Commitment Power in a Non-Stationary Durable-Good Market.

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Abstract

This paper derives and evaluates the decisions of a durable good monopolist in a context where demand for the services of the durable good changes over time. It shows that, if the size of the market decreases over time, social welfare may be higher when the monopolist has commitment ability than when she has not. Moreover, the equilibrium under a monopolist seller with commitment power may Pareto-dominate the equilibrium under a monopolist seller without commitment ability. The work also proves that these results obtain if there is uncertainty about future demand for the services of the durable good.

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1 Introduction

This work derives and evaluates the decisions of a durable good monopolist in a context where demand for the services of the good changes over time. The firm is the only producer of the good, perhaps due to a patent. The analysis compares the decisions of a monopolist seller with commitment ability and a monopolist seller that has no commitment power, in a context where the monopolist only decides production levels. An evaluation of these decisions with respect to producer's profits, consumers surplus and social welfare is also performed.

The decisions of the durable good monopolist in a context where demand for the durable good changes over time have not been analyzed. The literature on durable goods has, in general, assumed a stationary situation where demand for the services of the durable good does not change over time. An exception is Bhatt (1989), where there is uncertainty about future demand but expected demand in the future equals demand in the present. However, demand varies as more (or less) substitutes become available, population or incomes change, etc.[1] This paper shows the relevance of considering a changing environment in the study of the durable good monopolist decisions and their effects producer's profits, consumers surplus and social welfare.

A general result in the literature on durable goods monopolists (see Bulow (1982), Kahn (1986), Malueg, Solow and Kahn (1988) or, more recently, Chi (1999)) is that welfare under rentals (or under a monopolist seller with commitment ability) is lower than welfare under a monopolist seller that has no commitment power. Hence, attending to this general result, while the monopolist prefers to rent the durable-good, the sale of the units of the good would be recommended from a social welfare perspective.[2] This contradiction between producer preferences and society desires has been invoked to ask for regulation of the durable good monopolist. In the U.S. some producers of durable goods, as IBM and Xerox, that rented the good were required to also sell their output. However, it has also been shown in the literature that if the monopolist seller without commitment ability chooses, in addition to production levels, capacity or technology (Bulow (1982)), durability (Bulow (1986) and Malueg and Solow

[1]Cabral, Salant and Woroch (1999) consider network externalities in durable good consumption. In their analysis, demand for the services of the durable good increases endogenously with the number of buyers of the good.

[2]With some exceptions, the monopolist preference for rentals is very common (see Bulow (1982), Stokey (1981), Gul, Sonneschein and Wilson (1986), Kahn (1986) and Bucovetsky and Chilton (1986)).

(1987)) or investment in cost reducing innovations (Bond and Samuelson (1987)), she may incur in inefficiencies that lower social welfare under sales below social welfare under rentals. Also, it has been proved that social welfare may be higher under rentals if the demand function for the services of the durable good is non linear (Malueg and Solow (1989)) or if the demands of potential users are interdependent (Saracho (1997)). The analysis below shows how previous results are modified when demand of the durable good changes over time.

The work considers the case where the change in demand over time is certain and the case where this change in demand is uncertain. It is first noticed that a monopolist seller may attain the same level of profits (expected profits if there is uncertainty about demand) than a monopolist renter if she can commit to a future price (dependent on which happens to be future demand in the uncertainty case) and, hence, commit to sell or to buy units of the good in the future at that price.

When the change in demand for the services of the durable good is certain, the study differentiates, in a two period context, situations where the market size increases and where the market size decreases. Market size in a given period refers to the number of consumers that have a positive valuation of the durable good in that period. The results for the case where the size of the market increases are analogous to those in the literature on durable goods and may be derived in a similar way. A monopolist seller that has no commitment ability obtains lower profits than a monopolist with commitment power and welfare in the first case is higher than welfare in the latter case.

In the case where the size of the market decreases, however, important differences with previous results obtain. When the size of the market decreases the monopolist will be less interested in selling units of the durable good in the future. Hence, the Coase problem (Coase (1972)) will be less severe. Nevertheless, as the work will show, the different capacity of the monopolist seller without commitment ability and the monopolist renter to dispose of the good in the future maintains the preference of rentals over sales by the producer (however, as it has been mentioned, if the monopolist seller may commit to buy in the future, at the current price, any unit of the durable good she has sold previously, she may obtain as much profit as a leasing monopolist does).

When the size of the market decreases it is shown that welfare may be higher under a monopolist seller with commitment ability (or a monopolist renter) than under a monopolist seller without commitment power and the equilibrium under a monopolist seller with commitment ability may Pareto-dominate the equilibrium under a monopolist seller without commitment power. The paper derives the set of parameter values where each of these results occur.

When the change in demand for the services of the durable good is uncertain, the work focuses on the situation where in period 1 the agents don't know if market size will increase or decrease in the future. At the beginning of period 2 all agents learn which is the demand in that period and, with this information, take their period 2 decisions. In this context the paper shows that welfare may be higher under a monopolist seller with commitment ability (or a monopolist renter) than under a monopolist seller without commitment power. Moreover, the equilibrium under a monopolist seller with commitment ability may Pareto-dominate the equilibrium under a monopolist seller without commitment power. As in the case of certainty, the work obtains the set of parameter values where each result occurs.

The results derived in this paper ask for a specific analysis when commitment mechanisms or regulation of durable good production under a changing environment are being considered.

The paper is organized as follows: Section 2 presents the basic model. Section 3 analyzes, in the case where the change in demand is certain, the decisions of the monopolist renter, the monopolist seller with commitment ability and the monopolist seller that has no commitment power, and compares the monopolist profits in these three contexts. An evaluation of the consequences in terms of social welfare and consumers surplus of the decisions derived in Section 3 is performed in Section 4. Section 5 studies the situation where there is uncertainty about future demand for the services of the durable good. The last section summarizes the results. All proofs are included in the Appendix.

2 Model

Consider a durable-good produced by a monopolist and demanded by many price taking buyers in a two-period framework. The analysis below will be developed under the following set of assumptions:

A.1- Any unit of the durable-good produced in period 1 may be used again in period 2, with no depreciation.

A.2- The monopolist and the consumers have the same discount factor $s = \frac{1}{1+r}$, where r is the interest rate.

A.3- The monopolist and the consumers are completely informed about demand for the durable-good and production costs.

A.4- The potential consumers of the durable-good have perfect foresight.

A.5- If the monopolist sells the durable-good, there is a resale market in which the units of the durable-good bought during the first period can be resold to other consumers in the second period.

A.6- The cost of producing the good is zero and there is free disposal of the durable-good.

Assumption A.5 assures that the different units of the durable-good will be used in each period by those consumers that value it most. An alternative would be to assume that demand curves for the services of the durable-good have perfect rank correlation or to admit that any unit of the durable-good bought from a monopolist seller in period 1 may be sold to that monopolist in the future at the current price.

Under the free disposal assumption introduced in A.6 a monopolist renter may decide not to rent in period 2 some of the units of the durable good she has produced and leased in period 1.[3]

The results will be obtained in a context where the demand for the services of the durable-good is linear in each period and the monopolist chooses only her production levels.

Let us denote by q_i and y_i, respectively, the amounts of the durable good produced and used in period i. Also, let $e(y_1) = e - fy_1$ be the inverse demand function for the services of the durable good in period 1 and $g(y_2) = g - hy_2$ be the inverse demand function for the services of the durable good in period 2. We will say that the size of the market increases when $\frac{e}{f} < \frac{g}{h}$ and we will say that the size of the market decreases when $\frac{e}{f} > \frac{g}{h}$. Notice that willingness to pay for the first units of the good, that depends on the comparison between g and e, may change over time in a different direction than market size. In a market with a size that decreases over time we may have $g > e$ and, hence, willingness to pay for the first units of the good may increase. Moreover, we may have $g \neq e$ when $\frac{e}{f} = \frac{g}{h}$.

In an interpretation of this variation of demand over time we have consumers with different willingness to pay for the services of the good in a context where each consumer uses at most one unit of the durable good. Demand varies because willingness to pay for the services of the good changes over time. The willingness to pay of some consumers may change from zero to a positive number or conversely. This interpretation is implicit in the analysis presented in this

[3]Free disposal also implies that consumers may not use in period 2 some of the units of the durable good they have bought in period 1 from the monopolist seller.

paper. A different interpretation would consider that each consumer has the same decreasing demand curve for units of the durable good. With time, this demand curve, identical for all consumers, changes.

3 Monopolist decisions

The monopolist renter will maximize profits in each period. Hence, he will rent $\frac{e}{2f}$ in period 1 and $\frac{g}{2h}$ in period 2, and rental prices will be $p_1 = \frac{e}{2}$ and $p_2 = \frac{g}{2}$. As $q_1 = y_1$ and $q_2 = \max\{0, y_2 - y_1\}$ production under rentals will be $q_1 = \frac{e}{2f}$ and $q_2 = \max\left\{0, \frac{g}{2h} - \frac{e}{2f}\right\}$. If the size of the market decreases ($\frac{e}{f} > \frac{g}{h}$) the renter will dispose of $\frac{e}{2f} - \frac{g}{2h}$ units of the good in period 2. Under rentals, the monopolist profits are

$$\pi^{rent} = \frac{e^2}{4f} + s\frac{g^2}{4h}$$

Often, rentals are not feasible. It may be very costly to check if the good is returned in perfect conditions after a rental period. Or it may be difficult to get compensation from consumers if they have damaged the good. In these situations the monopolist may prefer to sell the durable good.

The decision of the monopolist seller will depend on her commitment ability. Consider first the decision of a monopolist seller that can commit to a future price level in a situation where this commitment is credible. If the size of the market does not decrease ($\frac{e}{f} \leq \frac{g}{h}$), commitment to a future price level implies commitment to a future production level and she will solve

$$\max_{q_1, q_2} q_1(e - fq_1 + s(g - hq_1 - hq_2)) + sq_2(g - hq_1 - hq_2)$$

Hence, the monopolist seller will select in each period the same production level than the monopolist renter. If the size of the market decreases ($\frac{e}{f} > \frac{g}{h}$), this monopolist may also obtain as much profit as a leasing monopolist does by committing to buy in period 2, at a price equal to $\frac{g}{2}$, any unit she has sold in period 1. In this case, $q_1 - \frac{g}{2h}$ units of the durable good will be resold to the monopolist in period 2, and the monopolist seller with commitment ability will solve

$$\max_{q_1} q_1(e - fq_1 + s\frac{g}{2}) - s\frac{g}{2}(q_1 - \frac{g}{2h})$$

The equivalence in profits with the monopolist renter requires the feasibility of these repurchase agreements. When the size of the market decreases over time it is not enough for that equivalence the ability to commit to a future production level by the monopolist seller. Moreover, best-price provisions is not an alternative similar to this repurchase agreement, as no sales in period 2 are required to attain the monopolist renter profits.[4]

The first period decision of a monopolist with ability to commit to those repurchase agreements is a flexible decision as she may undo that decision by repurchasing in period 2 units of the durable good she has sold in period 1. However, the first period decision of a monopolist without commitment power is an irreversible decision as any units sold in period 1 will remain available for use by consumers in period 2.[5]

Consider now the decision of the monopolist seller that has no commitment power. A future price is credible to the consumers in period 1 if and only if it is part of a subgame perfect equilibrium. Let us denote $M \equiv \frac{2eh}{\sqrt{4f(4f+sh)}}$. We have:

Proposition 1 *: Production decisions and profits of a monopolist seller that has no commitment power are:*[6]

i) If $g < M$*:*

$$q_1 = \frac{e}{2f} \text{ and } q_2 = 0$$

$$\pi_{nc}^{sale} = \frac{e^2}{4f}$$

ii) If $g > M$*:*

$$q_1 = \frac{2e}{4f+sh} \text{ and } q_2 = \frac{g}{2h} - \frac{e}{4f+sh}$$

$$\pi_{nc}^{sale} = \frac{e^2}{4f+sh} + s\frac{g^2}{4h}$$

When $g = M$ the monopolist seller that has no commitment power is indifferent between deciding $q_1 = \frac{e}{2f}$ and $q_2 = 0$ and deciding $q_1 = \frac{2e}{4f+sh}$ and $q_2 = \frac{g}{2h} - \frac{e}{4f+sh}$, as, from the proof of Proposition 1, $g = M \Leftrightarrow \frac{e^2}{4f} = \frac{e^2}{4f+sh} + s\frac{g^2}{4h}$.

[4]On best-price provisions in a context where demand does not change over time see Butz (1990).

[5]A decision is more flexible if it leaves more alternatives available for the future. As a consequence, a more flexible decision permits a better adjustment to any information arriving in the future. Often, a more flexible decision is a decision that implies less commitment. However, in the context of this work the ability to commit to a future price leaves more alternatives open in the future (among them, the alternative corresponding to the case where there is not ability to commit). See Jones and Ostroy (1984) and Usategui (1990) for development within the literature on flexibility.

[6]Subindex c is used for variables referred to the monopolist with commitment power and subindex nc is used for variables referred to the monopolist without commitment ability.

If the size of the market does not decrease ($\frac{e}{f} \leq \frac{g}{h}$) a monopolist seller without commitment ability produces less than a monopolist with commitment power in the first period, but more in the second period and in total. If the size of the market decreases ($\frac{e}{f} > \frac{g}{h}$) the decision of the monopolist seller that has no commitment power depends on the relationship between g and M, as it is shown in Proposition 1. When $g < M$, the monopolist sells $\frac{e}{2f}$ units of the good in period 1 and there is not demand left to be served in period 2 as $\frac{g}{h} < \frac{M}{h} < \frac{e}{2f}$. As in this case the monopolist decision implies no sales in period 2, production in period 1 is the same than under rentals (there are no sales in period 2 competing for consumers with the first period sales).

Clearly, we also have:

Corollary 1 *: It is*

$$\pi_c^{sale} = \pi^{rent} > \pi_{nc}^{sale}$$

Notice that when the monopolist seller without commitment ability decides the same production levels than a monopolist seller with commitment ability, or a leasing monopolist, does (i.e., $q_1 = \frac{e}{2f}$ and $q_2 = 0$ in a market that decreases in size) the profits she obtains are lower. The monopolist with commitment ability buys in period 2, at a price equal to $\frac{g}{2}$, $\frac{e}{2f} - \frac{g}{2h}$ units of the good she has sold in period 1 and only $\frac{g}{2h}$ units of the good are used in period 2. However, the monopolist seller without commitment power has sold already $\frac{e}{2f}$ units of the good in period 1 and only the buyers of these units can dispose of them in period 2. When this monopolist decides $q_1 = \frac{e}{2f}$ and $q_2 = 0$, it is $\frac{g}{h} < \frac{e}{2f}$ and the buyers use $\frac{g}{h}$ units of the durable good in period 2. In this case, the monopolist seller without commitment ability sells just $\frac{e}{2f}$ units of the good in period 1 at a price of $\frac{e}{2}$ per unit and her total profits are equal to the first-period profits of the monopolist seller with commitment power. We could say that the monopolist seller that has no commitment power can only commit to a future price equal to zero.

4 Welfare analysis

Let us measure social welfare as the present value of the sum of consumers and producer's surplus. Given the results in the previous section, we can prove the following relationship between social welfare under a monopolist seller with commitment ability (W_c^{sale}), or under a leasing monopolist ($W^{rent} = W_c^{sale}$), and social welfare with a monopolist seller that has no commitment power (W_{nc}^{sale}):

Proposition 2 *: i) If $g < M$ it is $W_c^{sale} < W_{nc}^{sale}$*

ii) If $g > M$ it is:

$$W_c^{sale} \gtreqless W_{nc}^{sale} \Leftrightarrow \frac{3eh}{4f} \gtreqless g \qquad (1)$$

When $g = M$ we know that the monopolist seller that has no commitment power is indifferent between deciding $q_1 = \frac{e}{2f}$ and $q_2 = 0$ and deciding $q_1 = \frac{2e}{4f+sh}$ and $q_2 = \frac{g}{2h} - \frac{e}{4f+sh}$. If the monopolist chooses $q_1 = \frac{e}{2f}$ and $q_2 = 0$ it will be $W_c^{sale} < W_{nc}^{sale}$. However, if the monopolist decides $q_1 = \frac{2e}{4f+sh}$ and $q_2 = \frac{g}{2h} - \frac{e}{4f+sh}$ it will be $W_c^{sale} > W_{nc}^{sale}$ as $M = \frac{he}{\sqrt{f(4f+sh)}} < \frac{3eh}{4f}$.

Therefore, when $g \in (M, \frac{3eh}{4f})$ welfare under a monopolist seller with commitment ability is higher than welfare under a monopolist seller that has no commitment ability. In these situations, the size of the market is decreasing over time and it is $y_{c1}^{sale} > y_{nc1}^{sale}$ and $y_{c2}^{sale} < y_{nc2}^{sale}$. However, the higher welfare under a monopolist seller without commitment ability in period 2 does not outweigh the higher welfare under a monopolist seller with commitment power in period 1. The result requires a change in the demand for the services of the durable good over time such that the monopolist that has no commitment power sells in both periods and there is a sufficient decrease in the size of the market ($\frac{3e}{4f} > \frac{g}{h}$).[7]

From Proposition 2 we have that it may be $W_c^{sale} > W_{nc}^{sale}$ in situations where demand curves in both periods are parallel ($e = 3$, $g = 2$, $f = h = 1$ and $s = 1$, for instance), have the same ordinate at the origin ($e = g = h = 2$, $f = 1$ and $s = 1$) or cross each other ($e = f = 1$, $g = 2$, $h = 3$ and $s = 1$).

We have shown that, when the size of the market decreases, welfare under a monopolist seller with commitment ability may be higher than welfare when the monopolist has no commitment power. Moreover, we can prove:

Proposition 3 *: If parameter values are such that $M < g < \frac{eh}{4f}$ the equilibrium under a monopolist seller with commitment power will Pareto-dominate the equilibrium under a monopolist seller without commitment ability.*

Let us use CS to denote consumers surplus, as in the proof of Proposition 3. When $W_c^{sale} > W_{nc}^{sale}$, we have also that $CS_c^{sale} > CS_{nc}^{sale}$ in the cases where the change in the demand for the services of the durable good over time is such that the monopolist that has no commitment power sells in both periods and there is an

[7] If $e = g$ and $f = h$, that is, if the demand function is the same in both periods, it will be, from (1), $W_c^{sale} < W_{nc}^{sale}$, as expected.

important decrease in the size of the market ($\frac{e}{4f} > \frac{g}{h}$). An example of a situation where $W_c^{sale} > W_{nc}^{sale}$ and $CS_c^{sale} > CS_{nc}^{sale}$ is $e = f = 1$, $g = 5$, $h = 21$ and $s = 1$.

Therefore, when $g \in (M, \frac{eh}{4f})$ it is $W_c^{sale} > W_{nc}^{sale}$, $\pi_c^{sale} > \pi_{nc}^{sale}$ and $CS_c^{sale} > CS_{nc}^{sale}$, and to establish a mechanism allowing the monopolist to get commitment ability may be advisable (analogously, a regulation inducing a monopolist without commitment ability to sell the durable good may be less advisable).

5 Uncertainty about future demand

In the context of the model of Section 2 let again $e(y_1) = e - fy_1$ be the inverse demand function for the services of the durable good in period 1, and assume that in period 1 all agents think there is a probability x that the inverse demand function in period 2 is $b(y_2) = b - cy_2$ and a probability $1 - x$ that the inverse demand function in period 2 is $v(y_2) = v - zy_2$, with $\frac{v}{z} < \frac{e}{f} < \frac{b}{c}$. Hence, they think there is a probability x of an increase in the size of the market and a probability $1 - x$ of a decrease in the size of the market. At the beginning of period 2 all agents learn which is the demand in that period and, with this information, take their period 2 decisions. Therefore, instead of assumption A.3 of Section 2 we have: The monopolist and the consumers are completely informed about demand for the durable good in period 1 and production costs, have the same beliefs in period 1 about demand in period 2, and learn at the beginning of period 2 which is the demand in that period. Moreover, consider that all agents participating in the market are risk neutral and that assumptions A.1, A.2, A.4, A.5 and A.6 of Section 2 hold.[8]

Denote $g \equiv xb + (1-x)v$ and $h \equiv xc + (1-x)z$. Therefore, g is the expected ordinate at the origin of the inverse demand curve in period 2 and h is the expected slope of that curve. The ratio $\frac{g}{h}$ will represent the market size corresponding to the curve with the expected ordinate at the origin and the expected slope. Let us call this curve the expected inverse demand curve in period 2.

The monopolist renter will maximize profits in each period. Hence, he will rent $\frac{e}{2f}$ in period 1, $\frac{b}{2c}$ in period 2 if $b(y_2)$ happens to be the inverse demand

[8]Bhatt (1989) considers risk aversion in a model where the slope of the demand curve is the same in both periods, the expected level of the ordinate at the origin is unchanged and production in period 2 is positive. The main results in this section require, however, a change over time in the slope of the demand curve or in the ordinate at the origin.

function in period 2 and $\frac{v}{2z}$ if that function is $v(y_2)$. Rental prices will be, respectively, $p_1 = \frac{e}{2}$, $p_{2b} = \frac{b}{2}$ and $p_{2v} = \frac{v}{2}$.[9] The monopolist renter will dispose in period 2 of units of the good produced in period 1 if the inverse demand function in period 2 is $v(y_2)$. Under rentals, the monopolist expected profits are

$$\pi^{rent} = \frac{e^2}{4f} + sx\frac{b^2}{4c} + s(1-x)\frac{v^2}{4z}$$

A monopolist seller may obtain profits equal to π^{rent} if she can commit to a future price or production level in case the inverse demand function in period 2 happens to be $b(y_2)$, and if she can also commit, in case the inverse demand function in period 2 happens to be $v(y_2)$, to buy in period 2 at a price equal to $\frac{v}{2}$ any unit she has sold in period 1. The argument proceeds as in Section 3.

Consider now the decision of a monopolist seller without commitment power. In this case we know that a future price is credible to the consumers in period 1 if and only if it is part of a subgame perfect equilibrium. Let us denote $N \equiv \frac{2ez}{\sqrt{(4f+sxc)(4f+sh)}}$. We have:

Proposition 4 : *Production decisions and profits of a monopolist seller that has no commitment power are:*

i) If $v < N$*:*

$$q_1 = \frac{2e}{4f+sxc} \text{ , } q_{2b} = \frac{b}{2c} - \frac{e}{4f+sxc} \textit{ and } q_{2v} = 0$$

$$\pi_{nc}^{sale} = \frac{e^2}{4f+sxc} + sx\frac{b^2}{4c}$$

ii) If $v > N$*:*

$$q_1 = \frac{2e}{4f+sh} \text{ , } q_{2b} = \frac{b}{2c} - \frac{e}{4f+sh} \textit{ and } q_{2v} = \frac{v}{2z} - \frac{e}{4f+sh}$$

$$\pi_{nc}^{sale} = \frac{e^2}{4f+sh} + sx\frac{b^2}{4c} + s(1-x)\frac{v^2}{4z}$$

When $v = N$ the monopolist seller that has no commitment power is indifferent between deciding $q_1 = \frac{2e}{4f+sxc}$, $q_{2b} = \frac{b}{2c} - \frac{e}{4f+sxc}$ and $q_{2v} = 0$ and deciding $q_1 = \frac{2e}{4f+sh}$, $q_{2b} = \frac{b}{2c} - \frac{e}{4f+sh}$ and $q_{2v} = \frac{v}{2z} - \frac{e}{4f+sh}$ as, from the proof of Proposition 4, $v = N \Leftrightarrow \frac{e^2}{4f+sxc} + sx\frac{b^2}{4c} = \frac{e^2}{4f+sh} + sx\frac{b^2}{4c} + s(1-x)\frac{v^2}{4z}$.

[9] Subindex b is used for variables referred to the case where the inverse demand function in period 2 is $b(y_2)$ and subindex v is used for variables referred to the case where the inverse demand function in period 2 is $v(y_2)$.

Notice from Proposition 4 that when $v < N$ we obtain that q_1 decreases with x. The intuition for this result is that the higher is x the greater is the probability that the monopolist will sell more units in period 2 and, hence, the incidence of the Coase problem is greater. As a consequence, the monopolist finds it profitable to reduce q_1 and in this way to make credible that in period 2 (when the inverse demand function is $b(y_2)$) the price will not be very low or the number of units in the market will not be very high. If $v > N$, q_1 is even lower. In this case the monopolist will always sell more units in period 2 and, hence, the impact of the Coase problem is greater (there will always be future sales competing for consumers with present sales).

Clearly, we also have

Corollary 2 *: It is*

$$\pi_c^{sale} = \pi^{rent} > \pi_{nc}^{sale}$$

Let us proceed now with the welfare analysis in this context of uncertainty about future demand. Given the result in Proposition 4, we can prove the following relationship between social welfare under a monopolist seller with commitment ability (W_c^{sale}), or under a leasing monopolist ($W^{rent} = W_c^{sale}$), and social welfare with a monopolist seller that has no commitment power (W_{nc}^{sale}):

Proposition 5 *: i) If $v < N$ it is $W_c^{sale} < W_{nc}^{sale}$*

ii) If $v > N$ it is:

$$W_c^{sale} \gtreqless W_{nc}^{sale} \Leftrightarrow \frac{3eh}{4f} \gtreqless g$$

When $v = N$ we know that the monopolist seller that has no commitment power is indifferent between the solutions in parts i) and ii) of Proposition 4. If the monopolist chooses the solution in part i) it will be $W_c^{sale} < W_{nc}^{sale}$. However, when the monopolist decides the solution in pat ii) it will be $W_c^{sale} > W_{nc}^{sale}$ if $\frac{3eh}{4f} > xb + (1-x)N$.

Therefore, welfare under a monopolist seller with commitment ability may be higher than welfare under a monopolist seller without commitment ability. In these situations, the expected size of the market is decreasing over time and it is $y_{c1}^{sale} > y_{nc1}^{sale}$, $y_{c2b}^{sale} < y_{nc2b}^{sale}$ and $y_{c2v}^{sale} < y_{nc2v}^{sale}$. However, the higher welfare under a monopolist seller without commitment ability in period 2 does not outweigh the higher welfare under a monopolist seller with commitment power in period 1. The result requires beliefs about future demand such that the monopolist that has no commitment power would sell in both periods even if there is a decrease in the size of the market in period 2 and the market size of the expected inverse demand curve in period 2 is sufficiently smaller than the market size in period 1 ($\frac{3e}{4f} > \frac{g}{h}$).[10]

We have shown that welfare under a monopolist seller with commitment ability may be higher than welfare when the monopolist has no commitment power. Moreover, we can prove:

Proposition 6 *: If parameter values are such that $v > N$ and $\frac{e}{4f} > \frac{g}{h}$ the equilibrium under a monopolist seller with commitment power will Pareto-dominate the equilibrium under a monopolist seller without commitment ability.*

When $W_c^{sale} > W_{nc}^{sale}$, we have also that $CS_c^{sale} > CS_{nc}^{sale}$ in the cases where there is an important decrease in the expected size of the market ($\frac{e}{4f} > \frac{g}{h}$). An example of a situation where $W_c^{sale} > W_{nc}^{sale}$ and $CS_c^{sale} > CS_{nc}^{sale}$ is $e = f = 1$, $b = 23$, $v = 3$, $c = 12$, $z = 22$, $x = 0.1$ and $s = 1$. When $W_c^{sale} > W_{nc}^{sale}$ and $CS_c^{sale} > CS_{nc}^{sale}$, it may be advisable, as in the case analyzed in the previous section, to establish a mechanism allowing the monopolist to get commitment ability (analogously, a regulation inducing a monopolist without commitment ability to sell the durable good may be less advisable).

6 Conclusion

In this paper we have analyzed the decisions of a durable good monopolist in a context where demand for the services of the durable good changes over time. The analysis has compared the decisions of a monopolist seller with commitment ability and a monopolist seller that has no commitment power, in a context where the monopolist only decides production levels. An evaluation of these decisions with respect to producer's profits, consumers surplus and social welfare has also been performed.

The work has considered the case where the change in demand over time is certain and the case where this change in demand is uncertain. It has been noticed that a monopolist seller may attain the same level of profits (expected profits if there is uncertainty about demand) than a monopolist renter would get if she can commit to a future price (dependent on which happens to be future demand in the

[10] An example of a situation where $W_c^{sale} > W_{nc}^{sale}$ is $e = 5$, $b = 6$, $v = 1$, $f = 1$, $c = 0.5$, $z = 1.5$, $x = 0.5$ and $s = 1$.

uncertainty case) and, hence, commit to sell or to buy units of the good in the future at that price.[11]

When the change in demand for the services of the durable good is certain, the study has differentiated, in a two period context, situations where the market size increases and where the market size decreases. The results for the case where the size of the market increases are analogous to those in the literature on durable goods. In the case where the size of the market decreases, however, important differences with previous results have been obtained. In this case it has been proved that welfare may be higher under a monopolist seller with commitment ability (or a monopolist renter) than under a monopolist seller without commitment power and the equilibrium under a monopolist seller with commitment ability may Pareto-dominate the equilibrium under a monopolist seller without commitment power. These results have also been obtained for the case where the change in demand for the services of the durable good is uncertain. The paper has derived the set of parameter values where each of these results occur.

This paper has shown the relevance of considering a changing environment in the study of the durable good monopolist decisions and their effects on producer's profits, consumers surplus and social welfare. The results obtained ask for a specific analysis when commitment mechanisms or regulation of durable good production under a changing environment are being considered.

7 Appendix

7.1 Proof of Proposition 1

The monopolist seller that has no commitment power will solve in period 2

$$\max_{q_2} q_2(g - hq_1 - hq_2)$$

subject to $q_2 \geq 0$. The solution to this maximization problem, if the nonnegativity restriction is satisfied, is $q_2 = \frac{g-hq_1}{2h}$. In this case $p_2 = \frac{g-hq_1}{2}$.

Since consumers have rational expectations, the sale price of period 1 equals the sum of the rental value for period 1 and the discounted sale price of period 2.

[11] If the monopolist seller cannot commit to buy in period 2 at a given price any unit she has sold in period 1 but she can commit to a future production level, it can be shown that, when the market size decreases over time, profits of this monopolist seller will be lower than profits of the monopolist renter. Moreover, when market size decreases over time welfare under rentals may be higher than welfare under this monopolist seller that cannot commit to repurchase agreements.

Hence, the monopolist will select q_1 that solves

$$\max_{q_1}(q_1(e - fq_1 + s\frac{g - hq_1}{2}) + s\frac{(g - hq_1)^2}{4h})$$

This monopolist problem solves to

$$q_1 = \frac{2e}{4f + sh} \qquad q_2 = \frac{g}{2h} - \frac{e}{4f + sh}$$

$$p_1 = s\frac{g}{2} + \frac{2fe}{4f + sh} \qquad p_2 = \frac{g}{2} - \frac{eh}{4f + sh}$$

This solution requires $q_2 = \frac{g}{2h} - \frac{e}{4f+sh} \geq 0$, i.e., $g \geq \frac{2he}{4f+sh}$. The monopolist's profits in this situation will be:

$$\pi_{nc}^{sale} = \frac{e^2}{4f + sh} + s\frac{g^2}{4h} \tag{2}$$

Let us consider the case where $g < \frac{2he}{4f+sh}$. If the monopolist chooses q_1 so that $q_1 \leq \frac{g}{h}$ and selects q_2 in order to maximize profits in period 2 it will be $q_2 = \frac{g-hq_1}{2h} \geq 0$. Therefore, this q_1 would solve

$$\max_{q_1} A \equiv (q_1(e - fq_1 + s\frac{g - hq_1}{2}) + s\frac{(g - hq_1)^2}{4h})$$

subject to $q_1 \leq \frac{g}{h}$. As we have $q_1 < \frac{2e}{4f+sh}$, it will be $\frac{dA}{dq_1} = e - q_1(2f + s\frac{h}{2}) > 0$. Hence, in this problem the monopolist would select $q_1 = \frac{g}{h}$ and $q_2 = 0$. However, this decision does not maximize the seller's profits. Notice that when there are sales only in period 1 and the sales price of period 2 is zero, the monopolist prefers to select $q_1 = \frac{e}{2f}$ (in period 2 consumers would use only $\frac{g}{h}$ units of the good: they would dispose of the rest of units bought in period 1). As $g < \frac{2he}{4f+sh}$ $\Rightarrow \frac{g}{h} < \frac{2e}{4f+sh} < \frac{e}{2f}$, we conclude that when $g < \frac{2he}{4f+sh}$ it will be $q_1 = \frac{e}{2f}$, $q_2 = 0$, $p_1 = \frac{e}{2}$ and $\pi_{nc}^{sale} = \frac{e^2}{4f}$.

The monopolist seller without commitment ability may decide $q_1 = \frac{e}{2f}$ and $q_2 = 0$ even if $g \geq \frac{2he}{4f+sh}$. It is easy to show that if $g < \frac{he}{\sqrt{f(4f+sh)}} (\equiv M)$ the value of π_{nc}^{sale} in (2) is lower than $\frac{e^2}{4f}$. As in this case $\frac{g}{h} < \frac{e}{\sqrt{f(4f+sh)}} < \frac{e}{2f}$, it is credible that $q_2 = 0$ when $q_1 = \frac{e}{2f}$ and the use of $p_1 = \frac{e}{2}$ is correct.

Moreover, as the value of π_{nc}^{sale} in (2) when $M < g$ is greater than $\frac{e^2}{4f}$, the proof is concluded.■

7.2 Proof of Proposition 2

We have

$$W_c^{sale} = \int_0^{\frac{e}{2f}} (e - f\theta)d\theta + s\int_0^{\frac{g}{2h}} (g - h\theta)d\theta = \frac{3}{8}(\frac{e^2}{f} + s\frac{g^2}{h})$$

and:

- if $g < M$:

$$W_{nc}^{sale} = \int_0^{\frac{e}{2f}} (e - f\theta)d\theta + s\int_0^{\frac{g}{h}} (g - h\theta)d\theta = \frac{3}{8}\frac{e^2}{f} + s\frac{g^2}{2h}$$

- if $M < g$:

$$W_{nc}^{sale} = \int_0^{\frac{2e}{4f+sh}} (e - f\theta)d\theta + s\int_0^{\frac{g}{2h}+\frac{e}{4f+sh}} (g - h\theta)d\theta = \frac{12e^2 + 4seg}{8(4f+sh)} + \frac{3}{8}s\frac{g^2}{h}$$

From the expressions above it is clear that $W_c^{sale} < W_{nc}^{sale}$ if $g < M$. To compare W_c^{sale} and W_{nc}^{sale} when $M < g$ notice that:

$$\frac{3e^2(4f+sh)}{8f(4f+sh)} = \frac{1}{8(4f+sh)}(12e^2 + \frac{3se^2h}{f})$$

Hence, it will be:

$$W_c^{sale} \gtreqless W_{nc}^{sale} \Leftrightarrow \frac{3se^2h}{f} \gtreqless 4seg \Leftrightarrow \frac{3eh}{4f} \gtreqless g \blacksquare$$

7.3 Proof of Proposition 3

Let CS represent consumers surplus. When $g \in (M, \frac{3eh}{4f})$ it is $W_c^{sale} > W_{nc}^{sale}$ and

$$CS_c^{sale} - CS_{nc}^{sale} = W_c^{sale} - \pi_c^{sale} - (W_{nc}^{sale} - \pi_{nc}^{sale}) = \frac{1}{8}\frac{e^2}{f} - \frac{e^2 + seg}{2(4f+sh)}$$

Hence,

$$CS_c^{sale} \gtreqless CS_{nc}^{sale} \Leftrightarrow \frac{eh}{4f} \gtreqless g$$

and the result is obtained.$\blacksquare$

7.4 Proof of Proposition 4

The monopolist seller that has no commitment power will solve in period 2

$$\max_{q_{2b}} q_{2b}(b - cq_1 - cq_{2b})$$

subject to $q_{2b} \geq 0$, if $b(y_2)$ is the inverse demand function in that period and

$$\max_{q_{2v}} q_{2v}(v - zq_1 - zq_{2v})$$

subject to $q_{2v} \geq 0$, if $v(y_2)$ is the inverse demand function in that period. The solutions to these maximization problems, if the nonnegativity restrictions are satisfied, are $q_{2b} = \frac{b-cq_1}{2c}$ ($p_{2b} = \frac{b-cq_1}{2}$) and $q_{2v} = \frac{v-zq_1}{2z}$ ($p_{2v} = \frac{v-zq_1}{2}$).

Since consumers have rational expectations and are risk neutral, the sale price of period 1 equals the sum of the rental value for period 1 and the discounted expected sale price of period 2. Hence, the monopolist will select q_1 that solves

$$\max_{q_1}(q_1(e-fq_1+sx\frac{b-cq_1}{2}+s(1-x)\frac{v-zq_1}{2})+sx\frac{(b-cq_1)^2}{4c}+s(1-x)\frac{(v-zq_1)^2}{4z})$$

This monopolist problem solves to

$$q_1 = \frac{2e}{4f+sh},\ q_{2b} = \frac{b}{2c} - \frac{e}{4f+sh} \text{ and } q_{2v} = \frac{v}{2z} - \frac{e}{4f+sh}$$

$$p_1 = \frac{2ef}{4f+sh} + s\frac{g}{2},\ p_{2b} = \frac{b}{2} - \frac{ec}{4f+sh} \text{ and } p_{2v} = \frac{v}{2} - \frac{ez}{4f+sh}$$

This solution requires $q_{2v} = \frac{v}{2z} - \frac{e}{4f+sh} \geq 0$, i.e., $v \geq \frac{2ez}{4f+sh}$ (notice that, as $\frac{b}{c} > \frac{v}{z}$, it is $q_{2v} > 0 \Rightarrow q_{2b} > 0$). The monopolist's expected profits in this situation will be:

$$\pi_{nc}^{sale} = \frac{e^2}{4f+sh} + sx\frac{b^2}{4c} + s(1-x)\frac{v^2}{4z} \tag{3}$$

Let us consider the case where $v < \frac{2ez}{4f+sh}$. If the monopolist chooses q_1 so that $q_1 \leq \frac{v}{z}$ and selects q_{2v} in order to maximize profits in period 2, when the inverse demand function is $v(y_2)$, it will be $q_{2v} = \frac{v-zq_1}{2z} \geq 0$. Therefore, this q_1 would solve

$$\max_{q_1} A \equiv (q_1(e-fq_1+sx\frac{b-cq_1}{2}+s(1-x)\frac{v-zq_1}{2})+sx\frac{(b-cq_1)^2}{4c}+s(1-x)\frac{(v-zq_1)^2}{4z})$$

subject to $q_1 \leq \frac{v}{z}$. As we have $q_1 \leq \frac{v}{z} < \frac{2e}{4f+sh}$, it will be $\frac{dA}{dq_1} = e - q_1(2f + \frac{sh}{2}) > 0$. Hence, in this problem the monopolist would select $q_1 = \frac{v}{z}$, $q_{2b} = \frac{b-cq_1}{2c} = \frac{\frac{b}{c}-\frac{v}{z}}{2}$ and $q_{2v} = 0$. However, this decision does not maximize the seller's profits as, if the monopolist is not going to sale any unit of the durable good in period 2 when the inverse demand function is $v(y_2)$, she would choose q_1 such that

$$\max_{q_1}(q_1(e - fq_1 + sx\frac{b-cq_1}{2}) + sx\frac{(b-cq_1)^2}{4c})$$

This monopolist problem solves to

$$q_1 = \frac{2e}{4f+sxc}\ , q_{2b} = \frac{b}{2c} - \frac{e}{4f+sxc} \text{ and } q_{2v} = 0$$

$$p_1 = \frac{2ef}{4f+sxc} + sx\frac{b}{2}\ , p_{2b} = \frac{b}{2} - \frac{ec}{4f+sxc} \text{ and } p_{2v} = 0$$

(notice that, as $\frac{2e}{4f+sxc} > \frac{2e}{4f+sh} > \frac{v}{z}$ consumers would use only $\frac{v}{z}$ units of the good in period 2, if $v(y_2)$ is the inverse demand function in that period, and they would dispose of the rest of units bought in period 1). The monopolist's profits in this situation will be:

$$\pi_{nc}^{sale} = \frac{e^2}{4f+sxc} + sx\frac{b^2}{4c} \tag{4}$$

The monopolist seller without commitment ability may decide $q_1 = \frac{2e}{4f+sxc}$, $q_{2b} = \frac{b}{2c} - \frac{e}{4f+sxc}$ and $q_{2v} = 0$ even if $v \geq \frac{2ez}{4f+sh}$. It is easy to show that if $v < \frac{2ez}{\sqrt{(4f+sxc)(4f+sh)}} (\equiv N)$ the value of π_{nc}^{sale} in (3) is lower than the value of π_{nc}^{sale} in (4). As in this case $\frac{v}{z} < \frac{2e}{\sqrt{(4f+sxc)(4f+sh)}} < \frac{2e}{4f+sxc}$, it is credible that $q_{2v} = 0$ when $q_1 = \frac{2e}{4f+sxc}$ and the use of $p_1 = \frac{2ef}{4f+sxc} + sx\frac{b}{2}$ is correct.

Moreover, as the value of π_{nc}^{sale} in (3) when $N < v$ is greater than the value of π_{nc}^{sale} in (4), the proof is concluded.■

7.5 Proof of Proposition 5

We have

$$W_c^{sale} = \int_0^{\frac{e}{2f}} (e - f\theta)d\theta + sx\int_0^{\frac{b}{2c}} (b - c\theta)d\theta + s(1-x)\int_0^{\frac{v}{2z}} (v - z\theta)d\theta =$$

$$= \frac{3e^2}{8f} + sx\frac{3b^2}{8c} + s(1-x)\frac{3v^2}{8z}$$

and:

- if $v < N$:

$$W_{nc}^{sale} = \int_0^{\frac{2e}{4f+sxc}} (e - f\theta)d\theta + sx\int_0^{\frac{b}{2c}+\frac{e}{4f+sxc}} (b - c\theta)d\theta + s(1-x)\int_0^{\frac{v}{z}} (v - z\theta)d\theta =$$

$$= \frac{1}{8}(\frac{12e^2 + 4sxbe}{4f+sxc}) + sx\frac{3b^2}{8c} + s(1-x)\frac{4v^2}{8z}$$

- if $N < v$:

$$W_{nc}^{sale} = \int_0^{\frac{2e}{4f+sh}} (e - f\theta)d\theta + sx\int_0^{\frac{b}{2c}+\frac{e}{4f+sh}} (b - c\theta)d\theta + s(1-x)\int_0^{\frac{v}{2z}+\frac{e}{4f+sh}} (v - z\theta)d\theta =$$

$$= \frac{1}{8}(\frac{12e^2 + 4seg}{4f + sh}) + sx\frac{3b^2}{8c} + s(1-x)\frac{3v^2}{8z}$$

From the expressions above it is clear that $W_c^{sale} < W_{nc}^{sale}$ if $v < N$ as

$$W_{nc}^{sale} - W_c^{sale} = \frac{sxe(4bf - 3ec)}{8f(4f + sxc)} + \frac{s(1-x)v^2}{8z} > 0$$

(remember that $\frac{b}{c} > \frac{e}{f}$). To compare W_c^{sale} and W_{nc}^{sale} when $N < v$ we have

$$W_{nc}^{sale} - W_c^{sale} = \frac{-3se^2h + 4sefg}{8f(4f + sh)}$$

Hence, it will be:

$$W_c^{sale} \gtreqless W_{nc}^{sale} \Leftrightarrow \frac{3eh}{4f} \gtreqless g. \blacksquare$$

7.6 Proof of Proposition 6

When $N < v$ and $\frac{3eh}{4f} \gtreqless g$ it is $W_c^{sale} > W_{nc}^{sale}$ and

$$CS_c^{sale} - CS_{nc}^{sale} = W_c^{sale} - \pi_c^{sale} - (W_{nc}^{sale} - \pi_{nc}^{sale}) = \frac{se^2h - 4sefg}{8f(4f + sh)}$$

Hence,

$$CS_c^{sale} \gtreqless CS_{nc}^{sale} \Leftrightarrow \frac{e}{4f} \gtreqless \frac{g}{h}$$

and the result is obtained.$\blacksquare$

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