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Information frictions and policy in DSGE models

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Ph.D Thesis

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Contents

General introduction

In this thesis I explore key aspects of general equilibrium models widely used in Central Banks, both in terms of theoretical assumptions and practical implications for policy makers. This document is divided in four chapters. In the first two chapters I study an alternative to the rational expectation assumption and its implications in the understanding of the business cycle, while in the last two chapters I explore policy questions related to the use of these models in policy.

General equilibrium models traditionally used in the design of monetary policy start from the premise that agents form their expectations about the economy in a rational manner. Under rational expectations, agents have full information about the true economic model, and use it accordingly to form their predictions. In particular, agents are capable of understanding the nature of macroeconomic shocks and their duration, and have the ability to consistently incorporate news about the expected evolution of the economy or changes in monetary policy into their expectations. The theory of rational expectations has been embedded not only into real business cycle models, but also into more realistic New Keynesian models, including many of the DSGE models that are used for policy purposes in central banks. However, in reality, agents are unlikely to have perfect ability to observe and process in an efficient manner all available information. Thus, on many occasions, the nature of the disturbances, or their transmission channels, are only imperfectly known by the agents or partially ignored. Alternatives to this hypothesis have been largely debated in the literature, the first two chapters explore different aspects of the adaptive learning approach as an alternative to rational expectations hypothesis, while the last two chapter explore modelling implications in the policy use of DSGE models.

More precisely, the first chapter assesses the importance of term structure and survey data information to the adaptive learning literature and the capability of macro-financial DSGE models with learning expectations to estimate a measure of the term premium associated with the 10-year US Treasury bond yield. The introduction of survey data adds a source of discipline in expectations under adaptive learning, which otherwise, are often criticized of arbitrary. In this context, this chapter finds that adding term structure information in agent's forecasting models improves the overall fit and does a great job in matching the expectations reported in the SPF across all forward-looking variables of the DSGE model. The rationale for this finding is that the SPF forecasts are based on real-time data and the term spread information included in our small forecasting models is also available in real time. These two pieces of real-time data—SPF and the yield curve— may therefore share important information in forecasting the economic outlook. This is consistent with the previous finding that the term structure contains useful information for forecasting real-time macroeconomic data. The second part of the chapter extends this model up to 10-years to estimate a measure of the term premium associated with the 10-year US Treasury bond yield from the medium-scale DSGE model under AL, showing that the inclusion of both term structure and survey data improves the estimation of the bond term premium, in line with the from no-arbitrage affine term structure models.

The second chapter looks at the anchoring of inflation expectations in the Euro Area and the performance of alternative monetary policy rules using a DSGE model with adaptive learning. The approach used allows a distinction to be drawn between which portion of the low inflation phenomenon might be due to temporary factors and which might be considered permanent. The results of the analysis for the euro area suggest that agents perceive the inflation rate's recent departure from the monetary policy objective to be predominantly temporary, although the deviations from target are marked by a considerable degree of persistence. Another relevant aspect is the impact of a prolonged period of low inflation in the effectiveness of monetary policy, and more importantly, under the presence of the effective lower bound. The second part of the chapter studies the properties of the monetary policy regime under the current expectations and studies the transitional effects caused by the change in inflation expectations of alternative regimes such as asymmetric inflation targeting and price-level targeting, now popular in the academic debate. The results show that while current expectations are curbing the effectiveness of monetary policy under the presence of the zero lower bound, alternative rules such asymmetric inflation targeting rules (that respond stronger when inflation is below trend) are beneficial to the economy. In addition, this chapter states the implications in the transition from one monetary policy rule to another, showing that changing the rule is not very effective until agents have had time to learn about it. The the announcement of the new rule has the maximum effect agents observe their implementation and learn about it, which requires time, which is very different from what the standard rational expectations models, where the announcement perfectly anchors agents' expectations and has immediate effects in the economy.

The third chapter devotes to the importance of real-time data and data revisions in the business cycle analysis. The main macroeconomic series are regularly revised relative to their real-time release to incorporate new information, which often, are significant and, if ignored, may lead to a bias in the study of the business cycle. This chapter provides a detailed analysis of the statistical properties of data revisions for the euro area and studies the appropriate modeling of real-time data and its revision in DSGE models for business cycle analysis. The first part of the chapter provides studies the statistical properties of data revisions in the euro area, showing that the series of GDP, consumption and inflation are predictable (they are correlated with the initial announcement) and have high volatility, suggesting that they are not well-behaved and studies the appropriate characterization of the data revision processes for its later inclusion in DSGE models. The second part of the paper details how to include real-time data and its revision in a DSGE model, by assuming that decisions related to GDP, Consumption and inflation are based on the initial announcement, and acknowledging that they are subject to revisions. This approach delivers two important results: first, it confirms the empirical findings from the reduced-form analysis and second, data revisions become an important source in the business cycle decomposition analysis. In the case of the Euro Area, they account for one third of the output variability, leading to the conclusion that DSGE models omitting real-time data and data revisions might be ignoring important sources of aggregate fluctuations.

Finally the fourth chapter has an important policy viewpoint, by assessing quantitatively the transmissions of macroprudential policies in the economy. Macroprudential policies are an important toolkit of central banks nowadays, this includes borrower-based macroprudential measures such as limits on loan-to-value and loan-to-income which their assessment has been traditionally in partial equilibrium models. By combining the model with information on the distribution of loan-to-value and loan-to-income ratios contained in the loan data, this paper tracks the impact of borrower-based measures from their impact on credit conditions at loan origination to the long-term macroeconomic effects on GDP, credit, real estate investment as well as mortgage defaults and mortgage spreads. The assessment reveals that borrower-based measures have sizable effects on credit amounts and can reduce long-run defaults. Its assessment is nevertheless limited to long-term effects, given limitation in the relatively simple way the real estate market is modeled. It opens up extension possibilities to develop additional models to shed light on the detailed working of the real estate market by focusing on additional sources of shocks and the role played by expectations for real estate prices.

Part I

Adaptive learning with term structure information

1 Introduction

Since the pioneering publications by Marcet and Sargent (1989) and Evans and Honkapohja (2001) a growing body of literature (including Preston, 2005; Milani, 2007, 2008, 2011; Eusepi and Preston, 2011; Slobodyan and Wouters, 2012a,b) has considered adaptive learning (AL) as an alternative to the rational expectations (RE) assumption in characterizing highly persistent macroeconomic dynamics. Recent papers (Sinha 2015, 2016) focus on some implications of AL in the yield curve, but there are still a few papers (e.g. Aguilar and Vázquez, 2019) that analyze how term structure information may interact with both learning and macroeconomic dynamics.

This paper considers the Euler-equation approach to AL suggested in Slobodyan and Wouters (2012a) to understand the contribution of term structure information in dealing with the incomplete knowledge issue addressed in the related AL literature.[1,2] Term structure

With Jesús Vázquez (UPV/EHU)

[1]There are two main approaches to AL in the recent literature. The Euler-equation approach focuses on short-sighted agents, for whom optimal current decisions are based on just one-period-ahead expectations that show up in the standard Euler equations (e.g. Milani, 2007; Slobodyan and Wouters, 2012a,b), while the other approach focuses on long-sighted agents (e.g. Preston, 2005; Eusepi and Preston, 2011; Sinha, 2015; and Sinha; 2016), taking into account infinite-horizon forecasts driven by their intertemporal decision problem. This distinction can be crucial because the second approach results in a much stronger source of persistent dynamics (see Eusepi and Preston, 2011). By including the term structure of interest rates, our approach certainly goes beyond the one-period-ahead expectations, but still follows the Euler-equation approach.

[2]More generally, the Euler-equation approach falls under the broad class of a restricted perceptions equilibrium, where agents use a small misspecified model but form their beliefs optimally given the misspecification (Sargent, 1991; Hommes and Sorger, 1998; Milani, 2007; Honkapohja, Mitra, and Evans, 2013). Other papers (Adam, 2005; Orphanides and Williams, 2005; Branch and Evans, 2006; Hommes and Zhu, 2014, Ormeño and Molnár, 2015) also provide support for the use of small forecasting models on several grounds, including their forecast performance, their usefulness for facilitating coordination, and their ability to approximate the Survey of Professional Forecasters well.

information is used in agents' forecasting models due to the ability of term spreads to forecast inflation (Mishkin, 1990) and real economic activity (Estrella and Hardouvelis, 1991; Estrella and Mishkin, 1997).[3]

We follow Aguilar and Vázquez (2019) by extending the medium-scale DSGE model used in Slobodyan and Wouters (2012a) to account for the term structure of interest rates. This is a behavioral DSGE model where (i) agents are boundedly rational in their economic decisions; (ii) agents form expectations using simple forecasting models; and (iii) standard no-arbitrage asset pricing laws hold.[4] Our focus, however, is on the small forecasting models (i.e. AR(2) processes) considered in Slobodyan and Wouters (2012a) augmented with term structure information rather than the small forecasting models based only on term structure information used in Aguilar and Vázquez (2019). By considering this minor deviation from the forecasting models of Slobodyan and Wouters (2012a), we seek to highlight the important role of term structure information in characterizing macroeconomic forecasts over and above the macroeconomic information used in AR forecasting models.

The estimation results show that adding term structure information in the small forecasting models results in a great improvement in the model fit. Moreover, term structure information helps to improve the AL performance in forecasting actual revised macroeconomic data used in the estimation procedure as well as in forecasting real-time (i.e. the first announcements of) macroeconomic data, which are not considered in the estimation. The latter finding suggests that the term structure of interest rates provides important information available in real time in forecasting aggregate variables in addition to that provided by revised macroeconomic data.

Beyond taking into account term structure information, we extend the analysis in Aguilar

[3] McCallum (1994) exclusively uses term spreads as simple predictors for future macroeconomic conditions for defining monetary policy rules.

[4] As pointed out by Eusepi and Preston (2018, footnote 10), expected yields (returns) under arbitrary subjective beliefs may not satisfy no-arbitrage with multiple bond maturities (assets). They impose the expectations hypothesis of the term structure to overcome this issue as we do in this paper. In contrast to this standard approach to AL, Adam and Marcet (2011) study a framework where the law of iterated expectations is not satisfied.

and Vázquez (2019) in three important directions. First, we assess the empirical ability of the AL formulation augmented with term structure information to match the macroeconomic forecasts reported in the Survey of Professional Forecasters (SPF). We find that small forecasting models which include term structure information do a much better job in matching the expectations reported in the SPF across all forward-looking variables of the DSGE model (that have an SPF counterpart) than the AR(2) processes used in Slobodyan and Wouters (2012a), thus showing the importance of term structure information for increasing the empirical validity of AL. The rationale for this finding is that the SPF forecasts are based on real-time data and the term spread information included in our small forecasting models is also available in real time. These two pieces of real-time data—SPF and the yield curve—may therefore share important information in forecasting the economic outlook. This finding is consistent with the above finding that the term structure contains useful information for forecasting real-time macroeconomic data.

Second, we further assess the performance of AL extended with term structure by disciplining AL expectations with SPF data in the estimation procedure. A comparison of the estimation results obtained with and without disciplining AL expectations with SPF data suggests that the need to discipline expectations is greatly reduced by including term structure information in the forecasting models.

Finally, we extend the analysis in Aguilar and Vázquez (2019) to estimate a measure of the term premium associated with the 10-year US Treasury bond yield from the medium-scale DSGE model under AL. Interestingly, our estimated AL bond term premium comoves with the corresponding measure estimated by Adrian, Crump and Moench (2013) using a no-arbitrage affine term structure model.

The rest of the paper is structured as follows. Section 2 briefly describes the DSGE model and the AL extension based on term structure information. Section 3 shows the estimation results and discusses their implications. Section 4 analyzes the empirical validity of model expectations when compared with actual revised data, real-time data, and the SPF forecasts.

Section 5 concludes.

2 An AL model with term structure

Our model builds on the Smets and Wouters (2007) model and its AL extension studied by Slobodyan and Wouters (2012a), henceforth called SlW. Seeking to understand the contribution of term structure (TS) information in dealing with the incomplete knowledge issue addressed by AL, we follow Aguilar and Vázquez (2019) by extending the medium-scale DSGE model to account for the TS of interest rates[5]. However, we focus on the small forecasting models—AR(2) processes—considered in SlW but augmented with TS information rather than the small forecasting models featuring only TS information used in Aguilar and Vázquez (2019). The consideration of this small deviation from the AR forecasting models considered in SlW enables us to identify the important contribution of TS information in the characterization of macroeconomic forecasts.

Next we present the main extensions of the model.[6]

2.1 The DSGE model

As in Smets and Wouters (2007), the model is characterized by the following sources of endogenous persistence: (i) Household preferences featuring external habits regarding consumption; (ii) labor supplied by households differentiated by a union with monopoly power setting sticky nominal wages à la Calvo (1983); (iii) capital investment decided by households and subject to capital adjustment costs; (iv) the degree of capital utilization, also determined by households, as a positive function of the rental rate of capital, which depends on the capital utilization adjustment costs; (v) intermediate firms setting the prices of their differentiated goods à la Calvo under monopolistic competition; and (vi) finally, wages and

[5]We also asume logarithmic utility function with constant risk aversion, which constraints from exepctations in employment.

[6]The remaining log-linearized equations of the model are presented in a supplementary appendix available from the authors upon request.

prices which are both partially indexed to lagged inflation when they are not re-optimized.

2.2 The term structure extension

Following Jermann (1998), among many others,[7] we extend the DSGE model under AL with term structure by considering the stochastic discount factor of households as the pricing kernel for bonds. Formally, the Euler equation of the representative household optimization problem associated with the demand for the n-period bond is given by the following no-arbitrage condition

$$P_{n,t}^{B} = E_t \left(m_{t,n} \times \$1 \right) = E_t \left[\beta^n \frac{U_C(C_{t+n}, N_{t+n})}{U_C(C_t, N_t)} \right], \tag{1}$$

which shows that the price at time t of a zero-coupon bond that makes a single payment of one dollar at time $t+n$, $P_{n,t}^{B}$, is equal to the (expected) n-period stochastic discount factor, $m_{t,n}$, times the payoff (one dollar). E_t denotes the AL subjective expectation operator,[8] β is the subjective discount factor, U_C denotes the marginal utility of consumption, and C_t and N_t stand for consumption and labor, respectively.

It is important to emphasize here that the SlW-DSGE model features standard optimality conditions for households and firms with the only difference that the expectations operator E_t represents AL subjective expectations and not RE. As in Sinha (2016) and Eusepi and Preston (2011, 2018), AL beliefs are assumed to be homogeneous across households and firms, though individual agents *do not* have knowledge of the beliefs of other agents—i.e. households (firms) are identical, but they do not know this to be so. As a consequence, agents do not know the equilibrium evolution of the aggregate variables. The AL expectations process is described below.

Equation (1) can be re-arranged so that the n-period bond price at t is related to the

[7]See, for instance, De Graeve, Emiris and Wouters (2009) and Bekaert, Cho and Moreno (2010) in the context of RE-DSGE models, and Sinha (2016) in a DSGE model under AL.

[8]In order to simplify notation, we do not distinguish between the expectation operators under AL and RE, since we focus exclusively on the former.

(expected) one-period stochastic discount factor, $m_{t,1}$, and the $(n-1)$-period bond price at $t+1$, $P^B_{n-1,t+1}$, as follows:

$$P^B_{n,t} = E_t\left[\beta \frac{U_C(C_{t+1},N_{t+1})}{U_C(C_t,N_t)}\beta^{n-1}\frac{U_C(C_{t+n},N_{t+n})}{U_C(C_{t+1},N_{t+1})}\right] = E_t\left[m_{t,1}E_{t+1}(P^B_{n-1,t+1})\right] = E_t\left(P^B_{1,t}P^B_{n-1,t+1}\right),$$

where the last equality holds under the law of iterated expectations. As described below, we use small forecasting models based on linear least squares projections in our AL approach, and this type of projection (regression) implies that the law of iterated expectations holds (Sargent, 1987, chapter X).[9]

After repeated substitutions, the last equation can be written in terms of the (expectations of the) one-period bond price:

$$P^B_{n,t} = E_t\left(P^B_{1,t}P^B_{1,t+1}P^B_{1,t+2}\cdots P^B_{1,t+(n-1)}\right).$$

The log-linear approximation of this equation results in

$$p^B_{n,t} = p^B_{1,t} + E_t\left(p^B_{1,t+1} + p^B_{1,t+2} + \cdots + p^B_{1,t+(n-1)}\right),$$

where lower-case variables denote variables expressed in logs. The last equation can be written in terms of the n-period bond yield, $r^{\{n\}}_t$, as follows

$$r^{\{n\}}_t = \frac{1}{n}\sum_{k=0}^{n-1} E_t r_{t+k}, \tag{2}$$

[9]We put aside asset pricing issues put forward by Adam and Marcet (2011), which arise from agents having heterogeneous information sets. As discussed in Honkapohja, Mitra and Evans (2013), standard formulations of AL assume that the law of iterated expectations holds for the subjective expectations of each individual agent. Moreover, it should be noted that agents are assumed to have homogeneous expectations in our framework and each agent considers itself the marginal trader when planning future asset allocations (as in Sinha, 2016; and Eusepi and Preston, 2018), so the law of iterated expectations in fact holds for the expectations of the marginal bond trader. Furthermore, the resulting symmetric equilibrium confirms these assumptions since all agents are identical (i.e. they face identical optimization problems and their beliefs are homogeneous).

since $r_t^{\{n\}} = -p_{n,t}^B/n$, and r_t denotes the interest rate associated with the one-period bond. Equation (2) represents the log pure version of the expectations hypothesis (EH) defined in terms of the AL expectations operator.

As is standard in empirical applications of term structure models, we augment equation (2) with a term premium term shock, $\xi_t^{\{n\}}$, as follows

$$r_t^{\{n\}} = \frac{1}{n}\sum_{k=0}^{n-1} E_t r_{t+k} + \xi_t^{\{n\}}. \tag{3}$$

This term premium shock is defined as the wedge between the model-implied yield given by (2) and the observed yield, and it can be interpreted as a measure of fluctuations in the risk premium (De Graeve, Emiris and Wouters, 2009).[10]

Our baseline estimated DSGE model relies on the EH hypothesis, equation (3) for $n = 4$, together with the set of log-linearized dynamic equations used in SlW—described in an appendix— which among others includes the log-linearized version of the optimality condition (1) for $n = 1$ augmented with a risk premium shock, ε_t^b.

As in SlW, we deviate from the monetary policy rule in the Smets and Wouters (2007) model by assuming that the monetary authorities follow a Taylor-type rule, reacting to inflation, output gap, and output gap growth, where the output gap is defined as the deviation of output from its underlying neutral productivity process. Moreover, the monetary policy rule assumed in SlW is slightly modified to include TS information as in Vázquez, María-Dolores and Londoño (2013). This assumption is in line with the TS information introduced in the small forecasting models of AL agents below. Formally,

$$r_t = \rho_r r_{t-1} + (1 - \rho_r)\left[r_\pi \pi_t + r_y \hat{y}_t\right] + r_{\Delta y}\Delta\hat{y}_t + r_{sp} sp_t^{\{4\}} + \varepsilon_t^r, \tag{4}$$

where the output gap is defined as $\hat{y}_t = y_t - \Phi\varepsilon_t^a$ (i.e. the output gap is defined as the

[10]Since we focus on government bonds in our empirical analysis, $\xi_t^{\{n\}}$ can also be understood as a *convenience yield term* (see, among others, Krishnamurthy and Vissing-Jorgensen, 2012; Greenwood et al., 2015; Del Negro et al., 2017) defined as a stochastic premium related to the safety and liquidity attributes of (US) government bonds relative to assets with the same payoff, but without such singular features.

deviation of output from its underlying neutral productivity process), and $sp_t^{\{4\}} = r_t^{\{4\}} - r_t$ denotes the term spread associated with the 1-year maturity yield.[11]

2.3 Adaptive learning with term structure information

This section provides a brief explanation of how AL expectation formation works.[12] A DSGE model can be represented in matrix form as follows:

$$A_0 \begin{bmatrix} y_{t-1} \\ w_{t-1} \end{bmatrix} + A_1 \begin{bmatrix} y_t \\ w_t \end{bmatrix} + A_2 E_t y_{t+j} + B_0 \epsilon_t = 0,$$

where y_t is the vector of endogenous variables at time t, $E_t y_{t+j}$ contains multi-period-ahead expectations, and w_t is a vector including eight exogenous shocks and the lagged innovations, ϵ_{t-1}, of the price- and wage-markup shocks since they are modeled as ARMA(1, 1) processes.

Agents are assumed to have a limited view of the economy under AL. Their so-called "perceived law of motion" (PLM) processes—i.e. their small forecasting models— are generally defined as follows:

$$y_{t+j} = X_t \beta_{t-1}^{\{j\}} + u_{t+j},\ for\ j =\ 1,\ 2,\ ...,\ n,$$

where y is the vector containing the forward-looking variables of the model, X is the matrix of regressors, $\beta^{\{j\}}$ is the vector of updating parameters, which includes an intercept, and u is a vector of errors. These errors are linear combinations of the true model innovations. The variance-covariance matrices, $\Sigma = E[u_{t+j} u_{t+j}^T]$, are therefore non-diagonal. Agents are further assumed to use simple econometric tools under AL. In particular, they use a linear least squares projection scheme in which the parameters are updated to form their expectations for each forward-looking variable: $E_t y_{t+j} = X_t \beta_{t-1}^{\{j\}}$. The updating parameter vector, β, which results from stacking all the vectors $\beta^{\{j\}}$, is further assumed to follow an autoregressive

[11] As in SlW, all but a few of the structural shocks follow AR(1) processes. The price- and wage-markup shocks follow ARMA(1,1) processes, and the AR(1) productivity shock allows for an interaction with the government spending shock.

[12] For a detailed explanation see Slobodyan and Wouters (2012a,b).

process where agents' beliefs are updated through a Kalman filter as described below. This updating expectation process can be represented as in SlW by the equation: $\beta_t - \bar{\beta} = F(\beta_{t-1} - \bar{\beta}) + v_t$, where F is a diagonal matrix with the learning parameter $\mid \rho \mid \leq 1$ on the main diagonal and v_t are i.i.d. errors with variance-covariance matrix V. This standard AL approach assumes that agents do not take into account the fact that their belief coefficients will be revised in the future (e.g. Sinha (2016) and Eusepi and Preston (2011, 2018)). This assumption can be rationalized by using an anticipated utility approach put forward in Kreps (1998) and Sargent (1999).[13]

Notice that each expectational horizon is estimated separately in our AL approach. This is in clear contrast to the maintained beliefs hypothesis suggested in Preston (2005)—an approach also followed in Eusepi and Preston (2011) and Sinha (2015, 2016)— which not only imposes an infinite forecast horizon, but also considers iterated forecasts used under the MSV approach. Nevertheless, our approach shares with other AL approaches the use of forecasting models based on linear least squares projections, which implies that the law of iterated expectations holds: $E_t\left(E_{t+h} y_{t+j}\right) = E_t y_{t+j}$, for any $j > h > 0$ (see Sargent (1987, chapter X, pp. 223-229) for a formal discussion), and this is consistent with the law of iterated expectations assumed in the derivation of the log pure version of the EH, equation (2), above.

Once the expectations of the forward-looking variables,$E_t y_{t+j}$, are computed they are plugged into the matrix representation of the DSGE model to obtain a backward-looking representation of the model as follows

$$\begin{bmatrix} y_t \\ w_t \end{bmatrix} = \mu_t + T_t \begin{bmatrix} y_{t-1} \\ w_{t-1} \end{bmatrix} + R_t \epsilon_t,$$

where the time-varying matrices μ_t, T_t and R_t are nonlinear functions of structural parameters

[13]The anticipated utility approach assumes that agents do not take into account future updates of beliefs when making current decisions but are otherwise fully optimal. This is in contrast to the Bayesian belief approach, which takes belief updates into account.

(entering into matrices A_0, A_1, A_2 and B_0) together with the learning coefficients, β. This representation of the model is called the *actual* law of motion (ALM).

The standard Kalman-filter updating and transition equations for the belief coefficients and their corresponding covariance matrix are given by

$$\beta_{t|t} = \beta_{t|t-1} + R_{t|t-1} X_{t-1} \Big[\Sigma + X_{t-1}^T R_{t|t-1}^{-1} X_{t-1} \Big]^{-1} \Big(y_t - X_{t-1} \beta_{t|t-1} \Big),$$

where $(\beta_{t+1|t} - \bar{\beta}) = F(\beta_{t|t} - \bar{\beta})$. $\beta_{t|t-1}$ is the estimate of β using the information up to time $t-1$ (but further considering the autoregressive process followed by β), $R_{t|t-1}$ is the mean squared error associated with $\beta_{t|t-1}$. Therefore, the updated learning vector $\beta_{t|t}$ is equal to the previous one, $\beta_{t|t-1}$, plus a correction term that depends on the previous forecast error, $\Big(y_t - X_{t-1} \beta_{t|t-1} \Big)$. The mean squared error, $R_{t|t}$, associated with this updated estimate is given by

$$R_{t|t} = R_{t|t-1} - R_{t|t-1} X_{t-1} \Big[\Sigma + X_{t-1}^T R_{t|t-1}^{-1} X_{t-1} \Big]^{-1} X_{t-1}^T R_{t|t-1}^{-1},$$

with $R_{t+1|t} = F R_{t|t} F^T + V$.

The initialization of this Kalman filter for the belief coefficients requires the specification of $\beta_{1|0} = \overline{\beta}$, $R_{1|0}$, Σ, and V. We follow Slobodyan and Wouters (2012a), where all these expressions are derived from the correlations between the model variables implied by the RE equilibrium evaluated at the corresponding structural parameter vector.

A PLM with term structure information

The baseline small forecasting models assumed in SlW are simple AR(2) processes. That is, the PLM of each forward-looking variable of the DSGE model is described by

$$E_t y_{t+j} = \theta_{y,t-1}^{\{j\}} + \beta_{y,1,t-1}^{\{j\}} y_t + \beta_{y,2,t-1}^{\{j\}} y_{t-1}, \tag{5}$$

where the intercept of the PLM, $\theta_{y,t-1}^{\{j\}}$, captures the low frequency movements of the corresponding forward-looking variable, $E_t y_{t+j}$, and the coefficients $\beta_{y,1,t-1}^{\{j\}}$ and $\beta_{y,2,t-1}^{\{j\}}$ together measure the persistence of beliefs.

We analyze the importance of introducing TS information by simply augmenting these PLM with the term spread $sp_t^{\{4\}}$:

$$E_t y_{t+j} = \theta_{y,t-1}^{\{j\}} + \beta_{y,1,t-1}^{\{j\}} y_t + \beta_{y,2,t-1}^{\{j\}} y_{t-1} + \beta_{y,3,t-1}^{\{j\}} sp_t^{\{4\}}, \tag{6}$$

where the coefficient $\beta_{y,3,t-1}^{\{j\}}$ captures agents' reaction to the term spread information while forecasting $E_t y_{t+j}$.

This small modification in the PLM enables us to clearly identify the contribution of TS information beyond that provided by current and lagged values of the forward-looking variables considered in SlW.[14]

3 Estimation results

We begin this section by describing the data and the estimation approach, then proceed to discuss the model fit, estimation results, a comparison of actual and simulated moments, the variance decomposition of shocks, and the estimate of the smoothed AL term premium.

3.1 Data and estimation approach

We estimate the AL model extended with TS for the alternative specifications of the PLM using US data for two sample periods: The whole sample period running from 1965:4 until 2009:1 and a subsample from 1981:4 until 2009:1. The set of observable variables used for the whole sample period estimation is the same one used by Slobodyan and Wouters (2012a) (i.e. the quarterly series of the inflation rate, the Fed funds rate, the log of hours worked, the quarterly log differences in real consumption, real investment, real wages, and real GDP)

[14]Aguilar and Vázquez (2019) also consider TS information but they deviate much further from the PLM assumed in SlW by considering only the term spread in the PLM. The approach followed here makes it easier to identify the contribution made by adding TS information over and above the information provided by AR processes.

plus the 1-year zero-coupon Treasury yield (i.e. a set of eight observable variables).[15]

The estimation of the shorter sample period extends the set of observables considered in the whole sample period to include six observable forecasts reported in the SPF. More precisely, we consider the SPF forecasts available, which have counterparts in forward-looking variables in the DSGE model: 1-quarter-ahead forecasts of inflation, 1-quarter-ahead forecasts of the consumption and investment growth rates, and 1-, 2- and 3-quarter-ahead forecasts of the short-term nominal interest rate[16,17] Analyzing this shorter sample period, but with a larger number of observables, enables us to assess the importance of TS information in disciplining model expectations by fitting SPF forecasts as well as assessing the robustness of results by studying a sample period featuring both milder aggregate fluctuations (the Great Moderation) and an inflation downtrend, which is in sharp contrast with the stagflation in the 1970s and early 1980s present in the first-half of the whole sample period.

[15]The zero-coupon Treasury bond yields come from the Gürkaynak, Sack and Wright (2007) data set available on the research data website of the Board of Governors of the Federal Reserve.

[16]Del Negro and Eusepi (2011) pioneer the use of SPF expectation data to discipline RE in DSGE models. A few more recent papers (Ormeño and Molnar, 2015; Aguilar and Vázquez, 2018) also use SPF data to discipline AL expectations in DSGE models.

[17]SPF forecasts were downloaded from the website of the Federal Reserve Bank of Philadelphia. Inflation forecasts are reported back to the late 1960's, but the rest of the forecast time series starts at 1981:3. Thus, data availability partially determines the choice of the first period in the short sample. Moreover, the initial quarter of the short sample roughly coincides with the start of a successful disinflation period.

The measurement equation is

$$X_t = \begin{bmatrix} dlGDP_t \\ dlCONS_t \\ dlINV_t \\ dlWAG_t \\ dlP_t \\ lHours_t \\ FEDFUNDS_t \\ 1-year\,TB\,yield_t \\ dlCONS_{t+1}^{SPF} \\ dlINV_{t+1}^{SPF} \\ dlP_t^{SPF} \\ r_t^{SPF\{1\}} \\ r_t^{SPF\{2\}} \\ r_t^{SPF\{3\}} \end{bmatrix} = \begin{bmatrix} \overline{\gamma} \\ \overline{\gamma} \\ \overline{\gamma} \\ \overline{\gamma} \\ \overline{\pi} \\ \bar{l} \\ \overline{r} \\ \overline{r}^{\{4\}} \\ \overline{\gamma}_c^{SPF} \\ \overline{\gamma}_i^{SPF} \\ \overline{\pi}^{SPF} \\ \overline{r}^{SPF} \\ \overline{r}^{SPF} \\ \overline{r}^{SPF} \end{bmatrix} + \begin{bmatrix} y_t - y_{t-1} \\ c_t - c_{t-1} \\ i_t - i_{t-1} \\ w_t - w_{t-1} \\ \pi_t \\ l_t \\ r_t \\ r_t^{\{4\}} \\ E_t\left(c_{t+1} - c_{t-1}\right) + \epsilon_{c,t} \\ E_t\left(i_{t+1} - i_{t-1}\right) + \epsilon_{i,t} \\ E_t \pi_{t+j} + \epsilon_{\pi,t} \\ E_t\left(r_{t+1}\right) + \epsilon_{r,t}^{\{1\}} \\ E_t\left(r_{t+2}\right) + \epsilon_{r,t}^{\{2\}} \\ E_t\left(r_{t+2}\right) + \epsilon_{r,t}^{\{3\}} \end{bmatrix}, \tag{7}$$

where l and dl represent the log and the log difference, respectively. $\overline{\gamma} = 100(\gamma - 1)$ is the common quarterly trend growth rate for real GDP, real consumption, real investment, and real wages. $\bar{l}$, $\overline{\pi}$, $\overline{r}$ and $\overline{r}^{\{4\}}$ are the steady-state levels of hours worked, inflation, the federal funds rate, and the 1-year (4-quarter) bond yield, respectively. The superscripts SPF and $\{j\}$ in the last six rows of the measurement equation denote actual forecasts from the SPF and the corresponding forecast horizon for $j = 1, 2, 3$; respectively. As in Ormeño and Molnár (2015), the measurement errors, ϵ, showing the deviations of model expectations from the actual forecasts reported in the SPF, are assumed to be i.i.d. processes. We also allow for differences in trend growth rates across SPF (consumption and investment) forecasts as well as differences between the steady-state levels of actual and SPF forecast data.

The measurement equation (7) reduces to the first eight equations when the alternative versions of the AL model are estimated for the whole sample period, whereas the complete system (7) is used for the short sample period when SPF data is considered in the estimation procedure.

We follow a Bayesian estimation procedure. First, the log posterior function is maximized by combining prior information on the parameters with the likelihood of the data. The prior assumptions are exactly the same as in Slobodyan and Wouters (2012a). In addition, we consider loose priors for the parameters characterizing both the 1-year yield dynamics and the measurement error processes. The Metropolis-Hastings algorithm is used to generate the posterior distribution and to compute the log density of the model.[18]

3.2 Posterior estimates

Our estimated AL model with TS (henceforth called the SlW-TS model) only differs from that of Slobodyan and Wouters (2012a) (henceforth called the SlW model) in the specification of the small forecasting models.

Table 1 shows the estimation results for the various PLM specifications and the various samples considered. Our sample period is almost identical to the one considered in SlW. Thus, the first column of Table 1 shows the estimation results of the SlW model for the whole sample period 1966:1-2009:1 using their original set of seven observable variables, whereas the second and third columns report the estimation results using the PLM of SlW and the PLM augmented with TS information (SlW-TS) as described by equations (5) and (6), respectively. The remaining two columns show the estimation results for the two PLM specifications for the short sample period running from 1981:4 until 2009:1, where the SPF time series are also included in the set of observables as described in the measurement equation (7).[19]

For each model estimated, Table 1 firstly reports the number of observable time series,

[18]The DSGE models are estimated using Dynare codes kindly provided by Sergey Slobodyan and Raf Wouters with a few modifications to accommodate the presence of TS information in both the structural model and the small forecasting models, as described above.

[19]For the short sample period, we find that simpler specifications of the two PLM built on AR(1) processes— i.e. imposing $\beta^{\{j\}}_{y,2,t-1} = 0$ in equations (5) and (6)— improve the model fit. The estimation results reported for the short sample period are based on these simpler specifications.

and the model fit based on the log data density. The remaining rows show the posterior mean and the corresponding 90 percent interval of the posterior distribution—in parentheses—for four groups of selected parameters. The first and second groups contain the parameters for real and nominal rigidities, respectively. The third group contains the parameters which describe the ARMA coefficients characterizing price and wage markup shocks. Finally, the fourth group contains the policy rule parameters.[20]

A comparison of column 1 in Table 1 with the figures reported in Slobodyan and Wouters (2012a, Table 1, p. 74) shows a similar fit and almost identical parameter estimates. This suggests that including or ignoring a few quarterly observations and assuming a logarithmic utility function has no impact on the estimation results.

The consequences of considering the 1-year Treasury bill

A comparison of columns 1 and 2 shows that including the 1-year Treasury bill as an observable in the SlW model decreases the importance of a few sources of endogenous rigidity, such as Calvo price and wage parameters, price and wage indexation parameters, and the parameter featuring the capital utilization adjusting cost, ψ. The rationale for this decrease in a few sources of endogenous persistence is that considering the EH of the term structure (equation (3)) brings with it additional persistence in (the expected path of) the short-term rate that is transmitted to other aggregate variables. Moreover, there is a large increase in persistence driven by the increase in the AR coefficients that describe the processes of price and wage markup shocks.

[20]All parameter estimates are reported in a supplementary appendix available from the authors upon request.

Table 1. Selected parameter estimates

	Without SPF data (1965:4-2009:1)			With SPF data (1981:4-2009:1)	
	SlW	SlW	SlW-TS	SlW	SlW-TS
Number of observables	7	8	8	14	14
log data density	-984.930	-1092.600	-1057.596	-1070.311	-853.960
Parameters associated with real rigidities					
habit formation	0.787	0.851	0.759	0.631	0.630
(h)	(0.742,0.833)	(0.842,0.878)	(0.745,0.788)	(0.607,0.643)	(0.611,0.643)
cost of adjusting capital	4.846	7.975	4.616	4.219	5.294
(φ)	(3.257,6.491)	(7.946,8.014)	(4.579,4.646)	(4.192,4.258)	(5.168,5.323)
capital utilization adjusting cost	0.611	0.151	0.092	0.163	0.050
(ψ)	(0.424,0.819)	(0.149,0.180)	(0.085,0.100)	(0.153,0.171)	(0.046,0.053)
Parameters associated with nominal rigidities					
price Calvo probability	0.612	0.472	0.545	0.617	0.715
(ξ_p)	(0.544,0.684)	(0.459,0.487)	(0.524,0.554)	(0.598,0.630)	(0.702,0.730)
wage Calvo probability	0.774	0.565	0.464	0.495	0.259
(ξ_w)	(0.721,0.831)	(0.549,0.589)	(0.456,0.482)	(0.485,0.511)	(0.245,0.268)
price indexation	0.372	0.178	0.377	0.896	0.820
(ι_p)	(0.169,0.566)	(0.151,0.192)	(0.325,0.401)	(0.884,0.928)	(0.796,0.863)
wage indexation	0.386	0.185	0.470	0.218	0.400
(ι_w)	(0.203,0.582)	(0.107,0.229)	(0.410,0.486)	(0.195,0.237)	(0.337,0.437)

Table 1. (*Continued*)

	Without SPF data (1965:4-2009:1)			With SPF data (1981:4-2009:1)	
	SlW	SlW	SlW-TS	SlW	SlW-TS
Parameters associated with price and wage markups					
markup price AR coef.	0.457	0.880	0.875	0.609	0.575
(ρ_p)	(0.130,0.786)	(0.860,0.904)	(0.875,0.911)	(0.584,0.690)	(0.558,0.602)
markup wage AR coef.	0.554	0.838	0.918	0.843	0.938
(ρ_w)	(0.287,0.827)	(0.827,0.853)	(0.909,0.928)	(0.833,0.858)	(0.929,0.950)
markup price MA coef.	0.476	0.608	0.693	0.590	0.747
(μ_p)	(0.224,0.742)	(0.591,0.635)	(0.676,0.711)	(0.547,0.638)	(0.741,0.769)
markup wage MA coef.	0.494	0.325	0.477	0.556	0.450
(μ_w)	(0.209,0.793)	(0.309,0.368)	(0.454,0.517)	(0.543,0.569)	(0.422,0.487)
Policy rule parameters					
inertia	0.880	0.884	0.886	0.834	0.835
(ρ_r)	(0.85,0.92)	(0.881,0.907)	(0.878,0.896)	(0.808,0.845)	(0.819,0.849)
inflation	1.692	1.662	1.617	2.373	1.854
(r_π)	(1.384,2.01)	(1.659,1.683)	(1.570,1.643)	(2.291,2.394)	(1.762,1.888)
output	0.101	0.075	0.038	0.080	0.082
(r_y)	(0.043,0.159)	(0.065,0.095)	(0.033,0.047)	(0.068,0.089)	(0.075,0.092)
output growth	0.118	0.122	0.144	0.075	0.040
($r_{\Delta y}$)	(0.087,0.150)	(0.104,0.131)	(0.132,0.154)	(0.068,0.090)	(0.036,0.053)
term spread	-	0.255	0.140	0.112	0.155
(r_{sp})	-	(0.218,0.284)	(0.118,0.159)	(0.086,0.145)	(0.129,0.174)

Notes: Parameter notation and 90% intervals of the posterior distribution in parentheses.

The consequences of considering TS information in the PLM

A comparison of the marginal likelihood values in columns 2 and 3 shows that the switch from the SlW learning scheme to our learning scheme augmented with TS information, SlW-TS, results in an appreciable improvement in model fit of $[-1057.596 - (-1092.600) =]35$ log-points. Regarding the posterior estimates of parameters, the SlW-TS model results in a reduction of the parameters associated with real rigidities and an increase in most of the parameters that include nominal rigidities (i.e. the Calvo price parameter and the price and wage indexation parameters), while for many of the rest the estimates are fairly similar across the two learning specifications.

The consequences of considering the Great Moderation period and SPF data

The improvement is much greater when SPF data is considered in the shorter sample (1981:4-2009:1), which mostly covers the Great Moderation, which features milder aggregate fluctuations, economic growth, and downtrend inflation. Thus, the improvement in the marginal likelihood of considering TS information in the PLM is 216.62 points, which results in a huge posterior odd of 1.19e+94 in favor of the AL specification augmented with TS information. As shown below, this huge difference is due to the impressive ability of the learning scheme with TS to match SPF expectations in this period.

We find that most posterior estimates of parameters in the short sample are fairly similar across learning specifications, though there are a few exceptions. Thus, the estimate of the elasticity of the cost adjusting capital, φ, is higher for the learning specification including TS information (5.3 versus 4.2), whereas the opposite occurs for the response of the nominal interest rate to inflation (1.9 versus 2.4). Moreover, a comparison of parameter estimates across the two samples studied suggests that the Calvo price probability and price indexation estimates (ξ_p and ι_p, respectively) increase during this downtrend inflation period as expected, whereas the estimates of Calvo wage probability, ξ_w, habit formation, h, and the elasticity of capital utilization adjusting cost, ψ, decrease further in relation to the estimates reported in the first column and in Slobodyan and Wouters (2012a).

3.3 Model fit

Along with the overall model fit based on the posterior log data density, we also analyze the performance of the two options for PLM specifications in reproducing selected second-moment statistics obtained from actual data as shown in Table 2. We focus on four types of moment: Standard deviations, contemporaneous correlations of each observable with output growth and with inflation, respectively, and first-order autocorrelations.

As for the actual size of fluctuations, we observe that the two AL formulations have trouble in matching the standard deviation of a few observable variables: Both models generate too much volatility, although this problem is clearly less severe for the PLM specification with TS information. Thus, this specification is able to match the volatility of the nominal variables reasonably well. For the other three types of second-moment statistics, the PLM specification including TS information also performs better in general than the SlW specification.

Table 2. Actual and simulated second moments

Actual	Δy	Δc	Δinv	Δw	Hours	π	r	$r^{\{4\}}$
Std. dev.	0.87	0.74	2.21	0.66	2.83	0.59	0.83	0.71
Corr. (Δy)	1.0	0.70	0.71	0.09	0.17	-0.23	-0.13	-0.03
Corr. (π)	-0.23	-0.28	-0.09	-0.13	-0.42	1.0	0.64	0.59
Autocorr.	0.30	0.27	0.56	0.03	0.95	0.88	0.94	0.94

SlW+TS	Δy	Δc	Δinv	Δw	Hours	π	r	$r^{\{4\}}$
Std. dev.	1.23	0.96	2.64^{+}	0.71*	4.06*	0.66*	0.85*	0.95*
	(1.08,1.38)	(0.84,1.09)	(2.25,3.11)	(0.64,0.77)	(2.48,6.16)	(0.48,0.86)	(0.60,1.16)	(0.65,1.33)
Corr. (Δy)	1.0	0.82	0.72*	0.41	0.09*	-0.04*	0.03*	0.18
	–	(0.76,0.87)	(0.63,0.79)	(0.30,0.51)	(0.02,0.17)	(-0.23,-0.15)	(-0.14,0.19)	(0.02,0.33)
Corr. (π)	-0.04*	-0.02	-0.05*	-0.01^{+}	0.11	1.0	0.62*	0.42*
	(-0.23,0.15)	(-0.20,0.16)	(-0.30,0.19)	(-0.12,0.10)	(-0.34,0.54)	–	(0.36,0.81)	(0.01,0.74)
Autocorr.	0.35*	0.42^{+}	0.53*	0.02*	0.97*	0.79*	0.92*	0.93*
	(0.20,0.48)	(0.28,0.55)	(0.39,0.67)	(-0.11,0.15)	(0.94,0.99)	(0.64,0.89)	(0.88,0.96)	(0.88,0.97)

SlW	Δy	Δc	Δinv	Δw	Hours	π	r	$r^{\{4\}}$
Std. dev.	1.68	1.45	3.07	0.92	3.93*	0.77*	1.28^{+}	1.28
	(1.45,1.93)	(1.25,1.69)	(2.58,3.61)	(0.84,1.01)	(2.76,5.37)	(0.57,1.04)	(0.86,1.82)	(0.87,1.80)
Corr. (Δy)	1.0	0.85	0.71*	0.35	0.15*	0.08	0.07	0.16
	–	(0.80,0.90)	(0.62,0.79)	(0.22,0.46)	(0.10,0.21)	(-0.11,0.26)	(-0.06,0.19)	(0.04,0.28)
Corr. (π)	0.08	0.06	0.11*	0.09	0.20*	1.0	0.61*	0.57*
	(-0.11,0.26)	(-0.12,0.24)	(-0.11,0.34)	(-0.01,0.19)	(-0.22,0.56)	–	(0.35,0.81)	(0.28,0.79)
Autocorr.	0.37*	0.44^{+}	0.59*	0.04*	0.95*	0.64^{+}	0.95*	0.93*
	(0.22,0.52)	(0.28,0.58)	(0.46,0.71)	(-0.08,0.17)	(0.92,0.98)	(0.35,0.84)	(0.91,0.98)	(0.87,0.97)

Note: Simulated statistics are computed from the state-space representation for 5,000 random draws from the posterior distributions. We report the mean and the 90% highest-posterior-density interval associated with each simulated moment statistic in parentheses. A star (cross) next to a simulated statistic indicates that the actual statistic lies within (is close to either the upper or the lower bound of) the posterior-density interval.

3.4 Variance decomposition

Table 3 shows the variance decomposition for most of the variables used in the estimation procedure for the AL specification with TS information at the 1-year (first component in each cell) and 10-year forecast horizons (second component in each cell). The estimated AL model shows that risk premium and exogenous spending shocks are the main contributors to explaining output growth fluctuations, while risk premium shocks explain almost all consumption growth fluctuations. Price markup shocks explain 72% (50%) of inflation variability at the 1-year (10-year) forecast horizon while wage markup shocks explain 91% of real wage fluctuations. The variance decompositions of output growth and inflation are somewhat similar to those reported in SlW. The only noteworthy differences are that risk premium shocks are more important for the SlW formulation whereas the opposite is true for the exogenous spending shock, and the role of productivity shocks in explaining output growth fluctuations becomes negligible when TS information is not considered in the SlW (8% in our model versus 2% in SlW).

Monetary policy and risk premium shocks are observed to make an important contribution to the variability of the short-term interest rate and the 1-year yield. Moreover, term premium shocks play an important role in explaining the short- and long-term fluctuations of the 1-year yield (26% and 45%, respectively) and the long-term variability of hours worked (21%). In contrast to the findings of Aguilar and Vázquez (2019), term premium shocks are found here to play a minor role in explaining the fluctuations of inflation and the short-term nominal interest rate.[21]

[21] They show that a PLM featuring only TS information implies that term premium shocks explain roughly 30% of the long-term variability of both inflation and the short-term interest rate.

Table 3. Variance decomposition

	Δy	Δc	Δinv	Δw	Hours	π	r	$r^{\{4\}}$
Productivity	8/8	0/1	0/0	0/0	16/5	10/18	8/7	6/4
Risk premium	45/44	94/90	13/14	5/4	51/46	2/8	20/33	17/20
Exogenous spending	36/34	0/1	0/0	0/0	21/9	0/0	7/6	6/4
Invest. specific tech.	7/8	0/0	83/81	0/0	7/2	0/0	3/3	2/3
Monetary policy	2/3	4/5	2/2	1/1	3/13	1/2	58/39	40/19
Price markup	0/0	0/1	0/0	2/2	0/2	72/50	3/6	2/3
Wage markup	1/1	0/0	0/0	91/91	0/2	10/13	1/3	1/1
Term premium	1/2	1/3	2/3	1/2	1/21	4/9	0/3	26/45

Notes: Each cell reports the contributions of the corresponding variable to the forecast error variance for the 1-year and the 10-year forecast horizons, respectively.

3.5 Term premium estimates

This subsection compares the AL term premium estimated using TS information with some term premium measures estimated in the literature. In particular, we consider the term premia estimated from two non-arbitrage affine models suggested by Adrian, Crump and Moench (2013)—henceforth called ACM— and Kim and Wright (2005)—referred as KW— respectively, and the term premium estimated from a DSGE model under RE by Drew-Becker (2014)—called DB. As is standard in the related literature, we focus on the long-term premia associated with the 10-year yield. Therefore, we estimate the AL-DSGE model also using the 10-year yield as an observable variable and the corresponding non-arbitrage condition (3) for $j = 40$. Considering a long term maturity yield such as the 10-year yield under the EH implies the need to characterize the expectations of the short-term interest rate up to the 39-quarter horizon. In our AL setup, a long forecasting horizon dramatically increases the number of expectation functions of the short-term interest rate, leading to a curse of dimensionality problem. To deal with this issue we assume the following simple recursive structure for the short-term rate on forecast horizons beyond the three-quarter horizon:

$$\begin{cases} E_t r_{t+4} = \frac{1}{4}\left(r_t + E_t r_{t+1} + E_t r_{t+2} + E_t r_{t+3}\right), & \\ E_t r_{t+j} = \mu_r E_t r_{t+j-1}, & j \geq 5, \end{cases} \tag{8}$$

where the initialization of this recursive structure, $E_t r_{t+4}$, is given by the 1-year yield implied by the EH. This structure builds on the forecasting rules described in equation (6) above which, among others, characterize $E_t r_{t+j}$, for $j = 1, 2, 3$. Preliminary estimation results showed that the parameter μ_r is poorly identified, so we set $\mu_r = 0.998$, which implies a very slow decay of short-term interest rate expectations over time that is in line with the high persistence of the short-term rate.

The AL term premium is given by $\left[\left(\overline{r}^{\{40\}} - \overline{r}\right) + \xi_t^{\{40\}}\right] \times 4$ when written in annualized levels—i.e. the estimate of the steady-state 10-year bond term premium, $\overline{r}^{\{40\}} - \overline{r}$, is added to the corresponding term-premium measured in deviation from its steady-state value, $\xi_t^{\{40\}}$, as described in equation (3). Figure 1 shows two AL term premium measures obtained from the estimated AL with TS information model. The dark-blue solid line shows the AL term premium without using the ACM term premium as observable, and the light-blue dashed line shows the AL term premium when the ACM term premium is included in the set of observables. Considering the ACM term premium as an observable variable in the estimation procedure clearly helps to close the gap between the levels of the AL and ACM term premia since it allows for an improvement in the identification of the steady-state 10-year bond term premium, $\overline{r}^{\{40\}} - \overline{r}$. Nevertheless, the two estimated AL term premia are highly correlated (0.86).

Figure 1 also shows the KW (purple short-dashed line) and DB (red dashed line) measures for the periods in which they are published. As discussed in the related literature (see, for instance, Swanson, 2007; Cohen, Hördahl, and Xia, 2018), it is rather common to find major discrepancies (around 200 basis points or even higher) between alternative measures for short periods of time. For instance, the discrepancy between the AL and ACM measures is roughly

200 basis points around 1984 for one of the AL measures, but the discrepancy between the ACM and DB term premia is much larger for the same period (roughly 450 basis points!). Another substantial discrepancy appears around the 2001-2002 recession, when the differences between the DB term premium and any other term premium take values close to 5%, whereas the differences between any pair of the rest of the term premium measures are around 1% in all cases. These discrepancies across models are not, however, explained by fitting errors implied by the alternative models, as all models tend to fit the yield data very well (as indeed our AL-DSGE model does (see Figure 2 below)). In spite of large discrepancies for a few periods, the differences between alternative term premia are in general less than 100 basis points.

Focusing on the comparison between the AL and ACM term premia, it can be observed that the fluctuations in the ACM term premium are slightly milder than those in the AL term premium: The standard deviations of these two term premium measures are 1.13 and 1.38, respectively (1.28 when the ACM term premium is not included in the set of observables). Moreover, both AL and ACM measures exhibit an upward trend during the Stagflation period and a downward trend during the disinflation period, which implies that the two measures are contemporaneously correlated (0.86 with the ACM measure in the set of observables and 0.65 without it). They also exhibit a high degree of persistence (the first-order autocorrelation coefficient is 0.96 for the ACM term premium and roughly 0.93 for the two AL term premia). Furthermore, the correlations between the ACM and AL and the cyclical measure of GDP obtained from the Hodrick and Prescott filter (Hodrick and Prescott, 1997) show a weak countercyclicality (-0.30, and -0.26, respectively) somewhat in line with the findings in the related literature (e.g. in Campbell and Cochrane, 1999; Cochrane and Piazzesi, 2005; Bauer, Rudebusch and Wu, 2014). This correlation is a little lower at -0.17 when the ACM term premium is removed from the set of observables.

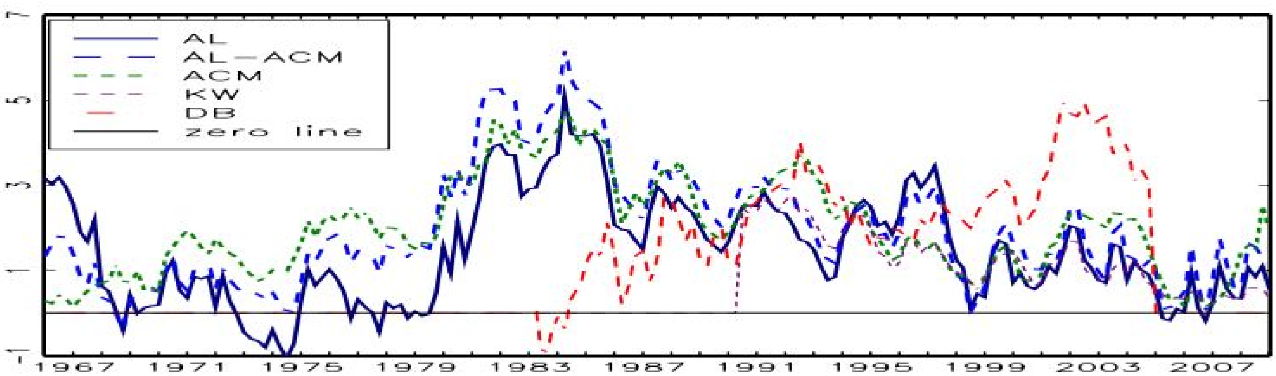

Figure 1. 10-year term premia

Note: The annualized AL term premia shown in this figure are computed as $\left[\left(\overline{r}^{\{40\}} - \overline{r}\right) + \xi_t^{\{40\}}\right] \times 4$.

Figure 2 shows the actual figure and the forecast for the 10-year yield based on the AL-DSGE model together with the estimated average of the expected path of the short-term rate over 10 years (i.e. the estimated 10-year yield implied by the EH of the term structure). It is clear that the estimated 10-year yield implied by the EH under AL shows great variability over the sample period, but it also shows a relatively small variability in the early 1980s when the 10-year yield shows the twin-peak fluctuations, which results in the large fluctuations of the estimated AL term premium shown in Figure 1.

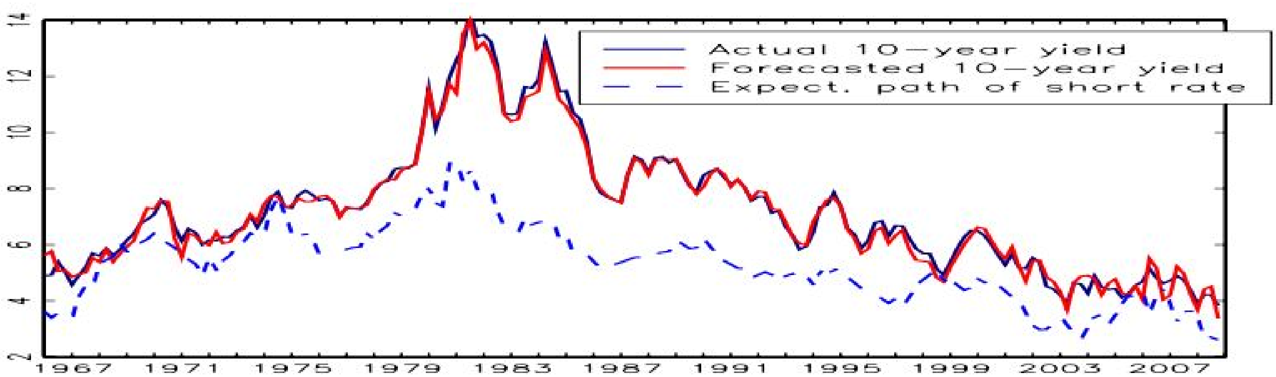

Figure 2. Actual and forecast 10-year yields, and the estimated average of the expected path of the short-term rate over 10 years

The relatively small variability of the expectations of the short-term rate in the early 1980's is confirmed in Figure 3, where the belief coefficients associated with a few forward-looking variables are shown. Thus, the bottom left graph in Figure 3 shows that the average of the short-term belief coefficients for the 1- 2- and 3-quarter ahead AL expectations does not capture the high variability of the federal funds rate in the early 1980s.

Figure 3 also shows that the term spread coefficients associated with the PLM of the alternative forward-looking variables exhibit great variability in general, capturing a strong reaction by agents to the spread while forecasting key macroeconomic variables. This is true in particular for investment beliefs, for which the term spread coefficient increases around recessions (in the mid and late 1970s, early 1980s, the 2001-2002 period, and before the Great Recession).

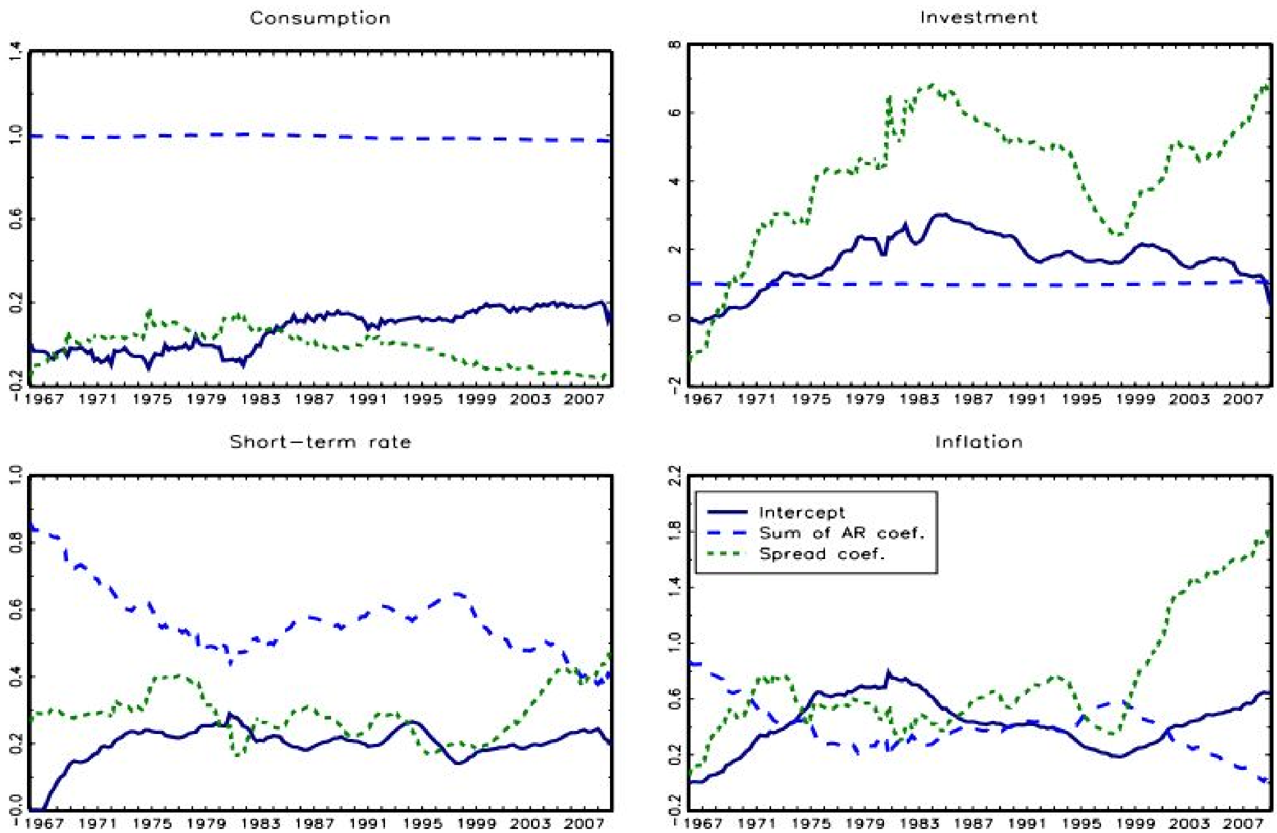

Figure 3. Time variation of belief coefficients

Note: The coefficients shown for the short-term interest rate are the averages of the corresponding belief coefficients for the 1- 2- and 3-quarter ahead AL expectations.

An analysis of the contemporaneous cross-correlations between the three types of belief coefficient (i.e. the intercept, the sum of the AR coefficients, and the term spread coefficient) also shows an interesting finding: There is a strong correlation between the term spread belief coefficient and the corresponding PLM's intercept of real variables (consumption and investment), indicating that the variability in the agent's reaction to the term spread is somewhat linked to the perception on the low-frequency movements of consumption and investment. Thus, the estimated correlation between the intercept and the term spread coefficient is negative at -0.73 for consumption beliefs, and positive at 0.79 for investment beliefs. Similarly, we also find a strong negative correlation between the intercept and the sum of the AR coefficients for consumption (-0.71), inflation (-0.85), and short-term rate (-0.77) beliefs indicating that both types of belief coefficient also compete to capture expectations about the low-frequency movements of these variables.

4 The empirical validity of the PLM

This section analyzes the empirical validity of the PLM implied by the two specification options in order to assess the contribution of TS information to both improve model fit and match SPF forecasts.

Table 4 shows the RMSE statistics from the PLM forecasts for the forward-looking variables that have observable counterparts. We also include the 1-year yield implied by the pure EH (i.e. the one-year yield implied by (2) where the term premium is restricted to zero). To assess the PLM (i.e. perceived law of motion) performance further, we also report the RMSE for the ALM (i.e. actual law of motion) for the observable variables. Notice that these statistics are all based on in-sample forecasts. ALM forecast errors are also minimized in the estimation procedure, so they provide a minimum bound against which the PLM performance can be assessed. Furthermore, since the log marginal density is a function of the ALM forecast errors, the RMSE statistics computed for alternative variables provide valuable information about the sources of the improvement in the model fit based on the log marginal density implied by introducing TS information into the PLM.

Table 4 has three panels. The first two show the RMSE statistics associated with the

ALM and the PLM for the whole sample—together with the PLM statistics associated with specific periods such as the Stagflation period (1966:1-1981:4), the disinflation period (1982:1-2009:1), and the contraction periods as dated by the NBER business cycle committee— for the DSGE model estimated under the two specifications of the PLM (equations (5) and (6)). The first two panels also show the RMSE statistics for the two PLM specifications using the first announcements (real-time data) instead of the actual (revised) data used in the estimation procedure. To facilitate discussion, we also report the RMSE statistics obtained from the estimation in the Slobodyan and Wouters (2012a) model (i.e. muting the TS part of the model and removing the 1-year yield from the set of observables) in the third panel.

Several important conclusions emerge from Table 4. First, the RMSE statistics associated with the ALM are lower for the learning specification that includes TS information across all three real variables (i.e. the growth rates of consumption, investment and the real wage), whereas the fit of the nominal variables is similar for the two AL specifications. A comparison of these statistics with those reported in the third panel suggests that including the 1-year yield as an observable variable and characterizing the 1-year yield in the model have only a slight effect on the model fit across variables, with a small improvement in the fit of consumption, investment, and inflation. Second, the RMSE statistics associated with the PLM are also lower for the learning specification with TS information across all variables but the 1-year yield. This outperformance by the PLM with TS is fairly robust across alternative subsample periods: The accelerating inflation period (1966:1-1981:4), the downtrend inflation period (1982:1-2009:1), and the periods of economic contraction. Interestingly, the PLM associated with the SlW specification does a much better job in forecasting the 1-year yield in the disinflation period than in the Stagflation period, but the opposite is true for the PLM with TS information. Finally, the outperformance by the PLM with TS extends to the case where the RMSE statistics are computed with real-time data as a reference instead of the actual revised data used in the rest of the table. This suggests that by helping to improve the forecasts of the first announcements of macroeconomic data, the yield curve (which is observed in real time) provides useful information for characterizing agents' expectations above and beyond that included in revised macroeconomic data.[22]

Table 4. RMSE comparison of PLM forecasts (1966:1-2009:1)

SlW-TS	Δc	Δinv	Δw	π	r	$r^{\{4\}}$
ALM	0.686	1.757	0.669	0.257	0.234	0.208
PLM	0.779	1.819	2.450	0.282	0.266	1.024
PLM (period 66:1-81:4)	0.823	2.145	2.682	0.379	0.365	0.484
PLM (period 82:1-09:1)	0.753	1.601	2.306	0.206	0.186	1.232
PLM (contraction periods)	1.353	2.889	2.181	0.305	0.391	0.411
PLM (real-time data)	0.775	4.320	–	0.325	–	–
SlW						
ALM	0.726	1.792	0.763	0.256	0.232	0.209
PLM	1.335	2.001	2.650	0.300	0.268	0.930
PLM (period 66:1-81:4)	1.227	2.201	2.601	0.369	0.354	1.439
PLM (period 82:1-09:1)	1.394	1.876	2.679	0.252	0.202	0.409
PLM (contraction periods)	1.827	3.121	2.099	0.297	0.358	0.439
PLM (real-time data)	1.335	4.478	–	0.348	–	–
SlW with 7 observables						
ALM	0.700	1.784	0.657	0.260	–	–
PLM	0.705	1.812	0.673	0.281	–	–

As pointed out by Slobodyan and Wouters (2012a), a sound performance by the expectation models in terms of RMSE may help obtain a good overall fit of the model, but it provides only indirect evidence on the empirical validity of those expectations. Next, we assess the forecasting performance of the two PLM specifications studied in this paper against

[22] See Croushore (2011) for an outstanding review of the literature on real-time macroeconomic data and the analysis of data revisions.

the forecasts reported in the SPF. Specifically, the SPF reports private sector quarterly expectations on consumption, investment, GDP deflator inflation, and the short-term interest rate (3-month TB yield) from late 1981 onward.[23] Table 5 shows the RMSE comparison of PLM forecasts with SPF data rather than the actual data used in the estimation procedure. Clearly, the PLM forecasts including TS information do a much better job in matching the expectations reported in the SPF across all forward variables than the PLM forecasts without TS information. The rationale for this finding is that the SPF forecasts are based on real-time data and the term spread information included in our PLM specification is also available in real time, whereas the PLM forecasts under the SlW specification are based only on ex-post revised data. The two pieces of real-time data (SPF and the yield curve) may thus share important information available in real time. This finding is consistent with our previous finding that TS information provides useful information for matching forecasts on real-time macroeconomic data. These findings suggest that the use of SPF data may help to discipline model expectations and improve the empirical fit of model expectations.

Table 5. RMSE comparison of PLM forecasts w.r.t. SPF data

Estimation period: 1966:1-2009:1 Comparison period : 1982:1-2009:1	Δc	Δinv	π	r
SlW-TS	0.363	1.240	0.442	1.007
SlW	1.358	2.106	1.206	1.580

The previous section looks at the implications for the parameters estimated of considering SPF data in the estimation procedure, as described in the measurement equation (7), for the

[23] Although SPF expectations on inflation are available for 1968 onward, we decided to focus on the period starting in the first quarter of 1982, when SPF expectations became available for all forward-looking variables considered in this analysis. 1982:1 also roughly coincides with the time when the rate of inflation started to go down. Furthermore, note that we consider the SPF forecasts of the 3-month TB rate as a good proxy of the expectations of the federal funds rate because the actual time series of these two short-term rates are almost perfectly correlated.

short sample period characterized by the Great Moderation. Table 6 shows the corresponding RMSE statistics of PLM forecasts obtained from the model estimated for the short sample period. As a reference, the first panel in this table shows the RMSE statistics for the SPF forecasts. The remaining two panels show the RMSE statistics for the two specifications of the small forecasting models associated with the ALM and the PLM. The numbers in parentheses below the RMSE statistics associated with the PLM forecasts indicate the percentage changes in the corresponding RMSE-statistics when model expectations are disciplined with SPF data (i.e. the percentage changes between the figures reported in the row labeled as "PLM (period 82:1-09:1)" in Table 4 and the corresponding figures in Table 6).

Table 6. RMSE comparison of PLM forecasts (1982:1-2009:1)

	Δc	Δinv	Δw	π	r	$r^{\{4\}}$
SPF	0.589	1.699	–	0.229	0.161	–
SlW-TS						
ALM	0.754	1.636	0.854	0.213	0.166	0.176
PLM	0.667	2.222	5.246	0.215	0.201	0.493
	(-11%)	(39%)	(127%)	(4%)	(8%)	(-60%)
SlW						
ALM	0.667	1.517	1.017	0.246	0.155	0.154
PLM	0.660	2.044	3.628	0.217	0.263	0.455
	(-53%)	(9%)	(35%)	(-14%)	(30%)	(11%)

Interestingly, the forecasts based on the ALM from the two AL specifications are as good as those reported in the SPF when SPF is considered in the set of observables. It is also important to highlight that the AL specification with TS results in similar RMSE statistics even when SPF is not used in the set of observables, as shown in Table 4. Interestingly, including SPF in the estimation procedure results in a greater improvement in the forecasts for those variables that perform worst when SPF data is not used. Thus, the improvement in the consumption growth forecast is greater for the SlW specification (a reduction in the RMSE of 53%) than for the specification that includes TS information (a reduction of 11%). Moreover, the performance of the PLM with TS information is lower for the rest of the variables, except for the 1-year yield, when SPF is included in the set of observables.[24] Thus, including SPF data improves the PLM forecasts of the 1-year yield when the PLM considers TS information (there is a 60% reduction in the RMSE). However, the opposite occurs (there is an increase of 11%) for the forecasting models based on the SlW formulation.

In line with the results shown in Table 5 for the DSGE models estimated using the whole sample period, Table 7 clearly shows that the PLM forecasts that include TS information (SlW-TS) do a better job than the SlW specification in matching the expectations reported in the SPF across most forward variables when the two AL specifications are estimated using the shorter sample (1982:1-2009:1) and SPF data is included in the estimation procedure. The figures in parentheses show the percentage changes in the RMSE-statistics when SPF data are considered in the set of observables (i.e. the percentage changes between the figures reported in Table 5 and the corresponding figures in Table 7). As expected, the forecasts from the two PLM specifications become closer to the SPF forecasts when those forecasts are used in the estimation procedure to discipline model expectations. Moreover, the improvement in the two PLM specifications is inversely related to their relative ability to match SPF forecasts when these forecasts are not used as observables in the estimation procedure (shown in Table 5). Put differently, the need to discipline expectations is greatly reduced for the real forward-looking variables (and to a lesser extent for the nominal variables) by including TS information in the small forecasting models.

[24]This deterioration observed for some variables may be due to the fact that learning requires time and information. That is, the RMSE-statistics computed for the period 1982:1-2009:1 using the whole sample period in the estimation procedure (those reported in Table 4) may be somewhat superior to those RMSE-statistics computed for the period 1982:1-2009:1 using the estimates for this shorter period (reported in Table 6) because the AL processes associated with the former take into account information predating 1982.

Table 7. RMSE comparison of PLM forecasts w.r.t. SPF data

Comparison period : 1982:1-2009:1	Δc	Δinv	π	r
SlW-TS	0.301	1.024	0.179	0.293
	(-17%)	(-17%)	(-60%)	(-71%)
SlW	0.374	1.180	0.175	0.338
	(-72%)	(-44%)	(-85%)	(-79%)

5 Conclusions

This paper considers an estimated DSGE model with adaptive learning (AL) in which the forecasting models of agents include term structure information. More precisely, we extend the AL model of Slobodyan and Wouters (2012a) by introducing the term structure of interest rates and then including term structure information observed in addition to the current and lagged values of the forward-looking variables.

The estimation results show that including term structure information in the agents' forecasting models results in an improvement in model fit. Moreover, the learning specification augmented with term structure information improves the performance of AL in forecasting actual revised macroeconomic data used in the estimation procedure as well as real-time (i.e. the first announcements of) macroeconomic data. The latter finding suggests that the yield curve contains important information available in real time, which is very useful in forecasting aggregate variables above and beyond that provided by revised macroeconomic data. In line with these findings, our estimation results also show that term structure information helps AL expectations to match the forecasts of aggregate variables reported in the Survey of Professional Forecasters, which are formed using information available in real time. Therefore, term structure information further contributes to the empirical validity of AL.

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Part II

Learning with ELMo: inflation expectations and monetary policy rules

1 Introduction

This paper studies the evolution of the inflation rate in the Euro Area (EA) since its creation through a model with learning expectations in order the shape of inflation expectations, showing that current inflation expectations are curbing the effects of the monetary policy and that alternative rules such as asymmetric inflation targeting rules may become more effective in escaping the zero lower bound once agents learn about them.

Most of the central banks of the advanced economies have received the institutional mandate to maintain price stability, defined by a medium-term inflation target. Specifically, in the case of the ECB, the mandate refers to keeping inflation rates close to, but below, 2% in the medium term. This objective is defined in terms of the Harmonised Index of Consumer Prices (HICP), whose basket includes both the components that comprise the core indicator (i.e. services and non-energy industrial goods) and energy and food. As a proxy for the overall indicator, in the model presented in this article the analysis of inflation in the medium term focuses on the core indicator. This has the disadvantage that the monetary policy objective is not strictly represented, but it does facilitates the study of the role of expectations in the deviation from the inflation objective, given that the trend of the core indicator is comparatively less volatile.

The core inflation rate hovered between 1.5% and 2% on average from the time the Euro came into being to the start of the global financial crisis. However, since then, and particularly since 2014, the Euro Area core inflation rate has stood for a prolonged period

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below its previous figures. For the 2009-2019 period, the rate of change of core inflation was 1.1%, 0.6 pp down on the phase prior to the global financial crisis. And further to the outbreak of COVID-19, this disinflationary process has tended to become more acute. Such a prolonged period of moderate inflation might be due either to temporary causes, albeit with high persistence, or, alternatively, to more structural reasons. The first group of explanatory factors, namely the temporary ones, would include elements such as the decline in energy prices or the durable presence over this period of a high degree of slack both in the Euro Area and global economies. The structural causes influencing long-term inflation movements relate to changes in certain fundamentals of the economy. These include most notably sectoral composition (with an increase in the weight of the services sector[1]), globalization (which would give rise to a greater interconnectedness of inflation rates across different economies, against the backdrop of the progressive incorporation into global trade of countries with lower production costs) and changes in consumption patterns linked to population aging.

A stable path of inflation expectations consistent with the price stability objective smooths monetary policy implementation, leading generally to a reduction in the volatility of the economic cycle. However, the prolongation over time of the current low-inflation phase has given rise to a debate on some deanchoring of inflation expectations in relation to the central bank's medium-term objective, and potential feedback between actual inflation and expectations. As a result, the diminished pace of price changes would be exerting a downward impact on economic agents' inflation expectations, which would in turn affect actual inflation in the same direction.

Most models traditionally used in monetary policy design start from the premise that agents form expectations about the economy rationally.[2] This hypothesis implies that, in the shaping of their expectations, agents observe and process efficiently all available information.

[1] In particular, there is a growing body of evidence indicating that services prices are adjusting with less frequency than in other sectors of the economy. See, for example, Bouakez, H., Cardia, E. and Ruge-Murcia, F. (2014), and Álvarez et al. (2006).

[2] For example, some of the general equilibrium models that are commonly used by the New York Federal Reserve (FRBNY DSGE) or the European Central Bank (EAGLE), mainly for conducting simulation exercises, are based on rational expectations.

In particular, agents are able to understand the nature of macroeconomic shocks and their duration, and have the capacity to consistently incorporate news on monetary policy changes or on expected developments in the economy into their expectations. However, in reality, it is unlikely that agents are able to observe and process all available information.[3] On numerous occasions, the nature of shocks and their transmission channels are only imperfectly known by agents and are difficult to identify. Alternatives to this hypothesis have been largely debated in the literature.[4] In this paper we explore the alternative of adaptive learning expectations. This alternative assumes that agents' expectations about future events are partly and progressively updated with the information they receive about developments in the main macroeconomic aggregates. It is further assumed that, when shaping their expectations, agents use a limited amount of information, which they incorporate every period upon the arrival of new information.

The model used in the paper is an Extended Learning Model (ELMo) version of Smets and Wouters (SW, 2007) as in Aguilar and Vazquez (2019) estimated for the EA. The model builds on the DSGE of Smets and Wouters (2007) under the assumption of adaptive learning expectations and the incorporation of the term structure of interest rates through multiple Euler equations associated with the different bond maturities. The extended model results in multi-period-ahead expectations appearing in the different Euler equations. More precisely, in this version of the model, agents form expectations on inflation (and consumption) from one quarter up to five years. The model, estimated for the Euro Area as a whole for the period from 1999 Q1 to 2019 Q4, combines macroeconomic information, (consumption and inflation, among others) with financial information relating to the yield curve. The inclusion of the yield curve enables financial-market information on the future course of the economy

[3] The empirical literature generally finds deviations in survey-based data from rational expectations. As it is explained in Coibion et al. (2018), surveys of expectations reveal that there are biases across different demographic groups, and that, for example, perceived inflation is affected by each agent's consumption basket, even if there is a commitment from a central bank.

[4] Since the pioneering publications by Marcet and Sargent (1989) and Evans and Honkapohja (2001) a growing literature (including Preston, 2005; Milani, 2007, 2008, 2011; Eusepi and Preston, 2011; Slobodyan and Wouters, 2012) , see the discussion in this regard in Aguilar and Vazquez (2019) and Vazquez and Aguilar (2021).

to be incorporated.[5] Accordingly, this specification allows a more complete characterization of expectations, by combining macroeconomic and financial information.

This paper focuses on the nature of the deviations from the inflation objective through a learning scheme, this allows us to understand to what extent agents perceive current deviations in the inflation rate as temporary or permanent and shed some light on the (de)anchoring of inflation expectations in the EA. The recent conclusions in the literature related to the EA point in two directions. On the one hand, Natoli and Sigalotti (2018) look at co-movements between short- and long-term inflation expectations and find higher correlation and negative shocks affecting short-run beliefs that impact long-run expectations, suggesting a risk of de-anchoring in the long-run. On the other hand, Grishchenko et al (2019) study the behavior of survey data for the US and EA in a dynamic factor model, finding that the expectations remain anchored in both economies.

Another aspect relevant is the presence of the Effective Lower Bound (ELB) during a prolonged period of low inflation and poor economic activity. The presence of the ELB curves the ability of the central bank to implement its monetary policy and has the risk of making low inflation episodes longer than in its absence. Alternatives to reduce the frequency and duration of ELB episodes with respect to the current framework are now in the debate in Bernanke (2017), and Mertens and Williams (2019) among others. These papers show that alternatives to the current framework such as, Inflation Targeting (IT) and Price-level Targeting (PLT), with the addition of an asymmetric version of each: Asymmetric Inflation Targeting (AsIT) and Temporary Price-level Targeting (TPLT), reduce the presence of ELB episodes, however, these results hinge on the assumption that the new rule is credible.

There is a bunch of papers studying the interaction between monetary policy and expectations under adaptive learning. Evans et al. (2008) argue that aggressive fiscal policy measures may reduce the severity of liquidity traps. Evans and Honkapohja (2005) study

[5] In particular, the breakdown of nominal interest rates into the real, risk-free interest rate, inflation expectations and a risk component enables the relationship between the implied yield on a bond and the inflation rate to be exploited

the ability of aggressive money supply rules to overcome ELB episodes. In Honkapohja and Mitra (2020), price-level targeting is a potent tool by means of escaping liquidity traps, even if the price-level targeting policy is imperfectly credible. Findings in Eusepi and Preston (2011, 2018) suggest that active fiscal theory may help stabilize inflation in economies with interest rate pegs and learning agents. Mertens and Ravn (2014) simulate an economy with learning agents and a one-time ELB episode and show that the learning economy can escape the ELB when expectations are not too pessimistic. In this matter this paper goes further and studies the transitional effects of new policy rules to inflation expectations.

The results show that current expectations are shaping the effects of monetary policy. An asymmetric inflation targeting rule, with a stronger response to inflation when it is below its trend, seems to be a robust alternative that provides improvements over standard inflation targeting, in terms of reducing the presence of ELB episodes, however there is one important consideration: changing the rule is not very effective until agents have had time to learn about it: in this model, the announcement of the new rule has no effect on agents' expectations; instead, they only update them as they see the central bank behaving in a different way and learn about it, which requires time. This is very different from what we observe in models with rational expectations, where the announcement perfectly anchors agents' expectations and has immediate effects in the economy.

The paper is structured as follows. Section 2 introduces the DSGE model with multi-period expectations estimated for the EA. Section 3 studies the determinants of inflation expectations in the EA since its creation. Section 4 analyses the transitional effects of alternatives to the current monetary policy framework, and section 5 concludes.

2 A DSGE model with multi-period expectations for the Euro Area

The model builds on the SW model and its AL extensions studied by Slobodyan and Wouters (2012) and Aguilar and Vázquez (2019). This standard medium-scale estimated DSGE model contains both nominal and real frictions affecting the choices of households and firms. The assumption of adaptive learning implies that expectations are based on a limited information set, meaning that agents use small forecasting models in forming their beliefs about future realizations of forward-looking variables, in this case by using simple autoregressive models, and that they adapt the coefficients of these forecasting models by a simple Kalman filter updating procedure. In addition, the extension of the model to account for the term structure of interest rates through the Euler equation results into a multi-period forecasting model, with expectations about the key macroeconomic variables ranging from one quarter to five years ahead.

More specifically, the expectations-formation mechanism of consumption, investment and inflation in the model rests, in each period, on simple learning rules that take into consideration the latest observed value and the size of the previous error forecasts to update the learning coefficients. In the concrete case of inflation, the rule for updating expectations is as follows:

$$E_t \pi_{t+i} = \alpha_{i,t-1} + \beta_{\pi_i,t-1} \pi_{t-1},$$

where π_{t-1} is the deviation from target observed in the last quarter and $\beta_{\pi_i,t-1}$ measures the degree of transmission of the observed deviation to expectations i (denoting a number) quarters ahead. That is to say, under this rule agents incorporate the latest available information on the deviation by inflation from target into their inflation expectations at different horizons (up to 5 years) target. Moreover, this learning rule captures through $\alpha_{i,t-1}$ the possibility that deviations from the inflation objective may have long-lasting effects on inflation expectations over a forecast horizon of i quarters. Three possible values are considered in the analysis for i: one, four and 20 quarters.

The greater the persistence of the deviations perceived by agents (π_{t-1}) is, for a given horizon i, the greater$\beta_{\pi_i,t-1}$will be and, therefore, the higher the pass-through of these deviations to expectations. By way of illustration, a perceived value of $\beta_{\pi_i,t-1}$ equals to 0.5 means that agents expect the latest observed deviation from target to halve in i quarter. Alternatively, a unit value for this coefficient would mean that agents expect the deviation to hold in full over the next i quarters. Moreover, if agents were to believe that deviations from target are permanent, which would be tantamount to a change in the inflation target, then the coefficient $\alpha_{i,t-1}$would be observed to be other than zero.

Testing the anchoring of expectations

Under this simple expectations-formation framework, it is possible to estimate both learning coefficients and, on the basis thereof, to analyze the degree of temporariness associated with the deviations from inflation assigned by agents in constructing their expectations. Under a scenario of fully credible monetary policy, agents would not perceive permanent deviations from target $\alpha_{i,t-1} = 0$ and temporary deviations would diminish over the course of the forecast horizon ($\beta_{\pi_1} > \beta_{\pi_4} > \beta_{\pi_{20}}$).

2.1 Estimation

The DSGE model is estimated for the sample period from 1999Q1:2019Q4, using the quarterly series of the inflation rate, the short term interest rate, the log of hours worked, and the quarterly log differences of real consumption, real investment, real wages, and real GDP with the addition of the 1, 3 and 5-year government benchmark bond yields. The measurement equation is

$$
X_t = \begin{bmatrix} dlGDP_t \\ dlCONS_t \\ dlINV_t \\ dlWAG_t \\ dlP_t \\ lHours_t \\ ECBrate_t \\ 1-year\,TB\,yield_t \\ 1-3year\,TB\,yield_t \\ 1-5year\,TB\,yield_t \end{bmatrix} = \begin{bmatrix} \overline{\gamma} \\ \overline{\gamma} \\ \overline{\gamma} \\ \overline{\gamma} \\ \overline{\pi} \\ \bar{l} \\ \overline{r} \\ \overline{r}^{\{4\}} \\ \overline{r}^{\{12\}} \\ \overline{r}^{\{20\}} \end{bmatrix} + \begin{bmatrix} y_t - y_{t-1} \\ c_t - c_{t-1} \\ i_t - i_{t-1} \\ w_t - w_{t-1} \\ \pi_t \\ l_t \\ r_t \\ r_t^{\{4\}} \\ r_t^{\{12\}} \\ r_t^{\{20\}} \end{bmatrix}, \tag{9}
$$

where l and dl represent the log and the log difference, respectively. $\overline{\gamma} = 100(\gamma - 1)$ is the common quarterly trend growth rate for real GDP, real consumption, real investment, and real wages. $\bar{l}$, $\overline{\pi}$, $\overline{r}$ and $\overline{r}^{\{j\}}$ are the steady-state levels of hours worked, inflation, the ECB interest rate, and the 1, 3, 5-year (ie. for j equal to 4, 12, 20 quarters) bond yields, respectively.

We follow a Bayesian estimation procedure. First, the log posterior function is maximized by combining prior information on the parameters with the likelihood of the data. The prior assumptions are exactly the same as in Slobodyan and Wouters (2012). In addition, we consider loose priors for the parameters characterizing both the $1, 3, 5$-year yield dynamics and the measurement error processes. The Metropolis-Hastings algorithm is used to generate the posterior distribution and to compute the log density of the model. We report the key parameter estimates in the model in Appendix A.1.

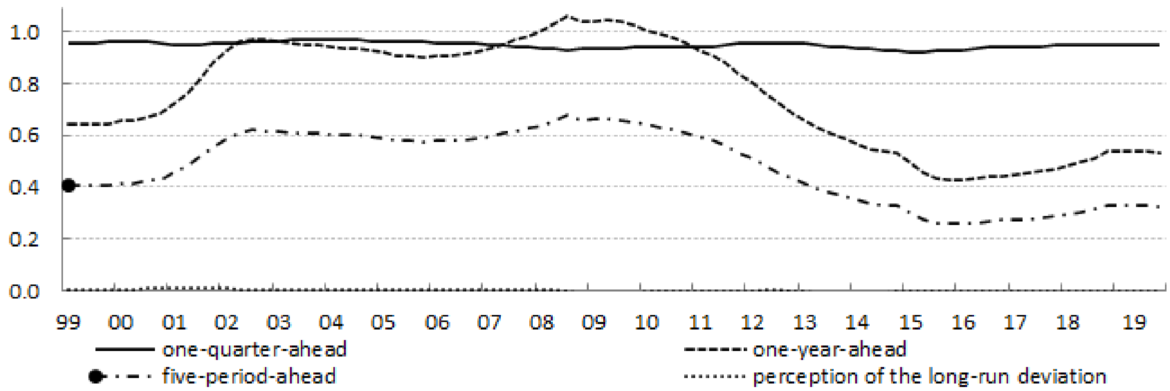
2.2 The evolution of expectations: cycle and trend

Figure 1 shows, for the different horizons analyzed, the estimated coefficients for the Euro Area for the period 1999-2019. As might be expected, the value of the coefficients indicates that, except for some isolated period, the weight assigned by agents to past inflation in their formation of expectations about price growth diminishes as the time horizon increases ($\beta_{\pi_1} > \beta_{\pi_4} > \beta_{\pi_{20}}$). The value of the coefficient at one quarter (β_{π_1}) is close to unity, suggesting that agents expect, at three months, that the deviations of inflation from target will hold unchanged. Moreover, this coefficient has been highly stable since the start of Economic and Monetary Union. In the case of medium-term expectations, i.e. four and 20 quarters ahead (β_{π_4} and $\beta_{\pi_{20}}$), the estimates suggest that agents reduce, as the time horizon increases, the weight they assign in their learning rule to the latest observed figure. The course of both coefficients shows a positive correlation with the behavior of actual inflation, indicating that, in periods with higher inflation rates (2001-2002 and 2007-2008), agents estimate that deviations have a higher persistence. This finding suggests that prices show a different degree of adjustment according to the level of the inflation rate through the cycle.[6] In any event, according to the model, in the longer run inflation would return, in the absence of fresh shocks, to the medium-term monetary policy objective, since the value estimated for ($\alpha_{i,t-1}$) is very close to zero at any forecast horizon.[7]

[6] One possible explanation is the greater ease with which firms can, in periods of excess demand, raise prices instead of increasing productive capacity. Conversely, in periods of low demand, they can opt to reduce their capacity temporarily. See Bobeica and Sokol (2019).

[7] The chart depicts the coefficient estimated when i= 20 quarters. In practice, the estimated value when i is equal to 1 or 4 is very similar, which can be explained by the fact that agents have the same information to estimate the long-term deviation by inflation from target irrespective of the horizon i at which they formulate their short or medium-term expectations.

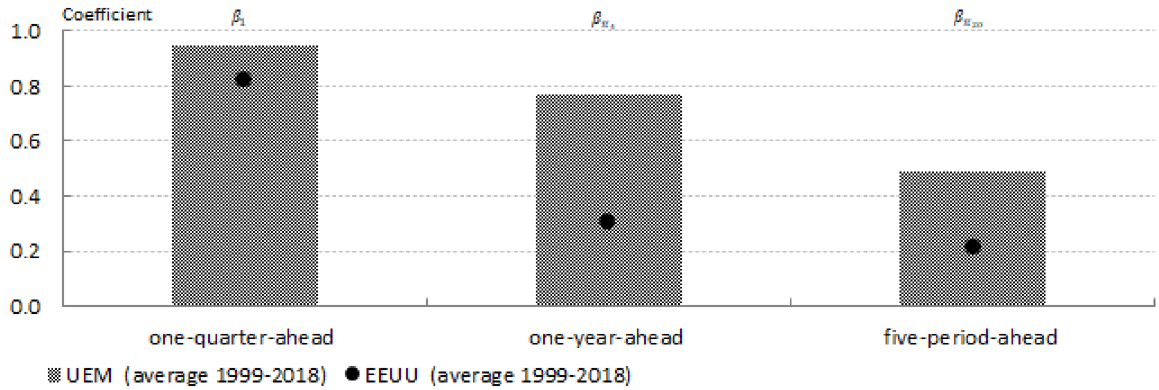
Figure 1. Inflation expectations coefficient's evolution



2.3 International comparison

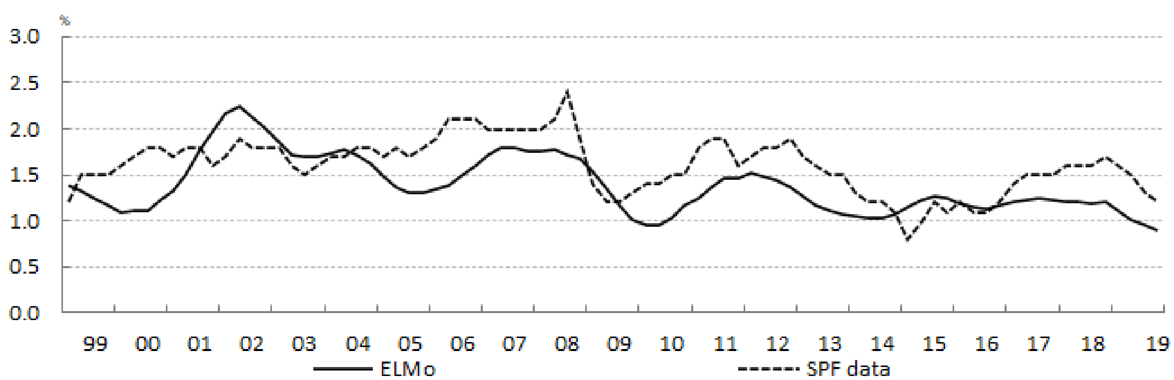
When comparing with the estimates from Aguilar and Vazquez (2019) for the United States (US), see figure 2 below, the degree of persistence of inflation over the past 20 years on average can be seen to be less in the US than in the Euro Area. That might be indicative of less nominal rigidities in the US economy. A shock to inflation will be more or less persistent depending on a series of factors which include, among others, the degree of wage inertia (depending on the degree to which wages are linked to the overall price index), price-setting rigidities and supply-side rigidities (which, in the model, are manifested via a limited capacity to adjust the use of productive factors). In the case of the model estimated for the US, the degree of wage indexation is comparatively lower, while the flexibility of prices is greater. Consequently, inflation expectations in the US economy are less sensitive to past inflation, mainly in the medium and long term. Specifically, the coefficients estimated for β_{π_4} and $\beta_{\pi_{20}}$(i.e. 1 and 5 years ahead) are approximately half those obtained for the Euro Area, meaning that the deviation by expectations in the face of a shock is less both in terms of level and duration.

Figure 2. Sensitivity of inflation expectation to last value observed: EA vs US



The estimation results can be somewhat sensitive to the model used. One way of assessing the estimates offered with is to compare the inflation expectations at the one-year forecast horizon obtained from the model and those drawn from the ECB's Survey of Professional Forecasters (SPF). This quarterly survey reflects the expectations of participant respondents – who are experts from financial and non-financial institutions alike in the Euro Area – about inflation rates, GDP growth and Euro Area unemployment at different horizons. The comparison between both sources of expectations shows that the dynamics captured in the model are consistent with the SPF series (see Chart 3), which supports the empirical validity of the estimates associated with the adaptive learning expectation formation.

Figure 3. ELMo vs SPF one-year-ahead inflation expectations in the EA



3 Monetary policy under learning: Transitional effects

3.1 Transitional excercise

In this section we focus on the evaluation of alternative monetary policy rules, and how the expectation formation of agents affects their performance, using the estimated version of ELMo for the EA.[8] We run a simulation exercise to see how often the economy hits the ELB, and how costly this is, under different monetary policy rules. Because the model is inherently time-varying, a nested Monte Carlo simulation is required.[9]

Starting from the current values of the $\alpha_{1,t=2018Q4}$ and $\beta_{1,t=2018Q4}$ parameters in the expectation rules (the ones that come out of the estimation of the model, for the end of 2018), we simulate 100 random paths of the economy, each with a horizon of 60 quarters; in these scenarios, the agents are constantly updating the parameters of their expectation rules according to the simulated evolution of the economy, which in turn depends on the specific realizations of the shocks. For each simulated scenario, and for each quarter, we run a longer simulation of million quarters *without additional learning* and see how often the economy hits the ELB and how costly this is; for doing so we run the Monte Carlo simulation with the learning parameters fixed with those corresponding to the learning process from the outer Monte Carlo simulation.[10]

We run this exercise, with the same set of shocks, under three alternative strategies: inflation targeting (IT, where the variable that enters the Taylor rule is the current year-on-year rate of inflation), asymmetric inflation targeting (AsIT, which has the same Taylor rule but with a higher response parameter when inflation is below its steady state level than when

[8] For this set of exercises, we use a version of the model with short-run expectations.

[9] In all simulations we use a weighted bootstrap from 1999-2018 to make sure the simulated evolution has characteristics that match those of the observed data; the weights are introduced to make sure that the simulations do not replicate too often an episode similar to the global financial crisis.

[10] A simulation with constant coefficients in the expectations equations does not intend to forecast how the economy would behave in the future, but to explore the distribution of outcomes of the model given the current coefficients. An alternative exercise where the inner Monte Carlo simulation also allows learning would yield results that would be independent of the initial conditions, and would not be informative about the transition dynamics of the economy when a new monetary policy rule is announced.

it is above: 3 instead of 2), and Price-Level Targeting (PLT, where the variable that enters the monetary policy rule is the deviation of the price-level from its long-run trend). Figure 4 shows the results. The left column of graphs corresponds to IT, the column on the center to AsIT and the column on the right to PLT; the first row of graphs shows the percentage of quarters at the ELB, the second row of graphs shows the average duration of ELB episodes, and the last two rows correspond to the GDP and inflation losses in the presence of ELB relative to a scenario where the ELB does not apply (i.e. when strongly negative interest rates are allowed).

3.2 Results

The first finding is that current expectations (the estimated values of the α and β coefficients for the end of 2018) are damaging: keeping the standard IT rule, simulating the model forward and letting agents update their beliefs tends to have very positive effects on the outcomes (the ELB becomes less common and the associated costs fall).The rationale for these simulation results is that, at some point, positive shocks start to arrive, agents see the economy recovering and the interest rate lifts off from the ELB (first row in figure 4), they update the learning coefficients, anchoring them again, and the model reaches a state in which the ELB is less common and less costly. Note that this is not because of the lift-off of the interest rates after those positive shocks appear, but because of how that evolution changes the parameters that govern the expectations of the agents.[11]

A second result is that changing the monetary policy rule is not very effective until agents have had time to learn about it: in this model, the announcement of the new rule has no instantaneous effect on agents' expectations; instead, they only update them as the new

[11]Simulating one million quarters with random shocks, with the standard IT rule and the estimated expectations rules of 2018, the model stays at the ELB more than 60% of the time; but after five years of random shocks and agents' learning, a simulation of one million quarters with the same random shocks, with the same IT rule but with updated expectations, makes the model stay at the ELB around 30% of the time (the range goes from almost 0% to around 40%, depending on the simulated scenario for those 5 years of learning: some scenarios lead to expectations rules that are still very damaging, whereas others end up with very benign expectations rules).

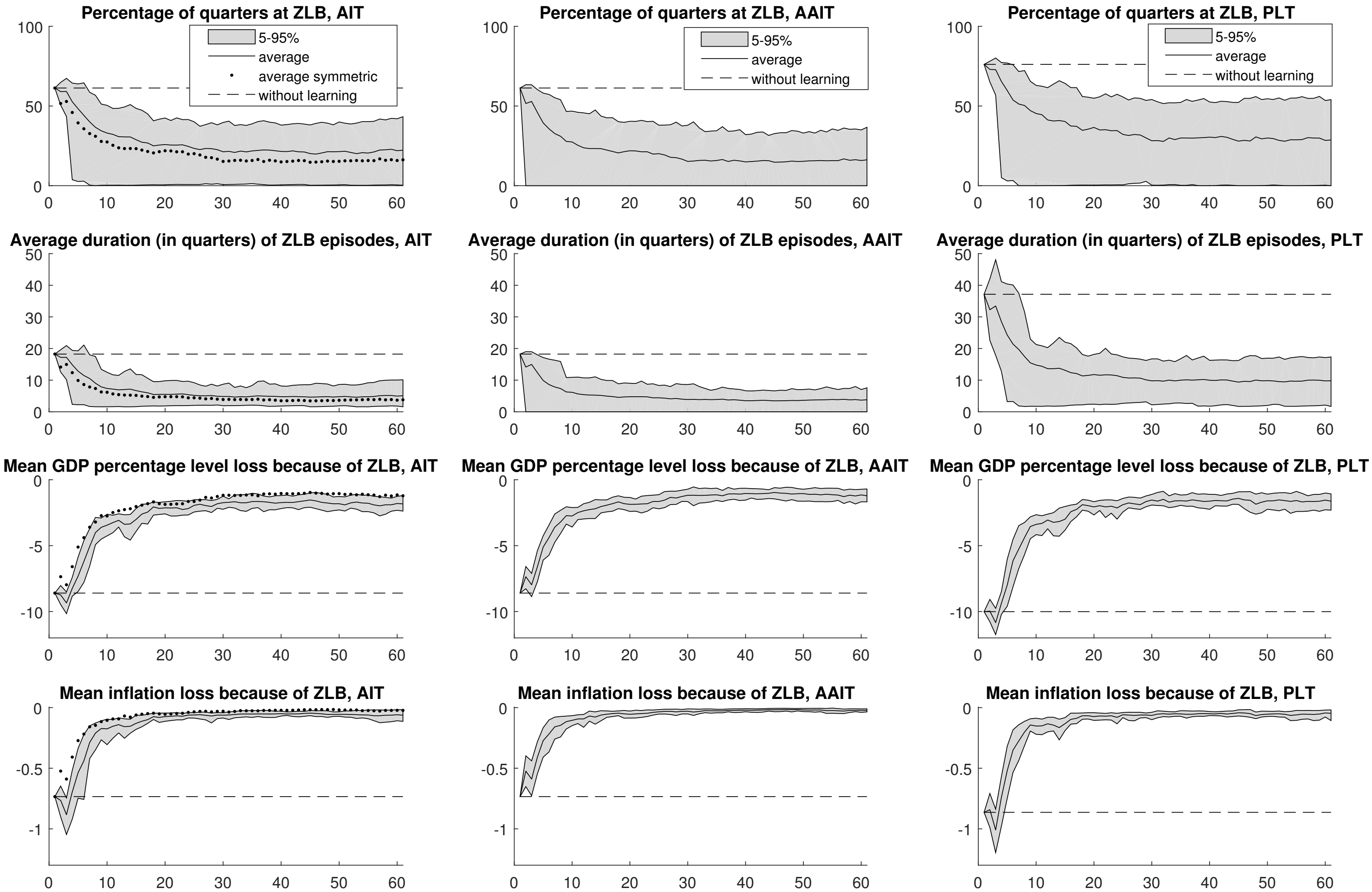
monetary policy changes the evolution of macroeconomic variables and agents learn about it, which requires time. This is very different from what we observe in rational expectations models, where the announcement perfectly anchors agents' expectations and has immediate effects in the economy.

Third, we observe that the asymmetric IT rule (AsIT) gives better results than the standard IT rule (left panel vs center panel, or, for an easier comparison, solid line vs dotted line in the left panel). The economy spends a bit less time in the ELB; the difference in this metric is small due to this stronger downwards reaction of interest rates, the rule itself makes them go faster towards the ELB. But, even with that, the GDP loss and inflation loss associated to the ELB are significantly reduced. And most of this improvement comes from the effect through the expectations of agents. Without learning (i.e. keeping the α and β coefficients of the expectations rules at their estimated values for 2018) the results are very similar with IT or AsIT, but when agents are allowed to adjust their beliefs, a faster reduction of interest rates when negative shocks hit the economy affects the expectations of agents, in a way that, even if the ELB is reached sooner, it also allows the economy to exit faster from the ELB.

Finally, the results under the price-level targeting rule are somewhat complex (right column of graphs). The PLT rule is as effective as the AsIT rule at reducing the cost of the ELB in terms of GDP and inflation, but the trade off generated by this make-up strategy is even stronger than was the case for AsIT, and the time spent at the ELB actually increases with respect to the results with IT or AsIT. When using PLT in ELMo, the ELB episodes become much longer: if negative shocks hit the economy and bring interest rates to their lower bound, the monetary policy rule keeps them there for a longer time (once inflation starts to rise, interest rates remain low, until the price-level actually recovers). In a model with rational expectations, the announcement that interest rates will remain lower for longer is effective at avoiding the ELB in the first place, because it anchors very efficiently agent's expectations. But in ELMo the announcement does not generate this change automatically: agents only change their expectations once they see the monetary authorities acting in a different way, which, in a ELB episode, happens when the economy starts to recover and they observe that interest rates remaining at the ELB for a longer time. After the lift off, if the economy is hit again by negative shocks relatively soon, agents will remember what happened the last time, and a new ELB episode can be avoided, or if it happens, it can be shorter; but as time passes and the economy evolves outside of the ELB, agents "forget" about their past experience at the ELB (they keep learning about how the economy behaves outside the ELB) which means that, if negative shocks strike again, a longer ELB episode may happen again. The results balance the negative direct effect of the PLT rule (a lower for longer make-up strategy) with the positive effect through agents' expectations (which become better anchored after those long ELB episodes); the overall result will depend on the parametrization used in the simulation exercise. Further exploration is needed regarding the parameters of the PLT rule and the speed at which agents learn and forget (e.g. faster learning makes the lower-for-longer strategy start to work sooner, but also makes agents forget sooner about what happened in previous ELB episodes).

In conclusion, these simulations showcase the importance of agents' expectations in the assessment of the effects of different monetary policy rules. An asymmetric inflation targeting rule, with a stronger response to inflation while the inflation rate is below its steady state level, seems to be a robust alternative that provides improvements over standard inflation targeting and does not have the communication and lower-for-longer costs of a price-level targeting strategy.

Figure 4. Simulations transitional effects of new regimes



4 Conclusions

This paper studies the anchoring of inflation expectations in the Euro Area in the context of a model with adaptive learning expectations and studies the transitional effects of alternative make-up strategies to the current monetary policy regime. The approach used allows a distinction to be drawn between which portion of the low inflation phenomenon might be due to temporary factors and which might be considered permanent. The first result, with respect to the evolution of inflation expectations in the Euro Area, suggest that agents perceive the inflation rate's recent departure from the monetary policy objective to be predominantly temporary, although the deviations from target are marked by a considerable degree of persistence. The second result is that expectations expectations are an important factor in shaping the effects of monetary policy. While current expectations are curbing the effectiveness of monetary policy under the presence of the ELB, alternative rules such asymmetric inflation targeting rules (that respond stronger when inflation is below trend) are benefitial in escaping the ELB. In addition, this paper states the implications in the transition from one monetary policy rule to another, showing that changing the rule is not very effective until agents have had time to learn about it. The the announcement of the new rule has the maximum effect agents observe their implementation and learn about it, which requires time, which is very different from what the standard rational expectations models, where the announcement perfectly anchors agents' expectations and has immediate effects in the economy.

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Part III

The importance of data revisions

1 Introduction

The existence of data revision must be acknowledged when macroeconomic series are used for business analysis. This chapter provides a detailed analysis of the statistical properties of data revisions for the euro area and studies the appropriate modeling of real-time data and its revision in DSGE models for business cycle analysis.

The main macroeconomic series are regularly revised relative to their real-time release to incorporate new information that was not available at the time of the initial announcement or to incorporate changes, such as in the definition of the indicator or the measurement of the variable. A distinction between whether the data comprises initial releases and/or final revised data must be taken into account by researchers when constructing datasets. If data revisions are not well-behaved, meaning that they can be forecasted, researchers who ignore this fact may suffer from a bias in their analysis. This chapter studies the properties of real-time data and their revisions for the euro area and proposes a modeling framework to incorporate this phenomenon into a DSGE model.

In one of the earliest studies of data revision properties, Mankiw, Runkle and Shapiro (1984) focus on the predictability of data revisions. They analyze whether the preliminary announcements of money stock are rational forecasts of the final announcements or observations containing a measurement error of the revised series. Mankiw and Shapiro (1986) extend this study to the series of GNP.[1] These two papers conclude that money stock revisions are predictable but GNP revisions are not. This led to a primary classification of revisions as adding news or reducing noise. Revisions add news when the initial announcement is an optimal forecast of the final data, in which case they are orthogonal to initial data and there-

[1] Other relevant papers of the matter during that periods are Mork (1987, 1990)

fore unpredictable. Revisions reduce noise when the initial announcement is an estimate of the final data with a measurement error. In that case the initial announcement is correlated with the revision, thus, becoming predictable. Diebold and Rudebusch (1991) subsequently highlight the importance of data revisions in macroeconomics. They show that the US index of leading economic indicators does a fine job at predicting recessions ex-post but fails in predicting future recessions. This is because the indicator is constructed to explain revised past data, and thus ignores the fact that initial data releases may look very different once they are revised.

The paper by Croushore and Stark (2001) increased the popularity of real-time data and their revisions by providing a regularly updated real-time dataset of the main macroeconomic variables. In particular, Croushore (2011) extensively reviews the literature and discusses the data implications of real-time data for data revisions, forecasts, monetary policy analysis, macroeconomic research, and current analysis of financial and business conditions. The use of real-time macroeconomic datasets appears to become more important for policy institutions with the development of new datasets by statistical agencies, such as the Federal Reserve Bank of Philadelphia, the European Central Bank, and the OECD.

More recently, Aruoba (2008) defines the desirable statistical properties of data revisions, namely i) the mean is expected to be zero; ii) small variance compared to that of the revised variable; and iii) unpredictability. He finds that these properties are not satisfied in the revisions of major macroeconomic variables in the United States, as they have a non zero mean, their volatility is large compared to the final data, and they can be predicted using the information set at the time of the initial announcement.[2]

Another relevant aspect is the impact of data revisions on the estimation of DSGE models, which are now popular for macroeconomic analysis at central banks. Casares and Vázquez (2016) introduce an extension of the Smets and Wouters DSGE model (2007) that includes both real-time and revised data from the U.S. economy. Their estimates show a level of both

[2]A similar study is present in Faust et al (2005) for the G7 countries

habit formation and price indexation which is lower than the standard model. They also find that shocks in data revisions explain roughly 10% of output variability. This means that omitting revisions may cause two problems: First, a bias in the parameter estimation; and second, overestimation in the sources of business cycle variability.

This chapter contributes to the literature on data revisions by providing a detailed analysis of the statistical properties of data revisions for the euro area and studying the appropriate modeling of real-time data and its revision in DSGE modeling. More precisely, following Casares and Vazquez (2016), the Smets and Wouters (2007) DSGE model is augmented to include real-time data (by assuming that indexation rules and the monetary policy rule are based on real-time data) and to incorporate data revisions. The aim is to pinpoint the source of data revisions (whether they reduce noise or add news) and to assess their macroeconomic implications. One of the main findings is that data revisions are not well-behaved, i.e. they are correlated with initial announcements and show high volatility. These empirical findings are confirmed in reduced-form regression analysis and in an estimated DSGE model augmented with data revisions. These findings are in line with those of Casares and Vázquez (2016) for the US. As a consequence, revisions become a major source in the business cycle decomposition. In the case of the Euro Area they account for one-third of the output variability, which is roughly three times the figure estimated for the US in Casares and Vázquez (2016). This finding leads to the conclusion that DSGE models for business cycle analysis which omit real-time data and data revisions may introduce a major source of bias into the estimated variance decomposition and encourages further improvements in the estimation of real-time data from the statistical agencies.

The rest of this chapter is structured as follows: Section Two introduces the concept of revisions, describes their main properties, and proposes a specific framework for the inclusion of data revisions in DSGE models. Section Three derives the real-time equations that enter into the extended DSGE model. Section Four presents the data and estimation procedure and Section Five discusses the main findings of the estimated DSGE-extended model. Section

Six concludes.

2 Data revisions

This section is divided into two parts and provides a rationale for the inclusion of data revisions in macro models. The first part defines the concept of data revisions and the main points to be considered when taking them into account, some of them often ignored in the literature. The second part studies the statistical properties of data revisions in the euro area and provides an empirical justification for their inclusion in DSGE models.

2.1 The concept of data revisions

Data revisions can be defined as the difference between the data initially announced and the final revised data . In the case of the euro area, the first announcements of quarterly real GDP, GPD deflator, and real consumption are generally released with a lag of one quarter, while the final revised data are published between four and twelve quarters later.[3] This definition can be expressed formally as follows:

$$y_t = y^r_{t,t+1} + rev^y_{t,t+S}, \tag{10}$$

where y_t refers to the final revised observation of GDP, $y^r_{t,t+1}$ represents the initial announcement with one quarter delay, and $rev^y_{t,t+S}$ captures the total value of revision after $t+S$ periods. A similar formula can be applied to the consumption and inflation revision processes.

The vintage matters

The literature abstracts from the importance of the vintage in defining data revisions.[4]

[3] Benchmark revisions may also occur during the revision process. They involve methodological changes, such as the concepts included in the definition of the variable or the reference year in the series.

[4] The paper by Croushore and Stark (2001) is an exception. They discuss the election of data vintages, but in the context of economic forecasting.

However, authors such as Croushore and Stark (2001) are an exception in that they focus their research on the choice of data vintages in the context of economic forecasting. The choice of the vintage, however, becomes highly important when it comes to variables expressed in growth rates, as revisions between vintages are a potential source of "noisy" revisions. Table 2.A shows the different vintage publications of US GDP, which help to illustrate the importance of the choice of vintage in computing output growth rates.

Table 2.A: GDP US

Period\Vintage	1990:Q2	1990:Q3	1990:Q4	1991:Q1
1990:Q1	4195.8	4150.6	4150.6	4150.6
1990:Q2		4163.2	4155.1	4155.1
1990:Q3			4173.6	4170
1990:Q4				4147.6

GDP: Billions of real Dollars

Depending on the choice of the vintage, output growth rates can be calculated in two ways: Across different vintages or within the same vintage. In the first case, the growth rate is obtained using the first quarterly data announcements from two consecutive vintages, while in the second growth rates are computed using the first vintage in which both quarterly variables are available. Formally, they can be expressed as follows:

$$g^1 = \left({}^{y^r_{t+1,t+2}}/_{y^r_{t,t+1}} - 1\right) \times 100,$$

$$g^2 = \left({}^{y^r_{t+1,t+2}}/_{y^r_{t,t+2}} - 1\right) \times 100.$$

Under the first option, g^1, the growth rates are always computed using the first release, while the second method, g^2, may already incorporate a revision in the first observation. However, using the same vintage avoids the impact of benchmark revisions between vintages.[5]

[5] For our US sample data, 1983Q1:2008Q1, there are in all five benchmark revisions (1985:Q3, 1991:Q3, 1995:Q4, 1999:Q3 and 2003:Q4), while for the euro area there was one main benchmark revision in 2005. Vázquez, María-Dolores, and Londoño (2012) adjust benchmark revisions by replacing them with the average

The quarterly growth rate of output in 1990Q2, depending on the method, would be either:

$$\Delta y_{1990Q2,g^1} = (y^r_{1990Q2,1990Q3}/y^r_{1990Q1,1990Q2} - 1)x100 = (4163.2/4195.8 - 1) \times 100 = -0.776\%.$$

$$\Delta y_{1990Q2,g^2} = (y^r_{1990Q2,1990Q3}/y^r_{1990Q1,1990Q3} - 1)x100 = (4150.6/4163.2 - 1) \times 100 = 0.303\%.$$

As illustrated in the example, different choices of vintage provide opposite-sign growth rates. The size of the revisions are therefore directly affected by this choice, so this research acknowledges the properties of data revisions under both alternatives.

2.2 Regression analysis of data revisions

This subsection studies the main statistical properties of data revisions in the euro area to explain why they are relevant and should be included in macro models. The analysis is divided into two parts: The first sets out the main descriptive statistics of data revisions under both approaches (g^1 and g^2) of the quarterly growth rates of real GDP, consumption, and inflation. The second part estimates the relationship between initial announcements and data revisions to determine whether revisions add news to the initial announcement or reduce errors, and provides an estimation of the process of data revisions.

Main descriptive statistics

According to Aruoba (2008), if data revisions are well-behaved they should have the following properties: First, the mean is expected to be zero. This would imply that the initial announcement is an unbiased estimate of the final revised value. Second, the variance should be small when compared to that of the revised value. This is measured by the noise to

value of the two observations before and after. In this paper, benchmark revisions are managed by replacing them with the value obtained from g^2 , which greatly simplifies the procedure.

signal ratio, which is the ratio between the standard deviation of the final revisions and the final data. Finally, the revision should be uncorrelated with the initial announcement, i.e. it should be unpredictable. Table 2.B shows the main descriptive statistics for data revisions according to the list of interest for the quarterly growth rates (g^1 and g^2) of quarterly real GDP, consumption and inflation.

Table 2.B Euro Area descriptive statistics of data revisions

	Revision in g^1			Revision in g^2		
	GDP	Consumption	Inflation	GDP	Consumption	Inflation
Mean	0.110	-0.111	-0.126	0.276	0.357	0.079
Absolute Mean	1.645	1.708	0.716	0.940	0.845	0.531
Median	0.172	-0.088	-0.088	0.478	0.413	0.095
Min	-4.640	-7.440	-3.003	-2.978	-2.204	-1.637
Max	4.827	4.830	2.055	2.479	3.131	2.202
Std. D. Revision	2.126	2.273	0.957	1.153	1.032	0.686
Noise/Signal	1.602	1.658	0.761	0.720	0.622	0.901
Correlation with Initial	-0.708	-0.712	-0.683	-0.376	-0.215	-0.414
Correlation with final	0.415	0.481	0.400	0.337	0.422	0.509
Correlation Initial-Final	0.348	0.272	0.395	0.744	0.793	0.571

The overall results suggest that revisions are not well-behaved. Data revisions of output, consumption, and inflation have statistically non-zero means. The output and consumption revisions also have a noise to signal ratio greater than one under both methods of computation. Finally, all variables show a relatively high level of (negative) correlation between revisions and the initial release.[6]

Regarding the choice of vintage in terms of the statistical properties of the revisions, it can be seen that when the second method of computing growth rates (i.e. using the same vintage)

[6] In the case of the US (see Appendix 1.C.2), the results are somewhat similar and the properties listed by Aruoba (2008) are not satisfied either.

is used in the case of the US (see Appendix 1.C.2) the results are somewhat similar; nor do they satisfy the properties listed by Aruoba (2008). g^2 reduces the variability of the revisions (and thus, the noise to signal ratio) and reduces correlation with the initial announcement, but the correlation with the final data remains low. In addition, the correlation between the initial announcement and the final data is closer when growth rates are computed for the same vintage. The fact that the mean is lower when g^1 is used than when g^2 is used is a consequence of large error offsetting signs, so the absolute mean is smaller when g^2 is used. These results may prompt the reader to use growth rates computed for the same vintage, but it must be realized that one revision may already be included in one of the observations used to compute the growth rate. Concerning the use of either method in the literature, Croushore and Stark (2001) rely on growth rates under the same vintage (although they mention the possibility of using g^1 too), Casares and Vázquez (2016) and Vázquez, Maria-Dolores and Londoño (2012) use the g^1 approach and make no specific mention of the method used in Aruoba (2008).

Noise or news?

We formally test the hypothesis of whether revisions reduce noise or add news. They reduce noise when the initial announcement is an early estimate of the revised variable with a measurement error. This implies that the revision is uncorrelated with the final value but correlated with the initial data release. By contrast, revisions add news when the initial announcement is an efficient estimate of the revised variable and the revision is correlated with the final data but uncorrelated with the initial announcement (as the revision is unpredictable). Following Aruoba (2008), we test both hypotheses under the two methods proposed for computing growth rates with real-time data:

-Noise: $y^r_{t,t+1} = \alpha_1 + \beta_1 y_t + u^1_t$

-News: $y_t = \alpha_2 + \beta_2 y^r_{t,t+1} + u^2_t$

where the first joint hypothesis $\alpha_1 = 0,\ \beta_1 = 1$ tests the noise hypothesis and the second hypothesis $\alpha_2 = 0$, $\beta_2 = 1$ tests the news hypothesis. If the first hypothesis is true, then the initial announcement, $y^r_{t,t+1}$, is equal to the final release plus a measurement error, u^1_t, which is uncorrelated with the final observation. So if $y_t - y^r_{t,t+1}$ is replaced in equation 1, the result is $rev^y_{t,t+S} = u^1_t$, showing that revisions are an estimation error which is uncorrelated with the final data. If, on the other hand, the second hypothesis is true, then the final observation is equal to the initial announcement plus a measurement error, u^2_t. In this case it can be shown that revisions are an estimation error which is uncorrelated with the initial announcement. Table 2.C shows the results for GDP, consumption, and inflation revisions under both growth rate computation methods for the euro area and the US.

Table 2.C Noise and News hypotheses

	Revision in g^1				Revision in g^2			
	Euro Area		US		Euro Area		US	
Variable\Test	Noise	News	Noise	News	Noise	News	Noise	News
GDP	0.00**	0.00**	0.021	0.00**	0.00**	0.00**	0.00**	0.00*
Consumption	0.00**	0.00**	0.775	0.00**	0.00**	0.00**	0.00*	0.00**
Inflation	0.00**	0.00**	0.876	0.00**	0.00**	0.00**	0.346	0.00**

Numbers in cells refer to p-value of each hypothesis.

Note:*, **, represent the significance at the 1 and 5% levels.

The news hypothesis is rejected under both approaches for the euro area and the US. Moreover, using g^1 the noise test cannot be rejected. In combination with the first results, this supports the idea that those data revisions are not well-behaved in the US. However, using the same vintage to compute growth rates the noise hypothesis is rejected for GDP and consumption. In the case of the euro area, the noise hypothesis is rejected under both methods, so no clear conclusions can be obtained. One potential issue in these estimations is the fact that the error term may contain a certain structure instead of being i.i.d., which may bias the results. To overcome this, the following section presents a more in-depth analysis of data revisions in the EA and the error term in the estimations.

A regression analysis on data revisions

This section runs a pool of regressions to provide a sound characterization of data revisions. We carry out a detailed set of regressions to ensure that the residuals are i.i.d. and conclude by verifying whether data revisions depend on the initial announcements, past revisions, and/or past errors. This analysis should also provide an empirical justification of how data revisions might be included in macro models. We set out from two options : i) Revisions depend only on the first announcement; ii) Revisions depend on the first announcement and on past revisions. We then analyze the properties of the error term to ensure that regressions are properly specified. More specifically, we take the following steps:

-For the estimation under the first alternative:

1) Estimate revisions depending on the initial announcement: $rev^{y}_{t,t+S} = \alpha_{y_1} + b_{y_1} y^{r}_{t,t+1} + \varepsilon^{y_1}_{t}$, where $rev^{y}_{t,t+S}$, is the estimation of the output revision without lagged revisions. α_{y_1} captures the mean on the revisions and ε^{y}_{t} corresponds to the error term. The coefficient b_{y_1}captures the correlation with the initial announcement of output. The subscript (1) refers to type of revisions (in this case without lagged revisions).

2) The error ε^{y}_{t} is computed from the residuals and regressed respect to its lagged values: $\varepsilon^{y_1}_{t} = \mu_{e_1} + \rho_{y_1} \varepsilon^{y_1}_{t-1}$. This step helps us to study the properties of the error, like the correlation of the revisions with the residual term

3) The first regression is re-estimated including the lagged $\varepsilon^{y_1}_{t-1}$ errors generated in 2. In this way we try to make sure that the new error, $\nu^{y_1}_{t}$, is well-behaved. The regression is $rev^{y}_{t,t+S} = \alpha'_{y_1} + b'_{y_1} y^{r}_{t,t+1} + \rho_{y_1} \varepsilon^{y_1}_{t-1} + \nu^{y_1}_{t}$.

-For the estimation under the second hypothesis we proceed likewise:

1) Estimate revisions as a function of the initial announcement and past revisions $rev^{y}_{t,t+S} = \alpha_{y_2} + b_{y_2} y^{r}_{t,t+1} + \delta_{y} rev^{y}_{t-1,t+S-1} + \varepsilon^{y_2}_{t}$.

2) The new error term is evaluated $\varepsilon^{y_2}_{t} = \mu_{e_2} + \rho_{y_2} \varepsilon^{y_2}_{t-1}$.

3) The revision equation is re-estimated including lagged errors $rev^{y}_{t,t+S} = \alpha'_{y_2} + b'_{y_2} y^{r}_{t,t+1} + \delta'_{y} rev^{y}_{t-1,t+S-1} + \rho_{y_2} \varepsilon^{y_2}_{t-1} + \nu^{y_2}_{t}$.

Table 2.E shows the results for data revision in the Euro Area.[7] The estimation under the first hypothesis shows that data revisions have an intercept (α_{y_1}) different from zero, they are highly negatively correlated with initial announcements (-0.72, -0.78, -0.68 in the revisions of output, consumption and inflation, respectively) and show a high correlation with the residual term (ρ_{y_1}, lies above 0.70 on average). Similar results are obtained if the first residuals are included in the regression (step 3). These results support the idea of noise in EA data revisions and provide a tentative structure for its inclusion in the medium-scale DSGE model below.

As regards the second alternative, an important result is that the coefficients relating revisions in period t to past revisions, δ_y, δ_c, δ_π, are significant. The negative sign suggests that high lagged revisions reduce the size of present revisions. Another important result is that including past revisions reduces the correlation between data revisions and the residual component (approximately from 0.70 to 0.50) in the cases of output and consumption revisions. These two results provide a solid justification for including past revisions in the data revision process when modeled. However, although the coefficient of lagged revision is significant ($\delta_y = -0.4$), the estimation of the revision of inflation does not improve and there is no clear advantage to including it.

3 Real-time data within a DSGE model

Introducing real-time data into the DSGE model relies on the assumption that the decisions of economic agents are based on the initial releases of output, consumption, and inflation variables. We formally introduce this assumption by using three channels present in the Smets and Wouters (2007) model and in Casares and Vázquez (2016).[8] First, it is assumed that indexation rules which affect firms and unions are formed using the real-time information set rather than final revised data on aggregate output and consumption. This

[7] Results for the US are also included in the Appendix (Table 2.E.1b).

[8] This can be extended to other DSGE models such as Christiano et al (2005) or more general macro models like Fagan et al (2005).

assumption is consistent with the fact that initial announcements are published with a one-quarter delay. Secondly, the monetary authority's decision is based on real-time as opposed to final data. Finally, the external habit in consumption is assumed to depend on real-time data on aggregate consumption. Under the representative agent model, the idea that the aggregate level of consumption does not belong to the agent's information set might sound illogical to the reader, but there is an argument to support it: The external habit in consumption plays the role of an externality in the model since it affects the agent's utility but is not controlled by the agent. Therefore, to keep the habit in consumption external, it seems reasonable to assume that final revised aggregate consumption does not belong to the agent's information set. These assumptions affect the following structural equations: The New Keynesian Phillips Curve, wages, monetary policy rule, and the Euler-consumption equation. This is detailed in the next section

3.1 An explicit specification of the revisions

Using the empirical analysis carried out in the previous section as a reference point, we propose the corresponding equations for data revisions to be included in the DSGE model. These equations permit a direct relationship between revisions and the initial announcements for output, consumption, and inflation. In the cases of output and consumption they also include past revisions.[9] More specifically, we include the three following equations:

$$rev^y_{t,t+S} = b_y y^r_{t,t+1} + \delta_y rev^y_{t-1,t+S-1} + \varepsilon^y_{t,t+S}, \tag{11}$$

$$rev^c_{t,t+S} = b_c c^r_{t,t+1} + \delta_c rev^c_{t-1,t+S-1} + \varepsilon^c_{t,t+S}, \tag{12}$$

$$rev^\pi_{t,t+S} = b_\pi \pi^r_{t,t+1} + \varepsilon^\pi_{t,t+S}, \tag{13}$$

where b_y, b_c, b_π are the coefficients capturing the correlation between revisions and initial

[9] Note that the constant coefficient (α_y) does not appear in the proposition for revisions in the DSGE model (equations 4 to 6). This is because these equations are introduced in a log-linearized model that is defined in deviations with respect to the steady-state. Although the econometrical approach is estimated with revisions in the growth rate and in the DSGE revisions in the level are involved, they should manifest the same properties. Since revisions in level do not affect the trend, they are expected to be stationary.

announcements, δ_y and δ_c account for the importance of past revisions in the revisions of output and consumption. Finally, $\varepsilon^y_{t,t+S}$, $\varepsilon^c_{t,t+S}$, $\varepsilon^\pi_{t,t+S}$, are the error terms associated with each revision, which we allow to be autocorrelated.

$$\varepsilon^y_{t,t+S} = \rho_y \varepsilon^y_{t-1,t+S-1}, \tag{14}$$

And likewise for consumption and inflation error term. The errors are assumed to follow an AR(1). Since revisions take S periods, the error in the final revision can be expressed as:

$$\varepsilon^y_{T,t+S} = \rho_y^S \varepsilon^y_{t-1,t+S-1}. \tag{15}$$

Finally, after some rearrangement, the precise characterization of revisions in output consumption and inflation is obtained:

$$rev^y_{t,t+S} = b_y(y^r_{t,t+1} + \delta_y y^r_{t-1,t}) + \rho_y^S \Big[\varepsilon^y_{t-1,t+S-1} + (\delta_y/\rho_y)\varepsilon^y_{t-2,t+S-2}\Big], \tag{16}$$

$$rev^c_{t,t+S} = b_c(c^r_{t,t+1} + \delta_c c^r_{t-1,t}) + \rho_c^S \Big[\varepsilon^c_{t-1,t+S-1} + (\delta_c/\rho_c)\varepsilon^c_{t-2,t+S-2}\Big], \tag{17}$$

$$rev^\pi_{t,t+S} = b_\pi \pi^r_{t,t+1} + \rho_\pi^S \varepsilon^\pi_{t-1,t+S-1}. \tag{18}$$

The innovation term in data revisions

The introduction of real-time variables is completed with the inclusion of three innovations in their revision shocks. As previously indicated (see equations 6 and 7), the shocks follow an AR(1) and they depend on the length of the revisions.[10] Like in Casares and Vázquez (2016), we allow shocks to contain an "innovation" component. Named as $\eta^y_{t,t+S}$, $\eta^c_{t,t+S}$ and $\eta^\pi_{t,t+S}$, they refer to output, consumption and inflation shock innovations respectively. As a consequence, the shock structure after including the number of periods in the revision becomes:

$$\varepsilon^y_{t,t+S} = \rho_{yr}^S \varepsilon^y_{t-1,t-1+S} + \eta^y_{t,t+S}, \tag{19}$$

[10] Aruoba (2008) uses S=12. Casares and Vázquez (2016) use S=3, 6 and 12 and find that their results are robust to the choice of S. In order to make results comparable, we take S=6.

$$\varepsilon^c_{t,t+S} = \rho^S_{cr}\varepsilon^c_{t-1,t-1+S} + \eta^c_{t,t+S}, \tag{20}$$

$$\varepsilon^\pi_{t,t+S} = \rho^S_{\pi r}\varepsilon^\pi_{t-1,t-1+S} + \eta^\pi_{t,t+S}. \tag{21}$$

Finally, for the rest of the shocks (please find the full set of equations in appendix), we follow those indicated in Smets and Wouters (2007). As a result, the model shocks are augmented up to a total of ten.

3.2 The new set of equations

The inclusion of past revisions in the definition of revisions for output and consumption requires the derivation of a new monetary policy rule and the Euler equation. For the rest of the extended model, NKPC, wage dynamics, and wage equation, the equations used are those derived by Casares and Vázquez (2016). They can be found in the appendix. This chapter uses the same notation as Smets and Wouters (2007). For a full description of the notation please see the Appendix 3.

3.2.1 The Euler equation

We start with the first order conditions of the household with respect to their consumption decision from the Smets and Wouters model

$$\left(C_t(i) - hC_{t-1,t}\right)^{-\sigma_c} \exp\left(\frac{\sigma_c - 1}{1+\sigma_l}\left(L_t(i)\right)^{1+\sigma_l}\right) - \Xi_t = 0, \tag{22}$$

As mentioned before, the external habit in consumption is now upon real-time announcements. After replacing and log-linearizng it, we get the following equation

$$\log \Xi_t = -\frac{\sigma_c}{1-h/\gamma}c_t(i) + \frac{(h/\gamma)\,\sigma_c}{1-(h/\gamma)}c^r_{t-1,t} + (\sigma_c - 1)\,L^{1+\sigma_l}l_t(i). \tag{23}$$

Besides, from the log-linearized FOC of the demand for bonds we obtain

$$\log \Xi_t = E_t \log \Xi_{t+1} + (R_t - E_t\pi_{t+1} + \varepsilon^b_t). \tag{24}$$

Taking (13) in t and $t+1$, so that we substitute $\log \Xi_t$ and $\log \Xi_{t+1}$ in (14) and aggregating across agents, the expression below is achieved

$$c_t = (h/\gamma)\, c^r_{t-1,t} - (h/\gamma)\, E_t c^r_{t,t+1} + E_t c_{t+1} + \tfrac{(1-h/\gamma)(\sigma_c-1)L^{1+\sigma_l}}{\sigma_c}\,(l_t - E_t l_{t+1}) -$$

$$\tfrac{1-h/\gamma}{\sigma_c}(R_t - E_t \pi_{t+1} + \varepsilon^b_t). \tag{25}$$

The $E_t c^r_{t,t+1}$ refers to the real-time announcement of aggregate consumption in $t+1$ which affects the external habit in t. The definition of this expression can be obtained using $c_t = E_t c^r_{t,t+1} + rev^c_{t,t+S}$ (as shown in equation 1, but for consumption) and substituting $rev^c_{t,t+S}$ by $rev^c_{t,t+S} = b_c(c^r_{t,t+1} + \delta_c c^r_{t-1,t}) + \rho^S_c\Big[\varepsilon^c_{t-1,t+S-1} + (\delta_c/\rho_c)\varepsilon^c_{t-2,t+S-2}\Big]$. Consequently we get

$$c_t = (1+b_c)c^r_{t,t+1} + b_c\delta_c c^r_{t-1,t} + \rho^S_c\Big[\varepsilon^c_{t-1,t+S-1} + (\delta_c/\rho_c)\varepsilon^c_{t-2,t+S-2}\Big],$$

and isolating $c^r_{t,t+1}$, an explicit term is obtained

$$c^r_{t,t+1} = \frac{c_t - b_c\delta_c c^r_{t-1,t} - \rho^S_c\Big[\varepsilon^c_{t-1,t+S-1} + (\delta_c/\rho_c)\varepsilon^c_{t-2,t+S-2}\Big]}{1+b_c}. \tag{26}$$

By substituting the latter equation into (16)

$$c_t + \frac{(h/\gamma)}{1+b_c}c_t = (h/\gamma)\, c^r_{t-1,t} + \frac{(h/\gamma)}{1+b_c}\Big\{b_c\delta_c c^r_{t-1,t} - \rho^S_c\Big[\varepsilon^c_{t-1,t+S-1} + (\delta_c/\rho_c)\varepsilon^c_{t-2,t+S-2}\Big]\Big\} +$$

$$E_t c_{t+1} + \tfrac{(1-h/\gamma)(\sigma_c-1)L^{1+\sigma_l}}{\sigma_c}\,(l_t - E_t l_{t+1}) - \tfrac{1-h/\gamma}{\sigma_c}(R_t - E_t \pi_{t+1} + \varepsilon^b_t),$$

and grouping $c^r_{t-1,t}$and isolating c_t we get to final expression for this new Euler equation:

$$c_t = c_1\big[1 + \delta_c(1+b_c)^{-1}\big]c^r_{t-1,t} + (1-c_1)E_t c_{t+1} + c_2\,(l_t - E_t l_{t+1}) - c_3(R_t - E_t\pi_{t+1} + \varepsilon^b_t) +$$

$$c_4\left[\varepsilon^c_{t-1,t+S-1} + (\delta_c/\rho_c)\varepsilon^c_{t-2,t+S-2}\right], \tag{27}$$

where:

$$c_1 = \tfrac{h/\gamma}{1+(h/\gamma)(1+b_{cc})^{-1}},\; c_2 = \tfrac{(\sigma_c-1)wL/(\phi_w C)}{\sigma_c(1+(h/\gamma)(1+b_{cc})^{-1})},\; c_3 = \tfrac{1-h/\gamma}{\sigma_c(1+(h/\gamma)(1+b_{cc})^{-1})},$$

$$and\; c_4 = \tfrac{(h/\gamma)\rho^S_{cr}}{(1+b_{cc})(1+(h/\gamma)(1+b_{cc})^{-1})}.$$

As a result, real-time data enters in the equation in the form of lagged values of external consumption. Moreover, shocks in the revision process of consumption do play a role. Later on, a specific structure will be provided to the shocks and its impact in the estimation discussed.

3.2.2 Monetary policy rule

We use equation (1) to rewrite output in the monetary policy rule as we assume that the monetary authority takes decisions based on the information available on output and inflation. This implies that for lagged values of output, the observation that the authority uses corresponds to the first announcement of aggregate output, which is published with a one-quarter delay. Similar reasoning is used in Casares and Vázquez (2016); however, the presence of lagged revisions affects the definition of revisions in output, and requires the derivation of a new monetary policy rule. Starting from the Smets and Wouters (2007) model, we have the following policy rule:[11]

$$R_t = \rho R_{t-1} + (1-\rho)[r_\pi \pi_{t-1} + r_y \left(y_{t-1} - y^p_{t-1}\right)] + r_{\Delta y}\left[\left(y_{t-1} - y^p_{t-1}\right) - \left(y_{t-2} - y^p_{t-2}\right)\right] + \varepsilon^R_t. \quad (28)$$

Including the equations for data revisions on output (7) and inflation (9) into the definition of real-time data (1), we can express define y_t and π_t as follows:

$$y_t = (1+b_y) y^r_{t,t+1} + b_y \delta_y y^r_{t-1,t} + \rho^S_y \left[\varepsilon^y_{t-1,t+S-1} + (\delta_y/\rho_y)\varepsilon^y_{t-2,t+S-2}\right],$$

$$\pi_t = (1+b_\pi)\pi^r_{t,t+1} + \rho^S_\pi \varepsilon^\pi_{t-1,t+S-1}.$$

Placing the last two expressions for y_{t-1}, y_{t-2}, π_{t-1} into (19) we have the following expression

$$R_t = \rho R_{t-1} + (1-\rho)\Bigg\{ r_\pi \left[(1+b_\pi)\pi^r_{t-1,t} + \rho^{S-1}_\pi \varepsilon^\pi_{t-2,t+S-2}\right] +$$

[11] Note that in this expression inflation and the output gap are lagged by one more period than in the original version. This makes it easier to introduce inflation in real-time.

$$r_y\Big[(1+b_y)y^r_{t-1,t}+b_y\delta_y y^r_{t-2,t-1}+\rho_y^{S-1}\big(\varepsilon^y_{t-2,t+S-2}+(\delta_y/\rho_y)\varepsilon^y_{t-3,t+S-3}\big)\Big]-y^p_{t-1}\Bigg\}+$$

$$r_{\triangle y}\Bigg\{\Big[(1+b_y)y^r_{t-1,t}+b_y\delta_y y^r_{t-2,t-1}+\rho_y^{S-1}\big(\varepsilon^y_{t-2,t+S-2}+(\delta_y/\rho_y)\varepsilon^y_{t-3,t+S-3}\big)-y^p_{t-1}\Big]-$$

$$\Big[(1+b_y)y^r_{t-2,t-1}+b_y\delta_y y^r_{t-3,t-2}+\rho_y^{S-2}\big(\varepsilon^y_{t-3,t+S-3}+(\delta_y/\rho_y)\varepsilon^y_{t-4,t+S-4}\big)-y^p_{t-2}\Big]\Bigg\}+\varepsilon^R_t.$$

Finally, operating with the terms measuring the change in the real-time output gap, a new monetary policy rule in real-time is obtained

$$R_t=\rho R_{t-1}+(1-\rho)\Bigg\{r_\pi\Big[(1+b_\pi)\pi^r_{t-1,t}+\rho_\pi^{S-1}\varepsilon^\pi_{t-2,t+S-2}\Big]+$$

$$r_y\Big[(1+b_y)y^r_{t-1,t}+b_y\delta_y y^r_{t-2,t-1}+\rho_y^{S-1}\big(\varepsilon^y_{t-2,t+S-2}+(\delta_y/\rho_y)\varepsilon^y_{t-3,t+S-3}\big)\Big]-y^p_{t-1}\Bigg\}+$$

$$r_{\triangle y}\Bigg\{(1+b_y)\Big[y^r_{t-1,t}-y^r_{t-2,t-1}\Big]+b_y\delta_y\Big[y^r_{t-2,t-1}-y^r_{t-3,t-2}\Big]+$$

$$\rho_y^{S-1}\Big[\varepsilon^y_{t-2,t+S-2}-(1/\rho_y)\varepsilon^y_{t-3,t+S-3}\Big]+\rho_y^{S-2}\delta_y\Big[\varepsilon^y_{t-3,t+S-3}-(1/\rho_y)\varepsilon^y_{t-4,t+S-4}\Big]+$$

$$(y^p_{t-1}-y^p_{t-2})\Bigg\}+\varepsilon^R_t. \tag{29}$$

4 Data and estimation procedure

This section seeks to study the impact of real-time data and data revisions on DSGE models through the three channels mentioned above. The sample period used for the euro area is 1995Q1-2008Q1. The set of observable variables comprises quarterly series of the inflation rate (expressed as the first difference in logs of the implicit GDP deflator), the ECB interest rate, the log of employment, and the quarterly log differences of real consumption, real investment, real wages, and GDP. In addition, for the extended model we incorporate real-time series of quarterly inflation (expressed as the first difference in logs of the real-time GDP deflator), and quarterly log differences of GDP and real consumption from the ECB Real-Time Database.[12] Variables displaying a long-run trend are expressed in log differences to remove the non-stationary component. The list of observable variables is measured as follows:

$$X_t = \begin{bmatrix} dlGDP_t \\ dlCONS_t \\ dlINV \\ dlWAG_T \\ dlEMPL_t \\ dlP_t \\ lECB\&lFEDFUNDS_t \\ dlGDPr_t \\ dlCONS_t \\ dlPr_t \end{bmatrix} = \begin{bmatrix} \overline{\gamma} \\ \overline{\gamma} \\ \overline{\gamma} \\ \overline{\gamma} \\ \bar{l} \\ \overline{\pi} \\ \overline{r} \\ \overline{\gamma} \\ \overline{\gamma} \\ \overline{\pi} \end{bmatrix} + \begin{bmatrix} y_t - y_{t-1} \\ c_t - c_{t-1} \\ i_t - i_{t-1} \\ w_t - w_{t-1} \\ l_t \\ \pi_t \\ r_t \\ y_t^r - y_{t-1}^r \\ c_t^r - c_{t-1}^r \\ \pi_t^r \end{bmatrix}$$

where l and dl respectively denote the log and the log difference. $\overline{\gamma} = 100(\gamma - 1)$, is the common quarterly trend growth rate for real GDP, consumption (also for real-time variables)

[12]Real time growth rates are computed using the g^1 approach. The extended model is also estimated using g^2 but is not shown here due to space constraints. The overall result using the latter approach is similar, but the identification of the revision processes becomes more sensitive.

investment and wages, which are the variables presenting a long run trend. $\bar{l}$, $\overline{\pi}$, $\overline{r}$ are the steady state the level of employment per capita, and the steady state values of inflation and the short-term interest rate.

The approach used is a two-step Bayesian estimation procedure in Dynare. First, the log posterior function is maximized by combining prior information on the parameters and the likelihood of the data. Then the Metropolis-Hastings algorithm is implemented, which runs a massive sequence of draws for all the possible realizations for each parameter to get a picture of the subsequent distribution. For the SW model this algorithm is executed using 3 blocks of 200,000 realizations each. The same is done for the extended model. The acceptance rates for both models for the US and euro area are between 20-30%.

5 Estimation results

This section discusses the main results of the DSGE estimation. The first part argues the main results in terms of parameter estimation for the euro area. The second provides the main findings in terms of second moments and variance decomposition.

5.1 Parameter estimates

The baseline SW for the EA vs the US

Table 3.A reports the mean and the 5th and 95th percentiles obtained from the Metropolis-Hastings estimation of both the extended and baseline model (without real-time data) parameters for the euro area. This subsection compares the baseline estimates (right-hand side of the table) with those estimated for the US, as reported in Table 3.B. 3.B.[13] The confidence intervals of the main group of parameters overlap to a great extent with those for the US. However, there are some noteworthy discrepancies. The degree of price and wage stickiness (ξ_p,ξ_w) is slightly smaller (0.45 and 0.63 for the euro area against 0.66 and 0.70 for the US). Regarding the indexation parameters (i_w and i_p), the price indexation is roughly the same

[13] We compare our results with those of Smets and Wouters (2007) since they are the main reference. Nonetheless these comments apply to a large extent to the estimates reported in Casares and Vázquez (2016)

(i_p= 0.24), while the wage indexation coefficient is less than one-half of the US estimate (0.21 versus 0.58). This makes the wage equation more forward-looking in the case of the euro area. Another important difference lies in investment decisions, namely the elasticity of capital utilization and the level of fixed costs (ψ and ϕ). The former, ψ, is significantly lower (0.19 versus 0.54), while the latter is estimated to be higher (1.90 versus 1.60). With respect to the parameter estimates in the policy rule, the smoothing parameter (ρ) and the coefficient measuring the change in output gap ($r_{\triangle y}$) are both rather similar to the US. However, the reaction to the inflation gap is smaller (1.72 versus 2.01) and the output gap is nearly zero. Finally, in terms of structural shocks, the results suggest a greater persistence of spending and risk premium shocks (ρ_b, ρ_R , 0.77 and 0.48 versus 0.22 and 0.15), while investment adjustment and price and wage mark-up shocks (ρ_i, μ_p and μ_w) are less persistent.

The SW model with EA real-time data

The assumption that agent's economic decisions are based on real-time data has two important effects in the Euler equation. It reduces the importance of the habit in consumption parameter (h drops from 0.68 to 0.4) and reduces the Frisch elasticity (σ_l increases from 1.09 to 3.07). In terms of nominal rigidities, it reduces the Calvo probability in wages (ξ_w drops from 0.65 to 0.44) but increases wage indexation (i_w increases from 0.21 to 0.45). Thus, wages are updated more frequently but are more backward-looking. Concerning the new monetary policy rule, the estimates show a similar reaction to real-time data as to the model with final data. Regarding the estimates of the structural shocks, the model with real-time data shows a smaller autocorrelation coefficient in the spending and risk premium shocks (ρ_g, ρ_R), and a lower estimate in the moving-average component of both prices and wage mark-up shocks (μ_p, μ_w).

The estimation of the data revision process supports the idea that data revisions are not well-behaved. In the case of output and consumption the initial announcement anticipates a future negative revision (b_y, b_c being -0.15 and -0.13 respectively) and they are correlated with past revisions. In the case of inflation the estimates show that the initial release anticipates an upward revision (b_π =2.2). The error component shows a high level of persistence for all the variables ($\rho_{yr} = 0.66$, $\rho_{cr} = 0.72$, and $\rho_{\pi r} = 0.94$) and the estimated volatility of the innovations (σ_{yr}, σ_{cr} and $\sigma_{\pi r}$) is on average twice as high as the rest of the shocks in the model. These results are in line with the regression analysis for the revisions of output and consumption. Both (reduced-form and structural) methodologies capture a negative correlation with the initial release, the persistence in the error term, and the negative coefficient associated with past revision. Concerning the inflation revision process, the estimations of the DSGE and the regression model show opposite signs in the coefficient relating revisions to the initial announcement, which might be due to the aforementioned differences between the Bayesian structural econometric approach and the reduced-form OLS approach. In any case, the overall conclusion (especially for output and consumption) remains robust, data revisions are correlated with their initial announcement, and their errors show high variance and persistence.

5.2 Second-moment statistics

Table 4 reports the main second-moment statistics: Standard deviation, contemporaneous correlation with output growth, and first-order autocorrelation. These statistics relate to actual and synthetic data from both the original and the extended SW models. In particular, Panel A of Table 4 reproduces the second-moment statistics for real-time variables (y^r, c^r, and π^r) as well as their revisions (rev^{ry}, rev^{rc}, and $rev^{r\pi}$). The extended model fulfills a moderate task when replicating them . First, serial autocorrelation is well approximated for all variables. Second, the estimated volatility of revisions in output and inflation is practically the same as in the actual data. However, the estimated variance of output and consumption growth is three times higher than in the actual data. This higher volatility is due in principle to the inclusion of new shocks (the specific impact is seen in subsections 5.3 and 5.4). Finally, with respect to the correlation with final output growth, the real-time estimates of y^r, c^r, and π^r are very close to the true values, while their revisions are less closely correlated (about half of the actual values).

Table 4, Panel B shows the same statistics for the remaining endogenous variables (y, c, i, w, l, R and π). The first conclusion that can be drawn is that the original model does a better job in terms of volatility. This reinforces the idea that the extended model with real-time data amplifies volatility. Concerning correlation with output, both models fit the data reasonably well. Finally, the model performs moderately well in regard to serial autocorrelation.

5.3 Variance decomposition

Table 5 shows the variance decomposition analysis for the EA baseline model and the model with real-time. The variability of output growth is driven by demand-side shocks in both models: the risk-premium shock (η^b), exogenous spending shock (η^g), investment adjustment cost shock (η^i) and interest rate shock (η^R) account for more than half of the total variability (61% in the original model and 57.2% in the extended model), with the risk premium shock as the main source (between 30-40% for both). With respect to supply shocks, the price mark-up shocks (η^p) are the main source of variation for wages, employment, interest rate and inflation.

The introduction of data revision shocks in the extended model (η^{ry}, η^{rc} and $\eta^{r\pi}$) accounts for roughly 30% of the variation in inflation and in consumption and output growth. In particular, shocks in the inflation revision process become the major source of business volatility in the model. This result highlights the importance of acknowledging the significance of data revisions.

6 Conclusions

This chapter provides a detailed analysis of the statistical properties of data revisions for the EA. It also studies what type of modeling is appropriate for real-time data and its revision in DSGE models.

From a practical standpoint, the statistical properties of data revisions are studied under different approaches, leading to this first conclusion: Revisions depend on initial announcements and show high volatility, which suggests that they are not well-behaved. In addition, a reduced-form regression analysis is carried out to propose an empirically based structure of the data revision processes. This characterization of data revisions is introduced into the Smets and Wouters (2007) DSGE model by assuming that the economic decisions of households, firms, and monetary authorities depend on real-time data. As a result, an extended version of the model is derived, enabling us to estimate the implications of real-time data and their revisions in the context of a DSGE model.

The estimates of the DSGE model corroborate that data revisions are correlated with their first release, highly volatile, and highly autocorrelated. In the euro area, for instance, a positive announcement of output and consumption is likely to lead to a negative future revision. In the case of inflation, the correlation between the initial release and the first revision is relatively close and positive. Furthermore, in terms of modeling, the incorporation of real-time data and data revisions affects the DSGE model in three relevant aspects. First, the estimated values of some of the main parameters vary, e.g. lower habit formation values are found. Second, revision shocks become a significant source in the business cycle decomposition. For instance, in the case of the euro area they account for up to one-third of output variability. Finally, the introduction of new shocks increases the volatility of the variables observed. In sum, these findings suggest that data revisions are not well-behaved, so DSGE models omitting real-time data and data revisions might be ignoring important sources of aggregate fluctuations. This work presents a way to accommodate this facts in business cycle analysis while it encourages further improvements in the estimation of real-time data from the statistical agencies.

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Tables and figures

Table 1.C. US descriptive statistics.

	Revision in g^1			Revision in g^2		
	GDP	Consumption	Inflation	GDP	Consumption	Inflation
Mean	0.176	0.141	-0.045	0.330	0.261	0.087
Absolute Mean	1.629	1.464	0.761	1.278	1.151	0.582
Median	-0.027	-0.024	0.033	0.325	0.256	0.204
Min	-7.340	-4.605	-3.541	-4.238	-2.604	-1.661
Max	6.360	6.471	3.668	3.565	4.759	2.018
Std. D. Revision	2.201	1.970	1.017	1.559	1.451	0.716
Noise/Signal	0.967	0.965	1.036	0.686	0.708	0.748
Correlation with Initial	-0.590	-0.695	-0.736	-0.176	-0.458	-0.556
Correlation with final	0.267	-0.003	-0.02	0.535	0.203	0.081
Correlation Initial-Final	0.623	0.720	0.695	0.737	0.776	0.783

Revisions of annualized quarterly growth series.

Table 2.E.1 Euro Area regressions without past revisions

Regression $rev^{y'}_{t,t+S} = \alpha_{y_1} + b_{y_1} y^r_{t,t+1} + \varepsilon^{y_1}_t$				Regression $rev^{y}_{t,t+S} = \alpha_{y_1'} + b_{y_1'} y^r_{t,t+1} + \rho_{y'} \varepsilon^{y_1}_{t-1} + \nu^{y_1}_t$.			
Coefficient	Variable			Coefficient	Variable		
	GDP	Consumption	Inflation		GDP	Consumption	Inflation
α_1	0.40	0.355	0.30	α'_1	0.40	0.34	0.30
	0.07	0.077	0.05		0.07	0.07	0.05
b_1	-0.70	-0.782	-0.68	b'_1	-0.73	-0.78	-0.67
	0.10	0.108	0.10		0.10	0.11	0.10
				ρ_1	0.06	-0.04	-0.21
					0.14	0.14	0.14
R^2	0.50	0.507	0.46		0.50	0.69	0.71
P-Value	0.00*	0.00*	0.00*		0.00*	0.00*	0.00*
Corr with Res.	0.70	0.70	0.70		0.70	0.69	0.71

The p value refer to the model signifcance.*, **, means that the model is significant at 1 or 5% respectively

The boxes with smaller numbers show the standard deviation

Table 2.E.2. Euro Area regressions with pasts revisions

Regression $rev^y_{t,t+S} = \alpha_{y2} + b_{y2} y^r_{t,t+1} + \delta_y rev^y_{t-1,t+S-1} + \varepsilon^{y2}_t$.				Regression $rev^y_{t,t+S} = \alpha_{y'_2} + b_{y'_2} y^r_{t,t+1} + \delta_{y'} rev^y_{t-1,t+S-1} + \rho_{y'_2} \varepsilon^{y2}_{t-1} + \nu^{y2}_t$			
Coefficient	Variable			Coefficient	Variable		
	GDP	Consumption	Inflation		GDP	Consumption	Inflation
α_2	0.42	0.33	0.29	α'_2	0.43	0.32	0.29
	0.06	0.07	0.05		0.06	0.06	0.05
b_2	-0.75	-0.76	-0.66	b'_2	-0.79	-0.80	-0.68
	0.09	0.09	0.09		0.08	0.09	0.09
δ_y	-0.31	-0.31	-0.34	δ_y	-0.52	-0.59	-0.42
	0.09	0.09	0.09		0.11	0.11	0.12
				ρ_1	0.49	0.52	0.20
					0.17	0.16	0.19
R^2	0.60	0.61	0.57		0.67	0.68	0.57
P-Value	0.00*	0.00*	0.00*		0.00*	0.00*	0.00*
Corr with Res.	0.62	0.69	0.65		0.57	0.55	0.65

The p value refer to the model signifcance.*, **, means that the model is significant at 1 or 5% respectively

The boxes with smaller numbers show the standard deviation

Table 2.E.1B. US regressions without past revisions

Regression $rev^{y'}_{t,t+S} = \alpha_{y_1} + b_{y_1} y^r_{t,t+1} + \varepsilon^{y_1}_t$				Regression $rev^{y}_{t,t+S} = \alpha_{y'_1} + b_{y'_1} y^r_{t,t+1} + \rho_{y'} \varepsilon^{y_1}_{t-1} + \nu^{y_1}_t$.			
Coefficient	Variable			Coefficient	Variable		
	GDP	Consumption	Inflation		GDP	Consumption	Inflation
α_1	0.41	0.43	0.33	α'_1	0.41	0.43	0.34
	0.06	0.05	0.03		0.06	0.05	0.03
b_1	-0.47	-0.48	-0.52	b'_1	-0.49	-0.49	-0.55
	0.05	0.05	0.05		0.07	0.05	0.05
				ρ_1	-0.04	0.04	0.14
					0.07	0.10	0.10
R^2	0.35	0.48	0.54		0.37	0.48	0.54
P-Value	0.00*	0.00*	0.00*		0.00*	0.00*	0.00*
Corr with Res.	0.80	0.71	0.67		0.79	0.71	0.67

The p value refer to the model signifcance. *, **, represent the significance at the 1 and 5% level.

The boxes with smaller numbers show the standard deviation

Table 2.E.2B. US regressions with past revisions

Regression $rev^{y}_{t,t+S} = \alpha_{y2} + b_{y2} y^{r}_{t,t+1} + \delta_y rev^{y}_{t-1,t+S-1} + \varepsilon^{y2}_{t}$.				Regression $rev^{y}_{t,t+S} = \alpha_{y'_2} + b_{y'_2} y^{r}_{t,t+1} + \delta_{y'} rev^{y}_{t-1,t+S-1} + \rho_{y'_2} \varepsilon^{y2}_{t-1} + \nu^{y2}_{t}$			
Coefficient	Variable			Coefficient	Variable		
	GDP	Consumption	Inflation		GDP	Consumption	Inflation
α_2	0.41	0.43	0.33	α'_2	0.46	0.43	0.38
	0.06	0.05	0.03		0.07	0.05	0.03
b_2	-0.48	-0.48	-0.52	b'_2	-0.54	-0.49	-0.62
	0.06	0.07	0.05		0.07	0.05	0.05
δ_y	-0.15	-0.11	-0.06	δ_y	-0.33	-0.22	-0.26
	0.08	0.07	0.06		0.12	0.10	0.10
				ρ_1	0.31	0.25	0.48
					0.18	0.15	0.15
R^2	0.39	0.49	0.54		0.41	0.52	0.59
P-Value	0.00*	0.00*	0.00*		0.00*	0.00*	0.00*
Corr with Res.	0.77	0.70	0.67		0.71	0.67	0.64

The p value refer to the model signifcance.*, **, means that the model is significant at 1 or 5% respectively

The boxes with smaller numbers show the standard deviation

Table 3.A.1 Priors and estimated posteriors of the structural parameters: Euro Area

	Priors			Posteriors					
				Extended model			SW model		
	Distr	Mean	Std D.	Mean	5%	95%	Mean	5%	95%
φ	Normal	4.00	1.50	7.22	5.24	9.11	6.00	4.22	7.80
h	Beta	0.70	0.10	0.40	0.28	0.53	0.68	0.60	0.77
σ_c	Normal	1.50	0.37	1.40	-	-	1.40	-	-
σ_l	Normal	2.00	0.75	3.07	1.95	4.20	1.09	-0.01	2.2
ξ_p	Beta	0.50	0.10	0.42	0.33	0.52	0.45	0.34	0.55
ξ_w	Beta	0.50	0.10	0.44	0.32	0.56	0.63	0.53	0.75
ι_w	Beta	0.50	0.15	0.45	0.21	0.65	0.21	0.06	0.36
ι_p	Beta	0.50	0.15	0.17	0.04	0.29	0.24	0.17	0.43
ψ	Beta	0.50	0.15	0.40	0.22	0.59	0.19	0.07	0.30
Φ	Normal	1.25	0.12	1.84	1.69	2.01	1.90	1.79	2.01
r_π	Normal	1.50	0.25	1.72	1.56	1.89	1.72	1.55	1.89
ρ	Beta	0.75	0.10	0.81	0.75	0.87	0.79	0.74	0.85
r_y	Normal	0.12	0.05	0.06	-0.01	0.12	-0.01	-0.05	0.05
$r_{\Delta y}$	Normal	0.12	0.05	0.07	0.01	0.12	0.20	0.12	0.27
π	Gamma	0.62	0.10	0.41	0.27	0.54	0.57	0.42	0.72
$100(\beta^{-1}-1)$	Gamma	0.25	0.10	0.33	0.16	0.50	0.30	0.17	0.43
l	Normal	0.00	2.00	-1.60	-3.28	0.81	0.86	-2.15	3.20
$100(\gamma-1)$	Normal	0.40	0.10	0.4	-	-	0.4	-	-
α	Normal	0.30	0.05	0.25	0.18	0.32	0.23	0.16	0.29

Table 3.A.2 Priors and estimated posteriors of the shock processes: Euro Area

	Priors			Posteriors					
				Extended model			SW model		
	Distr	Mean	Std D.	Mean	5%	95%	Mean	5%	95%
σ_a	Invgamma	0.10	2.00	0.08	0.064	0.11	0.07	0.05	0.08
σ_b	Invgamma	0.10	2.00	0.07	0.04	0.10	0.03	0.02	0.04
σ_g	Invgamma	0.10	2.00	0.10	0.08	0.13	0.10	0.08	0.12
σ_i	Invgamma	0.10	2.00	0.22	0.15	0.29	0.21	0.15	0.27
σ_R	Invgamma	0.10	2.00	0.07	0.05	0.08	0.09	0.07	0.11
σ_p	Invgamma	0.10	2.00	0.05	0.03	0.07	0.06	0.04	0.08
σ_w	Invgamma	0.10	2.00	0.06	0.04	0.08	0.05	0.03	0.06
ρ_a	Beta	0.50	0.20	0.97	0.94	0.99	0.96	088	0.99
ρ_b	Beta	0.50	0.20	0.92	0.81	0.99	0.77	0.66	0.89
ρ_g	Beta	0.50	0.20	0.82	0.74	0.90	0.92	0.86	0.98
ρ_i	Beta	0.50	0.20	0.43	0.17	0.67	0.32	0.08	0.55
ρ_R	Beta	0.50	0.20	0.39	0.23	0.54	0.48	0.33	0.62
ρ_p	Beta	0.50	0.20	0.99	0.98	0.99	0.99	0.98	0.99
ρ_w	Beta	0.50	0.20	0.96	0.93	0.99	0.91	0.86	0.96
μ_p	Beta	0.50	0.20	0.30	0.07	0.51	0.54	0.24	0.73
μ_w	Beta	0.50	0.20	0.39	0.16	0.61	0.51	0.25	0.76
ρ_{ga}	Beta	0.50	0.20	0.37	0.12	0.61	0.46	0.18	0.73

Table 3.A.3 Priors and estimated posteriors of revision processes parameters: Euro Area

	Priors			Posteriors					
				Extended model			SW model		
	Distr	Mean	Std D.	Mean	5%	95%	Mean	5%	95%
δ_y	Normal	0.00	2.00	0.24	-0.14	0.67	-	-	-
δ_c	Normal	0.00	2.00	-0.10	-0.36	0.14	-	-	-
b_y	Normal	0.00	2.00	-0.15	-0.22	0.11	-	-	-
b_π	Normal	0.00	2.00	2.20	1.35	3.02	-	-	-
b_c	Normal	0.00	2.00	-0.13	-0.14	-0.11	-	-	-
σ_{yr}	Invgamma	0.10	2.00	0.21	0.17	0.25	-	-	-
$\sigma_{\pi r}$	Invgamma	0.10	2.00	0.20	0.16	0.23	-	-	-
σ_{cr}	Invgamma	0.10	2.00	0.25	0.20	0.30	-	-	-
ρ_{yr}	Beta	0.50	0.20	0.66	0.33	0.95	-	-	-
$\rho_{\pi r}$	Beta	0.50	0.20	0.94	0.89	0.98	-	-	-
ρ_{cr}	Beta	0.50	0.20	0.72	0.56	0.89	-	-	-

Table 3.B.1 Priors and estimated posteriors of the structural parameters: US

	Priors			Posteriors					
				Extended model			SW model		
	Distr	Mean	Std D.	Mean	5%	95%	Mean	5%	95%
φ	Normal	4.00	1.50	5.30	3.43	7.13	5.93	4.06	7.85
h	Beta	0.70	0.10	0.13	0.10	0.16	0.57	0.45	0.67
σ_c	Normal	1.50	0.37	1.33	1.05	1.59	1.07	0.76	1.35
σ_l	Normal	2.00	0.75	1.79	0.79	2.75	1.95	0.99	2.85
ξ_p	Beta	0.50	0.10	0.66	0.57	0.75	0.72	0.63	0.81
ξ_w	Beta	0.50	0.10	0.51	0.37	0.65	0.59	0.45	0.72
ι_w	Beta	0.50	0.15	0.34	0.14	0.53	0.48	0.24	0.72
ι_p	Beta	0.50	0.15	0.09	0.03	0.15	0.33	0.14	0.51
ψ	Beta	0.50	0.15	0.76	0.57	0.75	0.72	0.57	0.88
Φ	Normal	1.25	0.12	1.43	1.30	1.57	1.48	1.34	1.61
r_π	Normal	1.50	0.25	1.86	1.58	2.15	2.09	1.78	2.42
ρ	Beta	0.75	0.10	0.84	0.81	0.87	0.83	0.80	0.87
r_y	Normal	0.12	0.05	-0.01	-0.03	0.01	0.04	0.01	0.08
$r_{\Delta y}$	Normal	0.12	0.05	0.09	0.07	0.11	0.18	0.13	0.22
π	Gamma	0.62	0.10	0.67	0.53	0.80	0.70	0.57	0.85
$100(\beta^{-1}-1)$	Gamma	0.25	0.10	0.17	0.07	0.27	0.20	0.10	0.31
l	Normal	0.00	2.00	-1.44	-3.75	0.86	0.18	-1.77	2.30
$100(\gamma-1)$	Normal	0.40	0.10	0.4	-	-	0.39	0.35	0.43
α	Normal	0.30	0.05	0.16	0.13	0.20	0.17	0.14	0.21

Table 3.B.2 Priors and estimated posteriors of the shock processes: US

	Priors			Posteriors					
				Extended model			SW model		
	Distr	Mean	Std D.	Mean	5%	95%	Mean	5%	95%
σ_a	Invgamma	0.10	2.00	0.39	0.34	0.44	0.38	0.34	0.43
σ_b	Invgamma	0.10	2.00	0.12	0.08	0.16	0.09	0.05	0.13
σ_g	Invgamma	0.10	2.00	0.39	0.35	0.44	0.40	0.35	0.45
σ_i	Invgamma	0.10	2.00	0.29	0.21	0.36	0.35	0.25	0.43
σ_R	Invgamma	0.10	2.00	0.12	0.11	0.14	0.13	0.11	0.14
σ_p	Invgamma	0.10	2.00	0.12	0.09	0.15	0.11	0.09	0.14
σ_w	Invgamma	0.10	2.00	0.33	0.25	0.40	0.30	0.23	0.36
ρ_a	Beta	0.50	0.20	0.91	0.87	0.95	0.92	0.87	0.97
ρ_b	Beta	0.50	0.20	0.84	0.77	0.90	0.74	0.55	0.93
ρ_g	Beta	0.50	0.20	0.98	0.96	0.99	0.97	0.96	0.99
ρ_i	Beta	0.50	0.20	0.82	0.70	0.94	0.70	0.57	0.84
ρ_R	Beta	0.50	0.20	0.09	0.01	0.16	0.27	0.13	0.40
ρ_p	Beta	0.50	0.20	0.87	0.78	0.97	0.81	0.68	0.95
ρ_w	Beta	0.50	0.20	0.97	0.94	0.99	0.96	0.93	0.99
μ_p	Beta	0.50	0.20	0.56	0.35	0.78	0.60	0.38	0.82
μ_w	Beta	0.50	0.20	0.64	0.44	0.87	0.66	0.46	0.86
ρ_{ga}	Beta	0.50	0.20	0.40	0.25	0.56	0.40	0.24	0.56

Table 3.B.3 Priors and estimated posteriors of revision processes parameters: US

	Priors			Posteriors					
				Extended model			SW model		
	Distr	Mean	Std D.	Mean	5%	95%	Mean	5%	95%
δ_y	Normal	0.00	2.00	-0.10	-0.32	0.09	-	-	-
δ_c	Normal	0.00	2.00	0	-	-	-	-	-
b_{yy}	Normal	0.00	2.00	0.25	0.06	0.43	-	-	-
$b_{\pi\pi}$	Normal	0.00	2.00	-0.13	-0.25	-0.1	-	-	-
b_{cc}	Normal	0.00	2.00	0.19	0.10	0.28	-	-	-
σ_{yr}	Invgamma	0.10	2.00	0.65	0.54	0.77	-	-	-
$\sigma_{\pi r}$	Invgamma	0.10	2.00	0.23	0.19	0.26	-	-	-
σ_{cr}	Invgamma	0.10	2.00	0.71	0.61	0.81	-	-	-
ρ_{yr}	Beta	0.50	0.20	0.91	0.85	0.97	-	-	-
$\rho_{\pi r}$	Beta	0.50	0.20	0.09	0.01	0.16	-	-	-
ρ_{cr}	Beta	0.50	0.20	0.80	0.73	0.87	-	-	-

Table 4. Second-moment statistics: Euro Area

Panel A	Δy^r	π^r	Δc^r	$rev^{\Delta y}$	rev^{π}	$rev^{\Delta c}$
Euro Area data:						
Stand. deviation (%)	0.22	0.02	0.22	0.23	0.13	0.23
Correlation with Δy	0.26	-0.056	0.19	0.63	-0.35	0.47
Autocorrelation	0.11	0.02	0.04	-0.05	0.25	-0.25
Extended model:						
Stand. deviation (%)	0.64	0.04	0.86	0.24	0.37	0.28
	(0.33,0.92)	(0.01,0.07)	(0.56,1.17)	(0.20,0.28)	(0.19,0.45)	(0.23,0.32)
Correlation with Δy	0.22	-0.03	0.24	0.31	-0.13	0.28
	(0,0.37)	(-0.10,0.01)	(0,0.39)	(0.05,0.55)	(-0.21,-0.06)	(0.14,0.4)
Autocorrelation	0.22	0.17	0.59	0.00	0.55	-0.13
	(0,0.38)	(0,0.32)	(0.43,0.90)	(-0.07,0.07)	(0.23,0.83)	(-0.19,-0.07)

Table 4. (Continued)

Panel B	Δy	Δc	Δi	Δw	l	R	π
Euro Area data:							
Stand. deviation (%)	0.14	0.14	0.45	0.13	0.03	0.31	0.13
Correlation with Δy	1	0.47	0.65	0.25	-0.02	-0.06	-0.32
Autocorrelation	0.50	0.01	-0.04	0.34	0.94	0.88	0.27
Extended:							
Stand. deviation (%)	0.53	0.64	1.05	0.27	2.11	0.19	0.23
	(0.28,0.75)	(0.36,0.92)	(0.47,1.62)	(0.08,0.44)	(0.47,3.70)	(0.04,0.35)	(0.10,0.37)
Correlation with Δy	1	0.24	0.10	0.19	-0.07	-0.12	-0.11
		(0,0.39)	(0,0.22)	(0,0.33)	(-0.12,0)	(-0.23,0)	(-0.19,0)
Autocorrelation	0.23	0.18	0.30	0.41	0.71	0.69	0.64
	(0,0.39)	(0,0.34)	(0,0.49)	(0,0.60)	(0,0.97)	(0,0.95)	(0,0.90)
SW model:	Δy	Δc	Δi	Δw	l	R	π
Stand. deviation (%)	0.24	0.16	0.82	0.12	10.6	0.55	0.56
	(0.14,0.34)	(0.09,0.24)	(0.43,1.23)	(0.05,0.18)	(1.21,20.3)	(0.07,1.17	(0.08,1.16)
Correlation with Δy	1	0.30	0.22	0.18	0	-0.08	-0.11
		(0,0.66)	(0,0.52)	(0,0.45)	(-0.02,0)	(-0.23,0)	(-0.32,0)
Autocorrelation	0.28	0.31	0.26	0.30	0.48	0.47	0.40
	(0,0.63)	(0,0.69)	(0,0.61)	(0,0.66)	(0,0.99)	(0,0.99)	(0,0.99)

The parenthesis refer to 95% posterior confidence intervals for second-moment statistics

Table 5. Variance decomposition (percent)

Innovations	Δy	Δy^r	Δc	Δc^r	Δi	Δw	l	R	π	π^r
Extended model										
Technology, η^a	1.3	1.2	1.2	0.9	0.2	0.5	0.7	1.5	0.6	0.4
Risk premium, η^b	32.8	30.5	37.3	29.5	7.8	22.4	8.3	53.5	25.2	17.2
Fiscal/Net exports, η^g	2.9	2.6	0.0	0.1	0.1	0.2	0.6	0.2	0.1	0.1
Investment adj. costs, η^i	11.9	11.0	0.0	0.5	75.6	3.3	4.8	7.4	3.2	2.1
Interest-rate, η^R	9.6	8.9	10.7	8.4	2.3	7.1	2.7	10.5	7.7	5.2
Wage-push, η^w	1.3	1.2	2.8	2.2	0.1	2.5	16.7	1.5	2.2	1.3
Price-push, η^p	8.7	8.1	9.7	7.7	0.1	37.5	53.8	19.0	30.9	21.2
Output revision, η^{ry}	0.1	7.0	0.1	0.1	0.1	0.1	0.1	0.2	0.1	0.1
Inflation revision, $\eta^{r\pi}$	23.2	21.6	22.4	17.4	9.8	20.7	11.1	2.6	28.3	51.2
Consumption revision, η^{rc}	7.9	7.4	14.5	32.8	1.4	5.4	1.5	3.4	1.5	1.0
SW model										
Innovations	Δy	Δy^r	Δc	Δc^r	Δi	Δw	l	R	π	π^r
Technology, η^a	18.5	-	11.0	-	0.9	0.8	1.4	0.4	0.2	-
Risk premium, η^b	29.8	-	39.3	-	18.0	4.5	3.4	27.5	11.2	-
Fiscal/Net exports, η^g	7.1	-	0.5	-	0.3	0.3	0.4	0.2	0.7	-
Investment adj. costs, η^i	8.4	-	2.5	-	43.1	0.2	0.6	1.0	0.4	-
Interest-rate, η^R	16.1	-	18.9	-	11.9	2.8	2.4	2.8	8.2	-
Wage-push, η^w	10.9	-	11.5	-	16.0	18.4	11.6	3.4	8.5	-
Price-push, η^p	25.5	-	28.7	-	9.5	73.0	80.0	64.0	71.2	-

Part IV

Borrower-based measures in a DSGE model

1 Introduction

Macroprudential policies are an important toolkit of central banks nowadays to ensure financial stability. Borrower-based macroprudential measures such as limits on loan-to-value and loan-to-income have been found to be effective to influence credit standards and flows through quantitative restrictions (see Claessens, Ghosh and Mihet (2014); BCBS (2010); JMCB special issue (2015)). Not only do they affect the credit flow for house purchases, but they are particularly important for financial stability by limiting risk-taking of borrowers and lenders. Given their transmission through quantities and directly affecting borrowers, these instruments are important to complement capital-based ones to counter the build-up systemic risks. While cross-country studies indicate that loan-to-value, loan-to-income or debt-servicing-to-income limits are effective to restrict credit, the individual measures differ in their transmission to counter risks and the way they influence financial stability, to this extent, this chapter uses a macro-financial DSGE model to address the feedback effects of macroprudential policies in ensuring financial stability.

Limits on LTV ratios limit leverage relative to the value of the collateral and primarily limit losses for the lender in the event of a borrower default. They thereby strengthen the resilience of lenders, mostly banks. In countries where mortgage debt is non-recourse debt, a tighter LTV ratio also reduces the incentives for strategic defaults and thereby reduces probability of defaults, which is especially relevant when house price volatility is structurally high and amortization rates are low. Strategic defaults have played an important role in the

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US during the financial crisis but their occurrence is limited in Europe due to full recourse loans. Caps on LTI and DSTI ratios, instead, focus more strongly on the affordability with respect to the income of the borrower. In this way it counters more directly the risks related to probabilities of default.

Borrower-based measures have been implemented in a number of countries and throughout history. More recently the financial crisis has focused on their financial stability implications and methodologies have been developed to quantify their impact, including DSGE models. Bruneau et al. (2016), Chen and Columba (2016) or Walentin (2014) have analyzed the role of borrower-based measures in the context of DSGE models.

The contribution of this paper is to quantitatively assess the transmission of limits to household leverage, being it relative to the value of housing or to the wage income, and to assess the role of shocks to income and to the value of housing for the macroeconomy under differing LTV/LTI restrictions. The work is based on a model considering three layers of default developed in the context of the Macro-prudential Research Network (MaRS) by Clerc et al. (2015)[1]. The model considers loans to households and Non-Financial Corporations (NFCs) and allows for defaults by banks, households and NFCs.

The modelling strategy to assess the impact of LTV and LTI limits consists in limiting these ratios for outstanding loans beyond the values borrowers and banks would negotiate among them. The assessment of the impact is intended to indicate the effects on real estate markets and on financial stability. The conducted analysis provides at this stage less of an assessment of the level of LTV and LTI limits from a normative perspective.

The quantitative implementation of the model uses data by the Eurosystem Household Finance and Consumption Survey (HFCS) in order, first, to calibrate the model and, second,

[1]The model in Clerc et al (2015) (the "3D" model) was developed in the context of the Macroprudential Research Network (MaRS) of the European System of Central Banks to provide a decision-support framework for the positive and normative analysis of macroprudential policy, with a specific focus on capital requirements. Mendicino et al (2018) extend the original 3D model and calibrate it to Euro Area (EA) in order to provide a quantitative assessment of increases in total and sectorial capital requirements in a stochastic environment. Building on both Clerc et al. (2015) and Mendicino et al (2018), the 3D model has been operationalized to all SSM countries in the context of the OMR task force.

to relate the steady state implications to the policy instrument which affects the distribution of the ratios. Indeed, as the policy instrument limits the distribution of LTV and LTI ratios at loan origination, the HFCS provides a relationship between the changes in the distribution of lending standards at origination and the average for outstanding loans. When macroprudential policy tightens LTV/LTI limits it affects the right tail of a market-supported LTV / LTI distribution at loan origination. The impact of such policy for the mean of the distribution of credit standards depends on the size of the policy change and on the number of affected individuals, i.e. the mass in the right tail of the distribution. The larger the change and the larger the affected mass, the stronger the expected impact of the policy.

By combining the model with information on the distribution of loans in data, this chapter tracks the impact of borrower-based measures from their impact on credit conditions at loan origination, the policy variable, to the variable affecting the economy, outstanding loans, as well as to the long-term macroeconomic effects on GDP, credit, real estate investment as well as mortgage defaults and mortgage spreads. The results reveals that LTV and LTI limits can have sizable effects on credit and leverage which have repercussions on real estate activity and on the defaults of households. Overall, borrower-based measures are effective in reducing credit flows and promoting household resilience through less mortgage defaults. The tighter loan to value limit induces a shift in household expenditure away from housing expenditure, resulting in a strong fall in housing investment, towards consumption, with an overall limit effect on GDP.

The paper is structured as follows. Section 2 introduces LTV and LTI limits into the 3D model, the workhorse model for capital-based macroprudential measures and covers the steady state effects of the two measures. Section 3 it relates the credit ratios for outstanding loans to the distributional changes at the right tail of distributions affected by policy instruments.

2 Macroprudential policies in the 3D model

The "3D" model in Clerc et al. (2015) introduces financial intermediation and default into an otherwise standard dynamic stochastic general equilibrium (DSGE) model and provides a clear rationale for banking regulation by introducing three types of distortions: limited liability by banks, limited participation in equity markets and bank funding cost externalities resulting in excessive risk-taking by banks. The model includes six types of representative agents: borrowers, savers, entrepreneurs, banks, bankers, and the macroprudential authority. However, because the focus of the model is on financial relations, the majority of the dynamics is concentrated to the banking sector. Banks finance their loans by raising equity (from bankers) and deposits (from savers). Deposits are formally insured by a deposit insurance agency that is funded by lump-sum taxes paid by savers and borrowers. When banks default (a non-linear event) depositors suffer some transaction costs despite the deposit insurance scheme. This feature effectively links bank risk to bank's funding cost

However, bank's cost of funding is not related to bank's individual risk taking. Instead, it is dependent on the system-wide risk pattern. This is due to two factors. First, safety-net guarantees insulate bank's cost of deposits from the effect of their individual risk taking. Second, the deposit premium is based on system-wide bank risk failure. This reduces the incentive of any individual bank to limit leverage and failure risk because it will get no funding cost premia (benefit) when depositors are assumed to be imperfectly informed.

Moreover, banks have an incentive to take as much risk as possible by leveraging up to the regulatory limit. This excessive leverage has two counter-acting effects on their funding costs in equilibrium. On one hand, default probability of banks increases, which exerts upward pressure on bank's funding costs. On the other, this results in higher bailout subsidy (and taxes), which puts downward pressure on their funding costs. The net effect depends on which of the two dominates. If bank failure risk is high, the first effect (higher deposit premium) dominates, and the excessive leverage depresses economic activity. If overall bank risk is low, excessive leverage will support economic activity. Economizing on expensive equity reduces overall bank funding costs, and higher leverage will increase economic activity.

Higher capital ratios tighten the supply of loans by reducing the incentives for banks to take on excessive leverage. At the same time, higher capital ratios reduce the cost of uninsured funds provided to banks, which in turn reduces the cost of credit. The final impact depends on which of the two channels dominates. Moreover, the heterogeneity in households means that there is a trade-off between the welfare of savers and borrowers. In the long run, savers benefit from tighter capital regulation due to the reduced likelihood of bank failures which implies safer bank deposits. Borrowers, meanwhile, lose out after a certain level of capital, as this leads to a reduced supply of loans.

This paper extends the macroprudential toolkit in the 3D model to include limits on LTV and LTI ratios as macroprudential policies to reduce household indebtedness and defaults. In the original model, loans to households result as efficient private contract between the household and the bank following the costly state verification by Bernanke, Gertler and Gilchrist (1999). Banks grant loans and charge a spread over their own funding costs which covers the losses from the expected default of households. A higher steady state rate of defaults implies higher spreads charged by banks.

2.1 Borrower-based measures: Modelling strategy

The defaults are a direct consequence from uncertainty on the value of the collateral. Furthermore, for given uncertainty a higher leverage ratio raises the risk of default by households. Hence, a macroprudential policy which limits household leverage in mortgage contracts raises the resilience of the household sector and reduces losses for banks. The model contains a measure of household leverage (loan-to-value ratio of outstanding loans) which is central for determining the mortgage spreads by banks. The equation for the loan contract in Clerc et al. (2015) is :

$$E_t\left[(1-\mathit{\Gamma}^H(\bar{w}_{t+1}))\left(\mathit{\Gamma}^m\left(\frac{\widetilde{x^m}_t}{R^H_{t+1}}\right)-\mu^m G^m\left(\frac{\widetilde{x^m_t}}{R^H_{t+1}}\right)R^H_{t+1}\right)\right]q^H_t h^m_t=\rho_t\phi^H_t b^m_t,$$

where the Participation constraint imposes a certain return on equity ρ_t over the total equity of the bank $\phi^H_t b^m_t$.

The return on equity is given by the left hand side of the equation, that takes into account the leverage of the bank by the term $(1-\mathit{\Gamma}^H(\bar{w}_{t+1})$, where $(\mathit{\Gamma}^H(\bar{w}_{t+1})$ is the share of bank assets belonging to the depositors, $\mathit{\Gamma}^m$represents the share of housing value belonging to the bank, $\mu^m$is the state verification cost, $G^m\left(\frac{\widetilde{x^m}_t}{R^H_{t+1}}\right)$is the rate of household defaults (the superscript m refers to borrowers), $\widetilde{x^m_t}=x^m_t R^m_t$ is the gross loan-to-value ratio (including the interest rate) and $\bar{w}_{t+1}$is the cut-off threshold between default and non-default for mortgage banks within the log-normal distribution distribution.$R^H_{t+1}$is the return on housing, which is given by:

$$R^H_{t+1}=\frac{q^H_{t+1}(1-\delta^h_{t+1})}{q^H_t}$$

The equation contains thus an implicit measure of household leverage (loan-to-value) defined as the mortgage debt (superscripted by m) in relation to the current value of the house:

$$\widetilde{x^m_t}=\frac{b^m_t}{q^h_t h^m_t},$$

where where, $b^m_t$is the mortgage debt, q^h_t is the current value of the house and h^m_t is the total stock of housing held by the borrower household.[2].

The equation for the loan contract equates the expected returns to extending loans to households, including losses for defaulting loans, to the applied interest rate spread. A higher household leverage results in a higher probability of default by households and thus requires

[2]Note that, given the structure of the model, loans are rolled over every period, LTV at origination and (outstanding) indexed LTV are hence equivalent, though when relating to the observed loan contracts, the LTV ratio in the model should be interpreted as the ratio referring to the average of outstanding loans.

a larger spread to compensate for the losses. Leverage enters in the budget constraint of the household, thus affecting their default decision and determines the spread (and hence the mortgage interest rates R_t^m) of loan contracts. In addition to the LTV ratio, an LTI ratio can be constructed in the model. It combines the amount of mortgage loans and borrower's annual income. The LTI ratio is defined as

$$LTI_t^m = \frac{b_t^m}{4 w_t^h l_t^m},$$

where b_t^m is the mortgage debt of the borrower, and the denominator stands for the annual income (w_t^h is the worker's wage and l_t^m is the quarterly labour supply). In order to compare to observable loan-to-income ratios we consider loans relative to annual income (four quarters) by multiplying the denominator by four. Note that borrowers in the model earn only labour income due to their time preference. Furthermore, the model does not explicitly include taxation, which implies that net and gross income are identical and that tax incentives related to mortgage debt are not explicitly considered. The calibration of the model considers the loan supply equation and computes the endogenous loan amounts and spreads. The counterfactual simulations which assess the impact of LTV or LTI restrictions substitutes the loan supply equation by an exogenous loan amount. The reduction in LTV or LTI ratios is implemented by reducing the exogenous mortgage loans until reaching the level of the desired LTV decline. Since the new value is below the amount privately negotiated by banks and households, it represents a constraint and is binding. The implementation accounts for the fact that macroprudential policies can only tighten.

2.2 Calibration in steady state

The quantitative version of the 3D model is based on a calibration that matches first and second key moments of individual-country macro-financial variables over the period 2001:1-2014:4. In addition it uses the first wave from 2010 of the Household Finance and Consumption Survey (HFCS) dataset to calibrate the share of borrowers, the wealth of borrowers and the LTV ratios. The original calibration of the model focused on macroeconomic and banking variables such as total capital, the default rate of banks and the returns on their equity. In order to account for household leverage in form of LTV ratios, the extended calibration strategy incorporates loan-to-value ratios of outstanding loans as a moment to match in addition to the variables in the original calibration. An alternative would be to use the LTI ratio as explicit target. We instead use the LTI ratios as variables to validate the model and compare the LTI ratio in the data to those obtained from the calibrated model.

The HFCS provides data on the financial situation of households in European countries. It provides the loan and house value at origination and at the time of the survey, as well as income at time of the interview. The data is used to construct the LTV ratio at the moment of survey for the calibration of LTV ratios of outstanding loans. Using the HFCS as main source for the LTV ratio is consistent with the data source of other two calibrated variables in the model, namely the fraction of borrowers and housing wealth held by borrowers.

The LTV ratio for outstanding loans of borrowers is computed by dividing mortgage loans for the household's main residence (HMR ,HB170x) by the current housing value (HB0900) multiplied by the share of home ownership (HB0500). The average across all borrowers is computed by weighting by mortgage size (HB170x)[3]. In order to limit the influence of outliers in the HFCS database, LTV ratios for individual borrowers are censored at 200% LTV. We proceed similarly for LTV ratios of at origination. We use loans for the household's main residence HMR at origination (HB140x) and divide it by the respondent's reply of the house value at origination (HB0800) and the weighted country mean is obtained by using the loan amount at origination. The LTV ratios for the 1st and 2nd wave are presented in Table 1, whereby wave 1 of the HFCS dataset, conducted in 2010, exhibits a smaller country coverage and wave 2 conducted in 2013 and 2014 provides a larger country coverage and allows assessing evolution over time.

[3]An additional weighting by survey weights has also been considered, but has not been applied because, first, it does not significantly alter the average value and, second, it is unclear to what degree the social weights help in raising representativity of the borrower's sample.

The values used in the original model calibration slightly differ as the original calibration was done based on a censoring of 150% instead of the 200% used in subsequent assessments. In addition to the LTV ratios, Table 1 also documents loan-to-income (LTI) ratios at origination and for outstanding mortgage loans. While the LTI ratio can be directly computed based on the current outstanding loan for the household's main residence HMR (HB170x) and the current income (DI2000). For the LTI ratio at origination we use the loan amounts at origination (HB140x) and deflate the current income using the aggregate consumption deflator[4]. The LTI ratios are censored at 20 times annual incomes (additional series are presented in the appendix).

As indicated, the model matches the LTV ratios relatively well and the model-implied LTI ratios are in the range of those in the data, indicating the broad fit of the model to the HFCS data. We do not expect that LTI ratios from the model and in data would fit perfectly given that the model does not account for taxation nor for capital incomes by households.

Table 1: LTV and LTI ratios at origination and for outstanding mortgage loans

		AT	BE	CY	DE	ES	GR	IT	LU	MT	NL	PT	SI	SK
LTV ratio at origination of mortgage loans (in %)														
200%	WM	80.7	93	86.2	83.3	87.2	87.4	83.4	-	90.8	103.8	93.7	71.9	84.8
LTV ratio of outstanding mortgage loans (in %)														
200%	WM	62.2	52.6	50.5	57.3	49.5	57.2	48.6	56.6	37.1	68.9	60.6	48.2	51.2
150%	Model	61.7	52.4	49.5	56.9	58.4	56.7	48.4	55.3	35.1	68.2	59.8	59.5	49.5
	LTI ratio of loans at origination of mortgage loans (in years of income)													
20	WM	4.0	3.3	4.9	3.0	4.1	3.9	3.8	3.6	3.2	4.0	4.5	2.8	4.0
LTI ratio of outstanding loans (in years of income)														
20	WM	3.5	3.1	4.5	2.5	3.7	3.3	3.0	3.2	2.7	3.9	4.0	1.9	3.5

As indicated, the model matches the LTV ratios relatively well and the model-implied

[4]This approach does not account for the income changes of each individual borrower between the origination of the loan and the current income. Still, an assessment by mean loans using national data for Portugal delivered comparable amounts for mean incomes using income at origination and using deflated income.

LTI ratios are in the range of those in the data, indicating the broad fit of the model to the HFCS data. We do not expect that LTI ratios from the model and in data would fit perfectly given that the model does not account for taxation nor for capital incomes by households.

2.3 Effects of varying household leverage in the 3D model

A 1 percentage point reduction in LTV ratios of outstanding loans

The credit standards in the 3D model are applied to the outstanding loans of the representative household. This section assesses the long-term (steady state) effects of changes to LTV and LTI ratios for outstanding loans. The next section sheds light on the implications for changes in the LTV and LTI ratios on the tail of the distribution of originating loans instruments. The quantification considered focuses on the implications when reducing LTV ratios by 1 p.p. and LTI ratios by one tenth of annual income, i.e. by 10 p.p.. The regulatory constraints are implied by substituting the endogenous loan contract between households and banks through an exogenous credit amount that is lowered from the calibrated value up to the point where it reaches the targeted LTV or LTI restriction

Figure 1. Steady state impact of a reduction of LTV ratios of outstanding loans by 1 percentage point

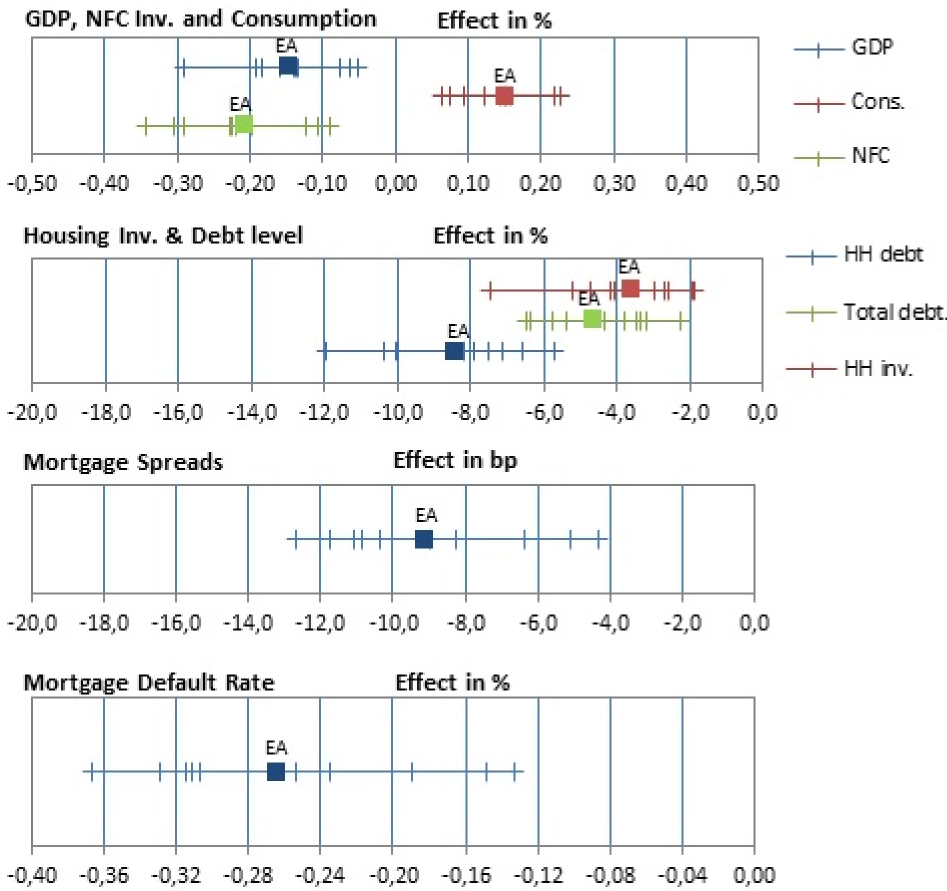


Table 1. Steady state impact of a reduction of LTV ratios of outstanding loans by 1 percentage point. Max and min effects

	LTV (Outs.)	LTI (Outs.)	GDP level	Consumption level	Mortgage debt level	Total debt level	Housing Inv. level	NFC Inv. level	Mortgage spreads bps	Mortgage defaults %
	p.p.	p.p.	%	%	%	%	%	%	bps	p.p.
Min	-1.00	-23.29	-0.29	0.06	-11.95	-6.50	-7.49	-0.34	-12.70	-0.37
Max	-1.00	4.12	-0.05	0.23	-5.70	-2.27	-1.86	-0.09	-4.31	-0.13

The effects of a 1 percentage point reduction in LTV ratios of outstanding loans on eight key model variables is depicted in Figure 1. The reduction in LTV ratios has overall a limited effect on aggregate long-term GDP, ranging from 0.02 to 0.30% of national GDP levels. The size of the effects depends on the share of borrowers in the economy and the relative size of the real estate construction sector as well as the initial LTV level. Indeed, an important element explaining the muted response in GDP is the shift in aggregate expenditure away from housing investment towards consumption. In addition, savers increase their expenditure of housing as housing appears relatively cheaper. An economy with sizable housing investment sees a relatively stronger fall in GDP, but also a stronger shift increase in consumption.

The LTV restrictions imply a reduction in aggregate debt levels by between 6 and 12% across countries, whereas the effects on aggregate credit are between 2.5 and 7%. The reduced leverage in the household sector implies also a reduction in the mortgage defaults because lower LTV ratios imply that future variations in housing value trigger less of defaults. Default rates decrease by between 0.03 up to 0.15 percentage points. Overall, banks face a reduction in losses from defaults which allow banks to reduce spreads on mortgage loans. These reductions amount to between 1 and 8 bps points.

A 10 percentage point reduction in LTI ratios of outstanding loans

An alternative to a reduction in LTV ratios consists in a decline in LTI limits. The decline in LTI ratios is implemented by reducing credit amounts by as much is necessary to reduce the imputed LTI ratios by the desired amounts, in line with the methodology for LTV ratios. The effects of a 10 percentage point reduction in the LTI ratio is depicted in Figure 2 and summarized in Table 2. A 10 percentage point reduction consists of a decline from e.g. 3.2 to 3.1 times the annual income.

The reduction in LTI ratio by the chosen value has, on average, a slightly larger effect compared to the considered 1 p.p. reduction in the LTV ratio. Nevertheless, the relative effects across countries depend on the relative initial indebtedness. The reduction in LTI ratios implies limited effects on GDP, accompanied by a shift towards consumption and a sizable fall in housing investment. Mortgage debt reduces by between 4 to 11% (17% for one country) reduces mortgage default rates by between 0.1 to 0.8 percentage points and 2.6 p.p.

to 26.9 p.p. for spreads.

Nevertheless, for some countries major quantitative differences exist. The reason for the differences resides especially in the lower level of mortgage loans and lower housing investment relative to GDP in these countries. This results in a higher percentage variation when reducing LTI ratios in percentage points.

Figure 2. Steady state impact of a reduction of loan-to-income ratios of outstanding loans

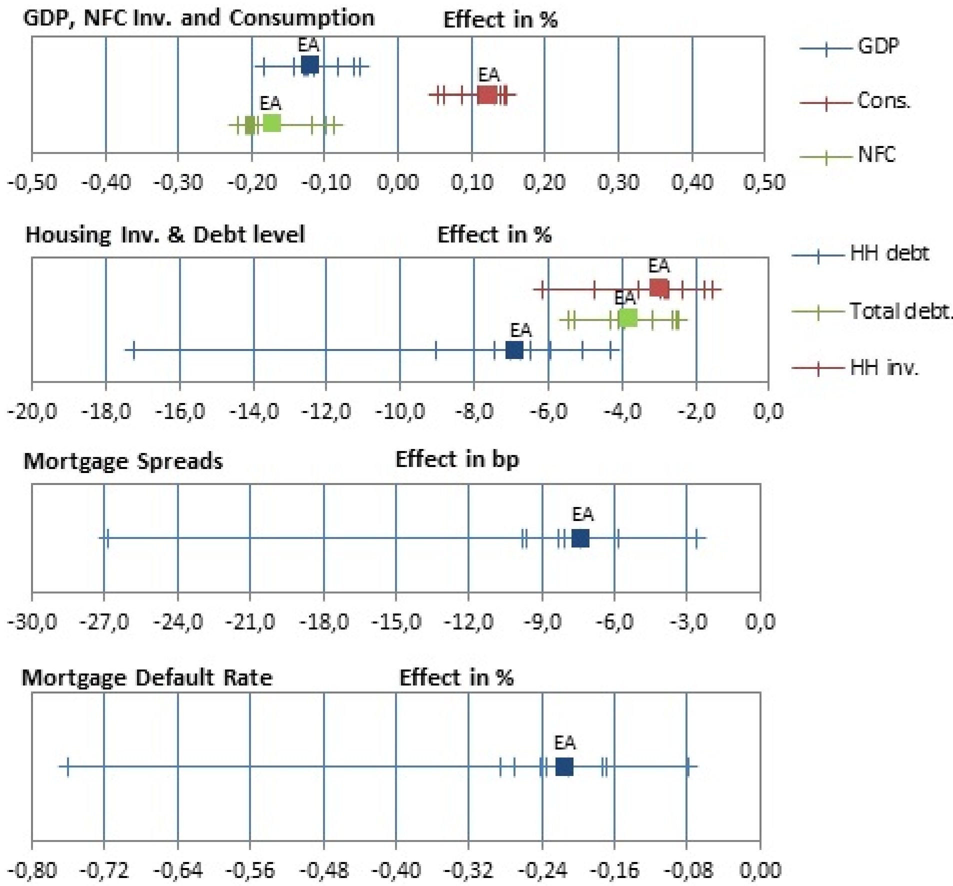


Table 2. Steady state impact of a reduction of LTI ratios of outstanding loans by 10 percentage points. Max and Min effects

	LTV (Outs.)	LTI (Outs.)	GDP level	Consumption level	Mortgage debt level	Total debt level	Housing Inv. level	NFC Inv. level	Mortgage spreads bps	Mortgage defaults %
	p.p.	p.p.	%	%	%	%	%	%	bps	p.p.
Min	-3.0	-10.0	-0.2	0.1	-17.2	-5.5	-6.2	-0.2	-26.9	-0.8
Max	-0.4	-10.0	-0.1	0.1	-4.3	-2.5	-1.5	-0.1	-2.7	-0.1

Caveats of the methodology

The simulations provided above are computations for steady state changes in LTV and LTI limits. The choice to focus on steady state impacts is due to the fact that loans in the 3D model are modeled as one-period loans with a roll-over of the entire loan mass every period. As a result, a reduction in LTV or LTI ratios would affect the entire stock of loans, whereas in reality, the policy instruments only affect the flow of loans. Furthermore, the model is set up in real terms. It hence neglects the possibility that inflation could make nominal more sustainable in times of high inflation. Likewise, it neglects the Fisherian debt deflation in consumer goods, while it does account for the effects of declining housing value.

An additional limitation is the fact that LTV or LTI ratios are modeled as permanently binding for the representative borrower. This does not allow relaxing credit conditions beyond those prevailing in the market. It is likely that this is a condition for any macroprudential policies. The default of mortgage loans in the 3D model occurs when loan size is larger than the housing value of the borrower (house values are subject to i.i.d shocks). In a situation in which house value shocks are the predominant source of defaults, such modelling is adequate. Instead, when income shocks are the main source of uncertainty and defaults, the 3D model only imperfectly captures the transmission of shocks. The shortcoming is conceptually more relevant when considering limits of LTI ratios instead of LTV ratios.

Finally, as mentioned, the LTV and LTI limits are applied to the representative borrower on outstanding loans. In reality, the available policy instruments act only on parts of the cross-sectional distribution of borrowers and only on the flow of lending. By acting on the cross-sectional distribution, the policy instruments curtail loans from the most risky ones. Instead, the model – by acting on the representative borrower – cannot overweight the riskier loans. As a result, the declines in defaults are likely to be an underestimation of what is achievable when reducing the high risk parts of the distribution.

In order to address the shortcoming, the next section provides the necessary steps to relate the policy instruments to the model values.

3 Relating LTV and LTI policies at loan origination to the dynamic model

The leverage considered in the 3D model relates to that of outstanding loans. Instead, policymakers use instruments that limit credit standards in the flow of loans. This section provides the information to relate credit standards at origination to those for outstanding loans. In order to relate the policy instrument to the model-relevant credit standard requires two steps:

1. Assessing the impact of the LTV or LTI policy limits at loan origination on the mean of the LTV(LTI) distributions at origination. The policy instrument limits the right-hand segment of the distribution and thereby reduces the mean LTV(LTI) ratio at origination.

2. Computing the effect on LTV (LTI) ratios of outstanding loans based on the mean LTV(LTI) ratio at origination. The quantitative effect is obtained by using a long-run relationship between credit conditions at origination and those for outstanding loans, assuming fixed-rate annuity mortgage contract.

3.1 LTV (LTI) limits and its effects on average credit standards at loan origination

The first step in computing the effects of the policy instrument to the model requires assessing the impact on the average credit condition at origination. The truncation/censoring of the right-hand tail of the LTV or LTI distribution reduces the mean of the distribution. The following assessment relies on two assumptions, discussed in more detail below. First, constrained borrowers are assumed to continue borrowing, but at a lower loan amount. Second, in line with implemented policies, we assume that the LTV limits can be applied only to a proportion of loans while a share of loans is exempted from the credit standard limit. In this case the loans exceeding the credit limit are uniformly reduced in order for their share to equate the imposed exemption share.

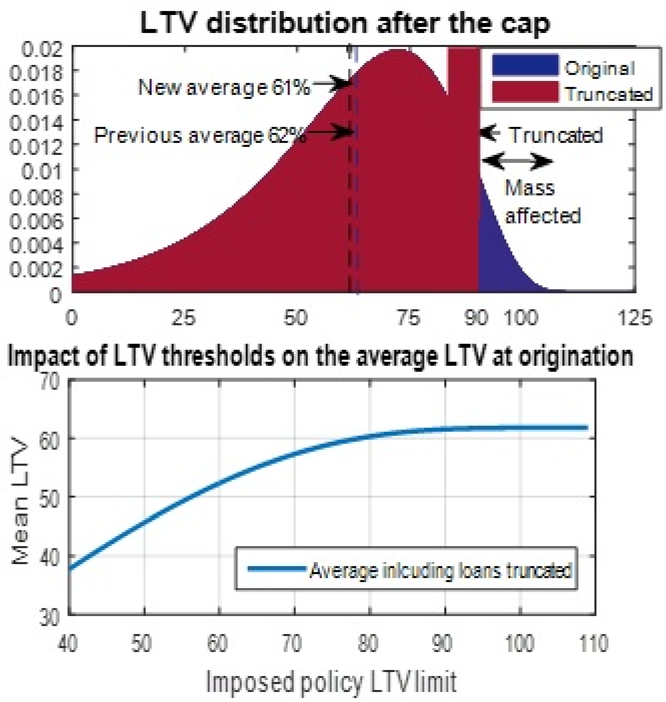
The upper hand panel of Figure 3 presents a stylized LTV distribution at loan origination. By imposing an LTV limit at a threshold of e.g. 90% implies that all loans above that threshold are curtailed. It is assumed that constrained borrowers continue borrowing, but the amount they borrow is limited to the regulatory LTV limit, resulting in an increase in loan amounts with an LTV ratio of 90%. This increase is identical to the originally affected mass to the right of the limit (blue area). As a result of the restriction, borrowers are now concentrated at the limited LTV ratio which is lower than their originally intended ratio and the average LTV ratio at origination declines (from 62% to 61% in the diagram).

The lower panel of Figure 3 illustrates the effects on the distributional mean (vertical axis) of imposing an LTV limit (horizontal axis)[5]. When reducing the LTV limit to e.g. 110% or 100%, the average mean LTV remains virtually unaffected (62%). This is because the market-based distribution features only very few borrowers at LTV ratios above 100%, given that the mass of the distribution is concentrated at LTV ratios around 75%. Reducing the limit further to 80% results in an average of about 60% and an LTV limit of 70% would imply an average LTV ratio of about 67%. The lower (tighter) the LTV limit, the more

[5]The example is based on a log-normal distribution.

borrowers are affected and the effects would eventually become proportional (a reduction of the LTV limit by 1 p.p. would reduce the mean by 1 p.p.).

Figure 3. Stylized distribution of LTV ratios at origination and implication of LTV limits on mean LTV at origination



Source: OMR Task Force calculation. Note: The stylized distribution is based on a log-normal distribution. Constrained borrowers are assumed to continue borrowing at the imposed regulatory LTV limit.

When setting limits on credit standard, policymakers have in practice also specified a proportion to which the limit applies, indicating that only a share of mortgage loans are required to comply with the limit whereas the remaining share of loans can exceed the limit. For example, the 2016 review of the LTV limits in Ireland imposes a 90% LTV limit from which 5% of loans to first time buyers and 20% to subsequent buyers are exempted. This provides the policymakers two margins of adjustment: the limit on the credit standard and the proportion to which the limit applies. It also implicitly offers a trade-off when adjusting the macroprudential between the limit of the credit standards and the exemption share.

In order to assess the implication of the exemption shares requires assuming how credit conditions behave for loans exceeding the limit. For modelling purposes, we assume that loans beyond the credit standard limit are reduced proportionally in order for their overall amount to match the exemption share. Hence, if e.g. originally 30% of loans exceeded the LTV limit considered by policymakers and the new exemption share amounts to 20%, we would reduce all loans exceeding the limit proportionally so as to sum to 20% of the entire distribution. This implies that one third of borrowers above the limit become constraint by the LTV limit.

The choice of applying the restriction proportionally and relocating the constrained borrowers to the limit has been influenced by two considerations. First, in the short-run constrained borrowers may decide to postpone an intended house purchase resulting in a drop-out out of the market. As the simulation is conducted for the long-run steady state, the individual delays would be compensated by new cohorts joining the housing market and result in a generally lower indebtedness. As a result, the reduction in the loan flow may be initially larger, but would recover in the longer-run. Second, the specific distribution of loans exempted from the limit is highly uncertain. The assumption that banks will grant the full exemption share at conditions beyond the limit is unlikely. The results are therefore to be seen as lower bound for the extension of credit.

Policymakers set the maximum values of macroprudential measures which banks have to comply with. These are the maximum LTV ratio at origination (LTVOmax). In addition, policymakers may exempt a share of loans from this limit, setting thereby an exemption share (ES). The impact of exemption shares on the mean of the distribution is illustrated in Figure A.1 in the appendix, which presents the average LTV ratio for different LTV limits and different exemption shares in some euro area countries based on HFCS data.

We distinguish overall five exemption shares (ES). The lowest line captures a situation where no exemption is granted to the imposed regulatory limit and all new loans with previously higher LTV ratios are constrained to the imposed LTV limit. This is the tightest application of an LTV limit. The four other cases allow for exemptions to the regulatory limit of 5, 10, 15 or 20% of total new loan production.

To provide an example, a reduction of the LTV limit to 90% in Spain (see figure A.1 in the appendix) implies a reduction in the mean LTV at origination by 12 p.p. (from 90% to 78%). When imposing the same limit but allowing for an exemption of 10% yields a reduction to an average LTV ratio by 8 p.p. (from 90% to 80%), indicating a looser stance. The different countries illustrate varying degree of dispersion. For countries with highly concentrated distributions, varying the exemption share does not imply major differences in the average LTV for a given LTV limit. Instead, a wider distribution of LTV ratios makes the average LTV ratio more dependent on the exemption share. In the extreme case where little mass is widely distributed in a thin tail, the necessary policy change to reduce the average LTV ratio is large. Instead, if a large mass is already accumulated close to the policy limit, only a small change in the policy instrument would suffice to achieve the same change in the average LTV.

When considering changes in the credit standards it is thus necessary to specify with which exemption share the credit standard is changed as the effect of a change in the policy limit would vary depending on the exempted right tail of the distribution. Conversely, if the policy goal is to reduce the average lending standards by a specific amount, the share of exempted loans matters in addition to the imposed LTV limit. Policymakers thus face a trade-off to make between imposing higher constraints with a lower exemption share, or instead imposing lower LTV limits but allowing for higher exemption shares.

Based on the available HFCS data it is possible to present the trade-off between LTV limit and exemption share. Figure A.2 in the appendix illustrates iso-mean LTV lines by combining the LTV limit (LTVOmax) on the horizontal axis to the exemption share (ES) on the vertical axis. Using Spain again as example, the LTVO distribution has its mean at 87%, as provided in the title. In addition, the percentiles of the full distribution with mean equal to 87% are also shown. The 95th percentile is equivalent to a 5% exemption share. Hence, the 80th percentile (20% exemption share) corresponds to an LTV ratio of 100% (illustrated by a square). When considering a reduction of the average LTV ratio from 87% to e.g. 84%, and allowing for an exemption share of 10%, the LTV ratio would need to be reduced from 118% to 99%. Alternatively, the same mean LTV ratio can be obtained if the LTV limit is reduced to 106% while exempting 5% of loans.

Figure A.3 and A.4 in the appendix provide the same concepts applied to LTI ratios based on the HFCS wave 1 dataset. It appears that LTI ratios are more uniformly distributed and unlike LTV ratios, the distributions do not exhibit a spike at a specific LTI value.

3.2 Linking credit standards at loan origination to outstanding loans

The previous section provided quantification between the cross-sectional distribution of lending standards at loan origination and the mean of the distribution. This section relates credit conditions at origination to those for outstanding loans. This is necessary as the 3D model operates with the average credit standard for outstanding loans and defaults on loans occur on outstanding loans and may endanger financial stability of banks.

To relate the HFCS data to the model we follow two approaches. The first approach computes the LTV ratio at origination and the same ratio for outstanding loans. The relative LTV ratio provides a conversation factor between LTV ratios at origination and for outstanding loans. This quotient may be subject to sizeable measurement error, especially because borrowers may over- or underestimate the value of their house when responding to the survey. This is especially the case if the house has been purchased already many years in the past and/or if the house owner is not actively following real estate developments. In addition, the quotient is prone to biases in the recent loan dynamics. If, for instance, the country has experienced a recent strong loan boom, the ratio would automatically be biased upwards. Instead, if such a boom had taken place in a more distant past, the quotient would have been lower. This makes this specific quotient excessively volatile across countries. We therefore also compute a second quotient based on more theoretical relationships.

The second approach exploits the annuity formula for loans to establish a correspondence between the loan size at origination and the average outstanding loan over the lifetime of the mortgage. Assuming that borrowers contract multi-annual mortgage loans with fixed annuity payments results in a clear relationship between interest rates, maturity, loan at origination and outstanding loan amounts. The quotient of LTV at origination and the average LTV for outstanding loans is determined by the annual interest rate and the overall maturity. A longer maturity implies higher interest payments and hence higher overall steady state debt levels and LTV ratios by borrowers. We further assume that underlying the mortgage market structure is a constant turnover of cohorts - with a constant inflow of new borrowers equated by a constant outflow of borrowers who have fully reimbursed their loans.

Table 3 provides an overview of the different LTV ratios and the quotients between LTVs at origination and for outstanding loans. For each HFCS wave, the first line is the average LTV at origination based on the HFCS data, the second line is the LTV ratio based on outstanding loans. The second block uses loan maturity, interest rates and the LTV ratios for outstanding loans in order to compute the implied LTVO ratio and ultimately the quotient between LTVs at origination and for outstanding loans. Overall, the first wave confirms the strong variation of quotients across euro area countries, whereas the ratios based on the annuity formula imply a ratio of 1.7 to 1.85. The 2nd wave overall confirms these ratios (1.69-1.87, see the last row of Table 3), but the empirical ones are generally lower, partly driven by the strong house price falls in some crisis during the crisis aftermath and due to the increased new recent loan contracts in other countries. For the calibration of the 3D model we use the average of the empirical quotients across all countries and those obtained from the annuity calculations which results in a factor of 1.71.

Table 3. LTV at origination and outstanding

	AT	BE	CY	DE	ES	GR	IT	LU	MT	NL	PT	SI	SK
Empirical ratio (LTV origination/LTV outstanding)													
Ratio	1.38	1.77	1.71	1.45	1.76	1.53	1.72	1.57	2.45	1.51	1.55	1.49	1.65
Other moments													
Maturity, years	23	19	20	13	22	20	18	21	25	28	26	14	19
Interest rate %	3.2	3.8	4.9	4.4	5.1	5.3	3.5	2.4	4.2	4.7	2.5	5.2	5.7
Implied LTV %	45	52	50	46	52	51	46	48	53	63	52	40	50
Implied ratio	1.79	1.79	1.73	1.83	1.69	1.71	1.82	1.85	1.71	1.65	1.81	1.79	1.70

Source: Source: Household Finance and Consumption Survey (HFCS) 1st wave, OMR Task Force calculation. Note: LTVO: LTV at origination. The average LTV ratio is the weighted mean. * The implied LTV ratio for outstanding loans is obtained by assuming a fixed-annuity loan contract over the average maturity of the loan contract assuming constant nominal interest rates over the entire horizon. The approach is consistent with a steady state view for the structural model, but does not account of past episodes of loan creation.

3.3 Policy simulation

Two policy simulations are considered. The first one reduces LTV limits at loan origination by 5 p.p. from 90% to 85% with an exemption share of 10%. The second simulation reduces LTI limits at loan origination by 50 p.p. from 500% to 450% of annual income, while exempting 10% of loans from this limit.

3.3.1 Reductions of LTV limits at origination

A standard calibration is considered in order to provide a reference quantification across countries. Such a calibration does not capture the relevant structural differences between countries and, when considering country-specific assessments would require to be recalibrated with caution to account for the national specification. The impact of such a standardized measure should therefore not be seen as a proposal for an implementation of borrower-based measures. This would require more specific assumptions based on information from each country. Nevertheless, the policy exercise provides a cross-country benchmark based on a standardized policy simulation and illustrates the importance of national real estate markets.

The general equilibrium long-term costs and benefits of the LTV limit are depicted in Table 4a and 4b. A reduction of the LTV limit from 90 to 85% (with 10% exemption share) results in a 1.2 – 3.2 p.p. decline in average LTV at origination and a long-term decline of 0.7 – 1.9 p.p. for the average LTV ratio of outstanding loans across euro area countries. The policy would imply only limited effects on GDP, but reduces long-term mortgage debt by 5.1 - 19.5% and reduces housing investment by between 1.7 and 14.2%, revealing strong adjustments in especially highly leveraged countries with large shares of borrowers. The tighter LTV limit induces a shift in household expenditure away from housing expenditure, resulting in a strong fall in housing investment, towards consumption[6]. The size of the fall in housing investment depends on the fraction of borrowers in the economy and on the original LTV ratio.

On the side of benefits, and importantly for financial stability, the resulting lower leverage in the household sector reduces mortgage defaults by between 0.2 and 0.4 p.p. compared with the historical averages used in the calibration. The lower default rates, in turn, allow banks to reduce spreads on mortgage loans by between 7.6 to 15.1 bps. Overall, the macroprudential instrument is effective in reducing credit flows and promoting household resilience through less mortgage defaults.

An assessment of possible over- and underestimation based on this methodology are in order. On the one hand, the effect on mortgage defaults may be overestimated in countries in which it has been found that housing valuations and housing defaults are poorly correlated. This would especially be the case in countries for which full recourse mortgages is a sufficiently strong deterrent to defaults. On the other hand, the effects may be underestimated, especially in those countries where high LTV loans exhibit much higher probabilities of default. In these

[6]The model is a closed economy model and does therefore not allow to consider adjustments in the balance of payments.

cases a reduction in the LTV limits implicitly targets the riskiest borrowers and would make them more resilient. In countries where default rates are strongly correlated with LTV ratios a smaller adjustment in the LTV limits would suffice to reduce aggregate defaults and bolster financial stability.

Especially as regards the substitution of housing expenditure with NFC investment and consumption, the macroeconomic effects may be overstated. The model assumes a reallocation between sectors which is only affected in the short-run by capital adjustment costs, but is not affected by skill mismatch of workers.

Table 4a. Impact of a 5 p.p. reduction in LTV ratios (from 95 to 90%) at loan origination

LTV Policy		Min	Max
LTV limit	p.p.	-5	-5
Average LTVO	p.p.	-3.2	-1.2
Average LTV ratio (outstanding)	p.p.	-1.9	-0.7
Average LTI (outstanding)	p.p.	-39.9	-2.9
GDP level	%	-0.55	-0.04
Consumption level	%	0.05	0.42
Mortgage debt level	%	-19.5	-5.1
Total debt level	%	-12.2	-1.6
Housing inv. level	%	-14.1	-1.9
NFC inv. level	%	-0.6	-0.1
Mortgage spread	bps	-15.1	-7.6
Mortgage default rate	p.p.	-0.4	-0.2

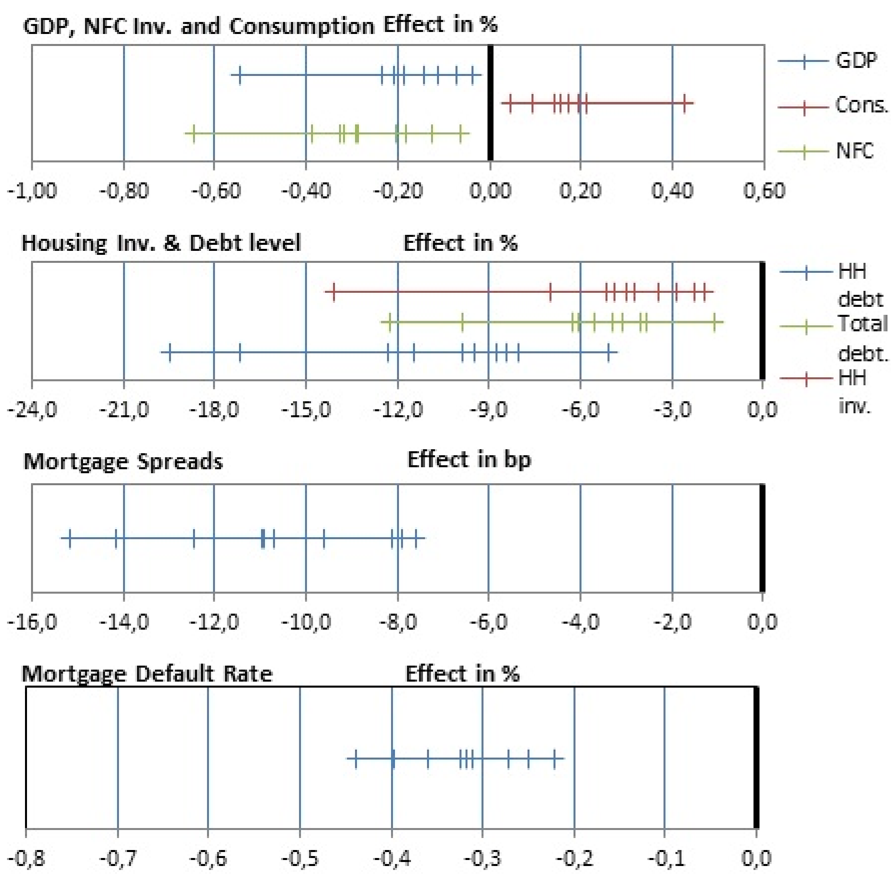
Source: OMR Task Force calculations based on HFCS data (1st wave) and on 3D model. Note: The reported changes are in percent of the long-term steady state value of the variable. Euro area countries not participating in the first wave of the HFCS are not considered. For some few small and open euro area countries, the model is assessed not to perform adequately and results are therefore excluded from this range.

Table 4b Impact of a 5 p.p. reduction in LTV at origination on macro-financial variables

		AT	BE	CY	DE	ES	GR	IT	NL	PT	SI
△mean LTV at origin	p.p.	-1.8	-2.5	-1.8	-1.7	-2.3	-2.4	-2.1	-3.2	-2.9	-1.2
△mean lLTV outst.	p.p.	-1.0	-1.5	-1.1	-1.0	-1.3	-1.4	-1.2	-1.9	-1.7	-0.7
GDP level	%	-0.1	-0.1	-0.2	-0.2	-0.2	-0.2	-0.1	-0.5	-0.2	0.0
Housing invest.	%	-4.9	-2.9	-4.5	-5.1	-3.5	-4.2	-2.2	-14.1	-7.0	-1.9
Mortgage debt	%	-12.3	-8.5	-8.0	-9.9	-8.8	-11.4	-9.5	-19.5	-17.1	-5.1
Mortgage spreads	bps	-10.7	-7.6	-9.6	-12.5	-10.9	-15.1	-14.2	-8.1	-10.9	-7.9
Mortgage default rate	p.p.	-0.3	-0.2	-0.3	-0.4	-0.3	-0.4	-0.4	-0.2	-0.3	-0.2

Source: OMR Task Force calculations based on HFCS data (1st wave) and on 3D model.

Figure 4. Impact of a 5 p.p. reduction in LTV at origination on macro-financial variables



3.3.2 Reductions of LTI limits at origination

Similarly to the policy simulations on LTV limits, we consider tighter limits on LTI ratios at loan origination to increase household resilience. The main policy exercise is a decline in LTI limits at loan origination by 50 p.p. from 5 to 4.5 times annual income, while exempting 10% of loans from this limit. The methodology first assesses the impact of the LTI constraint on the mean LTI conditions at loan origination. In a second step it converts the LTI ratio at loan origination into a ratio for outstanding loans. The results for such policy are presented in Table 5a and 5b and Figure 5.

Table 5a. Impact on macro-financial variables of a 50 p.p. reduction in LTI at origination from 5 to 4.5 times annual income

LTI Policy		Min	Max
LTV limit	p.p.	-0.5	-0.5
Average LTIO	p.p.	23.8	-4.39
Average LTV ratio (outstanding)	p.p.	-0.9	-0.4
Average LTI (outstanding)	p.p.	-13.9	-2.6
GDP level	%	-0.26	-0.03
Consumption level	%	0.04	0.2
Mortgage debt level	%	-9.1	-2.3
Total debt level	%	-5.7	-1.4
Housing inv. level	%	-6.6	-0.8
NFC inv. level	%	-0.3	-0.1
Mortgage spread	bps	-10.4	-2.1
Mortgage default rate	p.p.	-0.3	-0.1

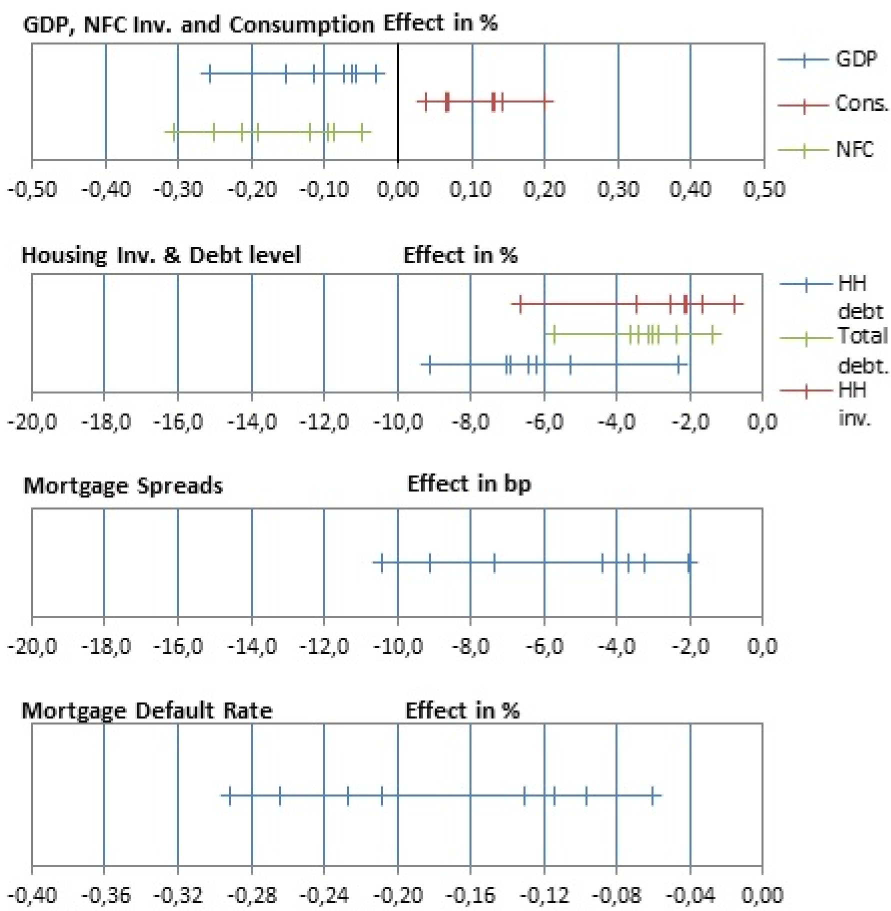
Source: OMR Task Force calculations based on HFCS data (1st wave) and on 3D model. Note: The reported changes are in percent of the calibrated steady state value of the variable. The value reported for mortgage default ratios and mortgage spreads are the calibrated value and the value after implementation, for ease of interpretation. A reduction in the LTI ratio from 5 to 4.5 has no impact in Germany and Slovenia, because the mass of the distribution is concentrated at lower LTI ratios.

Table 5b. Impact of a 50 p.p. reduction in LTI at origination on macro-financial variables

		AT	BE	CY	DE	ES	GR	IT	NL	PT	SI
△mean LTV at origin	p.p.	-12.9	-4.4	-20.9	-	-18.4	-15.8	-18.5	-23.9	-20.9	-
△mean ILTV outst.	p.p.	-7.5	-2.6	-12.2	-	-10.7	-9.3	-10.8	-14.0	-12.2	-
GDP level	%	-0.1	0.0	-0.2	-	-0.2	-0.1	-0.1	-0.3	-0.1	-
Housing invest.	%	-2.1	-0.8	-3.5	-	-2.5	-2.5	-1.7	-6.6	-2.2	-
Mortgage debt	%	-5.3	-2.3	-6.2	-	-6.4	-6.9	-7.0	-9.1	-5.3	-
Mortgage spreads	bps	-4.4	-2.1	-7.3	-	-8.0	-9.1	-10.4	-3.7	-3.2	-
Mortgage default rate	p.p.	-0.1	-0.1	-0.2	-	-0.2	-0.3	-0.3	-0.1	-0.1	-

Note: The reported changes are in percent of the calibrated steady state value of the variable. The value reported for mortgage default ratios and mortgage spreads are the calibrated value and the value after implementation, for ease of interpretation. A reduction in the LTI ratio from 5 to 4.5 has no impact in Germany and Slovenia, because the mass of the distribution is concentrated at lower LTI ratios.

Figure 5. Impact of a 50 p.p. reduction in LTI at origination on macro-financial variables



4 Conclusions

This chapter uses a macro-financial DSGE model where excessive risk behavior harms the economy introducing an incentive for macroprudential regulation, in which the benefit of tightening policies in terms of reducing risks may be offset by a reduction in bank activity and leading to a depression in the economy, providing a good set-up for analyzing the macroeconomic benefits of macroprudential regulation. By combining the model with information on the distribution of loans in data, this paper tracks the impact of borrower-based measures from their impact on credit conditions at loan origination, the policy variable, to the variable affecting the economy, outstanding loans, as well as to the long-term macroeconomic effects on GDP, credit, real estate investment as well as mortgage defaults and mortgage spreads. The assessment reveals that borrower-based measures have sizable effects on credit amounts and can reduce long-run defaults. For instance, a reduction of the loan to value limit from 90 to 85% leads to reductions of aggregate credit are between 2.5 and 7%, the resulting lower leverage in the household sector reduces mortgage defaults by between 0.2 and 0.4 p.p. compared with the historical averages used in the calibration. The lower default rates, in turn, allow banks to reduce spreads on mortgage loans by between 7.6 to 15.1 bps. Overall, the macroprudential instrument is effective in reducing credit flows and promoting household resilience through less mortgage defaults. The tighter loan to value limit induces a shift in household expenditure away from housing expenditure, resulting in a strong fall in housing investment, towards consumption, with an overall limit effect on GDP.

The assessment reveals that borrower-based measures have sizeable effects on credit amounts and can reduce long-run defaults. Its assessment is nevertheless limited to long-term effects, given limitation in the relatively simple way the real estate market is modeled. It opens up extension possibilities to develop additional models to shed light on the detailed working of the real estate market by focusing on additional sources of shocks and the role played by expectations of house prices.

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Figures

Figure A.1 Effect of LTV limits on the average of LTV ratio at loan origination (LTVO)

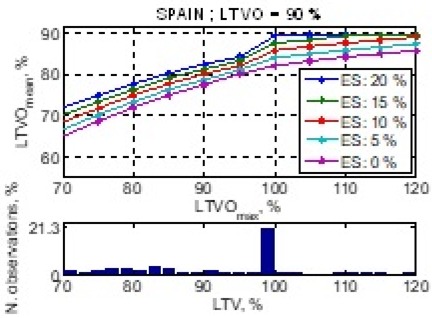


Figure A.2 Iso-mean for LTV ratios at loan origination relating LTV limits to exemption shares

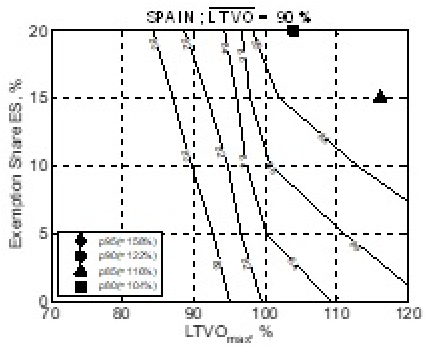


Figure A.3 Effect of LTI limits on the average of LTI ratio at loan origination (LTIO)

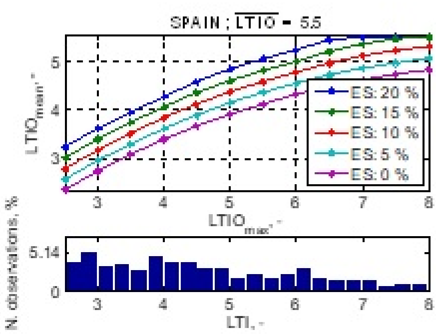
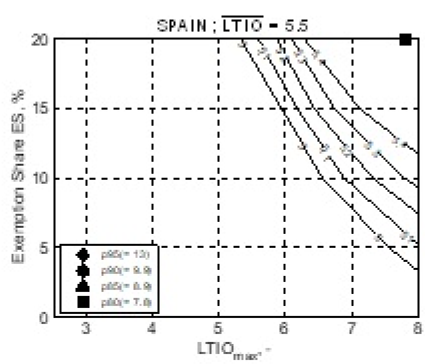


Figure A.4 Iso-mean for LTI ratios at loan origination relating LTI limits and exemption shares



Appendix Chapter 1

Supplementary appendix (Not intended for publication)

Log-linearized dynamic equations

In addition to equation (3) with $n = 4$ characterizing the 1-year bond yield, respectively, the set of the remaining log-linearized dynamic equations characterizing the estimated DSGE model are the following:

- Aggregate resource constraint:

$$y_t = c_y c_t + i_y i_t + z_y z_t + \varepsilon_t^g, \tag{30}$$

  where $c_y = \frac{C}{Y} = 1 - g_y - i_y$, $i_y = \frac{I}{Y} = (\gamma - 1 + \delta)\frac{K}{Y}$, and $z_y = r^k \frac{K}{Y}$ are steady-state ratios. As in Smets and Wouters (2007), the depreciation rate and the exogenous spending-GDP ratio are fixed in the estimation procedure at $\delta = 0.025$ and $g_y = 0.18$.

- Consumption equation:

$$x_t = E_t x_{t+1} - \left(\frac{1 - x_1}{\sigma_c}\right)\left[r_t - E_t \pi_{t+1} + \varepsilon_t^b\right], \tag{31}$$

  where : $x_t = c_t - x_1 c_{t-1}$, $x_1 = \frac{h}{\gamma}$, h denotes the habit formation parameter and γ denotes the balanced-growth rate.

- Investment equation:

$$i_t = i_1 i_{t-1} + (1 - i_1)\, E_t i_{t+1} + i_2 q_t + \varepsilon_t^i, \tag{32}$$

  where $i_1 = \frac{1}{1+\overline{\beta}}$, and $i_2 = \frac{1}{(1+\overline{\beta})\gamma^2\varphi}$ with $\overline{\beta} = \beta\gamma^{(1-\sigma_c)}$.

- Arbitrage condition (value of capital, q_t):

$$q_t = q_1 E_t q_{t+1} + (1 - q_1) E_t r_{t+1}^k - (R_t - E_t \pi_{t+1}) + c_3^{-1} \varepsilon_t^b, \tag{33}$$

  where $q_1 = \overline{\beta}\gamma^{-1}(1 - \delta) = \frac{(1-\delta)}{\left(r^k + 1 - \delta\right)}$.

- Log-linearized aggregate production function:

$$y_t = \Phi \left(\alpha k_t^s + (1-\alpha) l_t + \varepsilon_t^a \right), \tag{34}$$

where $\Phi = 1 + \frac{\phi}{Y} = 1 + \frac{\text{Steady-state fixed cost}}{Y}$ and α is the capital-share in the production function.[7]

- Effective capital (with one period time-to-build):

$$k_t^s = k_{t-1} + z_t. \tag{35}$$

- Capital utilization:

$$z_t = z_1 r_t^k, \tag{36}$$

where $z_1 = \frac{1-\psi}{\psi}$.

- Capital accumulation equation:

$$k_t = k_1 k_{t-1} + (1-k_1) i_t + k_2 \varepsilon_t^i, \tag{37}$$

where $k_1 = \frac{1-\delta}{\gamma}$ and $k_2 = \left(1 - \frac{1-\delta}{\gamma}\right)\left(1+\overline{\beta}\right)\gamma^2 \varphi$.

- Marginal cost:

$$mc_t = (1-\alpha) w_t + \alpha r_t^k - \varepsilon_t^a. \tag{38}$$

- New-Keynesian Phillips curve (price inflation dynamics):

$$\pi_t = \pi_1 \pi_{t-1} + \pi_2 E_t \pi_{t+1} - \pi_3 mc_t + \pi_4 \varepsilon_t^p, \tag{39}$$

where $\pi_1 = \frac{\iota_p}{1+\overline{\beta}\iota_p}$, $\pi_2 = \frac{\overline{\beta}}{1+\overline{\beta}\iota_p}$, $\pi_3 = \frac{A}{1+\overline{\beta}\iota_p}\left[\frac{\left(1-\overline{\beta}\xi_p\right)\left(1-\xi_p\right)}{\xi_p}\right]$, and $\pi_4 = \frac{1+\overline{\beta}\iota_p}{1+\overline{\beta}\iota_p}$. The coefficient of the curvature of the Kimball goods market aggregator, included in the definition of A, is fixed in the estimation procedure at $\varepsilon_p = 10$ as in Smets and Wouters (2007).

[7] From the zero profit condition in steady-state, it should be noticed that ϕ_p also represents the value of the steady-state price mark-up.

- Optimal demand for capital by firms:

$$-\left(k_t^s - l_t\right) + w_t = r_t^k. \tag{40}$$

- Wage markup equation:

$$\mu_t^w = w_t - mrs_t = w_t - \left(\sigma_l l_t + \tfrac{1}{1-h/\gamma}\left(c_t - (h/\gamma)\, c_{t-1}\right)\right). \tag{41}$$

- Real wage dynamic equation:

$$w_t = w_1 w_{t-1} + (1 - w_1)\left(E_t w_{t+1} + E_t \pi_{t+1}\right) - w_2 \pi_t + w_3 \pi_{t-1} - w_4 \mu_t^w + \varepsilon_t^w. \tag{42}$$

where $w_1 = \frac{1}{1+\overline{\beta}}$, $w_2 = \frac{1+\overline{\beta}\iota_w}{1+\overline{\beta}}$, $w_3 = \frac{\iota_w}{1+\overline{\beta}}$, $w_4 = \frac{1}{1+\overline{\beta}}\left[\frac{\left(1-\overline{\beta}\xi_w\right)(1-\xi_w)}{\xi_w((\phi_w-1)\varepsilon_w+1)}\right]$ with the curvature of the Kimball labor aggregator fixed at $\varepsilon_w = 10.0$ and a steady-state wage mark-up fixed at $\phi_w = 1.5$ as in Smets and Wouters (2007).

The rest of this appendix shows the complete set of parameter estimates for the two main specifications considered in the paper in Table A.1. Similarly, Table A.2 shows the parameter estimates considering SPF data in the set of observables.

Table A.1.A: Priors and estimated posteriors of the structural parameters

	Priors			Posteriors					
Sample period: 1965:4-2009:1				SlW-TS model			SlW model		
	Distr	Mean	Std D.	Mean	5%	95%	Mean	5%	95%
log data density					-1057.596			-1092.600	
φ: cost of adjusting capital	Normal	4.00	1.50	4.616	4.579	4.646	7.975	7.945	8.014
h: habit formation	Beta	0.70	0.10	0.759	0.749	0.788	0.851	0.842	0.878
σ_l : Frisch elasticity	Normal	2.00	0.75	1.767	1.745	1.798	1.744	1.744	1.801
ξ_p: price Calvo probability	Beta	0.50	0.10	0.563	0.550	0.583	0.472	0.459	0.487
ξ_w: wage Calvo probability	Beta	0.50	0.10	0.464	0.456	0.482	0.565	0.549	0.589
ι_w: wage indexation	Beta	0.50	0.15	0.470	0.401	0.486	0.185	0.107	0.229
ι_p: price indexation	Beta	0.50	0.15	0.378	0.325	0.401	0.178	0.151	0.192
ψ: capital utilization adjusting cost	Beta	0.50	0.15	0.092	0.085	0.100	0.151	0.149	0.180
Φ : steady state price mark-up	Normal	1.25	0.12	1.493	1.442	1.500	1.622	1.549	1.608
r_π: policy rule inflation	Normal	1.50	0.25	1.617	1.570	1.643	1.662	1.656	1.683
ρ_r: policy rule smoothing	Beta	0.75	0.10	0.886	0.878	0.896	0.884	0.881	0.907
r_y: policy rule output gap	Normal	0.12	0.05	0.038	0.032	0.047	0.075	0.065	0.095
$r_{\Delta y}$: policy rule output gap growth	Normal	0.12	0.05	0.144	0.132	0.154	0.122	0.104	0.131
r_{sp}: policy rule term spread	Normal	0.12	0.05	0.140	0.118	0.159	0.255	0.218	0.284
π: steady-state inflation	Gamma	0.62	0.10	0.658	0.628	0.706	0.610	0.589	0.656
$100(\beta^{-1}-1)$: steady-state rate of disc.	Gamma	0.25	0.10	0.174	0.151	0.182	0.324	0.234	0.324
l: steady-state labor	Normal	0.00	2.00	-0.449	-0.509	-0.396	1.903	1.792	1.903
$\overline{r}^{\{4\}}$: steady-state 1-year yield	Normal	0.00	2.00	0.844	0.844	0.967	2.397	2.381	2.463
γ: one plus st-state rate of output growth	Normal	0.40	0.10	0.393	0.386	0.403	0.408	0.405	0.415
α: capital share	Normal	0.30	0.05	0.169	0.153	0.188	0.182	0.152	0.200
ρ: learning parameter	Beta	0.50	0.28	0.992	0.988	0.997	0.998	0.996	1.000

Table A.1.B: Priors and estimated posteriors of the structural shock process parameters

	Priors			Posterior					
				SlW-TS model			SlW model		
	Distr	Mean	Std D.	Mean	5%	95%	Mean	5%	95%
σ_a: Std. dev. productivity innovation	Invgamma	0.10	2.00	0.480	0.454	0.501	0.488	0.476	0.525
σ_b: Std. dev. risk premium innovation	Invgamma	0.10	2.00	0.396	0.312	0.446	0.669	0.669	0.749
σ_g: Std. dev. exogenous spending innovation	Invgamma	0.10	2.00	0.483	0.452	0.520	0.508	0.506	0.534
σ_i: Std. dev. investment innovation	Invgamma	0.10	2.00	0.433	0.426	0.446	0.604	0.599	0.692
σ_r: Std. dev. monetary policy innovation	Invgamma	0.10	2.00	0.230	0.218	0.248	0.216	0.199	0.226
σ_p: Std. dev. price mark-up innovation	Invgamma	0.10	2.00	0.134	0.121	0.140	0.170	0.164	0.197
σ_w: Std. dev. wage mark-up innovation	Invgamma	0.10	2.00	0.693	0.677	0.742	0.801	0.788	0.859
$\sigma_\eta^{\{4\}}$: Std. dev. 1-year yield innovation	Invgamma	0.10	2.00	0.170	0.161	0.170	0.167	0.156	0.182
ρ_a: Autoregressive coef. productivity shock	Beta	0.50	0.20	0.954	0.946	0.965	0.911	0.895	0.926
ρ_b: Autoregressive coef. risk-premium shock	Beta	0.50	0.20	0.439	0.384	0.471	0.544	0.498	0.544
ρ_g: Autoregressive coef. exog. spending shock	Beta	0.50	0.20	0.988	0.983	0.993	0.964	0.960	0.979
ρ_i: Autoregressive coef. investment shock	Beta	0.50	0.20	0.526	0.503	0.539	0.685	0.668	0.696
ρ_r: Autoregressive coef. monetary policy shock	Beta	0.50	0.20	0.057	0.014	0.061	0.046	0.030	0.053
ρ_p: Autoregressive coef. price markup shock	Beta	0.50	0.20	0.875	0.875	0.910	0.880	0.860	0.904
ρ_w: Autoregressive coef. wage markup shock	Beta	0.50	0.20	0.918	0.909	0.928	0.838	0.827	0.853
ρ_{tp}: Autoregressive coef. 1-yr term premium shock	Beta	0.50	0.20	0.973	0.965	0.980	0.961	0.949	0.969
μ_p: MA coef. price markup shock	Beta	0.50	0.20	0.693	0.676	0.711	0.608	0.591	0.635
μ_w: MA coef. wage markup shock	Beta	0.50	0.20	0.477	0.454	0.517	0.325	0.309	0.368
ρ_{ga}: Interact. betw. product. and spending shocks	Beta	0.50	0.25	0.506	0.465	0.557	0.589	0.567	0.620

Table A.2.A: Priors and estimated posteriors of the structural parameters using SPF in the set of observables

	Priors			Posteriors					
Sample period: 1981:4-2009:1				SlW-TS model			SlW model		
	Distr	Mean	Std D.	Mean	5%	95%	Mean	5%	95%
log data density				-853.96			-1070.311		
φ: cost of adjusting capital	Normal	4.00	1.50	5.294	5.168	5.323	4.219	4.192	4.258
h: habit formation	Beta	0.70	0.10	0.633	0.611	0.643	0.631	0.607	0.643
σ_l : Frisch elasticity	Normal	2.00	0.75	2.151	2.097	2.219	2.234	2.225	2.370
ξ_p: price Calvo probability	Beta	0.50	0.10	0.715	0.702	0.730	0.617	0.598	0.630
ξ_w: wage Calvo probability	Beta	0.50	0.10	0.259	0.245	0.268	0.495	0.485	0.511
ι_w: wage indexation	Beta	0.50	0.15	0.400	0.337	0.437	0.218	0.195	0.237
ι_p: price indexation	Beta	0.50	0.15	0.820	0.796	0.863	0.896	0.884	0.928
ψ: capital utilization adjusting cost	Beta	0.50	0.15	0.050	0.046	0.053	0.163	0.153	0.171
Φ : steady state price mark-up	Normal	1.25	0.12	1.575	1.504	1.595	1.199	1.152	1.246
r_π: policy rule inflation	Normal	1.50	0.25	1.854	1.762	1.888	2.373	2.291	2.394
ρ_r: policy rule smoothing	Beta	0.75	0.10	0.835	0.819	0.849	0.834	0.808	0.845
r_y: policy rule output gap	Normal	0.12	0.05	0.0823	0.075	0.092	0.080	0.068	0.089
$r_{\Delta y}$: policy rule output gap growth	Normal	0.12	0.05	0.040	0.035	0.053	0.075	0.068	0.090
r_{sp}: policy rule term spread	Normal	0.12	0.05	0.155	0.129	0.174	0.112	0.086	0.145
π: steady-state inflation	Gamma	0.62	0.10	0.839	0.826	0.857	0.819	0.809	0.841
$100(\beta^{-1}-1)$: steady-state rate of disc.	Gamma	0.25	0.10	1.240	1.219	1.263	1.468	1.454	1.476
l: steady-state labor	Normal	0.00	2.00	9.410	9.269	9.585	9.954	9.954	10.102
$\overline{r}^{\{4\}}$: steady-state 1-year yield	Normal	0.00	2.00	2.096	2.070	2.138	3.037	3.037	3.310
γ: one plus st-state rate of output growth	Normal	0.40	0.10	0.481	0.477	0.487	0.445	0.440	0.454
α: capital share	Normal	0.30	0.05	0.192	0.166	0.208	0.138	0.115	0.160
ρ: learning parameter	Beta	0.50	0.28	0.961	0.956	0.964	0.973	0.969	0.975

Table A.2.B: Priors and estimated posteriors of the structural shock process parameters using SPF in the set of observable

	Priors			Posterior					
				SlW-TS model			SlW model		
	Distr	Mean	Std D.	Mean	5%	95%	Mean	5%	95%
σ_a: Std. dev. productivity innovation	Invgamma	0.10	2.00	0.429	0.405	0.448	0.592	0.579	0.611
σ_b: Std. dev. risk premium innovation	Invgamma	0.10	2.00	0.276	0.253	0.287	0.557	0.511	0.603
σ_g: Std. dev. exogenous spending innovation	Invgamma	0.10	2.00	0.480	0.454	0.513	0.568	0.523	0.564
σ_i: Std. dev. investment innovation	Invgamma	0.10	2.00	0.736	0.708	0.741	0.607	0.568	0.636
σ_r: Std. dev. monetary policy innovation	Invgamma	0.10	2.00	0.110	0.093	0.116	0.130	0.120	0.136
σ_p: Std. dev. price mark-up innovation	Invgamma	0.10	2.00	0.143	0.128	0.153	0.136	0.127	0.143
σ_w: Std. dev. wage mark-up innovation	Invgamma	0.10	2.00	0.880	0.859	0.933	0.784	0.762	0.785
$\sigma_\eta^{\{4\}}$: Std. dev. 1-year yield innovation	Invgamma	0.10	2.00	0.116	0.106	0.130	0.128	0.120	0.146
ρ_a: Autoregressive coef. productivity shock	Beta	0.50	0.20	0.884	0.872	0.896	0.834	0.804	0.853
ρ_b: Autoregressive coef. risk-premium shock	Beta	0.50	0.20	0.408	0.388	0.443	0.665	0.657	0.678
ρ_g: Autoregressive coef. exog. spending shock	Beta	0.50	0.20	0.990	0.987	0.994	0.992	0.990	0.993
ρ_i: Autoregressive coef. investment shock	Beta	0.50	0.20	0.845	0.841	0.855	0.541	0.521	0.545
ρ_r: Autoregressive coef. monetary policy shock	Beta	0.50	0.20	0.385	0.365	0.414	0.415	0.383	0.452
ρ_p: Autoregressive coef. price markup shock	Beta	0.50	0.20	0.575	0.558	0.602	0.609	0.584	0.690
ρ_w: Autoregressive coef. wage markup shock	Beta	0.50	0.20	0.938	0.929	0.950	0.843	0.833	0.858
ρ_{tp}: Autoregressive coef. 1-yr term premium shock	Beta	0.50	0.20	0.970	0.959	0.979	0.916	0.885	0.950
μ_p: MA coef. price markup shock	Beta	0.50	0.20	0.744	0.741	0.769	0.590	0.547	0.638
μ_w: MA coef. wage markup shock	Beta	0.50	0.20	0.450	0.422	0.487	0.556	0.543	0.569
ρ_{ga}: Interact. betw. product. and spending shocks	Beta	0.50	0.25	0.661	0.627	0.695	0.661	0.601	0.721

Table A.2.C: Priors and estimated posteriors of parameters characterizing the measurement errors of SPF observables

	Priors			Posterior					
				SlW-TS model			SlW model		
	Distr	Mean	Std D.	Mean	5%	95%	Mean	5%	95%
π_{SPF}: SPF steady-state inflation	Gamma	0.62	0.10	0.823	0.811	0.838	0.773	0.767	0.789
$\bar{r}_{SPF}$: SPF steady-state nominal interest rate	Normal	0.00	0.10	2.430	2.414	2.445	2.550	2.520	2.580
$\gamma_{c,SPF}$: one plus SPF st-state rate of consp. growth	Normal	0.40	0.10	0.673	0.662	0.682	0.643	0.628	0.667
$\gamma_{i,SPF}$: one plus SPF st-state rate of inves. growth	Normal	0.40	0.10	0.374	0.329	0.402	0.826	0.798	0.850
$\sigma_{\epsilon,\pi}$: Std. dev. of inflation forecast error	Invgamma	0.10	2.00	0.169	0.164	0.180	0.171	0.169	0.175
$\sigma_{\epsilon,c}$: Std. dev. of consp. growth forecast error	Invgamma	0.10	2.00	0.213	0.205	0.221	0.621	0.569	0.645
$\sigma_{\epsilon,i}$: Std. dev. of inves. growth forecast error	Invgamma	0.10	2.00	1.084	1.064	1.129	1.057	1.042	1.094
$\sigma_{\epsilon,r}^{\{1\}}$: Std. dev. of 1-p-a int. rate forecast error	Invgamma	0.10	2.00	0.098	0.086	0.102	0.097	0.093	0.101
$\sigma_{\epsilon,r}^{\{2\}}$: Std. dev. of 2-p-a int. rate forecast error	Invgamma	0.10	2.00	0.545	0.526	0.578	0.358	0.341	0.365
$\sigma_{\epsilon,r}^{\{3\}}$: Std. dev. of 3-p-a int. rate forecast error	Invgamma	0.10	2.00	0.597	0.497	0.614	1.193	1.159	1.243

Appendix Chapter 3

Supplementary appendix (Not intended for publication)

List of new equations in the extended model:

1) Equations derived with a new structure in the error term

-Euler equation

$$c_t = c_1\left[1+\delta_c(1+b_c)^{-1}\right]c^r_{t-1,t} + (1-c_1)E_t c_{t+1} + c_2\left(l_t - E_t l_{t+1}\right) - c_3(R_t - E_t\pi_{t+1} + \varepsilon^b_t) +$$

$$c_4\left[\varepsilon^c_{t-1,t+S-1} + (\delta_c/\rho_c)\varepsilon^c_{t-2,t+S-2}\right],$$

where:

$$c_1 = \tfrac{h/\gamma}{1+(h/\gamma)(1+b_{cc})^{-1}}, c_2 = \tfrac{(\sigma_c-1)wL/(\phi_w C)}{\sigma_c(1+(h/\gamma)(1+b_{cc})^{-1})}, c_3 = \tfrac{1-h/\gamma}{\sigma_c(1+(h/\gamma)(1+b_{cc})^{-1})},$$

$$and\ c_4 = \tfrac{(h/\gamma)\rho^S_{cr}}{(1+b_{cc})(1+(h/\gamma)(1+b_{cc})^{-1})}.$$

-Monetary policy rule

$$R_t = \rho R_{t-1} + (1-\rho)\Bigg\{ r_\pi\left[(1+b_\pi)\pi^r_{t-1,t} + \rho^{S-1}_\pi \varepsilon^\pi_{t-2,t+S-2}\right] +$$

$$r_y\left[(1+b_y)y^r_{t-1,t} + b_y\delta_y y^r_{t-2,t-1} + \rho^{S-1}_y\left(\varepsilon^y_{t-2,t+S-2} + (\delta_y/\rho_y)\varepsilon^y_{t-3,t+S-3}\right)\right] - y^p_{t-1}\Bigg\} +$$

$$r_{\triangle y}\Bigg\{(1+b_y)\left[y^r_{t-1,t} - y^r_{t-2,t-1}\right] + b_y\delta_y\left[y^r_{t-2,t-1} - y^r_{t-3,t-2}\right] +$$

$$\rho^{S-1}_y\left[\varepsilon^y_{t-2,t+S-2} - (1/\rho_y)\varepsilon^y_{t-3,t+S-3}\right] + \rho^{S-2}_y\delta_y\left[\varepsilon^y_{t-3,t+S-3} - (1/\rho_y)\varepsilon^y_{t-4,t+S-4}\right] +$$

$$(y^p_{t-1} - y^p_{t-2}) + \varepsilon^R_t\Bigg\}.$$

2) Remaining equations, as in Casares and Vazquez (2012)

-NKPC

$$\pi_t = \frac{\iota_p}{1+\overline{\beta}\iota_p B}\pi^r_{t-1,t} + \frac{\overline{\beta}}{1+\overline{\beta}\iota_p B}E_t\pi_{t+1} - \left[\frac{A\left(1-\overline{\beta}\xi_p\right)(1-\xi_p)}{\left(1+\overline{\beta}\iota_p B\right)\xi_p}\right]\mu^p_t + \frac{1+\overline{\beta}\iota_p}{1+\overline{\beta}\iota_p B}\varepsilon^p_t + \frac{\overline{\beta}\iota_p B}{1+\overline{\beta}\iota_p B}\rho^S_\pi\varepsilon^\pi_{t-S,t}.$$

$$with\, \overline{\beta} = \beta\gamma^{(1-\sigma_c)}$$

-Wage dynamics

$$\begin{aligned} w_t &= w_1 w_{t-1} + (1-w_1)\left(E_t w_{t+1} + E_t\pi_{t+1}\right) - w_1\left(1+\overline{\beta}\iota_w B\right)\pi_t + w_2\pi^r_{t-1,t} - w_3\mu^w_t \\ &\quad + w_1\overline{\beta}\iota_w B\rho^S_\pi\varepsilon^\pi_{t-S,t} + \varepsilon^w_t. \end{aligned}$$

where:

$$w_1 = \frac{1}{1+\overline{\beta}}, w_2 = \frac{\iota_w}{1+\overline{\beta}}, and\, w_3 = \frac{1}{1+\overline{\beta}}\left[\frac{\left(1-\overline{\beta}\xi_w\right)(1-\xi_w)}{\xi_w\left((\phi_w-1)\varepsilon_w+1\right)}\right].$$

-Wage mark-up equation

$$\mu^w_t = w_t - mrs_t = w_t - \left(\sigma_l l_t + \frac{1}{1-h/\gamma}\left(c_t - (h/\gamma)\, c^r_{t-1,t}\right)\right).$$

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List of parameters A.1. Model parameter description

γ	gamma1	Steady-State growth rate
δ	delta	Capital depreciation rate
g_y	gy	Steady-state exogenous spending-output ratio
σ_w	phiw	Steady-state labor mark-up
ϵ_p	epsilonp	Curvature of the Kimball labor good aggregator
ϵ_w	epsilonw	Curvature of the Kimball labor market aggregater
φ	varphi	Steady-state elasticity of the capital adjusment function
h	lambda1	Habit formation paramater
σ_c	sigmac	Inv. Elasticity of the intertemporal substitution between leisure and work
σ_l	sigmal	Inv. Elasticity of labor supply with respect to real wages
ξ_p	xip	Calvo probability in prices
ξ_w	xiw	Calvo probability in wages
ι_w	iotaw	wage indexation coefficient
ι_p	iotap	price indexation coefficient
ψ	psi	Elasticity of capital utilization
Φ	phip	Level of fixed cost (1+level)
r_π	rhopi	Inflation coefficient in the MPR
ρ	rho	Smoothing paramater in the MPR
r_Y	rhoy	Output gap coefficient in the MPR
π	constpi	Constant inflation coefficient
$100(\beta^{-1}-1)$	beta1	Personal discount factor
l	constl	Constant labor coefficient
$100(\gamma-1)$	gamma	Steady-state growth rate
α	alpha	Capital share

List of parameters A.2 Shock parameters

ε_t^a	epsa	Technology shock
ε_t^b	epsb	Risk premium shock
ε_t^g	epsg	Expenditure shock
ε_t^i	epsi	Investment adjustment shock
ε_t^R	epsr	Monetary policy shock
ε_t^w	epsw	Wage mark-up shock
ε_t^p	epsp	Price mark-up shock
$\varepsilon_{t,t+S}^y$	eyr	Output revision shock
$\varepsilon_{t,t+S}^\pi$	epir	Inflation revision shock
$\varepsilon_{t,t+S}^c$	ecr	Consumption revision shock
σ_a	stderr e_a	Standard deviation of productivity innovation
σ_b	stderr e_b	Standard deviation of risk premium innovation
σ_g	stderr e_g	Standard deviation of exogenous spending innovation
σ_i	stderr e_i	Standard deviation of investment-specific innovation
σ_R	stderr e_r	Standard deviation of monetary policy rule innovation
σ_p	stderr e_p	Standard deviation of price mark-up innovation
σ_w	stderr e_w	Standard deviation of wage mark-up innovation
σ_y^r	stderr e_yr	Standard deviation of output revision innovation
σ_π^r	stderr e_pir	Standard deviation of inflation revision innovation
σ_c^r	stderr e_cr	Standard deviation of consumption revision innovation
ρ_a	rhoa	AR coefficient of productivity shock
ρ_b	rhob	AR coefficient of risk premium shock
ρ_g	rhog	AR coefficient of exogenous spending shock
ρ_i	rhoi	AR coefficient of investment-specific shock
ρ_R	rhor	AR coefficient of policy rule shock
ρ_p	rhop	AR coefficient of price mark-up shock
ρ_w	rhow	AR coefficient of wage mark-up shock
μ_p	cmap	MA coefficient of price mark-up shock
μ_w	cmaw	MA coefficient of wage mark-up shock

List of variables B.1. Endogenous variables

y_t	y	Output
c_t	c	Consumption
i_t	i	Investment
z_t	z	Capital utilization rate
l_t	l	Employment level
R_t	r	Interest rate
π_t	pi	Inflation
q_t	q	Capital value
r_t^k	rk	Rental rate of capital
k_t^s	ks	Capital supply
k_t	k	Capital
μ_t^w	muw	Wage mark-up
μ_t^p	mup	Prices mark-up
w_t	w	Wages
y_t^r	yr	Real-time output
π_t^r	pir	Real-time inflation
c_t^r	cr	Real-time consumption
r_t^y	ry	Output revision
r_t^c	rc	Consumption revision
r_t^π	rpi	Inflation revision
y_t^p	yp	Potential Output
i_t^p	ip	Potential interest rate
z_t^p	zp	Potential capital utilization rate
l_t^p	lp	Potential employment
R_t^p	rp	Potential interest rate
π_t^p	pip	Potential inflation
q_t^p	qp	Potential capital value
$r_t^{k,p}$	rkp	Potential capital interest rate
$k_t^{s,p}$	ksp	Potential capital supply

List of variables B.3 Predeterminated variables

c_{t-1}, i_{t-1}, k_{t-1}, π_{t-1}, w_{t-1}, R_{t-1}, y_{t-1}, y^r_{t-1}, π^r_{t-1},
c^r_{t-1}, r^y_{t-1}, r^π_{t-1}, r^c_{t-1}, c^p_{t-1}, i^p_{t-1}, k^p_{t-1}, r^p_{t-1}