Asset pricing: analysis of multi-factor models using Fama and French factors

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30th July 2021

Master in Economics: Empirical Applications and Policies

University of the Basque Country UPV/EHU

ACKNOWLEDGMENTS

First and foremost, I would like to thank my supervisor, Miguel Ángel Martínez Sedano, for his incredible help and commitment with this paper. He has guided me all the way from the very first day, until the very last step of the Master's thesis. He is not only a talented researcher, but he also is a close and assisting person who I highly admire. His patience and politeness empower him to transmit all his knowledge and experience in such a peaceful and fulfilling way, that I have received all his positive energy towards this area of study. I could never have done this paper without him.

My sincere thanks to Iñaki Erauskin Iurrita, professor of the University of Deusto (San Sebastián), with whom I had the opportunity to work with. Had not I worked with him, for sure I would not have studied a Master's in Economics, and thus, I would have never found my passion and profession. I am incredibly thankful for having the chance of being professor assistant in the University and for his full dedication and commitment to his profession.

Shall I not miss, I would like to express my thankfulness to Jon Mikel Zabala Iturriagagoitia, also professor of the University of Deusto (San Sebastián), for being my Guardian Angel all this way from my very first year of undergraduate. I am deeply grateful for having him in my path. Thank you for every single teaching hour, and above all, for every piece of advice you gave me, that led me to be, today, a professional, willing, and qualified economist and woman.

Last but not least, I would like to thank all the teaching body of the *Master in Economics: Empirical Applications and Policies*, as I consider them highly prepared, well known and qualified professors of the University of the Basque Country. Special thanks to Javier Gardeazabal, who has not only been the director of the Master, but also my tutor during this academic year.

July 2021, Donostia

ABSTRACT

The present paper considers the five factors proposed by Fama and French (1992, 2015), plus the factor proposed by Carhartt (1997): market premium, size, book-to-market ratio, profitability, investment, and momentum. The aim of this thesis is to analyze the behavior of these factors and test the ability to explain cross-sectional variations in the data. Other authors have done the similar analysis with different databases, such as, Asgharian & Hansson (2002) with Swedish data, and Beltratti & Di Tria (2002) with Italian market data, both with the same results as this paper. The data used for this purpose is monthly European market from July 1990 to March 2021: twenty-five portfolios of European returns formed on size and book-to-market value and European based six returns of factors. Using Fama & MacBeth (1973) methodology and OLS regression, results show rarely significant and different from zero coefficients in the cross-sectional analysis for the six factors, even if sporadically some coefficients are positive and different from zero.

Sections I and II serve as an introduction to the topic of Financial Economics and asset pricing models. Section III explains the methodology that will be used for the research and the data to be taken. The procedure step by step and the results are shown in Section IV and Section V serves as a conclusion of the whole paper. Additional information can be found in Section VII, appendix.

Keywords: Fama-French factors, cross-section, asset pricing, Fama-MacBeth, factor model

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1. Introduction

Financial Economics is the study of the behaviour of investors in the intertemporal decision making of their resources, under uncertainty and risk situations. This field also focuses on the organization of financial markets by applying economic theories in models. The main difference with traditional economics relies on the idea of focusing only on capital markets.

The principal fields of study of Financial Economics can be summed up in three. The first one is determining the price of financial assets as well as their risk, by using pricing models of financial assets, derivatives, and efficiency theories. Assets pricing is the most analysed field of Financial Economics and this paper focuses on models of asset pricing. It is important to understand that this economic field focuses only on financial investment, understood as the acquisition of rights over some real investment, like firms (Marín & Rubio, 2010). For instance, an investor acquires rights of a firm in the form of stocks, that gives her/him the right to perceive gaining (or losses), like capital or dividends.

To understand how capital markets work, Rubio and Marin (Marín & Rubio, 2010) explain a processual sequence of cash flows in the market: firms issue financial assets as a financing form of their investments by using capital markets; which are bought by investors in the market and give financial resources to firms, which then repay investors.

Second field of Financial Economics is related to financial intermediation with capital markets, as investors and financial agents interact in the capital markets with each other, but also with financial institutions (regulators, banks and so on). Thirdly, Financial Economics focuses on firms and how they efficiently take optimal decisions of investing, organization, and financing.

The portfolio selection theory is considered the overall field of Financial Economics for investors, viewed as a combination of the previous three fields of study. This theory establishes all the areas of decision making that should be considered in Financial Economics for investors, as the efficient allocation of resources, risk management, and opportunity cost analysis.

As said, this paper will focus on asset pricing techniques and models. Asset pricing models can be structured in two areas, depending on the techniques used: no arbitrage opportunities (APT model) and equilibrium conditions (CAPM model). But before going deeper in these two models, it is important to clear the historical background of Financial Economics and all the literature and academia that has stood up.

Asset pricing is a relatively new field. The contribution of this paper to the academia world will be oriented to more research in the area, being one of the first analytic papers done related to the testing of asset pricing models of Fama and French (1992, 2015) factors, even there are some authors, such as, Asgharian & Hansson (2002) and Beltratti & Di Tria (2002) that have already tested the cross-sectional explanation power of these factors. The combination of different factors in a multi-factor asset pricing model is a relatively new analytic study of which factors have, indeed, an influence on prices in the market. Furthermore, in the changing world we nowadays live, the stability of financial markets is essential for the well-functioning of economies, and therefore, it is of extremely importance to do research on the best asset pricing predicting models, and what is more, to find which are the factors influencing the prices.

The research questions of this paper are basically two, even if they involve more questions to consider. Firstly, whether Fama & French five factors (1992,2015), together with *momentum* factor (Carhartt, 1997) are good estimators of the expected return of assets. That is, whether including more factors in the classical unifactorial model, improve the goodness of fit of the model.

Afterwards, it is essential to question whether those factors are truly risk factors. Because of the risk aversion economic theory, we know that more risk is rewarded with more return. Then, if the factors are, in fact, appropriate risk factors, we should expect that the higher the risk of assets regarding these factors, the higher the expected return of the assets. This is what is called market premium. Hence, the question is whether the factors included in the previous research question are risk factors and imply a market risk premium, as one should expect that the higher the risk assumed in an investment, the higher the return of it, as a consequence of the market risk premium.

Throughout this paper, and while answering the research questions, the reader will come across descriptive statistics of the risk factors -explanatory variables- with the idea of exploring their behaviour through time and making some thoughts out of it.

2. Theoretical background

Since Financial Economics is a relatively new field in Economics, its basis was established just one century ago. Before the efficient market theory, portfolio selection theory, and even risk management methods; Louis Bachelier is considered the first footprint of this brand-new field. His PhD paper about the speculation theory in 1900 opened the door to the scientific research of Financial Economics, even if it was not until some decades after, that his work was recognized. The next big step forward in Financial Economics has been the portfolio theory development in the middle of the XX century.

2.1. Modern Portfolio Theory

Markowitz (1952) uses geometric algebra to explain the portfolio selection process, to later discuss the efficient portfolio selection. Using the mean-variance hypothesis, he stated that this maxim implied diversification, as investors should choose a portfolio with a high number of individual assets but of different sectors. That is, Markowitz (1952) argued that investors should focus on trying to avoid high covariances among individual assets, as it will be the way for making the variance of the portfolio small.

The expected return on a portfolio is a weighted average of the expected returns of the individual assets. The variance is the expected value of the deviations of the returns from the mean. The variance of a portfolio of two assets is formed with the individual variances and the covariance of the two. In general terms, Markowitz (1952) expressed the return and the variance as:

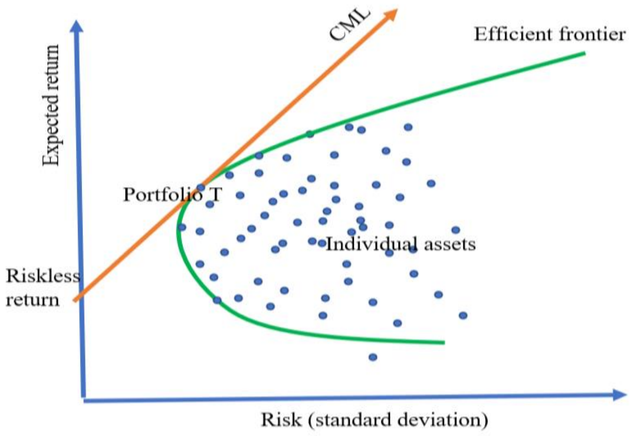
$$R_P = \sum_{i=1}^{N} X_i R_i \qquad and \qquad \sigma_P^2 = \sum_{i=1}^{N} \sum_{j=1}^{N} \sigma_{ij} X_i X_j \tag{1}$$

This paper, Markowitz (1952), is what in the field of Financial Economics is called the basis of the Modern Portfolio Theory, of which its basic idea is that investors should choose portfolios with minimum risk given the expected returns, or conversely, maximum expected return given a level of risk. This opened the door to diversification of portfolios, using a combination of risky and riskless assets in the portfolio, and thus, creating efficient portfolios, showing that diversified portfolios are less volatile (standard deviation) than individual portfolios.

Markowitz (1952) referred to the efficient frontier as the set of portfolios with the highest expected return levels for the lowest risk level possible, creating optimal return maximizing portfolios. Following the mean-variance analysis, and under the assumption that we allow short selling and riskless assets combination, we should draw an upward sloping line of the riskless assets and find the tangency point with the efficient frontier, creating the minimum variance portfolio line, or likewise, the efficient frontier with risk free assets.

Depending on the risk aversion of each individual investor, they will individually allocate their portfolio investment in the straight line starting in the vertical axis from the riskless asset, connecting with the tangency point of the frontier, creating the capital market line (CML). Portfolio T is the optimal combination of risky assets, and the CML line is the combination of risky and riskless assets that are the most efficient given the mean-variance assumptions and the risk aversion of investors. Figure 1 clearly shows how the combination of risky and riskless assets creates a more efficient portfolio selection line than the previous efficient frontier (with only risky assets).

Figure 1: efficient frontier



Having said this, the expected return and the risk can be easily replicable by using a combined portfolio formed only by two individual assets, the risk free asset and the market portfolio, that include all the existing assets in the market as a weighted sum of their market value. In the Figure 1 presented above, the market portfolio is identified as the portfolio T. Using the market portfolio enables the investor to diversify the risk due to the idea of Markowitz (1952) that the combination of all individual assets is less risky than individual assets by themselves.

It is important to distinguish between systematic and no-systematic risk. The unsystematic risk can be diversified by combining individual assets, as it is assumed this will eliminate the idiosyncratic risk of individual assets. However, the systematic risk (or market risk) cannot be eliminated by diversification of the portfolio as it affects all individual assets. We will return to the determination of risk later in the paper.

2.2. Capital Asset Pricing Model

As Markowitz (1952) established the basis of the portfolio analysis, many economists continued the study of the capital asset pricing model: Treynor(1962), Sharpe (1964), Lintner (1965), Mossin (1965), Fama (1968), Modigliani and Miller (1958), or Black (1972), among others. It is to say, Capital Asset Pricing Model and sub models are based on the equilibrium condition. That is, in the analysis of the efficient portfolio selection, Markowitz (1952), demand of portfolio assets must equal supply of portfolio assets in the capital market. This implies that for market clearing conditions to occur, the efficient portfolio (T portfolio) must be the market portfolio, as mentioned before.

The most basic equilibrium condition model is the standard capital asset pricing model (CAPM), also referred to as the one-factor capital asset pricing model. According to Elton and Gruber, after years of CAPM analysis and according to the assumptions in the paper of Sharpe (1963), they wrapped up the assumptions of the model in their book in the following way (Elton & Gruber, 1995*)*:

1. There are no transaction costs, that is, no friction costs when buying/selling assets. This assumption is of minor importance, given that the transaction costs are, in general terms, of small size.
2. Financial assets are infinitely divisible. Investors can, thus, take any position in their investments.
3. There is absence of personal individual income tax, which means that the investor is indifferent between getting gains in dividend form or in capital form when the return on investment is received. This assumption holds given that if income tax and capital gains taxes are equal size.
4. Investors cannot affect the price of an asset by her/his actions individually. Thus, perfect competition assumption holds, as it is investors in total (N) that determine prices, and individual actions do not affect prices.
5. Investors take their investment decision based on expected returns and standard deviations on their portfolios, relying on mean-variance analysis.
6. Unlimited short sales are allowed, which involves taking a negative position in a security (selling a security investor does not own). For this to happen, we need the brokerage firm to borrow the security or to directly lend it to the investor.
7. There is also unlimited lending and borrowing opportunities at riskless rate, any amount of funds can be lent or borrowed at the interest rate equal to the risk-free securities' interest rates.
8. There is homogeneity of expectations in the sense that all investors are concerned with the mean and variance of the returns. We also find homogeneity of expectations because all investors have the same information and expectations with respect to the inputs of the portfolio decision. This is an important assumption as investors are considered rational actors of the market and are expected to make the same decision given the situation of the market and do not focus on anything else but mean and variance.
9. Lastly, all assets are marketable, including human capital, that can be sold and bought in the market.

These assumptions are considered to be a "*too much*" simplification of the real world. However, the CAPM model has been widely used among investors for its simplicity and its straightforward relation with the efficient portfolio selection Markowitz (1952). It is important to understand the connection between the asset pricing models and the portfolio theory presented by Markowitz (1952). Consider that the most used asset pricing valuation tool has been the replication portfolio of the future cash flows of the portfolio we are analysing, considering that, without arbitrage, both our portfolio and the replicated portfolio must have the same value (Marín & Rubio, 2010).

For this to happen, it is essential that investors focus on the mean variance analysis (assumption nº5), which implies that the distribution of the probability of the returns of the assets is normal, or similarly, that investors are risk averse.

Under a scenario of uncertainty, it is essential to understand the interrelation between the replication portfolio and diversification (Marín & Rubio, 2010). CAPM model uses replication portfolio, as said, as a combination of riskless assets and the market portfolio, which consequently erases unsystematic risk because of the diversification effect of the market portfolio. From the theory of Markowitz (1952), we know that the market portfolio is efficient (as equilibrium condition holds) and therefore, it is in the minimum variance portfolio line (MVPL).

The standard CAPM model finds a positive and lineal relationship between the return of an asset and its beta, understood as the covariance between the asset and the market portfolio, which is, as said, the tangency and efficient portfolio. Beta coefficient will, thus, be understood as the contribution of our portfolio assets to the risk of the market portfolio. Many economists worked on this model, but the most remarkable ones are Sharpe (1965) and Lintner(1965) in their contribution to a market portfolio as a tangency point portfolio when there is a riskless / secure asset; and Black(1973) for his contribution to the zero-beta CAPM model considering there is not a riskless asset.

Sharpe and Lintner considered a CAPM static model, taking the supply of financial assets as given. They also assumed risk free assets can be unlimitedly lent and borrowed by investors, and investors make their decisions based on the expected return and the variance with homogeneous expectations (Sharpe, 1963). In short, they followed the same assumptions as the standard CAPM model we mentioned before. As we have seen, the non-systematic risk can be diversified through the creation of portfolios rather than doing individual assets analysis, but that other inherited risk is what Sharpe (1963) modelled and created a "market equilibrium theory of asset prices" (Sharpe, 1963). In his papers, he presented the investment opportunity curve by combining the utility of investors expressed in expected returns, and the set of investment opportunities. In the end, in equilibrium, there exists a linear relationship between the expected return and the standard deviation, matching with what Markowitz (1952) stated, but now referring to it as systematic risk. The remaining risk is not correlated with the portfolio return and that is why it is called systematic risk. The interpretation Sharpe (1963) offers is that portfolios that are more responsive to changes in the market portfolio return, that is, with a higher value of beta, will have higher expected return than those with lower value of betas.

$$E(R_j) = r + \beta_{mj}[E(R_m) - r] + \varepsilon_j, \tag{2}$$

Where $E(R_j)$ is the expected return of asset j, r is the return of the risk free asset, $E(R_m)$ is the expected return of the market portfolio, and β_{mj} is the sensitivity of asset j to changes in the market risk premium $[E(R_m) - r]$. Previous equation wraps up all the theory of Sharpe (1963): the expected return of a portfolio is a combination of the risk-free asset return and the expected return of risky and uncertain assets. These risky assets are calculated by using risk premium theory, the difference between the expected return of the market portfolio and the risk-free asset, which are considered to be for its own merits of the portfolio. Therefore, beta coefficient represents the covariance between the market portfolio and our portfolio, which describes the systematic risk that cannot be diversified. The higher the beta coefficient, the higher the sensitivity of our portfolio to variations in the market portfolio

In any version of the CAPM model, beta coefficient represents the unifactorial risk of the portfolio with respect to the market portfolio. Because of that, it is assumed that beta coefficient equal to one means that the portfolio behaves exactly the same way as the market portfolio does. Beta coefficients higher than one are considered aggressive procyclical betas, as the portfolio is more sensitive to changes in the market than the market portfolio itself: if the returns of the market portfolio increase by one unit, the returns of the portfolio will increase more than one unit. Conversely, beta coefficients lower than one will indicate defensive betas: if the market portfolio increases by one unit, our portfolio will increase less than a unit.

Market model as the return generating process

In the CAPM model it is assumed that agents only accept systematic risk as specific risk disappears with diversification of portfolio, because they consider the market portfolio as the optimal. Thus, the CAPM model assumes individual risk is completely diversified with the creation of an optimal portfolio, considering there are no other individual factors affecting assets.

A factor model with only one factor is what CAPM model considers a market model. This unifactorial return generating process assumes market portfolio is the unique aggregated risk factor, which is considered a strong assumption. Statistically, the unifactorial market model has bivariate normal distribution, thus, the error term and the dependent variable (*Rm*) are independent. In short, this model expresses the return of a particular asset by the inherent component of the asset and the market component. (Marín & Rubio, 2010).

Moreover, it is essential for the factor model that the returns of individual assets are not correlated with each other. That is, $E\ (\varepsilon_j, \varepsilon_h) = 0$, so that the principle of diversification is satisfied. This assumption, which must be true, implies that the covariance between the return of the portfolio (consisting of diversified individual assets) and the return on the market portfolio should tend to zero in the limit. The higher the number of individual assets in the portfolio, the lower the idiosyncratic risk, expressed as the standard deviation, is; due to the fact that the error terms of individual assets are not correlated, which makes the idiosyncratic risk disappear in a diversified portfolio.

2.3. APT model

Arbitrage Pricing Theory (APT) model, Ross (1976) is an asset pricing model under the assumption of no arbitrage, a model that imposes a given behaviour in the return-generating process. We should first discuss what is arbitrage and what it means not to allow arbitrage conditions.

Arbitrage is an investment strategy that allows the investor to earn money without needing to invest any quantity in the present time and it neither requires a future payment. Arbitrage opportunity consists of buying a cheap asset and selling it more expensive in the market, for instance, arbitrage happens when two markets sell the same identical asset at different prices. (Marín & Rubio, 2010). Thus, it is considered arbitrage because it ends up creating an equilibrium condition of prices in both markets. It is based on the law of one price (Elton & Gruber, 1995), that states that two assets with the same characteristics regarding risk and return, cannot be sold at different prices.

To make arbitrage possible, it is essential to have two portfolios/assets mutually replicable, creating equivalent and replicable assets. Assuming there are no transaction costs, i.e. there are no frictions in the market, if there are two mutually replicable investments but are sold at different prices, we would conclude there is an arbitrage opportunity which will consist of buying the cheaper investment and short selling the expensive one. It is essential to be a short selling action as it is based on a speculative decline in the price of an asset and does not imply having the ownership of it.

In short, the underlying assumptions of the model are: (1) the returns of assets are generated by using a factor model of K risk factors, (2) there are no arbitrage opportunities, (3) markets are competitive and there are no frictions or transaction costs, and (4) because of diversification, there is no idiosyncratic risk. Therefore, we find a factor model of return-generating process without idiosyncratic risk, in which we use portfolios that replicate those K factors.

In all asset pricing models no arbitrage opportunities are considered, or more generally, these opportunities do not exist systematically in the market, assuming arbitrage would not last if it existed (Marín & Rubio, 2010).

APT model also establishes a lineal and positive relationship between the yield and the risk. However, APT does not explicitly state which are the K risk factors, even if we consider risk is measured using covariance. Anyhow, the APT model does not rely on equilibrium conditions, and, therefore, we cannot guarantee market clears (demand and supply equal in equilibrium). Therefore, it cannot be ensured that idiosyncratic risk disappears, and it is not important, so, the idea that idiosyncratic risk is equal to zero in the factor model is assumed.

Going back to the return generating process, APT considers more than one factor, and thus, it is referred to it as a multifactorial return generating process. The observed yield is composed of two parts: the previously expected yield and the innovation component. In turn, this innovation can be decomposed in two. In the first part, the entrance of new economic information about markets (inflation shocks, changes in interest rates…), which is called systemic innovation as it affects all agents/firms. It is true, however, that it does not equally affect all companies in the market, for instance, a shock in the demand of petroleum will affect much more Repsol than Viscofan. This innovation component cannot be eliminated by diversification, as it is considered as macroeconomic changes/innovations.

The second part of the innovation component is the so-called idiosyncratic risk, an inherent component of individual assets, that affects individually and exclusively each asset. This component can be eliminated by diversification. Thus, the factor model, or the return generating process can be expressed as:

$$E(R_j) = \alpha_j + \beta_{j1}F_1 + \beta_{j2}F_2 + \dots + \beta_{jK}F_K + \varepsilon_j, \tag{3}$$

Where ε_j represents the idiosyncratic risk, which is not diversifiable, F represents the risk factors affecting the expected return on the asset $E(R_j)$. The beta coefficients express the sensitivity to the risk factors, which are the covariances between the returns of the portfolio and the systematic risk factors. Remember that with this covariance measures the reaction of the returns of a portfolio to unexpected variations of a systematic risk factor, that of course, cannot be diversified.

Therefore, the return generating process of K risk factors is expressed as the expected component and the innovation component, systematic or idiosyncratic. Then, if the portfolio is well-diversified, the idiosyncratic risk will disappear as the covariance between the error term and the aggregate variable of systematic innovation, that is, the risk factors, is zero, ($cov(F, error) = 0$). This is the strong assumption also needed in the APT to get the exact model.

Which are the risk factors is a question yet not answered in the Financial Economics, but certainly, there is common consensus on some. For instance, (i) growth rate of industrial production index, (ii) changes in unexpected inflation rate, (iii) difference between long run and short run interest rates, (iv) yield of market portfolio, and so on. Many economists worked on this new model, considering different risk factors: Chen, Roll and Ross (1986) considered only macroeconomic factors, Connor & Korajzcyk (1986,1988&1991) did a statistic approximation, and Fama & French (1992) created new risk factors that affected the yield on portfolios. We will come back to the papers of Fama & French and their APT model proposition later.

Before going on with the study, it is important to highlight, firstly, why the CAPM model became obsolete and the APT came as a solution, theoretically and statistically speaking, and which are the main differences between the two models.

CAPM vs. APT

Recent past years have demonstrated the new principles of Financial Economics that can be wrapped up in the following ideas (Marín & Rubio, 2010). Firstly, returns of financial assets cannot be explained by exclusively one factor, and thus the CAPM model is obsolete. Multi-factor models appeared as a solution to the standard CAPM model, as they are more precise in the association of diverse risk factors that affect returns. Then, the yield of assets will depend on more risk factors than just the covariance between its returns and the return of the market portfolio.

Another questionable hypothesis of the CAPM model, that is widely spread in the financial world, is the positive relationship between risk and the ex-ante return, as the expected returns and the risk cannot be directly observed. The hypothesis is that the expectations on the returns are correct, and thus, the expected return equals the true return. Assuming this hypothesis is satisfied, then agents can contrast the model with ex post data.

Furthermore, the CAPM model is a static model in one unique period, which implies stationarity on the risk premium and beta coefficients. Last main problem of the CAPM is the impossibility of observing and checking the true market portfolio, which disables agents to check on its efficiency.

All these problems make the CAPM model quite difficult to test. As the CAPM model assumes a replicate portfolio for the market portfolio, testing CAPM implies testing whether the true market portfolio is efficient, based on its assumptions of the efficiency of the market portfolio in the mean-variance model.

Setting problems aside, the main and principal difference between CAPM and APT models is the idea of market equilibrium conditions that are held in the CAPM model, but not necessarily (even if assumed) in the APT model. In fact, APT models do not require the identification of the market portfolio, but this implies the APT to be an approximation of the expect return, which makes the APT model more difficult to test in practice, as the risk factors are not identified.

2.4. Fama & French APT model

Until now, the two main models in the asset pricing field of Financial Economics have been presented, and how the unifactorial models are not sufficient to estimate the returns on assets as we should consider more risk factors in the models. The APT model is proposed based on the absence of arbitrage condition and multi factor return generating process, where economists proposed some factors that should be included.

The father of the APT model, Steve Ross, proposed macroeconomic factors as risk factors that influenced the return on assets. Alternatively, Fama & French (1993) suggested an APT model with three risk factors that could be replicable by creating portfolios of the real-world economy (Marín & Rubio, 2010). They proposed three factors: the market premium, size of firms and book-to-market ratio, and they created replicated portfolios for each of the factors. Thus, these factors are the return of portfolios which mimic size and book-to-market ratio characteristics.

The factor of the market premium is relatively simple as it is the factor associated with the CAPM model. Fama & French (1992) replicated market premium using zero cost portfolio formed by long positions in market portfolio and short position in risk free assets. For the creation of the other factors, Fama & French (1992) created six portfolios based on the size and book-to-market equity ratio to try to mimic the underlying risk factors related to them. The combination of those portfolios allows for the creation of factors replicating size and book-to-market ratio.

In the end, Fama & French (1992) aimed to create portfolios built based on several factors, like size and book-to-market, to then estimate the following regression:

$$R_{jt} = \alpha_j + \beta_{jm}(R_{mt} - r_t) + \beta_{jsmb}SMB_t + \beta_{jhml}HML_t + \varepsilon_{jt}\ ;\ t = 1,\ldots,T. \quad (4)$$

Where ε_j represents the idiosyncratic risk, which is not diversifiable. *SMB* is the factor mimicking size (small minus big), *HML* is the factor mimicking the book-to-market ratio (high minus low), $R_{mt} - r_t$ represents the market risk premium, and the betas are the coefficients of the sensitivities of the return of asset j in time t, to the factors. Fama & French (1992) showed that the R^2 value increases around 20% when including more risk factors in the regression, and thus, they conclude there are more factors than the market portfolio that explain variations in the returns on assets.

However, they fail to continue with the cross-sectional analysis. Other economists (Jagannathan & Wang, 1996) have shown that Fama & French (1992) factors are not valid for explaining the risk premium of the return of assets, even if these factors are good estimates of the return. Hence, the betas of the respective risk factors are not able to explain, in a cross-sectional analysis, the return on assets, as beta coefficients are not sensitive to the true systematic risk factors (Marín & Rubio, 2010).

Five-factor asset pricing model of Fama & French (2015)

After years of discussion and analysis in the academia, Fama & French defended the three factors model was not enough for predicting the estimated return because the factors missed a lot of variation. Because of that, they included profitability (*RMW*) and investment (*CMA*) as two new factors that could explain the expected returns on assets.

$$R_{jt} = \alpha_j + \beta_{jm}(R_{mt} - r_f) + \beta_{jsmb}SMB_t + \beta_{jhml}HML_t + \beta_{jRMW}RMW_t + \beta_{jCMA}CMA_t + \varepsilon_{jt} \; ; \; j = 1,2,\dots,N \; and \; t = 1,2,\dots,T \tag{5}$$

Where ε_j represents the idiosyncratic risk. SMB is the factor mimicking size (small minus big), HML is the factor mimicking the book-to-market ratio (high minus low), $R_{mt} - r_t$ represents the market risk premium. RMW is the factor mimicking the profitability (robust minus weak), CMA is the factor mimicking the investment (conservative minus aggressive), and the betas are the coefficients of the sensitivities of the return of asset j in time t, relative to the factors. This approach of Fama & French will be used for this study, and thus, it will be deeper analysed in the chapter referring to the methodology.

$$R_{jt} = \gamma_{0t} + \gamma_{mt}\hat{\beta}_{jmt} + \gamma_{SMBt}\hat{\beta}_{jSMBt} + \gamma_{HMLt}\beta_{jHMLt} + \gamma_{RMWt}\hat{\beta}_{jRMWt} + \gamma_{CMAt}\hat{\beta}_{jCMAt} + \gamma_{WMLt}\hat{\beta}_{jWMLt} + \varepsilon_{jt} \tag{6}$$

In the recent years, economists and several agents in Financial Economics have been working, not only on other factors that should be included for the estimation of returns; but also in testing Fama & French model to demonstrate whether their factors are, in fact, risk factors that the market rewards, checking on the consistency of equation 6. For instance, Asgharian & Hansson (2002) showed that beta coefficients (equation 6) are never significantly different form zero in the Swedish stock market between 1980-1990, Beltratti & Di Tria (2002) shown the positive response of the Fama & French model in a time series analysis, but those factors do not work on explaining cross-sectional returns in Italian market, Strong & Xu (1997) concluded that Fama & French factors combination have very low explanatory power for average returns, and Kubote & Takehara (2017) showed that investment and profitability factors were not significant in Japanese market. On the other hand, other authors worked on which other factors should be considered: Roy & Shijin (2018) introduced human capital to the five factor Fama & French model.

<u>Carhartt Momentum factor</u>

Carhartt (1996) went one step further including another risk factor based on the Fama & French (1992) three-factor-model, which he referred to as *momentum*. It is basically the equal-weighted average of those assets of firms with the highest and lowest returns, hence, *Momentum* factor is the difference between the lowest accumulated returns and the highest accumulated returns, considering the size of the firms.

3. Data and methodology

3.1. Construction of Fama & French five factors

Since in the three-factor model, the factors of book-to-market and size were not considering the effect of profitability and investment, Fama & French (2015) constructed again the factors of size (*SMB*), book-to-market (*HML*), profitability (*RMW*) and investment (*CMA*).

Fama & French (2015) five factors are built considering six value weighted portfolios (2x3) of size and of book-to-market value (*S/L, S/M, S/H, B/L, B/M, B/L)*, six value weighted portfolios of size and profitability (*S/R, S/M, S/W, B/R, B/M, B/W)* and six value weighted portfolios of size and investment *(S/C, S/M, S/A, B/C, B/M, B/A)*: 18 value weighted portfolios in total.

What Fama & French (2015) did was to group assets according to size (small and big) and according to book-to-market value (three categories: high, neutral, or low), operating profitability (three categories: robust, neutral or weak) and investment (three categories: conservative, neutral or aggressive) groups. The breakpoints for the division in groups is done considering the median according to NYSE for the size factor, while when dividing in three groups using the 30^{th} and 70^{th} percentiles for the rest of the factors.

Thus, for instance, for constructing the factor of size, Fama & French (2015) first divide all the firms of their dataset in two groups, big and small, according to their market capitalization. At the same time, they classify all firms in three groups (high, medium, low) according to their book-to-market ratio. They follow the same classification of three subcategories considering the profitability and investment. Once with the four classifications, Fama & French combine them to create 18 portfolios based on size and book-to-market, size and profitability, and size and investment. In the end, Fama & French (2015) create the size factor with the difference in returns of the big and small classified firms, taking an average of the three subcategories used. The combination of the other factors enables the elimination of their effect on size. They follow the same rationale with all the factors, mathematically:

- *SMB* (Small Minus Big) is calculated as the difference in average returns of big and small portfolios, weighting size and book-to-market, size and profitability, and size and investment:

$$SMB = 1/3\left(SMB_{B/M} + SMB_{OP} + SMB_{INV}\right)\ where,$$
$$SMB_{B/M} = 1/3\ (Small\ Value + Small\ Neutral + Small\ Growth) - 1/3\ (Big\ Value + Big\ Neutral + Big\ Growth)$$
$$SMB_{OP} = 1/3\ (Small\ Robust + Small\ Neutral + Small\ Weak) - 1/3\ (Big\ Robust + Big\ Neutral + Big\ Weak)$$
$$SMB_{INV} = 1/3\ (Small\ Conservative + Small\ Neutral + Small\ Aggressive) - 1/3\ (Big\ Conservative + Big\ Neutral + Big\ Aggressive) \quad (7)$$

It is expected that smaller firms give higher expected return than the big ones, because of the risk of being more vulnerable and smaller. Hence, when the value in the size factor is big, small companies are expected to have higher returns than big firms, small companies outperform better relative to big firms in the stock market.

- *HML* (High Minus Low) is calculated as difference in average returns between the value portfolios (high) and the growth portfolios (low), considering their size. Value portfolios are those with a high book-to-market ratio:

$$HML = 1/2\ (Small\ High + Big\ High) - 1/2\ (Small\ Low + Big\ Low) \quad (8)$$

The factor associated to the book-to-market value, *HML*, is the difference between the simple average of the returns of the highest book-to-market value (S/H and B/H, also referred to as "value") and the returns of the lowest book-to-market values (S/L and B/L, also referred to as "growth") (Fama & French, 1992). A high value in the book-to-market ratio will indicate a higher risk for investors, as the book value of the firm is much higher than the market value, which means the company is not well reflected in the market. These types of firms are more vulnerable to financial stress situations (crisis, booms…) and, thus, investors require a risk premium, that is why the value of the factor would be positive.

- *RMW* (Robust Minus Weak) is calculated as the difference in average returns between the robust portfolios and the weak portfolios, considering their size:

$$RMW = 1/2\ (Small\ Robust + Big\ Robust) - 1/2\ (Small\ Weak + Big\ Weak) \quad (9)$$

It is expected that companies with higher expected future earnings have higher returns in the stock market, and hence, the value of this factor will be positive. It measures the outperformance of robust companies in the stock market relative to those weaker ones.

- *CMA* (Conservative Minus Aggressive) is calculated as the difference in average returns between the conservative investments and the aggressive investments, considering their size:

$$CMA = 1/2\,(Small\ Conser + Big\ Conser) - 1/2\,(Small\ Aggr + Big\ Aggr) \quad (10)$$

Investment variable is growth level in the book to equity ratio of portfolios. CMA indicates that firms that invest profits internally in the company are expected to experience lower returns in the stock market (Fama & French, 2015): the higher the growth in book-to-equity ratio, the lower the expected returns of portfolios. Fama & French (2015) defined investment factor as the growth of a firm divided by the total assets of that firm. Hence, theoretically, we should expect negative influence on the expected returns of portfolios.

- WML (Momentum factor, winners minus losers): is calculated as the difference in average between the Firms are classified in two groups according to size, and then are subclassified according to their accumulated return/performance in the last eleven months (30% lowest are the *losers*, the 30% highest are the *winners* and in between they are *neutral*). Using the intersections between *size* and *momentum*, Carhartt (1996) gets six portfolios based on: S/L, S/N, S/W, B/L, B/N, and B/W:

$$WML = 1/2\,(Sm.\ Winner + Big\ Winner) - 1/2\,(Sm. Loser + Big\ Loser) \quad (11)$$

For this paper, six factors will be analysed as possible risk factors: market premium, size, book-to-market ratio, profitability, investment, (Fama & French, 2015) and momentum (Carhartt, 1996).

3.2. Database

For this concrete paper, dataset of 25 portfolios will be used, formed on size and book-to-market for developed market, precisely, European market. Therefore, for consistency, the factors are also calculated using the same returns database. The database consists of monthly returns data from July 1990 to March 2021. All the returns are in US dollars, which also include dividends and capital gains, and are not continuously compounded. The portfolios are built considering breakpoints for size and book-to-market value:

The portfolios are built considering breakpoints for size and book-to-market value, Fama & French (2014) divide the firms in five groups according to size (using market capitalization), from small to big. In each of the five groups, they subdivide each one in five according to its book-to-market ratio value, creating intersections of portfolios between size and book-to-market. The combination of those 5x5 subgroups leads to 25 portfolios. Hence, portfolio 21, for instance, will be formed by the biggest firms on size and the lowest capitalization of that group of big firms.

Fama & French constructed portfolios instead of working with individual assets because of the principle of diversification. It is essential that the individual assets are not correlated between them, $E(\varepsilon_j \varepsilon_h) = 0$ and thus, the covariance between the return of two individual assets should only depend on risk factors' behaviour, making the variance of the portfolio tend to zero in the limit (Marín and Rubio, 2010). It is known from Financial Economics theory that the higher the number of assets included in the portfolio, the idiosyncratic risk diminishes, and hence the standard deviations, as the inherent component of assets (disturbances) are not correlated.

From the market model section in this paper, it is concluded that the inherent component can be divided in two: the systematic innovation and the idiosyncratic innovation. The later can be diversified by the combination of individual assets constructing a portfolio. The idea of diminishing the idiosyncratic risk is what guides the creation of portfolios.

Having that said, the European countries included in the dataset are Austria, Belgium, Switzerland, Germany, Denmark, Spain, Finland, France, Great Britain, Ireland, Italy, Netherlands, Norway, Portugal, and Sweden.

3.3. Methodology: Fama & MacBeth (1973) approach with rolling windows

One could test the implications of the CAPM model using cross sectional regression methodology, due to the linear relationship between the returns and the market beta, to test whether it is an appropriate asset pricing model. Fama & MacBeth published in 1973 a new idea of testing the CAPM model, based on the idea of projecting the returns with the betas and then aggregating them in T time periods. The implications of asset pricing models (Fama & MacBeth, 1973) are basically three: (1) the relationship between return and risk is linear, (2) beta is the measure of risk of a certain asset of the portfolio, and (3) higher risk should lead to higher return, assuming a risk aversion situation. The assumptions of their methodology imply a perfect market competition situation, and homogeneous expectations, which in fact imply that the market portfolio is efficient, and correspondence between the ex-ante and ex-post returns. They proposed (Fama & MacBeth, 1973) a stochastic model for returns, as a stochastic generalisation of the

CAPM model, imposing period-by-period analysis of the CAPM model to test the implications (1)-(3).

$$R_{pt} = \gamma_{op} + \gamma_{1t}\beta_p + \xi_{pt} \qquad t = T-1, T-2, \dots, T-60 \; p = 1,2,\dots,P \tag{12}$$

"*which allow coefficients gamma to vary stochastically from period to period"* (Fama & MacBeth, 1973).

They proposed a two-step contrast procedure, in which the first stage consists of estimating the beta coefficient of each portfolio using the market model and OLS regressions for the T periods. That is, using the unifactorial factor model in a time series process, Fama & MacBeth (1973) regress portfolios individually against the factor of the CAPM model, for each period being analysed, obtaining a sequence of estimated betas for each of the portfolios. This could be applied to the APT model by carrying out individual unifactorial regressions for each of the factors considered, which is precisely what will be done in this paper.

On a second stage, Fama & MacBeth (1973) proposed a cross sectional regression for each time observation (monthly) in the data, regressing the returns of portfolios against the previously estimated betas (include more betas if referring to an APT multifactor model), in order to test the time series of the first step. This second step will show if the factor being analysed is, indeed, a risk factor, and therefore, has a risk premium.

$$R_{pt} = \gamma_{0t} + \gamma_{1t}\widehat{\beta_{pt}} + \eta_{pt} \qquad t = 1,2,\dots,T \; and \; p = 1,2,\dots,P \tag{13}$$

Where β_{pt} is the beta coefficient of the portfolio p, which was estimated in the first step for the period t; and η_{pt} is the disturbance error. We expect, from the implications of Fama & MacBeth (1973), that the estimated gamma coefficients of the previous cross-sectional regressions are strictly positive and a gamma constant equal to zero, and we should check them using t-statistics ($\gamma_{0t} = 0 \; and \; \gamma_{1t} > 0$). As we have assumed the returns to follow a normal distribution and be independent and identically distributed, the estimated gammas are expected to be normal and IID as well, with T degrees of freedom. Moreover, to solve for heteroskedasticity problem in the error terms that appear in the one step regression method (OLS), gamma estimates should be obtained by and must be tested using the t-statistics of:

$$t(\widehat{\gamma_i}) = \frac{\widehat{\gamma_i}}{\widehat{\sigma_{\gamma i}}}; i = 1,2 \dots N \tag{14}$$

$$\widehat{\gamma_i} = \frac{1}{T}\sum_{t=1}^{T}\widehat{\gamma}_{it} \; and \quad \widehat{\sigma^2_{\gamma i}} = \frac{1}{T(T-1)}\sum_{t=1}^{T}(\widehat{\gamma_{it}} - \widehat{\gamma_i})^2 \tag{15}$$

Which implies that the variance-covariance matrix of the disturbances in the monthly regression does not influence in the t-statistics. The main econometric problem of Fama & MacBeth (1973) cross-sectional methodology is that the betas are not known, and thus, regressions are done using estimated betas. To solve for the problem of errors in variables (EIV), that any error or deviation in the estimation of any beta in the first step can make the second step regression of the estimation of gamma to be inconsistent, it is essential to use portfolios and not individual assets, (Fama & MacBeth, 1973). Shaken (1992) proposed asymptotic correction of the bias by adjusting the standard error of the estimates, which corrects the EIV bias, but it does not correct the possible entrance of other variables, because betas are unknown (Campbell, Lo & McKinlay, 1997). Newey & West (1987) proposed a covariance matrix estimator to solve for serial correlation and heteroscedasticity and error in terms problems, widely used in time series data regressions.

In this paper, the rolling window method will be applied, using 60 observations per window (60 months, thus, 5 years). Hence, having 6 factors, 25 portfolios (and not individual assets) and a period 369 months (from July 1990 to March 2021), estimations of 46,350 betas will be done (7,725 betas/factor), in a total of 309 windows (369 – 60). [1]

The fact of using rolling windows instead of static estimations is because of the idea that the betas of the factors, in the first step of Fama & MacBeth (1973), change over time depending on the financial markets' situation, and hence, we cannot rely on static regressions for estimations of coefficients for the factors. With the rolling window methodology, instability of the factors over time is assumed, and hence, their non-constant behaviour and time variance.

Thus, using time series regressions with rolling windows in the first step, the dependent variable will be the return of each portfolio for each one of the six factors, for each of the rolling window time, doing unifactorial analysis. In general forms, the regression is written as:

$$R_{pt} = \alpha_p + \sum_{n=1}^{N} \beta_{pn}\, Factor_{nt} \quad p = 1,2,\dots,25 \quad t = 1,2,\dots,369 \quad n = 1,2,\dots,6 \qquad (16)$$

Where R_{pt} represents the vector of return of each portfolio j at each period t of the rolling window, β_{pn} represents the vector of sensitivity of each portfolio j with respect to each factor N, $Factor_{nt}$ represents the vector of each of the six factors for all periods t, α_p represents the constants coefficients and ε_p the disturbance error for each portfolio p.

[1] Because of the rolling window of 60 observations, the first 60 observations when doing the beta estimations are lost.

$$\begin{bmatrix} R_{1,1} \\ R_{1,2} \\ R_{1,3} \\ \dots \\ R_{1,60} \end{bmatrix} = \alpha_1 + \beta_{1SMB} \begin{bmatrix} SMB_{1,1} \\ SMB_{1,2}, \\ SMB_{1,3} \\ \dots \\ SMB_{1,60} \end{bmatrix} + \begin{bmatrix} \varepsilon_{1,1} \\ \varepsilon_{1,2}, \\ \varepsilon_{1,3} \\ \dots \\ \varepsilon_{1,60} \end{bmatrix} \tag{17}$$

In the equation above, we find that for the portfolio number 1 and the factor of size, we run a time series regression of the first rolling window, from the months 1 to 60.

In short, one should run this time series regression for each portfolio for each of the factors, for each of the 309 windows; so that at the end one obtains 309 beta estimates for each of the factors and for each of the portfolios. These estimated betas will express the sensitivity of the portfolio with respect to each of the factors, which means that when the return of a certain factor increases/decreases by 1%, the return of the portfolio is expected to increase/decrease the % of the beta estimated coefficient.

Once we have all the estimated betas, the second step goes on with a cross-sectional regression for each of the period. Remember that form the first step we have obtained T-60, that is, 309 observations of beta estimates. We regress the returns of each portfolio against the estimated betas (multifactorial regression) for each of the 309 observations. Thus, it is a multifactorial regression including the six factors, in general terms:

$$R_{pt} = \gamma_{0t} + \gamma_{mt}\hat{\beta}_{pmt} + \gamma_{SMBt}\hat{\beta}_{pSMBt} + \gamma_{HMLt}\hat{\beta}_{pHMLt} + \gamma_{RMWt}\hat{\beta}_{pRMWt} + \gamma_{CMAt}\hat{\beta}_{pCMAt} + \gamma_{WMLt}\hat{\beta}_{pWMLt} + \varepsilon_{pt} \quad for\ t = 60,61,\dots,309\ and\ p = 1,2,\dots,25 \tag{18}$$

Where $\hat{\beta}_{pt}$ is the beta coefficient estimated in the first step of the methodology, for each of the portfolios and for each of the factors in each period. The example below shows the regression at period t=60, the first one, for each of the factors and for the 25 portfolios.

$$\begin{bmatrix} R_{1,60} \\ R_{2,60} \\ \dots \\ R_{25,60} \end{bmatrix} = \gamma_{0,60} + \gamma_{m,60} \begin{bmatrix} \hat{\beta}_{1,m,60} \\ \hat{\beta}_{2,m,60} \\ \dots \\ \hat{\beta}_{25,m,60} \end{bmatrix} + \gamma_{SMB,60} \begin{bmatrix} \hat{\beta}_{1,SMB,60} \\ \hat{\beta}_{2,SMB,60} \\ \dots \\ \hat{\beta}_{25,SMB,60} \end{bmatrix} + \gamma_{HML,60} \begin{bmatrix} \hat{\beta}_{1,HML,60} \\ \hat{\beta}_{2,HML,60} \\ \dots \\ \hat{\beta}_{25,HML,60} \end{bmatrix} + \gamma_{RMW,60} \begin{bmatrix} \hat{\beta}_{1,RMW,60} \\ \hat{\beta}_{2,RMW,60} \\ \dots \\ \hat{\beta}_{25,RMW,60} \end{bmatrix} + \gamma_{CMA,60} \begin{bmatrix} \hat{\beta}_{1,CMA,60} \\ \hat{\beta}_{2,CMA,60} \\ \dots \\ \hat{\beta}_{25,CMA,60} \end{bmatrix} + \gamma_{WML,60} \begin{bmatrix} \hat{\beta}_{1,WML,60} \\ \hat{\beta}_{2,WML,60} \\ \dots \\ \hat{\beta}_{25,WML,60} \end{bmatrix} + \quad for\ each\ t = 60,61,\dots,369 \tag{19}$$

The interpretation of these gammas is the risk premium the investor gets for changes in the estimated risk unit (beta coefficient). That is, when the estimated beta of one factor increases/decreases by 1%, the expected extra return of the portfolio, or the risk premium, is expected to increase/decrease by % gamma coefficient.

One should expect significant and strictly higher than zero gamma coefficients, as we expect the risk factors to be rewarded in the market. Because of the basis of Financial Economics, it is known that the higher the risk, the higher the return an investor should get, as a premium for assuming that risk. Then, when the unit of risk increases, there should be a market reward, to which we refer as market risk premium, for the risk taken.

4. Empirical analysis: the model

Considering all the research and methodology from former authors and economists, the aim of these paper is to test whether those factors proposed by Fama & French (1992, 2015) and Carhartt (1996) are, in fact, risk factors, by using the Fama & MacBeth (1973) methodology, as a two-step procedure with rolling windows, to later on carry out an individual hypothesis testing to confirm the results.

4.1. Descriptive statistics

Before starting with the model per se, a description and summary of the variables in the dataset is presented, which includes, the values of the six factors and twenty-five portfolios' returns available in the website of Fama & French. Hence, observations go from July 1990 on monthly basis until March 2021.

Table 1 shows the descriptive statistics of the 25 portfolios constructed by Fama & French. They all follow the same characteristics according to mean, standard deviation, and minimum and maximum values: positive and lower than one mean values, standard deviation around 5 and minimum and maximum values -25 and 20, respectively. However, it is true that the last portfolios of the list (referring to the biggest firms in size) seem to have higher mean values, but also higher variability.

Table 1: descriptive statistics of 25 portfolios

Variable	Obs	Mean	Std. Dev.	Min	Max
Portfolio1	369	.299	5.607	-24.47	18.76
Portfolio2	369	.64	5.373	-26	16.97
Portfolio3	369	.709	5.161	-26.85	19.19
Portfolio4	369	.812	4.959	-25.26	17.78

Portfolio5	369	.96	4.997	-26.73	18.56
Portfolio6	369	.561	5.737	-26.74	17.57
Portfolio7	369	.762	5.319	-25.93	17.62
Portfolio8	369	.757	5.199	-24.98	20.98
Portfolio9	369	.886	5.228	-27.39	20.46
Portfolio10	369	.96	5.47	-26.48	20.29
Portfolio11	369	.675	5.848	-27.19	18.52
Portfolio12	369	.85	5.254	-26.56	15.24
Portfolio13	369	.765	5.316	-27.04	20.48
Portfolio14	369	.774	5.312	-26.25	19.8
Portfolio15	369	.903	5.706	-27.31	22.88
Portfolio16	369	.78	5.335	-25.14	15.5
Portfolio17	369	.788	5.045	-24.36	16.23
Portfolio18	369	.762	5.118	-23.33	17.37
Portfolio19	369	.751	5.468	-25.27	21.14
Portfolio20	369	.837	6.015	-27.62	24.22
Portfolio21	369	.632	4.776	-20.16	18.07
Portfolio22	369	.759	4.725	-17.21	14.64
Portfolio23	369	.767	5.287	-18.48	17.67
Portfolio24	369	.823	5.427	-19.02	20.2
Portfolio25	369	.723	6.569	-31.28	26.62

Table 2: descriptive statistics of the factors

	Mean	Std. Dev.	variance	skewness	kurtosis	min	max
SMB	.085	2.136	4.563	-.067	3.952	-7.33	8.83
HML	.229	2.536	6.429	.218	6.53	-11.3	11.16
RMW	.375	1.581	2.5	-.295	3.941	-5	6.4
CMA	.107	1.799	3.236	.355	6.532	-7.3	8.77
WML	.883	3.978	15.824	-1.374	10.846	-26.09	13.65
RF	.209	.182	.033	.34	1.803	0	.68
MKT	.518	4.958	24.58	-.554	4.658	-22.02	16.62

Table 2 above shows the descriptive statistics of the factors. *SMB* refers to size factor, *HML* to book-to-market factor, *RMW* to profitability factor, *CMA* to investment factor, and *WML* to momentum factor. The variable *MKT* refers to the difference between the market and the risk free (*RF*) rate, that is, the market premium. *MKT* is what will be used in the analysis, to eliminate the effect of the risk-free returns. See also, the fact that all factors have a mean higher than zero. Analysing the factors, we find differences among them, being for instance, the market premium factor with the highest variance and the risk-free factor, obviously, the lowest variance. According to the distribution of the factors, HML and WML show excess of kurtosis, meaning a peaked bell and tiny tails distribution, while WML shows the highest negative skewness.

It is also interesting to see how these variables are intercorrelated among them. Table 3 shows the pairwise correlations of the first ten portfolios, for simplicity, the rest are not included. The high correlations among them show how these portfolios are constructed considering the same individual asset database, in this case, European market.

Table 3: pairwise correlation of the first 10 portfolios

Variables	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
(1) Portf1	1.000									
(2) Portf2	0.957	1.000								
(3) Portf3	0.943	0.973	1.000							
(4) Portf4	0.903	0.947	0.968	1.000						
(5) Portf5	0.855	0.908	0.937	0.973	1.000					
(6) Portf6	0.948	0.944	0.934	0.898	0.843	1.000				
(7) Portf7	0.920	0.941	0.944	0.943	0.910	0.939	1.000			
(8) Portf8	0.888	0.932	0.952	0.962	0.947	0.910	0.947	1.000		
(9) Portf9	0.847	0.897	0.929	0.958	0.963	0.861	0.927	0.958	1.000	
(10)Portf10	0.810	0.863	0.899	0.938	0.967	0.819	0.891	0.942	0.964	1.000

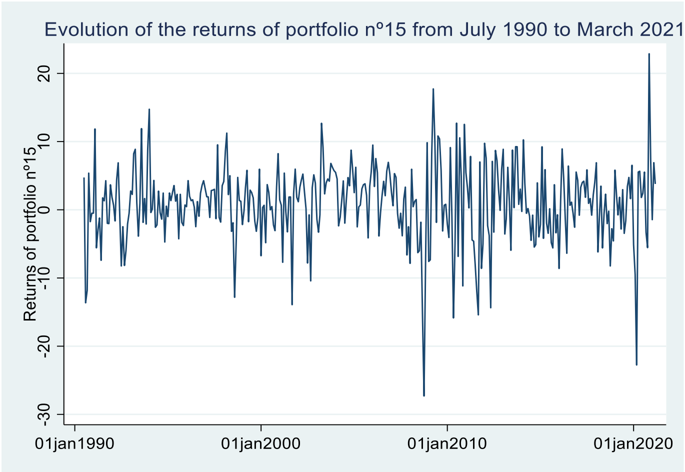
The correlation between the portfolios and the factors is low and not significant, that is why the table is not included in the paper. However, there is a high correlation between the market factor and all the portfolios, a positive correlation of 0.89 approximately. This indicates the similarity between the portfolios and the market factor, indicating high diversification of portfolios. Regarding the rest of the factors, Table 4 shows the pairwise correlations between the factors. No correlation between them to avoid multicollinearity is expected, and data demonstrates that.

Table 4: pairwise correlations of the factors

Variables	(1)	(2)	(3)	(4)	(5)	(6)	(7)
(1) SMB	1.000						
(2) HML	0.012	1.000					
(3) RMW	-0.007	-0.566	1.000				
(4) CMA	-0.013	0.568	-0.210	1.000			
(5) WML	0.061	-0.355	0.440	-0.016	1.000		
(6) RF	-0.163	0.078	0.036	0.050	0.031	1.000	
(7) MKT	-0.113	0.234	-0.298	-0.259	-0.350	-0.050	1.000

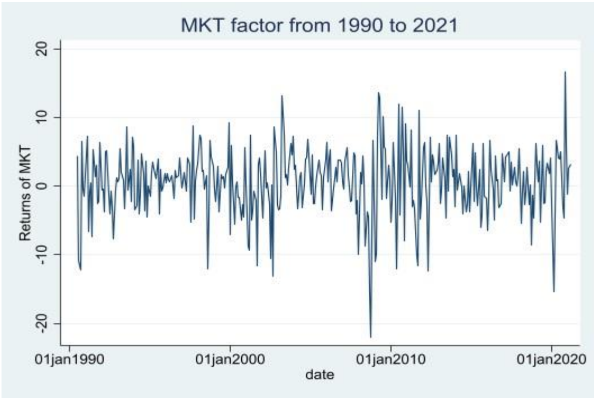
Figure 2 below shows the evolution of the time series of the returns of a concrete portfolio, number 15, for easier illustration. The Figure 2 shows the variation of the returns mainly between -10% and 10% of return, but there are some shocks. The first shock happened the 1st of October 2008, when the financial crisis started, and in fact, the 25 portfolios experienced a negative return of an average of -25%. It was not until the 1st April 2009 that assets recovered positively from the shock experiencing an average of a 16% return. As we have recent data, we can also see the effect of the CoVid-19 and the global lockdown. The 25 portfolios experienced, again, a negative shock causing returns of, on average, -16% on 1st March 2020. It has not been until 1st November 2020, when lockdown ended, that assets experienced a positive increase of, on average, 16%.

Figure 2: evolution of returns of portfolio nº15 from July 1990 to March 2021



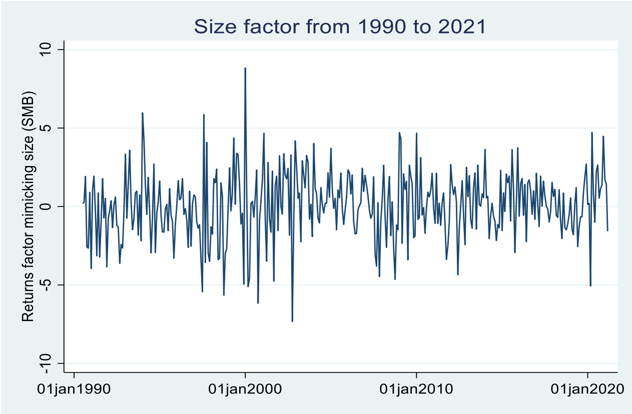
Because of the high and positive correlation between the returns of portfolios and the market premium returns, when driving both variables, very similar behaviour is expected. Indeed, it is what it is obtained, the figure below, Figure 3, shows fluctuations between -10% and 10% returns, and it is easily appreciable the shocks in 2008 and 2020, as happened with the returns of portfolio.

Figure 3: evolution of market premium factor from 1990 to 2021



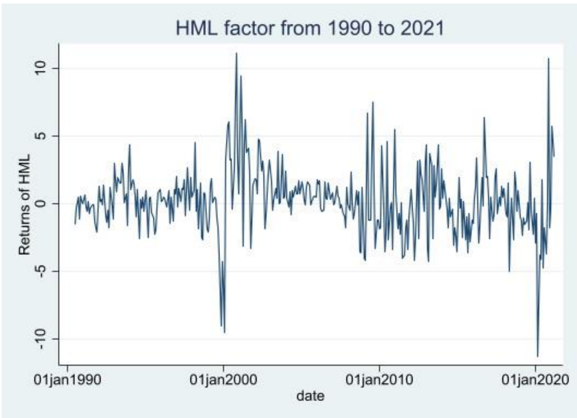
Same analysis with the factor variables could be done. For instance, Figure 4 shows the evolution over time of the returns of the factor mimicking the size from 1990 to 2021. The evolution of the size factor does not behave as the returns of portfolios, in this case, we find more stability between -5% and 5% returns regarding the size factor. The last years of the 20th century and the first years of the 21st century seem to be more volatile, but stability is again soon recovered.

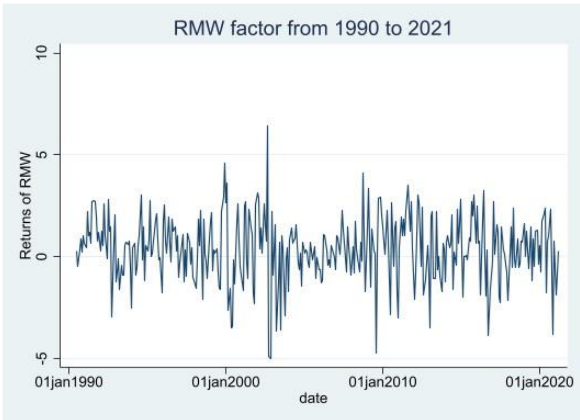
Figure 4: returns of factor mimicking size from 1990 to 2021

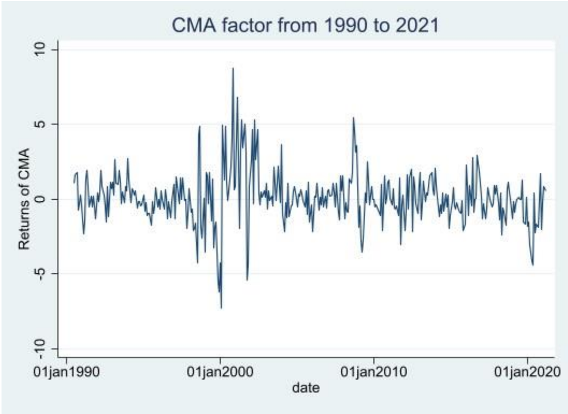


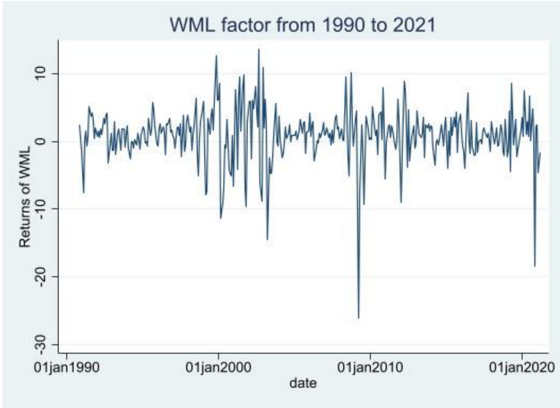
The rest of the factors and their evolution over time from July 1990 to March 2021 are plotted below in Figure 5. According the factor mimicking the book-to-market ratio, we clearly see three shocks: one in the beginning of the 21st century, another smaller one in the 2008 financial crisis, and the last shock in March 2020 due to the Covid-19 crisis. The factor mimicking the profitability of firms is more stable over time, and we can only appreciate some negative peaks in approximately 2002, and a short negative shock in around 2008. The factor mimicking the investment level of firms also fluctuates stable around the mean, except for a big shock in the beginning of the 21st century, and two other smaller shocks in the financial crisis of 2008 and the Covid-19 crisis in 2020. Finally, the factor mimicking the momentum of firms suffers the worst negative shock in the financial crisis of 2008, while the shocks of the beginning of the century and the Covid-19 crisis in 2020 were not that big.

Figure 5: returns of factors of book-to-market, profitability, investment, and momentum from 1990 to 2021









4.2. Applying the model

Before starting with the methodology of Fama & MacBeth (1973) proposed for this paper, it would be interesting to analyse how good model the unifactorial returns generating model is. Furthermore, it would be interesting to see how this unifactorial model would have worked without considering the rolling window method, that is, estimating just one beta considering it does not vary stochastically along time. For this purpose, let us create an average of the 25 portfolio returns for each observation, to generate the dependent variable of the regression, being the market premium factor the independent one. It is an appropriate first insight into what is the aim of this paper, it gives an overview on how the market factor affects the expected returns in financial markets. Results are shown in Table 5.

$$E(R) = \alpha + \beta MKT + \varepsilon \tag{20}$$

Table 5: unifactorial return generating process

Mean(portfolios)	Coef.	St. Err.	t-value	p-value	[95% Conf	Int.]	Sig
MKT	.987	.013	78.72	0	.963	1.012	***
Constant	.246	.062	3.95	0	.124	.369	***

Mean dependent var	0.757	SD dependent var	5.037
R-squared	0.944	Number of obs	369.000
F-test	6196.574	Prob > F	0.000
Akaike crit. (AIC)	1179.249	Bayesian crit. (BIC)	1187.071

**** p<.01, ** p<.05, * p<.1*

From the pairwise highly positive correlation between the returns of portfolios and the market premium, we should expect a beta coefficient of the standard CAPM model close to one, indicating assets and market behave the same way in the same direction. In other words, with such a high coefficient (0.987), the estimated return of the asset, in this case the average of the

25 portfolios, equals to the expected return of the market. Due to this close to one beta coefficient estimate, the R2 value will also be high, as seen in Table 5: with a goodness of fit of 94.4%, including more factors as possible estimates of the expected return, should improve the goodness of fit, but the improvement cannot be tremendous.

As a sense of curiosity, the return generating process including more factors should also be checked and compare what happens to the estimated coefficient of the market premium in the previous regression and the goodness of fit.

$$E(R) = \alpha + \beta_{MKT}MKT + \beta_{SMB}SMB + \beta_{HML}HML + \beta_{RMW}RMW + \beta_{CMA}CMA + \beta_{WML}WML + \varepsilon \quad (21)$$

Table 6: multifactorial return generating process

mean port	Coef.	St. Err.	t-value	p-value	[95% Conf	Int]	Sig
MKT	1.001	.003	287.04	0	.994	1.008	***
SMB	.543	.007	82.47	0	.53	.556	***
HML	.023	.009	2.68	.008	.006	.041	***
RMW	-.027	.011	-2.34	.02	-.049	-.004	**
CMA	-.031	.011	-2.85	.005	-.053	-.01	***
WML	-.014	.004	-3.34	.001	-.022	-.006	***
Constant	.211	.015	13.76	0	.181	.241	***

Mean dependent var	0.796	SD dependent var	4.980
R-squared	0.997	Number of obs	365.000
F-test	21237.629	Prob > F	0.000
Akaike crit. (AIC)	75.430	Bayesian crit. (BIC)	102.729

**** p<.01, ** p<.05, * p<.1*

With a multifactor return generating process using Fama & French factors and Carhartt factor, shown in Table 6, we find out that, considering the average returns of the 25 portfolios each observation, all the factors included in the regression are significant at 1% significance level. Moreover, the goodness of fit increases up to 99.7%, hence, this model seems to almost predict the average returns of portfolios perfectly. It is also remarkable the beta coefficient of the market premium factor (1.001), indicating the unitary sensitivity of the returns regarding any change in the market premium factor. The negative estimated beta coefficients for profitability (RMW), investment (CMA) and momentum (WML), indicating a negative sensitivity of portfolios to changes in those factors, is due to the idea that firms that keep their profits, or internally reinvest, are expected to respond negatively in the market.

These results question whether the multifactor return generating processes are, in fact, a better estimator than the unifactorial return generating process models. That is, if it is necessary to include more factors as possible estimators. To clarify this question, and deeply check whether these factors are appropriate or not, we should proceed on doing regressions for each portfolio using the unifactorial market premium model, and the multifactor model with the rest of the factors proposed. Checking on the goodness of fit will clarify whether the inclusion of more factors benefits the asset pricing estimation.

Table 7: goodness of fit of 25 unifactorial and multifactorial regressions

	Portf. 1	Portf. 2	Portf. 3	Portf. 4	Portf. 5	Portf. 6	Portf. 7
R2 with only MKT	69.12	74.81	78.18	79.46	76.16	74.58	81.16
R2 with more factors	92.15	93.74	94.96	95.86	96.32	94.74	95.40
	Portf. 8	Portf. 9	Portf. 10	Portf. 11	Portf. 12	Portf. 13	Portf. 14
R2 with only MKT	82.23	81.85	79.26	78.12	85.42	86.18	86.14
R2 with more factors	96.26	96.38	96.75	93.64	94.13	94.09	94.75
	Portf 15	Portf 16	Portf 17	Portf 18	Portf 19	Portfolio 20	Portf. 21
R2 with only MKT	83.59	84.39	89.28	92.11	88.47	84.66	82.43
R2 with more factors	95.14	92.81	91.61	94.45	93.81	94.22	93.44
	Portf 22	Portf. 23	Portf. 24	Portf. 25			
R2 with only MKT	91.32	94.94	91.65	85.73			
R2 with more factors	94.76	95.63	94.19	95.50			

Table 7 above shows the goodness of fit of each of the regressions. Surprisingly enough, and with higher values for the goodness of fit than what Fama & French (1992b) got in their analysis, the goodness of fit of the unifactorial return generating process varies around the 80%, with an average value of 83.11%. However, it is true that including more factors, doing a multifactorial return generating process, improves the goodness of fit up to 94.59%. Furthermore, the multicollinearity check, using Variance Inflation Factor (VIF) suggest a value of 1.68, not indicating multicollinearity.

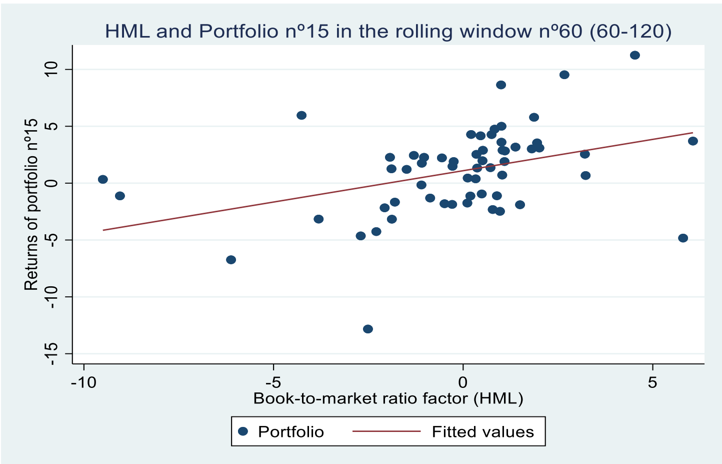
4.2.1. First step of Fama & MacBeth (1973): time series analysis with rolling windows

As explained in the methodology part, this first step consists of analysing the relationship of each of the factors with the returns of each of the portfolios. In that sense, the aim is to find the sensitivity of the factors proposed by Fama & French with respect to the expected returns of portfolios, by doing individual unifactorial regressions for each of the 25 portfolios and for each of the rolling windows.[2]

Graphically, one should take one by one windows of 60 time observations and plot the return of one concrete portfolio in the vertical axis, and the returns of one of the factors in the horizontal axis, creating a point cloud of 60 observations each time. The fitted line of that point cloud will give the slope, which is precisely the estimated beta. We should do this graphical analysis for each portfolio, with each factor with each rolling window, 46,350 times.

For instance, Figure 6 shows the relationship between the book-to-market ratio factor and the returns of portfolio number 15 in the rolling window number 60, that is, time observations from 60 to 120, corresponding to from May 1995 to June 2000. In this case, we find a positive relationship, which indicates that, the higher the returns regarding the factor of book-to-market ratio, the higher the returns of that concrete portfolio. Estimated beta will, in this example, be positive.

Figure 6: HML and Portfolio nº15 in the rolling window nº60



[2] STATA16 has commands for this methodology and automatically does all the 46,350 beta estimations, giving as well the R2 and R2 adjusted values and standard errors, from which we can also calculate the t-statistics.

It could happen, and it does if one checks the dataset, that even if in general forms the relationship between one factor and one portfolio is positive, during some time periods, or in some concrete rolling windows, that relationship inverses. This is a normal situation and it is because of the idea that financial markets fluctuate, the economies are volatile and changing over time.

Once the unifactorial regressions for each portfolio and for each factor are done, as explained in the methodology part, the dataset has now 150 new variables (25x6) corresponding to the estimated betas. As averages cannot be explained because we are working with separate portfolios, but considering portfolios behave very similarly -due to high correlation-, plotting the summary statistics of one concrete portfolio purely reflects the behaviour of the estimated betas of the rest of the portfolios[3].

Table 8: summary statistics of estimated betas for portfolio nº10

	Mean	Std. Dev.	variance	min	max
bSMB10	.594	.442	.196	-.053	2.056
bHML10	1.216	.805	.648	-.241	3.25
bRMW10	-1.394	.861	.742	-3.124	-.451
bCMA10	-.17	.985	.97	-2.712	1.916
bWML10	-.39	.392	.154	-.969	.595
b MKT10	.926	.188	.035	.563	1.191

Table 8 shows the summary statistics for the estimated betas of the portfolio nº 10. Considering the mean values along the whole time series, the estimated beta of the factors mimicking profitability, investment and momentum are negative, as happened in the multifactorial return generating process model. On the other hand, the estimated beta for the market premium factor (0.926) is very close to one, indicating that changes in the values of the market premium factor affect almost the same way to the returns of portfolio nº10, on average. This conclusion is the same as the ones obtained in the unifactorial and multifactorial return generating process, due to the fact that the market behaviour and the behaviour of the portfolios are very similar. Furthermore, the greatest variance is founded in the estimated beta for the factors mimicking book-to-market ratio, profitability and investment, and the distribution is, in general terms, normal and not skewed. Finally, the estimated beta coefficient for the market premium factor has very low volatility, indicating low volatility across time. However, estimated beta coefficients of the factors of investment, profitability, and book-to-market a variance close to one, showing a high volatility.

[3] The list of all the estimated betas for the six factors is not included in the appendix because of its length. If interested in checking it, ask the author. However, appendix I includes the mean values of the returns and estimated betas for each of the 25 portfolios.

If we do the same summary of statistics for another portfolio, randomly chosen, we approximately obtain the same results. Indeed, results should not be exactly the same, since we are considering different portfolios, and even if they are highly correlated among them, they are not the same.

Table 9: summary statistics of estimated betas for portfolio nº20

	Mean	Std. Dev.	variance	min	max
b SMB20	.111	.508	.258	-.754	1.965
b HML20	1.381	.935	.874	-.358	3.252
b RMW20	-1.595	.919	.845	-3.454	-.34
b CMA20	-.264	1.158	1.341	-3.287	2.428
b WML20	-.496	.493	.243	-1.216	.689
b MKT20	1.072	.179	.032	.692	1.293

As said, results shown in Table 9 are approximately the same. We could highlight the mean value for the estimated beta of the factor mimicking size, as it has gone from 0.594 in the portfolio nº10, to 0.111 in portfolio nº20. Should not be worried about this since it is a cause of the high variability of the estimated beta values. The rest of the values are very similar from one portfolio to the other, and hence, we could apply these values as a general behaviour of all the estimated betas of factors of the 25 portfolios.

Before conducting a time series analysis, an appropriate check would be to test the normality of the distribution of these estimated beta time series. For that purpose, Jarque-Bera test, asymmetry and kurtosis tests will be done, as well as histograms[4] for each of the six series, taking one of the portfolios as reference (Portfolio nº10).

Table 10: normality tests of estimated betas

	Beta SMB	Beta MKT	Beta HML	Beta RMW	Beta CMA	Beta WML
J-B, Chi (2)	3.9e-16	2.0e-08	0.0027	1.5e-10	1.4e-04	9.4e-05
Pr(Skewness)	0.0000	0.0001	0.3013	0.0000	0.0001	0.0042
Pr(Kurtosis)	0.0057	0.0000	0.0000	0.0000	0.4318	0.0000

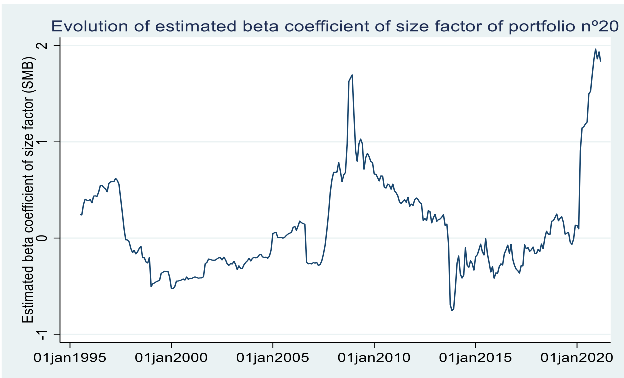
Results from Table 10 above show that, being the p-value (0.05) in the six cases higher than the Chi (2) value, the null hypothesis is rejected and therefore estimated betas are not normally distributed. The measure of asymmetry of the probability distribution of the estimated betas suggests an asymptotically normal distribution of the estimated betas of book-to-market factor. Lastly, from the probability of kurtosis, defining the shape of the distribution, kurtosis test

[4] Figures of histograms for the estimated betas of the six factors can be found in appendix II.

suggests and asymptotic distribution of the estimated beta for the investment factor. The general conclusion is that the estimated betas do not follow a normal distribution.

Focusing on the time series analysis of this first part of Fama & MacBeth (1973) methodology, we should analyse the behaviour of the estimated betas in the time series to check for stationarity and structural shocks that could have affected the financial markets in that period. For instance, Figure 7 shows the time series evolution of the estimated beta for the factor mimicking the size. Taking into account that the factor mimicking the size indicates that the higher its value the more return investors gets because of investing in small firms (small firms have higher returns than big firms, as small – big), Figure 7 shows how these sensibilities (referring to beta values) increased during the creation of the real-state bubble and decreased when the financial crisis began. Same rationale applies in 2020 when Covid-19 hit the economies of the world. This is totally consistent with historical events and Financial Economics theory: investors were willing to take more risk in expanding situations, and that risk involves more returns, just until the bubble explodes.

Figure 7: evolution of the estimated beta coefficient of size factor of portfolio nº20



A straightforward visual conclusion from the Figure 7 is that the estimated beta for the size factor is not stationary, which is totally consistent if we consider that beta estimates are done using the previous sixty observations, and thus, it would be obvious that the results are lagged and correlated. A deeper time series analysis leads to check the stationarity of the estimated betas. The reason for doing rolling window methodology in the estimations is that the value for beta changes over time, because of its non-stationarity, and hence, one unique beta estimate for all the time series data will not be enough to be considered a proper estimator. Because of that, one should check stationarity of the estimated beta coefficients to see if the rolling window method was the solution to get a better asset pricing model. It is to say, this analysis will be done considering the portfolio nº20 for each of the six factors, assuming the results apply to the rest of the portfolios. An extensive analysis will be done for the beta coefficients corresponding to size factors, while the others will be shortly done, because of the similarity of results.

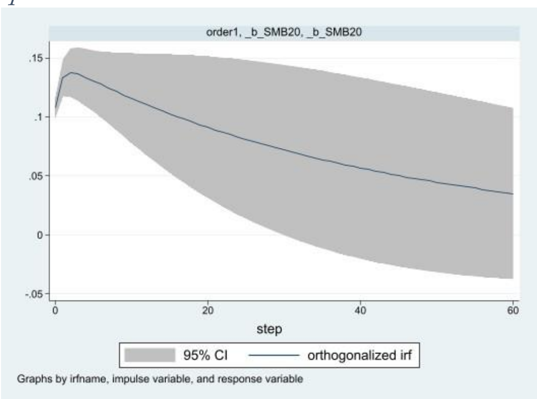
The *varsoc* command in STATA is useful to identify the appropriate lags for the time series analysis considering different criterion, such as, the Akaike Information Criteria, the Schwartz Bayesian Information Criteria (SBIC), or the Hannan Quin Information Criterion. (HQIC) Focusing my analysis based on AIC, the optimum lags suggested by STATA is two, 2, for the time series of the estimated beta coefficients for the size factor, as it indicates the lower value for AIC. Moreover, SBIC and LR (Likelihood Ratio) also suggest two lags. This indicates that, even if each beta contains 59 same observations as the previous estimated beta for the calculation, its lags only affect two observations.

The Dicky Fuller test for the estimated betas corresponding to the size factor suggests that, with a -0.965 value for the t-statistics, we happen to not to reject the null hypothesis that the process has a unit root, because the Z(t) is lower in absolute values than any critical value of the test. Hence, we conclude the process of the estimated betas for the size factor is not stationary. Phillip-Perron stationarity test also reaches the same conclusion, but as Perron (1989) argued, structural breaks could be a cause of bias in the testing for stationarity, and even more when the dataset considers an economic bubble and only 30 years of observations.

Because of that, it is important to ensure the presence or absence of structural breaks in the estimated betas, using the Chow test or the Wald test, in order to conclude if the rolling windows were a good choice. The Wald test suggests a structural break on April 2017, which is not the greatest shock founded graphically, but it is true there is a change in the tendency of the time series in April 2017. Therefore, we happen to reject the null hypothesis that there is not structural breaks in the data, and thus, estimated beta for the size factor is not stable over the time, and hence, it was necessary to use the rolling window method.

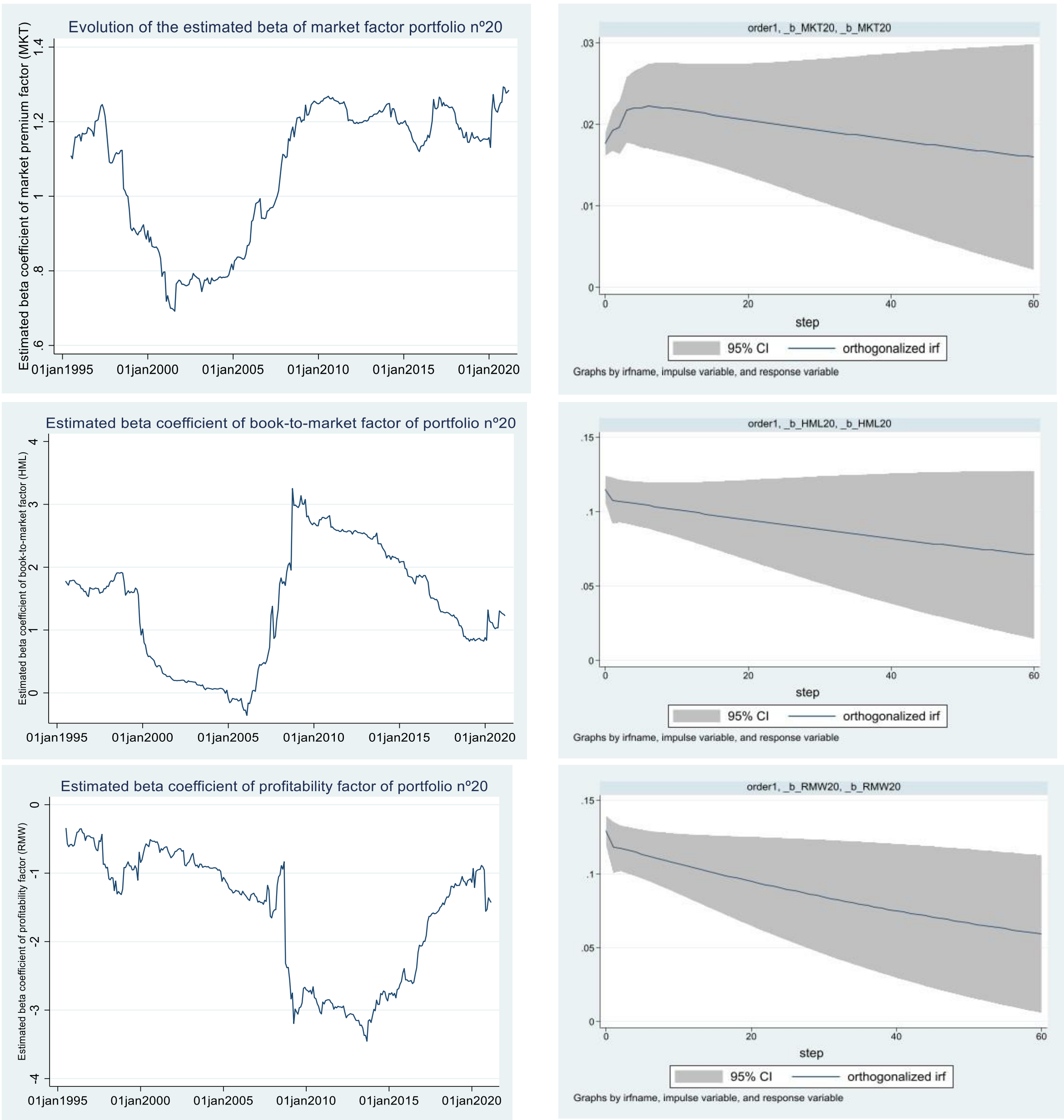
Finally, Figure 8 below shows the impulse response function of order one for the estimated betas of the size factor, and it clearly explains how the unpredictability and non-stationary affects on the shocks and long run stability.

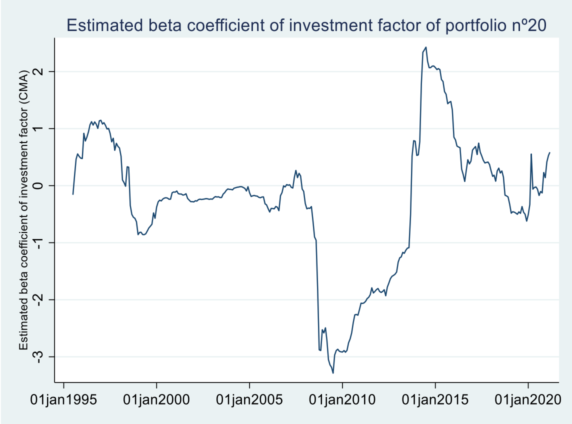
Figure 8: impulse response function of estimated betas for size factor portfolio nº20

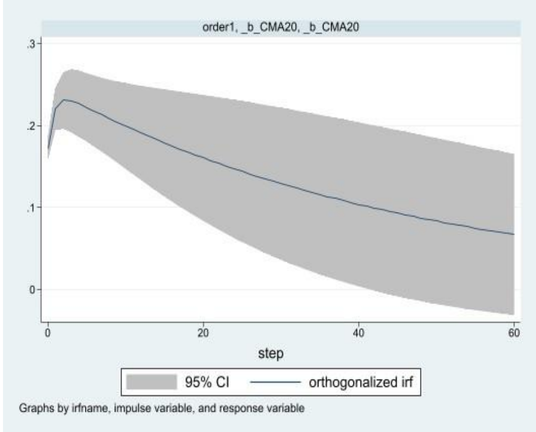


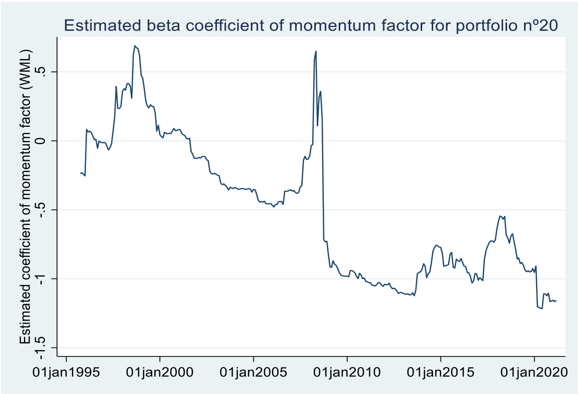
Once we have done a deep time series analysis of the estimated betas of the factor mimicking size, we can proceed on doing the same for the other estimated betas of the rest of the factors. Figure 9 below builds up the estimated beta coefficients of the other five factors being analysed, showing the time series evolution of the estimated betas and their corresponding response function.

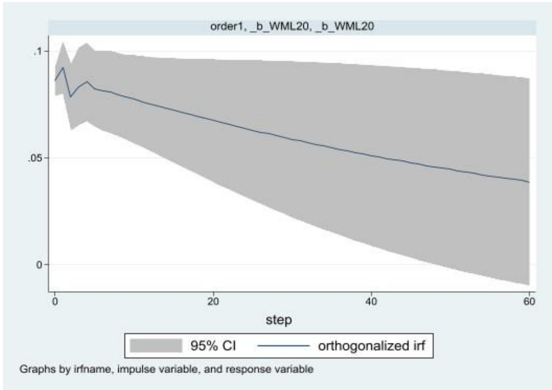
Figure 9: evolution and impulse responses of estimated betas for MKT, HML, RMW, CMA and WML factors











The optimal lag periods for the estimated beta coefficients for the factors corresponding to market premium and momentum are four (based on the *varsoc* test in Stata), while the optimum lags for the estimated beta coefficients for the factors of book-to-market, profitability and investment, are two, extremely few lags. The Dicky-Fuller test and the Phillip-Perron test suggest, for the five of the estimated beta coefficients corresponding to the factors time series, not rejecting the null hypothesis that the process has a unit root. Hence, this acceptance implies the conclusion that the process of the estimated betas for the five factors are not stationary.

Regarding the estimated betas for the market premium factor, Wald test suggests a structural break in the time series in September 2001. Visual observation of the evolution of the estimated betas over time for the market premium factor can lead us to conclude a valley that starts, approximately, by the end of 2001, and makes the difference in the evolution. The impulse response function of order one for the estimated betas of the market premium factor suggests the effect of the first and fourth lag, and how the time series is not able to stabilize over the periods, showing its non-stationarity behaviour.

Considering now the time series of the estimated betas of the factor mimicking the book-to-market ratio, structural break test suggests a break in the time series on March 2006, just before the financial crisis started, when the bubble was being created. Graphically, a valley from 2001 to 2006 is observed, and then, when the structural break, the estimated beta values increase unexpectedly. This non-stationarity is reflected in the impulse response function of order one of the estimated betas for the book-to-market ratio factor.

As for the time series analysis of the estimated betas for the factor mimicking the profitability, the structural break test suggests a break in August 2016. This is not easily appreciable in the visual observation of the evolution of the estimated betas, indeed because the greatest shock happens in a decrease of the value of the estimated betas around April 2009, when values dropped down to -3. However, there is a change in the tendency of the evolution around 2016, from a valley to an increasing trend. The impulse response function Figure of order one of the estimated betas for the profitability factor reflects a small shock in the first lag, and the huge gap of the 95% confidence of interval reflects the unpredictability of the estimated betas.

Regarding the time series analysis of the estimated betas of the factor mimicking investment, the structural break test suggests a break in the process in August 2009. Graphical analysis of the evolution of the estimated betas of investment factor over time suggests a very scarce stationarity because of the peaks and valleys over time. There is a great shock before the financial crisis of 2008, and it can be graphically seen how in 2009 that decreasing trend stops and values increase again, and that is why Wald test suggest there a structural break. The impulse response function of order one for the estimated betas of investment factor plots the first lag as the one with the highest response to a shock. Then, responses tend to decay to zero, but the confidence interval gap is too big to conclude anything.

Finally, as for the time series analysis of the estimated betas of the factor mimicking the momentum, the structural break test suggests a break in June 2008. As this momentum factor measures how well firms are expected to do in the short run, before the financial crisis and with the creation of the bubble and expansionary economies, the value for the estimated betas of the factor increased exponentially (indicating higher sensitivity of portfolios to this momentum factor). However, when it all broke, firms were not doing a good performance, and neither were expected to perform better in the short run. Thus, the estimated betas of the momentum factor suffer a structural shock just some month before the financial crisis exploded in 2008 (assuming it exploded with the insolvency of Leman Brothers in September 2008). Moreover, graphically, these values were not able to recover from the shock, and even nowadays, they do not have reached the values of the beginning of the time series. This is also reflected in the impulse response function of order one of the estimated betas of momentum factor, where the responses shift up and down, depending on the sign. The enormous gap of the 95% confidence interval reveals about the unpredictability of the estimated beta coefficient.

As a brief summary of this time series analysis of the estimated betas for the factors, and considering only one of the portfolios as a reflect of the others, one can conclude these estimated betas are not stationary, and hence, their predictability is almost impossible. There are structural breaks in the six-time series being analysed, and Figures shows peaks and valleys all over the time series, explaining the unpredictability and non-stationarity.

4.2.2. Second step of Fama & MacBeth (1973): cross-sectional regression analysis

Once all the estimated betas for each of the factors and for each of the time observation are saved, and once the time series analysis has been done, we can proceed on the second step of the methodology. The cross-sectional regressions allow to see our variables at each specific point of time, helping us avoid the serial correlation of residuals. The aim of this second part is to test, according to Fama & MacBeth (1973), whether higher risk is associated with higher expected return, assuming a risk-averse behaviour of investors in the market.

The easiest and firstly used approach in this second step consists of calculating average values and running one single regression with averages. This was the first approach when the first empirical analysis of Fama & MacBeth (1973) was done: a unique regression with vectors of 25 observations, corresponding to the average values of the returns and beta coefficients of each portfolio. The results are six static gamma coefficients for each of the average estimated betas for each of the factors. The static one-single gamma coefficient can give serve as a first clue of how the results using cross-sectional analysis should look like.

$$mean\ R_p = \gamma_{SMB}(mean\hat{\beta}_{SMB}) + \gamma_{MKT}(mean\hat{\beta}_{MKT}) + \gamma_{HML}(mean\hat{\beta}_{HML}) + \gamma_{RMW} mean\hat{\beta}_{RMW} + \gamma_{CMA}(mean\hat{\beta}_{CMA}) + \gamma_{CMW}(mean\hat{\beta}_{CMW}) \quad (22)$$

Table 11: static cross-sectional regression

Mean portfolio	Coef.	St. Err.	t-value	p-value	[95% Conf	Intl]	Sig
Mean betaSMB	-.027	.069	-0.38	.705	-.173	.119	
Mean betaMKT	0	0	-0.38	.707	0	0	
Mean betaHML	0	0	-0.53	.603	0	0	
Mean betaRMW	0	0	-1.38	.185	0	0	
Mean betaCMA	0	0	0.62	.545	0	0	
Mean betaWML	-.23	.494	-0.47	.647	-1.268	.808	
Constant	.673	.148	4.55	0	.363	.983	***

Mean dependent var	0.757	SD dependent var	0.134
R-squared	0.276	Number of obs	25.000
F-test	1.142	Prob > F	0.379
Akaike crit. (AIC)	-24.550	Bayesian crit. (BIC)	-16.018

**** p<.01, ** p<.05, * p<.1*

As seen in Table 11 none of the coefficients of the average estimated betas are significant. What is more, if they were, the coefficients are zero for all the estimated betas except for the estimated beta of size factor and momentum factor. This can give a clue of what results we should expect: non significance of gamma coefficients.

Going deeper in the cross-sectional methodology, Fama & MacBeth (1973) proposed a period-by-period cross-sectional regression, in which each of the $\hat{\beta}$ is a vector containing the 25 estimated betas for each of the factors at a specific time observation, corresponding to the 25 portfolios. As we have estimated the betas with rolling windows of 60 observations, the first 60 observations are empty, and thus, are not considered in this second step, having a total of 309 cross-sectional regressions.

$$R_t = \gamma_{0t} + \gamma_{MKTt}\hat{\beta}_{MKTt} + \gamma_{SMBt}\hat{\beta}_{SMBt} + \gamma_{HMLt}\hat{\beta}_{HMLt} + \gamma_{RMWt}\hat{\beta}_{RMWt} + \gamma_{CMAt}\hat{\beta}_{CMAt} + \gamma_{WMLt}\hat{\beta}_{WMLt} + \varepsilon_t \quad for\ t = 60,61,\ldots,309 \tag{23}$$

Because these second step regressions are done taking estimated of betas as independent variables, to solve for heteroscedasticity, serial correlation, and errors in variables (EIV), the covariance matrix estimator of Newey & West (1987) is used for calculating the standard errors. This enables to obtain more accurate t-statistics value for individual hypothesis testing.

As said in the beginning of the paper, the research question is whether the factors proposed by Fama & French (1992) and Carhart (1996) are, indeed, risk factors regarding the market premium. That is, the higher the risk associated to each of the factors, the higher the expected returns lead by that factor should be. Therefore, one should expect a positive and significant value for each of the gammas[5].

For instance, the following Table 12 shows the cross-sectional regression when the time observation is 90, that is, December 1997. Considering the t-values of each of the coefficients of the estimation, it is clear the non-significance of estimated betas of the factors. At this specific point of time of our panel data, what this second step of Fama & MacBeth suggests is that none of the factors proposed as return' estimators are, in fact, risk factors. The reason is that, being the coefficients of the estimated betas, the gammas, non-significant, implies there is no relationship between the risk factors and the expected returns of assets. Hence, the higher the risk these factors express with the sensitivities of their betas, does not necessarily imply the expected returns will be higher, because of the higher risk assumed.

[5] The gamma coefficients of all cross-sectional regressions and the values of the t-statistics are not included in the Appendix section because of the length. If required, please ask the author.

Table 12: cross-sectional regression on December 1997

Portfolio	Coef.	St. Err.	t-value	p-value	[95% Conf	Int.]	Sig
_b_SMB	1.578	2.007	0.79	.442	-2.639	5.795	
_b_MKT	9.458	6.424	1.47	.158	-4.039	22.95	
_b_HML	-1.35	4.556	-0.30	.77	-10.921	8.221	
_b_RMW	-3.929	3.775	-1.04	.312	-11.86	4.002	
_b_CMA	-2.207	1.93	-1.14	.268	-6.263	1.848	
_b_WML	3.549	2.767	1.28	.216	-2.265	9.362	
Constant	-7.213	3.307	-2.18	.043	-14.162	-.265	**
	Mean dependent var	0.734	SD dependent var			2.269	
	R-squared	0.868	Number of obs			25.000	
	F-test	19.78	Prob > F			0.000	
	Akaike crit. (AIC)	74.20	Bayesian crit. (BIC)			82.736	

**** p<.01, ** p<.05, * p<.1*

A deeper analysis suggests analysing cross-sectional regression at that time observation considering unifactorial CAPM model, then constructing the 3-factor model by Fama & French (1992), and finally analysing the multifactor model with 6 factors. Table 13 below shows the results.

Table 13: unifactorial and multifactorial cross-sectional regression in December 1997

Variables	CAPM Model	3FF Model	6FF Model [6]
_b_MKT	11.75***	9.845*	9.458
	(1.171)	(5.031)	(6.424)
_b_SMB		-0.582	1.578
		(1.716)	(2.007)
_b_HML		0.560	-1.350
		(1.040)	(4.556)
_b_RMW			-3.929
			(3.775)
_b_CMA			-2.207
			(1.930)
_b_WML			3.549
			(2.767)
Constant	-9.137***	-7.95**	-7.213**
	(1.003)	(3.552)	(3.307)
R-squared	0.814	0.817	0.868

Obs: 25. Standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

[6] Average value of VIF: 41.40. High multicollinearity detected.

Unifactorial CAPM cross-sectional analysis suggests that the coefficient for the estimated betas of unifactorial regressions corresponding to market premium factor is significant at a 1% level, and it has positive value. Moreover, the 3-factor model of Fama & French (1992) does not generate significant gamma coefficients for the estimated betas of the factors, indicating, when grouping them together, that the risk this factor express by the sensitivities of their corresponding betas is not absolutely rewarded with returns by the market. Same happens when including the six factors model: more risk factor expressed by the beta coefficients does not imply more expected returns in the market.

Another example to certify the significance of the gammas, is, for instance, when the time observation is 300, that is, June 2015, shown in Table 14 below. This specific regression also gives different results to the unifactorial regressions of December 1997. In this case, considering the unifactorial CAPM analysis, the estimated betas corresponding to market premium factor is not significant. When running the 3-factor model regression, we happen to see, curiously, that the only significant gamma is the one for the estimated beta of the size factor. Considering the 6-factor model, the only value we could consider significant at a 10% level would be the gamma coefficient corresponding to the estimated beta of the factor mimicking profitability, but its negative sign denies any logical expected relation.

Table 14: unifactorial and multifactorial cross-sectional regression on June 2015

Variables	CAPM Model	3 FF Model	6 FF Model[7]
_b_MKT	-1.836	0.306	-5.873
	(1.937)	(3.064)	(5.734)
_b_SMB		2.123***	0.747
		(0.408)	(0.846)
_b_HML		0.0398	-9.817
		(0.944)	(7.491)
_b_RMW			-10.84*
			(5.777)
_b_CMA			-0.896
			(1.591)
_b_WML			3.698
			(5.659)
Constant	0.109	-2.364	-2.144
	(1.935)	(2.047)	(2.515)
R-squared	0.038	0.615	0.690

Obs: 25. Standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

[7] Average value of VIF: 135.96. Incredibly high multicollinearity detected.

We could also run a general regression of this second step, which the results are given as an average, Table 15 below shows the results of this regression. The results show that none of the variables are significant in neither of the three models presented, except for the constant value. Thus, the conclusions are straightforward: none of the factors can be considered as actual risk factors, and therefore, none of them are good estimators of risk.

Table 15: FMB cross-sectional regression of average gammas

Variables	CAPM Model	3 FF Model	6 FF Model[8]
_b_MKT	-0.791	0.376	1.210
	(0.517)	(0.564)	(0.781)
_b_SMB		0.0399	0.0747
		(0.156)	(0.190)
_b_HML		-0.0597	0.161
		(0.258)	(0.517)
_b_RMW			0.403
			(0.330)
_b_CMA			0.206
			(0.350)
_b_WML			-0.141
			(0.528)
Constant	1.54***	0.95***	0.834**
	(0.405)	(0.313)	(0.336)
R-squared	0.211	0.523	0.646

Standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

The key point in this step is to check the t-statistics of all the regressions to see which estimated betas of each of the factors are significant and when. That is, when the gammas are significant and positive. Computing the t-statistics as Fama & MacBeth (1973) proposed to eliminate heteroscedasticity and creating dummies for counting the significant regressions will be enough.

As there is no distinction in the gamma coefficients between portfolios, that is, all portfolios at the same time observation have the same estimated gamma coefficients, one should count the significance of gamma coefficients (out of 306 observations[9]). Not only should we expect, theoretically, significant gamma coefficient, but we should also expect them to be positive.

[8] Uncentered VIF analysis: average value 3.66. Some hints of multicollinearity appear in the second step.
[9] The WML factor time series data starts in November 1990. That is why some regressions are lost in the second step. Instead of having 309 regressions, we have 306.

Table 16: Significance of gamma coefficients for each portfolio

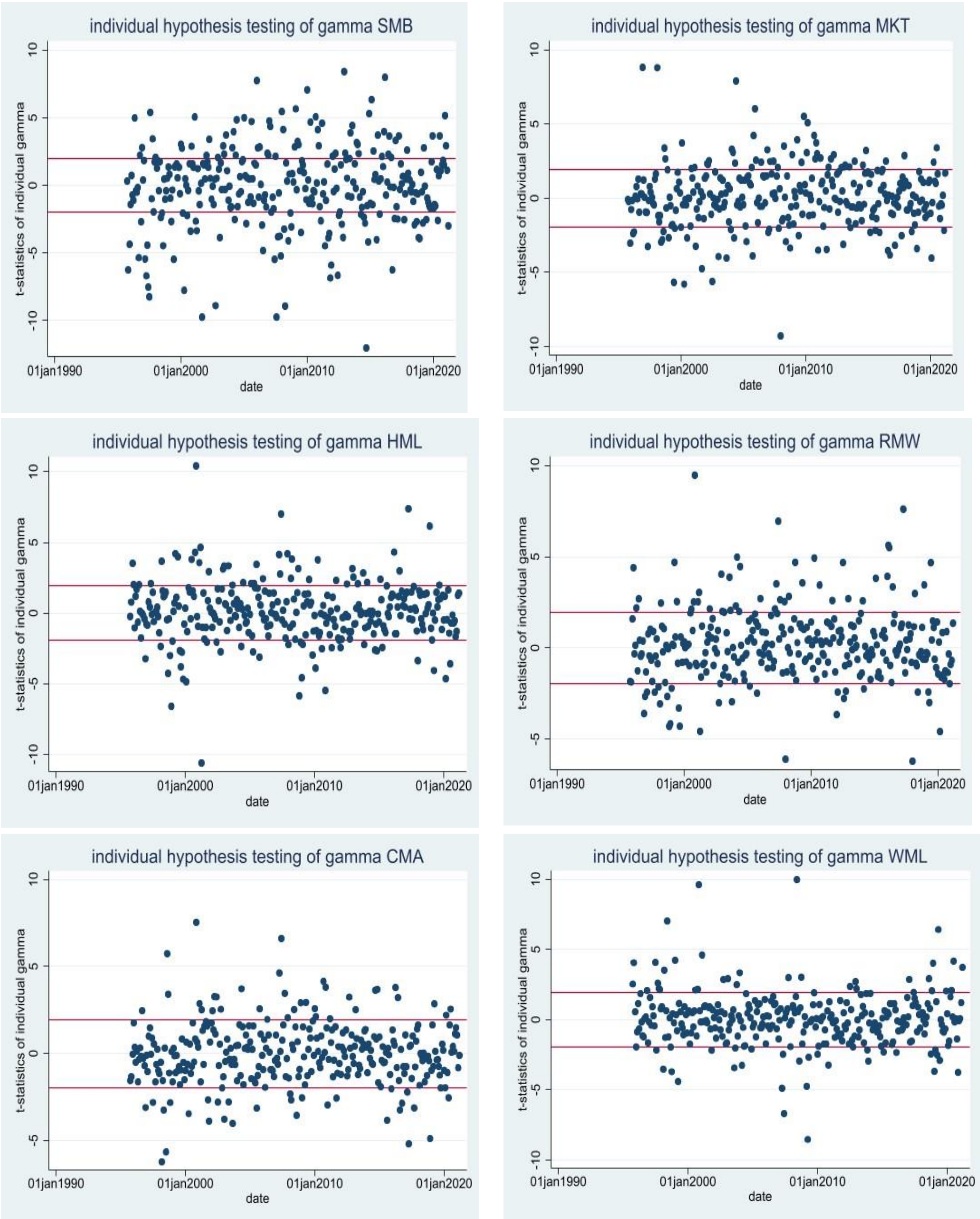
	Gamma SMB	Gamma MKT	Gamma HML	Gamma RMW	Gamma CMA	Gamma WML
Significant	138	92	81	74	65	62
Among which, positive:	70	49	44	45	33	32
Not significant	168	214	225	232	241	244

What it is observed in Table 16 is that clearly those not significant gammas outweigh the significant ones. Furthermore, we should only consider those significant and positive gamma coefficients, which is, in percentage terms: 22.8% for size, 16% for market premium, 14,4% for book-to-market ratio, 14.7% for profitability, 10.8% for investment, and 10.4% for momentum.

And what is more, if we check when those significant gammas took place, the distribution is equally along the years, with around 3-4 significant regression per year, considering all the portfolios. This means that there is not a concrete period where the factors proposed by Fama & French were indeed risk factors, but it is more related to some random significance along the years. Other interesting thing is that the factor mimicking the size could be the most appropriate, as it is the one with the most significant gammas, around a 46% of the estimations are significant. Thus, on average, the only factor, if any, which we could consider risk factor, and hence, an appropriate estimator of the expected returns assuming a risk-aversion situation, is the factor mimicking the size proposed by Fama & French (1992).

In order to graphically see the insignificance of our regressions, one could plot the t-statistics along time for each of the gammas. Figure 10 below shows the individual hypothesis testing of the gammas corresponding to the estimated betas of each of the factors (size, market premium, book-to-market ratio, profitability, investment, and momentum).

Figure 10: individual hypothesis testing of the gammas corresponding to SMB, MKT, HML, RMW, CMA, WML



The individual hypothesis testing of the gamma coefficient corresponding to the estimated beta of the book-to-market ratio factor shows some very high outliers (t-stat > 10) corresponding to November 2000, (t-stat >7) in June 2007 and April 2017. For the case of individual hypothesis testing of the gamma coefficient corresponding to the estimated beta of the profitability factor, the outliers happen in November 2000 and April 2017. The individual hypothesis testing of the gamma coefficient corresponding to the estimated beta of the investment factor shows more dispersed data, but we also find an outlier in November 2000, in June 2007. For the case of the t-statistics of gamma coefficient corresponding to the estimated beta of the momentum, we also find positive outliers in November 2000 and June 2008.

Overall, considering all the t-statistics of the gamma coefficients of the estimated betas of each of the factors, the cross-sectional regression in November 2000 and in June 2007 was an outlier and significant for all.

Summing up all this section, first step of Fama MacBeth (1973) involved the estimation of the betas of the factors, as measures of sensitivity of the portfolios with respect to each of the factors. These estimations have been done individually and with rolling windows, using unifactorial regressions, to capture the variability of the estimated betas. On the second step, the cross-sectional analysis involves estimating the gamma coefficients of the previously estimated betas of the six factors, using multifactorial cross-sectional analysis. The positive significance of these t-statistics would indicate data is able to explain cross-sectional variation, but the results in this paper state the opposite. The extra risk associated to the factors is not rewarded in the market.

5. Conclusions and final remarks

The two steps of Fama MacBeth procedure are the guideline for answering the two questions of this research paper. It is true that the regression of the average returns of portfolios against market premium factor, as the unique independent variable, got a high goodness of fit, (94.4%) but, when including the other factors, size, book-to-market, profitability, investment, and momentum, the goodness of fit improved to 99.7%, almost a perfect regression with no residuals.

Furthermore, when running individual regressions being the dependent variable the returns of each portfolio, and the independent variables the returns mimicking the six factors, the goodness of fit for the unifactorial return generating process, on average the 25 regressions, was of 83.11%. For the case of multifactorial return generating process regressions, on average, the goodness of fit increased to 94.59%. This increase indicates an improvement because of adding factors to the estimation, but the unifactorial process with market premium factor also had a high goodness of fit.

Hence, answering to the first research question of whether Fama & French (1992,2015) five factors, together with momentum factor of Carhartt (1997) improve the estimation of expected returns of assets: yes, the estimation improves, even if it is true the unifactorial estimation was a good estimator per se, adding more factors to the estimation improves the goodness of fit by around 10%. It is probable that considering other database, for instance, developing countries' data, the results vary. What Fama & French (1992) got in their paper was that the goodness of fit improved from around a 60% up to 80%.

The second step of Fama & MacBeth (1973) methodology enables to answer the second research question of this paper: test if, indeed, these factors proposed by Fama & French 81992, 2015) and Carhartt (1997) are risk factor. That is, whether higher risk regarding these factors, is rewarded in the market with higher returns. Because the majority of the cross-sectional regressions did not result in significant t-statistics, from the hypothesis testing is concluded that more risk regarding those factors is not rewarded in the market, and thus, does not generate higher returns.

As it has been empirically shown that those factors are not risk factors, one should wonder on the reasons lying behind this phenomenon, on how could it be possible that some factors are improvers of the expected returns estimations, but are not risk factors rewarded by the market. The fundamentals factors, earnings of companies and valuation, have been introduced, so perhaps what is missing to be introduced are those technical factors.

It is true, however, that some of those technical/external factors are common for all firms, thus referring to the systematic risk, which is almost impossible to identify and eliminate. Some of the external factors that could also have an influence on asset prices are: (1) the track and strength of peers, referring to the co-movement between similar firms, rather than only considering individual performance, (2) assets substitutes that firms compete in the stock market with, (3) news and trends that happen in globalized world, which is in some sense captured by momentum factor, (4) liquidity, reflecting the interest of investors on firms, which could be proxied by size, and/or (5) behaviour of investors and psychology of participants in the market, that could lead to the so called irrational exuberance of financial markets and investors. Based on Shiller R.J. (2000), it is my opinion that the irrational exuberance, the unexpected irrational behaviour of investors, herd behaviour, and many other issues analysed in psychological finance, are factors that influence the returns of assets and that, in fact, cannot be predicted nor ex-ante measured.

One should also consider other factors, for instance, the appearance of digital currencies, crypto assets, the easiness of investing thanks to digitalization, brand new public platforms of investing that lead to new young non-expert investors to participate as agents in stock markets… Digitalization in the past few decades has opened the doors of the financial system not only to ordinary people that can now easily participate in investment decisions, but also to a faster and more dynamic way of functioning. That speed of transactions and the entrance of new agents as investors are factors that may be influencing the prices in the stock market.

To sum up all the previous ideas, the main conclusion of this research paper is that including more factors improve the estimation of expected returns, as it is understood there are more factors apart from the market premium included in the CAPM model, that have an effect on the returns of assets. However, it is true that those factors suggested by Fama & French (1992, 2015) and Carhartt (1997) are not risk factors, as the risk associated to each of the factors does not lead to higher expected returns. The reason why could rely on the fact that the globalization, and/or unstoppable trends and events of todays' word also affect the expected returns of assets, thus the list of factors that one should include in the model would be infinite.

6. Bibliography

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7. Appendix

01. Appendix I: average values of the returns of portfolio, and beta coefficients of size, market premium, book-to-market, profitability, investment, and momentum factors, for each portfolio.

N.º Porfolio	*Mean returns*	*mean beta SMB*	*mean beta MKT*	*mean beta HML*	*mean beta RMW*	*mean beta CMA*	*mean beta WML*
1	,299	.78	.923	.326	-.848	-.939	-.251
2	,640	.73	.905	.465	-.845	-.813	-.253
3	,709	.679	.873	.567	-.943	-.714	-.289
4	,812	.68	.838	.74	-.941	-.478	-.308
5	,960	.699	.824	.992	-1.157	-.175	-.326
6	,561	.679	.983	.272	-.713	-1.138	-.265
7	,762	.623	.929	.517	-.851	-.883	-.342
8	,757	.658	.9	.719	-.93	-.642	-.312
9	,886	.632	.889	.951	-1.108	-.488	-.338
10	,960	.594	.926	1.216	-1.394	-.17	-.39
11	,675	.411	1.028	.199	-.704	-1.311	-.246
12	,850	.411	.955	.55	-.831	-.83	-.283
13	,765	.407	.95	.767	-.941	-.699	-.329
14	,774	.386	.944	.949	-1.112	-.572	-.374
15	,903	.343	1.016	1.255	-1.475	-.196	-.448
16	,780	.056	1	.258	-.696	-1.13	-.25
17	,788	.065	.952	.591	-.865	-.758	-.306
18	,762	.06	.972	.778	-.935	-.584	-.35
19	,751	.115	.995	1.01	-1.23	-.555	-.414
20	,837	.111	1.072	1.381	-1.595	-.264	-.496
21	,632	-.56	.902	-.008	-.388	-.906	-.168
22	,759	-.503	.931	.356	-.494	-.661	-.194
23	,767	-.5	1.046	.654	-.919	-.599	-.303
24	,823	-.485	1.06	.974	-1.22	-.505	-.373
25	,723	-.498	1.213	1.516	-2.011	-.344	-.605

02. Appendix II: Figure with the histograms corresponding to the six-time series of the estimated beta coefficients for size, market premium, book-to-market ratio, profitability, investment, and momentum factors corresponding to portfolio Nº10, as a general representation of the rest of portfolios.