustering and two high-clustering agents. Hence, this network enables us to study whether short network cycles have any implications on long-run inequality even if people are not segregated by the density of their neighborhoods. Such «no-segregation» condition is important because of the steady-state spatial correlations in employment status across directly and indirectly connected individuals, shown in Calvó-Armengol and Jackson (2004). Such long-run employment correlations across network paths reduce the inequality across nodes in the same component and may thus may potentially eliminate the negative effect of short cycles if high- and low-clustering individuals are close to each other in an network. We therefore test whether differing clustering patterns can still generate inequality in the presence of such a tradeoff.

Figure 1.8. Regular network with two types of nodes: high and low-clustering agents

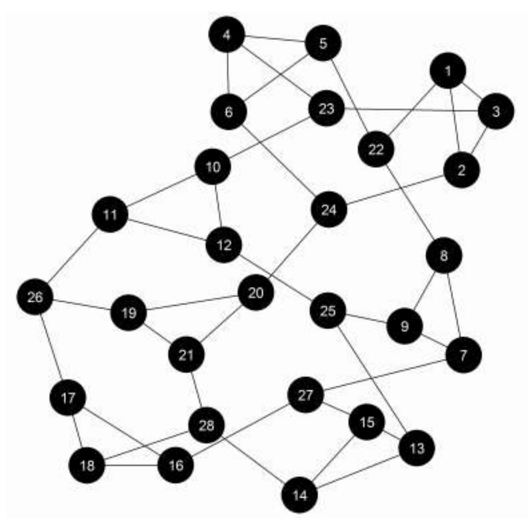


Table 1.3 summarizes the long-run labor-market outcomes for the whole network (denoted All) as well as disaggregated for the low- and high-clustering individuals. The structure of the table is the same as in Table 1.2. To ensure the comparability of all delivered statistics across the two node types, we deliver the statistics of 100 independent realizations for each type. Due to the differing number of the two types in the network, we

[41] This conclusion only holds generally in qualitative terms. For other values of a and b, the exact cycles lengths for which we (do not) observe differences may change.

[42] The effects of squares and pentagons in the reported simulation exercises are again qualitatively similar but quantitatively weaker than those of triangles.

simulated 300 economies/networks and report the results of 100 economies for the high-clustering individuals and all the 300 economies for the low-clustering agents. This way, we compare a total of 2,100 low- vs. 2,100 high-clustering individuals in the last 1,000 periods of the simulated Markov chain.

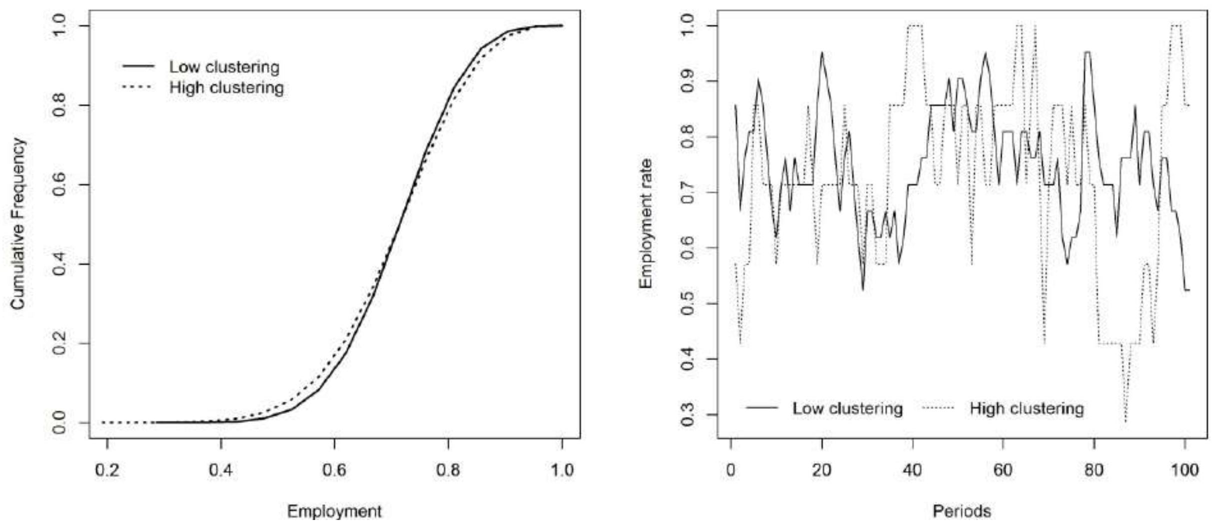
Table 1.3. Steady-state labor-market statistics in the network from Figure 1.8 (last 1000 periods)

	Type of node		
	All	Low clust.	High clust.
A. Employment Statistics			
Num. nodes:	28	7	21
Num. economies/networks:	300	300	100
Employment rate (E):	0.7313888	0.7340754	0.7314381
St. deviation of E:	0.1023162	0.09790657	0.1119343
Coef. variation of E:	0.139893	0.133374	0.1530332
B. Time and spatial correlations			
Cor(E_{t-1}, E_t):	0.7951071 (p=0)	0.7180069	0.7762561
95% confidence intervals of Cor(E_{t-1}, E_t):		[0.7149923; 0.7209948]	[0.7737822; 0.7787063]
Cor(E_{t-2}, E_t):	0.634352	0.5197073	0.6058818
Simple-Matching Coef. (1st neighb.):	0.6446468		
Simple-Matching Coef. (2nd neighb.):	0.6205101		
C. Transition rates			
EE:	0.6750431	0.6772819	0.6742968
UU:	0.2122656	0.2091224	0.2133133
EU:	0.0563369	0.05679	0.05618587
UE:	0.0563544	0.05680571	0.05620397
Conditional: EE/(EU+EE)	0.9229718	0.9226369	0.9230839
Conditional: UU/(UU+UE)	0.0563544	0.786387	0.791464
D. Kolmogorov-Smirnov, Wilcoxon, and Fligner-Killeen tests			
	D (p)	W (p)	χ^2 (p)
Low vs. high clustering	0.03461 (0)	5020228825 (0.1137)	20.309 (0.2587)

From Table 1.3, we observe that the two types of agents exhibit important differences in their long-run employment prospects. First, low-clustering individuals are somehow more likely to be employed and enjoy smaller employment volatility; these differences are small and statistically weak though ($p = 0{,}1137$ and 0,2587 for the mean and the variance, respectively). The difference corresponds to roughly 2,637 unemployed individuals out of 1,000,000. However, we reject the equality of the two distributions at any reasonable significance level, using the Kolmogorov-Smirnov test of equality of the distributions ($p < 0{,}00001$). This suggest that the combined effect of the average employment and its variability are statistically strong. Figure 1.9(a) visualizes the comparison. We observe that the steady-state employment distribution of the low-clustering individuals second-order stochasticly dominates the employment distribution of the non-zero-clustering nodes.

The two types of nodes further differ in the persistence of the employment status. Note that short cycles simultaneously *increase* the employment variance−despite slightly−as well as the serial correlations of employment (see the reported 95 % confidence intervals of Cor(E_t, E_{t-1})). That is, although the correlation of employment between two consecutive periods is considerable higher if one is embedded in more correlated neighborhoods, the standard deviation of employment is higher for the nodes with higher clustering. As a result, the employment cycles exhibit a different structure in case of each type. Figure 1.9(b) provides an example of employment cycles of both types in one simulated economy. It illustrates that high clustering maintains the employment state more stable across consecutive periods but, once we escape a state, the troughs and peaks of employment cycles are lower and higher in close-knit network environments. This is an effect that a simple variance of the distribution cannot capture. These features result from a combination of different network effects. Since short network cycles increase spatial correlation, connected individuals are more likely to be in the same state across periods and netowrk links, and the effect «drags» the employment of most members of a clustered community up or down, toward a new common employment status. The higher unemployment and higher employment persistence among high-clustering individuals also affects labor-market transitions. While high-clustering individuals are less likely to preserve their employment, they are slightly more likely to remain unemployed across periods.

Figure 1.9. Cumulative density function of employment and employment fluctuations with two types of nodes (Figure 1.8)



Summarizing, network integration of low and high clustering nodes quantitatively and qualitatively attenuates but does not eliminates the detrimental impact of short cycles. As a consequence, a policy aiming at the integration of communities with differing clustering patterns does not necessarily eliminates labor-market disadvantages generated by network close-knittedness. Furthermore, such integration do not seem to eliminate the effect of short cycles on the volatility and the time and spatial correlations of employment.

Clustering in a real-world network. In contrast to the network in Figure 1.8, typical social networks exert certain degree of segregation of clustering patterns and certain correlations between clustering and connectivity. Therefore, the following exercise simulates our model on a real-life friendship network elicited in Brañas et al. (2010). We use this particular network because it is not too large−and thus computationally not too demanding−and exhibits typical features of real-life social networks, including a large variability in the joint distribution of degrees, second-order degrees and the clustering, positive assortativity, and negative clustering-degree correlation (see Brañas et al. (2010) for details). We use the giant component of their network with $n = 76$, depicted in Figure A1.1 in Appendix A.3, and again simulate 100 independent networked economies over $10,000$ periods. The average steady-state employment rate is 72,37 % ($E = 0{,}7237$; $sd(E) = 0{,}06$) and the employment again exhibits large serial and spatial correlations.

Our main interest is to determine how these patterns at the individual level correlate with network positioning. Table 1.4 reports three regressions, differing in the dependent variable and whether they control for the average individual employment in the steady state. More precisely, we regress the steady-state average employment rate (E_i; column (1)), standard deviation of employment (sd(E_i); column (2)), and the autocorrelation ($Cor(E_{it}, E_{it-1})$; column (3)) of each network member in the last $1,000$ periods on her first- and second-order degree, clustering coefficient and a constant. In columns (2) and (3), we further control for the average employment of each node. To provide a clean effect of the clustering coefficient, the regressions only include individuals with $n_i(g) \geq 2$ so that they have a well defined clustering. Standard errors are clustered at the network level (the smallest independent unit in the simulated data).

The results corroborate the theoretical hypotheses. The average employment, its volatility, and the serial correlations all change systematically with individual degree, second-order degree, and the clustering coefficient. The estimates are statistically strong ($p < 0{,}00001$). Most importantly, holding the first- and second-order degree constant, the clustering coefficient decreases individual employment and increases simultaneously its volatility and autocorrelation.[43]

[43]Tables A1.7 - A1.9 in Appendix A.3 illustrate the importance of our *ceteris paribus* condition. The estimated effects of the regressors frequently switch signs depending on their combination in the model. Moreover, the tables show that the results are robust to controlling for global centrality of each node.

Table 1.4. Real-world network: OLS. People with $n_i(g) \geq 2$ who have well defined clustering coefficient.

	Dependent variable:		
	E_i	sd(E_i)	Cor(E_{it}, E_{it-1})
	(1)	(2)	(3)
Degree	0.029***	−0.001***	−0.011***
	(0.0003)	(0.0001)	(0.0004)
Second-order degree	−0.008***	0.001***	0.004***
	(0.0004)	(0.0001)	(0.0004)
Clustering coef.	−0.006***	0.001***	0.012***
	(0.001)	(0.0003)	(0.001)
Average E_{it}		−0.535***	−0.383***
		(0.002)	(0.010)
Constant	0.662***	0.830***	1.005***
	(0.002)	(0.002)	(0.007)
Observations	6,300	6,300	6,300
R^2	0.648	0.964	0.653
Adjusted R^2	0.648	0.964	0.653
Residual Std. Error	0.035 (df = 6296)	0.006 (df = 6295)	0.028 (df = 6295)
F Statistic	3,866.398*** (df = 3; 6296)	41,584.200*** (df = 4; 6295)	2,962.656*** (df = 4; 6295)

Note: robust st. errors clustered at network level in parentheses. *p<0.1; **p<0.05; ***p<0.01

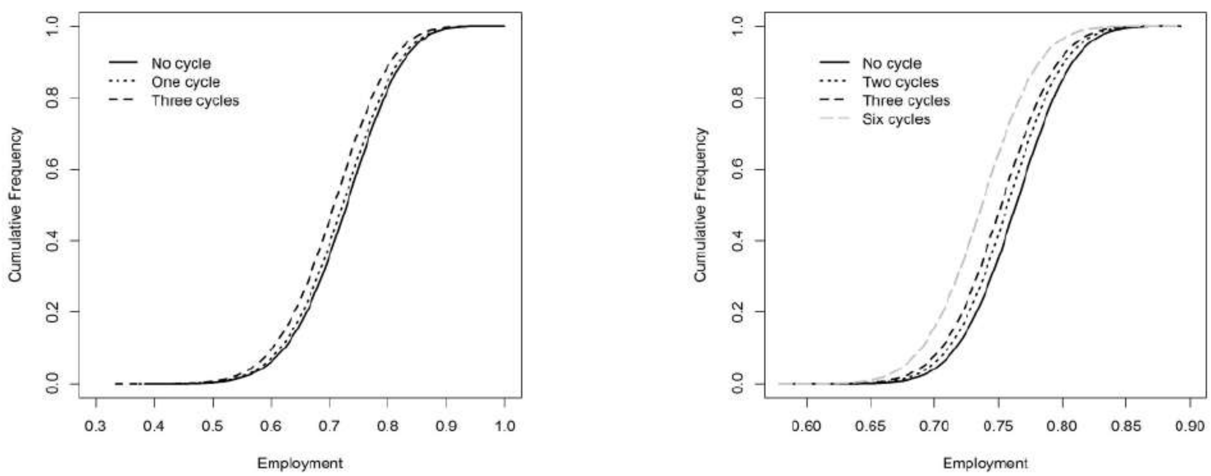
1.4.3. Vertex-transitive networks

In this section, we propose an exercise that resembles a first-order stochastic dominance shift of the distribution of the clustering coefficient. To that aim, we design a series of vertex-transitive networks a là Figure 1.1, in which all nodes occupy identical positions. More precisely, we hold the first- and second-order degree distributions constant across the networks to the extent possible but vary systematically the number of triangles each node is embedded in. We perform this exercise for degree-three and degree-four networks. Figures A1.2 and A1.3 in Appendix A.3 illustrate the networks under our comparisons; Tables A1.10 and A1.11 deliver the steady-state labor-market statistics.

Figure 1.10 summarizes the main findings of this subsection. Independently of whether we focus on $n_i(g) = 3$ or 4 for each node, the steady-state probability distributions of employment dominate each other as we decrease the number of triangles, in which each individual is involved. In all cases, the distributions and the means are significantly different ($p < 0{,}00001$), while the variances do not differ systematically. In quantitative terms, the steady-state employment rate decreases from 73,62 % to 72,95 % under $n_i(g) = 3$ as we mode from a cycle-free network to the case when each node is involved in exactly one triangle. This would correspond to over $6,660$ unemployed in a one-million-people economy. The figure further decreases to 71,57 % in a network with $C(g) = 1$, corresponding to $20,525$ more unemployed with respect to the zero-clustering network. The average employment is naturally higher in networks where $n_i(g) = 4$ for each node, but the ranking with respect to triangles is preserved: the employment rates are 76,5 %, 75,88 %, 75,36 %, and 73,8 % as we move from the zero-clustering network to a fully clustered architecture in our four networks under study. We thus conclude that, *ceteris paribus*, first-order stochastic dominance shifts of the distribution of the clustering coefficient organize the distribution of employment in the sense of the first-order stochastic dominance.

In line with the previous sections, short network cycles again induce larger serial correlations in the steady-state employment status in the networks analyzed here. As a consequence, since the variance is similar across the networks but the time correlations increase steadily with the number of triangles, the peaks and troughs of the employment cycles are somehow higher and lower as we increase the clustering of the networks under study. This−jointly with the lower employment prospects in more close-knit networks−again affects labor-market transitions: the likelihood of maintaining a job is virtually unaffected across the networks as we increase their close-knittedness. In contrast, the probability of remaining unemployed between two consecutive periods increases steadily. Correlations in employment of linked people increase in the limit distributions but they decrease between two-links-away individuals. All these observations corroborate the conclusions from the previous subsections.

Figure 1.10. Cumulative distributions of employment in vertex-transitive networks.



(a) $n_i(g) = 3$ for each $i \in N$ (b) $n_i(g) = 4$ for each $i \in N$

1.5. Wages

In this section, we briefly analyze the impact of three-cycles on wages. To that purpose, we analyze an extension of the static model in Section 1.2, in which each job offer comes with a wage.

1.5.1. Information transmission with wages

Let $W_i(g)$ be a random variable denoting the wage of the position occupied by individual i in network g. For simplicity, we assume that there are two wage levels in the economy: low-paying positions with wage w_0 and high-paying positions with wage $w_1 > w_0$. Initially, all people are employed in a high-paying job.[44] Again, each worker may lose her job with probability $b \in (0,1)$. Then, each individual hears about a low- or a high-paying job with probabilities a_0 and a_1, respectively, with $a_0 + a_1 = a \in (0,1)$. At this stage, each worker can find herself in one out of six situations (status):

Status 1: with probability $\alpha_0 = a_0(1-b)$, she is employed in a high-paying job and possesses information about a low-paying job,

Status 2: with probability $\alpha_1 = a_1(1-b)$, she is employed in a high-paying job and possesses information about a high-paying job,

Status 3: with probability $\beta = b(1-a_0-a_1)$, she is unemployed and has no information about any vacancy,

Status 4: with probability $\delta_0 = a_0 b$, she is unemployed but has heard about a low-paying job,

Status 5: with probability $\delta_1 = a_1 b$, she is unemployed but has heard about a high-paying job,

Status 6: with probability $\gamma = (1-b)(1-a_0-a_1)$, she is employed with no offer to pass to her contacts.

where $\alpha{=}\alpha_0{+}\alpha_1$. Let us write $\bar{y}_i^s(g) = 1$ if agent i is in status s.[45] At this stage, unemployed workers who learn about a vacancy (agents in status 4 or 5) immediately accept the offer, regardless of whether the job is high- or low-paying. Employed workers who learn about a low-paying job (status 1) or a high-paying

[44]Once again, this assumption is inconsequential. All the results are qualitatively robust as long as all people occupy a high-paying job with the same probability.

[45]Bear in mind that there are four status in Sections 1.2 - 1.4, while there are six of them in this extended version of the model.

job (status 2) pass the offer uniformly at random onto one of their unemployed contacts (in status 3) who accepts the offer.

Individuals in status 1 and 2 are potential providers; we call them *low providers* and *high providers*, respectively. Second-order neighbors in status 3 will be called *competitors*. As in Section 1.2, it is possible that an unemployed individual receives multiple offers simultaneously. In such a case, she accepts the job with the highest wage, while the other positions remain unfilled.

1.5.2. The incidence of triangles on wages

We first show that adding a triangle to a network a là Proposition 2 reduces the expected wage of the nodes involved in the triangle:

Proposition 3. *Consider networks g and g^t defined in Proposition 2. Then, $E[W_h(g)] > E[W_h(g^t)]$ for $h \in \{i, j, k\}$ and $E[W_z(g)] = E[W_z(g^t)]$ for all $z \neq i, j, k$.*

Proposition 3 complements Proposition 2 by showing that lower employment prospects of clustered individuals and networks translate into lower expected wages. However, this result raises a question: Is this finding driven by the unemployment channel or does higher clustering affect wages through additional mechanisms? To answer this question, the following proposition focuses on the expected wage conditional on ending up employed and asks whether close-knit neighborhoods benefit or hurt employed individuals:

Proposition 4. *Consider networks g and g^t defined in Proposition 2. Then, $E[W_h(g^t) \mid E_h(g^t) = 1] > E[W_h(g) \mid E_h(g) = 1]$ for $h \in \{i, j, k\}$ and $E[W_z(g^t) \mid E_h(g^t) = 1] = E[W_z(g) \mid E_h(g) = 1]$ for all $z \notin \{i, j, k\}$.*

We know from Propostion 2 that forming part of short network cycles decreases one's employment probability. Therefore, it is not surprising that in turn decreases one's expected wage. Nevertheless, Proposition 4 shows that the negative effect is driven by the unemployment channel. If we compare the wages of two *employed* individuals whose local positioning only differs in the cohesion of their networks, the presence of triangles actually benefits people. The intuition behind this findings is closely related to intuition behind Proposition 2. The lack of independence of information flows from different neighbors persists, leading to higher probability of receiving multiple offers. However, while multiple offers do not increase one's employment likelihood because each agent can only accept one job, they do increase the probability of hearing about at least one high-paying job. As a result, receiving multiple offers is not redundant any longer and the expected wage conditional on being employed is higher in clustered neighborhoods.

This result provides an additional channel for how network cycles contribute to the persistence and widening of income inequalities across communities and over time periods. Well-off communities or economic crises with high employment rates are benefited by close-knittedness, while bad neighborhoods and periods of economic unease that suffer from high unemployment rates are actually hurt by the same network feature.

1.6. Conclusions

This chapter analyzes systematically the role of short network cycles in labor market outcomes. We show formally that densely-knit neighborhoods lead to the affiliation in information diffusion with important micro- and macro-economic consequences on expected unemployment rates, wages, inequality, and employment fluctuations. In particular, network cycles lead to lower expected employment rates both at the individual and the population level and both in the short and long run. Moreover, clustering leads to employment fluctuations with higher volatility and more persistence (higher time correlation). Clustering results also in lower expected wages. This effect is, however, driven by the lower probability of employment; for employed workers, expected wages are higher if they belong to short cycles. The reason is that detected affiliation can benefit workers because they may benefit from receiving multiple offers by selecting better-paying jobs. The main direction for future research stemming from our work is the empirical test of the theoretical results.

1.7. Appendix

A.1. Proofs

A.1.1 Proof of Proposition 1

Part (a). *Three cycle.* Consider a node $i \in N$ such that $y_i^2(g) = 1$. Assume that $S_S^i(g) = \{i, j, k\}$. Then, there is no $s \neq i, j, k$ such that $\{i, j, s, k\} \in S_S(g)$. The probabilities of i receiving different combination of job offers from j and k, $f(I_j(g), I_k(g))$, thus are:

$$\begin{aligned}
f(1,0) &= \alpha(1-\alpha-\beta)\big[1-q_j(n_j-1 \mid 0)\big] + \alpha\beta\big[1-q_j(n_j-1 \mid 1)\big] + \alpha^2\big[1-q_j(n_j-1 \mid 0)\big]q_k(n_k-1 \mid 0) \\
f(0,1) &= \alpha(1-\alpha-\beta)\big[1-q_k(n_k-1 \mid 0)\big] + \alpha\beta\big[1-q_k(n_k-1 \mid 1)\big] + \alpha^2\big[1-q_k(n_k-1 \mid 0)\big]q_j(n_j-1 \mid 0) \\
f(0,0) &= (1-\alpha^2) + \alpha(1-\alpha-\beta)q_j(n_j-1 \mid 0) + \alpha(1-\alpha-\beta)q_k(n_k-1 \mid 0) + \alpha\beta q_j(n_j-1 \mid 1) \\
&\quad + \alpha\beta q_k(n_k-1 \mid 1) + \alpha^2 q_j(n_j-1 \mid 0)q_k(n_k-1 \mid 0) \\
f(1,1) &= \alpha^2\big[1-q_j(n_j-1 \mid 0)\big]\big[1-q_k(n_k-1 \mid 0)\big]
\end{aligned}$$

Then,

$$[f(1,0) * f(0,1)] - [f(1,1) * f(0,0)] =$$

$$\alpha^2\beta\Bigg[\Big(q_j(n_j-1 \mid 0) - q_j(n_j-1 \mid 1)\Big)\Big[\Big(1-q_k(n_k-1 \mid 0)\Big) - \beta\Big(q_k(n_k-1 \mid 1) - q_k(n_k-1 \mid 0)\Big)\Big]$$

$$+\Big(q_k(n_k-1 \mid 0) - q_k(n_k-1 \mid 1)\Big)\Big[\Big(1-q_j(n_j-1 \mid 0)\Big) - \beta\Big(q_j(n_j-1 \mid 1) - q_j(n_j-1 \mid 0)\Big)\Big]\Bigg] < 0,$$

since by assumption $\alpha, \beta > 0$, and by Claim 1 $q_m(n_m-1 \mid 0) - q_m(n_m-1 \mid 1)\Big) < 0$ and

$$\Big[\Big(1-q_m(n_m-1 \mid 0)\Big) - \beta\Big(q_m(n_m-1 \mid 1) - q_m(n_m-1 \mid 0)\Big)\Big] > 0 \text{ for } m \in \{j,k\}$$

Therefore, $I_j^i(g)$ and $I_k^i(g)$ are strictly affiliated.

Four cycle. If rather $S_S^i(g) = \{i, j, z, k\}$, $X_2^j(g)$ and $X_2^k(g)$ both depend on the status of z and $I_j^i(g)$ and $I_k^i(g)$ are not independent. More precisely, $f(I_j^i(g), I_k^i(g))$ are as follows:

$$\begin{aligned}
f(1,0) &= \alpha(1-\alpha)\Big[\beta(1-q_j(n_j-1 \mid 1)) + (1-\beta)(1-q_j(n_j-1 \mid 0))\Big] \\
&+\alpha^2\Big[\beta(1-q_j(n_j-1 \mid 1))q_k(n_k-1 \mid 1) + (1-\beta)(1-q_j(n_j-1 \mid 0))q_k(n_k-1 \mid 0)\Big]
\end{aligned}$$

$$\begin{aligned}
f(0,1) &= \alpha(1-\alpha)\Big[\beta(1-q_k(n_k-1 \mid 1)) + (1-\beta)(1-q_k(n_k-1 \mid 0))\Big] \\
&+\alpha^2\Big[\beta(1-q_k(n_k-1 \mid 1))q_j(n_j-1 \mid 1) + (1-\beta)(1-q_k(n_k-1 \mid 0))q_j(n_j-1 \mid 0)\Big]
\end{aligned}$$

$$\begin{aligned}
f(0,0) &= (1-\alpha)^2 + \alpha(1-\alpha)\Big[\beta q_k(n_k-1 \mid 1) + (1-\beta)q_k(n_k-1 \mid 0)\Big] \\
&+\alpha(1-\alpha)\Big[\beta q_j(n_j-1 \mid 1) + (1-\beta)q_j(n_j-1 \mid 0)\Big] \\
&+\alpha^2\Big[\beta q_k(n_k-1 \mid 1)q_j(n_j-1 \mid 1) + (1-\beta)q_j(n_j-1 \mid 0)q_k(n_k-1 \mid 0)\Big]
\end{aligned}$$

$$f(1,1) = \alpha^2 \Big[\beta\Big(1 - q_k(n_k - 1 \mid 1)\Big)\Big(1 - q_j(n_j - 1 \mid 1)\Big) + (1-\beta)\Big(1 - q_k(n_k - 1 \mid 0\Big)\Big(1 - q_j(n_j - 1 \mid 0)\Big)\Big]$$

Then,

$$[f(1,0) * f(0,1)] - [f(1,1) * f(0,0)] =$$
$$\alpha^2\beta\Big(q_k(n_k - 1 \mid 0) - q_k(n_k - 1 \mid 1)\Big)\Big(q_j(n_j - 1 \mid 1) - q_j(n_j - 1 \mid 0)\Big)(1-\beta) < 0$$

by Claim 1. As a result, $I_j(g)$ and $I_k(g)$ are strictly affiliated.

Part (b). Let $X_2^{j-k\setminus k}$ be the random variable of the agents in $N_{j-k}(g) \setminus \{k\}$ who are in state 2. If $jk \in E$,

$$p[I_j^i(g) = 1 \mid x_2^{jk}(g), y_k^2(g)] = \frac{\alpha}{X_2^{j-k\setminus k} + x_2^{jk}(g) + y_k^2(g)} \tag{1.7.1}$$

and $p[I_j^i(g) = 0 \mid x_2^{jk}(g), y_k^2(g)] = 1 - p[I_j^i(g) = 1 \mid x_2^{jk}(g), y_k^2(g)]$. If rather $jk \notin E$,

$$p[I_j^i(g) = 1 \mid x_2^{jk}(g)] = \frac{\alpha}{X_2^{j-k\setminus k}(g) + x_2^{jk}(g)} = \frac{\alpha}{X_2^{j-k}(g) + x_2^{jk}(g)} \tag{1.7.2}$$

Note from (1.7.1) and (1.7.2) that, when conditioned on the status of neighbors of j and k who belong to three- or four-cycles with i, $I_j^i(g)$ and by symmetry $I_k^i(g)$ depend on $X_2^{j-k\setminus k}$ and $X_2^{k-j\setminus j}$, two events that are independent of each other. As a result, $I_j^i(g)$ and $I_k^i(g)$ are independent conditional on the status of agents who form three- and/or four-cycles with i. ■

A.1.2 Proof of Proposition 2

The proof of Proposition 2 relies on Lemmas 1 and 2.

Lemma 1. *Let $g = (N, E)$ and $g^t = (N^t, E^t)$ be two networks such that $i \in N \cap N^t$, $j, k \in N_i(g) \cap N_i(g^t)$, and $n_h(g) = n_h(g^t)$ for $h \in \{j, k\}$. If $S_S^i(g^t) = S_S^i(g) \cup \{i, j, k\}$, then $P_{jk}^i(g) > P_{jk}^i(g^t)$.*

Case (a) With probability α^2, $y_j^1(g) = y_k^1(g) = 1$ and, in such a case, the probability with which i receives an offer neither from j nor from k in g is

$$R_{jk}^i\big(g|y_j^1(g) = 1, y_k^1(g) = 1\big) = \sum_{h=0}^{\eta} \binom{\eta}{h} \beta^h (1-\beta)^{\eta-h} q_j(n_j - \eta \mid h)\, q_k(n_k - \eta \mid h) \tag{1.7.3}$$

where

$$q_j(n_j - \eta \mid x_2^{jk}) = \sum_{h=0}^{n_j-\eta} \binom{n_j - \eta}{h} \beta^h (1-\beta)^{n_j-\eta-h} \left(\frac{h + x_2^{jk} - 1}{h + x_2^{jk}}\right)$$

is the probability that j does not transmit any offer to i, conditional on $X_2^{jk} = x_2^{jk}$, and analogously for $q_k(n_k - \eta \mid x_2^{jk})$.

For g^t,

$$R_{jk}^i\big(g^t|y_j^1(g^t) = 1, y_k^1(g^t) = 1\big) = \sum_{h=0}^{\eta} \binom{\eta}{h} \beta^h (1-\beta)^{\eta-h} \Big[q_j(n_j - \eta - 1 \mid h)\, q_k(n_k - \eta - 1 \mid h)\Big] \tag{1.7.4}$$

Subtracting expressions (1.7.4) and (1.7.3) and multiplying by their corresponding probabilities we get

$$\begin{aligned} &\alpha^2 \Big[R_{jk}^i\big(g^t|y_j^1(g^t) = 1, y_k^1(g^t) = 1\big) - R_{jk}^i\big(g|y_j^1(g) = 1, y_k^1(g) = 1\big)\Big] = \\ &= \alpha^2 \sum_{h=0}^{\eta} \beta^h (1-\beta)^{\eta-h} \Big[q_j(n_j - \eta - 1 \mid h)\, q_k(n_k - \eta - 1 \mid h) - q_j(n_j - \eta \mid h)\, q_k(n_k - \eta \mid h)\Big] \end{aligned} \tag{1.7.5}$$

Case (b). If $y_j^1(g) = 1$ but $y_k^1(g) = 0$, the probability that k does not transmit any offer to i is equal to 1 while the probability with which j does not transmit information to i in g, conditional on k not being a provider, is

$$R^i_{jk}\big(g|y_j^1(g) = 1, y_k^1(g) = 0\big) = q_j(n_j) = \sum_{h=0}^{\eta} \binom{\eta}{h} \beta^h (1-\beta)^{\eta-h} q_j(n_j - \eta \mid h) \tag{1.7.6}$$

Observe that it does not matter in g whether k needs a job or not. Then, the probability that i does not receive information neither from j nor from k is

$$\begin{aligned} \alpha(1-\alpha)R^i_{jk}\big(g|y_j^1(g) = 1, y_k^1(g) = 0\big) = \alpha(1-\alpha)q_j(n_j) \ &= \ \alpha(1-\alpha)\sum_{h=0}^{\eta} \binom{\eta}{h} \beta^h (1-\beta)^{\eta-h} q_j(n_j - \eta \mid h) \\ &= \alpha \sum_{h=0}^{\eta} \binom{\eta}{h} \beta^h (1-\beta)^{\eta-h} \Big[q_j(n_j - \eta \mid h) - \alpha q_j(n_j - \eta \mid h) \Big] \end{aligned} \tag{1.7.7}$$

In network g^t, on the contrary, it matters whether $y_k^2(g^t) = 0$ or $y_k^2(g^t) = 1$ because $k \in N_j(g^t)$ and k can thus compete with i for job information from j. Then,

$$\begin{aligned} &\alpha(1-\alpha)R^i_{jk}\big(g^t|y_j^1(g^t) = 1, y_k^1(g^t) = 0\big) \\ &= \alpha\beta R^i_{jk}\big(g^t|y_j^1(g^t) = 1, y_k^1(g^t) = 0, y_k^2(g^t) = 1\big) + \alpha\big(1-\alpha-\beta\big)R^i_{jk}\big(g^t|y_j^1(g^t) = 1, y_k^1(g^t) = 0, y_k^2(g^t) = 0\big) \\ &= \alpha\beta q_j(n_j - 1 \mid y_k^2(g^t) = 1) + \alpha(1-\alpha-\beta)q_j(n_j - 1 \mid y_k^2(g^t) = 0) \end{aligned} \tag{1.7.8}$$

being the first term the probability that j does not transmit information to i when $y_k^2(g^t) = 1$ (k competes with i) whereas the second term corresponds to the probability that provider j does not transmit information to i when either $y_k^3(g^t) = 1$ or $y_k^4(g^t) = 1$. Since $q_j(n_j) = \beta q_j(n_j - 1 \mid y_k^2(g^t) = 1) + (1-\beta)q_j(n_j - 1 \mid y_k^2(g^t) = 0)$, (8) is equal to:

$$\begin{aligned} &\alpha\beta q_j(n_j - 1 \mid y_k^2(g^t) = 1) + \alpha(1-\alpha-\beta)q_j(n_j - 1 \mid y_k^2(g^t) = 0) = \\ &\alpha\beta q_j(n_j - 1 \mid y_k^2(g^t) = 1) + \alpha(1-\beta)q_j(n_j - 1 \mid y_k^2(g^t) = 0) - \alpha^2 q_j(n_j - 1 \mid y_k^2(g^t) = 0) = \\ &\quad \alpha q_j(n_j) - \alpha^2 q_j(n_j - 1) = \alpha \sum_{h=0}^{\eta} \binom{\eta}{h} \beta^h (1-\beta)^{\eta-h} \Big[q_j(n_j - \eta \mid h) - \alpha q_j(n_j - \eta - 1 \mid h) \Big] \end{aligned} \tag{1.7.9}$$

The difference between expressions (1.7.9) and (1.7.7) leads to:

$$\begin{aligned} &\alpha(1-\alpha)\Big[R^i_{jk}\big(g^t|y_j^1(g^t) = 1, y_k^1(g^t) = 0\big) - R^i_{jk}\big(g|y_j^1(g) = 1, y_k^1(g) = 0\big) \Big] \\ &= \alpha^2 \sum_{h=0}^{\eta} \binom{\eta}{h} \beta^h (1-\beta)^{\eta-h} \Big[q_j(n_j - \eta \mid h) - q_j(n_j - \eta - 1 \mid h) \Big] \end{aligned} \tag{1.7.10}$$

Case (c). If $y_j^1(g) = 0$ but $y_k^1(g) = 1$, the equivalent of (1.7.10) has by symmetry the following form:

$$\begin{aligned} &\alpha(1-\alpha)\Big[R^i_{jk}\big(g^t|y_j^1(g^t) = 0, y_k^1(g^t) = 1\big) - R^i_{jk}\big(g|y_j^1(g) = 0, y_k^1(g) = 1\big) \Big] \\ &\alpha^2 \sum_{h=0}^{\eta} \binom{\eta}{h} \beta^h (1-\beta)^{\eta-h} \Big[q_k(n_k - \eta \mid h) - q_k(n_k - \eta - 1 \mid h) \Big] \end{aligned} \tag{1.7.11}$$

Case (d). If $y_j^1(g) = y_k^1(g) = 0$, $R^i_{jk}(g^t|y_j^1(g^t) = 0, y_k^1(g^t) = 0) - R^i_{jk}(g|y_j^1(g) = 0, y_k^1(g) = 0) = 0$.

The sum of expressions (1.7.5), (1.7.10), and (1.7.11) reflects the difference in the probabilities, with which i does not receive any offer from either j or k in networks g^t and g. Formally,

$$R_{jk}(g^t) - R_{jk}(g) = \alpha^2 \sum_{h=0}^{\eta} \binom{\eta}{h} \beta^h (1-\beta)^{\eta-h} \Bigg[\Big(q_j(n_j - \eta \mid h) - q_j(n_j - \eta - 1 \mid h) \Big) \Big(1 - q_k(n_k - \eta - 1 \mid h) \Big) \tag{1.7.12}$$

$$+ \Big(q_k(n_k - \eta \mid h) - q_k(n_k - \eta - 1 \mid h) \Big) \Big(1 - q_j(n_j - \eta \mid h) \Big) \Bigg] > 0$$

by Claim 1. Consequently, $P^i_{jk}(g) > P^i_{jk}(g^t)$. ■

Lemma 2. *Let $g = (N, E)$ and $g^s = (N^s, E^s)$ be two networks such that $i \in N \cap N^s$, $j, k \in N_i(g) \cap N_i(g^s)$, and $n_h(g) = n_h(g^s)$ for $h \in \{j, k\}$. If $S_S(g^s) = S_S(g) \cup \{i, j, z, k\}$, $P^i_{jk}(g) > P^i_{jk}(g^s)$.*

Proof of Lemma 2. Let $y^2_i(g) = y^2_i(g^s) = 1$. The probability with which i receives no offer from j or k if $j(k)$ is a provider but $k(j)$ is not is independent of the four-cycles that i shares with j and k. As a result, $R^i_{jk}(g|y^1_j(g) \neq y^1_k(g)) = R^i_{jk}(g^s|y^1_j(g^s) \neq y^1_k(g^s))$. Similarly, $R^i_{jk}(g|y^1_j(g) = y^1_k(g) = 0) = R^i_{jk}(g^s|y^1_j(g^s) = y^1_k(g^s) = 0) = 1$. Hence, the only difference between g and g^s arises when $y^1_j(g) = y^1_k(g) = y^1_j(g^s) = y^1_k(g^s) = 1$. We focus on this case below.

In network g^s, node i forms $n_{jk}(g^s) = \eta$ four-cycles with j and k. Denote z the node such that $\{z\} \in N_{jk}(g^s) \setminus N_{jk}(g)$. Define $X^{jk\setminus z}_2(g^s)$ as the random variable of the agents in $N_{jk}(g^s) \setminus \{z\}$ who are in state 2, and $x^{jk\setminus z}_2(g^s)$ as a realization of this variable. The expected probability that i does not receive any offer from providers j and k can be expressed considering whether $z \in N_{jk}(g^s)$ is in status 2 (with probability β) or not:

$$R^i_{jk}\big(g^s|y^1_j(g^s) = y^1_k(g^s) = 1\big) =$$

$$\sum_{h=0}^{\eta-1} \binom{\eta-1}{h} \beta^h (1-\beta)^{\eta-1-h} \Big[\beta q_j(n_j - \eta - 1 \mid h+1) q_k(n_k - \eta - 1 \mid h+1) + (1-\beta) q_j(n_j - \eta - 1 \mid h) q_k(n_k - \eta - 1 \mid h) \Big] \tag{1.7.13}$$

where

$$q_j\big(n_j - \eta - 1 \mid x^{jk\setminus z}_2(g^s) + y^2_z(g^s)\big) = \sum_{h=0}^{n_j-\eta-1} \binom{n_j - \eta - 1}{h} \beta^h (1-\beta)^{n_j-\eta-1-h} \left(\frac{h + x^{jk\setminus z}_2(g^s) + y^2_z(g^s) - 1}{h + x^{jk\setminus z}_2(g^s) + y^2_z(g^s)} \right)$$

is the probability that j does not transmit any offer to i, conditional on $x^{jk}_2(g^s) = x^{jk\setminus z}_2(g^s) + y^2_z(g^s)$, and analogously for $q_k\big(n_k - \eta - 1 \mid x^{jk\setminus z}_2(g^s) + y^2_z(g^s)\big)$. Note that (1.7.13) depends on the number of agents in $N_{jk}(g^s) \setminus \{z\}$ in status 2 (captured by the terms multiplying the expression in brackets) as well as on the status of z.

In network g, node i forms $\eta - 1$ four-cycles with j and k. Since $S_S(g^s) = S_S(g) \cup \{i, j, k, z\}$ and $n_m(g^s) = n_m(g)$ for all $m \in N \cap N^s$, there exist two nodes that we label s, l such that $N_j(g) \setminus N_j(g^s) = \{l\}$ and $N_k(g) \setminus N_k(g^s) = \{s\}$, $l \neq s$.[46] Then, the probability that i does not receive any offer from providers j and k in g can be expressed in function of whether $y^2_s(g) = 1$ and $y^2_l(g) = 1$ as follows:

$$R^i_{jk}\big(g|y^1_j(g) = y^1_k(g) = 1\big) = \sum_{h=0}^{\eta-1} \binom{\eta-1}{h} \beta^h (1-\beta)^{\eta-1-h}$$

$$= \Big[\beta^2 q_j(n_j - \eta - 1 \mid h+1) q_k(n_k - \eta - 1 \mid h+1) + \beta(1-\beta) q_j(n_j - \eta - 1 \mid h+1) q_k(n_k - \eta - 1 \mid h)$$

$$+ \beta(1-\beta) q_j(n_j - \eta - 1 \mid h) q_k(n_k - \eta - 1 \mid h+1) + (1-\beta)^2 q_j(n_j - \eta - 1 \mid h) q_k(n_k - \eta - 1 \mid h) \Big] \tag{1.7.14}$$

[46] Note that it is possible that $s = z$ or $l = z$.

The difference between expressions (1.7.14) and (1.7.13) is

$$R_{jk}\big(g|y_j^1(g) = y_k^1(g) = 1\big) - R_{jk}\big(g^s|y_j^1(g^s) = y_k^1(g^s) = 1\big) = \sum_{h=0}^{\eta-1}\binom{\eta-1}{h}\beta^h(1-\beta)^{\eta-1-h}$$

$$\beta(1-\beta) * \Big[q_k(n_k-\eta-1 \mid h+1) - q_k(n_k-\eta-1 \mid h)\Big] * \Big[q_j(n_j-\eta-1 \mid h) - q_j(n_j-\eta-1 \mid h+1)\Big] < 0 \tag{1.7.15}$$

Thereby, $R_{jk}\big(g^s|y_j^1(g^s) = y_k^1(g^s) = 1\big) > R_{jk}\big(g|y_j^1(g) = y_k^1(g) = 1\big)$. Since $P[y_j^1(g^s) = y_k^1(g^s) = 1] > 0$ by model assumptions, $P_{jk}^i(g) > P_{jk}^i(g^s)$. ■

Proof of Proposition 2. Since the only difference across networks g, g^t, and g^s arises from network transmission toward the agents involved in the additional triangle in g^t and square in g^s, we can focus on the probabilties of receving information from neighbors, $P^i(g)$, when $y_i^2(g) = y_i^2(g^t) = y_i^2(g^s) = 1$.

Part (i). By Lemma 1, $P_{jk}^i(g) > P_{jk}^i(g^t)$ and $P_m^i(g) = P_m^i(g^t)$ for any $m \notin \{j,k\}$. As a result, $P^i(g) > P^i(g^t)$. Analogously, $P^j(g) > P^j(g^t)$ and $P^k(g) > P^k(g^t)$. Since the remaining nodes have the same positioning in terms of degree, second-order degree, and short cycles in g and g^t, $P^s(g) = P^s(g^t)$ for any $s \neq i,j,k$.
Part (ii). By Lemma 2, $P_{jk}^i(g) > P_{jk}^i(g^s)$ and $P_m^i(g) = P_m^i(g^s)$ for any $m \notin \{j,k\}$, implying that $P^l(g) > P^l(g^s)$ for $l \in \{i,j,k,z\}$, while $P^t(g) = P^t(g^s)$ for $t \neq i,j,k,z$.
Part (iii). Denote $\eta = n_{jk}(g) = n_{jk}(g^t)$ the number of four-cycles, in which i,j and k are involved in both g and g^t. Since $S_S(g^s) = S_S(g) \cup \{i,j,k,z\}$, $\eta' = \eta+1$ is the number of four-cycles that i,j and k form in g^s. In what follows, we relate the probabilities of (not) receiving information in g^t and g^s conditional on the status of j and k case by case:
Case (a). If $y_j^1(g^t) = y_j^1(g^s) = 1$ and $y_k^1(g^t) = y_k^1(g^s) = 1$, the probability that i does not receive any offer from j and k in g^s is

$$\alpha^2 R_{jk}\big(g^s|y_j^1(g^s) = 1, y_k^1(g^s) = 1\big) = \alpha^2\sum_{h=0}^{\eta'}\binom{\eta'}{h}\beta^h(1-\beta)^{\eta'-h}\Big[q_j(n_j-\eta' \mid h)\, q_k(n_k-\eta' \mid h)\Big], \tag{1.7.16}$$

where

$$q_j(n_j-\eta' \mid x_2^{jk}) = \sum_{h=0}^{n_j-\eta'}\binom{n_j-\eta'}{h}\beta^h(1-\beta)^{n_j-\eta'-h}\left(\frac{h+x_2^{jk}-1}{h+x_2^{jk}}\right) \tag{1.7.17}$$

reflects the probability that j does not transmit any offer to i, conditional on $X_2^{jk} = x_2^{jk} \geq 1$, and analogously for $q_k(n_k-\eta \mid x_2^{jk})$. Since $\eta' = \eta+1$, (1.7.16) can be rewritten as

$$\alpha^2 R_{jk}\big(g^s|y_j^1(g^s) = 1, y_k^1(g^s) = 1\big) = \alpha^2\sum_{h=0}^{\eta+1}\binom{\eta+1}{h}\beta^h(1-\beta)^{\eta+1-h}\Big[q_j(n_j-\eta-1 \mid h)\, q_k(n_k-\eta-1 \mid h)\Big]$$

$$= \alpha^2\sum_{h=0}^{\eta}\binom{\eta}{h}\beta^h(1-\beta)^{\eta-h}\Big[\beta q_j(n_j-\eta-1 \mid h+1) q_k(n_k-\eta-1 \mid h+1) + (1-\beta) q_j(n_j-\eta-1 \mid h) q(n_k-\eta-1 \mid h)\Big] \tag{1.7.18}$$

In g^t, $jk \in E^t$. Hence, $y_j^1(g^t) = y_k^1(g^t) = 1$ implies that one neighbor of j (i.e. agent k) does not compete with i for information and viceversa for k. Therefore, the probability that i receives an offer from neither j nor k in g^t is

$$\alpha^2 R_{jk}(g^t|y_j^1(g^t) = 1, y_k^1(g^t) = 1) = \alpha^2\sum_{h=0}^{\eta}\binom{\eta}{h}\beta^h(1-\beta)^{\eta-h}\Big[q_j(n_j-\eta-1 \mid h)\, q_k(n_k-\eta-1 \mid h)\Big]$$

$$= \alpha^2\sum_{h=0}^{\eta}\binom{\eta}{h}\beta^h(1-\beta)^{\eta-h}\Big[\beta q_j(n_j-\eta-1 \mid h)\, q_k(n_k-\eta-1 \mid h) + (1-\beta) q_j(n_j-\eta-1 \mid h)\, q_k(n_k-\eta-1 \mid h)\Big] \tag{1.7.19}$$

Then (1.7.19)-(1.7.18),

$$\alpha^2\Big[R_{jk}\big(g^t|y_j^1(g^t)=1,y_k^1(g^t)=1\big)-R_{jk}\big(g^s|y_j^1(g^s)=1,y_k^1(g^s)=1\big)\Big]$$

$$=\alpha^2\sum_{h=0}^{\eta}\binom{\eta}{h}\beta^h(1-\beta)^{\eta-h}\Big[q_j(n_j-\eta-1\mid h)q_k(n_k-\eta-1\mid h)-q_j(n_j-\eta-1\mid h+1)q(n_k-\eta-1\mid h+1)\Big] \tag{1.7.20}$$

Case (b). Let $y_j^1(g^s)=y_j^1(g^t)=1$ and $y_k^1(g^s)=y_k^1(g^t)=0$. Since j is a provider but k is not, the probability that i does not receive any offer from k is one and independent of the number of common neighbors of j and k in status 2. In such a case, the number of four-cycles that i forms with j and k is irrelevant and information flows from j and k are equal in g^s and in g^t. Then, the difference in the probability that i gets an offer in g^t with respect to g^s neither from j nor from k when j is a provider and k is not is given by expression (1.7.10) (see the proof of Lemma 1). That is,

$$\begin{aligned}\alpha(1-\alpha)\Big[R_{jk}\big(g^t|y_j^1(g^t)=1,y_k^1(g^t)=0\big)-R_{jk}\big(g^s|y_j^1(g^s)=1,y_k^1(g^s)=0\big)\Big]\\=\alpha^2\sum_{h=0}^{\eta}\binom{\eta}{h}\beta^h(1-\beta)^{\eta-h}\Big[q_j(n_j-\eta\mid h)-q_j(n_j-\eta-1\mid h)\Big]\end{aligned} \tag{1.7.21}$$

Case (c). When rather $y_j^1(g^s)=y_j^1(g^t)=0$ while $y_k^1(g^s)=y_k^1(g^t)=1$, by analogy with Case (b):

$$\begin{aligned}\alpha(1-\alpha)\Big[R_{jk}\big(g^t|y_j^1(g^t)=0,y_k^1(g^t)=1\big)-R_{jk}\big(g^s|y_j^1(g^s)=0,y_k^1(g^s)=1\big)\Big]\\=\alpha^2\sum_{h=0}^{\eta}\binom{\eta}{h}\beta^h(1-\beta)^{\eta-h}\Big[q_k(n_k-\eta\mid h)-q_k(n_k-\eta-1\mid h)\Big].\end{aligned} \tag{1.7.22}$$

Case (d). If $y_j^1(g^t)=0$ and $y_k^1(g^t)=0$, the probabilities are identical in g^t and g^s.

Adding up the relevant expressions (1.7.20), (1.7.21), and (1.7.22) weighted by their corresponding probabilities, we get the difference in the probability that i does not receive any offer neither from j nor from k in the two networks as follows:

$$\begin{aligned}&R_{jk}(g^t)-R_{jk}(g^s)=\\&\alpha^2\sum_{h=0}^{\eta}\binom{\eta}{h}\beta^h(1-\beta)^{\eta-h}\Big[-\beta q_k(n_k-\eta-1\mid h+1)\Big(q_j(n_j-\eta-1\mid h+1)-q_j(n_j-\eta-1\mid h)\Big)\\&\qquad-\beta q_j(n_j-\eta-1\mid h)\Big(q_k(n_k-\eta-1\mid h+1)-q_k(n_k-\eta-1\mid h)\Big)\\&\qquad+\Big(q_j(n_j-\eta\mid h)-q_j(n_j-\eta-1\mid h)\Big)+\Big(q_k(n_k-\eta\mid h)-q_k(n_k-\eta-1\mid h)\Big)\end{aligned}$$

Given that $q_j(n_j-\eta\mid x_2^{jk})=\beta q_j(n_j-\eta-1\mid x_2^{jk}+1)+(1-\beta)q_j(n_j-\eta-1\mid x_2^{jk})$,

$$\begin{aligned}&R_{jk}(g^t)-R_{jk}(g^s)=\\&\alpha^2\sum_{h=0}^{\eta}\binom{\eta}{h}\beta^h(1-\beta)^{\eta-h}\Big[\Big(q_j(n_j-\eta\mid h)-q_j(n_j-\eta-1\mid h)\Big)\Big(1-q_k(n_k-\eta-1\mid h+1)\Big)\\&+\Big(q_k(n_k-\eta-1\mid h+1)-q_k(n_k-\eta-1\mid h)\Big)\Big(1-q_j(n_j-\eta-1\mid h)\Big)\Big]>0\end{aligned}$$

As each case occurs with positive probability, $P_{jk}^i(g^s)>P_{jk}^i(g^t)$. Since $P_m^i(g^s)=P_m^i(g^t)$ for $m\neq\{j,k\}$, $P^i(g^s)>P^i(g^t)$. By symmetry and Part (ii), $P^h(g^s)>P^h(g^t)$ for $h\in\{j,k,z\}$. ■

A.1.3 Proof of Proposition 3

Since the difference in i's expected wage between g and g^t only arises from network transmission by j and k while the expected wage conditional on receiving it from any $s\in N_i(g)\setminus\{j,k\}$ is the same in both networks, we can focus on the expected wage of an employed agent i who has received her job from either j or k.

Denote $\eta \in \{0\} \cup \mathbb{N}$ the number of four-cycles, in which the three i, j and k are involved. In what follows, we analyze the expected wage conditional on whether i has received a job from any $s \in N_i(g) \setminus \{j, k\}$:

Case (a). Assume first that i has received no offer from any $s \in N_i(g) \setminus \{j, k\}$. In such a case, i's expected wage depends exclusively on information flows from j and k in both networks. The rows of Table A1.1 list all the situations that can arise depending on the status of j and k. The second column (denoted $g^t - g$) contains the difference in the expected wage of i between g^t and g, multiplied by the probability of the occurrence of each case. As a result, the sum of all the elements in the second column of Table A1.1 is the difference in i's expected wage between g and g^t. After some simplification, we get

$$
\begin{aligned}
E[W_i(g^t)] - E[W_i(g)] = \sum_{h=0}^{\eta} \binom{\eta}{h} \beta^h (1-\beta)^{\eta-h} \\
-(\alpha_0^2 w_0 + \alpha_1^2 w_1)\Big[\Big(q_k(n_k \mid h) - q_k(n_k - 1 \mid h)\Big) * \Big(1 - q_j(n_j \mid h)\Big) + \Big(q_j(n_j \mid h) - q_j(n_j - 1 \mid h)\Big) * \Big(1 - q_k(n_k - 1 \mid h)\Big)\Big] \\
-\alpha_0\alpha_1 * w_0\Big[\Big(q_j(n_j \mid h) - q_j(n_j - 1 \mid h)\Big) * \Big(2 - q_k(n_k \mid h) - q_k(n_k - 1 \mid h)\Big) \\
+ \Big(q_k(n_k \mid h) - q_k(n_k - 1 \mid h)\Big) * \Big(2 - q_j(n_j \mid h) - q_j(n_j - 1 \mid h)\Big)\Big].
\end{aligned}
\tag{1.7.23}
$$

Using that $q_j(n_j) = \sum_{h=0}^{\eta} \binom{\eta}{h} \beta^{\eta}(1-\beta)^{\eta-h} q_j(n_j - \eta \mid h)$, we get after some algebra that $E[W_i(g)] > E[W_i(g^t)]$ if i receives no information from any $s \in N_i(g) \setminus \{j, k\}$.

Case (b). Assume that i receives at least one offer of a low-paying but no high-paying job from agents $s \in N_i(g) \setminus \{j, k\}$. Then, $E[W_i(g)] > E[W_i(g^t)]$ if the probability of receiving a high-paying job from either j or k or both is greater in g than in g^t. Agent i receives information about a high-paying job from j and k only if at least one of them is a high provider, corresponding to cases (2 - 4) and (7 - 8) in Table A1.1. Operating, the difference in these probabilities between g and g^t is

$$
\begin{aligned}
&\sum_{h=0}^{\eta} \binom{\eta}{h} \beta^h (1-\beta)^{\eta-h} \\
&\alpha_0^2\Big[\Big(q_k(n_k \mid h) - q_k(n_k - 1 \mid h)\Big)\Big(1 - q_j(n_j - 1 \mid h)\Big) + \Big(q_j(n_j \mid h) - q_j(n_j - 1 \mid h)\Big)\Big(1 - q_k(n_k - 1 \mid h)\Big)\Big] > 0
\end{aligned}
$$

by Claim 1. Hence, $E[W_i(g)] > E[W_i(g^t)]$ conditional on receiving at least one low offer from an $s \in N_i(g) \setminus \{j, k\}$.

Case (c). If i receives at least one high-payoff offer from $\{s \in N \mid s \in N_i(g) \setminus \{j, k\}\}$, her expected wage is w_1 in both networks, independently on the information flows from j and k.

Since all cases occur with positive probability, $E[W_i(g)] > E[W_i(g^t)]$. ■

Table A1.1 Expected wage of agent i in function of the offers she receives from j, k if i, j, and k are involved in η four-cycles.

Cases	$g^t - g$
(1) $\overline{y}_j^1 = 1,\, \overline{y}_k^1 = 1$	$\sum_\eta \alpha_0^2 w_0 \Big[q_j(n_j - \eta \mid h) q_k(n_k - \eta \mid h) - q_j(n_j - 1 - \eta \mid h) q_k(n_k - 1 - \eta \mid h) \Big) \Big]$
(2) $\overline{y}_j^2 = 1,\, \overline{y}_k^2 = 1$	$\sum_\eta \alpha_1^2 w_1 \Big[q_j(n_j - \eta \mid h) q_k(n_k - \eta \mid h) - q_j(n_j - 1 - \eta \mid h) q_k(n_k - 1 - \eta \mid h) \Big) \Big]$
(3) $\overline{y}_j^2 = 1,\, \overline{y}_k^1 = 1$	$\sum_\eta \alpha_0 \alpha_1 \Big[q_j(n_j - 1 - \eta \mid h) \Big[\Big(1 - q_k(n_k - 1 - \eta \mid h)) \Big) w_0 - w_1 \Big] - q_j(n_j - \eta \mid h)) \Big[\Big(1 - q_k(n_k - \eta \mid h) \Big) w_0 - w_1 \Big] \Big] \Big]$
(4) $\overline{y}_j^1 = 1,\, \overline{y}_k^2 = 1$	$\sum_\eta \alpha_0 \alpha_1 \Big[q_k(n_k - 1 - \eta \mid h) \Big[\Big(1 - q_j(n_j - 1 - \eta \mid h)) \Big) w_0 - w_1 \Big] - q_k(n_k - \eta \mid h)) \Big[\Big(1 - q_j(n_j - \eta \mid h) \Big) w_0 - w_1 \Big] \Big] \Big]$
(5) $\overline{y}_j^1 + \overline{y}_j^2 = 0,\, \overline{y}_k^1 = 1$	$-\alpha_0 w_0 \alpha \Big[q_k(n_k) - q_k(n_k - 1) \Big]$
(6) $\overline{y}_j^1 = 1,\, \overline{y}_k^1 + \overline{y}_k^2 = 0$	$-\alpha_0 w_0 \alpha \Big[q_j(n_j) - q_j(n_j - 1) \Big]$
(7) $\overline{y}_j^1 + \overline{y}_j^2 = 0,\, \overline{y}_k^2 = 1$	$-\alpha_1 w_1 \alpha \Big[q_k(n_k) - q_k(n_k - 1) \Big]$
(8) $\overline{y}_j^2 = 1,\, \overline{y}_k^1 + \overline{y}_k^2 = 0$	$-\alpha_1 w_1 \alpha \Big[q_j(n_j) - q_j(n_j - 1) \Big]$
(9) $\overline{y}_j^1 + \overline{y}_j^2 + \overline{y}_k^1 + \overline{y}_k^2 = 0$	0

Note: $\sum_\eta = \sum_{h=0}^{\eta} \binom{\eta}{h} \beta^h (1-\beta)^{\eta - h}$ is an abbreviated form of an expression that conditionates on the status of the common neighbors of providers j and k.

A.1.4 Proof of Proposition 4

Since the expected wage before the network transmission is independent of the network and i's expected wage conditional on receiving it from a neighbor $s \in N_i(g) \setminus \{j,k\}$ is equal in both networks, the only difference between the two networks for a node i can arise when i receives her job either through i or j. We thus focus on the expected wage of an individual i who found her job through either j or k, conditional on being employed through i or j. There are eight possible cases in function of the status of j and k listed in Table A1.1. Overall, such conditional expected wage in of i in a network g is

$$E\big[W_i(g) \mid \; I^i_j(g) + I^i_k(g) \geq 1\big] =$$

$$\alpha_0^2 E\big[W_i(g) \mid \; I^i_j(g) + I^i_k(g) \geq 1, \bar{y}^1_j(g) = 1, \bar{y}^1_k(g) = 1\big] + \alpha_1^2 E\big[W_i(g) \mid \; I^i_j(g) + I^i_k(g) \geq 1, \bar{y}^2_j(g) = 1, \bar{y}^2_k(g) = 1\big]$$

$$\begin{aligned}
+\alpha_1\alpha_0 E\big[W_i(g) \mid \; I^i_j(g) + I^i_k(g) \geq 1, \bar{y}^2_j(g) = 1, \bar{y}^1_k(g) = 1\big] + \alpha_1\alpha_0 E\big[W_i(g) \mid \; I^i_j(g) + I^i_k(g) \geq 1, \bar{y}^1_j(g) = 1, \bar{y}^2_k(g) = 1\big] \\
+ \alpha_0(1-\alpha) E\big[W_i(g) \mid \; I^i_j(g) + I^i_k(g) \geq 1, \bar{y}^1_j(g) + \bar{y}^2_k(g) = 0, \bar{y}^1_k(g) = 1\big] \\
+ \alpha_0(1-\alpha) E\big[W_i(g) \mid \; I^i_j(g) + I^i_k(g) \geq 1, \bar{y}^1_j(g) = 1, \bar{y}^1_j(g) + \bar{y}^2_k(g) = 0\big] \\
+ \alpha_1(1-\alpha) E\big[W_i(g) \mid \; I^i_j(g) + I^i_k(g) \geq 1, \bar{y}^1_j(g) + \bar{y}^2_k(g) = 0, \bar{y}^2_k(g) = 1\big] \\
+ \alpha_1(1-\alpha) E\big[W_i(g) \mid \; I^i_j(g) + I^i_k(g) \geq 1, \bar{y}^2_j(g) = 1, \bar{y}^1_j(g) + \bar{y}^2_k(g) = 0\big].
\end{aligned}$$

Table A1.2 contains the expected wages $E\big[W_i(g) \mid \; I^i_j(g) + I^i_k(g) \geq 1, \bar{y}^s_j, \bar{y}^s_k\big]$ conditional on each possible state combination of j and k in networks g and g^t. For example, the expected wage of an employed individual i if she received the job either from her neighbor j or k who both possess information about a high paying job satisfies the following:

$$E\big[W_i(g) \mid \; I^i_j(g^t) + I^i_k(g^t) \geq 1, \bar{y}^1_j(g^t) = 1, \bar{y}^1_k(g^t) = 1\big] = \frac{\sum_\eta \Big(1 - q_j(n_j - \eta - 1 \mid h) q_k(n_k - \eta - 1 \mid h)\Big) w_0}{\sum_\eta \Big(1 - q_j(n_j - \eta - 1 \mid h) q_k(n_k - \eta - 1 \mid h)\Big)}$$

$$= E\big[W_i(g) \mid \; I^i_j(g) + I^i_k(g) \geq 1, \bar{y}^1_j(g) = 1, \bar{y}^1_k(g) = 1\big] = \frac{\sum_\eta \Big(1 - q_j(n_j - \eta \mid h) q_k(n_k - \eta \mid h)\Big) w_0}{\sum_\eta \Big(1 - q_j(n_j - \eta \mid h) q_k(n_k - \eta \mid h)\Big)} = w_0$$

where $\eta = n_{jk}(g) = n_{jk}(g^t)$ is the number of four-cycles in which i, j and k are involved in both g and g^t, and $\sum_\eta = \sum_{h=0}^{\eta} \binom{\eta}{h} \beta^h (1-\beta)^{\eta-h}$ is an abbreviated form of an expression that conditions on the status of the common neighbors of j and k.

Note that the only difference between g and g^t arises in cases (3) and (4). To compare these cases, note first that the probability that i receives at least one offer either from j of from k in g satisfies:

$$\begin{aligned}
P^i_{jk}(g) = P^i_j(g) + \Big(1 - P^i_j(g)\Big) P^i_k(g) = \sum_\eta \Big[1 - q_j(n_j - \eta \mid h) q_k(n_k - \eta \mid h)\Big] = \\
\sum_\eta \Big[\Big(1 - q_j(n_j - \eta \mid h)\Big) + q_j(n_j - \eta \mid h)\Big(1 - q_k(n_k - \eta \mid h)\Big)\Big]
\end{aligned} \tag{1.7.24}$$

where $P^i_j(g) = \sum_\eta 1 - q_j(n_j - \eta \mid h)$ is the probability that i receives information from j. Using the expression (1.7.24), we can express i 's expected wage in case (3) in g as follows:

$$\begin{aligned}
E\big[W_i(g) \mid \; I^i_j(g) + I^i_k(g) \geq 1, \bar{y}^2_j(g) = 1, \bar{y}^1_k(g) = 1\big] = \\
w_0 + \sum_\eta \frac{1 - q_j(n_j - \eta \mid h)}{1 - q_j(n_j - \eta \mid h) q_k(n_k - \eta \mid h)} (w_1 - w_0)
\end{aligned} \tag{1.7.25}$$

and that in case (4) as follows:

$$\begin{aligned}
E\big[W_i(g) \mid \; I^i_j(g) + I^i_k(g) \geq 1, \bar{y}^2_k(g) = 1, \bar{y}^1_j(g) = 1\big] = \\
w_0 + \sum_\eta \frac{1 - q_k(n_k - \eta \mid h)}{1 - q_j(n_j - \eta \mid h) q_k(n_k - \eta \mid h)} (w_1 - w_0)
\end{aligned} \tag{1.7.26}$$

Since both cases occur with the same probabilty, we can add (1.7.25) and (1.7.26) up to obtain:

$$\begin{aligned} &E\big[W_i(g) \mid \ I_j^i(g) + I_k^i(g) \geq 1, \bar{y}_j^2(g) = 1, \bar{y}_k^1(g) = 1\big] + E\big[W_i(g) \mid \ I_j^i(g) + I_k^i(g) \geq 1, \bar{y}_k^2(g) = 1, \bar{y}_j^1(g) = 1\big] \\ &\qquad = 2w_0 + \sum_\eta \frac{2 - q_j(n_j - \eta \mid h) - q_k(n_k - \eta \mid h)}{1 - q_j(n_j - \eta \mid h) q_k(n_k - \eta \mid h)}(w_1 - w_0) \end{aligned} \tag{1.7.27}$$

Analogously, the corresponding expression to (1.7.27) in network g^t is

$$\begin{aligned} &E\big[W_i(g^t) \mid \ I_j^i(g^t) + I_k^i(g^t) \geq 1, \bar{y}_j^2(g^t) = 1, \bar{y}_k^1(g^t) \geq 1\big] + E\big[W_i(g^t) \mid \ I_j^i(g^t) + I_k^i(g^t) \geq 1, \bar{y}_k^2(g^t) = 1, \bar{y}_j^1(g^t) = 1\big] \\ &\qquad = 2w_0 + \sum_\eta \frac{2 - q_j(n_j - \eta - 1 \mid h) - q_k(n_k - \eta - 1 \mid h)}{1 - q_j(n_j - \eta - 1 \mid h) q_k(n_k - \eta - 1 \mid h)}(w_1 - w_0). \end{aligned} \tag{1.7.28}$$

Note that (1.7.28) is greater than (1.7.27) if:

$$\begin{aligned} &\sum_\eta \Big[2 - q_j(n_j - \eta \mid h) - q_k(n_k - \eta \mid h)\Big] * \Big[1 - q_j(n_j - \eta - 1 \mid h) q_k(n_k - \eta - 1 \mid h)\Big] \\ &- \Big[2 - q_j(n_j - \eta - 1 \mid h) - q_k(n_k - \eta - 1 \mid h)\Big] * \Big[1 - q_j(n_j - \eta \mid h) q_k(n_k - \eta \mid h)\Big] \leq 0 \end{aligned} \tag{1.7.29}$$

Operating in (1.7.29):

$$\begin{aligned} &\sum_\eta \Bigg[q_j(n_j - \eta - 1 \mid h) - q_j(n_j - \eta \mid h)\Bigg] * \Bigg[1 - q_k(n_k - \eta - 1 \mid h)\Big(2 - q_k(n_k - \eta \mid h)\Big)\Bigg] \\ &+ \Bigg[q_k(n_k - \eta - 1 \mid h) - q_k(n_k - \eta \mid h)\Bigg] * \Bigg[1 - q_j(n_j - \eta \mid h)\Big(2 - q_j(n_j - \eta - 1 \mid h)\Big)\Bigg] \leq 0 \end{aligned} \tag{1.7.30}$$

since $q_j(n_j - \eta - 1 \mid h) \leq q_j(n_j - \eta \mid h) \forall j \in N$ by Claim 1, $\Big[1 - q_k(n_k - \eta - 1 \mid h)\Big(2 - q_k(n_k - \eta \mid h)\Big)\Big] \geq 0$ and $\Bigg[1 - q_j(n_j - \eta \mid h)\Big(2 - q_j(n_j - \eta - 1 \mid h)\Big)\Bigg] \geq 0$. Consequently, (1.7.28) is greater than (1.7.27), and $E\big[W_i(g^t) \mid \ I_j^i(g^t) + I_k^i(g^t) \geq 1] \geq E\big[W_i(g) \mid \ I_j^i(g) + I_k^i(g) \geq 1]$. ■

Table A1.2. Expected wage of i in function of the status of her neighbors j and k, conditional on $E_i(g) = E_i(g_t) = 1$.

Cases	g^t	g
(1) $\overline{y}_j^1 = 1$, $\overline{y}_k^1 = 1$	w_0	w_0
(2) $\overline{y}_j^2 = 1$, $\overline{y}_k^2 = 1$	w_1	w_1
(3) $\overline{y}_j^2 = 1$, $\overline{y}_k^1 = 1$	$\sum_\eta \frac{\left[\left(1-q_j(n_j-1-\eta\|h)\right)w_1+q_j(n_j-1-\eta\|h)\left(1-q_k(n_k-1-\eta\|h)\right)w_0\right]}{[1-q_j(n_j-1-\eta\|h)q_k(n_k-1-\eta\|h)]}$	$\sum_\eta \frac{\left[\left(1-q_j(n_j-\eta\|h)\right)w_1+q_j(n_j-\eta\|h)*\left(1-q_k(n_k-\eta\|h)\right)w_0\right]}{[1-q_j(n_j-\eta\|h)q_k(n_k-\eta\|h)]}$
(4) $\overline{y}_j^1 = 1$, $\overline{y}_k^2 = 1$	$\sum_\eta \frac{\left[\left(1-q_k(n_j-1-\eta\|h)\right)w_1+q_k(n_k-1-\eta\|h)\left(1-q_j(n_j-1-\eta\|h)\right)w_0\right]}{[1-q_j(n_j-1-\eta\|h)q_k(n_k-1-\eta\|h)]}$	$\sum_\eta \frac{\left[\left(1-q_k(n_k-\eta\|h)\right)w_1+q_k(n_k)*\left(1-q_j(n_j-\eta\|h)\right)w_0\right]}{[1-q_j(n_j-\eta\|h)q_k(n_k-\eta\|h)]}$
(5) $\overline{y}_j^1 + \overline{y}_j^2 = 0$, $\overline{y}_k^1 = 1$	w_o	w_o
(6) $\overline{y}_j^1 = 1$, $\overline{y}_k^1 + \overline{y}_k^2 = 0$	w_o	w_o
(7) $\overline{y}_j^1 + \overline{y}_j^2 = 0$, $\overline{y}_k^2 = 1$	w_1	w_1
(8) $\overline{y}_j^2 = 1$, $\overline{y}_k^1 + \overline{y}_k^2 = 0$	w_1	w_1

Note: $\sum_\eta = \sum_{h=0}^{\eta} \binom{\eta}{h} \beta^h (1-\beta)^{\eta-h}$ is an abbreviated form of an expression that conditionates on the status of the common neighbors of providers j and k.

A.2 Examples

A.1.2 The role of the initial state

In this subsection, we provide an example showing that the results from the main text are robust to relaxing the assumption of initial full employment. We focus on an initial situation, in which each node is unemployed with probability one half.

To this aim, Table A1.3 lists the employment probabilities of node 1 for each initial combination of employment status of all the network members in networks g_b and g_c in Figure 1.2. Assuming that each node is (un)employed with probabilty one half and all the initial states thus have the same probability (i.e., $\frac{1}{8}$ for network g_b and $\frac{1}{16}$ for network g_c), the expected employment probability of node 1 is, in g_b:

$$E_i(g_b) = 4 - 4b + a(1+b)\Big[7 - 3a(-1+b)^2 - (4-b)b - 2a^2(1-b)b\Big],$$

while in network g_c :

$$E_i(g_c) = \frac{1}{2} - \frac{1}{2}b + \frac{1}{16}a\Big[14 - a\Big(\frac{21}{4} + (\frac{3}{2} - \frac{3}{4}a)a\Big) + \Big(6 + (6-4a)a\Big)b$$
$$-\Big(6 - a(\frac{9}{2} + (3 - \frac{3}{2}a)a)\Big)b^2 + \Big(2 - a(6-4a)\Big)b^3 + \frac{3}{4}(1-a)^2ab^4\Big].$$

It is easy to show that $E_i(g_b) < E_i(g_c)$ for any $a, b \in (0,1)$. Moreover, since all nodes occupy an identical position in both networks, $E(g_b) < E(g_c)$ for any $a, b \in (0,1)$.

Table A1.3. Employment probability of node 1 for each initial state in networks g_b and g_c in Figure 1.2

Triangle (g_b)		Square (g_c)	
Initial state:	Employment probability	Initial state:	Employment probability
(1,1,1)	$(1-\beta)+\alpha\beta(2-\alpha-\beta)$	(1,1,1,1)	$(1-\beta)+\alpha\beta\Big(2-\alpha-\beta(1-\frac{3}{4}\alpha)\Big)$
		(1,1,1,0)	$(1-\beta)+\alpha\beta\Big(2-\alpha-(1-a)(1-\frac{3}{4}\alpha)\Big)$
(1,1,0)	$(1-\beta)+\alpha\beta\frac{1}{2}(1+a)$	(1,1,0,1)	$(1-\beta)+\alpha\beta\Big(1-\frac{1}{2}\beta\Big)$
		(1,1,0,0)	$(1-\beta)+\alpha\beta\frac{1}{2}\Big(1+a\Big)$
(1,0,1)	$(1-\beta)+\alpha\beta\frac{1}{2}(1+a)$	(1,0,1,1)	$(1-\beta)+\alpha\beta\Big(1-\frac{1}{2}\beta\Big)$
		(1,0,1,0)	$(1-\beta)+\alpha\beta\frac{1}{2}\Big(1+a\Big)$
(0,1,1)	$a+\alpha(1-a)(2-\alpha-\beta)$	(0,1,1,1)	$a+\alpha(1-a)\Big(2-\alpha-\beta(1-\frac{3}{4}\alpha)\Big)$
		(0,1,1,0)	$a+\alpha(1-a)\Big(2-\alpha-(1-a)(1-\frac{3}{4}\alpha)\Big)$
(1,0,0)	$(1-\beta)$	(1,0,0,1)	$(1-\beta)$
		(1,0,0,0)	$(1-\beta)$
(0,1,0)	$a+\alpha(1-a)\frac{1}{2}(1+a)$	(0,1,0,1)	$a+\alpha(1-a)\Big(1-\frac{1}{2}\beta\Big)$
		(0,1,0,0)	$a+\alpha(1-a)\frac{1}{2}\Big(1+a\Big)$
(0,0,1)	$a+\alpha(1-a)\frac{1}{2}(1+a)$	(0,0,1,1)	$a+\alpha(1-a)\Big(1-\frac{1}{2}\beta\Big)$
		(0,0,1,0)	$a+\alpha(1-a)\frac{1}{2}\Big(1+a\Big)$
(0,0,0)	a	(0,0,0,1)	a
		(0,0,0,0)	a

A.2.2 Example 1

Consider agent 1 in networks g_b, g_c, g_e, and g_f depicted in Figure 1.2. Focus first on networks g_b and g_e; in both networks, $n_1(g_b) = n_1(g_e) = 2$ and $n_1^2(g_b) = n_1^2(g_e) = 2$. Hence, 1's neighborhood and second-order neighborhoods are equally sized. However, the link 23 generates a three-cycle in g_b, which has important consequences for the flow of information from nodes 2 and 3 to agent 1. More precisely, the probability that node 1 receives an offer from one particular neighbor can be expressed as the dot product of two vectors: (i) a vector of the probabilities of the different status combinations of 1's neighbors 2 and 3, and (ii) a vector of the probabilities that 1 receives an offer for each combination. Let

$$\Phi = [\alpha^2, \alpha\beta, \beta\alpha, \alpha(1-\alpha-\beta), (1-\alpha-\beta)\alpha, (1-\alpha-\beta)^2, \beta^2, \beta(1-\alpha-\beta), (1-\alpha-\beta)\beta]$$

denote the first vector. For example, α^2 is the probability that both neighbors are i's potential providers.

Then, the expected probability of receiving information about a vacancy from, say, *neighbor* 2 in networks g_b and g_e is

$$P_2^1(g_b) = P_2^1(g_e) = \Phi * [1, \frac{1}{2}, 0, 1, 0, 0, 0, 0, 0]' = \alpha^2 + \frac{1}{2}\alpha\beta + \alpha(1-\alpha-\beta) = \alpha(1-\frac{\beta}{2}),$$

where Φ are the probabilities of the combined states of nodes 2 and 3 in g_b and 2 and 4 in g_e. Analogously, the probability of getting an offer from *neighbor* 3 is:

$$P_3^1(g_b) = P_3^1(g_e) = \Phi * [1, \frac{1}{2}, 0, 1, 0, 0, 0, 0, 0]' = \alpha(1-\frac{\beta}{2})$$

where Φ are the probabilities of the combined states of 3 and 2 in g_b and 3 and 5 in g_e. The probabilities P_2^1 and P_3^1 are the identical in networks g_b and g_e. However, when we compute the expected probability that agent 1 receives *at least one offer* from her contacts 2 and 3, $P^1(g_b) \neq P^1(g_e)$. First,

$$P^1(g_b) = \Phi * [1, \frac{1}{2}, \frac{1}{2}, 1, 1, 0, 0, 0, 0]' = \alpha^2 + \alpha\beta + 2\alpha(1-\alpha-\beta) = \alpha(2-\alpha-\beta)$$

where Φ are the probabilities of the combined states of 2 and 3 in g_b.

Note that $P_3^1(g_b)(1-P_2^1(g_b)) + P_2^1(g_b)(1-P_3^1(g_b)) + P_2^1(g_b)P_3^1(g_b) = P^1(g_b) + \alpha^2(\beta - \frac{\beta^2}{4}) > P^1(g_b)$. That is, we cannot compute $P^1(g_b)$ directly from $P_2^1(g_b)$ and $P_3^1(g_b)$ because information flows from 2 and 3 to 1 are not independent in g_b.

In contrast, in network g_e,

$$\begin{aligned} P^1(g_e) &= \alpha^2\Phi * [1, 1, 1, 1, 1, 1, \frac{3}{4}, 1, 1]' + \alpha(1-\alpha)\Phi * [1, 1, \frac{1}{2}, 1, 1, 1, \frac{1}{2}, \frac{1}{2}, 1]' + \\ &+ (1-\alpha)\alpha\Phi * [1, \frac{1}{2}, 1, 1, 1, 1, \frac{1}{2}, 1, \frac{1}{2}]' = \alpha(1-\frac{\beta}{2})[2-\alpha(1-\frac{\beta}{2})], \end{aligned}$$

where Φ are the probabilities of the combined states of nodes 4 and 5 in g_e. In such a case, the information flows are independent as $P_3^1(g_e)(1-P_2^1(g_e)) + P_2^1(g_e)(1-P_3^1(g_e)) + P_2^1(g_e)P_3^1(g_e) = P^1(g_e) = 2\alpha(1-\frac{\beta}{2}) - \alpha^2(1-\frac{\beta}{2})^2$.

Most importantly, note that $P^1(g_e) - P^1(g_b) = \alpha^2(\beta - \frac{\beta^2}{4}) > 0$. That is, g_b provides a lower expected probability of receiving a job offer through the network than g_e as a consequence of the lack of independence in information flows.

Consider network g_f, in which node 1 has one competitor mode in this network, node 6, than in g_e. Formally, $P_2^1(g_f) = P_2^1(g_b) = P_2^1(g_e) = \alpha(1-\frac{\beta}{2})$, but

$$P_3^1(g_f) = \alpha\Phi * [1, \frac{1}{2}, \frac{1}{2}, 1, 1, 1, \frac{1}{3}, \frac{1}{2}, \frac{1}{2}]' = \alpha[1-\beta(1-\frac{\beta}{3})],$$

where Φ are the probabilities of the combined states of nodes 5 and 6 in g_f. Consequently, $P_3^1(g_f) < P_3^1(g_b) = P_3^1(g_e) = \alpha(1-\frac{\beta}{2})$. Moreover, since the information flows from nodes 2 and 3 to agent 1 are independent,

$$P^1(g_f) = \alpha(1-\frac{\beta}{2}) + [\alpha - \alpha^2(1-\frac{\beta}{2})][1-\beta(1-\frac{\beta}{3})],$$

implying that $P^1(g_f) < P^1(g_e)$.

In general, $P^1(g_f)$ may be higher or lower than $P^1(g_b)$. Table A1.4 illustrates the probabilities, with which node 1 receives job offers from her neighbors in the four networks for different parameter values; note that, as a consequence of the triangle in g_b, node 1 may have better employment prospects in g_f than in g_b even though node 1 has more competitors in g_f.

Table A1.4. Employment prospects for parameter values $\beta = 0.01$, $\alpha = 0.9$ (panel A on the left) and $\beta = 0.1$, $\alpha = 0.8$ (panel B on the right)

A	g_b	g_f	g_c	g_e	B	g_b	g_f	g_c	g_e
P_2^1	0.8955	0.8955	0.8955	0.8955	P_2^1	0.7600	0.7600	0.7600	0.7600
P_3^1	0.8955	0.8909	0.8955	0.8955	P_3^1	0.7600	0.7109	0.7600	0.7600
P^1	0.9810	0.9886	0.9871	0.9891	P^1	0.8800	0.9306	0.9280	0.9424

Finally, let us compare networks g_c and g_e. Again, $n_1(g_c) = n_1(g_e) = 2$ but $n_1^2(g_c) = 1 < n_1^2(g_e) = 2$, which in principle should yield better employment prospects for node 1 in g_c than in g_e due to the lower number of potential competitors. The probability of getting information about a job offer *from neighbor 2* in g_c is

$$P_2^1(g_c) = \Phi * [1, \frac{1}{2}, 0, 1, 0, 0, 0, 0, 0]' = \alpha(1 - \frac{\beta}{2}),$$

where Φ are the probabilities of the combined states of nodes 2 and 4 in g_c. Thus, $P_2^1(g_f) = P_2^1(g_b) = P_2^1(g_e) = P_2^1(g_c)$. Similarly, $P_3^1(g_b) = P_3^1(g_e) = P_3^1(g_c) = \alpha(1 - \frac{\beta}{2})$. Therefore, $P^1(g_c) = \beta\Phi * [\frac{3}{4}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, 0, 0, 0, 0] + (1-\beta)\Phi * [1, 1, 1, 1, 1, 0, 0, 0, 0] = \alpha(2 - \alpha - \beta + \frac{3}{4}\alpha\beta)$, being Φ the vector of the probabilities of the combined states of nodes 2 and 3 in g_c.

Thus, $P_1(g_c) < P_1(g_e)$, i.e. in contrast to the results in Calvó-Armengol (2004), node 1 is more likely to receive a job offer through the network in g_e despite the lower number of indirect contacts in g_c. Table A1.5 shows the probabilities of receiving two, one or no offers.

Table A1.5. Distribution of job offers in the different networks

Example 1	pr. 2 offers in network		
Φ	g_b	g_e	g_c
α^2	1	(1-$\frac{\beta}{2}$)^2	$(1-\beta) + \frac{\beta}{4}$
$\alpha\beta$	0	0	0
$\beta\alpha$	0	0	0
$\alpha(1-\alpha-\beta)$	0	0	0
(1-$\alpha - \beta)\alpha$	0	0	0
(1-α)^2	0	0	0

Example 1	pr. 1 offer in network		
Φ	g_b	g_e	g_c
α^2	0	β(1-$\frac{\beta}{2}$)	$\frac{\beta}{2}$
$\alpha\beta$	$\frac{1}{2}$	(1-$\frac{\beta}{2}$)	(1-$\frac{\beta}{2}$)
$\beta\alpha$	$\frac{1}{2}$	(1-$\frac{\beta}{2}$)	(1-$\frac{\beta}{2}$)
$\alpha(1-\alpha-\beta)$	1	(1-$\frac{\beta}{2}$)	(1-$\frac{\beta}{2}$)
(1-$\alpha - \beta)\alpha$	1	(1-$\frac{\beta}{2}$)	(1-$\frac{\beta}{2}$)
(1-α)^2	0	0	0

Example 1	pr. 0 offers in network		
Φ	g_b	g_e	g_c
α^2	0	$\frac{\beta^2}{4}$	$\frac{\beta}{4}$
$\alpha\beta$	$\frac{1}{2}$	$\frac{\beta}{2}$	$\frac{\beta}{2}$
$\beta\alpha$	$\frac{1}{2}$	$\frac{\beta}{2}$	$\frac{\beta}{2}$
$\alpha(1-\alpha-\beta)$	0	$\frac{\beta}{2}$	$\frac{\beta}{2}$
(1-$\alpha - \beta)\alpha$	0	$\frac{\beta}{2}$	$\frac{\beta}{2}$
(1-α)^2	1	1	1

A.2.3 Example 2

Consider networks g and g' in Figure 1.4. Both networks have the same degree distribution and the distribution of second-order degree, but the joint first- and second-order distribution differ. For example, $n_1(g) = n_1(g') = 2$ but $n_1^2(g) = 4$ but $n_1^2(g') = 3$. Moreover, $S(g') = \{1,2,3\}$ but $S(g) = \emptyset$. That is, $S(g') = S(g) \cup \{1,2,3\}$ as in Proposition 2.

(i) **Network g**. Since $R_3^1(g) = R_4^1(g) = \alpha q_3(n_3) + (1-\alpha) = 1-\alpha+\alpha\Big[(1-\beta)\beta + \frac{2\beta^2}{3})\Big]$, the probability with which 1 receives at least one offer from her neighbors in network g is:

$$P^1(g) = 1 - \left[1-\alpha+\alpha\left[(1-\beta)\beta + \frac{2\beta^2}{3}\right]\right]^2$$

The probability that node 2 does not receive information from her neighbor $8 \in N_2(g)$ is $R_8^2(g) = (1-\alpha)$, while the probability she does not receive information from $4 \in N_2(g)$ is $R_4^2(g) = R_3^1(g)$. As a result,

$$P^2(g) = 1 - R_8^2(g)R_4^2(g) = 1-(1-\alpha)\left[1-\alpha+\alpha\left[(1-\beta)\beta + \frac{2\beta^2}{3}\right]\right]$$

Consider agent 3. The probability that she does not receive information from her neighbor 1 is $R_1^3(g) = \alpha q_1(n_1) + (1-\alpha) = 1-\alpha\Big[(1-\frac{1}{2}\beta)\Big]$, while $R_5^3(g) = R_6^3(g) = (1-\alpha)$. Then, agent 3 receives information from her contacts with probability:

$$P^3(g) = 1 - R_1^3(g)R_5^3(g)R_6^3(g) = 1-(1-\alpha)^2\left[1-\alpha+\frac{\alpha\beta}{2}\right]$$

Agent 4 does not receives information from $7 \in N_4(g)$ with probability $R_7^4(g) = (1-\alpha)$, while $R_1^4(g) = R_2^4(g) = R_1^3(g)$. Thereby,

$$P^4(g) = 1 - R_1^4(g)R_2^4(g)R_7^4(g) = 1-(1-\alpha)\left[1-\alpha+\frac{\alpha\beta}{2}\right]^2$$

The probability that node 5 does not receive information from her only neighbor $3 \in N_5(g)$ is $1 - R_3^5(g)$, with $R_3^5(g) = R_3^1(g)$. Henceforth, $P^5(g) = \alpha - \alpha\Big[(1-\beta)\beta + \frac{2\beta^2}{3}\Big]$ and $P^5(g) = P^6(g) = P^7(g)$. Similarly, since $R_2^8(g) = R_2^4(g) = R_1^3(g)$, $P^8(g) = 1 - R_2^8(g) = \alpha - \frac{\alpha\beta}{2}$ and $P^8(g) = P^{11}(g) = P^{12}(g) = P^{13}(g) = P^{14}(g)$. Last, $P^9(g) = P^{10}(g) = 1-(1-\alpha)^2$.

(ii) Network g'. Information flows from nodes 2 and 3 to 1 are affiliated, since $\{1,2,3\} \in S(g')$. The probability that provider 2 does not transmit information to 1 is $\frac{1}{2}$ when $y_3^2(g') = 1$ (3 is i's competitor) and 0 when $y_3^2(g') = 0$. Conditional on $y_2^2(g') = 1$ (2 is a competitor), the probability that node 3 does not transmits information to 1 is $q_3(n_3 - 1 \mid y_2^2(g') = 1) = \frac{2}{3}\beta + (1-\beta)\frac{1}{2} = \frac{1}{2} + \frac{\beta}{6}$. Analogously, $q_3(n_3 - 1 \mid y_2^2(g') = 0) = \frac{1}{2}\beta$ is the probability that provider 3 does not transmit information to 1, conditional on $y_2^2(g') = 0$.

Let $\Phi = [\alpha^2,\ \alpha\beta,\ \beta\alpha,\ \alpha(1-\alpha-\beta),\ (1-\alpha-\beta)\alpha,\ (1-\alpha-\beta)^2,\ \beta^2,\ \beta(1-\alpha-\beta),\ (1-\alpha-\beta)\beta]$ be the vector of probabilities of all relevant combined states of 2 and 3. Then, the probability that 1 does not receive any offer from 2 or 3 can be expressed as the dot product of Φ and the vector of the probabilities that 1 does not receive any offer in each joint state: $R_{23}^1(g') = \Phi * [0, \frac{1}{2}, q_3(n_3 - 1 \mid 1), 0, q_3(n_3 - 1 \mid 0), 1, 1, 1, 1]' = 1 + \alpha^2\Big(1 - \frac{\beta}{2}\Big) - \alpha\Big(2 - \frac{3}{2}\beta + \frac{1}{3}\beta^2\Big)$. Therefore, the probability that 1 receives at least one offer in g' is:

$$P^1(g') = 1 - R_{23}^1(g') = \alpha\left(2 - \frac{3}{2}\beta + \frac{1}{3}\beta^2\right) - \alpha^2\left(1 - \frac{\beta}{2}\right) = P^2(g')$$

by symmetry of 1's and 2's positions.

Consider agent 3. The probability that provider 1(2) does not pass information to 3 when 2(1) is in state 2 is $\frac{1}{2}$ and 0 when 2(1) is not in such a state. Then,

$$R^3_{12}(g') = \Phi * [0, \frac{1}{2}, \frac{1}{2}, 0, 0, 1, 1, 1, 1]' = \alpha\beta + (1-\alpha)^2$$

Since $R^3_4(g')$=$R^1_3(g)$,

$$P^3(g') = 1 - R^3_{12}(g')R^3_4(g') = 1 - \left[(1-\alpha)^2 + \alpha\beta\right]\left[1 - \alpha + \alpha\left((1-\beta)\beta + \frac{2\beta^2}{3}\right)\right]$$

Observe that $R^4_3(g')$=$R^1_3(g)$, and $R^4_5(g')$=$R^6_5(g')$=$(1-\alpha)$. Hence,

$$P^4(g') = 1 - R^4_3(g')R^4_5(g')R^4_6(g') = 1 - (1-\alpha)^2\left[(1-\alpha) + \alpha\left((1-\beta)\beta + \frac{2\beta^2}{3}\right)\right].$$

As for agent 9, the likelihood that she does not receive information from $10 \in N_9(g')$ is $R^9_{10}(g')$=$R^3_1(g)$, while the probability that she does not receive information from $11 \in N_9(g')$ is $R^{11}_9(g)$=$(1-\alpha)$. This implies that

$$P^9(g') = 1 - R^9_{10}(g)R^9_{11}(g') = 1 - (1-\alpha)\left[1 - \alpha\left(1 - \frac{\beta}{2}\right)\right]$$

and $P^9(g')$=$P^{10}(g')$ by symmetry. Finally, note that $P^5(g')$=$P^6(g')$=$P^5(g)$=$P^6(g)$, $P^7(g')$=$P^8(g')$=$P^{13}(g')$=$P^{14}(g')$=α, and $P^{11}(g')$=$P^{12}(g')$=$P^{11}(g)$=$P^{12}(g)$.

Table A1.6 summarizes the employment probabilities of each node for $a = 0.1$ and $b = 0.2$, the values in Example 2. The employment probability of node i is computed as $E^i(g) = (1-\beta) + \beta P^i(g)$, while $E(g) = \frac{1}{n}\sum_{i \in N} E_i(g)$.

Table A1.6. Employment probability for a =0.1, b = 0.2 in Example 2

Node	$P^i(g)$	$E^i(g)$	$P^i(g')$	$E^i(g')$
1	0.128511	0.843132	0.133440	0.844019
2	0.141147	0.845406	0.133440	0.844019
3	0.215218	0.858739	0.196412	0.855354
4	0.209076	0.857634	0.209855	0.857773
5	0.066464	0.831964	0.066464	0.831964
6	0.066464	0.831964	0.066464	0.831964
7	0.066464	0.831964	0.080000	0.834400
8	0.072800	0.833104	0.080000	0.834400
9	0.153600	0.847648	0.146968	0.846454
10	0.153600	0.847648	0.146968	0.846454
11	0.072800	0.833104	0.072800	0.833104
12	0.072800	0.833104	0.072800	0.833104
13	0.072800	0.833104	0.080000	0.834400
14	0.072800	0.833104	0.080000	0.834400
Aver.	1.564544	0.840116	1.565610	0.840129

A.2.4 Computation of probabilities for Table 1.1

First, note that since $S_S(g) = \emptyset$, $R_{jk}(g) = R_j(g)R_k(g)$. Formally,

$$R_{jk}(g) = \alpha^2 q_j(n_j)q_k(n_k) + \alpha(1-\alpha)q_j(n_j) + \alpha(1-\alpha)q_k(n_k) + (1-\alpha)^2$$

considering the four possible cases (both j and k are providers, only j is a provider, only k is a provider and none of them is a provider) and their corresponding probabilities.

If $S_S(g) = \{i, j, k\}$ as in g^t, the probability that i does not receive any offer from provider j conditional on y_k^2 is

$$q_j(n_j - 1 \mid y_k^2) = \sum_{h=0}^{n_j-2} \binom{n_j-2}{h} \beta^h (1-\beta)^{n_j-2} \left(\frac{h + y_k^2}{h + y_k^2 + 1}\right)$$

With this expression in and, we compute $R_{jk}(g^t)$. The probability that i does not receive an offer from j when j is a provider but k is not, depends on whether k is a competitor or not, and the same holds for he probability that i does not receive an offer from k when k is a provider but j is not. There are six cases to consider: (1) both j and k are providers; (2) j is a provider and k a competitor, (3) k is a provider and j a competitor; cases (4) and (5) in which one is a provider and the other neither a provider nor a competitor (i.e. in state 3 or 4, event that occurs with probability $\delta + \gamma = 1 - \alpha - \beta$); and (6) neither j nor k are providers:

$$\begin{aligned}
R_{jk}(g^t) &= \alpha^2 q_j(n_j - 1 \mid y_k^2 = 0) q_k(n_k - 1 \mid y_j^2 = 0) + \alpha\beta q_j(n_j - 1 \mid y_k^2 = 1) + \alpha(1-\alpha-\beta) q_j(n_j - 1 \mid y_k^2 = 0) \\
&\quad + \alpha\beta q_k(n_k - 1 \mid y_j^2 = 1) + \alpha(1-\alpha-\beta) q_k(n_k - 1 \mid y_j^2 = 0) + (1-\alpha)^2 \\
&= \alpha^2 q_j(n_j - 1 \mid y_k^2 = 0) q_k(n_k - 1 \mid y_j^2 = 0) \\
&\quad + \alpha\beta q_j(n_j - 1 \mid y_k^2 = 1) + \alpha(1-\beta) q_j(n_j - 1 \mid y_k^2 = 0) - \alpha^2 q_j(n_j - 1 \mid y_k^2 = 0) \\
&\quad + \alpha\beta q_k(n_k - 1 \mid y_j^2 = 1) + \alpha(1-\beta) q_k(n_k - 1 \mid y_j^2 = 0) - \alpha^2 q_k(n_k - 1 \mid y_j^2 = 0) + (1-\alpha)^2
\end{aligned}$$

Given that $\beta q_j(n_j - 1 \mid y_k^2 = 1) + (1-\beta) q_j(n_j - 1 \mid y_k^2 = 0) = q(n_j)$ and $q_j(n_j - 1 \mid y_k^2 = 0) = q_j(n_j - 1)$, the above probability simplifies to

$$R_{jk}(g^t) = [\alpha^2 q_j(n_j - 1) q_k(n_k - 1)] + [\alpha q_j(n_j) - \alpha^2 q_j(n_j - 1)] + [\alpha q_k(n_k) - \alpha^2 q_k(n_k - 1)] + [(1-\alpha)^2]$$

where each term in brackets corresponds to the probability that i does not receive any offer in each of the four cases in column labeled as g^t in Table 1.1.

Adding up all the rows of the last column in Table 1.1, we obtain that

$$R_{jk}(g^t) - R_j(g) R_k(g) = \alpha^2 \Big[\Big(q_j(n_j) - q_j(n_j - 1)\Big)\Big(1 - q_k(n_k - 1)\Big) + \Big(q_k(n_k) - q_k(n_k - 1)\Big)\Big(1 - q_j(n_j)\Big) \Big] > 0$$

because both $q_j(n_j)$ and $q_k(n_k)$ increase in their arguments by Claim 1. Therefore, $P^i(g) > P^i(g^t)$. ■

A.3 Additional material for the dynamic analysis

A.3.1 Real-life network

Figure A1.1 Giant component of the friendship network from Brañas et al. (2010).

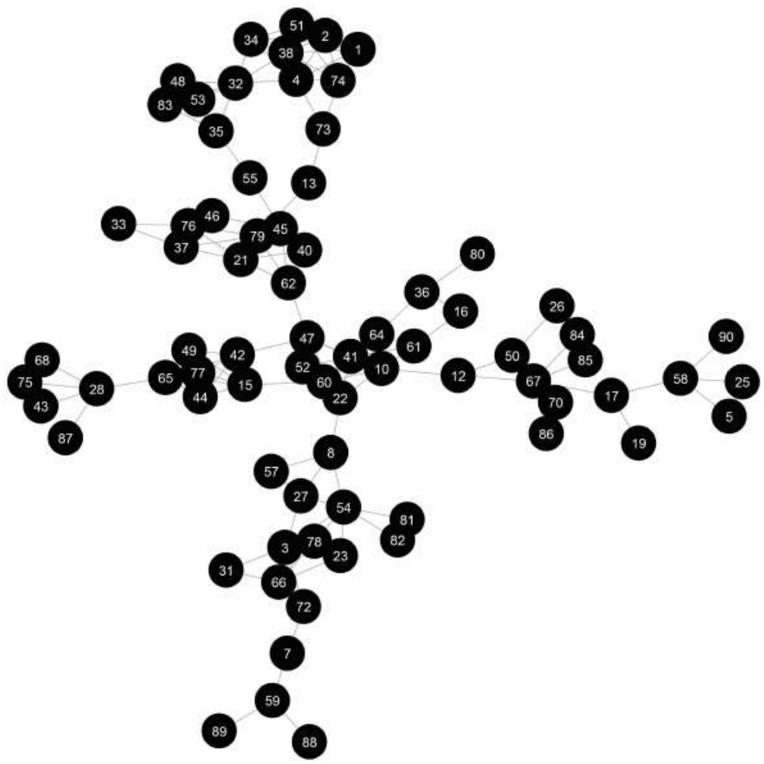


Table A1.7. Average unemployment in the last 1000 periods. Real-world network: OLS. People with well defined clustering

	Dependent variable:					
	average.e					
	(1)	(2)	(3)	(4)	(5)	(6)
degree		0.027***		0.029***	0.028***	0.029***
		(0.0003)		(0.0003)	(0.0003)	(0.0003)
second_degree			0.006***	−0.008***	−0.008***	−0.007***
			(0.001)	(0.0004)	(0.0004)	(0.0004)
clustering	−0.032***	−0.018***	−0.041***	−0.006***	−0.004*	−0.005***
	(0.002)	(0.001)	(0.002)	(0.002)	(0.002)	(0.002)
Betweenness					0.00000**	
					(0.00000)	
eigenvector						−0.010***
						(0.002)
Constant	0.767***	0.639***	0.744***	0.662***	0.662***	0.658***
	(0.001)	(0.002)	(0.003)	(0.002)	(0.002)	(0.002)
Observations	6,300	6,300	6,300	6,300	6,300	6,300
R^2	0.033	0.629	0.045	0.648	0.648	0.650
Adjusted R^2	0.033	0.629	0.045	0.648	0.648	0.650

Note: $^{*}p<0.1$; $^{**}p<0.05$; $^{***}p<0.01$

Table A1.8. Standard deviation of (un)employment in the last 1000 periods. Real-world network: OLS. People with well defined clustering

	Dependent variable:					
	st.dev.E					
	(1)	(2)	(3)	(4)	(5)	(6)
degree		-0.001^{***}		-0.001^{***}	-0.001^{***}	-0.001^{***}
		(0.0001)		(0.0001)	(0.0001)	(0.0001)
second_degree			0.0004^{***}	0.001^{***}	0.001^{***}	0.001^{***}
			(0.0001)	(0.0001)	(0.0001)	(0.0001)
clustering	0.003^{***}	0.003^{***}	0.002^{***}	0.001^{***}	0.0005^{+}	0.001^{***}
	(0.0003)	(0.0003)	(0.0003)	(0.0003)	(0.0003)	(0.0003)
betweenness					-0.00000^{***}	
					(0.00000)	
eigenvector						-0.00003
						(0.0003)
average.e	-0.561^{***}	-0.541^{***}	-0.562^{***}	-0.535^{***}	-0.535^{***}	-0.535^{***}
	(0.001)	(0.002)	(0.001)	(0.002)	(0.002)	(0.002)
Constant	0.847^{***}	0.836^{***}	0.847^{***}	0.830^{***}	0.830^{***}	0.830^{***}
	(0.001)	(0.001)	(0.001)	(0.002)	(0.002)	(0.002)
Observations	6,300	6,300	6,300	6,300	6,300	6,300
R^2	0.962	0.963	0.962	0.964	0.964	0.964
Adjusted R^2	0.962	0.963	0.962	0.964	0.964	0.964

Note: robust standard errors clustered at realizations; $^{*}p<0.1$; $^{**}p<0.05$; $^{***}p<0.01$; $^{+}p = 0.1139$

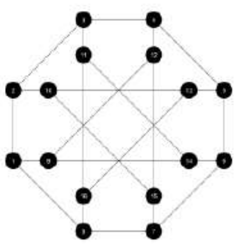
Table A1.9. Serial correlation in the last 1000 periods. Real-world network: OLS. People with well defined clustering.

	Dependent variable:					
	time.corr					
	(1)	(2)	(3)	(4)	(5)	(6)
degree		−0.009***		−0.011***	−0.011***	−0.011***
		(0.0003)		(0.0004)	(0.0004)	(0.0004)
second_degree			−0.0002	0.004***	0.004***	0.003***
			(0.0004)	(0.0004)	(0.0004)	(0.0004)
clustering	0.015***	0.017***	0.015***	0.012***	0.009***	0.012***
	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)
betweenness					−0.00000***	
					(0.00000)	
eigenvector						0.005***
						(0.002)
average.e	−0.620***	−0.405***	−0.620***	−0.383***	−0.382***	−0.381***
	(0.007)	(0.010)	(0.007)	(0.010)	(0.010)	(0.010)
Constant	1.150***	1.030***	1.151***	1.005***	1.005***	1.005***
	(0.005)	(0.007)	(0.005)	(0.007)	(0.007)	(0.007)
Observations	6,300	6,300	6,300	6,300	6,300	6,300
R^2	0.607	0.648	0.607	0.653	0.654	0.654
Adjusted R^2	0.607	0.648	0.607	0.653	0.653	0.653
Note:						*p<0.1; **p<0.05; ***p<0.01

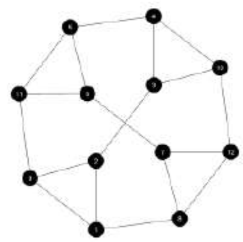
A.3.2 Steady state results for vertex-transitive networks

Here, we present more details regarding the analysis of Section 1.4.3. First of all, Figure A1.2 presents the three network under study if $n_i(g) = 3$ and Figure A1.3 those for $n_i(g) = 4$.

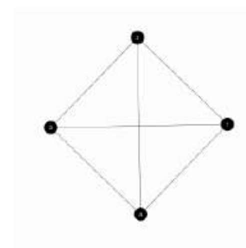
Figure A1.2. Vertex-transitive networks with $n_i(g) = 3$ for each i but varying clustering



(a) No triangle

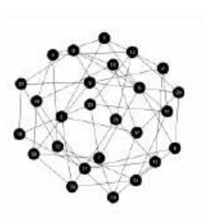


(b) One triangle

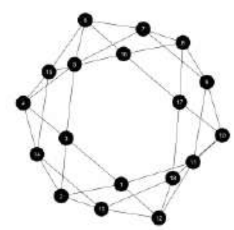


(c) Three triangles; $n = 4$

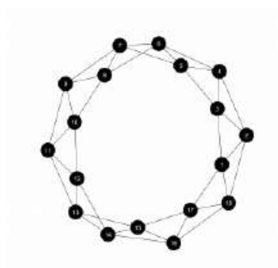
Figure A1.3.Vertex-transitive networks with $n_i(g) = 4$ for each i but varying clustering



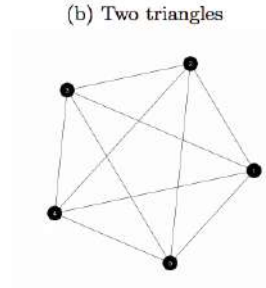
(a) No triangle



(b) Two triangles



(c) Three triangles



(d) Six triangles

In Figure A1.2, each node has three links. In (a) and (b), each node has six neighbors of neighbors while $n_i^2(g_{(c)}) = 3$. As for cycles, there is neither any triangle or square in (a), each node is involved in one triangle but no square in (b), whereas $C_i(g_{(c)}) = 1$ for each i and everybody belongs to one four-cycle. Network (c) presents the only feasible degree-three vertex-transitive network with more than one triangle per person. Hence, any differences between (a) and (b) can be attributed to short cycles, but the comparison of (a) and (b) with (c) is more complex. People belong to more short cycles, but they have three competitors less. Table A1.10 reports the results of the simulations using these networks.

In Figure A1.3, each node has degree four and, in networks (a), (b), and (c), second-order degree equal to eight while $n_i^2(g_{(c)}) = 8$. Network (c) is not vertex-transitive. All members have identical local position, but the nodes in the interior of the circle are slightly more globally central than those in the periphery in the figure. We constructed a corresponding vertex-transitive network satisfying the above conditions, but the network (c) resembles more the networks (a) and (b) in the presence of cycles of longer length. We thus opted for network (c) in our comparison. There is neither any triangle or square in (a), each node is involved in two or three triangles but no square in (b) and (c), whereas $C_i(g_{(d)}) = 1$ (six triangles) for each i and everybody belongs to three four-cycles and one five-cycle. Network (d) presents the only feasible network with $n_i(g) = 4$ and $C(g) = 1$. Hence, networks (a), (b), and (c) enable a clean comparison with respect to short network cycles, whereas the comparison of these networks with (d) is more complex. Table A1.11 reports the results of the simulations using these networks.

Table A1.10. Labor-market statistics in vertex-transitive networks with $n_i(g) = 3$ but varying clustering

	Networks		
	No cycle	One cycle	Three cycles
A. Employment statistics			
Net. nodes:	48	48	48
Num. economies:	100	100	100
Num. networks:	100	100	100
Employment rate:	0.7362184	0.7295554	0.7156931
St.Dev.	0.07832863	0.07858979	0.07843081
Coef.variation.	0.1063932	0.1077229	0.1095872
B. Time and spatial correlations			
Correlation(t-1,t):	0.7953321	0.7979369	0.8014949
Correlation(t-2,t):	0.6326194	0.6390886	0.6469422
Simple-Matching coef. (1st neighb.):	0.6454542	0.6470625	0.6579889
Simple-matching coef. (2nd neighb.):	0.62935	0.62145	-
C. Transition rates in the last 1000 periods (out of 10000):			
Fraction keeping employed:	0.6796644	0.673396	0.6607165
Fraction keeping unemployed:	0.2072279	0.2143017	0.2293419
Fraction lost job:	0.05654521	0.05615208	0.05497229
Fraction found job:	0.0565625	0.05615021	0.05496938
Conditional: EE/(EU+EE)	0.9231941	0.9230317	0.9231897
Conditional: UU/(UE+UU)	0.7855779	0.7923838	0.8066578
D. Kolmogorov-Smirnov, Wilcoxon and, Fligner-Killeen tests			
	D (p)	W (p)	χ^2 (p)
No vs. one cycle	0.03589 (0)	5244320870 (0)	35.63115 (0.2204011)
No vs. three cycles	0.10981 (0)	5744366450 (0)	41.54416 (0.09777024)
One vs. three cycles	0.07392 (0)	5501532003 (0)	25.54077 (0.7430232)

Table A1.11. Labor-market statistics in vertex-transitive networks with $n_i(g) = 4$ but varying clustering

	Networks			
	No cycle	Two cycle	Three cycles	All cycles
A. Employment statistics				
Net. size:	234	234	234	235
Num. economies:	100	100	100	100
Employment rate:	0.7650278	0.7587585	0.7536416	0.7384366
St.Dev.	0.03560841	0.03538005	0.03573074	0.03592831
Coef.variation.	0.04654525	0.04662887	0.04741078	0.04865456
B. Time and spatial correlations				
Correlation(t-1,t):	0.7898028	0.7923561	0.7947033	0.8038305
Correlation(t-2,t):	0.6258672	0.6277517	0.6335188	0.6492123
Correlation(t-3,t):	0.4965343	0.4975933	0.5048997	0.5265075
Correlation(t-4,t):	0.3966029	0.395039	0.4047934	0.4280346
Correlation(t-5,t):	0.3157266	0.3160765	0.324128	0.3494497
Correlation(t-6,t):	0.2500541	0.2535421	0.2592487	0.285112
Correlation(t-7,t):	0.1988207	0.2038812	0.2068884	0.2317018
Correlation(t-8,t):	0.1580622	0.1639143	0.1647087	0.1892681
Correlation(t-9,t):	0.1268962	0.1324563	0.1309997	0.153785
Correlation(t-10,t):	0.1008872	0.1095631	0.1058485	0.126705
Average SM (1st neighbors):	0.6623812	0.641397	0.6663983	0.6701957
Average SM (2nd neighbors):	0.6496121	0.6433835	0.643141	-
C. Transition rates:				
Fraction keeping employed:	0.7081917	0.7025405	0.6977068	0.6836624
Fraction keeping unemployed:	0.1781347	0.1850209	0.1904169	0.206792
Fraction lost job:	0.05683855	0.05622128	0.0559394	0.0547697
Fraction found job:	0.05683504	0.05621735	0.05593697	0.05477591
Conditional: EE/(EU+EE)	0.9257042	0.9259039	0.925775	0.9258297
Conditional: UU/(UE+UU)	0.7581176	0.7669634	0.7729406	0.790586
D. Kolmogorov-Smirnov, Wilcoxon, and Fligner-Killeen tests				
	D (p)	W (p)	χ^2 (p)	
No vs. two cycles	0.07307 (0)	5501702434 (0)	70.28512 (0.3681783)	
No vs. three cycles	0.12932 (0)	5895907351 (0)	61.71775 (0.7494325)	
No vs. all cycles	0.30189 (0)	6932782427 (0)	79.68325 (0.224715)	
Two vs. three cycles	0.05809 (0)	5404198927 (0)	69.79693 (0.4843591)	
Two vs. all cycles	0.2363 (0)	6488505634 (0)	95.06107 (0.0299015)	
Three vs. all cycles	0.18001 (0)	6102184061 (0)	72.37386 (0.4323282)	

Capítulo 2

Clustering in Network Games

1. Introduction

Social networks have an incidence on individuals' behavior (Goyal, 2012; Jackson and Zenou, 2015). An agent may decide to wear a sanitary mask in Covid-19 times only if people in her network do. In other contexts, the same agent may perform a task, such as a providing a public good, only if none of her neighbors does.

The study of such strategic interactions from a social network perspective has grown over the last decades, motivated by the empirical evidence that social networks exert enormous influence on human behavior (Topa, 2001; Conley and Udry, 2005; Centola, 2010, 2011; Bond et al. 2012; Breza and Chandrasekhar, 2019). Literature on network games has aimed to understand which features of social networks affect behavior and how in different contexts, such as criminality (Calvó-Armengol and Zenou, 2004; Ballester et al. 2010), labor (Calvó-Armengol and Jackson, 2004, 2007), public good provision (Bramoullé and Kranton, 2007), or security investment (Acemoglu et al. 2016) among others.[1]

One of the most prevalent structural properties of real-life social networks is network cliquishness: individuals tend to form close-knit networks, where network cycles are highly present (Holland and Leinhardt, 1971; Watts and Strogatz, 1998, Ravasz and Barabási, 2003; Vega-Redondo, 2007; Leskovec et al. 2008; Jackson and Rogers, 2007; Jackson, 2010; Seshadhri et al. 2012). Network knittedness is typically quantified through the clustering coefficient, which captures the frequency with which neighbors of a node are mutually connected. For instance, Newman (2003) finds average clustering coefficients of 0.496 in coauthor networks, meaning that around 50 percent of the coauthors of the researchers in the network are, on average, coauthors of each other.[2]

Clustering has been pointed as a driver of human behavior through different mechanisms. One is information-based: news flow faster and more reliably though clustered networks, what may discourage misbehavior because of reputation effects (Merry, 1984; Raub and Weesie, 1990; Burt, 2005, 2008; Lippert and Spagnolo, 2011). Another related mechanism is the emergence of collective sanctioning systems: people may coordinate efforts to punish an individual more easily if the network they are part of is tightly clustered (Coleman, 1988a, 1988b). These types of mechanisms constitute the basis of theories of social capital (Coleman, 1990, Putnam, 2000; Burt, 2001 2005) that posit that clustering might prevent the emergence of free-rider attitudes and foster the emergence of cooperation. Recent research has found support for the positive effects of network cliquishness on cooperation (Bloch et al. 2008, Righi, and Takács, 2014; Vega-Redondo, 2005; Ali and Miller, 2012, 2016; Melamed et al. 2018), both through communication and ostracism patterns (Jackson, et al. 2012, Ali and Miller, 2016) as well as though social collateral (Karlan et al. 2009).[3]

[1]See also Ballester, Calvó-Armengol and Zenou (2004), Calvó-Armengol et al. (2009), and Leduc et al. (2017).

[2]Different explanations have been proposed to explain the tendency of people to form transitive relationships. Levine and Kurzman (2006) argue how humans may have evolved cognitive mechanisms designed to exploit the positive externalities that derive from network clustering, while Kovářík and Van der Leij (2014) relate it to risk aversion. See also Rapoport, (1953) or Jin et al. (2001).

[3]Beyond their incidence on behavior, clustering has also been relevant when explaining other economic phenomena, such as employment (Granovetter, 1973) or gender inequality (Ductor, et al. 2018).

This chapter explores the impact of clustering in two canonical forms of strategic interactions: games of strategic substitutes and games of strategic complements.[4] Under strategic complements, the benefits that a player obtains from taking an action are greater as more contacts of her do the same. This typically occurs because of the positive effects of having compatible products or undertaking similar behaviors (e.g. buying compatible technologies or speaking a common language) or because of peer pressures (in the case for example of smoking habits). Under strategic substitutes the opposite occurs: the benefits of a player from taking an action are lower as more contacts of her do the same. This is often due to the positive externalities that emerge from people's actions, that enables others to free ride from them. Games of strategic substitutes cover applications such as public good provision, costly experimentation or information adquisition, etc.

The main challenge while studying games played on networks stems from the intrinsic complexity of network architectures. The two main difficulties are the following. First, it is problematic and sometimes impossible to isolate the *ceteris paribus* incidence that each network feature has on behavior, because changing one network property affects the whole network architecture and thus any network measure that depends on the whole structure. Second, even if one focuses on a particular network, a bewildering range of equilibrium outcomes is possible when players have complete information about the network they are embedded in (see e.g. Bramoullé and Kranton, 2007).

One approach to solve these fundamental problems is the introduction of incomplete network information. Under incomplete information, behavior may only depend on anticipated interaction patterns, simplifying the analysis (De Martí and Zenou, 2015; Jackson and Zenou, 2015). Moreover, in many real-life situations, people indeed have incomplete information about their network and, even if complete information is available, people exhibit cognitive limitations while recalling and processing information about the whole network architecture (Jannick and Larrick, 2005; Dessi, et al. 2016).

Galeotti et al. (2010) take this approach. In their model, each player knows her degree and the degree distribution of the network, but not the degree of other players.[5] The ignorance of players about other players' degrees defines a Bayesian game where the degree of each player is interpreted as her type. An important contribution of these authors is the prediction of a player's optimal action on the sole basis of one of her network attributes, her degree. They show that every symmetric equilibrium is monotone non-increasing (non-decreasing) in players' degrees under strict strategic substitutes (complements).[6] This implies that social connections create personal advantages−a recurrent idea in networks literature (Burt, 1992, Granovetter, 2018)−as the expected payoffs of players with higher degrees are greater.[7] However, the incidence that other features of networks have on behavior, such as network clustering, is set aside from their analysis.

This chapter builds on the incomplete information framework of Galeotti et al. (2010), but extends it to incorporate players' information about (positive) network clustering. In our setup, each player has private information about her own degree, which defines her type. Besides, each player has information about the degree distribution, as well as about the extent to which her neighbors can be connected.[8] To be precise, players are informed about the maximal number of triangles they may be involved in, information that we refer as *perceived clustering.* However, no player knows neither her neighbors' identities nor the triangles she forms with them. We particularly explore how behavior changes as perceived clustering varies, *ceteris paribus,* and why.

Our information structure applies to many real life situations. A newcomer to a small village (or a prospective member of a fraternity) may expect that the people with whom she is going to interact know

[4]These games cover many of the game-theoretic applications studied in economics (see Bulow et al. 1985 or Potters and Suetens, 2009).

[5]This information setup−also considered in Jackson and Yariv, (2007)−applies for situations in which people have to take decisions without knowing with whom they are going to interact. For example, an agent who has to decide whether to get a vaccination may anticipate the volume of agents with whom she is going to have contact, but not the identity of these people. Although this is the main information setup in their paper, the authors also explore the effects of endowing players with richer network information.

[6]This result holds for games that satisfy a property: having an additional neighbor playing 0 is payoff equivalent to not having such a neighbor. This property holds for instance when players' payoffs depend on the sum of their neighbors' actions, but not when they depend, say, on the average.

[7]Consider for example a binary game of strategic substitutes (complements): action 1 is interpreted as paying for a public good (as adopting a complementary technology in the case of strategic complements), and action 0 not doing so. High-degree players may derive higher payoffs than low-degree ones, since they can free-ride at the expense of low degree-players under strategic substitutes. Under strategic complements, high-degree players may enjoy the benefits derived from compatibility.

[8]As we show below, the information structure of Galeotti et al. (2010) can be considered as a special case of our model when players believe that their neighbors do not know each other (clustering is zero).

each other. On the contrary, a first-year student (or a person starting a new job in a new firm) may believe that her future colleagues come from distant places and are not linked. In most real-life situations, people's expectations about network clustering lies in between these two extremes.

Our main result shows that equilibrium behavior of people can change depending on perceived clustering. Under strategic substitutes, equilibrium contribution to the public good does not decrease (does not increase) as perceived clustering increases (decreases). The opposite occurs under strategic complements: technology adoption in equilibrium does not increase (does not decrease) as perceived clustering increases (decreases).[9]

The intuition behind this result is the following. As in Galeotti et al. (2010), the popularity of each player (her degree) determines her equilibrium action, and thereby players' expectations about their neighbors' actions correspond to their expectations about their neighbors' degrees. Players know that there exist some correlation in their neighbors' degrees that derives from the fact that their neighbors may be linked.[10] The more neighbors a player expects to be mutually connected, the more neighbors she expects to have with correlated degrees. This implies that a player believes more likely to have *similar* types of neighbors (in terms of popularity) when she knows that many of her neighbors can be linked, and a greater *diversity* of neighbors when she knows that only a few of them can. Consequently, a player believes more likely to have at least one type of neighbor playing action 1 (i.e. a neighbor that contributes to the public good under strategic substitutes or adopts the technology under strategic complements) when perceived clustering is lower than when it is higher. Since in our game of strategic substitutes (complements) a player wants to play action 0(1) if at least one of her neighbors plays 1(1) and action 1(0) otherwise, a greater perceived clustering neither reduces public good contribution nor increases technology adoption in equilibrium.

The contribution of this chapter is to uncover a novel mechanism by which clustering influences equilibrium in games played on networks: the influence of this network feature in the perceived heterogeneity of players. We show that people's choices are not only driven by the number of connections they have, but also by their expectations about the diversity of their contacts in terms of popularity, which are shaped by their information about network clustering. Our findings are in line with those predicted by Coleman (1988a, 1988b), as greater perceived clustering discourages people to free ride. However, our result is not driven by the emergence of collective sanctions as suggested by him, but by the effects of perceived clustering on the perceived heterogeneity of players. Lower perceived clustering traduces in greater expectations about neighbors diversity, what creates personal advantages in a similar way as personal connections do. In fact, increasing perceived clustering has the same effect as inducing a shift in the sense of first order stochastic dominance in the degree distribution (see Galeotti et al. 2010) under strategic substitutes and the opposite effect under strategic complements.

The closest chapter to ours is Lamberson (2015), that also builds on the framework of Galeotti et al. (2010) to analyse the incidence of clustering in binary games of strategic substitutes and of strategic complements. Their model is however different: the network is regular and the beliefs of each player about her neighbors' actions are based on their past actions.[11] This introduces local correlation in players' actions, increasing both technology adoption and public good provision in the Bayes-Nash equilibrium.

The chapter is organized as follows. Sections 2.1 and 2.2 present some basic definitions and the games considered in this work, respectively. Section 2.3 provides the results. In Section 2.4, we discuss the results and offer directions for future research.

2.1. Background definitions

Consider a social network $g = (N, E)$ composed of a set of nodes $N = \{1, .., n\}$ and a set of edges or links E between them. Each node $i \in N$ represents one of the $n = |N|$ agents in the network. We write $g_{ij} = 1$ if

[9] Under strategic complements, there is always a trivial equilibrium where all types of players play action 1(0). Since these equilibria are always possible, we set them aside when exploring the incidence of perceived clustering on equilibria.

[10] Players know that nodes in the network do not have degrees with independent probabilities, but the degree of connected nodes are correlated even if links are formed in a fully random way. Consider the simplest example of a network integrated by node i and her neighbors j and l. If each each edge forms randomly and independently with probability p, i knows that there exist perfect correlation in their neighbors' degrees: either both j and l have degree 1 (with probability p) or they both have degree 0 (with probability $1 - p$). While this correlation is always present, it tends to vanish as the network gets very large, as we discuss.

[11] For example, if an agent contributes to the provision of a public good, she may not know whether their neighbors will contribute in a subsequent period, but she may expect them to be less likely to contribute because they can count on her to do it.

agents $i \in N$ and $j \in N$ are directly linked in g and $g_{ij} = 0$ otherwise, with $g_{ij} = g_{ji}, \forall i, j \in N$. The network is represented by a $n \times n$ symmetric adjacency matrix $A = (g_{ij})_{i,j \in N}$, with $g_{ii} = 0$.

The neighborhood of node $i \in N$ is the set of agents directly connected to i, $N_i(g) = \{j \in N : g_{ij} = 1\}$. The degree of node i is denoted $k_i(g) = |N_i(g)|$.

Let $P_g(k)$ be the probability degree distribution. Namely, $P_g(k)$ is the probability that each $i \in N$ has degree k, for all $k \in \{0, 1, ..., n-1\}$.

In our analysis, we will refer to a model of network formation: the Erdös-Renyi model (1959; 1960). In this model, each link between any two nodes occurs randomly with probability p, independently of any other link. The degree of each node $i \in N$ is then given by a random variable $K_i(g)$, which takes value k with probability:

$$P[K_i(g) = k] = \binom{n-1}{k} p^k (1-p)^{n-1-k} \tag{2.1.1}$$

A three—cycle $Z_i^3(g)$ in a network g is a set of nodes $\{i, j, k\}$ such that $g_{ij} g_{jk} g_{ki} = 1$. We define $S_i^3(g) = \{Z_i^3(g) : i \in Z_i^3(g)\}$ as the set of all three cycles in g to which agent i belongs. We refer a three-cycle also as a triangle.

The clustering coefficient of $i \in N$ captures the proportion of three-cycles (triangles) that she forms with her neighbors:

$$C_i(g) = \frac{2\left|S_i^3(g)\right|}{k_i(g)\big(k_i(g)-1\big)}$$

In words, $C_i(g)$ is the number of pairs of neighbors of i that are linked divided by the maximal number of links that could exist among $i's$ neighbors. The average clustering coefficient of network g is $C(g) = \frac{1}{n}\sum_{i \in N} C_i(g)$ (Barabasi, 2016). In Erdös-Renyi networks, $C(g) = p$.

2.2. Network games

2.2.1. Network and players

Throughout all our analysis, we consider a random network, where each link is formed independently with probability $p \in (0, 1)$ as in the Erdös-Renyi model. In such a network, each node has degree k as given by (2.1.1).[12] Nodes are the players of the game.

2.2.2. Payoffs

We analyse two types of strategic interactions: games of strategic substitutes and games of strategic complements.

2.2.2.1. Strategic substitutes

We consider a game similar to that in Bramoullé and Kranton (2007). Each player chooses simultaneously and independently an action in $X = \{0, 1\}$.[13] Action 1 represents an activity that generates positive externalities to people. For instance, action 1 may be interpreted as contributing to a public good or investing in a non-rival technology. Players pay a cost c when they play action 1, with $0 < c < 1$. Action 0 bears no cost.

Let x_i be the action of player $i \in N$, and $\mathsf{x_N} = (x_1, ..., x_n)$ the action profile of all players. The utility of each i is $u_i(x_i, \overline{x}_{N_i(g)})$, with $\overline{x}_{N_i(g)} = \sum_{j \in N_i(g)} x_j$. It takes the following values:

$$u_i(0, \overline{x}_{N_i(g)}) = 0 \quad \text{if} \quad \overline{x}_{N_i(g)} = 0$$

$$u_i(0, \overline{x}_{N_i(g)}) = 1 \quad \text{if} \quad \overline{x}_{N_i(g)} \geq 1$$

[12] Galeotti et al. (2010) do not consider a specific network in their model. However, the assumption of degree independence on which many of their results are based is interpreted as holding asymptotically in random networks, as we discuss below. We thereby consider a random network as well when presenting their results.

[13] In Bramoullé and Kranton (2007), players can play any action in $[0, \infty)$. We consider here a simpler version of this model.

$$u_i(1, \overline{x}_{N_i(g)}) = 1 - c \quad \text{for any} \quad \overline{x}_{N_i(g)}$$

It is readily seen that each player prefers her neighbors to take action 1 rather than taking this action herself. However, if none of her neighbors plays 1, she prefers playing 1 than playing 0.

2.2.2.2. Strategic complements

Jackson (2010) provides the example of a game where players' actions are strategic complements. Imagine a situation where each $i \in N$ has to decide independently and simultaneously whether to take an certain action ($x_i = 1$) or not ($x_i = 0$), and the enjoyment of this activity is higher if at least one of her neighbors plays also this action. For example, i has to decide whether to attend an event (action 1) or not (action 0), and c accounts for the travel expenses, $0 < c < 1$. Another example is the decision to adopt a certain technology: the benefit that an individual derives from the use of a technology often depends on the number of neighbors that use compatible items (Katz and Shapiro, 1986). In this case:

$$u_i(1, \overline{x}_{N_i(g)}) = 1 - c \quad \text{if} \quad \overline{x}_{N_i(g)} \geq 1$$

$$u_i(1, \overline{x}_{N_i(g)}) = -c \quad \text{if} \quad \overline{x}_{N_i(g)} = 0$$

$$u_i(0, \overline{x}_{N_i(g)}) = 0 \quad \text{for any} \quad \overline{x}_{N_i(g)}$$

2.2.3. Information

We write $I_i(g)$ to denote the whole information set that each $i \in N$ has about the network.

2.2.3.1. Galeotti et al. (2010).

Information and types. Agents' private information corresponds to their degree, while the probability degree distribution of the network is common knowledge. These two aspects constitute the whole information set that each $i \in N$ has about the network, $I_i(g) = \{k_i(g), p, n\}$. The games are analysed within the framework of Bayesian games of incomplete information by identifying the type of each player with her degree.

Players' beliefs. Given $I_i(g)$, each $i \in N$ knows that the probability that $j \in N_i(g)$ has degree k is the probability that j forms $k - g_{ij} = k - 1$ additional links with the remaining $n - 2$ agents in the network. Namely,

$$P[K_j(g) = k \mid I_i(g)] = \binom{n-2}{k-1} p^{k-1}(1-p)^{n-k-1} \qquad \forall j \in N_i(g),\ \forall i \in N \tag{2.2.1}$$

Define $F(g) = \{1, 2, ..., \overline{k}\}$ as the set of feasible neighbors' degrees: the set of degrees that each neighbor of any node can have in network g, where $\overline{k}$ is the maximal degree in this set. Equivalently, it could be assumed that the information set of each i is $I'_i(g) = \{k_i(g), p, F(g)\}$ and $\overline{k} = n - 1$; players have the same beliefs about the connectivity of their neighbors given $I_i(g)$ and given $I'_i(g)$,

$$P[K_j(g) = k \mid I'_i(g)] = \binom{\overline{k}-1}{k-1} p^{k-1}(1-p)^{\overline{k}-k} = P[K_j(g) = k \mid I_i(g)], \qquad \forall j \in N_i(g),\ \forall i \in N \tag{2.2.2}$$

We stress the equivalence between both information sets here, as it will be relevant in certain parts of our analysis.

Degree independence. Although links are formed randomly and independently, nodes do not have degrees with independent probabilities, but their degrees are to some extent correlated.[14] While this correlation is always present, it tends to vanish as the network gets large, since the possibility of an edge between each pair of nodes is only 1 out of the $n - 1$ that each might have (Jackson, 2010). Then, it is reasonable to assume that nodes have degrees with probabilities that are (roughly) independent when $n \to \infty$. The assumption of degree independence made by Galeotti et al. (2010) is interpreted in this manner, as holding (approximately) for infinitely large random networks (often referred as *Poisson networks*).[15]

[14] See footnote 10.

[15] Galeotti et al. (2010) make use of this assumption to characterize the equilibrium generally (see their Proposition 2). However, they also show how some of their results for strategic complements (substitutes) maintain when the degree of connected nodes are positively (negatively) correlated.

Clustering. In Poisson networks, the probability that two specific nodes i and j are connected is insignificant. This implies not only that nodes have degrees with (roughly) independent probabilities, but also that the probability that two neighbors of a node are connected is negligible in these networks.[16] Galeotti et al. (2010) presume then not only that players' believe that nodes have degrees with independent probabilities, but implicitly also that players believe that the network has a tree-like structure.[17]

2.2.3.2. Our Setting.

Information and types. Players are privately informed about their degree, while the degree distribution of the network is common knowledge. Besides, each player privately knows the extent to which her neighbors can be connected. In particular, each player is exogenously informed about the maximal number of triangles in which she may be involved, denoted τ. We refer to τ as *perceived clustering*. The information set of each $i \in N$ is then $I_i(g) = \{k_i(g), p, F(g), \left|S_i^3(g)\right| \leq \tau\}$, with $F(g) = \{1, 2, ..., \overline{k}\}$.[18] Note that i does not have information about the identity of her neighbors nor about the triangles that she forms with them; even if she knows that the cardinality of $S_i(g)$ lies in the interval $[0, \tau]$, both $S_i^3(g)$ and $\left|S_i^3(g)\right|$ are unknown for her. For example, if $N_i(g) = \{j, l, m\}$ and $\tau = 1$, i knows that she may form one triangle with her neighbors. However, she does not know whether this triangle is $\{i, j, l\}$, $\{i, j, m\}$, or $\{i, l, m\}$. It might be also the case that she does not form any triangle with her neighbors.

Perceived clustering is the same for all players, regardless of their degree. It is an exogenous information that they may have obtained from the context, in a similar way as they know the degree distribution. Imagine for example a company representative who is considering to attend a networking event, say, a business congress. Before attending the congress, this agent may know the volume of clients of her company, and have one idea about the volume of clients of the other participating companies. Besides, she may know the companies that participated in a past edition of the congress, the number of them that operate within the same area or have collaborated previously in any project, etc. These exogenous signals may inform the agent about the number of firms that might have a prior relationship, and in turn, about the maximal number of connections that might exist among her potential partners (τ).

The private information of each i, $k_i(g)$ and τ, define her type. However, since perceived clustering is the same for all players, we will assume that the type of each player is only given by her degree, since this is the only information that can be different across players. Hence, every symmetric strategy σ is a mapping that specifies each player's action as a function of her type, $\sigma(k_i) \in \{0, 1\}$.

Finite random network. The analysis of clustering requires setting aside Poisson networks, since clustering is very small in these graphs. We thereby consider a random network of finite size throughout our analysis. Unlike Poisson networks, degrees of connected nodes are (not negligibly) positively correlated in this type of networks (Newman 2003; Jackson and Rogers 2007).

2.3. Results

This section analyses the relation between clustering and behavior. We start by presenting the main insights in Galeotti et al. (2010) (Section 2.3.1). Then, we show how a variation in perceived clustering may have consequences in players' equilibrium behavior (Section 2.3.2).

2.3.1. Degree and behavior

Under the information setup in Galeotti et al. (2010), the (pure) strategy of each player can be identified with a mapping σ that specifies her action as a function of her degree, $\sigma(k_i) \in \{0, 1\}$, $\forall i \in N$. Galeotti et

[16] In fact, the probability that a pair of nodes are connected is always p in Poisson networks, regardless on whether they share a common neighbor. Then, for fixed p, the clustering coefficient becomes vanishing small as the number of nodes increases (see Vega-Redondo, 2007).

[17] In Section 2.3.2.1, we will show that the equilibrium setup in Galeotti et al. (2010) is rougly equal that the one that emerges when players know that the network has clustering equal to zero.

[18] As we explained in the previous section, players form the same beliefs about the degree of their neighbors from the knowledge of n and p as well as from the knowledge of $F(g)$ and p: in both cases, they believe that each neighbor of them has degree k with the probability in (2.2.1). However, players can derive some additional information about their neighbors' connectivity from the joint knowledge of τ and n, while they cannot from the joint knowledge of τ and $F(g)$, as we discuss in a subsequent section. Then, in order to isolate the effects of perceived clustering from those that may be caused by changes in players' beliefs about their neighbors' degrees we assume $I_i(g) = \{k_i(g), p, F(g), \left|S_i^3(g)\right| \leq \tau\}$ for each $i \in N$.

al. (2010) focus on the symmetric equilibria of the games, where players with the same types play the same strategies. In fact, these are the only equilibria that are possible under the assumption of degree independence, as they discuss.[19]

2.3.1.1. Strategic substitutes.

Suppose all players follow a strategy σ. The expected utility of each $i \in N$ of playing 0 corresponds to the probability that at least one $j \in N_i(g)$ plays 1 (i.e. the probability that $\exists j \in N_i(g) : \sigma(k_j) = 1$). Under degree independence, $i's$ expected utility of playing 0 is:

$$E_{U_i(g)}\left(0, \sigma, I_i(g)\right) = 1 - \left[\sum_{k_j : \sigma(k_j)=0} p[K_j(g) = k_j \mid I_i(g)]\right]^{k_i(g)} \tag{2.3.1}$$

while $E_{U_i}\left(1, \sigma, I_i(g)\right) = 1 - c$ is the expected utility of i of playing 1. In equilibrium, each agent plays 0 if $E_{U_i}\left(0, \sigma, I_i(g)\right) \geq E_{U_i}\left(1, \sigma, I_i(g)\right) = 1 - c$, and action 1 otherwise.

Independence in degrees guarantees that $p[K_j(g) = k_j \mid I_i(g)]$ is the same for all $i \in N$, implying that $E_{U_i}\left(0, \sigma, I_i(g)\right)$ is increasing in $k_i(g)$, whenever players are uncertain about their neighbors actions.[20] As a consequence, if i is best responding with action 0, each player l with degree $k_l(g) > k_i(g)$ must be responding with action 0 as well, since $E_{U_l}\left(0, \sigma, I_l(g)\right) > E_{U_i}\left(0, \sigma, I_i(g)\right) \geq 1 - c$. It follows then that any equilibrium of the game is and characterized by a degree threshold. Such an equilibrium threshold is the value t for which following equality satisfies:

$$1 - P\Big[K_j(g) > t \mid I_i(g)\Big]^t = 1 - c \tag{2.3.2}$$

Any equilibrium strategy σ of the game satisfies σ: $\sigma(k) = 1$ if $k < t$, $\sigma(k) = 0$ if $k > t$ and $\sigma(k) \in \{0, 1\}$ for $k = t$.[21]

2.3.1.2. Strategic complements.

Reasoning is similar for strategic complements. Under degree independence, the expected utility of i of playing 1 is increasing in $k_i(g)$:

$$E_{U_i(g)}\left(1, \sigma, I_i(g)\right) = -c + 1 - \left[\sum_{k_j : \sigma(k_j)=0} p[K_j(g) = k_j \mid I_i(g)]\right]^{k_i(g)} \tag{2.3.3}$$

while $E_{U_i}\left(0, \sigma, I_i(g)\right) = 0$. Therefore any equilibrium of this game is non-decreasing, and characterized by a threshold.

Let t the value for which:

$$1 - \Big[P[K_j(g) < t \mid I_i(g)]\Big]^t = c \tag{2.3.4}$$

Analogously as for strategic substitutes it is readily seen that any (non-trivial)[22] equilibrium strategy σ satisfies σ: $\sigma(k) = 0$ if $k < t$, $\sigma(k) = 1$ if $k > t$ and $\sigma(k) \in \{0, 1\}$ for $k = t$.

[19] Under degree independence, players with the same degree face the same probability distribution over their neighbors' actions, and consequently the same decision problem. Then, their best response to their neighbors' actions is the same.

[20] Clearly, if all agents play the same action, say, action 0(1), $E_{U_i(g)}\left(0, \sigma, I_i(g)\right)$ equal to 0(1) and does not depend on $k_i(g)$.

[21] Strategy σ_1: $\sigma_1(k) = 1$ for $k \leq t$ and $\sigma_1(k) = 0$ for $k > t$ is always an equilibrium strategy, since $E_{U_i}\left(0, \sigma_1, I_i(g)\right) = 1 - P[K_j(g) > t \mid I_i(g)]\Big]^{k_i(g)} \leq 1 - c$, for all i with $k_i(g) \leq t$, and $E_{U_i}\left(0, \sigma_1, I_i(g)\right) = 1 - P[K_j(g) > t \mid I_i(g)]\Big]^{k_i(g)} > 1 - c$ for all i with $k_i(g) > t$. However, strategy σ_2: $\sigma_2(k) = 1$ for $k < t$ and $\sigma_2(k) = 0$ for $k \geq t$ might not. Suppose $P[K_j(g) \geq t \mid I_i(g)] > P[K_j(g) > t \mid I_i(g)]$. For $k_i(g) = t$, $E_{U_i}\left(0, \sigma_2, I_i(g)\right) = 1 - P[K_j(g) \geq t \mid I_i(g)]\Big]^t$ would be lower than $1 - P[K_j(g) > t \mid I_i(g)]\Big]^t = 1 - c$, and i would not best responding with action 0 but with action 1. On the contrary, if $P[K_j(g) \geq t \mid I_i(g)] = P[K_j(g) > t \mid I_i(g)] = 1 - c$, strategy σ_2 is an equilibrium strategy, for the same reasons as strategy σ_1 is.

[22] See footnote 9.

2.3.2. Clustering and behavior.

Section 2.3.2.1 shows that the equilibrium in Galeotti et al. (2010) is roughly equal to that in an information setup where players know that the network is a tree, in addition to their own degree and the degree distribution. Section 2.3.2.2 analyses the equilibria that arise under our information setup, presented in Section 2.2.3.2.

2.3.2.1. Triangle-free networks

Consider a random network g where each link between any pair of nodes is formed randomly with probability p. Conditioned on g_{jl}, the probability that $j \in N$ has degree k is independent from the probability that $l \in N$ has degree k', as the following Lemma highlights.

Lemma 1. *If each link in g is formed randomly with probability p,*

$$p[K_j(g) = k, K_l(g) = k' \mid g_{jl}] = p[K_j(g) = k \mid g_{jl}] * p[K_l(g) = k' \mid g_{jl}]$$

for any $j, l \in N$.

Proof. By construction, each link between $j(l) \in N$ and any agent in $N \setminus \{j, l\}$ occurs with probability p, independently of any other edge. Then,

$$p[K_j(g) = k \mid K_l(g) = k', g_{jl}] = \binom{n-2}{k-g_{jl}} p^{k-g_{jl}}(1-p)^{n-2-k+g_{jl}} = p[K_j(g) = k \mid g_{jl}]$$

and K_j and K_l are conditionally independent given g_{jl}, $\forall j, l \in N$. Hence, $p[K_j(g) = k, K_l(g) = k' \mid g_{jl}] = p[K_j(g) = k \mid g_{jl}] * p[K_l(g) = k' \mid g_{jl}]$, $\forall j, l \in N$. ■

Lemma 1 stresses an important fact: the lack of independence in degrees of $i's$ neighbors derives from the fact that these agents may be linked. Conditioning on the possible links among $i's$ neighbors, the degree of these agents depend on independent events, thereby they are (conditionally) independent.

Galeotti et al. (2010) analyse the games under the assumption that the network is that large that players' disregard the probability that their neighbors' are connected when choosing their actions, which is implicit in their assumption of degree independence. The equilibrium that arise under their information setup is similar to the one that would emerge in a setting where the information set of each $i \in N$ is $I_i(g) = \left\{k_i(g), p, n, \left|S_i^3(g)\right| = 0\right\}$. When the information set of each $i \in N$ is $I_i(g) = \left\{k_i(g), p, n, \left|S_i^3(g)\right| = 0\right\}$, each i believes that $j \in N_i(g)$ has degree k with probability:

$$P[K_j(g) = k \mid I_i(g)] = \binom{n-k_i-1}{k-1} p^{k-1}(1-p)^{n-k_i-k} \qquad \forall j \in N_i(g),\ \forall i \in N \tag{2.3.5}$$

Applying the reasoning in Section 2.3.1, it can be easily checked that any equilibrium is monotone non-increasing (non-decreasing) in players' degrees under strategic substitutes (complements) and characterized by a threshold t. Such a threshold t is the value for which (2.3.2) satisfies under strategic substitutes, and the value for which (2.3.4) holds under strategic complements. Since for $n \to \infty$, (2.2.1) is roughly equal to (2.3.5), the equilibrium in Galeotti et al. (2010) is practically equal that the one that emerges when the information set of each i is $I_i(g) = \left\{k_i(g), p, n, \left|S_i^3(g)\right| = 0\right\}$.

2.3.2.2. Clustered networks

We first show that under our information setup, every symmetric equilibrium is characterized by a degree threshold. This means that the monotonicity property of equilibrium maintains, even when the assumption of degree independence is relaxed.

Proposition 1. *Let $I_i(g) = \{k_i(g), p, F(g), \left|S_i^3(g)\right| \leq \tau\}$, $\forall i \in N$. Under strategic substitutes (complements), every symmetric equilibrium is characterized by a degree threshold and it is non-increasing (non-decreasing).*

We now compare the equilibria that arise in two random networks g_1 and g_2 under the information setup presented in Section 2.2.3.2. The information set of each i in g_1 is $I_i(g_1) = \{k_i(g_1), p, F(g_1), \left|S_i^3(g_1)\right| \leq \tau\}$, while the information set of each i in g_2 is $I_i(g_2) = \{k_i(g_2), p, F(g_2), \left|S_i^3(g_2)\right| \leq \tau + 1\}$. We assume that $F(g_1) = F(g_2) = F = \{1, 2, ..., \bar{k}\}$. Hence, the only difference in the information that players have in the two networks is their perceived clustering: in g_1, each i knows that she can form a maximum of τ triangles with her neighbors, while in g_2 she knows that she can form a maximum of $\tau + 1$.

As we introduced in Section 2.2.3.1, players can calculate the probability that each neighbor of them has degree k whether they know p and n or they know p and F. We assume that players in network g_x, $x \in \{1, 2\}$, know p and F but not n because the joint knowledge of the maximum value of $\left|S_i^3(g_x)\right|$ and n can provide them information about their neighbors' degrees. In such a case, players in g_1 and in g_2 might have different beliefs about their neighbors' degrees, and any difference in their behavior might be due to this fact and not to their different perceived clustering, *per se*.[23] To isolate the effects of perceived clustering from those that might be caused by the different beliefs of players about their neighbors' degrees, we assume that players are not informed about the network size: they only know that the feasible degrees of their neighbors are those in set $F(g_1) = F(g_2) = F$. Then, both in g_1 as well as in g_2 each player i believes that her neighbor j has degree $k \in F(g_x)$ with probability:

$$P[K_j(g_1) = k \mid I_i(g_1)] = P[K_j(g_2) = k \mid I_i(g_2)] = \binom{\bar{k} - 1}{k - 1} p^{k-1}(1 - p)^{\bar{k}-k} \tag{2.3.6}$$

Since perceived clustering is τ in g_1 and $\tau+1$ in g_2, we will assume that players have degree equal or greater than $2(\tau + 1)$ in the two networks.[24]

Proposition 2. *Consider two networks* $g_1 = (N, E)$ *and* $g_2 = (N', E')$, *with* $N \subseteq N'$. *Let* $I_i(g_1) = \left\{k_i(g_1), p, F(g_1), \left|S_i^3(g_1)\right| \leq \tau\right\}$ $\forall i \in N$, $I_i(g_2) = \left\{k_i(g_2), p, F(g_2), \left|S_i^3(g_2)\right| \leq \tau + 1\right\}$ $\forall i \in N'$, $F(g_1) = F(g_2) = F$, $k_i(g_1) = k_i(g_2) \geq 2(\tau + 1)$ $\forall i \in N \cup N'$, $\tau \geq 0$ *and* $0 < p < 1$. *If* σ_x *is a symmetric equilibrium strategy in network* g_x, $x \in \{1, 2\}$*:*

1. $\sigma_2(k) \geq \sigma_1(k)$, *under strategic substitutes, for all* k.

2. $\sigma_2(k) \leq \sigma_1(k)$, *under strategic complements, for all* k, *where* $\sigma_x(k) \neq 0$ *and* $\sigma_x(k') \neq 1$ *for some* k *and some* k'.[25]

Proposition 2 shows how players' behavior may change depending on their beliefs about network knittedness: under strategic substitutes, the range of degree values for which players act as a free-rider decreases as their perceived clustering increases. As for strategic complements, the opposite occurs: people play action 0 for a greater range of degree values if their their perceived clustering is greater. As a consequence, the situations where players coordinate to play the action that may provide them the highest payoff $-$action $0(1)$ under strategic substitutes (complements)$-$ reduces.

Intuitively, the greater the number of links between $i's$ neighbors, the greater the number of neighbors of i with correlated degrees. This is known by i when she chooses her action. Since i expects to have a greater (lower) diversity of types of neighbors when her perceived clustering is lower (greater), she expects to have *at least one* neighbor playing 1 with a greater probability in g_1 than in g_2. This means that $E_{U_i(g_1)}(0, \sigma, I_i(g_1)) > E_{U_i(g_2)}(0, \sigma, I_i(g_2))$ under strategic substitutes and $E_{U_i(g_1)}(1, \sigma, I_i(g_1)) > E_{U_i(g_2)}(1, \sigma, I_i(g_2))$ under strategic complements, and the result follows.

To conclude, our results show that an increase in the perceived clustering has a similar effect to an overall increase in connectivity under strategic substitutes, and the opposite effect under strategic complements. To be precise, Galeotti et al. (2010) show how inducing a shift in the degree distribution in the sense of first order stochastic dominance (specifically, moving from p to p' where $p' > p$) does neither reduce overall contribution to public good provision nor technology adoption. Since players' expectations about

[23] Consider a for instance a network g and suppose that each $i \in N$ knows $\left|S_i^3(g)\right| \leq \tau$. If i knows n, she knows that the maximum degree that each $j \in N_i(g)$ can have is $n - 1 - k_i + \tau$, since j may form a maximum of τ links with agents in $N_i(g)$, and a maximum of $|N \setminus \{j\}| - k_i(g) = n - 1 - k_i(g)$ links with agents outside $N_i(g)$. On the contrary, if i knows $\left|S_i^3(g)\right| \leq \tau + 1$, she knows that the maximum degree of j is $n - 1 - k_i + \tau + 1$.

[24] Notice that it would not be reasonable to assume that every player expects to be immersed in a maximum of, say, one triangle if any player does not have at least two neighbors.

[25] As mentioned above, we are interested in comparing the equilibria in the two networks in which not all types play the same actions.

their neighbors' popularity is higher as p increase, the probability that any of them plays action 1 does not increase (does not reduce) under strategic substitutes (complements), and the best response of each particular player is based in a threshold higher (lower) than t. This suggest that, if greater expectations about network clustering are accompanied by greater expectations about neighbors' connectivity as it often occurs,[26] our results on strategic substitutes might reinforce, while those under strategic complements might depend on the tradeoff of the effects of these two network aspects (clustering and degree).

2.4. Concluding Remarks

Network clustering is an ubiquitous property of real-life networks. Although the importance of clustering has broadly discussed in social capital theory (Burt, 2001, 2005; Coleman, 1988a, 1988b; 1990), there is a scarcity of work that analyses its impact on the strategic interactions among networked agents. This chapter provides novel predictions about clustering, and evinces how this network property may be a driver of behavior: irrespective of whether the game exhibits strategic complements or substitutes, players with the same degree may behave differently depending on their perceived clustering. Specifically, an increase in perceived clustering does not reduce public good provision under strategic substitutes, while it does not increase technology adoption under strategic complements.

In this chapter, we have focussed on games in which players only require that one neighbor plays a certain action to be best responding with the same action (under strategic complements) or with the opposite one (under strategic substitutes). In certain contexts, however, people may require social reinforcement from multiple neighbors to adopt particular behaviors, and empirical evidence shows that clustering foster the spread of behaviors in these specific contexts (Centola, 2010). When such a social reinforcement is required, do our results maintain? Could the correlation in degrees induced by clustering explain the advantage of clustered networks to spread behaviors that require social reinforcement to be adopted? We leave these questions for future research.

2.5. Appendix

Claim 1. *$p[K_j(g) = k \mid K_l(g) = k']$ is independent on g_{lr} , $\forall l, r \neq j$.*

The probability that j has degree k does not depend on any link involving agents different from j. Likewise, $p[K_j(g) = k \mid K_l(g) = k']$ does not depend on links between agents in $N \setminus \{j\}$, nor even on those involving l. In fact, although K_j is not independent of K_l, the lack of independence between these two variables is exclusively due to the fact that j and l may link (see footnote 10). However, K_j is independent of any link between l and any node distinct from j, since each link forms randomly and independently with probability p. Analogously, $p[K_j(g) = k \mid K_l(g) = k', ..., K_m = k'']$ is independent of any link that does not involve j.

Proof of Proposition 1. Intuitively, since $F(g)$, p and τ is the same for all $i \in N$, the probability that a neighbor of i is a type that plays 1 is the same for all $i \in N$. However, since players with greater degrees have more neighbors, they face a higher probabilility of having at least one neighbor playing action 1, and the result follows.

A. Strategic substitutes. We first note that there is not any equilibrium in pure strategies at which all types choose the same actions. Observe that when $\sigma(k) = 1(0)$ for all k, $E_{U_i(g_1)}\big(0, \sigma, I_i(g_1)\big) = 1(0)$ and i is not best responding with action 1(0) but with action 0(1). This implies that, if σ is an equilibrium strategy, $\sigma(k) \neq 0$ and $\sigma(k') \neq 1$ for some k and some k'. Let $D = \{k \in F(g) : \sigma(k) = 0\}$ be the set of (feasible) degree values for which a symmetric strategy σ specifies action 0, assuming $\sigma(k) \neq 0$ and $\sigma(k') \neq 1$ for some k and for some k'.

The expected utility of i of playing 0 is the probability that at least one of her neighbors plays 1. If neighbors of i play the symmetric strategy σ, $E_{U_i(g)}\big(0, \sigma, I_i(g)\big)$ is the probability that at least one of her neighbors does not have a degree in set D. This probability is

[26] Note that the average clustering of a random network is simply p. Hence, an increment in p increases both players' expectations about their neighbors' degrees and about network clustering.

$$E_{U_i(g)}\big(0,\sigma,I_i(g)\big) = 1 - p[K_1(g),K_2(g),...,K_{k_i(g)} \in D \mid I_i(g)] \tag{2.5.1}$$

where $K_1(g),K_2(g),...,K_{k_i(g)}(g)$ are the random variables of the degrees of the agents in $N_i(g)$=$\{1,2,...,k_i(g)\}$, and $p[K_1(g),K_2(g),...,K_{k_i(g)} \in D\Big]$ is the joint probability degree distribution of $i's$ neighbors, given $I_i(g)$.

If perceived clustering is equal to τ, i knows that she may be involved in a maximum of τ triangles. That is, the maximum number of agents in her neighborhood that may be linked is 2τ. This means that ther are $k_i(g)-2\tau$ agents in $N_i(g)$ that are not linked to any other agent in $N_i(g)$, and thereby these agents have degrees with independent probabilities (see Lemma 1). Then, (2.5.1) can be expressed as:

$$E_{U_i(g)}\big(0,\sigma,I_i(g)\big) = 1 - p[K_1(g),K_2(g),...,K_{2\tau}(g) \in D] * p[K_j(g) \in D]^{k_i(g)-2\tau} \tag{2.5.2}$$

To see the equivalence between (2.5.2) and (2.5.1), suppose that $\tau = 1$ and $N_i(g) = \{1,2,3\}$. Agent i knows that she may be involved in one triangle at most. However, she does not know whether this triangle is $\{i,1,2\}$, $\{i,1,3\}$ or $\{i,2,3\}$. It may also occur that none of her neighbors are linked and she forms no triangle. Recall from Lemma 1 that, conditional on not being linked, nodes have degrees with independent probabilities. Since p is the probability that two neighbors of i are linked, and $(1-p)$ the probability that they are not, the probability that all neighbors of i have a degree in set D can be expressed as:

$$\begin{aligned} p[K_1(g),K_2(g),K_3(g) \in D \mid I_i(g)] &= \frac{p}{3} * p[K_1(g),K_2(g) \in D \mid g_{12}=1] * p[K_3(g) \in D] \\ \frac{p}{3} * p[K_1(g),K_3(g) \in D \mid g_{13}=1] * p[K_2(g) \in D] &+ \frac{p}{3} * p[K_2(g),K_3(g) \in D \mid g_{23}=1] * p[K_1(g) \in D] \\ &(1-p) * p[K_1(g) \in D] * p[K_2(g) \in D] * p[K_3(g) \in D] \end{aligned} \tag{2.5.3}$$

All probabilities in (2.5.3) are conditioned on $I_i(g)$. Since the value of $p[K_j,K_l \in D \mid g_{jl}=1]$ is the same for all $j,l \in N$ and the value of $p[K_j \in D]$ is the same for all $j \in N$, Expression (2.5.3) is equal to:

$$\begin{aligned} p[K_1(g),K_2(g),K_3(g) \in D \mid I_i(g)] &= p[K_3(g) \in D] * \Big[p * p[K_1(g),K_2(g) \in D \mid g_{12}=1] + (1-p) * p[K_1(g)]p[K_2(g)]\Big] \\ &= p[K_3(g) \in D] * \Big[p * p[K_1(g),K_2(g) \in D \mid g_{12}=1] + (1-p) * p[K_1(g),K_2(g) \mid g_{12}=0]\Big] \\ &= p[K_3(g) \in D] * p[K_1(g),K_2(g) \in D] \end{aligned}$$

Hence,

$$E_{U_i(g)}\big(0,\sigma,I_i(g)\big) = 1 - p[K_1(g),K_2(g) \in D] * p[K_3(g) \in D]$$

which only reflects the fact two neighbors of i can be linked (i.e. they have degrees with non-independent probabilities) while a third one is not linked to the other two. Generally, for other values of $k_i(g)$ and τ, $E_{U_i(g)}\big(0,\sigma,I_i(g)\big)$ is given by (2.5.2).

Consider (2.5.2). Given that $p[K_j(g) \in D]^{k_i(g)-2\tau}$ is decreasing in $k_i(g)$ and $p[K_1(g),K_2(g),...,K_{2\tau}(g) \in D]$ does not depend on $k_i(g)$, $E_{U_i(g)}\big(0,\sigma,I_i(g)\big)$ is increasing in $k_i(g)$. This means that $E_{U_i(g)}\big(0,\sigma,I_i(g)\big)$ is greater for player with greater degrees. It follows then that, if a player of degree k is best responding with action 0, a player of degree $k' > k$ must be best responding with this action as well. Then, any equilibrium is non-increasing in players' degrees, and it is characterized by a degree threshold t. Such a threshold t is the value of $k_i(g)$ for which $E_{U_i(g)}\big(0,\sigma,I_i(g)\big) = E_{U_i(g)}\big(1,\sigma,I_i(g)\big) = 1-c$. That is, it is the value t for which the following equality holds

$$1 - \Big[p[K_1(g),K_2(g),...,K_{2\tau_x}(g) > t\Big] * \Big[p[K_j(g) > t]\Big]^{t-2\tau} = 1-c \tag{2.5.4}$$

Strategy σ: $\sigma(k)=1$ for $k \le t$, and $\sigma(k)=0$ for $k > t$ is always an equilibrium strategy in g. Indeed, $i's$ expected utility of playing 0 when all agents play σ is:

$$1-\Big[p[K_1(g),K_2(g),...,K_{2\tau}(g)>t]\Big]*\Big[p[K_j(g)>t]\Big]^{k_i(g)-2\tau}$$

which is greater (lower) than $1-c$ if $k_i(g)$ is greater (lower) than t, and equal to $1-c$ for $k_i(g)=t$. Note that, if the left part of (2.5.4) is equal to:

$$1-\Big[p[K_1(g),K_2(g),...,K_{2\tau}(g)\geq t]\Big]*\Big[p[K_j(g)\geq t\mid I_i(g)]]\Big]^{t-2\tau}$$

(what happens if t is a non-integer), $E_{U_i(g)}\big(0,\sigma,I_i(g)\big)$ is the same regardless on the action played by agents with degree t. In this case, the strategy σ': $\sigma'(k)=1$ for $k<t$, and $\sigma'(k)=0$ for $k\geq t$ is also an equilibrium strategy. Thus, while all players with degree below (above) t must play 1(0) in equilibrium, there may exist equilibria at which players with degree t play any of these two actions, $\{0,1\}$.

B. Strategic complements. Reasoning is analogous as for strategic substitutes. In this case,

$$E_{U_i(g)}\big(1,\sigma,I_i(g)\big)=-c+1-\Big[p[K_1(g),K_2(g),...,K_{2\tau_x}(g)\in D\Big]*\Big[p[K_j(g)\in D]\Big]^{k_i(g)-2\tau}$$

Any equilibrium strategy σ satisfies $\sigma(k)=0$ for $k<t$, $\sigma(k)=1$ for $k>t$, and $\sigma(k)\in\{0,1\}$ for $k=t$, where t is the value for which the following equality holds:

$$1-\Big[p[K_1(g),K_2(g),...,K_{2\tau_x}(g)<t]\Big]*\Big[p[K_j(g)<t]\Big]^{t-2\tau}=c$$

■

Lemma 2. *Consider two networks $g_1=(N,E)$ and $g_2=(N',E')$, where each link was formed randomly and independently with probability $0<p<1$, as in the Erdös-Renyi model. Let $I_i(g_1)=\{k_i(g_1),p,F(g_1),g_{jl}=0\}$ be the information set of each $i\in N$, and $I_i(g_2)=\{k_i(g_2),p,F(g_2)\}$ the information set of each $i\in N'$, with $F(g_1)=F(g_2)$. If $D\subseteq F(g_1)$ is a subset of degree values,*

$$p[K_j(g_2),K_l(g_2)\in D\mid I_i(g_2)]>p[K_j(g_1),K_l(g_1)\in D\mid I_i(g_1)]=\Big[p[K_j(g_1)\in D\mid I_i(g_1)]\Big]^2\qquad\forall j,l\in N_i(g_1)\cap N_i(g_2)$$

Proof.

1. **Network g_2.** Recall that $F(g_2)=\{1,2,...,\bar{k}\}$ is the set of feasible degrees that each neighbor of a player can have in network g_2, and $\bar{k}$ is the maximal number of agents to which such a neighbor can be linked in g_2. Each i does not know whether $j,l\in N_i(g_2)$ are linked in g_2. Since $j\in N_i(g_2)$ can be linked to a maximum of $\bar{k}$ agents (including i and l), the probability that j has a degree in set D, conditioned on $g_{jl}=1$ and on $I_i(g_2)$, is the probability that j forms $k-g_{ij}-g_{jl}=k-2$ additional links with some of the $\bar{k}-|\{i,l\}|=\bar{k}-2$ remaining agents:

$$p[K_j(g_2)\in D\mid I_i(g_2),g_{jl}=1]=\sum_{k\in D}\binom{\bar{k}-2}{k-2}p^{k-2}(1-p)^{\bar{k}-k}$$

and $p[K_j(g_2)\in D\mid I_i(g_2),g_{jl}=1]=p[K_l(g_2)\in D\mid I_i(g_2),g_{jl}=1]$.

Conditioned on $g_{jl}=0$, the probability that the realization of $K_j(g_2)$ falls inside D is, given $I_i(g_2)$:

$$p[K_j(g_2)\in D\mid I_i(g_2),g_{jl}=0]=\sum_{k\in D}\binom{\bar{k}-2}{k-1}p^{k-1}(1-p)^{\bar{k}-k-1}$$

with $p[K_j(g_2)\in D\mid I_i(g_2),g_{jl}=0]=p[K_l(g_2)\in D\mid I_i(g_2),g_{jl}=0]$.

Agent i knows that each link is formed randomly and independently with probability $p\in(0,1)$. Then, $K_j(g_2)$ and $K_l(g_2)$ are conditionally independent given g_{jl} (see Lemma 1), and $P[K_j(g_2),K_l(g_2)\in D\mid I_i(g_2)]$ can be expressed as a sum of two conditional probabilities: the probability that both j and l

have a degree in set D conditioned on $g_{jl}=1$, and the probability that they both have a degree in D conditioned on $g_{jl}=0$:

$$P[K_j(g_2),K_l(g_2)\in D\mid I_i(g_2)] = p*\Big[p[K_j(g_2)\in D\mid I_i(g_2),g_{jl}=1]*p[K_l(g_2)\in D\mid I_i(g_2),g_{jl}=1]\Big]$$
$$+(1-p)*\Big[p[K_j(g_2)\in D\mid I_i(g_2),g_{jl}=0]*p[K_l(g_2)\in D\mid I_i(g_2),g_{jl}=0]\Big]$$

Since $p[K_j(g_2)\in D\mid I_i(g_2),g_{jl}]=p[K_l(g_2)\in D\mid I_i(g_2),g_{jl}]$:

$$=p*\Big[p[K_j(g_2)\in D\mid I_i(g_2),g_{jl}=1]\Big]^2+(1-p)*\Big[p[K_j(g_2)\in D\mid I_i(g_2),g_{jl}=0]\Big]^2 \tag{2.5.5}$$

2. **Network** g_1. As in network g_2, i knows that $j\in N_i(g_1)$ can be linked to a maximum of $\bar{k}$ agents (included i). However, i knows that none of these agents is l, since $g_{jl}=0$. Hence, the probability that each $j\in N_i(g_1)$ has a degree in set D given $I_i(g_1)$ is the probability that j forms $k-g_{ij}=k-1$ additional links with some of the $\bar{k}-|\{i\}|=\bar{k}-1$ remaining agents to which j can be linked:

$$p[K_j(g_1)\in D\mid I_i(g_1)]=\sum_{k\in D}\binom{\bar{k}-1}{k-1}p^{k-1}(1-p)^{\bar{k}-k}$$

In network g_1, each $i\in N$ knows that $g_{jl}=0$, what implies that j and l have degrees with independent probabilities (see Lemma 1). Hence, player i believes that both j and l have a degree in set D with probability:

$$\begin{aligned}p[K_j(g_1),K_l(g_1)\in D\quad\mid I_i(g_1)]&=p[K_j(g_1)\in D\mid I_i(g_1)]*p[K_l(g_1)\in D\mid I_i(g_1)]\\&=p[K_j(g_1)\in D\mid I_i(g_1)]^2\end{aligned}$$

since $p[K_j(g_1)\in D\mid I_i(g_1)]=p[K_l(g_1)\in D\mid I_i(g_1)]$.
Given that $\binom{n}{k}=\binom{n-1}{k-1}+\binom{n-1}{k}$:

$$\begin{aligned}p[K_j(g_1)\in D\mid I_i(g_1)]&=\sum_{k\in D}\binom{\bar{k}-1}{k-1}p^{k-1}(1-p)^{\bar{k}-k}\\
&=\sum_{k\in D}\Big[\binom{\bar{k}-2}{k-2}+\binom{\bar{k}-2}{k-1}\Big]p^{k-1}(1-p)^{\bar{k}-k}\\
&=\Big[\sum_{k\in D}\binom{\bar{k}-2}{k-2}p^{k-1}(1-p)^{\bar{k}-k}\Big]+\Big[\sum_{k\in D}\binom{\bar{k}-2}{k-1}p^{k-1}(1-p)^{\bar{k}-k}\Big]\\
&=p*\Big[\sum_{k\in D}\binom{\bar{k}-2}{k-2}p^{k-2}(1-p)^{\bar{k}-k}\Big]+(1-p)*\Big[\sum_{k\in D}\binom{\bar{k}-2}{k-1}p^{k-1}(1-p)^{\bar{k}-k-1}\Big]\\
&=p*p[K_j(g_1)\in D\mid I_i(g_1),g_{jl}=1]+(1-p)*p[K_j(g_1)\in D\mid I_i(g_1),g_{jl}=0]\end{aligned}$$

Then, the probability that both j and l have a degree in set $k\in D$ given $I_i(g_1)$ is:

$$p[K_j(g_1),K_l(g_1)\in D\mid I_i(g_1)]=p[K_j(g_1)\in D\mid I_i(g_1)]^2$$
$$=\Big[p*p[K_j(g_1)\in D\mid I_i(g_1),g_{jl}=1]+(1-p)*p[K_j(g_1)\in D\mid I_i(g_1),g_{jl}=0]\Big]^2$$
$$=p^2*\Big[p[K_j(g_1)\in D\mid I_i(g_1),g_{jl}=1]\Big]^2+2p(1-p)\Big[p[K_j(g_1)\in D\mid I_i(g_1),g_{jl}=1]*p[K_j(g_1)\in D\mid I_i(g_1),g_{jl}=0]\Big]$$
$$+(1-p)^2\Big[p[K_j(g_1)\in D\mid I_i(g_1),g_{jl}=0]\Big]^2 \tag{2.5.6}$$

3. Subtracting (2.5.5)-(2.5.6):

$$= p(1-p)\Big[p[K_j(g_1) \in D \mid I_i(g_1), g_{jl} = 1] - p[K_j(g_1) \in D \mid I_i(g_1), g_{jl} = 0]\Big]^2 > 0 \qquad (2.5.7)$$

■

Proof of Proposition 2. We write g_x to denote network x, with $x \in \{1, 2\}$.

A. Strategic substitutes. Under strategic substitutes, there is not any equilibrium in which all types play the same actions. If $\sigma(k) = 1(0)$ for all k, $E_{U_i(g_1)}\big(0, \sigma, I_i(g_1)\big) = 1(0)$ and i is not best responding with action 1(0) but with action 0(1). Hence, if σ is an equilibrium strategy, $\sigma(k) \neq 0$ and $\sigma(k') \neq 1$ for some k and some k'. We define $D = \{k \in F : \sigma(k) = 0\}$ as the set of feasible degree values for which a symmetric strategy σ specifies action 0, assuming $\sigma(k) \neq 0$ and $\sigma(k') \neq 1$ for some k and for some k'.

The expected utility of i of playing 0 is the probability that at least one of her neighbors plays 1. When all agents play the symmetric strategy σ, $i's$ expected utility of playing 0 is the probability that at least one of her neighbors does not have a degree in set D. Namely, in network g_x, $x \in \{1, 2\}$,

$$E_{U_i(g_x)}\big(0, \sigma, I_i(g_x)\big) = 1 - p[K_1(g_x), K_2(g_x), ..., K_{k_i(g_x)} \in D \mid I_i(g)\Big] \qquad (2.5.8)$$

where $K_1(g_x), K_2(g_x), ..., K_{k_i(g_x)}(g_x)$ are the random variables of the degrees of the agents in $N_i(g_x)$=$\{1, 2, ..., k_i(g_x)\}$, and $p[K_1(g_x), K_2(g_x), ..., K_{k_i(g_x)} \in D \mid I_i(g_x)]$ is the joint probability degree distribution of $i's$ neighbors, conditioned on $I_i(g_x)$.

Comparison between g_1 and g_2. Applying the probability chain rule (also known as the general product rule, Schum, 2001; Klugh, 2013), we can rewrite (2.5.8) as:

$$E_{U_i(g_x)}\big(0, \sigma, I_i(g_x)\big) = 1 - \Bigg[p\Big[K_1(g_x)K_2(g_x) \in D\Big] * p\Big[K_3(g_x) \in D \mid K_1(g_x), K_2(g_x) \in D\Big] *$$
$$p\Big[K_4(g_x) \in D \mid K_1(g_x), K_2(g_x), K_3(g_x) \in D\Big] * ... * p\Big[K_{k_i(g_x)}(g_x) \in D \mid K_1(g_x), K_2(g_x), ..., K_{k_i(g_x)-1}(g_x) \in D\Big]\Bigg] \qquad (2.5.9)$$

where all the probabilities in (2.5.9) are conditional on $I_i(g_x)$.

Perceived clustering is equal to τ in network g_1, and equal to $\tau + 1$ in network g_2. This means that $i \in N$ knows that she may form at most τ triangles, while $i \in N'$ knows that she may form a maximum of $\tau + 1$. This is the only different information that player i has in g_1 and in g_2. Then, the different value of $E_{U_i(g_1)}\big(0, \sigma, I_i(g_1)\big)$ and $E_{U_i(g_2)}\big(0, \sigma, I_i(g_2)\big)$ comes from the fact that $i \in N$ knows that there are at most τ pairs of linked agents in $N_i(g_1)$, while $i \in N'$ knows that there are at most $\tau + 1$ in $N_i(g_2)$.

a) Agent i does not know the identity of the agents that may be in a triangle with her. For example, if i has three neighbors, $1, 2$ and 3 and $\tau = 1$, she knows that she may form one triangle with her neighbors, but she does not know whether this triangle is $\{i, 1, 2\}$, $\{i, 1, 3\}$ or $\{i, 2, 3\}$. Imagine however that she knew the set of triangles that she may form in g_1, denoted $T_i(g_1)$. If $\{i, 1, 2\} \notin T_i(g_1)$, the probability that $1 \in N_i(g_1)$ has a degree in set D is independent of the probability that $2 \in N_i(g_1)$ does (see Lemma 1), since i knows that 1 and 2 are not linked:

$$\begin{aligned} p[K_1(g_1), K_2(g_1) \in D] &= p[K_1(g_1) \in D] * p[K_2(g_1) \in D] \qquad (2.5.10) \\ &= p[K_1(g_1) \in D]^2 \end{aligned}$$

Analogously, imagine $i \in N'$ knew $T(g_2)$ (the set of triangles that in which she may be in g_2). Suppose $T(g_2) = T(g_1) \cup \{i, 1, 2\}$. In $i's$ beliefs, the probability that $1 \in N_i(g_2)$ has a degree in D would not be independent of the probability that $2 \in N_i(g_2)$ does, since these two neighbors might be linked in g_2:

$$p[K_1(g_2), K_2(g_2) \in D] \neq p[K_1(g_2) \in D]^2 \tag{2.5.11}$$

Observe that all other probabilities in $E_{U_i(g_2)}\big(0, \sigma, I_i(g_2)\big)$ are equal as their corresponding ones in $E_{U_i(g_2)}\big(0, \sigma, I_i(g_2)\big)$. That is,

$$p\Big[K_3(g_1) \in D \mid K_1(g_1), K_2(g_1) \in D\Big] = p\Big[K_3(g_2) \in D \mid K_1(g_2), K_2(g_2) \in D\Big],$$

$$p\Big[K_4(g_1) \in D \mid K_1(g_1), K_2(g_1), K_3(g_1) \in D\Big] = p\Big[K_4(g_2) \in D \mid K_1(g_2), K_2(g_2), K_3(g_2) \in D\Big]$$

and so on, since each of these probabilities only depend on p, $F(g_1) = F(g_2)$, and on $T(g_1) = T(g_2) \setminus \{i, 1, 2\}$. For example, $p\Big[K_3(g_1) \in D \mid K_1(g_1), K_2(g_1) \in D\Big]$ depends on p, $F(g_1)$, and on the possible triangles among 3, 1 and 2, but not on whether $g_{12} = 1$ or $g_{12} = 0$ (see Claim 1 in the Appendix). Hence, $E_{U_i(g_1)}\big(0, \sigma, I_i(g_1)\big) - E_{U_i(g_2)}\big(0, \sigma, I_i(g_2)\big)$ corresponds to the difference between (2.5.11) and (2.5.10), which is positive by Lemma 2 (in the Appendix). As a result, $\sigma_2(k) \geq \sigma_1(k)$.

b) The above shows that when the only difference in $i's$ beliefs in g_1 and in g_2 is that in g_2 the triangle $\{i, 1, 2\}$ may exist while in g_1 it cannot, $E_{U_i(g_1)}\big(0, \sigma, I_i(g_1)\big) > E_{U_i(g_2)}\big(0, \sigma, I_i(g_2)\big)$. Observe that i has the same beliefs about other network aspects in g_1 and in g_2, including the other triangles that she might form with her neighbors, $T(g_1) = T(g_2) \setminus \{i, 1, 2\}$.

In our setup, integrants of g_1 and g_2 only know the maximal number of triangles that they might form with their neighbors, $\big|S_i^3(g_1)\big| \leq \tau$ and $\big|S_i^3(g_2)\big| \leq \tau + 1$, respectively, but they know neither $T(g_1)$ nor $T(g_2)$. However, the result in (a) holds regardless on the identity of the agents in triangle Z_3, $T(g_2) = T(g_1) \cup Z_3$. As a result, $E_{U_i(g_1)}\big(0, \sigma, I_i(g_1)\big) > E_{U_i(g_2)}\big(0, \sigma, I_i(g_2)\big)$, and $\sigma_2(k) \geq \sigma_1(k)$.

B. Strategic complements. As for strategic complements,

$$E_{U_i(g_x)}\big(1, \sigma, I_i(g_x)\big) = -c + 1 - \Bigg[p\Big[K_1(g_x)K_2(g_x) \in D\Big] * p\Big[K_3(g_x) \in D \mid K_1(g_x), K_2(g_x) \in D\Big] *$$
$$p\Big[K_4(g_x) \in D \mid K_1(g_x), K_2(g_x), K_3(g_x) \in D\Big] * ... * p\Big[K_{k_i(g_x)}(g_x) \in D \mid K_1(g_x), K_2(g_x), ..., K_{k_i(g_x)-1}(g_x) \in D\Big]\Bigg] \tag{2.5.12}$$

Applying the same reasoning as for strategic substitutes, it is readily seen that $E_{U_i(g_1)}\big(1, \sigma, I_i(g_2)\big) > E_{U_i(g_2)}\big(0, \sigma, I_i(g_2)\big)$, and consequently $\sigma_2(k) \leq \sigma_1(k)$. ∎

Capítulo 3

Network Perception in Network Games

3.1. Introduction

It is widely documented that peers exert a great influence on human behavior (Goyal, 2012; Jackson and Zenou, 2015). Research on network games has particularly modeled the interaction of people when their choices are influenced by those of their network neighbors in a variety of contexts (Bala and Goyal, 1998; Jackson and Yariv, 2007; Galeotti et al. 2010; Bramoullé et al. 2014; Bourlès et al. 2017).

Regarding the information that players' have about the underlying network architecture, network games have primarily been analysed through two approaches. One approach assumes that players have complete information about the network they are embedded in (e.g. Goyal and Moraga-González, 2001; Ballester et al. 2007; Bramoullé and Karton, 2007). This approach presents a fundamental drawback: under complete information, a wide range of equilibrium outcomes are possible, what makes difficult to draw general conclusions on the incidence that each specific network feature has on behavior.[1] A second approach assumes that players have incomplete information about the network they are part of and they take decisions without knowing with whom they are going to interact. Because of their usefulness to solve the equilibrium selection problem, different papers take this approach (Jackson and Yariv, 2007; Galeotti et al. 2010; Lamberson, 2015). Among these papers, the most closely related to ours is Galeotti et al. (2010), who consider a setup where each player has private information about her degree and the probability degree distribution of the network is common knowledge. Players' beliefs about the network are fully identified with the degree distribution, as they are not able to learn finer aspects about the network from the information they have. They show that, under such an information setup, every symmetric equilibrium is monotone non-decreasing (non-increasing) in players' degrees under strategic complements (substitutes) when certain assumptions on the network assortativity patterns hold: nodes have degrees either with independent probabilities or with probabilities that are positively (negatively) correlated. Such a monotonicity property of equilibria implies that social connections create personal advantages, since the expected payoffs of players choosing higher (lower) actions are greater than those of players choosing lower (higher) ones under strategic complements (substitutes).[2]

The information setup of Galeotti et al. (2010) applies for many real-life situations. A person may decide to learn a language, to start a business, to get a vaccination, etc. on the sole basis of her expected volume of future interactions, without necessarily knowing the identity of their future contacts. In such a situation, the unique network feature that influences her decision may be the *number* of individuals she expects to interact with from a relatively unbounded population (e.g. a country). A natural way of modeling this type of situations (where the network influencing people's behavior is typically very large) is to identify players'

[1] For instance, Bramoullé and Kranton (2007) show that people who contribute to the provision of a public good in equilibrium compose a maximal independent set (namely, their are not linked). However, how do different features of people's networks affect their equilibrium behavior? Do people with greater degree (or clustering, or betweenness centrality, etc.) contribute systematically more (or less) to the provision of the good in equilibrium?.

[2] The idea that social connections create personal advantages is fundamental in social capital theory (Granovetter, 1994; Burt, 1994) and it is backed by a vast number of empirical studies. See for instance Montgomery (1991) and Beaman (2012) for the effects of networks on employment, Ahuja (2000) and Gulati (1995) for the role of links to generate competitive advantages in markets or Baum et al. (2000), for the impact of connections on firm performance.

beliefs about the network with a probability degree distribution, setting aside beliefs about other network aspects. This assumption is reasonable in these particular environments, in which the following conditions apply: (i) people have a good sense of their volume of future interactions and (ii) they have neither information about the identity of their contacts not about any other aspects of the network topology.[3]

Notwithstanding this, in most real-life situations people indeed have incomplete−typically local−network information but they know with whom they interact, how popular their opponents might be, whether they know each other, etc. (Killduf et al. 2008). Such more detailed information in turn allows network members to estimate further properties of their local and global networks. Indeed, research in social psychology documents that people entering into a social group tend to form a cognitive map of the existing network−a mental picture of connections capturing who is connected with whom in the group (Hecker, 1993; Kilduff and Tsai, 2003). For example, a newcomer to a firm may have a certain idea about other people in the company: she may notice who is popular, who shares office with whom, who eats with whom, who holds which position in the company, etc. All this information provides signals enabling to form mental representations about the firm network. Such mental representations then influence one's behavior. Since in contexts where such a mental representations of networks emerge−such as a firm, a class, or a faculty department[4]−people's beliefs about the network are not usually restricted to one's degree and a probability degree distribution, this issue raises novel research questions:

- From the perspective of network perception, we ask: How can we model players' beliefs about the network in this kind of contexts? What can people deduce about the underlying network from such incomplete local information? Which network features affect how people view their social networks?
- From the game-theoretical perspective, are the unicity and monotonicity results of Galeotti et al. (2010) robust to relaxing their information assumptions? If not, how does deeper network information shape players' behavior and payoffs and the aggregate welfare in network games? Can we link the depth of network information to the multiplicity of equilibria?

This chapter aims to answer these questions. We present different information settings which differ in the depth of information that agents have about the network they are embedded in. The information settings range from an extreme case of incomplete network information (a setting similar to that in Galeotti et al. 2010) to the complete information setup (considered for instance in Ballester et al. 2006 or in Bramoullè and Kranton, 2007). We analyse how, and to what extent, each agent can infer different aspects of the interaction structure in function of the information that she possesses even if she is not directly informed about these aspects. To that aim, we illustrate how exploring the role of information requires to distinguish two types of information: information about links (related to connectivity) and information about nodes (relevant to learn who is connected with whom). Building up on this, we develop a theoretical framework linking incomplete network information to agents' beliefs about the properties of the underlying network architecture and explore how such beliefs shape the behavior of players in games played on networks.

We first show that even a minimal knowledge of the social environment enables people to learn both the network geometries that are compatible with their information as well as their probability distribution, which in turn allows them to estimate any network feature (e.g. the assortativity patterns, the expected clustering coefficient, the expected number of components, their variances, etc.). We find that, in contexts of limited network information (such as those in Galeotti et al. (2010)), all agents have identical perceptions of the underlying network architecture: they have identical beliefs about the feasible network geometries and their probabilities. Nevertheless, this symmetry in beliefs disappears once we depart from such a limited information setup, as agents may have different beliefs about set of feasible geometries and/or about its probability distribution. Furthermore, we show that individuals with complete and incomplete network knowledge can coexist under any information setup−even when their level of network information is limited and equal across agents−and this may have important consequences on behavior.

We particularly relate individuals' beliefs about the network with notions of equivalence among nodes, such as *structurally equivalence* and *automorphic equivalence* (Wasserman and Faust, 1994; Easley and Kleinberg, 2010). These measures are canonical in social network analysis (Lorrain and White, 1971; Everet, 1985;

[3]Think for instance about a wholeseller: from survey data, she may get information about the shopping habits of her potential customers (e.g. the links that they may have to other firms), but not about other features of the network they compose. Such an application is considered in Nermuth et al (2009).

[4]This usually occurs when the network is not very large. As pointed by Killduf et al. (2008), even a network integrated by 20 people requires an agent to monitor hundreds of possible links, what may constitute a cognitive challenge.

Hanneman and Riddle, 2003; Newman, 2004, Leicht, et al. 2006; Casse et al. 2013; Jin et al. 2014; Prota and Doreian, 2016; Audenaert, et al. 2019) and identify nodes that occupy identical positions in the network.[5] Based on these notions, we show how people exhibit a cognitive biases towards asymmetric networks structures in all these information setups, giving more probabilistic weight to network architectures with an automorphism group of a lower order. This implies, for example, that a recently hired employee believes more likely that most of their colleagues occupy heterogeneous positions in the network firm rather than equivalent ones.

In the second part, we explore the effects that network perception of agents have on Bayesian games of strategic substitutes and strategic complements played on networks. We find that the equilibrium structure in Galeotti et al. (2010) breaks even when players' network information is limited to their own degree and the degree distribution: symmetric equilibria are not necessarily monotone, but they can exhibit different patterns in the absence of further assumptions on the degree assortativity patterns.[6] Besides, although the set of symmetric equilibria is the same across all networks with the same degree distribution, the welfare that these equilibria yield may be different depending on their respective geometries. To this respect, we provide a sufficient condition for a network to be *efficient*: a network is efficient if there is not any other network with the same degree distribution in which equilibrium welfare is higher.[7] Such a condition establishes a positive link between network efficiency and the degree of symmetry of the network, suggesting that under certain conditions the only networks that are efficient are the more asymmetric ones. Last, we illustrate how a subtle variation in the network information of players (the knowledge of their neighbors' *neighborhoods* vs. the knowledge of their neighbors' *degrees*) can modify their equilibrium behavior. The reason is that players' observation of certain network features−such as the three- or four-cycles that they form with their neighbors−depends on which information they possess (neighborhood or degree). Such an observation shapes players' beliefs about the network, which in turn determine their behavior. As a consequence, the equilibrium structure may change dramatically even if the network information varies only slightly, but the direction in which it changes is network-specific.

The first contribution of this chapter is the development of a theoretical framework for how rational agents form mental representations of the networks they are embedded in from network information. The information assumptions in many papers on network games (such as Galeotti et al. 2010) imply that players' beliefs about the network confine to one particular network aspect: connectivity. In contrast, full network information does not require developing a theory regarding how people view the networks. However, in real-life people typically possess limited information about the interaction patterns. Our framework thus provides a first step toward modeling more real-life strategic and non-strategic network interactions under incomplete information about the underlying network. The proposed framework complements the large literature on network cognition in social psychology and sociology (Krackhardt; 1987, Carley; 1986; Michaelson and Contractor, 1992; Freeman, 1992; Kumbasar et al. 1994; Casciaro, 1998; Johnson and Orbach; 2002; Janicik and Larrick, 2005)[8] and recently in economics (Dessi et al. 2016), by characterizing formally the formation of people's cognitive network maps. In this sense, we uncover a relation between some canonical notions of symmetry among nodes and network perception. Although such notions have been fundamental in structural theory, they have not been theoretically related, to the best of our knowledge, to network perception.[9] The uncovered relation between these notions and agents' network perception reflects a cognitive biases towards asymmetric structures that may imply a misperception of agents' social environments, since empirical evidence shows that a certain degree of symmetry is ubiquitous in real-life social networks (MacArthur et al. 2008; Ball and Geyer-Schulz, 2018a, 2018b).[10]

[5]These concepts were initially studied in sociology (Boorman and White, 1976; White et al. 1976; Sailer, 1978; Doreian, 1988; Winship, 1988; Burt, 1976, 1990; Borgatti and Everett, 1992; Doreian et al. 2005) to explain the role that a person plays within a society on the basis on how she is connected to others according to the network topology (e.g. the role of a student, of a politician, of a boss, of a employee, etc.). Given their applicability to other settings, their study has extended to more general domains (see e.g. Rossi et al. 2014 or Vega et al. 2016).

[6]The key insight is that, under our framework, players' beliefs about other players' degrees may not adjust to the assumptions in Galeotti et al. (2010). For example, a degree-k player may expect to have a particular type of neighbor (say, a neighbor of degree $\bar{k}$) with a greater probability than a degree-k' one, even if $k' \geq k$. If this happens, the assumption of degree independence in Galeotti et al. (2010) fails to hold, and the degree-k player has an advantage over the degree-k' one (a greater probability of having a particular type of neighbor). As a consequence, equilibria are not necessarily monotone.

[7]The condition focuses on the symmetric equilibria in pure-strategies.

[8]For a review, see Brands (2003).

[9]Some empirical papers study how perceived similarity relates to these notions of equivalence among nodes (e.g. Michaelson and Contractor, 1992), yet, the context of this paper is different to ours.

[10]Wang et al. (2009) also find a certain degree of symmetry in their analysis of the world trade network. On the contrary,

Second, we contribute to the literature on network games in two ways. On the one hand, we provide a first step toward bridging the two extreme assumptions regarding network knowledge: extremely limited information (Galeotti, et al. 2010; Jackson and Yariv, 2005; 2007; Sundararajan, 2008) and complete information (Goyal and Moraga-González, 2001; Ballester et al. 2006; Bramoullé and Karton, 2007). We characterize the intermediate information setups and exploit their role in network games. Most importantly, we show that incomplete information is not panacea to solve the equilibrium selection problem. In fact, the introduction of incomplete information as a way of solving the problem of equilibrium multiplicity has faced a major critique: the equilibrium achieved depends on the way incomplete information is introduced (Weinstein and Yildiz, 2007). While this critique applies generally to all incomplete information games, we show that it is particularly relevant for those played on networks, given (i) the wide range of network aspects that may shape players' behavior and welfare and (ii) the variety of network characteristics that players can learn from the knowledge of particular network aspects, given the intrinsic interdependency among different network features. This explains why, as we illustrate in this chapter, manipulating information of players has largely non-monotonic effects on the structure and number of equilibria. In a similar vein, we show that a subtle variation of the information setup in Galeotti et al. (2010) may break the equilibrium symmetry of games of strategic substitutes and strategic complements: while for strategic complements the equilibrium predictions of Galeotti et al. (2010) tend to maintain, non-systematic predictions can be done under strategic substitutes.

Last, while most network applications in economics focus on connectivity, centrality, and network density (see Jackson, et al. 2017), we point to a potential role of one particular feature of network symmetry−the network automorphism group–in agents' perception, behavior, and welfare. In fact, this network property is receiving increasing attention in mathematics and physics (MacArthur and Anderson, 2006; Xiao, et al. 2008a, 2008b, 2008c; Wang et a. 2009; Dehmer et al. 2020), given its impact on the dynamics of processes that take place on networks (Golubitsky and Stewart, 2003), on the network's eigenvalue spectrum (Cvetkovíc et al. 1979) or because of its utility to simplify the network topology by collapsing redundant information (Xiao et al 2008b), among other applications.[11] This chapter points out the importance of this network property on behavior and welfare, suggesting that behavior of people might be shaped in a greater extent by the network characteristics associated to asymmetric structures in incomplete information contexts as a consequence of the greater probabilistic weight that players assign to these network architectures.

The chapter is organized as follows. Section 3.2 presents some background definitions. Section 3.3 presents different setups of network information. Section 3.4 provides the results on network perception, which are the bases of our theoretical framework. Results on network games are presented in Section 3.5. Section 3.6 concludes.

3.2. Background Definitions

Let $g = (N, E)$ be a social network characterized by a set of nodes $N = \{1, .., n\}$ and a set of edges or links E between them. Each node in g represents one agent and there are $n = \left|N\right|$ agents in the network. Let g_{ij} denote the link between $i \in N$ and $j \in N$; $g_{ij} = 1$ if individuals $i \in N$ and $j \in N$ are directly linked in g and $g_{ij} = 0$ otherwise, with $g_{ij} = g_{ji}, \forall i, j \in N$. The network is represented by a $n \times n$ symmetric adjacency matrix $A = (g_{ij})_{i,j \in N}$, with $g_{ii} = 0$. Equivalently, we sometimes denote a link between i and j by ij.

We distinguish two network characteristics:

- The *neighborhood* of node i is the set of agents directly connected to i, $N_i(g) = \{j \in N : g_{ij} = 1\}$.
- The *degree* of node i is the cardinality of $N_i(g)$, $k_i(g) = |N_i(g)|$. It is the number of neighbors of i.

Although both characteristics are similar, the level of network information that they capture is different: the degree reflects with how many other agents one interacts without providing any information about their identities, while the neighborhood reflects both their number and their identities. Considering one or another feature has important consequences in certain parts of our analysis.

The set of i's second-order neighbors is $N_i^2(g) = \{s \in N : g_{ij}g_{js} = 1 \text{ for some } j \in N,\ i \neq s\}$.

almost all random graphs are asymmetric (Erdös-Renyi, 1963).

[11] See also Soicher, (2004) and Kocay, (2007), for its application to simplify the computational complexity of network algorithms.

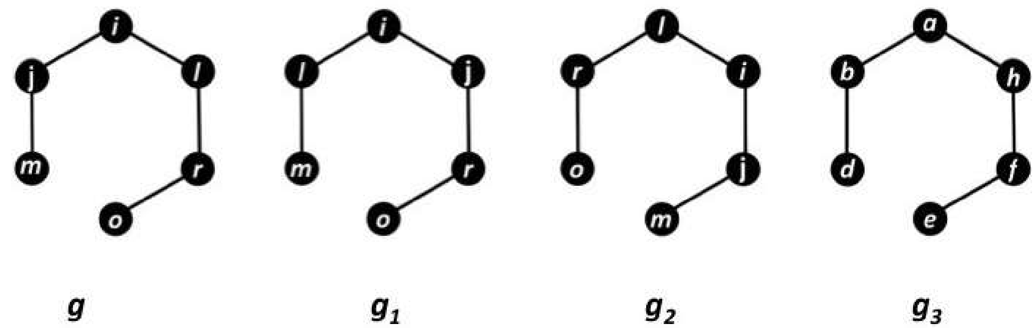
The degree distribution of network g, denoted $F_g(k)$, specifies, for all $k \in \{0, 1, ..., n-1\}$, the fraction of nodes that have degree k in this network:[12]

$$F_g(k) = \frac{1}{n} \left|\{i \in N : k_i(g) = k\}\right|$$

The degree counts in network g, denoted $D_g(k) = n\, F_g(k)$, are the numbers of nodes that have degree k in this network. The degree sequence of a nodes is the sequence of the node degrees; two networks g and g' have the same degree sequence if $D_g(k) = D_{g'}(k)$, $\forall k$.

The geometry of a network correspond to its structure: the network architecture created by its edges. Two networks have the same geometry if and only if they are *isomorphic:* there exists a bijection (an isomorphism) $f : N \to N'$, such that $ij \in E$ if and only if $f(i)f(j) \in E'$ (see Borgatti and Everett, 1992). Thus, f just relabels the nodes, but their network structure is the same. For example, the four networks in Figure 3.1 are isomorphic. We use the symbol $\cong$ to denote an isomorphism; $g \cong g'$ means that g and g' are isomorphic.

Figure 3.1



Network $g = (N, E)$ is different from network $g' = (N', E')$ if and only if their respective adjacency matrices differ, i.e. if $g_{ij} \neq g'_{ij}$, for at least one $ij \in E \cup E'$. The adjacency matrix of a network depends on two aspects: (i) the network geometry and (ii) the distribution of labels among the nodes (how agents are distributed within the network). Hence, networks g and g' can be different if either (i) or (ii) (or both) is different in the two networks. For example, network g and g_1 in Figure 3.1 are isomorphic. However, since agents are distributed differently in g and in g_1 both networks are distinct, as reflected in their respective adjacency matrices:

$$g = \begin{pmatrix} g_{ii} & g_{ij} & g_{il} & g_{im} & g_{io} & g_{ir} \\ g_{ji} & g_{jj} & g_{jl} & g_{jm} & g_{jo} & g_{jr} \\ g_{li} & g_{lj} & g_{ll} & g_{lm} & g_{lo} & g_{lr} \\ g_{mi} & g_{mj} & g_{ml} & g_{mm} & g_{mo} & g_{mr} \\ g_{oi} & g_{oj} & g_{ol} & g_{om} & g_{oo} & g_{or} \\ g_{ri} & g_{rj} & g_{rl} & g_{rm} & g_{ro} & g_{rr} \end{pmatrix} = \begin{pmatrix} 0 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 \end{pmatrix} \neq g_1 = \begin{pmatrix} 0 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 \end{pmatrix}$$

Similarly, g_3 is different from the other three networks in Figure 3.1, since it is integrated by different agents. In our analysis, the only isomorphic networks to g that will play a role are those integrated by the agents in N.

An isomorphism f of a graph with itself that preserves the adjacency matrix is known as an *automorphism*: $f : N \to N$, where $ij \in E$ if and only if $f(i)f(j) \in E$. That is, an automorphism is a permutation of the labels of the nodes in g that results in a network $g' = g$. For example, $f : f(i) = l$, $f(j) = r$, $f(l) = i$, $f(m) = o$, $f(r) = j$, $f(o) = m$ is an automorphism of g, as it results in network $g_2 = g$ (see Figure 3.1). The set composed of all the automorphisms of g is the *automorphism group* of g, denoted $Aut(g)$ (Chartrand et al. 2010). Note that all graphs in an automorphism group represent the same network, since they all have the same adjacency matrix. The order of the automorphism group of g is the number of elements in $Aut(g)$, $|Aut(g)| \geq 1$. It captures the degree of symmetry of the network: the greater $|Aut(g)|$, the more symmetric network g is (Xiao et al., 2008b).

[12]In contexts of random networks, $F_g(k)$ is naturally interpreted as the probability that a randomly selected node has degree k (Vega-Redondo, 2007), while here it is the distribution of degree frequencies in the network.

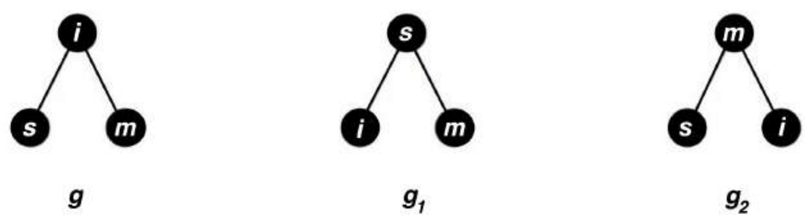
Two nodes $i \in N$ and $j \in N$ are *automorphically equivalent* if they are identical in terms of all network measures (degree, second-order degree, centrality, number of cycles to which they belong, etc.). Namely, i and j are automorphically equivalent if and only if there exists an automorphism $f : N \to N$ such that $f(i) = l$. We write $i \equiv l$ to indicate that i and l are automorphically equivalent. In network g of Figure 3.1, $i \equiv l$, $j \equiv r$ and $m \equiv o$. Other pairs of agents are not automorphically equivalent, since their number of second-order neighbors is different. Two nodes occupy the same position in the network if and only if they are automorphically equivalent, regardless on whether the identity of their direct and indirect neighbors is distinct.

An important property of automorphically equivalent nodes is that we can exchange their labels to form a new network that is identical to the original one (Friedkin and Johnsen, 1997). For example, starting from graph g in Figure 3.1, we can exchange the labels of i and l and relabel all other nodes to obtain network g_2 (in the same Figure) with the same adjacency matrix as g, $g_2 = g$. Notice that this is possible because $i \equiv l$. If we swap the labels of two nodes that are not automorphically equivalent, say, i and j in network g, we cannot obtain a network equal to g.

The *orbit* of node i is the set composed of all nodes that occupy the same position as her, $O_i(g) = \{l \in N : l \equiv i\}$. In graph g of Figure 3.1, $O_i(g) = O_l(g) = \{i, l\}$, $O_j(g) = O_r(g) = \{j, r\}$, $O_m(g) = O_o(g) = \{m, o\}$.

Structural equivalence is a particular form of automorphic equivalence. Node $i \in N$ is structurally equivalent to $l \in N$ if and only if both agents are connected to the same nodes, $N_i(g) \setminus \{l\} = N_l(g) \setminus \{i\}$.[13] We write $i \equiv_s l$ to indicate that i and l are structurally equivalent. Structural equivalence is more demanding than automorphic equivalence: it not only requires that the nodes occupy indistinguishable structural locations in the network, but also that the identities of the agents connected to them are the same. Thus, structural equivalent nodes must be automorphically equivalent, but the opposite is not true. For example, in network g in Figure 3.2, nodes $s \equiv_s m$ are structurally equivalent and therefore $s \equiv m$. However, there is no pair of structurally equivalent nodes in network g of Figure 3.1, despite the fact that $i \equiv l$, $j \equiv r$ and $m \equiv o$. The set of nodes that are structurally equivalent to i is denoted $S_i(g) = \{j \in N : j \equiv_s i, j \neq i\}$.

Figure 3.2



In certain parts of our analysis, we need to identify a set of nodes with the same degree as node i but a distinct network position:

DEFINITION 1. *An equal-degree set to i (i-ed set hereafter) is a subset of nodes $K_i(g) \subseteq N$ such that if $j \in K_i(g)$ and $l \in K_i(g)$ and $j \neq l$, then (i) $k_j(g) = k_l(g) = k_i(g)$ and (ii) $j \not\equiv l$, $\forall j, l \in N$. $K_i(g)$ is maximal if and only if it is not a proper subset of any other i-ed set in g.*

In network g of Figure 3.1, the maximal i-ed sets in N are $\{i, j\}$, $\{i, r\}$, $\{l, j\}$ and $\{l, r\}$. Notice that an i-ed set only includes nodes with a different network position; since $i \equiv l$ and $j \equiv r$ in g, there is not any i-ed set $K_i(g)$ such that $i, l \in K_i(g)$ and/or $j, r \in K_i(g)$.

Let G be the set of all feasible networks integrated by the agents in N. Set $G_{F_g} \subseteq G$ is the subset of different networks in G with degree distribution $F_g(k)$ and size n:

$$G_{F_g} = \{g \in G : |\{i \in N : k_i(g) = k\}| = D_g(k), \forall k\}$$

[13] According to the standard definition, i an l are structurally equivalent if and only if $N_i(g) = N_l(g)$ (Burt, 1976). Since this definition is too strict, different relaxations have been proposed (see e.g. Everett et al. 1990). Our relaxed definition increases the set of structurally equivalent nodes; while the three nodes $\{i, j, k\}$ composing a network triangle are not structurally equivalent according to the standard definition, they are structurally equivalent according to ours. Observe that i and l with $N_i(g) = \{l\}$ and $N_l(g) = \{i\}$ are structurally equivalent, since $N_i(g) \setminus \{l\} = N_l(g) \setminus \{i\} = \emptyset$. However, neither i nor l are structurally equivalent to an isolated node m, since $N_i(g) \setminus \{m\} = \{l\} \neq N_m(g) \setminus \{i\} = \emptyset$, and $N_l(g) \setminus \{m\} = \{i\} \neq N_m(g) \setminus \{l\} = \emptyset$.

Suppose for example that $N = \{i, s, m\}$, $D_g(1) = 2$ and $D_g(2) = 1$. Then, G_{F_g} is integrated by three networks in Figure 3.2, $G_{F_g}(g) = \{g, g_1, g_2\}$. In this example, all networks in G_{F_g} are isomorphic. However, G_{F_g} can generally contain networks with different geometries.

Last, we define network cycles. A Q-cycle, denoted $Z^Q(g)$ is a sequence of distinct nodes $i_1, i_2, ..., i_{Q-1}, i_Q$ such that $Q > 2$, $g_{i_q,i_{q+1}} = 1$ for each $q \in (1, ..., Q-1)$ and $g_{i_1 i_Q} = 1$. For example, i, j, k is a three cycle if $g_{ij} = g_{jk} = g_{ki} = 1$; i, j, k, l is a four cycle if $g_{ij} = g_{jl} = g_{lk} = g_{ki} = 1$, and so on.

3.3. Network knowledge

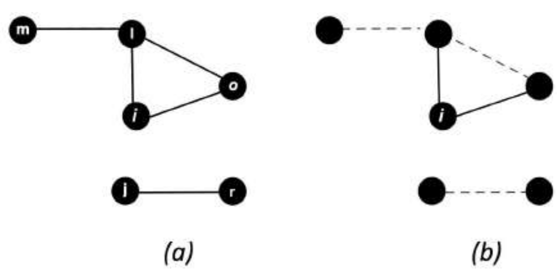
This section introduces several settings that differ in the information that people possess about the network they are embedded in (network g). In each setting, we use $I_i(g)$ to denote the information set that each $i \in N$ has about network g. Agents have private information which is different depending on the setting, while $F_g(k)$ and n are common knowledge in all our setups.

3.3.1. Setting A

In this setting, $I_i(g) = \{k_i, F_g(k), n\}$.[14] That is, people know their degree, the degree distribution and the network size. However, although people know their proclivity to interact with others, they do not know the identity of the people with whom they are going to interact. This setting is similar to that in Galeotti, et al. (2010).[15] Such a setting applies, for example, to a recently hired employee: she may know the number of people with whom she is going to interact in the company and their popularity, but not necessarily who these people will be at the moment she is hired. Similarly, an actor considering to take part of a movie may anticipate the status (popularity) of other actors involved in the project, but not the identity of these agents when accepting the role.

Consider agent i in Figure 3.3, where network (a) displays the whole network architecture whereas network (b) represents how i views the network under Setting A. The dashed lines in network (b) represent the links in g that are not observed by i, while the solid lines represent the links that are observed by her. Note that all nodes but i are unlabelled in graph (b) because i does not have information about the identity of these agents.

Figure 3.3 Network g and $I_i(g)$ under Setting A



3.3.2. Setting B

Under Setting B, the private information of each $i \in N$ corresponds to her neighborhood and the degree of each of her neighbors. Let $\mathtt{k}_{N_i(g)}=(k_1, k_2, ..., k_{k_i})$ be the vector of degrees of all agents in $N_i(g)$, where k_j is the degree of neighbor $j \in N_i(g)$ $(j = 1, 2, ..., k_i)$. Elements in this vector are indexed according to a decreasing order, $k_j \geq k_{j+1}$. Under setting B, $I_i(g) = \left\{N_i(g), \mathtt{k}_{N_i(g)}, F_g(k), n\right\}$.

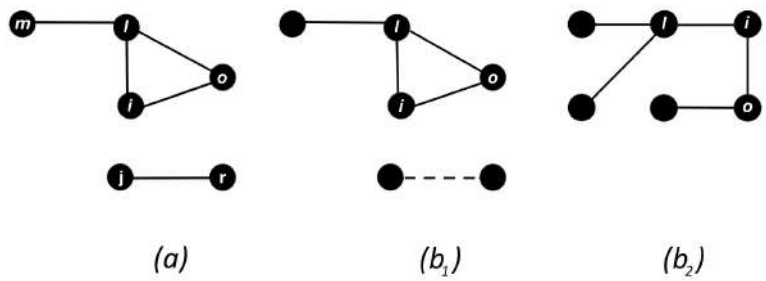
Imagine network g is the network in Figure 3.4(a). From $I_i(g) = \{N_i(g), (k_l, k_o), [F_g(1), F_g(2), F_g(3)], 6\} = \left\{\{l, o\}, (3, 2), [\frac{1}{2}, \frac{1}{3}, \frac{1}{6}], 6\right\}$, agent i knows that her neighbors are l and o, and $k_l = 3$ and $k_o = 2$. Nevertheless, i does not know *who* are the agents in $N_l(g)$ and in $N_o(g)$. Hence, she does not know the geometry created by

[14] Equivalently, we could assume that the information set of each i is $I_i(g) = \{k_i, D_g(k), \forall k\}$.

[15] The difference between our Setting A and the setup in Galeotti et al. (2010) is that players know the distribution of degree frequencies in our setup, while in Galeotti et al. (2010) they know the probability degree distribution. Our assumption makes sense for bounded populations of small size (e.g. a firm), while theirs does for large ones (e.g. a country).

her neighbors' links. Figure 3.4(b_1) and 3. 4(b_2) represents the two possibilities in which links of $i's$ neighbors can be disposed given $I_i(g)$.[16] Agent i does not know which of these two configurations is the true one, hence, $I_i(g)$ is jointly represented by Figures 3.4(b_1) and 3.4(b_2).

Figure 3.4. Network g and $I_i(g)$ under Setting B



In certain situations, an individual may have information about the popularity of her contacts without necessarily knowing the identity of their respective neighbors. Think for instance about a faculty department. In such a social environment, an agent may receive (directly or through three parties) information about her colleagues: she may know how they usually spend the weekend, whether they are married or not, the number of followers they have in their virtual networks, etc. These "signals" inform the agent about the relative popularity of her coworkers, but not necessarily about the identity of the people composing their neighborhoods.

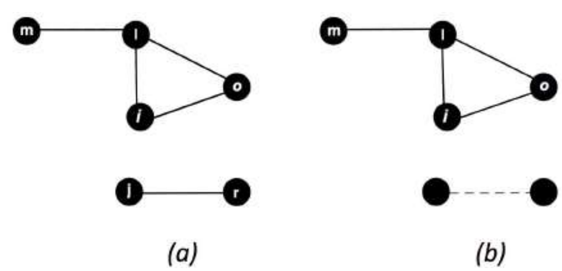
3.3.3. Setting C

Under Setting C, the private information of each $i \in N$ corresponds to her neighborhood and the neighborhood of each of her neighbors. Define $\mathtt{N}_{N_i(g)} = (N_j(g), ..., N_m(g))$ as the vector integrated by the neighborhoods of $i's$ neighbors, $N_i(g) = \{j, ..., m\}$. Under Setting C, $I_i(g) = \big\{N_i(g), \mathtt{N}_{N_i(g)}, F_g(k), n\big\}$.

The difference between Setting C and Setting B is that under Setting C each i knows the identity of her second-order neighbors, while she does not under Setting B. This informs her about the three- and four-cycles that she forms with her neighbors, what in turn provides her information about the degree of her second-order neighbors.

Consider for example network (a) in Figure 3.5, where $I_i(g) = \big\{\{l, o\}, \big\{\{i, m, o\}, \{i, l\}\big\}, [\frac{1}{2}, \frac{1}{3}, \frac{1}{6}], 6\big\}$. Since i knows $N_l(g) = \{i, m, o\}$ and $N_o(g) = \{i, l\}$, she knows that l and o are linked. Since neighbors l and o are simultaneously second-order neighbors of i, the degree of two second-order neighbors of i is known by her.[17]

Figure 3.5. Network g and $I_i(g)$ under Setting C



Setting C applies for many real-life circumstances. A close familiar of an agent may be able to identify the agents who comprise the circle of friends of her familiar. Similarly, a researcher may able to identify who are her coauthors' coauthors, even when she never met these people.

[16] In Figures 3.4(b_1) and 3.4(b_2), nodes different from i, j and l are unlabeled because i does not have information about their identities. The dashed line in Figure 3.4(b_1) represents the link that is not directly observed by i. In a subsequent section, we show that i can learn such a link from $I_i(g)$ (i.e. she can deduce that the degree-one agents integrate a dyad), despite not being provided such an information directly.

[17] In a similar way, i could obtain some information about the degree of her second-order neighbors by observing the four-cycles that she forms with her neighbors: if i observes that her second-order neighbor z is linked both to $j \in N_i(g)$ and to $k \in N_i(g)$, i knows that $k_z \geq 2$. Analogously, if z is a common neighbor of x neighbors of i, i knows that $k_z \geq x$.

3.3.4. Setting Z

Under Setting Z, all agents agents have complete network information. That is, $I_i(g) = \{g\}$. This is the information setting considered in the main bulk of literature on network games (see e.g. Goyal and Moraga-González, 2001; Ballester et al. 2006; Bramoullé and Karton, 2007, among others), and implies that all network integrants have identical network knowledge.

3.4. Network perception

3.4.1. Network beliefs

From $I_i(g)$, each $i \in N$ forms beliefs about the network she is embedded in. Define $B_i(g) \subseteq G_{F_g}$ as the set of networks that are compatible with $I_i(g)$ in each information setting, with $b_i(g) = |B_i(g)|$. Namely, $B_i(g)$ is the set of feasible networks in $i's$ beliefs; each network in $B_i(g)$ could be network g according to the information of i. Depending on the information setup, $B_i(g)$ is different:

- Under Setting A, $B_i(g) = \{g' \in G_{F_g} : |N_i(g')| = k_i(g)\}$
- Under Setting B, $B_i(g) = \{g' \in G_{F_g} : N_i(g') = N_i(g), \mathtt{k}_{\mathtt{N_i(g')}} = \mathtt{k}_{\mathtt{N_i(g)}}\}$
- Under Setting C, $B_i(g) = \{g' \in G_{F_g} : N_i(g') = N_i(g), \mathtt{N}_{\mathtt{N_i(g')}} = \mathtt{N}_{\mathtt{N_i(g)}}\}$

Set $B_i(g)$ may differ across agents under any information setup. Furthermore, it is possible the coexistence of individuals with complete and incomplete network knowledge. This can occur even within one information setup (where the level of network information of all network integrants is the same), since the content of the the information of different agents can be distinct. The following example illustrates.

Example 1. Consider network g in Figure 3.2. Given $F_g(1) = \frac{2}{3}$, $F_g(2) = \frac{1}{3}$ and $n = 3$, G_{F_g} is integrated by the three networks in Figure 3.2, $G_{F_g} = \{g, g_1, g_2\}$. Under Setting A, $I_i(g) = \{k_i(g), [F_g(1), F_g(2)], n\} = \{2, \left[\frac{2}{3}, \frac{1}{3}\right], 3\}$, thereby, $B_i(g) = \{g\}$. On the contrary, $B_s(g) = \{g, g_2\}$ and $B_m(g) = \{g, g_1\}$, since $I_s(g) = I_m(g) = \{1, \left[\frac{2}{3}, \frac{1}{3}\right], 3\}$ under this information setup. Hence, i knows the whole network, while the network knowledge of s and m is incomplete. ■

Since the private information of i does not include the probability distribution of the networks in $B_i(g)$, we assume that the networks in $B_i(g)$ follow an uniform distribution. This means that each network in $B_i(g)$ is equally likely to be network g, in $i's$ beliefs.

Feasible network geometries. So far we provide the example of a simple network (Example 1), where all networks in $B_i(g)$ have the same geometry, $\forall i \in N$. However, some networks in $B_i(g)$ may have different network geometries, while others may be different but isomorphic. We say that a particular network geometry is a *feasible geometry* if some graph in $B_i(g)$ have such a geometry. Let $\Omega_i(g) = \{1, 2, ..., \omega_i(g)\}$ be the set of feasible geometries in $i's$ beliefs, with $\omega_i(g) = |\Omega_i(g)|$. The set of (isomorphic) networks in $B_i(g)$ with geometry $z \in \Omega_i(g)$ is denoted $B_i^z(g)$, and $b_i^z(g) = |B_i^z(g)|$. Networks in $B_i^z(g)$ differ in how agents are allocated within the network, but they all have the same network geometry.

Each i can infer the probability that network g has a particular geometry z by counting the number of (isomorphic) networks in $B_i(g)$ with this geometry and dividing this number by the total number of feasible networks, $b_i(g)$. In other words, agent i believes that network g has geometry z with probability $\frac{b_i^z(g)}{b_i(g)}$. Agent i may assign more probability to some network geometries than to others, as we show in the following example.

Example 2. Suppose we are under Setting A, and $I_i(g){=}\{k_i(g), [F_g(1), F_g(2), F_g(3)], n\}{=}\{k_i(g), \left[\frac{1}{2}, \frac{1}{3}, \frac{1}{6}\right], 6\}$ From $I_i(g)$, a fully rational i can infer that there are three feasible network geometries, depicted in Figure 3.6. Depending on how agents are allocated in the network, there are different networks with each of these geometries. In particular, there exist $b_i(g) = 450$ different networks that are feasible in the beliefs of an i with $k_i(g) = 1$: $b_i^1(g) = 180$ networks have geometry 1, $b_i^2(g) = 90$ have geometry 2, and $b_i^3(g) = 180$ have geometry 3, as we show in the following sections.

Figure 3.6. Feasible geometries in Example 2

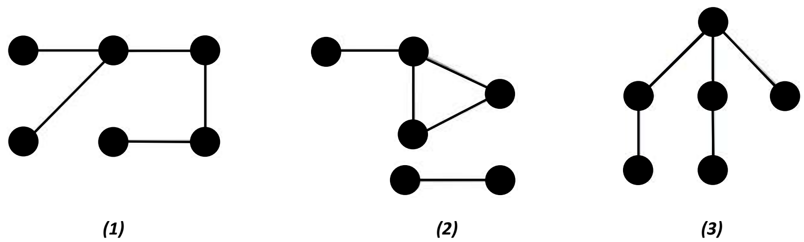


Table 3.1 provides the number of feasible networks for other values of $k_i(g)$. Observe that the value of $b_i^z(g)$ is relevant as it affects the *probabilistic weight* that i assigns to each network geometry. Thus, i believes that geometries 1 and 2 are more likely than geometry 3, since $\frac{b_i^1(g)}{b_i(g)} = \frac{b_i^3(g)}{b_i(g)} = \frac{180}{450} = 40\,\% > \frac{b_i^2(g)}{b_i(g)} = \frac{90}{450} = 20\,\%$.

Table 3.1. Network beliefs of each $i \in N$

$k_i(g)$	$b_i^1(g)$	$b_i^2(g)$	$b_i^3(g)$	$b_i(g)$
$k_i(g) = 1$	180	90	180	450
$k_i(g) = 2$	120	60	120	300
$k_i(g) = 3$	60	30	60	150
$\sum$	360	180	360	$\left\|G_{F_g}\right\| = 900$

■

Notice that in Example 2 all agents have identical beliefs about the feasible geometries regardless of their degree: they assign probability $\frac{180}{450} = \frac{120}{300} = \frac{60}{150} = 0{,}4$ to geometry 1, probability $\frac{90}{450} = \frac{60}{300} = \frac{30}{150} = 0{,}2$ to geometry 2, and probability 0,4 to geometry 3. As we show below, this not by chance. Rather, it is a general property of agents' beliefs under Setting A.

Under other information setups, on the contrary, network integrants may have different beliefs about the set of feasible geometries and their probabilities. Consider, for instance, network g in Figure 3.5(a). Under Setting B, $i \in N$ knows that she has a degree-two neighbor (agent o). Hence, the only geometries that are feasible given $I_i(g)$ are geometry 1 and geometry 2 in Figure 3.6.[18] Conditional on $I_l(g)$, on the contrary, the feasible geometries are geometries 2 and 3 in Figure 3.6, while conditional on $I_j(g)$ the only feasible one is geometry 2 in the same figure.

3.4.2. Isomorphisms of a graph

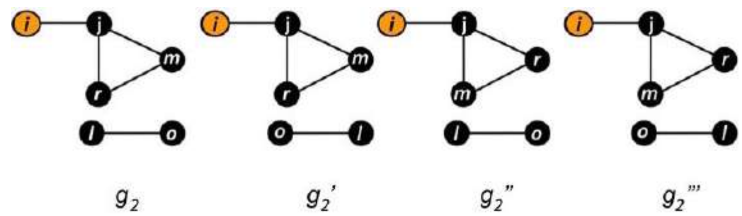
Let $\bar{N} \subseteq N$ be a subset of nodes in a network g, with $\bar{n} = \left|\bar{N}\right|$. We compute the number of distinct labelings of the nodes in $N \setminus \bar{N}$. In other words, we compute the number of distinct isomorphic networks to g that can be obtained by permuting *exclusively* the labels of the nodes in $N \setminus \bar{N}$. We denote this number $y(g \mid \bar{N})$.[19]

Labels of the nodes in $\bar{N}$ are not permuted, they are maintained fixed. Notice that in some cases we may permute the labels of some nodes in $N \setminus \bar{N}$ without any incidence in the adjacency matrix of the network: given g we may permute the labels of some nodes in $N \setminus \bar{N}$ and get a network $g' = g$. The set of different ways in which we can (exclusively) permute the labels of the nodes in $N \setminus \bar{N}$ without affecting the adjacency matrix of g is given by the *stabilizer of* $\bar{N}$, $stab(\bar{N})$. The stabilizer of a subset of nodes $\bar{N}$ is characterized as the set of all automorphisms that map each node in $\bar{N}$ into itself, $stab(\bar{N}) = \{f \in Aut(g) : f(v) = v, \forall v \in \bar{N}\}$ (Erwin and Harary, 2006). Consider for example network g_2 in Figure 3.7. Since $r \equiv_s m$ and $l \equiv_s o$, $Stab(i) = \{f, f', f'', f'''\}$, resulting in g_2, g_2', g_2'' and g_2''' in Figure 3.7, respectively.

[18] Observe that no degree-two agent has a degree-two neighbor in the third network of Figure 3.6. Hence, geometry 3 is not a feasible geometry given $I_i(g)$.

[19] For example, suppose network g is the network in Figure 3.2, and $\bar{N} = \{s\}$. Maintaining fixed the position of agent s in that network, there are two different networks that are isomorphic to g: network g and network g_2 in Figure 3.2. Hence, $y(g \mid \bar{N}) = y(g \mid \{s\}) = 2$.

Figure 3.7. Actions of the automorphisms in $Stab(i)$ in network g_2.



Lemma 1. *Let $g = (N, E)$. The total number of distinct isomorphic networks to g that can be obtained by exclusively permuting the labels of the nodes in $N \setminus \bar{N}$ is:*

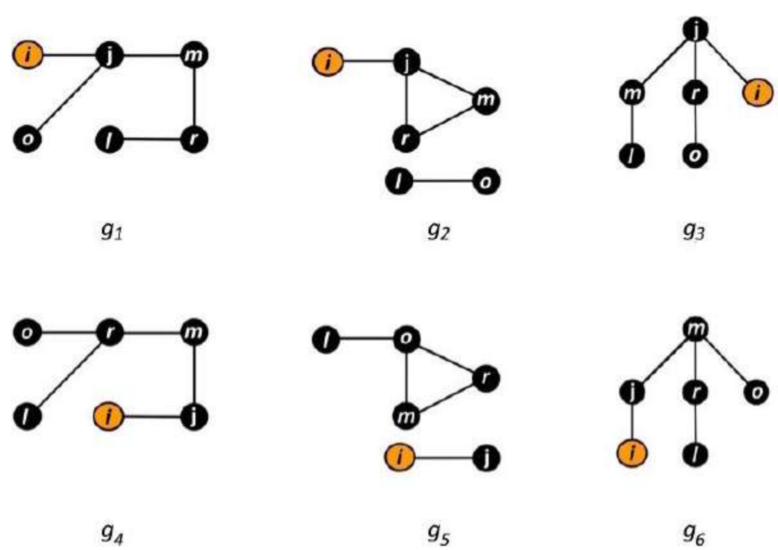
$$y(g \mid \bar{N}) = \frac{(n-\bar{n})!}{\left|stab(\bar{N})\right|}$$

Proof. There are $(n-\bar{n})!$ possible permutations of the labels of the nodes in $N \setminus \bar{N}$. For each of these $(n-n)!$ possible labelings, there are $\left|stab(\bar{N})\right|$ that are equal, as there are $\left|stab(\bar{N})\right|$ different ways in which we can permute the labels of the nodes in $N \setminus \bar{N}$ without any incidence in the adjacency matrix of the network. Thereby, $y(g \mid \bar{N}) = \frac{(n-\bar{n})!}{\left|stab(\bar{N})\right|}$. ■

Example 3. Suppose $\bar{N} = \{i\}$, $\bar{n} = 1$.

(a) Consider network g_1 in Figure 3.8. Since there is no pair of automorphically equivalent nodes in $N \setminus \bar{N}$, each permutation of the labels of the nodes in $N \setminus \bar{N}$ gives rise to a different network. Therefore, $Stab(i) = \{f\}$, where $f(i) = i$ for all i, and $|Stab(i)| = 1$. Hence, $y(g_1 \mid \{i\}) = \frac{(n-1)!}{1} = 120$. Similarly, $y(g_6 \mid \{i\}) = 120$.[20]

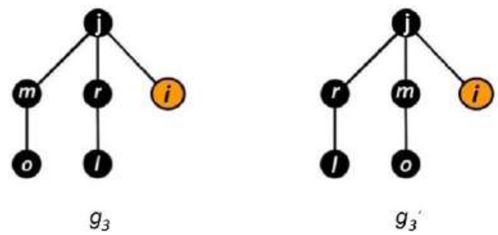
Figure 3.8. Networks from Example 3.



(b) Consider now network g_2 in Figure 3.8. Since $r \equiv_s m$ and $l \equiv_s o$, $Stab(i) = \{f, f', f'', f'''\}$, resulting in g_2, g_2', g_2'', and g_2''' in Figure 3.7, respectively. Since $|Stab(i)| = 4$, $y(g_2 \mid \{i\}) = \frac{(n-1)!}{4} = 30$.
(c) As for network g_3 in Figure 3.8, since $m \equiv r$ and $l \equiv o$, $|Stab(i)| = 2$, as shown in Figure 3.9. Therefore, $y(g_3 \mid \{i\}) = \frac{(n-1)!}{2} = 60$. Analogously, $y(g_4 \mid \{i\}) = y(g_5 \mid \{i\}) = 60$.

[20] Recall from Section 3.2 that automorphically equivalent nodes have the property that their labels can be interchanged to form a new network that is identical to the original one. Since $j \equiv r$ in g_6, we could interchange the labels of these two nodes and relabel *all other nodes* in the network to obtain a network that is identical to g_6 . Notwithstanding, there is no way to exchange the labels of any nodes different from i and obtain network g_6 maintaining fixed the label of i, since all nodes different from i are located at a different distance from her.

Figure 3.9. Actions of the automorphisms in $Stab(i)$ in network g_3



■

Example 3 calculates the number of distinct isomorphic networks that exist conditional on a given position of i making use of Lemma 1. For instance, there exist 120 different networks with the geometry of g_1 in Figure 3.8 in which i occupies her position in this network; all these networks differ in how agents different from i are allocated. Lemma 1 is necessary to compute $b_i^z(g)$ and $b_i(g)$, as we show in the next section.

3.4.3. Beliefs about the network geometry

In our incomplete information setups, no i is informed about all the details of her network position. Her information set only describes some aspects of her positioning in the network, but there may exist distinct positions compatible with such an information. We say that a position is *feasible* in $i's$ beliefs if it is consistent with $I_i(g)$: if it exists a positive probability that i occupies this position according to her beliefs.

Suppose for example that we are under Setting A, and $I_i(g) = \{k_i(g), [F_g(1), F_g(2), F_g(3)], n\}$ $= \left\{k_i(g), \left[\frac{1}{2}, \frac{1}{3}, \frac{1}{6}\right], 6\right\}$. All the feasible geometries given $I_i(g)$ are depicted in Figure 3.6. Agents i and l in network $g_1 \in B_i(g)$ of Figure 3.8 occupy a feasible position of i, and constitute a maximal i-ed set in g_1, $K_i(g_1) = \{i, l\}$. The same applies for i and l in network $g_2 \in B_i^2(g)$ in Figure 3.8, and for i and l in network $g_3 \in B_i^3(g)$ in the same figure.

The following proposition calculates $b_i^z(g)$ and $b_i(g)$ under Setting A. It takes into account that, if $g_z \in B_i^z(g)$ and $K_i(g_z)$ is a maximal i-ed set in g_z, then each agent in $K_i(g_z)$ occupies a feasible position of i under Setting A.

Proposition 1. *Let $g_z \in B_i^z(g)$. Under Setting A,*

$$b_i^z(g) = \frac{(n-1)! D_g(k_i)}{|Aut(g_z)|} \quad \textit{and} \quad b_i(g) = \sum_{z \in \Omega_i(g)} b_i^z(g)$$

Proof. Let $K_i(g_z) = \{j, l, ..., m\}$ be a maximal i-ed set in g_z. Each agent in $K_i(g_z)$ occupies a (distinct) feasible position of i. That is, there exist at least one network in $B_i(g)$ in which i occupies the position of $j \in K_i(g_z)$ in g_z.[21] Precisely, there are $y(g_z \mid \{j\})$ (isomorphic) networks in $B_i^z(g)$ in which i occupies the position of j in g_z; all these networks differ in how agents different from i are allocated. Similarly, there are $y(g_z \mid \{l\})$ (isomorphic) networks in $B_i^z(g)$ where i occupies the position of l in g_z, an analogously for other agents in $K_i(g_z)$. Hence, if $K_i(g_z) = \{j, l, ..., m\}$, $b_i^z(g) = y\big(g_z \mid \{j\}\big) + y\big(g_z \mid \{l\}\big) + ... + y\big(g_z \mid \{m\}\big)$.
By Lemma 1:

$$b_i^z(g) = y\big(g_z \mid \{j\}\big) + y\big(g_z \mid \{l\}\big) + ... + y\big(g_z \mid \{m\}\big) = \frac{(n-1)!}{\big|Stab(\{j\})\big|} + \frac{(n-1)!}{\big|Stab(\{l\})\big|} + ... + \frac{(n-1)!}{\big|Stab(\{m\})\big|}$$

Applying the Orbit-Stabilizer Theorem (in the Appendix A1) this is equal to:

$$\frac{(n-1)!}{\big|Aut(g_z)\big| / \big|O_j(g_z)\big|} + \frac{(n-1)!}{\big|Aut(g_z)\big| / \big|O_l(g_z)\big|} + ... + \frac{(n-1)!}{\big|Aut(g_z)\big| / \big|O_m(g_z)\big|}$$
$$= \frac{(n-1)!\Big[\big|O_j(g_z)\big| + \big|O_l(g_z)\big| + ... + \big|O_m(g_z)\big|\Big]}{\big|Aut(g_z)\big|} = \frac{(n-1)! D_g(k_i)}{\big|Aut(g_z)\big|}$$

[21] Note that the number of distinct positions that i can occupy if network g has geometry z is $|K_i(g_z)|$.

■

The following example illustrates Proposition 1.

Example 4. Consider the information structure of Example 2. Networks $g_1 \in B_i^1$, $g_2 \in B_i^2$ and $g_3 \in B_i^3$ are depicted in Figure 3.8. Assume $K_i(g_1) = K_i(g_2) = K_i(g_3) = \{i, l\}$ and $k_i = 1$. Then:

$$b_i^1(g) = y\big(g_1 \mid \{i\}\big) + y\big(g_1 \mid \{l\}\big) = 120 + 60 = 180^{22}$$

$$b_i^2(g) = y\big(g_2 \mid \{i\}\big) + y\big(g_2 \mid \{l\}\big) = 30 + 60 = 90$$

$$b_i^3(g) = y\big(g_3 \mid \{i\}\big) + y\big(g_3 \mid \{l\}\big) = 60 + 120 = 180$$

Hence, $b_i(g) = b_i^1(g) + b_i^2(g) + b_i^3(g) = 450$. ■

Example 4 shows that, an agent i with information set $I_i(g) = \left\{1, \left[\frac{1}{2}, \frac{1}{3}, \frac{1}{6}\right], 6\right\}$ believes that there are 450 feasible networks: 180 have geometry 1 in Figure 3.7, 90 have geometry 2, and 180 have geometry 3, as we introduced in Example 2. Following the same procedure as for the degree-one agent, it can be obtained the number of feasible networks in the beliefs of each $i \in N$ with $k_i \neq 1$ (see Table 3.1).

Proposition 1 provides an expression for $b_i^z(g)$ and another for $b_i(g)$. Dividing both expressions, we get the probabilistic weight that i assigns to geometry $z \in \Omega_i(g)$. Corollary 1 gives such a probability, and shows its relation with the order of the network automorphism group.

Corollary 1. *Let $g_z \in B_i^z(g)$ be a network with geometry $z \in \Omega_i(g)$. Under Setting A, each $i \in N$ believes that network g has geometry z with probability $\rho_z = \frac{b_i^z(g)}{b_i(g)} = \frac{1}{1+\sum_{x \in \Omega_i(g)\setminus\{z\}} \frac{|Aut(g_z)|}{|Aut(g_x)|}}$.*

Under Setting A, the set of all feasible geometries $\Omega_i(g)$ is the same for all agents: the feasibility of a network geometry only depends on $F_g(k)$ and on n. Since all agents have the same information regarding $F_g(k)$ and n, the set of feasible geometries is identical for all network integrants. Corollary 1 further shows that all network integrants assign the same probability to each feasible geometry under Setting A, even if they have different degrees. In contrast, their beliefs about the feasible geometries and their probabilities generally differs under Settings B and C, because the private information can provide information that is not conveyed by the common knowledge one. Consider for example network g_1 in Figure 3.8. The fact that $k_j = 3$ and $k_r = 2$ implies that i and l have different information about their neighbors' degrees. As a consequence, their beliefs about the network under Setting B are distinct: the three network geometries in Figure 3.6 are feasible given $I_i(g_1)$, while only geometry 1 and 3 are feasible given $I_l(g_1)$.

The second implication of Corollary 1 is that each $i \in N$ believes more likely to be immersed in a network with a more asymmetric structure than in a network with a more symmetric one. The degree of symmetry of a network g_z is captured $|Aut(g_z)|$: the lower $|Aut(g_z)|$, the lower the number of nodes that occupy identical positions in g_z and the more asymmetric the network is. Observe in Corollary 1 that $\frac{b_i^z(g)}{b_i(g)}$ decreases as $|Aut(g_z)|$ increases, which means that $\frac{b_i^z(g)}{b_i(g)} < \frac{b_i^y(g)}{b_i(g)}$ if $|Aut(g_z)| > \big|Aut(g_y)\big|$, $g_z \in B_i^z(g)$ and $g_y \in B_i^y(g)$. This results holds also under settings B and C. Under Setting B(C), the probabilistic weight that i assigns to geometry z is $\rho_z = \frac{b_i^z(g)}{b_i(g)}$, where $b_i^z(g)$ and $b_i(g)$ are the sizes of $B_i^z(g)$ and $B_i(g)$ under Setting B(C). Note that $B_i^z(g)$ under Setting B(C) is a subset of $B_i^z(g)$ under Setting A, and $b_i^z(g)$ under Setting A is strictly decreasing in $|Aut(g_z)|$ (see Proposition 1). Then, $b_i^z(g)$ must be also decreasing in $|Aut(g_z)|$ under Setting B(C), what means that $\rho_z < \rho_y$ under Setting B(C) if $|Aut(g_z)| > |Aut(g_y)|$.

The following Lemma stress the relation between the automorphism group of a network and the notions of equivalence among nodes presented in Section 3.2. By construction, the automorphism group of a network increases as the number of automorphically equivalent nodes in the network increases, *ceteris paribus*. Yet, maintaining constant the orbit of each node in the network, $|Aut(g)|$ increases as the number of structurally equivalent nodes increases. The following lemma captures this result.

[22] In Example 3 we showed $y(g_4 \mid \{i\}) = y(g_5 \mid \{i\}) = 60$ and $y(g_6 \mid \{i\}) = 120$. Clearly, $y(g_4 \mid \{i\}) = y(g_1 \mid \{l\})$, $y(g_5 \mid \{i\}) = y(g_2 \mid \{l\})$, and $y(g_6 \mid \{i\}) = y(g_3 \mid \{l\})$

Lemma 2. *Let $g = (N, E)$ and $g' = (N', E')$ two networks such that (i) $N = N'$, (ii) $\big|O_i(g)\big| \geq \big|O_i(g')\big|$ and $\big|S_i(g)\big| \geq \big|S_i(g')\big|$, $\forall i \in N$, (ii) $\exists m \in N : \big|O_m(g)\big| > \big|O_m(g')\big|$ and (or) $\big|S_m(g)\big| > \big|S_m(g')\big|$, then $\big|Aut(g)\big| > \big|Aut(g')\big|$.*

In words, given two networks g and g' such that (i) the orbit of each node in g is not lower than its orbit in g' and (ii) the set of structurally equivalent nodes of each node in g is no lower than such a set in g', then $\big|Aut(g)\big| > \big|Aut(g')\big|$ if either the orbit of any node is higher in g than in g' and/or the set of structurally equivalent notes to any node in g' is higher than in g'. Consider for instance g_2 and g_3 in Figure 3.8. Although $\big|O_i(g_2)\big| = \big|O_i(g_3)\big|\ \forall i \in N$, $\big|S_r(g_2)\big| > \big|S_r(g_3)\big|$ for $r \in \{r, m, o, l\}$ and $\big|S_j(g_2)\big| = \big|S_j(g_3)\big|$ for $j \in \{j, i\}$. Hence, $\big|Aut(g_2)\big| > \big|Aut(g_3)\big|$.

To conclude, we illustrate how the knowledge of the set of feasible geometries and their probability distribution enables agents to learn different network features, such as the probability that their neighbors have particular degrees. As we show below, this will be relevant for the strategic interactions that take place within the network.

Example 5. Assume we are under Setting A, and $I_i(g) = \big\{k_i(g), \big[F_g(1), F_g(2), F_g(3)\big], n\big\} = \big\{1, \big[\frac{1}{2}, \frac{1}{3}, \frac{1}{6}\big], 6\big\}$. Figure 3.6 shows the set of feasible geometries, given $I_i(g)$. Table 3.1 displays $b_i(g)$ as a function of $k_i(g)$. A degree-one agent i expects to have degree-three neighbor if she occupies either her position in g_1 in Figure 3.8, her position in g_2 in the same figure, or her position in g_3. Hence, she expects to have a degree-three neighbor with probability:

$$\frac{y\big(g_1 \mid \{i\}\big) + y\big(g_2 \mid \{i\}\big) + y\big(g_3 \mid \{i\}\big)}{b_i(g)} = \frac{210}{450} = \frac{7}{15}$$

In a different way, degree-two agent m expects to have a degree-three neighbor if she occupies either the position of m in network g_1 in Figure 3.8, the position of m in g_2, or the position of this agent in g_3 in the same figure. That is, with probability:

$$\frac{y\big(g_1 \mid \{m\}\big) + y\big(g_2 \mid \{m\}\big) + y\big(g_3 \mid \{m\}\big)}{b_i(g)} = \frac{240}{300} = \frac{8}{10} = \frac{4}{5}$$

■

The example further illustrates how, in finite networks, the probability that i has a neighbor of degree k is not independent of $k_i(g)$. This fact will be key to understand the equilibrium patterns of games played on networks, as we illustrate in the next section.

3.5. Network Games

In this section, we exploit the belief formation framework developed in the previous section and its implications on behavior in network games. In particular, we show how the fact that people can estimate−and sometimes even learn−different network aspects, together with the cognitive bias they have towards asymmetric structures, has an incidence on their equilibrium behavior.

3.5.1. Players' types and strategies

We analyse the strategic interactions that take place in the network under the information settings presented above. Network integrants correspond to the players of a Bayesian Game in which they have partial information about the network position of other players. The type of each player is a particular aspect of her network position contained in her private information (for instance, her degree). Depending on the information setting, the type of each player is defined differently. Later on, we provide a precise definition of the type of each player in each information setting. The type of each $i \in N$ is assigned by the function $\tau_i : G \to T_i$; each network in G can be interpreted as a feasible state of nature. The set of feasible types of i, denoted T_i, is the set of different types that i can be considering all networks in G, with $|T_i| = \eta$. In every setting, $T_i = T_j = T, \forall i, j \in N$. We write t_i to denote the type of player i under each setting.

A strategy σ is a mapping that specifies which action is chosen as a function of each player's type. Strategy σ is symmetric if $\sigma(t_i) = \sigma(t_j)$ as long as $t_i = t_j$. Following Galeotti et al. (2010), we focus on the symmetric equilibria of the game in most part of our analysis; we only consider asymmetric strategies when exploring the effects of varying players' network information on equilibria (Section 3.5.4.2). Since we analyse binary games (where players choose an action in $\{0,1\}$), $\sigma(t_i)$ is the probability that player i takes action 1. Strategy σ is *non-decreasing* if $\sigma(k')$ first-order stochastically dominates (FOSD) $\sigma(k)$, for all $k' > k$. Analogously, σ is *non-increasing* if $\sigma(k)$ FOSD $\sigma(k')$, for all $k' > k$.

In our games, players only have incomplete information about the network; all other ingredients of the games (e.g. their payoff functions) are common knowledge.

3.5.2. A game of strategic substitutes

Each player has to choose independently and simultaneously an action in the set $X = \{0,1\}$. Action 1 may be interpreted as acquiring a certain technology, acquiring a piece of information, or contributing to the provision of a public good, and action 0 as not doing so. Agents incur in a cost c when they play action 1, with $0 < c < 1$, but not when they play action 0. This game is a simplified version of the public good game in Bramoullé and Kranton (2007). Define x_i as the action played by $i \in N$, and $x_{N_i(g)} = \sum_{i\in N} x_i$ as the sum of her neighbors actions. The utility of each i is $u_i(x_i, x_{N_i(g)})$, and takes the following values:

$$u_i\big(0, x_{N_i(g)}\big) = 1 \quad \text{if} \quad x_{N_i(g)} \geq 1$$

$$u_i\big(0, x_{N_i(g)}\big) = 0 \quad \text{if} \quad x_{N_i(g)} = 0$$

$$u_i\big(1, x_{N_i(g)}\big) = 1 - c \quad \text{for any} \quad x_{N_i(g)}$$

It is readily seen that each player prefers that any of her neighbors takes action 1 rather than taking this action herself. However, if none of her neighbors plays 1, she prefers playing 1 than playing 0.

Assume $i \in N$ has k_i neighbors, $N_i(g) = \{j, ..., v\}$. When all agents play the pure symmetric strategy σ, the expected utility of i of taking action 0 is the probability that at least one of her neighbors plays action 1. That is, the probability that at least one neighbor of i is a type for which σ assigns action 1:

$$E_{U_i}\big(0, \sigma, B_i(g)\big) = \frac{|\{g \in B_i(g) : \exists j \in N_i(g) : \sigma(t_j) = 1\}|}{b_i(g)} \tag{3.5.1}$$

In equilibrium, each type of i plays 0 if her expected utility of playing 0 is at least as high as her expected utility of playing 1, $E_{U_i}\big(1, \sigma, B_i(g)\big) = 1 - c$.

3.5.3. A game of strategic complements

Consider the game in Jackson (2010), where each $i \in N$ has to choose independently and simultaneously an action in $X = \{0,1\}$. Action 1 represents an activity that is enjoyed if at least one other person participates, and has a cost c. For example, i has to choose whether to book a tennis court (action 1) or not (action 0), and c is the booking fee, $0 < c < 1$. Or i has to choose whether to buy a particular software or not, and the software is only enjoyed if at least one neighbor uses it. The utility of i depends on the same arguments as in the previous game, but it takes different values:

$$u_i\big(1, x_{N_i(g)}\big) = 1 \quad \text{if} \quad x_{N_i(g)} \geq 1$$

$$u_i\big(0, x_{N_i(g)}\big) = 0 \quad \text{for any} \quad x_{N_i(g)}$$

$$u_i\big(1, x_{N_i(g)}\big) = -c \quad \text{if} \quad x_{N_i(g)} = 0$$

The expected utility of playing 1 when all agents play the symmetric strategy σ is:

$$E_{U_i}\big(1, \sigma, B_i(g)\big) = -c + \frac{|\{g \in B_i(g) : \exists j \in N_i(g) : \sigma(t_j) = 1\}|}{b_i(g)} \tag{3.5.2}$$

Each type of i is best responding with action 1 if $E_{U_i}\big(1, \sigma, B_i(g)\big) \geq E_{U_i}\big(0, \sigma, B_i(g)\big) = 0$.

3.5.4. Equilibrium

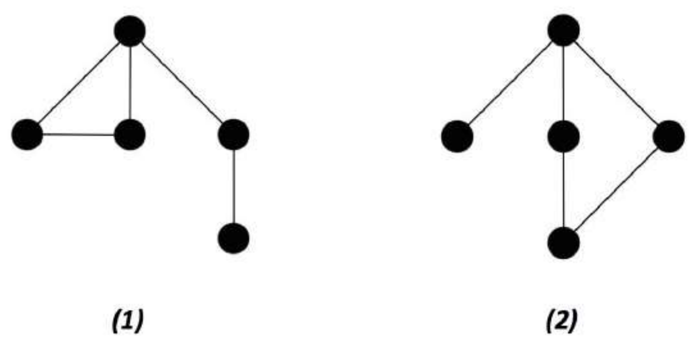
3.5.4.1. Setting A

As in Galeotti et al. (2010), the type of each player corresponds to her degree. Each i believes that $j \in N$ is type $t_j = k_j$ with probability $p_i(t_j) = p_i(k_j) = \frac{|\{g \in B_i(g): \tau_j(g) = k_j\}|}{b_i(g)}$.
Galeotti et al. (2010) prove the existence of a unique symmetric equilibrium in both games. They show that such an equilibrium is monotone non-decreasing (non-increasing) in players' degrees under strict strategic complements (substitutes),[23] if nodes have degrees either with independent probabilities or with probabilities that are positively (negatively) correlated.[24] Their result is intuitive: under degree independence i and z have the same beliefs about the degree of each of their respective neighbors, even when $k_i > k_z$. This implies that i and z face the same probability distribution over the action of each of her respective neighbors, since the action of each player is determined by her type (her degree). However, since i has a more neighbors, the probability that at least one neighbor of her plays action 1 is greater than the probability that at least one neighbor of z does. As a result, the incentives of i to play action 1 under strict complements (substitutes) are higher (lower) than the incentives of z, and the equilibrium is necessarily non-decreasing (non-increasing) in players' degrees.[25] If there is not degree independence but degrees are positively (negatively) correlated, the monotonicity property of equilibrium maintains under strategic complements (substitutes): high degree players are more likely to have at least one neighbor playing action 1, since they are more likely to be have at least one high degree (low degree) neighbor, and the result follows.

Under our Setting A, players' do not posses information about the assortativity patterns in the network. But, each i can learn the probability that a neighbor of her has degree k from $I_i(g)$ (see Example 5). Since such a probability may vary positively with $k_i(g)$ (nodes may have degrees neither with independent probabilities nor with probabilities that are negatively correlated) the equilibrium predictions in Galeotti et al. (2010) under strategic substitutes may fail to hold, as the following example illustrates.

Example 6. Consider the game of strategic substitutes. For each $i \in N$, let $I_i(g) = \{k_i(g), [F_g(1), F_g(2), F_g(3)], n\} = \{k_i(g), [\frac{1}{5}, \frac{3}{5}, \frac{1}{5}], 5\}$ and $T_i = \{k_i, k_i', k_i''\} = \{1, 2, 3\}$, referred as types 1, 2 and 3, respectively. Figure 3.10 shows the two feasible geometries, in the beliefs of each $i \in N$.

Figure 3.10. Feasible geometries of network g in Example 6.



There are three strategies that constitute a pure-strategy symmetric equilibrium:

(a) σ_1: $\sigma_1(2) = 0$ and $\sigma_1(k_i) = 1$ for $k_i \in \{1, 3\}$, $\forall i \in N$ and $\frac{1}{6} \leq c \leq \frac{1}{2}$.
(b) σ_2: $\sigma_2(3) = 1$ and $\sigma_2(k_i) = 0$ for $k_i \in \{1, 2\}$, $\forall i \in N$ and $c \geq \frac{1}{2}$.
(c) σ_3: $\sigma_3(1) = 1$ and $\sigma_3(k_i) = 0$ for $k_i \in \{2, 3\}$, $\forall i \in N$ and $c \geq \frac{5}{6}$.

Table 3.2 lists the expected utility of playing 0 of each type of $i \in N$ when all agents follow each of these equilibrium strategies.[26]

[23] Strictness is important for their result: when players are indifferent between actions, non-monotone equilibria are possible.
[24] Under degree independence, the probability that i has a high-degree neighbor does not depend on $k_i(g)$. Under positive (negative) degree correlation, the probability that i has a high-degree neighbor depends positively (negatively) on $k_i(g)$.
[25] Recall that in games of strategic complements (substitutes), players incentives to play higher actions are higher (lower) as their neighbors' actions are higher.
[26] See the Appendix for details.

Table 3.2. Expected utilities of playing 0 for each type

Type of $i \in N$	$E_{U_i}\big(0, \sigma_1, B_i(g)\big)$	$E_{U_i}\big(0, \sigma_2, B_i(g)\big)$	$E_{U_i}\big(0, \sigma_3, B_i(g)\big)$
$k_i = 1$	$1/2$	$1/2$	0
$k_i = 2$	$5/6$	$5/6$	$1/6$
$k_i = 3$	$1/2$	0	$1/2$

The equilibria structure in Galeotti et al. (2010) does not maintain. First, equilibria are not unique, but multiple equilibria can coexist depending on the cost value. Second, equilibria can be monotone non-increasing, monotone non-decreasing as well as non-monotone under strategic substitutes, even when players are only informed about their degree and the degree distribution of the network. ■

Example 6 shows that the equilibrium predictions for strategic substitutes in Galeotti et al. (2010) are not robust to relaxing their assumptions: (i) nodes have degrees with independent probabilities or (ii) with probabilities that are negatively correlated.[27] For the sake of simplicity, suppose that the only agents that play action 1 are those with degree k. If (i) does not hold, a degree-k_1 player may expect to have a neighbor of degree k with a greater probability than a degree-k_2 does, even if $k_2 > k_1$. This implies that the expected utility of playing 0 may be greater for the degree-k_1 player than for the degree-k_2 one. As a result, the high-degree player may be best responding with a higher action than the low degree one, and the equilibrium may be increasing in players' degrees under strategic substitutes. The intuition is similar if we relax the assumption of negative degree correlation. Suppose that the agents playing action 1 are the "low degree players", say, those with degree equal or lower than k, while "high degree players" (with degree greater than k) play action 0. If (ii) does not hold, low degree players may expect to have low degree neighbors (neighbors playing 1) with a higher probability than high degree players. As a consequence, high degree players may be best responding with higher actions than low degree ones, and equilibria are not necessarily monotone non-increasing under strategic substitutes.

As for strategic complements, the results of Galeotti et al. (2010) tend to maintain. The reason is that high degree players have a greater tendency to be connected to high degree nodes, even if links are formed in a fully random way (Newman, 2003; Jackson and Rogers, 2007; Jackson, et al. 2010). Since Galeotti et al. (2010) guarantee that equilibria are monotone non-decreasing under strategic complements under positive degree assortativity, their results for this type of games tend to maintain.

Efficient network structures. Under Setting A, the set of equilibria is the same in all networks with the same degree sequence.[28] However, the payoffs that players receive may differ across these networks, if their respective geometries are different. Consider, for example, networks: g_1 and g_2 in Figure 3.10. Under Setting A, the three strategies in Example 6 are equilibrium strategies. However, when all agents play σ_1, the sum of their payoffs is $2(1-c)+3$ in network g_1, while this sum is $2(1-c)+2$ in network g_2. Thus, although players' choices are identical, the aggregate welfare is not.
We want to understand how the equilibrium welfare relates to the geometry of the network. To that purpose, we adopt the utilitarian approach: welfare in network g when all agents play the strategy σ corresponds to the sum of the payoffs obtained by all players when they all follow such a strategy:

$$W(\sigma, g) = \sum_{i \in N} u_i(x_i, x_{N_i(g)})$$

where x_i and $x_{N_i(g)}$ are the action of i and the sum of the actions of $i's$ neighbors when all agents follow the strategy σ, respectively.

Network g is said to be *efficient* if there is not any other network g' with the same degree sequence that yields higher welfare at any pure-strategy symmetric equilibrium. Formally, g is efficient if $\nexists g' : D_g(k) = D_{g'}(k)$ $\forall k$ such that $W(\sigma^*, g') > W(\sigma^*, g)$, for any σ^* that constitutes a pure-strategy symmetric equilibrium under Setting A.[29] Proposition 2 provides a *sufficient* condition for a network to be efficient, both for the game of

[27] As we show in the Appendix, beliefs of each i about her neighbors' degrees depends on $k_i(g)$ but there is not a monotone relation between both issues: the likelihood of having a neighbor of degree k does not vary monotonously with $k_i(g)$.

[28] Under Setting A, each i with degree $k_i(g)$ has the same beliefs about the network in all networks with the same degree sequence (see Section 3.4.3). Hence, her best response to her neighbors' actions is identical across all these networks.

[29] The concept is similar to that in Goyal and Vega-Redondo (2007). However, our definition focus on equilibrium welfare.

strategic substitutes as well as for the game of strategic complements. Such a condition depends on ρ_z, which is the probability that all agents assign to geometry z under Setting A (see Corollary 1).

Proposition 2. *Assume network g has geometry z, $k =$arg max $D_g(k)$ and we are under Setting A:*
1. For the game of strategic substitutes, network g is efficient if $c < \frac{\rho_z}{D_g(k)}$
2. *For the game of strategic complements, network g is efficient if $1 - c < \frac{\rho_z}{D_g(k)}$.*

To illustrate Proposition 2, consider the information structure in Example 6, and let g_1 and g_2 be two networks with the geometry of networks 1 and 2 in Figure 3.10, respectively. Since $|Aut(g_1)| = |Aut(g_2)| = 2$, all agents in g_1 believe that the network they are immersed in has geometry 1 with probability $\rho_1 = \frac{1}{2}$, and geometry 2 with the same probability, $\rho_2 = \frac{1}{2}$. Since *arg max* $D_{g_1}(k)$=*arg max* $D_{g_2}(k) = 2$ *and* $D_{g_1}(2) = 3$, Proposition 2 implies that network g_1 is efficient for $c < \frac{\rho_1}{D_g(2)} = \frac{1}{6}$ under strategic substitutes and analogously for g_2. Note that for $c \geq \frac{1}{6}$, network g_2 is not efficient: when all agents play the equilibrium strategy σ_1, welfare is $2(1 - c) + 2$ in this network, while it is equal to $2(1 - c) + 3$ in network g_1.

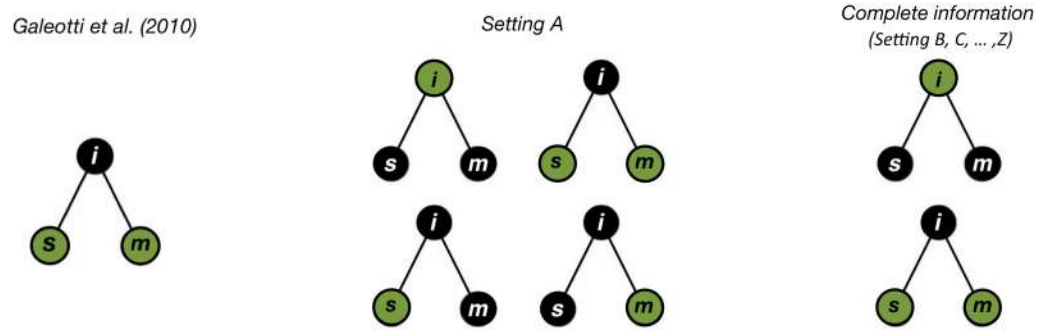
Under strategic substitutes (complements), the set of values of c $(1-c)$ for which Proposition 2 guarantees that g is efficient decreases as the automorphism group of g increases. The intuition of this result is the following. When players choose their actions, they infer the feasible allocations of types that can exist in the network by considering all the network geometries that are feasible; each feasible geometry has associated a particular allocation of types. For example, a degree-one player in Example 2 infers that her neighbor is type 1 if network g has geometry 2, but not if g has geometry 1 or geometry 3. However, all feasible geometries are not equally likely, but the probability that g has the geometry of $g_z \in B_i^z(g)$ decreases as $|Aut(g_z)|$ increases. This means that the allocations of types associated to asymmetric network geometries have a higher weight in the decision of agents when they choose their strategies. Since players aim to maximize their payoffs when choosing their strategies and assymetric geometries are more likely in their beliefs, networks with more assymetric geometries are guaranteed to be efficient for a greater range of values of c $(1 - c)$ under strategic substitutes (complements) in Proposition 2.

3.5.4.2. Information effects

This section explores the effects of varying the depth of players' network information on equilibria. We first compare the equilibria that arise under different information setups, and show how increasing players' network information neither has a monotone effect on the number of equilibria nor on their structure (Example 7). We then illustrate how a subtle variation in players' network information (the knowledge of neighbors' *neighborhoods* vs. the knowledge of neighbors' *degrees*) can largely affect players' equilibrium behavior (Example 8).

Example 7. Players are embedded in network g of Figure 3.2. Figure 3.11 shows the pure-strategy equilibria that arise under strict strategic substitutes under different information setups. The green (black) nodes in Figure 3.11 represent agents playing action 1(0).

Figure 3.11. Pure-strategy equilibria in Example 7



a) Galeotti et al. (2010). For all $i \in N$, let $I_i(g) = \{k_i(g), P_g(k)\}$, where $P_g(k)$ is the probability degree distribution. Suppose that, according $P_g(k)$ each node has degree $k = 1$ with probability p_1 and degree $k = 2$ with probability $p_2 = 1 - p_1$, independently on the degree of other nodes.
Under strict strategic substitutes, the only equilibrium strategy is $\sigma_1 : \sigma_1(k_i) = 1$, $\forall k_i = 1$ and $\sigma_1(k_i) = 0$, $\forall k_i = 2$. Clearly,

$$E_{U_i}\big(0, \sigma_1, I_i(g)\big) = 1 - p_1^{k_i(g)}$$

which is greater than $E_{U_i}\big(1, \sigma_1, I_i(g)\big) = 1 - c$ for $k_i(g) \geq 2$ and $p_1^2 < c < p_1$ and lower than $1 - c$ for $k_i(g) = 1$ and the same cost values. On the contrary, strategy $\sigma_2 : \sigma_2(k_i) = 0$, $\forall k_i = 1$ and $\sigma_2(k_i) = 1$, $\forall k_i = 2$. is not an equilibrium strategy: since the expected utility of playing zero is always greater for the degree-two players than for the degree-one ones, for any cost value for which the second agents are best responding with action 0, the former ones must be best-responding with this action as well. Then, σ_1 is the unique equilibrium strategy under strict strategic substitutes.[30]

b) Setting A. For all $i \in N$, $I_i(g) = \{k_i(g), [F_g(1), F_g(2)], n\} = \{k_i(g), \left[\frac{2}{3}, \frac{1}{3}\right], 3\}$. Hence, $B_s(g) = \{g, g_2\}$, $B_m(g) = \{g, g_1\}$ and $B_i(g) = \{g\}$, where g, g_1 and g_2 are depicted in Figure 3.2.
Since each i can infer the unique feasible geometry from $I_i(g)$, both σ_1 and σ_2 in (a) are equilibrium strategies for any cost value.
Suppose all players follow an asymmetric strategy σ_3: $\sigma_3(k_s) = 1$ for k_s and $\sigma_3(k_m) = \sigma_3(k_i) = 0$. Agent i knows that $s \in N_i(g)$, and since $E_{U_i}\big(0, \sigma_3, B_i(g)\big) = 1$, i is best responding with action 0. The opposite occurs for s: when all agents play σ_3, $E_{U_s}\big(0, \sigma_3, B_s(g)\big) = 0$, and she is best responding with action 1. Last, the probability that at least one neighbor of m plays 1 when all agents follow the strategy σ_3 is $E_{U_m}\big(0, \sigma_3, B_m(g)\big) = \frac{|\{g_1\}|}{b_m(g)} = \frac{1}{2}$. Then, under srict strategic substitutes ($c > \frac{1}{2}$), σ_3 and σ_4 are equilibrium strategies, where σ_4: $\sigma_4(k_m) = 1$ for k_m and $\sigma_4(k_s) = \sigma_4(k_i) = 0$.

c) Complete information. Under Setting B, C and Z, all players have complete network information. Each i is best responding with action 0 if she is linked to at least one neighbor playing 1, and with action 1 otherwise. Then, the only possible equilibrium strategies are σ_1 and σ_2. ■

Example 7 illustrates how reducing players' information does not necessarily solve the equilibrium selection problem: although the shift from Setting Z to the information setting in Galeotti et al. (2010) eliminates any ambiguity in behavior, the shift from Setting Z to Setting A does not. Notice that the set of equilibria under our Setting A and the information setup in Galeotti et al. (2010) is markedly different even when the difference between both setups is subtle (under our Setting A, players' know the distribution of degree frequencies in the network while in Galeotti et al. (2010) they know the probability degree distribution).

Example 8. Consider the networks in Figure 3.12. The three networks have the same degree distribution, whereas only networks (b) and (c) have the same *joint degree distribution*.[31] We analyse (i) whether players' behavior may change depending on whether they have information about their second-order neighbors' identities or not (Setting B *vs.* Setting C) and (ii) whether players with the same degree and neighbors' degrees may behave differently depending on the observed geometry of links in their local network (Setting C). To that aim, the type of each i is jointly defined by her degree and her neighbors' degrees under settings B and C, $(k_i, \mathtt{k}_{\mathtt{N}_\mathtt{i}}(g))$, and we compare the symmetric equilibria that arise in different networks within and across these settings.

Table 3.3 contains the symmetric equilibrium strategies in the networks of Figure 3.12 for the game of strategic substitutes. The range of cost values for which each of these strategies constitutes an equilibrium in each network is displayed in Figure 3.13. As can be seen in this figure, equilibria can be different within and across setups, since the different information of players traduce in different beliefs about the network and in turn in different equilibrium choices.

[30]Note that there is not equilibrium where all players play the same actions. If all agents play 1, $E_{U_i}\big(0, \sigma, I_i(g)\big) = 1 > 1 - c$ for all i, so each player wants to deviate and play action 0. The same applies if all they play 0; since $E_{U_i}\big(0, \sigma, I_i(g)\big) = 0 < 1 - c$, each player wants to change her action to 1.

[31]Networks $g{=}(N, E)$ and $g'{=}(N', E')$ with $N{=}N'$ have the same joint degree distribution if $(k_i; \mathtt{k}_{N_i(g)}){=}(k'_i; \mathtt{k}'_{N_i(g)})$ $\forall i \in N$.

Figure 3.12. Networks in Example 8.

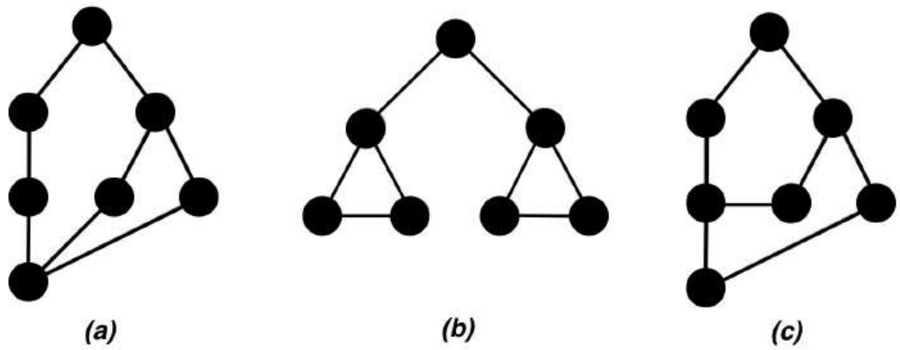
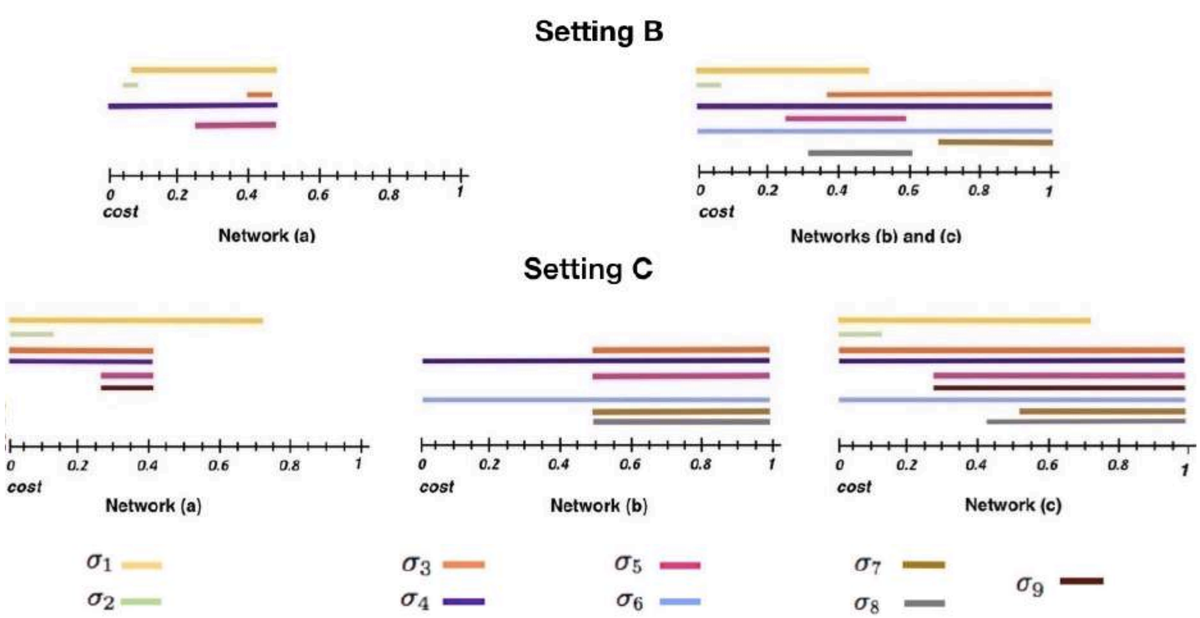


Table 3.3. Symmetric equilibrium strategies in Example 8

Type of $i \in N$	$\sigma_1(t_x)$	$\sigma_2(t_x)$	$\sigma_3(t_x)$	$\sigma_4(t_x)$	$\sigma_5(t_x)$	$\sigma_6(t_x)$	$\sigma_7(t_x)$	$\sigma_8(t_x)$	$\sigma_9(t_x)$
$t_1 = (2;(2,2))$	0	0	1	1	1	0	0	0	1
$t_2 = (2;(3,2))$	1	1	0	0	0	0	0	0	0
$t_3 = (2;(3,3))$	1	1	0	0	1	0	0	1	1
$t_4 = (3;(2,2,2))$	0	0	1	1	0	1	1	0	0
$t_5 = (3;(3,2,2))$	0	1	0	1	1	1	0	1	0

Figure 3.13. Symmetric equilibria in Example 8



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Examples 7 and 8 provide a crucial message: although the framework of Galeotti et al. (2010) eliminates the problem of multiplicity of equilibria in network games, their uniqueness and monotonicity results are largely not robust to relaxing their information assumptions. As a result, their approach fails to refine the set of predictions in many situations in which people still have local network information. Such situations abound.

The introduction of incomplete information as a way of solving the problem of multiplicity of equilibria has faced a major critique: the equilibrium achieved depends on the way incomplete information is introduced (Weinstein and Yildiz, 2007). While this critique applies generally to all incomplete information games, it seems particularly relevant for those played on networks, given the variety of network aspects that players can infer from the information they are given.[32] Examples 7 and 8 illustrate that behavior is particularly sensitive to subtle changes in players' network information. This implies that, even when it is clear the network aspects that are unknown for players in specific contexts (e.g. the degree of agents that are two-link separated from them), fine differences in the information assumptions (e.g. assuming that players know the identity of their second-order neighbors or not) can bias the results in a particular direction.

3.6. Concluding Remarks

Cognitive network research has showed how people form mental representations about their networks that influence their behavior (e.g. Brewer, 2011 or Pittinsky and Carolan, 2008). Literature on network games has primarily opted for a simplification of people's network perception, setting aside players' beliefs about finer details of the network structure (e.g. Galeotti et al. 2010). The principal innovation of this chapter is the modeling of richer cognitive maps of networks, and their placement in relation with sociological notions of equivalence among nodes. Although these notions have broadly been studied by sociologists, they have not been theoretically related, to the best of our knowledge, to network perception.

We identify a bias in people's perception towards asymmetric network structures. If we order a set of networks with the same degree distribution according to the size of their automorphism group, such an order reflects a likelihood ranking of network geometries in people's beliefs. An implication for the analysis of network games under incomplete network information is that people's choices are to a great extent shaped by the features of more asymmetric structures, as these structures have a higher probabilistic weight in their network beliefs. Network symmetry can also have an incidence in people's welfare, as we have shown.

Our theoretical framework provides a way of capturing players' beliefs about a variety of network features that are absent in canonical models of network analysis (e.g. in random-graph models), which allows to analyse their incidence on behavior in incomplete information contexts. Yet, it presents two major drawbacks. First, since players' infer a variety of network features from the information they are given, a subtle variation in players' network information can change completely the spectrum of equilibria. This requires a deeper understanding of the network knowledge that people actually have in different contexts, and calls for experimental research analysing this issue. Second, the great range of equilibria that emerge under each information setup makes it hard to draw conclusions on the incidence that each specific feature has on behavior. A prospective way to address this matter might be to impose some *ceteris paribus* restrictions on the set of feasible geometries (similarly as in Espinosa et al. 2020) in such a way that players' are only uncertain about one specific network feature, while they have a founded knowledge of its probability distribution. We leave this for future research.

[32] The interdependency between different network features implies that players' information does not end in their information set, but they may learn different network aspects from the network information they are given. Observe that such a learning process may not occur in other Bayesian games.

3.7. Appendix

A. Network Perception.

A1. The Orbit Stabilizer Theorem.

ORBIT-STABILIZER THEOREM. *Let* $Stab_v(g) = \{g \in Aut(g) : f(v) = v\}$ *be the stabilizer of node* $v \in N$*: the set of all automorphisms of* g *that map node* v *to itself. Then,*

$$\big|Aut(g)\big| = \big|O_v(g)\big| * \big|Stab_v(g)\big|$$

A2. Proof of Lemma 2.

Proof of Lemma 2. Since neither the orbit nor the number of structurally equivalent nodes to any node in g is higher than in g', for each automorphism $f : N \to N$ exists there is an identical automorphism $f : N' \to N'$. If $\exists m \in N : \big|O_m(g)\big| > \big|O_m(g')\big|$, then there exist at least one automorphism in g that does not exist in g'. Thereby, $\big|Aut(g)\big| > \big|Aut(g')\big|$.
We still have to prove that if $\big|O_i(g)\big| = \big|O_i(g')\big|\ \forall i \in N$, and $\exists m \in N : \big|S_m(g)\big| > \big|S_m(g')\big|$, $\big|Aut(g)\big| > \big|Aut(g')\big|$. Suppose $\exists m \in N : |S_m(g)| > |S_m(g')|$. Since $N_m(g) \setminus \{r\} = N_r(g) \setminus \{m\}\ \forall r \in S_m(g) \setminus S_m(g')$, there exists an automorphism $f : N \to N$ between m and each $r \in S_m(g) \setminus S_m(g')$ such that:

$$f(w) \begin{cases} r & if\ w = m \\ m & if\ w = r \\ w & otherwise \end{cases}$$

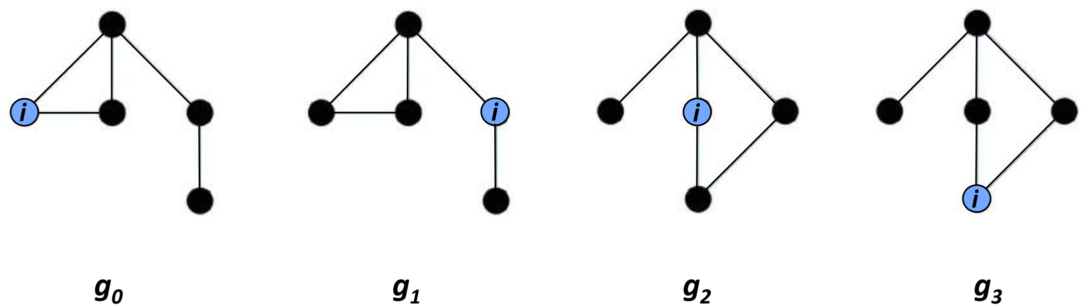
On the contrary, since $r \notin S_m(g')$ it does not exist such an automorphism between m and r in network g'. As a result, $\big|Aut(g)\big| > \big|Aut(g')\big|$. ∎

B. Network Games

B1. Examples

Example 6. Let $N = \{i, j, l, m, r\}$, and $k_i = 2$. The feasible positions of i are depicted in Figure A3.1. By Lemma 1, $y\big(g_0 \mid \{i\}\big) = y\big(g_2 \mid \{i\}\big) = 24$, $y\big(g_1 \mid \{i\}\big) = y\big(g_3 \mid \{i\}\big) = 12$, $b_i^1(g) = b_i^2(g) = 24 + 12 = 36$, and $b_i(g) = 72$, so i assigns the same probability to both network geometries, $\frac{b_i^1(g)}{b_i(g)} = \frac{b_i^2(g)}{b_i(g)} = \frac{1}{2}$.

Figure A3.1. Feasible positions of an i with $k_i = 2$ in Example 6



Agent i has at least one neighbor of type 1 or of type 3 if she occupy either her position in g_0, g_1 or in g_2. Therefore, $E_{U_i}\big(0, \sigma_1, B_i(g)\big) = \frac{y(g_0|\{i\})+y(g_1|\{i\})+y(g_2|\{i\})}{b_i(g)} = \frac{24+12+24}{72} = \frac{5}{6}$. Since she has at least one neighbor of type 3 if she occupies either her position in g_0, g_1 or in g_2, $E_{U_i}\big(0, \sigma_2, B_i(g)\big) = \frac{y(g_0|\{i\})+y(g_1|\{i\})+y(g_2|\{i\})}{b_i(g)} = \frac{24+12+24}{72} = \frac{5}{6}$. Analogously, $E_{U_i}\big(0, \sigma_3, B_i(g)\big) = \frac{y(g_1|\{i\})}{b_i(g)} = \frac{12}{72} = \frac{1}{6}$, given that i has at least one neighbor of type 1 if she occupies her position in g_1.
By Corollary 1, each $i \in N$ assigns the same probability to each feasible network geometry, $\frac{b_i^1(g)}{b_i(g)} = \frac{b_i^2(g)}{b_i(g)} = \frac{1}{2}$.
Agent i with $k_i = 1$ believes that her neighbor has degree 3 if network g has the geometry 2, and degree 2 otherwise. Hence, if $k_i = 1$, $E_{U_i}\big(0, \sigma_1, B_i(g)\big) = E_{U_i}\big(0, \sigma_2, B_i(g)\big) = \frac{1}{2}$, and $E_{U_i}\big(0, \sigma_3, B_i(g)\big) = 0$. Similarly, an i with $k_i = 3$ expects to have a neighbor of type 1 if network g has geometry 1; otherwise, all her neighbors are expected to have degree-two. Hence, if $k_i = 3$, $E_{U_i}\big(0, \sigma_1, B_i(g)\big) = E_{U_i}\big(0, \sigma_3, , B_i(g)\big) = \frac{1}{2}$, and $E_{U_i}\big(0, \sigma_2, B_i(g)\big) = 0$.
Table A3.1 shows the probability that i has a neighbor of each type as a function of $k_i(g)$. As can be seen from this table, the probability that i has a neighbor of degree k varies with $k_i(g)$, but it does not vary monotonously with $k_i(g)$.

Table A3.1 Probability that i has a neighbor of degree k

$p[\exists j \in N_i(g) : k_j = k]$	$k = 1$	$k = 2$	$k = 3$
$k_i(g) = 1$	0	1/2	1/2
$k_i(g) = 2$	1/6	5/6	5/6
$k_i(g) = 3$	1/2	1	0

■

Example 7.
A) Symmetric equilibria under Setting B.
Network (a). Network (a) is integrated by types 1, 2, 3 and 4. Hence, there is an equilibrium if all these types are playing their best response to their neighbors' actions.

When all agents play the strategy σ_1, the expected utility of playing 0 of each type of i is the probability that she has at least one neighbor that plays action 1, i.e. the probability that she has either a neighbor of type 2 or a neighbor of type 3:

$$E_{U_i}(0, \sigma_1, B_i(g)) = \frac{\big|\{g \in B_i(g) : \exists j \in N_i(g) : \tau_j(g) = \big(2; (3, 2\big) \vee \tau_j(g) = \big(2; (3, 3\big)\}\big|}{b_i(g)}$$

Similarly,

$$E_{U_i}(0,\sigma_2,B_i(g)) = \frac{\left|\{g \in B_i(g) : \exists j \in N_i(g) : \tau_j(g) = \big(2;(3,2\big) \vee \tau_j(g) = \big(2;(3,3\big) \vee \tau_j(g) = \big(3;(3,2,2\big)\}\right|}{b_i(g)}$$

$$E_{U_i}(0,\sigma_3,B_i(g)) = \frac{\left|\{g \in B_i(g) : \exists j \in N_i(g) : \tau_j(g) = \big(2;(2,2\big) \vee \tau_j(g) = \big(3;(2,2,2\big)\right|}{b_i(g)}$$

$$E_{U_i}(0,\sigma_4,B_i(g)) = \frac{\left|\{g \in B_i(g) : \exists j \in N_i(g) : \tau_j(g) = \big(2;(2,2\big) \vee \tau_j(g) = \big(3;(2,2,2\big) \vee \tau_j(g) = \big(3;(3,2,2\big)\right|}{b_i(g)}$$

$$E_{U_i}(0,\sigma_5,B_i(g)) = \frac{\left|\{g \in B_i(g) : \exists j \in N_i(g) : \tau_j(g) = \big(2;(2,2\big) \vee \tau_j(g) = \big(2;(3,3\big) \vee \tau_j(g) = \big(3;(3,2,2\big)\right|}{b_i(g)}$$

Tables A3.2 and A3.3 provide the expected utility of playing 0 of each type of $i \in N$ when all agents play the same strategy: a strategy in $\{\sigma_1, \sigma_2, \sigma_3, \sigma_4, \sigma_5\}$. From these tables, it can be checked that each of these strategies is an equilibrium strategy for the range of cost specified in Table A3.4.
Note that players of types 1 and 2 must play different actions in equilibrium. If both types play 1(0), the expected utility of playing 0 of a type-one player is 1(0), since each neighbor of her is either type 1 or type 2 with probability 1. Therefore, i is not best responding with action 1(0) but with action 0(1).
Similarly, there cannot exist an equilibrium in which type-two players and type-four players take both action 1: if type-two players play 1, best response of type-four players is playing 0, since type-four players have a type-two neighbor with probability 1 (see Figure A3.5 in this Appendix). Analogously, there is not an equilibrium such that type-three players and type-four players play both action 1 (type-four players have a type-three neighbor with probability 1).
Note that there is neither an equilibrium in which types 3, 4 and 5 take action 0(1): if all these types play 0(1), the expected utility of playing 0 of each type-three player is 0(1), since each neighbor of her is either type 4 or type 5 with probability 1. Hence, best response of each type-three player is not playing 0(1) but playing 1(0).

Considering this, the only additional equilibrium strategies that could exist would be the followings:

- $\sigma_9 : \sigma_9(t_x) = 0$, for $x \in \{2,4,5\}$ (types 2, 4 and 5), and $\sigma_9(t_x) = 1$ for $x \in \{1,3\}$ (types 1 and 3), $\forall i \in N$.
- $\sigma_{10} : \sigma_{10}(t_x) = 0$, for $x \in \{1,3,4\}$ and $\sigma_{10}(t_x) = 1$ for $x \in \{2,5\}$, $\forall i \in N$.
- $\sigma_{11} : \sigma_{11}(t_x) = 0$, for $x \in \{1,3,4\}$ and $\sigma_{11}(t_x) = 1$ for $x \in \{1,5\}$, $\forall i \in N$.

where,

$$E_{U_i}(0,\sigma_9,B_i(g)) = \frac{\left|\{g \in B_i(g) : \exists j \in N_i(g) : \tau_j(g) = \big(2;(2,2\big) \wedge \tau_j(g) = \big(2;(3,3\big)\}\right|}{b_i(g)}$$

$$E_{U_i}(0,\sigma_{10},B_i(g)) = \frac{\left|\{g \in B_i(g) : \exists j \in N_i(g) : \tau_j(g) = \big(2;(3,2\big) \wedge \tau_j(g) = \big(3;(3,2,2\big)\}\right|}{b_i(g)}$$

$$E_{U_i}(0,\sigma_{11},B_i(g)) = \frac{\left|\{g \in B_i(g) : \exists j \in N_i(g) : \tau_j(g) = \big(2;(2,2\big) \wedge \tau_j(g) = \big(3;(3,2,2\big)\}\right|}{b_i(g)}$$

for all $i \in N$.

Table A3.5 shows $E_{U_i}(0,\sigma_9,B_i(g))$, $E_{U_i}(0,\sigma_{10},B_i(g))$ and $E_{U_i}(0,\sigma_{11},B_i(g))$ for all $i \in N$. Observe that, when all agents play the strategy σ_9, type-one players are best responding with action 1 if $1-c \geq 0{,}529$, while type-two players are best responding with action 0 if $1-c \leq 0{,}5$. Since both things cannot occur simultaneously, strategy σ_9 is not an equilibrium strategy. Analogously, when all agents play the strategy σ_{10}, type-two players are best responding with action 1 if $1-c \geq 0{,}916$, while type-three players are best responding with action 0 if $1-c \leq 0{,}4$. Hence, strategy σ_{10} is not an equilibrium strategy. The same happens for strategy σ_{10}: in this case type-one players are best responding with action 1 if $1 - c \geq 0{,}529$, while type-three players are best responding with action 0 if $1 - c \leq 0{,}4$. Then, σ_{10} is not an equilibrium strategy.

Networks (b) and (c). Networks (b) and (c) are integrated by types 2, 3 and 4; there exist an equilibrium if all these types are playing their best response to their neighbors' actions.
As explained above, there cannot exist an equilibrium such that $\sigma(t_1) = \sigma(t_2) = 1$, for all $i \in N$. However, since there are not type-one players neither in network (b) nor in network (c), strategies σ_6, σ_7 and σ_8 are equilibrium strategies for certain values of c in both networks. Table A3.6 provides $E_{U_i}(0, \sigma_6, B_i(g))$, $E_{U_i}(0, \sigma_7, B_i(g))$ and $E_{U_i}(0, \sigma_8, B_i(g))$ for all $i \in N$. Table A3.7 provides the symmetric equilibria in pure strategies that exist for each range of c.

Table A3.2 in Example 7

Type of i	F.P. of i	$y(g_x \mid \{i\})$	$E_{U_i}(0, \sigma_1, B_i(g))$	$E_{U_i}(0, \sigma_2, B_i(g))$	$E_{U_i}(0, \sigma_3, B_i(g))$
t_1	Figure A3.2	$y(g_0 \mid \{i,j,l\}) = 6$	$\frac{\sum_{x=1}^{5} y(g_x\mid\{i,j,l\})}{b_i(g)=\sum_{x=0}^{5} y(g_x\mid\{i,j,l\})} = \frac{96}{102} = 0{,}941$		$\frac{\sum_{x=0}^{2} y(g_x\mid\{i,j,l\})}{b_i(g)=\sum_{x=0}^{5} y(g_x\mid\{i,j,l\})} = \frac{54}{102} = 0{,}529$
		$y(g_x \mid \{i,j,l\}) = 24$ for $x \in \{1,2,3\}$			
		$y(g_x \mid \{i,j,l\}) = 12$ for $x \in \{4,5\}$.			
t_2	Figure A3.3	$y(g_x \mid \{i,j,l\}) = 24$ for $x \in \{0,3,4,5\}$	$\frac{\sum_{x\in\{2,4,5,7\}} y(g_x\mid\{i,j,l\})}{b_i(g)=\sum_{x=0}^{7} y(g_x\mid\{i,j,l\})} = \frac{72}{144} = 0{,}5$	$\frac{144-y(g_6\mid\{i,j,l\})}{b_i(g)=\sum_{x=0}^{7} y(g_x\mid\{i,j,l\})} = \frac{132}{144} = 0{,}916$	$\frac{144-\sum_{x\in\{2,4\}} y(g_x\mid\{i,j,l\})}{b_i(g)=\sum_{x=0}^{7} y(g_x\mid\{i,j,l\})} = \frac{108}{144} = 0{,}75$
		$y(g_x \mid \{i,j,l\}) = 12$ for $x \in \{1,2,6,7\}$			
t_3	Figure A3.4	$y(g_0 \mid \{i,j,l\}) = 4$	0	$\frac{\sum_{x=0}^{1} y(g_x\mid\{i,j,l\})}{b_i(g)=\sum_{x=0}^{4} y(g_x\mid\{i,j,l\})} = \frac{28}{70} = 0{,}4$	$\frac{\sum_{x=2}^{4} y(g_x\mid\{i,j,l\})}{b_i(g)=\sum_{x=0}^{4} y(g_x\mid\{i,j,l\})} = \frac{42}{70} = 0{,}6$
		$y(g_x \mid \{i,j,l\}) = 24$ for $x \in \{1,3\}$			
		$y(g_2 \mid \{i,j,l\}) = 6$			
		$y(g_4 \mid \{a,b,c\}) = 12$			
t_4	Figure A3.5	$y(g_x \mid \{i,j,l,m\}) = 6$ for $x \in \{0,1,2,3,4,5\}$	1	1	0
		$y(g_x \mid \{i,j,l,m\}) = 3$ for $x \in \{6,7,8\}$			

Table A3.3 in Example 7

Type of i	F.P. of i	$y(g_x \mid \{i\})$	$E_{U_i}(0, \sigma_4, B_i(g))$	$E_{U_i}(0, \sigma_5, B_i(g))$
t_1	Figure A3.2	$y(g_0 \mid \{i,j,l\}) = 6$	$\frac{\sum_{x=0}^{2} y(g_x\mid\{i,j,l\})}{b_i(g)=\sum_{x=0}^{5} y(g_x\mid\{i,j,l\})} = \frac{54}{102} = 0{,}529$	
		$y(g_x \mid \{i,j,l\}) = 24$ for $x \in \{1,2,3\}$		
		$y(g_x \mid \{i,j,l\}) = 12$ for $x \in \{4,5\}$.		
t_2	Figure A3.3	$y(g_x \mid \{i,j,l\}) = 24$ for $x \in \{0,3,4,5\}$	1	$\frac{144-\sum_{x\in\{5,7\}} y(g_x\mid\{i,j,l\})}{b_i(g)=\sum_{x=0}^{7} y(g_x\mid\{i,j,l\})} = \frac{108}{144} = 0{,}75$
		$y(g_x \mid \{i,j,l\}) = 12$ for $x \in \{1,2,6,7\}$		
t_3	Figure A3.4	$y(g_0 \mid \{i,j,l\}) = 4$	1	$\frac{\sum_{x=0}^{1} y(g_x\mid\{i,j,l\})}{b_i(g)=\sum_{x=0}^{4} y(g_x\mid\{i,j,l\})} = \frac{28}{70} = 0{,}4$
		$y(g_1 \mid \{i,j,l\}) = 24$ for $x \in \{1,3\}$		
		$y(g_2 \mid \{i,j,l\}) = 6$		
		$y(g_4 \mid \{a,b,c\}) = 12$		
t_4	Figure A3.5	$y(g_x \mid \{i,j,l,m\}) = 6$ for $x \in \{0,1,2,3,4,5\}$	0	1
		$y(g_2 \mid \{i,j,l,m\}) = 3$ for $x \in \{6,7,8\}$		

Table A3.4. Symmetric equilibria in network (a) under Setting B

Equilibrium strategies	c
σ_1	$0{,}059 \leq c \leq 0{,}5$
σ_2	$0{,}059 \leq c \leq 0{,}084$
σ_3	$0{,}4 \leq c \leq 0{,}471$
σ_4	$c \leq 0{,}471$
σ_5	$0{,}25 \leq c \leq 0{,}471$

Table A3.5 in Example 7

Type of i	F.P. of i	$y(g_x \mid \{i\})$	$E_{U_i}(0, \sigma_9, B_i(g))$	$E_{U_i}(0, \sigma_{10}, B_i(g))$	$E_{U_i}(0, \sigma_{11}, B_i(g))$
t_1	Figure A3.2	$y(g_0 \mid \{i,j,l\}) = 6$ $y(g_x \mid \{i,j,l\}) = 24$ for $x \in \{1,2,3\}$ $y(g_x \mid \{i,j,l\}) = 12$ for $x \in \{4,5\}$.	$\frac{\sum_{x=0}^{2} y(g_x \mid \{i,j,l\})}{b_i(g)=\sum_{x=0}^{5} y(g_x \mid \{i,j,l\})} = \frac{54}{102} = 0{,}529$	$\frac{\sum_{x=1}^{5} y(g_x \mid \{i,j,l\})}{b_i(g)=\sum_{x=0}^{5} y(g_x \mid \{i,j,l\})} = \frac{96}{102} = 0{,}941$	$\frac{\sum_{x=0}^{2} y(g_x \mid \{i,j,l\})}{b_i(g)=\sum_{x=0}^{5} y(g_x \mid \{i,j,l\})} = \frac{54}{102} = 0{,}529$
t_2	Figure A3.3	$y(g_x \mid \{i,j,l\}) = 24$ for $x \in \{0,3,4,5\}$ $y(g_x \mid \{i,j,l\}) = 12$ for $x \in \{1,2,6,7\}$	$\frac{\sum_{x \in \{0,1,3,6\}} y(g_x \mid \{i,j,l\})}{b_i(g)=\sum_{x=0}^{7} y(g_x \mid \{i,j,l\})} = \frac{72}{144} = 0{,}5$	$\frac{144-\sum_{x \in \{6\}} y(g_x \mid \{i,j,l\})}{b_i(g)=\sum_{x=0}^{7} y(g_x \mid \{i,j,l\})} = \frac{132}{144} = 0{,}916$	$\frac{144-\sum_{x \in \{5,7\}} y(g_x \mid \{i,j,l\})}{b_i(g)=\sum_{x=0}^{7} y(g_x \mid \{i,j,l\})} = \frac{108}{144} = 0{,}75$
t_3	Figure A3.4	$y(g_0 \mid \{i,j,l\}) = 4$ $y(g_1 \mid \{i,j,l\}) = 24$ for $x \in \{1,3\}$ $y(g_2 \mid \{i,j,l\}) = 6$ $y(g_4 \mid \{a,b,c\}) = 12$	0	$\frac{\sum_{x=0}^{1} y(g_x \mid \{i,j,l\})}{b_i(g)=\sum_{x=0}^{4} y(g_x \mid \{i,j,l\})} = \frac{28}{70} = 0{,}4$	
t_4	Figure A3.5	$y(g_x \mid \{i,j,l,m\}) = 6$ for $x \in \{0,1,2,3,4,5\}$ $y(g_2 \mid \{i,j,l,m\}) = 3$ for $x \in \{6,7,8\}$	1	1	0

Table A3.6 in Example 7

Type of i	F.P. of i	$y(g_x \mid \{i\})$	$E_{U_i}(0, \sigma_6, B_i(g))$	$E_{U_i}(0, \sigma_7, B_i(g))$	$E_{U_i}(0, \sigma_8, B_i(g))$
t_2	Figure A3.3	$y(g_x \mid \{i,j,l\}) = 24$ for $x \in \{0,3,4,5\}$	1	$\frac{\sum_{x\in\{5,6,7\}} y(g_x\|\{i,j,l\})}{b_i(g)=\sum_{x=0}^{7} y(g_x\|\{i,j,l\})} = \frac{48}{144} = 0{,}333$	$\frac{144-\sum_{x\in\{5,6,7\}} y(g_x\|\{i,j,l\})}{b_i(g)=\sum_{x=0}^{7} y(g_x\|\{i,j,l\})} = \frac{96}{144} = 0{,}666$
		$y(g_x \mid \{i,j,l\}) = 12$ for $x \in \{1,2,6,7\}$			
t_3	Figure A3.4	$y(g_0 \mid \{i,j,l\}) = 4$	1	$\frac{\sum_{x=2}^{4} y(g_x\|\{i,j,l\})}{b_i(g)=\sum_{x=0}^{4} y(g_x\|\{i,j,l\})} = \frac{42}{70} = 0{,}6$	$\frac{\sum_{x=0}^{1} y(g_x\|\{i,j,l\})}{b_i(g)=\sum_{x=0}^{4} y(g_x\|\{i,j,l\})} = \frac{28}{70} = 0{,}4$
		$y(g_x \mid \{i,j,l\}) = 24$ for $x \in \{1,3\}$			
		$y(g_2 \mid \{i,j,l\}) = 6$			
		$y(g_4 \mid \{a,b,c\}) = 12$			
t_4	Figure A3.5	$y(g_x \mid \{i,j,l,m\}) = 6$ for $x \in \{0,1,2,3,4,5\}$	0	0	1
		$y(g_2 \mid \{i,j,l,m\}) = 3$ for $x \in \{6,7,8\}$			

Table A3.7 Symmetric equilibria in networks (b) and (c) under Setting B

Equilibrium strategies	c
σ_1	$c \leq 0{,}5$
σ_2	$c \leq 0{,}084$
σ_3	$c \geq 0{,}4$
σ_4	$0 \leq c \leq 1$
σ_5	$0{,}25 \leq c \leq 0{,}6$
σ_6	$0 \leq c \leq 1$
σ_7	$c \geq 0{,}67$
σ_8	$0{,}33 \leq c \leq 0{,}67$

Figure A3.2. Feasible positions of an i of type $\big(2;(2,2)\big)$ under Setting B

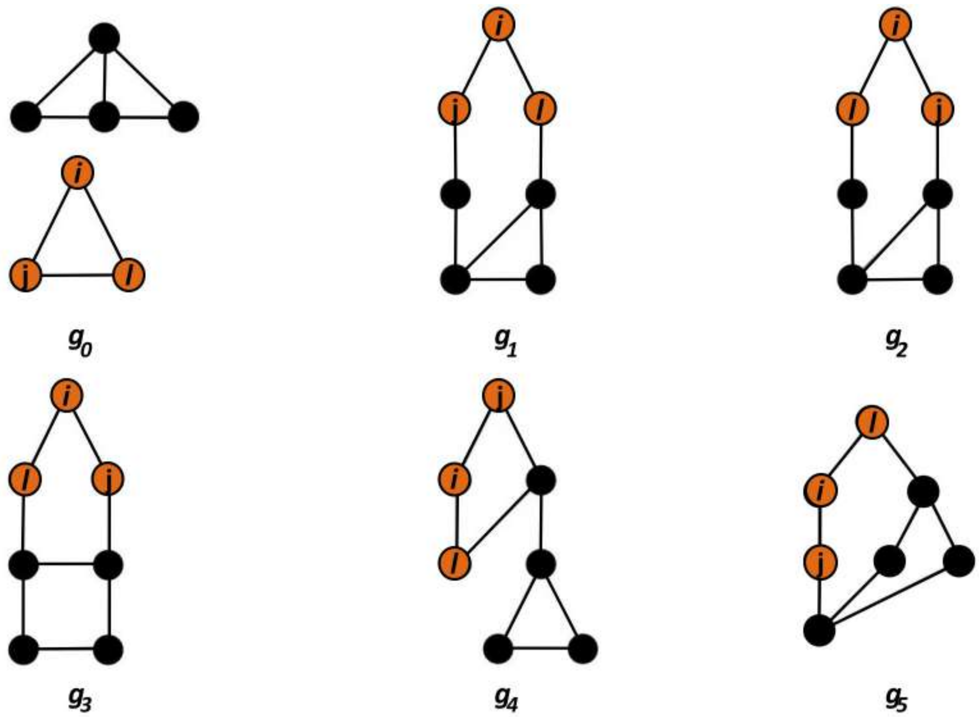


Figure A3.3. Feasible positions of an i of type $\big(2;(3,2)\big)$ under Setting B

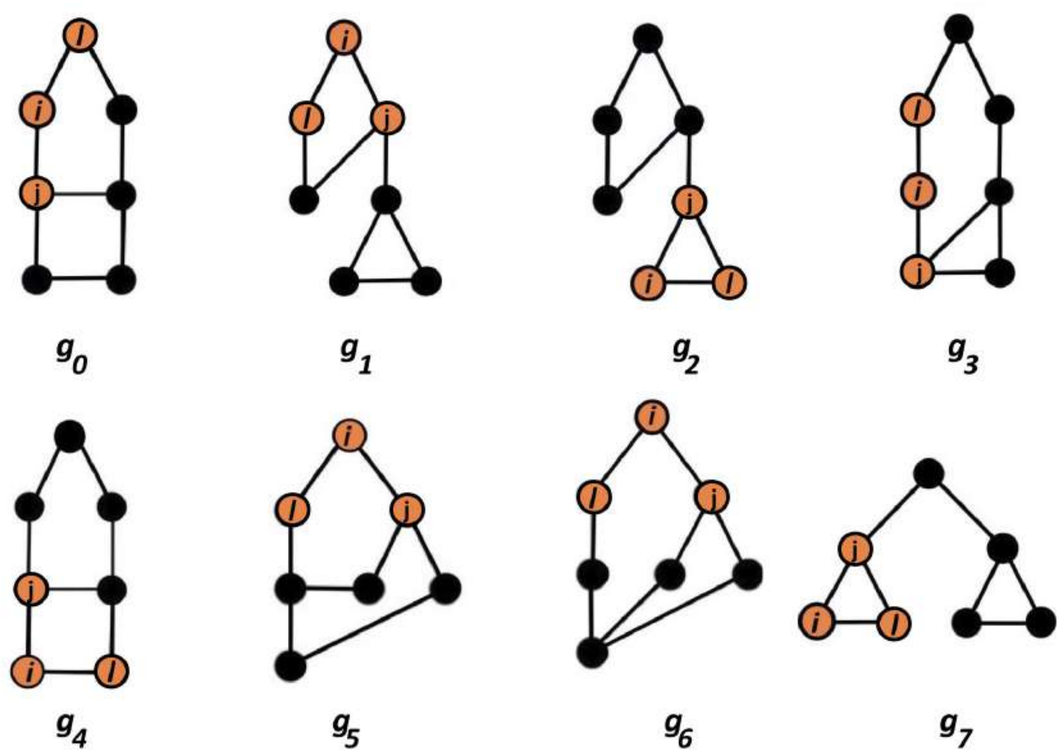


Figure A3.4 Feasible positions of an i of type $\big(2;(3,3)\big)$ under Setting B

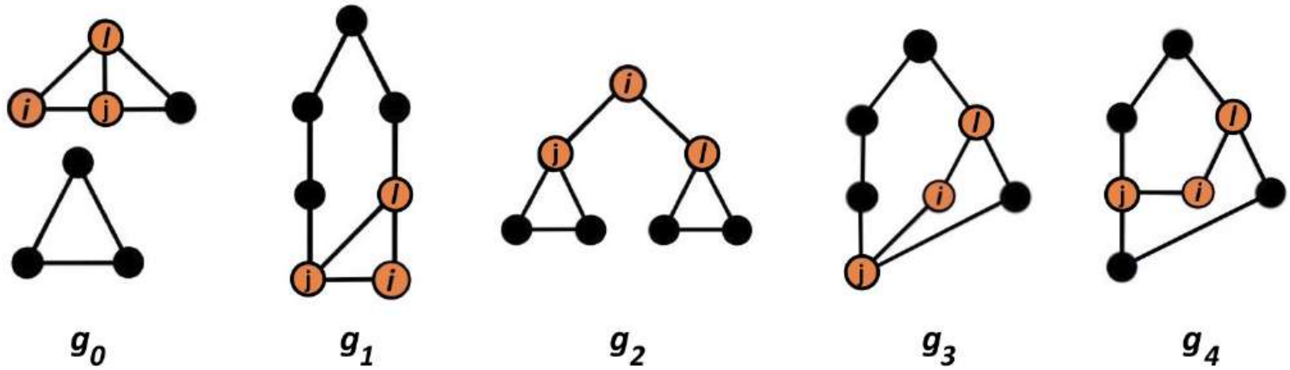
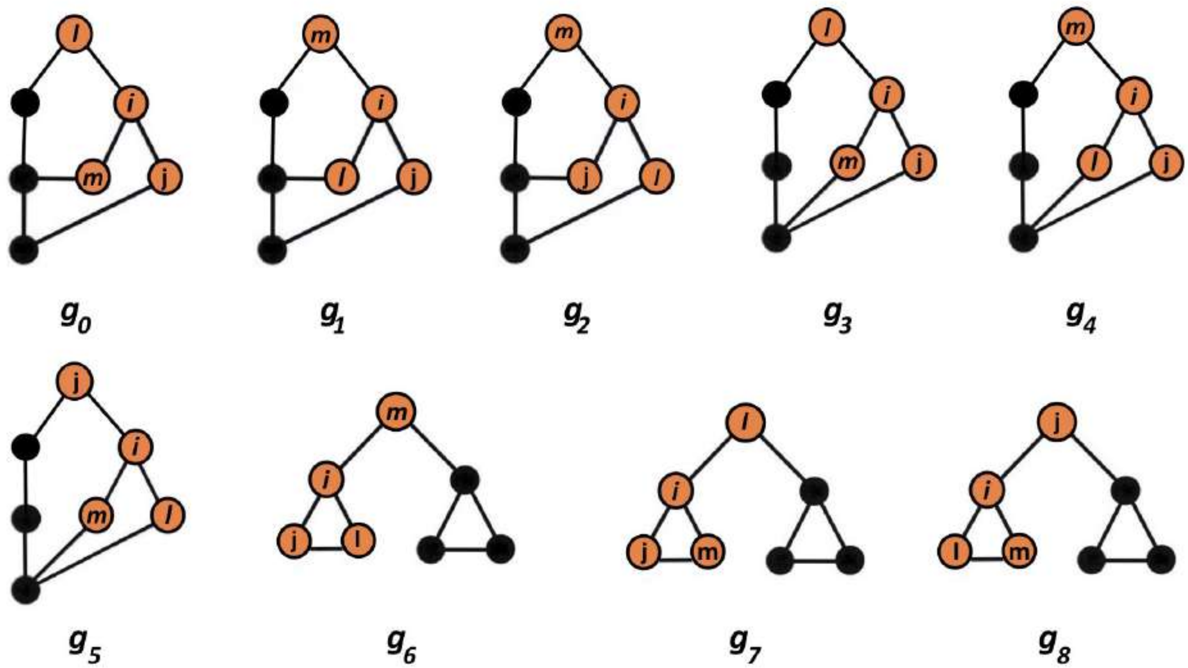


Figure A3.5 Feasible positions of an i of type $\big(3;(2,2,2)\big)$ under Setting B



B) Symmetric equilibria under Setting C.

Network (a). Tables A3.8 and A3.9 show the expected utility of playing 0 of types 1 and 2 for each equilibrium strategy. Under Setting C, type-three players observe that her two neighbors are not linked. Thereby, they know that their neighbors are type 4. If all type-four players play 1, best response of type-three players is playing 0 and viceversa. Consequently, the strategies σ_{10} and σ_{11} (defined in point A) are not equilibrium strategies for any value of c.

Each type-four player can deduce the whole network from the information she is given. Hence, she can deduce that two neighbors of her are type 2 and one neighbor of her is type 3. Then, a type-four player is best responding with action 1 if neither type-two players nor type-three players play action 1, and with action 0 if either type-two players or type-three players (or both) play action 1.

Table A3.10 provides the symmetric equilibria in pure strategies that exist for each range of c. Note that strategies σ_6, σ_7 and σ_8 are not equilibrium strategies: as explained above, if all players of type 1 and 2 play 0, best response of a type-one player is not playing 0 but playing 1.

Figure A3.6. Feasible positions of an i of type $\big(2;(2,2)\big)$ under Setting C

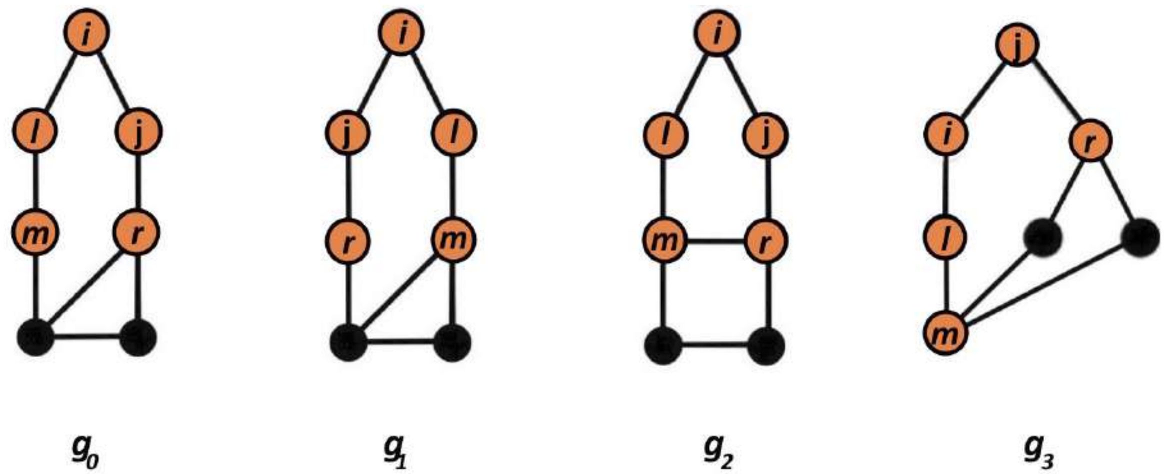


Figure A3.7. Feasible positions of an i of type $\big(2;(3,2)\big)$ under Setting C

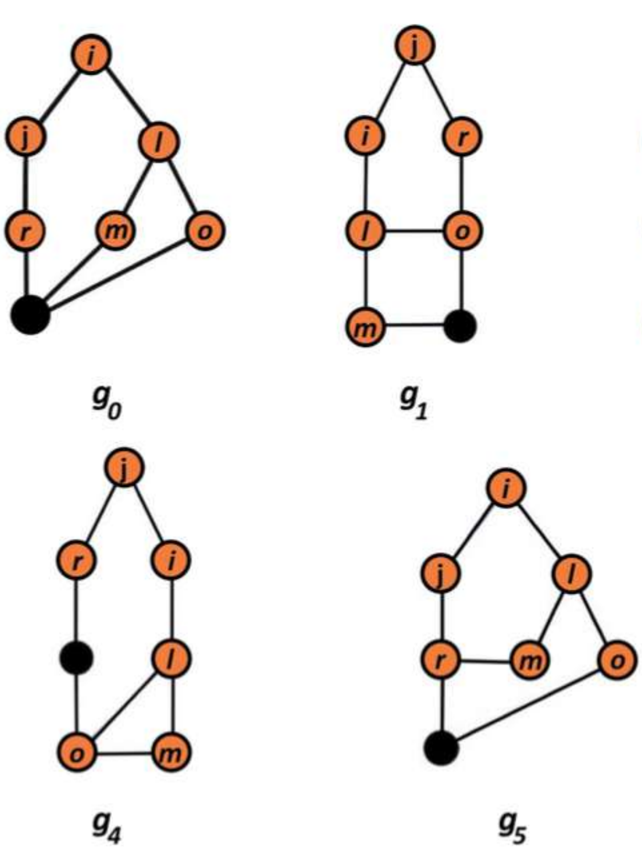


Table A3.8 in Example 7

Type of i	F.P. of i	$y(g_x \mid \{i\})$	$E_{U_i}(0, \sigma_1, B_i(g))$	$E_{U_i}(0, \sigma_2, B_i(g))$	$E_{U_i}(0, \sigma_3, B_i(g))$
t_1	Figure A3.6	$y(g_x \mid \{i,j,l,m,r\}) = 2$ for $x \in \{0,1,2\}$ $y(g_3 \mid \{i,j,l,m,r\}) = 1$	1		$\frac{\sum_{x=0}^{1} y(g_x\|\{i,j,l,m,r\})}{b_i(g)=\sum_{x=0}^{3} y(g_x\|\{i,j,l,m,r\})} = \frac{4}{7}$ $= 0{,}571$
t_2	Figure A3.7	$y(g_x \mid \{i,j,l,m,r,o\}) = 1$ for $x \in \{0,1,2,3,4,5,6\}$	$\frac{\sum_{x=5}^{6} y(g_x\|\{i,j,l,m,r,o\})}{b_i(g)=\sum_{x=0}^{6} y(g_x\|\{i,j,l,m,r,o\})} = \frac{2}{7}$ $= 0{,}2857$	$\frac{7-y(g_6\|\{i,j,l,m,r,o\})}{b_i(g)=\sum_{x=0}^{6} y(g_x\|\{i,j,l,m,r,o\})} = \frac{6}{7}$ $= 0{,}857$	1

Table A3.9 in Example 7

Type of i	F.P. of i	$y(g_x \mid \{i\})$	$E_{U_i}(0, \sigma_4, B_i(g))$	$E_{U_i}(0, \sigma_5, B_i(g))$	$E_{U_i}(0, \sigma_9, B_i(g))$
t_1	Figure A3.6	$y(g_x \mid \{i,j,l,m,r\}) = 2$ for $x \in \{0,1,2\}$ $y(g_3 \mid \{i,j,l,m,r\}) = 1$	$\frac{\sum_{x=0}^{1} y(g_x\|\{i,j,l,m,r\})}{b_i(g)=\sum_{x=0}^{3} y(g_x\|\{i,j,l,m,r\})} = \frac{4}{7}$ $= 0{,}571$		
t_2	Figure A3.7	$y(g_x \mid \{i,j,l,m,r,o\}) = 1$ for $x \in \{0,1,2,3,4,5,6\}$	1	$\frac{7-\sum_{x\in\{5,6\}} y(g_x\|\{i,j,l,m,r,o\})}{b_i(g)=\sum_{x=0}^{6} y(g_x\|\{i,j,l,m,r,o\})} = \frac{5}{7}$ $0{,}714$	

Table A3.10. Symmetric equilibria in network (a) under Setting C

Equilibrium strategies	c
σ_1	$c \leq 0{,}714$
σ_2	$c \leq 0{,}143$
σ_3	$c \leq 0{,}429$
σ_4	$c \leq 0{,}429$
σ_5	$0{,}286 \leq c \leq 0{,}429$
σ_9	$0{,}286 \leq c \leq 0{,}429$

Network (b). Recall that network (b) is exclusively integrated by players of types 2, 3 and 4, and there exist an equilibrium as long as these types are playing their best response to their neighbors' actions.

Each type-two player observes that her degree-two neighbor is type 3. As a consequence, there is not any symmetric equilibrium in which type-two players play action 1: if all players of type 2 play action 1, the expected utility of playing 0 of each type-two player is 1, so this player is not best responding with action 1 but with action 1. Note that σ_9 is not an equilibrium strategy in this network: since $E_{U_i}(0, \sigma_9, B_i(g)) = 0$ for each i with type 2, best response of i is playing action 1 and not action 0.

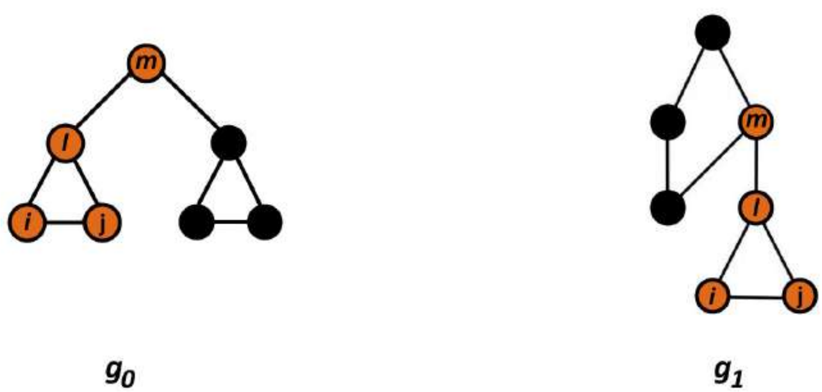
Figure A3.8 shows the feasible positions of each i of type 2, according to $B_i(g)$. Considering these positions:

$$E_{U_i}(0, \sigma_3, B_i(g)) = E_{U_i}(0, \sigma_7, B_i(g)) = \frac{y(g_0 \mid \{i,j,l,m\})}{b_i(g) = \sum_{x=0}^{1} y(g_0 \mid \{i,j,l,m\})} = \frac{3}{6} = 0{,}5$$

$$E_{U_i}(0, \sigma_5, B_i(g)) = E_{U_i}(0, \sigma_8, B_i(g)) = \frac{y(g_1 \mid \{i,j,l,m\})}{b_i(g) = \sum_{x=0}^{1} y(g_0 \mid \{i,j,l,m\})} = \frac{3}{6} = 0{,}5$$

and $E_{U_i}(0, \sigma_4, B_i(g)) = E_{U_i}(0, \sigma_6, B_i(g)) = 1$, for each i of type 2.

Figure A3.8. Feasible positions of an i of type $\big(2; (3,2)\big)$ under Setting C.



As for network (a), players of type 3 and 4 deduce her neighbors' types from $I_i(g)$. Players of type 3 are best responding with action 0(1) if players of type 4 play 1(0), while players of type 4 are best responding with action 1(0) if neither players with type 2 nor players with type 3 take this action.

Table A3.11. Symmetric equilibria in network (b) under Setting C

Equilibrium strategies	c
σ_3	$c \geq 0{,}5$
σ_4	$0 \leq c \leq 1$
σ_5	$c \geq 0{,}5$
σ_6	$0 \leq c \leq 1$
σ_7	$c \geq 0{,}5$
σ_8	$c \geq 0{,}5$

Network (c). As in the previous case, network (c) is exclusively integrated by players of types 2, 3 and 4; if these types are playing their best response to their neighbors' actions there exist an equilibrium.

Under Setting C, type-two players have identical beliefs about their neighbors' types in network (c) as in network (a), since the geometry created by their neighbors' links is identical in both networks. Hence, the expected utility of playing 0 of a type-two player when all agents play a strategy in $\{\sigma_1, \sigma_2, \sigma_3, \sigma_4, \sigma_5, \sigma_9\}$ is given by Table A3.8 and Table A3.9. As in network (a), players of type 3 and type 4 deduce their neighbors' types from $I_i(g)$.

Table A3.12 lists the expected utility of playing 0 of each type-two player when all agents play either the strategy σ_6, σ_7 or σ_8. Tables A3.13 provides the symmetric equilibrium strategies in network (c) for each range of cost under Setting C.

Table A3.12 in Example 7

Figure	Feasible positions of i of type 2	$E_{U_i}(0, \sigma_6, B_i(g))$	$E_{U_i}(0, \sigma_7, B_i(g))$	$E_{U_i}(0, \sigma_8, B_i(g))$
$C3.2$	$y(g_x \mid \{i,j,l,m,r,o\}) = 1$ for $x \in \{0,1,2,3,4,5,6\}$	1	$\frac{\sum_{x\in\{0,5,6\}} y(g_x\mid\{i,j,l,m,r,o\})}{b_i(g)=\sum_{x=0}^{6} y(g_x\mid\{i,j,l,m,r,o\})} = \frac{3}{7}$ $= 0{,}428$	$\frac{7-\sum_{x\in\{0,5,6\}} y(g_x\mid\{i,j,l,m,r,o\})}{b_i(g)=\sum_{x=0}^{6} y(g_x\mid\{i,j,l,m,r,o\})} = \frac{4}{7}$ $= 0{,}571$

Table A3.13. Symmetric equilibria in network (c) under Setting C

	Network (c)
	c
σ_1	$c \leq 0{,}714$
σ_2	$c \leq 0{,}143$
σ_3	$0 \leq c \leq 1$
σ_4	$0 \leq c \leq 1$
σ_5	$c \geq 0{,}286$
σ_6	$0 \leq c \leq 1$
σ_7	$c \geq 0{,}572$
σ_8	$c \geq 0{,}429$
σ_9	$c \geq 0{,}286$

■

B2. Equilibrium welfare under Setting A

Lemma A. *Let $g = (N, E)$ and $g' = (N', E')$ be two networks such that $D_g(k) = D_{g'}(k)$, $\forall k$. Let σ be a symmetric strategy under Setting A in g and in g'.*

1. *For the game of strategic substitutes,*

$$\theta_k(\sigma, g) = |\{i \in N : \sigma(k_i) = 0 \wedge \sigma(k_j) = 1, \mathit{for\ some}\ j \in N_i(g), k_i = k\}|$$

2. *For the game of strategic complements,*

$$\theta_k(\sigma, g) = |\{i \in N : \sigma(k_i) = 1 \wedge \sigma(k_j) = 1, \mathit{for\ some}\ j \in N_i(g), k_i = k\}|$$

3. *In both games, let $\theta(\sigma, g) = \sum_{k=1}^{n-1} \theta_k(\sigma, g)$. If $W(\sigma, g) < W(\sigma, g')$, then $\theta_k(\sigma, g) < \theta_k(\sigma, g')$, for at least one $k \in \{1, ..., n-1\}$.*

Proof. Since $D_g(k) = D_{g'}(k)$ $\forall k$, when all integrants of g and all integrants of g' play the strategy σ, the number of agents who play action 1(0) is the same in g as in g'. This means that:

1. For the game of strategic substitutes, $\sum_{i\in N:\sigma(k_i(g))=1} u_i\big(1, x_{N_i(g)}\big) = \sum_{i\in N':\sigma(k_i(g'))=1} u_i\big(1, x_{N_i(g')}\big)$

2. For the game of strategic complements, $\sum_{i\in N:\sigma(k_i(g))=0} u_i\big(0, x_{N_i(g')}\big) = \sum_{i\in N':\sigma(k_i(g'))=0} u_i\big(0, x_{N_i(g')}\big) = 0$

Hence, welfare in g can only be lower than in g' if $\theta(\sigma, g) < \theta(\sigma, g')$. If $\theta_k(\sigma, g) \geq \theta_k(\sigma', g)$ $\forall k$, then $\theta(\sigma, g) \geq \theta(\sigma, g')$, and $W(\sigma, g) \geq W(\sigma, g')$. Therefore, if $W(\sigma, g) < W(\sigma, g')$, $\theta_k(\sigma, g)$ must be strictly lower than $\theta_k(\sigma, g')$ for at least one $k \in \{1, ..., n-1\}$. ■

Proof of Proposition 2.
1. *Game of strategic substitutes.*
Recall that $\Omega_i(g) = \{1, 2, ..., \omega_i(g)\}$ is the set of feasible geometries, and this set is identical for all $i \in N$ under Setting A (see Section 4.1). Hence, $\Omega_i(g) = \Omega$ and $\omega_i(g) = |\Omega_i(g)| = \omega$ under Setting A. We assume that $g_z \in B_i^z(g)$, $\forall z \in \Omega_i(g)$ (i.e. g_1 is a feasible network with geometry 1, g_2 is a feasible geometry with geometry 2, and so forth).

Step 1. We first calculate $E_{U_i}\big(0, \sigma^*, B_i(g)\big)$.

Let $K_i(g_z) = \{j, l, ..., m\}$ be a maximal i-ed set in $g_z \in B_i^z(g)$. Each agent in $K_i(g_z)$ occupies a feasible position of i under Setting A; given $I_i(g)$ there exist a positive probability that i occupies the position that each agent occupies in $K_i(g_z)$. Let $\bar{K}_i(g_z) \subseteq K_i(g_z)$ be a subset of agents in $K_i(g_z)$:

$$\bar{K}_i(g_z) = \{j \in K_i(g_z) : \exists q \in N_j(g_z) : \sigma^*(k_q) = 1\}$$

Agent i deduces that, if network g (the network she is embedded in) has geometry z, she has a neighbor that plays action 1 if she occupies the position that an agent in $\bar{K}_i(g_z)$ occupies in g_z. Assume $\bar{K}_i(g_z) = \{j, l, ..., y\}$. Applying Lemma 1 and the Orbit-Stabilizer Theorem, there exist $y(g_z \mid \{j\}) = \frac{(n-1)!}{|Stab(\{j\})|} = \frac{(n-1)!|O_j(g_z)|}{|Aut(g_z)|}$ distinct networks in $B_i(g)$ in which i occupies the position that j occupies in g_z, $y(g_z \mid \{l\}) = \frac{(n-1)!}{|Stab(\{l\})|} = \frac{(n-1)!|O_l(g_z)|}{|Aut(g_z)|}$ distinct networks in $B_i(g)$ in which i occupies the position that l occupies in g_z, and analogously for all other agents in $\bar{K}_i(g_z) = \{j, l, ..., y\}$. Hence, there are $\sum\limits_{j \in \bar{K}_i(g_z)} \frac{(n-1)!|O_j(g_z)|}{|Aut(g_z)|}$ (isomorphic) networks in $B_i^z(g) \subseteq B_i(g)$ in which i has a neighbor that plays action 1.

The expected utility of i of playing 0 when all agents play the symmetric strategy σ^* is the probability that at least one of her neighbors is a type that plays action 1, considering all feasible network geometries $\Omega_i(g) = \{1, 2, ..., \omega\}$:

$$E_{U_i}\big(0, \sigma^*, B_i(g)\big) = \frac{\sum\limits_{j \in \bar{K}_i(g_1)} \frac{(n-1)!|O_j(g_1)|}{|Aut(g_1)|} + \sum\limits_{j \in \bar{K}_i(g_2)} \frac{(n-1)!|O_j(g_2)|}{|Aut(g_2)|} + ... + \sum\limits_{j \in \bar{K}_i(g_\omega)} \frac{(n-1)!|O_j(g_\omega)|}{|Aut(g_\omega)|}}{b_i(g)} \tag{3.7.1}$$

When all agents play the stategy σ^*, $\theta_k(\sigma^*, g)$ is the total number of agents in g with degree $k = k_i$ that play 0 and are linked to some agent that plays 1 (see Lemma A point 1 in the Appendix B2). Then, the right side of (3.7.1) is equal to:

$$= \frac{(n-1)!}{b_i(g)} \left[\frac{\theta_k(\sigma^*, g_1)}{|Aut(g_1)|} + \frac{\theta_k(\sigma^*, g_2)}{|Aut(g_2)|} + ... + \frac{\theta_k(\sigma^*, g_\omega)}{|Aut(g_\omega)|} \right] \tag{3.7.2}$$

Note that $b_i(g) = b_i^1(g) + b_i^2(g) + ... + b_i^\omega(g) = \frac{(n-1)!D_g(k_i)}{|Aut(g_1)|} + \frac{(n-1)!D_g(k_i)}{|Aut(g_2)|} + ... + \frac{(n-1)!D_g(k_i)}{|Aut(g_\omega)|}$ (see Proposition 1). Taking into account this and operating, (3.7.2) becomes:

$$\frac{\alpha_1 \theta_k(\sigma^*, g_1) + \alpha_2 \theta_k(\sigma^*, g_2) + ... + \alpha_\omega \theta_k(\sigma^*, g_\omega)}{D_g(k_i)\left[\alpha_1 + \alpha_2 + ... + \alpha_\omega\right]} \tag{3.7.3}$$

where $\alpha_z = \frac{1}{|Aut(g_z)|}$, $\forall z \in \Omega$. In equilibrium, i plays action 0 if $E_{U_i}\big(0, \sigma^*, B_i(g)\big) \geq 1 - c$, that is, if (3.7.4) holds:

$$1 - \frac{\alpha_1 \theta_k(\sigma^*, g_1) + \alpha_2 \theta_k(\sigma^*, g_2) + ... + \alpha_\omega \theta_k(\sigma^*, g_\omega)}{D_g(k_i)\left[\alpha_1 + \alpha_2 + ... + \alpha_\omega\right]} \leq c \tag{3.7.4}$$

Step 2. We show now that, when all agents play σ^*, network g is efficient if $\frac{\rho_z}{D_g(k)} > c$.

Suppose network g has geometry 1, and there exist a network g_2 (with geometry 2) with the same degree sequence as g at which $W(\sigma^*, g_2) > W(\sigma^*, g)$. By Lemma A (in the Appendix B2), this can only occur if

$\theta_k(\sigma^*, g_2) > \theta_k(\sigma^*, g) = \theta_k(\sigma^*, g_1)$ for at least one $k \in \{1, ..., n-1\}$, say, $k = k_i$. Imagine this is the case, $\theta_k(\sigma^*, g_1) = \theta_k(\sigma^*, g_2) - \pi$, $\pi > 0$. Then the left part of (3,7,4) can be expressed as:

$$1 - \frac{\alpha_1 \left(\theta_k(\sigma^*, g_2) - \pi\right) + \alpha_2 \theta_k(\sigma^*, g_2) + ... + \alpha_\omega \theta_k(\sigma^*, g_\omega)}{D_g(k_i)\left[\alpha_1 + \alpha_2 + ... + \alpha_\omega\right]} =$$

$$1 - \frac{(\alpha_1 + \alpha_2)\theta_k(\sigma^*, g_2) + ... + \alpha_\omega \theta_k(\sigma^*, g_\omega)}{D_g(k_i)\left[\alpha_1 + \alpha_2 + ... + \alpha_\omega\right]} + \frac{\pi \alpha_1}{D_g(k_i)\left[\alpha_1 + \alpha_2 + ... + \alpha_\omega\right]} \tag{3.7.5}$$

Observe that $\frac{\alpha_1}{D_g(k_i)[\alpha_1+\alpha_2+...+\alpha_\omega]} = \frac{\rho_z}{D_g(k_i)}$ (see Corollary 1). If $\frac{\rho_z}{D_g(k_i)} = \frac{\rho_z}{D_g(k)} > c$, $E_{U_i}\big(0, \sigma^*, B_i(g)\big) < 1 - c$ for all $i \in N$, and no player is best responding with action 0. Hence, it cannot exist a symmetric equilibrium at which welfare is greater in g_2 than in g.

2. *Game of strategic complements.*
Reasoning is analogous for the game of strategic complements. In this case,

$$E_{U_i}\big(1, \sigma^*, B_i(g)\big) = -c + \frac{\alpha_1 \theta_k(\sigma^*, g_1) + \alpha_2 \theta_k(\sigma^*, g_2) + ... + \alpha_\omega \theta_k(\sigma^*_w, g)}{D_g(k_i)\left[\alpha_1 + \alpha_2 + ... + \alpha_\omega\right]}$$

where $\theta_k(\sigma^*, g_x)$ $(x = 1, 2, ..., \omega)$ is defined in Lemma A (point 2) in Appendix B2. Player i is best responding with action 1 if $E_{U_i}\big(1, \sigma^*, B_i(g)\big) \geq 0$.
Suppose network g has geometry 1. Consider a network g_2 with geometry 2 and the same degree sequence as g. By Lemma A, $W(g_2, \mathtt{x}^*_\mathtt{N}) > W(g, \mathtt{x}^*_\mathtt{N}) =$ if $\theta_k(\sigma^*, g_2) > \theta_k(\sigma^*, g) = \theta_k(\sigma^*, g_1)$ for some $k \in \{1, ..., n-1\}$. Suppose this is the case, $\theta_k(\sigma^*, g_1) = \theta_k(\sigma^*, g_2) - \pi$, $\pi > 0$. Then,

$$\begin{aligned} E_{U_i}\big(1, \sigma^*, B_i(g)\big) &= -c + \frac{\alpha_1 \left(\theta_k(\sigma^*, g_2) - \pi\right) + \alpha_2 \theta_k(\sigma^*, g_2) + ... + \alpha_\omega \theta_k(\sigma^*, g_\omega)}{D_g(k_i)\left[\alpha_1 + \alpha_2 + ... + \alpha_\omega\right]} \\ &= -c + \frac{(\alpha_1 + \alpha_2)\theta_k(\sigma^*, g_2) + ... + \alpha_\omega \theta_k(\sigma^*, g_\omega)}{D_g(k_i)\left[\alpha_1 + \alpha_2 + ... + \alpha_\omega\right]} - \frac{\pi \alpha_1}{D_g(k_i)\left[\alpha_1 + \alpha_2 + ... + \alpha_\omega\right]} \end{aligned}$$

and i is best responding with action 1 if

$$\frac{(\alpha_1 + \alpha_2)\theta_k(\sigma^*, g_2) + ... + \alpha_\omega \theta_k(\sigma^*, g_\omega)}{D_g(k_i)\left[\alpha_1 + \alpha_2 + ... + \alpha_\omega\right]} - \frac{\pi \alpha_1}{D_g(k_i)\left[\alpha_1 + \alpha_2 + ... + \alpha_\omega\right]} \geq c \tag{3.7.6}$$

Note that the left side of (3.7.6) cannot be greater than $1 - \frac{\rho_z}{D_g(k_i)} = 1 - \frac{\alpha_1}{D_g(k_i)[\alpha_1+\alpha_2+...+\alpha_\omega]}$. Then, if $\frac{\rho_z}{D_g(k_i)} > 1 - c$, (3.7.6) does not hold, and no i is best responding with action 1. Hence, it does not exist a symmetric equilibrium at which welfare is greater in network g_2 than in network g. ■

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