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Do jumps and cojumps matter for electricity price forecasting? Evidence from the German-Austrian day-ahead market

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ABSTRACT

This paper analyzes the potential for including jumps and cojumps in electricity price forecasting models. The study is carried out on the German–Austrian day-ahead electricity market with a multivariate framework in which each hour of the day is treated as an individual time series. Three models are specified: The ARX model, the ARX-J model (which includes jumps), and the ARX-J-CJ model (which also includes cojumps). Prices are transformed using several variance stabilizing transformations. The forecasting performance of the three models with original and transformed prices is compared using several forecast horizons running from one day-ahead to one week-ahead. Results show that the forecast horizon is crucial in determining whether jumps and cojumps should be included in electricity price forecasting. Jumps and cojumps add important information to forecast prices for horizons longer than 4 days, but there is no gain in forecast accuracy for shorter horizons. The results are of interest to market participants for taking optimal decisions and pricing base week futures contracts.

1. Introduction

With the liberalization of electricity markets, a need to understand electricity price formation has arisen. Once prices are modeled appropriately, they are predicted. Market participants need different price forecast horizons depending on the economic decisions that they have to make at any given time. A better understanding of electricity price dynamics is therefore crucial. However, electricity prices have unique characteristics not found in other commodities that make forecasting them difficult. Electricity cannot be stored, at least on a large scale, and it must be available and managed upon demand. This makes electricity prices very volatile and leads to frequent spikes. Recent large scale deployment of smart grid communication networks enables suppliers and customers to manage their power based on metered data, thus reducing volatility and shaving spikes [2].

Taking into account these features, many research papers have tackled the modeling and forecasting of electricity prices in different markets. Modeling approaches can be divided into four categories: Fundamental models, artificial intelligence-based models, hybrid models, and statistical models [3]. Given the nature of electricity price formation, incorporating jumps into these categories is a leap forward towards improving the forecasting ability of the models. [4] analyze the role of jumps in electricity price modeling. Since their paper, jump tests have been widely used to detect spikes in electricity price series data.[1] Fundamental models require an extensive representation of the electricity system to simulate the market clearing mechanism before estimation and forecasting take place [5]. Artificial intelligence models include machine-learning models and deep-learning models (examples are [6], and [7]). Hybrid models combine different models with the goal of achieving higher accuracy in forecasting (examples are [8,9], and [10]). Finally, statistical models use historical price data and some external price-related information (mainly weather and load forecasts) to predict prices for different time frames. Examples include [3]. Our work belongs to this last category. Statistical models have the advantage of enabling an interpretation of their components to be obtained. They therefore help market participants to understand the relationship between electricity prices and their determinants, and to take their decisions accordingly. In particular, we estimate autoregressive models with exogenous variables (ARX), where prices depend on their past and other exogenous factors.

The role of cojumps, defined as jumps occurring at the same time in different time series, has also been studied in the literature, especially in financial markets. For instance, [11] find evidence of cojumps in

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[1] Properties of different jump tests are analyzed in [1].

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stock prices using different approaches, and [12] show that the forecast accuracy of asset returns variance improves when jumps and cojumps are considered. However, to the best of our knowledge, the relevance of cojumps in electricity markets has not been analyzed.

This paper analyzes whether jumps and cojumps add relevant information to electricity price forecasting. To that end, univariate and multivariate frameworks can be adopted. The former considers one time series for all prices, while the latter divides the whole time series into several series, one for each load period. As an example, [13] focus on ARX models to compare univariate and multivariate frameworks and conclude that there is a slight gain in forecast accuracy using the latter. We use German–Austrian electricity day-ahead auction prices in a multivariate framework, so 24 price series are constructed, one for each hour. Following [14], we transform prices applying different variance stabilizing transformations, which are intended to smooth series and improve forecasts. We specify three ARX models: First, the ARX model (which includes no jumps or cojumps); second, the ARX-J model (which includes jumps); and third, the ARX-J-CJ model (which includes jumps and cojumps).

Jumps and cojumps are detected in the residuals of the ARX model because they are seasonally adjusted. This prevents spikes that are purely seasonal effects from being flagged as jumps or cojumps. As far as we know, there are no other analyses that use this approach to detect jumps and cojumps. Among the several widely-used jump tests applied in the literature, we use the one proposed by [15], hereinafter LM, which is applicable for daily data. Cojumps are constructed following [11].

We assess the role of jumps and cojumps in electricity price forecasting. To that end, jumps and cojumps are embedded in ARX statistical models. To the best of our knowledge, in electricity markets jumps have mostly been used to forecast electricity price volatility and there is little research into their use for predicting electricity prices directly. Furthermore, cojumps have not been analyzed in electricity markets to date.

ARX models are usually over-parameterized, which makes them hard to estimate using OLS. Estimation methods with a shrinkage property have therefore been applied in the literature. [16] proposes the lasso estimation method, which allows variable selection, and [17] propose the elastic net estimation method, which imposes the so-called lasso and ridge penalties on the OLS estimation to reduce the number of variables. [18] apply different estimation methods with the shrinkage property in electricity price forecasting, and conclude that the elastic net is the best-performing method.

Therefore, we estimate the three ARX models using the elastic net for the original and transformed price series using a rolling window. Forecasting is then carried out with the following seven days of each window being forecast for all 24 h of each day. Finally, the mean absolute error (MAE) and the root mean squared error (RMSE) out-of-sample criteria are used to assess the forecasting performance. The difference in forecasting performance between models in pairs is compared using a multivariate approach via the [19] test. Interesting results are obtained regarding the role of jumps and cojumps and price transformations in electricity price forecasting depending on the forecast horizon.

The day-ahead market is the one with the highest liquidity in the German–Austrian zone of the European Power Exchange (EPEX). The participating agents need signposts to decide their bidding strategies optimally. Those signposts are the forecasts made for the following days' prices. This highlights the importance of forecasting as accurately as possible. Moreover, in the European Energy Exchange (EEX) there is trading of base, peak and off-peak products for electricity with cash settlement in the German–Austrian zone known as Phelix (Physical Electricity Index).[2] The underlying prices of these future products are based on the German–Austrian EPEX day-ahead hourly prices.[3] The Phelix base price product is calculated as the mean of all hourly prices in the delivery period. For weekly products it is calculated as the mean of the 168 hourly prices. Therefore, we reasonably concentrate on forecasting the corresponding underlying prices for different time horizons. Note that to forecast weekly products it is necessary to forecast prices for intermediate horizons. It is in this context that we explore the role of jumps and cojumps in price forecasting.

In summary, our contribution to the literature is the following:

- We build an ARX model that embeds jumps or/and cojumps.
- We detect jumps and cojumps in the seasonally adjusted residuals of the ARX model.
- We measure the accuracy of the models in forecasting prices from one day to one week ahead.
- We analyze the forecast for Phelix base week futures contracts including jumps and cojumps in different forecasting horizons.

The rest of the paper is organized as follows. Section 2 explains the methodology used. Section 3 describes the data used in the analysis. Section 4 shows the estimation and forecasting results for all models and transformations. Section 5 summarizes and concludes.

2. Methodology

Price forecasting follows several steps. In the first, the price series is divided into 24 series, one for each hour, which are transformed using several variance stabilizing transformations (VST). These transformations make the time series smoother, thus improving forecast performance. In the second step, an ARX model is specified and estimated using the elastic net method. Next, jumps are detected in the residuals of the estimated model by applying the LM test. Once jumps are detected in each of the 24 time series of the original and transformed prices, cojumps are detected as per [11]. The ARX model is then expanded including only jumps and both jumps and cojumps, resulting in the ARX-J and the ARC-J-CJ models, respectively, also estimated via the elastic net. Finally, price forecasting accuracy is assessed in each model and for original and transformed prices using RMSE and MAE criteria. The forecasting performance of the different pairs of models is compared using the multivariate approach in the [19] test, hereinafter DM.

The subsections below provide a detailed explanation of each step.

2.1. Variance stabilizing transformations

Based on [14], different VSTs are applied. All the transformations used are applicable with negative prices. The objective of these transformations is to obtain transformed price series which are easier to forecast and then to apply the inverse of the transformation to recover the forecast prices. In total 6 different transformations are used: 3σ, logistic, area hyperbolic sine, mirror-logarithmic, and probability integral transformation using both normal and Student-t cumulative distributions. In the first four transformations standardized prices are obtained before the transformation is applied. In addition to the common standardization that uses the standard deviation of prices, a second standardization that uses the median absolute deviation of prices and is more robust to outliers, is also applied.[4] Once prices are forecast, the standardization process is undone.

[2] See https://www.eex.com/en/markets/trading-ressources/indices.

[3] In EPEX, hourly, half-hourly and quarter-hourly prices are set in the intraday continuous market.

[4] For the rest of the paper we use subscripts 1 and 2 after the name of the VST to indicate that prices have been standardized using the standard deviation and the median absolute deviation, respectively. For the sake of simplifying the notation, we denote as p both the original and standardized prices.

The 3σ transformation smooths the series, thus decreasing the effect of outliers in price forecasting. Following [14], the transformation is made as follows:

$$y_{d,h} = \begin{cases} 3\text{sign}(p_{d,h}) & \text{if} \quad |p_{d,h}| > 3 \\ p_{d,h} & \text{if} \quad |p_{d,h}| \le 3 \end{cases}$$

where $p_{d,h}$ denotes the price at day d of the time series corresponding to hour h. By construction, the 3σ transformation does not have an inverse.

The logistic transformation has often been applied in data analytics, but as far as we know, it has only been applied as a VST in electricity price forecasting by [14]. The transformation is:

$$y_{d,h} = (1 + e^{-p_{d,h}})^{-1}$$

After forecasting, the inverse transformation is used to recover the forecast of the original price as:

$$p_{d,h} = \log\left(\frac{y_{d,h}}{1 - y_{d,h}}\right)$$

The area hyperbolic sine (asinh) has been used as a VST in electricity data when modeling negative prices (see [13,20], and [14]). It preserves the behavior of the logarithmic transformation for positive prices but is also defined for negative prices. The transformed prices are calculated as:

$$y_{d,h} = \text{asinh}\left(p_{d,h}\right) = log\left(p_{d,h} + \sqrt{p_{d,h}^2 + 1}\right)$$

with the corresponding inverse transformation:

$$p_{d,h} = \sinh\left(y_{d,h}\right)$$

The mirror-logarithmic (mlog) transformation is a generalization of the logarithmic transformation to make it applicable for negative prices (see [14]). The transformation is constructed as:

$$y_{d,h} = \text{sign}\left(p_{d,h}\right)\left[\log\left(|p_{d,h}| + \frac{1}{c}\right) + \log\left(c\right)\right]$$

The mlog transformation depends on the constant c, which is set to $c = \frac{1}{3}$ following [14]. Consequently, the inverse transformation is:

$$p_{d,h} = \text{sign}\left(y_{d,h}\right)\left[e^{|y_{d,h}| - \log(c)} - \frac{1}{c}\right]$$

The last transformation considered is based on the so-called probability integral transformation (PIT), constructed using the empirical cumulative distribution as an approximation of the unknown true distribution of the time series (see [14]):

$$y_{d,h} = \Phi^{-1}(\widehat{F_p}(p_{d,h}))$$

where Φ^{-1} is the inverse cumulative distribution and $\widehat{F_p}$ is the empirical cumulative distribution of the price series p. Both the normal (N-PIT) and the Student-t with eight degrees of freedom (T-PIT) cumulative distributions are considered. The inverse of the transformation is:

$$p_{d,h} = \widehat{F_p}^{-1}(\Phi(y_{d,h}))$$

2.2. Models

Three different ARX-type models are estimated. The first is the ARX model, based on the fARX model proposed by [13]:

$$p_{d,h} = \underbrace{\beta}_{\text{Constant}} + \underbrace{\sum_{h=1}^{24}\sum_{i=1}^{7}\beta_{i,h}p_{d-i,h}}_{\text{Autoregressive effects}} + \underbrace{\sum_{j=1}^{6}\gamma_{0,j}\text{W}_d^j}_{\text{Day-of-the-week effects}} + \epsilon_{d,h} \quad (1)$$

where p refers to the original price or the transformed price (y), W_d^j is a dummy variable for day j of the week, and $\epsilon_{d,h}$ is the error term with mean 0 by construction. The second term accounts for up to seventh order autoregressive and cross-period effects (effects of each hour from up to 7 days ago). The third term accounts for seasonality.

Jumps are included in model (1), resulting in the ARX-J model. The sign of the jumps might differ depending on the hour of the day, so this model considers both positive and negative jumps:

$$p_{d,h} = \underbrace{\beta}_{\text{Constant}} + \underbrace{\sum_{h=1}^{24}\sum_{i=1}^{7}\beta_{i,h}p_{d-i,h}}_{\text{Autoregressive effects}} + \underbrace{\sum_{j=1}^{6}\gamma_{0,j}\text{W}_d^j}_{\text{Day-of-the-week effects}} + \underbrace{\sum_{i=1}^{7}\theta_i^p PJ_{d-i,h}}_{\text{Positive jumps}} + \underbrace{\sum_{i=1}^{7}\theta_i^n NJ_{d-i,h}}_{\text{Negative jumps}} + \epsilon_{d,h} \quad (2)$$

where $PJ_{d,h}$ is a dummy variable that takes a value of one if there is a positive jump on day d in time series h, and zero otherwise. Analogously, $NJ_{d,h}$ is a dummy variable that takes a value of one if there is a negative jump on day d at hour h, and zero otherwise. The ARX-J model is expected to capture the behavior of prices more accurately at the tails of the distribution.

The third model proposed, ARX-J-CJ, includes jumps and cojumps:

$$p_{d,h} = \underbrace{\beta}_{\text{Constant}} + \underbrace{\sum_{h=1}^{24}\sum_{i=1}^{7}\beta_{i,h}p_{d-i,h}}_{\text{Autoregressive effects}} + \underbrace{\sum_{j=1}^{6}\gamma_{0,j}\text{W}_d^j}_{\text{Day-of-the-week effects}} + \underbrace{\sum_{i=1}^{7}\theta_i^p PJ_{d-i,h}}_{\text{Positive jumps}} + \underbrace{\sum_{i=1}^{7}\theta_i^n NJ_{d-i,h}}_{\text{Negative jumps}} + \underbrace{\theta^c CJ_{d-1}}_{\text{Cojumps}} + \epsilon_{d,h} \quad (3)$$

where CJ_d is a dummy variable that takes a value of one if a cojump is detected on day d and 0 otherwise. Note that it is equal for all time series. The ARX-J-CJ model accounts for correlation between jumps by considering cojumps, which are jumps that occur on the same day across different hours. The ARX-J-CJ model not only accounts for correlation by cross-period effects; it also takes into account correlation in the tails, through the cojump variable.

Finally, we also consider the following naive model as a benchmark: $p_{d,h} = p_{d-1,h} + \epsilon_{d,h}$, where price at a given hour is determined by the price at the same hour of the previous day.

It should be noted that other factors such as load and weather forecasts might also affect prices. However, these factors are not included in the models because the focus of the paper is to forecast prices up to seven days ahead and data are available day-ahead, so they cannot be used to predict prices beyond horizon one.

2.3. Jump and cojump detection

The LM jump test is applied to the residuals of the ARX model estimated (Eq. (1)), thus avoiding spikes that could be explained by seasonal effects. The test compares the size of a standardized observation to a threshold so that it can be assessed whether a significant jump has occurred or not.

First, a window size must be selected. According to LM, the optimal choice of the window size is $K = 20$.[5] Thus, the local variation at day d and for hour h is estimated as:

$$\widehat{\sigma_{d,h}}^2 = \frac{1}{K-2}\sum_{j=d-K+2}^{d-1}|\epsilon_{j,h}||\epsilon_{j-1,h}|,$$

where $\epsilon_{j,h}$ is the residual from the estimated ARX model for day j and hour h.

[5] Each hour of the day is analyzed separately and the data frequency is daily, so $K = \lceil\sqrt{365}\rceil$.

The standardized residual is $z_{d,h} = \frac{\epsilon_{d,h}}{\hat{\sigma}_{d,h}}$. The asymptotic distribution of the maximums of the test statistic in the absence of jumps converges to a Gumbel variable.[6] The LM test identifies significant jumps but does not indicate their sign. Hence, the sign of the corresponding price is checked to determine the jump sign.

The jump detection procedure is usually applied in a single iteration. However in this paper an iterative jump detection procedure is followed, because if jumps are close together the detection of the second jump may be affected. Jumps detected in the iteration are therefore set to the mean of the previous K observations and the jump test is rerun until no more jumps are detected or a maximum of five iterations is reached.

Cojumps over different hours of the same day are detected following the approach proposed by [11].[7] Specifically, day d is classified as a cojump day, i.e. $CJ_d = 1$, if a jump is detected in at least two hours of day d, i.e. if

$$\sum_{h=1}^{24} J_{d,h} \geq 2,$$

where $J_{d,h} = 1$ if a jump is detected on day d and at hour h, and 0 otherwise.

2.4. Estimation

Models (1), (2) and (3) are estimated using the elastic net method introduced by [17], thus solving the poor estimation of OLS when the number of parameters to be estimated is large.

Estimation is carried out using a rolling window of size D. The size of the window has to be large enough to properly estimate the model but not too large, as the effect of the variables might change over time. The window is then moved one day forward and the estimation procedure is repeated. In total, there are N different windows of size D.

The elastic net estimator is obtained by solving the following optimization problem (see [17]):

$$\hat{\tilde{\beta}}_h = \underset{\beta \in \mathbb{R}^L}{\operatorname{argmin}} \left[\underbrace{\sum_{d=1}^{D} \left(\tilde{p}_{d,h} - \tilde{\boldsymbol{X}}_{d,h}\beta \right)^2}_{\text{OLS estimation term}} + \underbrace{\lambda \left(\frac{1-\alpha}{2} \sum_{i=1}^{L} \beta_{i,h}^2 + \alpha \sum_{i=1}^{L} |\beta_{i,h}| \right)}_{\text{Penalty term}} \right],$$

where $\tilde{p}_{d,h}$ and $\tilde{\boldsymbol{X}}_{d,h}$ are the scaled price and the scaled regression matrix on day d and at hour h, respectively, so that $\tilde{p}_{d,h}$ and each column of $\tilde{\boldsymbol{X}}_{d,h}$ have zero mean and standard deviation one. L is the number of parameters to be estimated and λ and α are the tuning parameters, which take values between zero and one, and characterize the penalty term for including variables.

When $\alpha = 1$ the elastic net estimation method is identical to the lasso penalty proposed by [16], while if $\alpha = 0$ the elastic net results in the ridge penalty first introduced by [21]. Following [18], α is set at 0.5.[8]

The optimum value of λ is selected by 10-fold block cross-validation (see [22]). [23] show that the number of observations, the number of parameters, the variance and the correlation are taken into consideration when selecting the tuning parameter.

[6] Using a 10% significance level, the threshold for the test statistic is $-\log(-\log(0.9)) = 2.25$. See [15] for more detail.

[7] The authors propose two different approaches to detect cojumps. However, one of them considers intraday prices and cannot therefore be applied to daily observations. Thus, only one method is included in this paper.

[8] We also use lasso, $\alpha = 1$, but the forecast results do not change significantly. Results are available upon request.

Once the parameters are estimated by solving the optimization problem, the unscaled elastic net estimations $\hat{\beta}_h$ are obtained by rescaling $\hat{\tilde{\beta}}_h$.

Finally, we compare the goodness of fit of the models using the adjusted R-squared for each rolling window in the estimation of the three models (using original and transformed prices).

2.5. Forecast

Once models (1), (2) and (3) are estimated, prices for each hour are predicted for the following 7 days. The MAE and RMSE out-of-sample criteria are used to assess forecasting performance over the N rolling windows and the 7 horizons. Both criteria are widely used in the literature on forecasting in electricity markets, for instance in [13,14,18,24], and [25]. By construction, the MAE criterion is optimal for median forecasts while the RMSE is optimal for mean forecasts.

The MAE criterion for horizon k and hour h is calculated as follows[9]:

$$MAE_{h,k} = \frac{1}{N} \sum_{d=1}^{N} |p_{d,h,k} - \hat{p}_{d,h,k}|,$$

where $p_{d,h,k}$ and $\hat{p}_{d,h,k}$ are the observed and predicted price on day d, at hour h and horizon k, respectively. The mean error across all the hours of the day is calculated as in [14]:

$$MAE_k = \frac{1}{24N} \sum_{h=1}^{24} \sum_{d=1}^{N} |p_{d,h,k} - \hat{p}_{d,h,k}| \tag{4}$$

Analogously, the RMSE measure for the horizon k and hour h is calculated using the square error instead of the absolute error as[10]:

$$RMSE_{h,k} = \sqrt{\frac{1}{N} \sum_{d=1}^{N} (p_{d,h,k} - \hat{p}_{d,h,k})^2},$$

and the corresponding error across all the hours of the day (see [14]) is:

$$RMSE_k = \sqrt{\frac{1}{24N} \sum_{h=1}^{24} \sum_{d=1}^{N} (p_{d,h,k} - \hat{p}_{d,h,k})^2} \tag{5}$$

To determine whether the differences in forecasting performance of the models is significant the multivariate version of the DM test is applied, using the absolute errors for the MAE criterion and the squared errors for the RMSE criterion as loss functions.

3. Data description

The data used in this paper are day-ahead prices from the former German–Austrian electricity market.[11] This is a fully integrated market which sets a single price for both countries. On day $d-1$ the market sets prices for the 24 h of day d according to the following mechanism: First, market agents submit electricity sale and purchase bids up to 12 pm on day $d-1$. Then the system aggregates the bids to demand and supply functions, and finally the intersection between the supply and demand curves determines the quantity traded and the market price for each hour of day d. This was the market with the highest level of liquidity in the EPEX power exchange market. The data run from 1st January 2014 to 30th September 2018, which was the last day on which the German–Austrian day-ahead market operated.[12] In total, there are 41,616 hourly prices and 1734 days in the sample period.

[9] See for example [26].

[10] See for example [26].

[11] Data available on the ENTSOE transparency platform.

[12] After this date, the German and Austrian energy regulators agreed to split their combined day-ahead market zone. This came as a result of frequent transmission congestion between the two grids and the resulting costly redispatching to deal with it. The Luxembourg electricity market subsequently joined the German market to form a single zone.

Table 1
Descriptive statistics for original and transformed prices.

Transformation	Mean	Median	Minimum	Maximum	Std. Dev.	Skewness	Ex. Kurtosis
Original	33.50	32.34	−130.09	163.52	15.04	−0.12	6.42
$3\sigma_1$	0.10	0.00	−3.00	3.00	1.17	0.15	0.30
Logistic_1	0.52	0.50	0.00	1.00	0.23	0.05	−0.70
Asinh_1	0.07	−0.00	−3.35	3.13	0.89	0.05	−0.37
Mlog_1	0.02	0.00	−1.75	1.57	0.32	0.06	0.74
N-PIT	0.00	0.00	−4.06	4.06	1.00	0.00	−0.00
T-PIT	0.00	0.00	−7.89	7.89	1.15	0.01	1.39

Descriptive statistics for original and transformed price series using the standard deviation for standardization. Std. Dev. and Ex. Kurtosis stand for Standard Deviation and Excess Kurtosis, respectively.

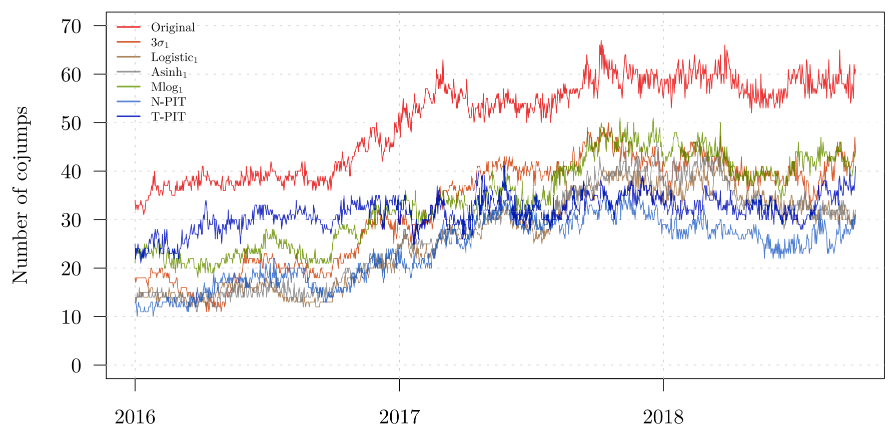

Fig. 1. Number of cojumps detected in each rolling window.

Table 1 reports the main descriptive statistics for the original and transformed price series.[13] As expected, the variability of prices decreases significantly when transformed data are used. Specifically, original prices range from €−130.09 to €163.52/MWh with a standard deviation of €15.04/MWh. By contrast, the largest spread in transformed prices is 15.78 and occurs for the T-PIT transformation. The original prices show negative skewness and excess kurtosis. The heavy tails of the distribution might indicate the presence of jumps. The distribution of the transformed prices is close-to-normal except for the N-PIT, whose distribution is normal.

The original and transformed price series are divided into 24 time series, one for each hour of the day. All the series are stationary according to the ADF unit root test. To estimate models (1), (2), and (3), a rolling window of size $D = 730$ (two years) is used. The initial rolling window starts on 1st January 2014 and ends on 31st December 2015. 7 horizons are forecast in each window, and $N = 998$ different rolling windows are considered. Note that the first 730 observations of the sample are used to forecast the first price.

According to the LM test, there are significant jumps in both original and transformed prices in most of the 24 h and each rolling window.

Fig. 1 shows the number of cojumps detected in the residuals of the ARX model in each rolling window for the original and transformed prices.[14] As expected, the largest number of cojumps is detected in the original price series. Observe that the number of cojumps detected rises to almost 70 at the end of the period in which the market was in place. During periods of highly variable renewable generation, negative electricity price spikes occur because prices drop sharply to feed that renewable generation into the grid. In periods of low renewable generation when thermal units fill the demand gap, prices may spike as a response to unexpected fossil fuel fluctuations, as happened in 2018. The spikes observed may happen in adjacent hours, so cojumps are also identified. This seems to be particularly so in early 2017 and late 2018.

For the transformed series cojumps are detected in each rolling window, although they are fewer in number. In general, the T-PIT is the transformed price series with the largest number of cojumps, especially during the first half of the sample. This result is expected, as the probability distribution of T-PIT transformed prices is heavy-tailed. By contrast, the number of cojumps detected is lower in the N-PIT transformed price series.

4. Estimation and forecast results

4.1. Estimation results

Models (1), (2), and (3) are estimated and the corresponding adjusted R-squared is calculated for each rolling window. Table 2 reports the mean values of the adjusted R-squared for all hours and each model, and for original and transformed prices. The comparison within each transformation shows that the best model in terms of goodness of fit is the ARX.

4.2. Forecast results

Models (1), (2), and (3) are used to forecast prices for 1 to 7 days ahead for each rolling window.

[13] For 3σ, logistic, asinh, and mlog transformations the common standardization has been applied to prices. Statistics for standardized prices with the median absolute deviation do not change significantly and are not shown. They are available upon request.

[14] These results are for standardized prices using the standard deviation. Results for standardized prices using the median absolute deviation are available upon request.

Table 2
Mean adjusted-R^2 for all hours.

	Original	$3\sigma_1$	$3\sigma_2$	Logistic$_1$	Logistic$_2$	Asinh$_1$	Asinh$_2$	Mlog$_1$	Mlog$_2$	N-PIT	T-PIT
ARX	0.655	0.683	0.675	0.686	0.688	0.688	0.688	0.683	0.679	0.686	0.680
ARX-J	0.653	0.681	0.674	0.684	0.687	0.687	0.687	0.682	0.678	0.685	0.679
ARX-J-CJ	0.653	0.681	0.674	0.684	0.687	0.687	0.687	0.682	0.678	0.685	0.679

Mean adjusted R-squared criterion for all hours, models, and price transformations. Subscripts 1 and 2 indicate that prices are standardized using the standard deviation and the median absolute deviation, respectively. A heat map is used to indicate higher (green) and lower (red) values within each transformation.

Table 3
Mean MAE for all hours.

Model	Transf.	H1	H2	H3	H4	H5	H6	H7
Naive		8.396	10.754	11.297	11.530	11.603	10.706	9.389
ARX	Original	5.598	7.327	7.845	8.116	8.311	8.475	8.563
	$3\sigma_1$	5.584	7.153	7.639	7.908	8.119	8.290	8.382
	$3\sigma_2$	5.530	7.167	7.675	7.951	8.156	8.315	8.401
	Logistic$_1$	5.722	7.270	7.734	7.993	8.180	8.343	8.444
	Logistic$_2$	5.544	7.178	7.672	7.941	8.144	8.310	8.405
	Asinh$_1$	5.558	7.229	7.718	7.989	8.187	8.351	8.445
	Asinh$_2$	6.254	7.310	7.734	7.991	8.229	8.392	8.483
	Mlog$_1$	5.500	7.190	7.703	7.976	8.181	8.346	8.444
	Mlog$_2$	5.507	7.198	7.705	7.982	8.188	8.351	8.447
	N-PIT	5.529	7.134	7.638	7.919	8.128	8.308	8.402
	T-PIT	5.523	7.125	7.627	7.910	8.129	8.304	8.404
ARX-J	Original	5.623	7.316	7.848	8.114	8.274	8.428	8.520
	$3\sigma_1$	5.615	7.174	7.658	7.925	8.123	8.281	8.378
	$3\sigma_2$	5.553	7.167	7.671	7.943	8.115	8.261	8.361
	Logistic$_1$	5.746	7.324	7.787	8.042	8.219	8.374	8.471
	Logistic$_2$	5.579	7.207	7.701	7.967	8.159	8.314	8.409
	Asinh$_1$	5.590	7.275	7.770	8.025	8.214	8.366	8.460
	Asinh$_2$	6.285	7.349	7.785	8.038	8.259	8.411	8.496
	Mlog$_1$	5.533	7.226	7.728	7.982	8.177	8.331	8.435
	Mlog$_2$	5.532	7.209	7.716	7.973	8.169	8.319	8.424
	N-PIT	5.550	7.171	7.672	7.959	8.164	8.338	8.445
	T-PIT	5.548	7.137	7.631	7.911	8.120	8.305	8.413
ARX-J-CJ	Original	5.626	7.316	7.847	8.116	8.281	8.436	8.536
	$3\sigma_1$	5.614	7.178	7.666	7.931	8.129	8.288	8.385
	$3\sigma_2$	5.553	7.169	7.673	7.946	8.122	8.268	8.368
	Logistic$_1$	5.747	7.326	7.786	8.039	8.218	8.373	8.472
	Logistic$_2$	5.579	7.211	7.700	7.969	8.164	8.314	8.410
	Asinh$_1$	5.593	7.277	7.770	8.027	8.216	8.370	8.466
	Asinh$_2$	6.281	7.354	7.785	8.040	8.257	8.411	8.497
	Mlog$_1$	5.534	7.218	7.727	7.986	8.176	8.336	8.442
	Mlog$_2$	5.539	7.217	7.725	7.989	8.180	8.331	8.438
	N-PIT	5.555	7.175	7.674	7.964	8.166	8.341	8.451
	T-PIT	5.547	7.138	7.624	7.904	8.117	8.296	8.405

Mean MAE criterion (Eq. (4)) for all hours per horizon (H1 to H7), model, and price transformation. Subscripts 1 and 2 indicate that prices are standardized using the standard deviation and the median absolute deviation, respectively. A heat map is used to indicate lower (green) and higher (red) forecast errors within each horizon.

Model selection involves two criteria. First, we measure the forecasting performance of the models by sorting them according to the value of the MAE and RMSE criteria and then we choose the ones with the lowest values for each criterion and forecast horizon. Second, we run the multivariate approach of the DM test to determine whether forecasts for each pair of models are significantly better in one of them.

Tables 3 and 4 show the forecasting errors of each model for original and transformed prices, for the naive model and for the seven horizons (H1 to H7) using the MAE and RMSE criteria across all hours of the day, i.e. Eqs. (4) and (5), respectively.[15]

Regardless of the forecasting horizon, results show that the naive model provides the largest errors, which means that at least terms accounting for correlation between prices at a given hour and day and their lags should be considered as explanatory variables. For the rest of the models, the VST results show that none of the models are selected with the original prices, so it is important to smooth price series so as to obtain more accurate forecasts. However, not all the transformations are equally good: logistic$_1$, logistic$_2$, and asinh$_1$ transformations do not provide better forecasts as they are not selected for the best models. By contrast, $3\sigma_2$, mlog$_1$, mlog$_2$, and T-PIT are, in general, the transformations with the best forecast performances. These are the transformations for which price distribution has excess kurtosis, so it is important to accurately capture the behavior at the tails because jumps are observations that fall at the tails of the distribution.

Market participants take decisions that depend on the time horizon under consideration. Forecasting is relevant in day-to-day market operations of EPEX and risk management in the EEX futures markets for different delivery periods. We discuss results ranging from the closest-to-delivery one day-ahead forecast to a one week-ahead price forecast.

- Horizon 1: Day-ahead forecasting is important for electricity trading and plant operation scheduling decisions. The ARX model

[15] The results of the DM test for the MAE criterion are reported in Appendix and those using the RMSE criterion are available from the authors upon request.

Table 4
Mean RMSE for all hours.

Model	Transf.	H1	H2	H3	H4	H5	H6	H7
Naive		13.351	16.535	17.444	17.803	17.905	16.727	15.195
ARX	Original	8.959	11.235	11.859	12.163	12.397	12.558	12.636
	$3\sigma_1$	9.397	11.225	11.801	12.133	12.376	12.530	12.613
	$3\sigma_2$	9.121	11.141	11.776	12.113	12.360	12.513	12.592
	Logistic$_1$	9.506	11.399	11.981	12.313	12.537	12.688	12.773
	Logistic$_2$	9.148	11.234	11.840	12.178	12.423	12.574	12.660
	Asinh$_1$	9.030	11.245	11.850	12.195	12.446	12.601	12.695
	Asinh$_2$	9.623	11.252	11.769	12.084	12.349	12.527	12.603
	Mlog$_1$	8.891	11.150	11.771	12.106	12.359	12.514	12.613
	Mlog$_2$	8.894	11.143	11.755	12.094	12.342	12.505	12.599
	N-PIT	9.081	11.211	11.830	12.185	12.435	12.598	12.688
	T-PIT	8.981	11.159	11.786	12.133	12.388	12.543	12.642
ARX-J	Original	9.007	11.277	11.900	12.187	12.380	12.541	12.631
	$3\sigma_1$	9.445	11.264	11.840	12.159	12.386	12.525	12.614
	$3\sigma_2$	9.161	11.166	11.785	12.114	12.324	12.461	12.557
	Logistic$_1$	9.550	11.490	12.057	12.373	12.580	12.719	12.806
	Logistic$_2$	9.205	11.289	11.886	12.216	12.442	12.583	12.673
	Asinh$_1$	9.083	11.318	11.922	12.245	12.481	12.621	12.715
	Asinh$_2$	9.676	11.316	11.824	12.131	12.379	12.544	12.612
	Mlog$_1$	8.941	11.212	11.813	12.123	12.355	12.506	12.617
	Mlog$_2$	8.941	11.191	11.796	12.101	12.338	12.486	12.594
	N-PIT	9.116	11.264	11.879	12.226	12.470	12.625	12.729
	T-PIT	9.032	11.195	11.801	12.133	12.383	12.540	12.659
ARX-J-CJ	Original	9.022	11.273	11.904	12.196	12.390	12.558	12.651
	$3\sigma_1$	9.448	11.269	11.846	12.164	12.391	12.534	12.619
	$3\sigma_2$	9.163	11.165	11.789	12.120	12.329	12.467	12.568
	Logistic$_1$	9.569	11.499	12.057	12.373	12.582	12.720	12.813
	Logistic$_2$	9.207	11.292	11.888	12.221	12.449	12.584	12.670
	Asinh$_1$	9.085	11.322	11.924	12.251	12.488	12.626	12.725
	Asinh$_2$	9.655	11.324	11.826	12.134	12.376	12.542	12.617
	Mlog$_1$	8.940	11.207	11.814	12.126	12.362	12.508	12.624
	Mlog$_2$	8.942	11.198	11.810	12.122	12.357	12.503	12.612
	N-PIT	9.126	11.273	11.886	12.237	12.477	12.632	12.740
	T-PIT	9.039	11.200	11.799	12.133	12.378	12.538	12.650

Mean RMSE criterion (Eq. (5)) for all hours per horizon (H1 to H7), model, and price transformation. Subscripts 1 and 2 indicate that prices are standardized using the standard deviation and the median absolute deviation, respectively. A heat map is used to indicate lower (green) and higher (red) forecast errors within each horizon.

with mlog transformed prices outperforms the rest. Furthermore, models that include jumps and/or cojumps are not selected. Thus, the inclusion of jumps as explanatory factors does not improve the forecast.

- Horizon 2: The results are not that conclusive. Under the MAE criterion, ARX, ARX-J, and ARX-J-CJ are candidate models for selection using T-PIT transformation. However, the DM test finds no significant differences between them or with respect to mlog and N-PIT transformations. For the RMSE criterion, in general the ARX model gives the lowest error and the DM test never selects models with jumps and cojumps. These results are in line with those for horizon 1, so information on jumps does not help to forecast prices two days ahead.
- Horizon 3: The results differ depending on the criterion. Under MAE the best performing model is the ARX-J-CJ with T-PIT transformed prices, followed by the ARX model with the same transformation. However, the DM test finds no significant differences between them. By contrast, RMSE selects the ARX model for the mlog$_2$ transformed prices. DM results show that for the ARX model the difference in error measures between the mlog$_2$ and the asinh$_2$ transformations is not significant. These results are qualitatively similar to previous horizons, so there is no clear gain from including jumps or cojumps.
- Horizon 4: Under the MAE criterion, ARX-J-CJ with the T-PIT transformation is the best model, but the error difference with respect to the ARX model for $3\sigma_1$ and T-PIT transformations is not significant according to the DM test results. The results for the RMSE criterion differ because the model with the smallest error is the ARX with the asinh$_2$ transformed prices. However, DM results show no significant differences between the forecasting accuracy in this case and that of the three models for $3\sigma_2$, mlog$_1$ and mlog$_2$ transformations. Therefore, results are qualitatively similar to those for previous horizons.
- Horizon 5: MAE and RMSE criteria select the ARX-J model with the $3\sigma_2$ transformation as the best model. However, the DM test using MAE shows that it does not outperform the ARX-J-CJ model with the T-PIT transformation. Moreover, the same test using RMSE shows no clear evidence of superiority for any of the three models. Five days ahead there is some evidence that models that incorporate jumps and/or cojumps as explanatory variables provide better price forecasts. Factors that contribute to the occurrence of price shocks are expected to become more likely as the forecast horizon becomes longer.
- Horizon 6: MAE and RMSE criteria select the ARX-J model with the $3\sigma_2$ transformation as the best model. This result is confirmed by the DM test using MAE, which shows this model to be significantly superior to the ARX-J-CJ model. However, the DM test using RMSE shows that it does not outperform the ARX-J-CJ model with the same transformation. Nor is it superior to any of the models with the mlog transformation. Information on jumps and/or cojumps is therefore more significant in forecasting. These results reinforce the findings for horizon 5 and show that hedging is important in longer horizons too.
- Horizon 7: MAE and RMSE criteria select the ARX-J model with the $3\sigma_2$ transformation as the best model. This result is confirmed by DM testing using either criterion, which shows it to be significantly superior to the ARX and ARX-J-CJ models. As expected, these results are in line with those for horizons 5 and 6, and

highlight the gain from taking into account the information on jumps and/or cojumps in managing risk. Given that the Phelix base product is calculated as the mean of all 168 hourly forecast prices, including jumps in the models improves the Phelix base weekly product.

To summarize, in terms of forecast accuracy, the ARX model outperforms models that incorporate jumps and/or cojumps for the shortest horizons. Therefore, electricity trading and plant operation scheduling decisions do not benefit from information on jumps and/or cojumps. However, as the forecast horizon lengthens, incorporating jumps and/or cojumps into the estimation of the models improves forecasting accuracy. This could be because jumps are extremely rare, short-lived events, so the likelihood of their occurring increases with time. These results have implications for pricing weekly products in futures markets. For instance, the way in which the Phelix base product for one-week delivery is calculated shows the importance of models that incorporate jumps and cojumps as explanatory variables.

5. Summary and conclusions

Price modeling and forecasting have become challenging since electricity markets were liberalized. This is especially relevant with the large-scale deployment of renewable energy production, integration with neighboring markets, and increased use of financial products. Electricity prices also exhibit unique characteristics that make these tasks more complex. One of those characteristics is the presence of spikes.

We use day-ahead prices from the German-Austrian electricity market for the period from January 1, 2014 to September 30, 2018 to analyze the role of jumps and cojumps in price forecasting. It should be noted that this is the market with the greatest liquidity in Germany and Austria, even after market decoupling. It is therefore important to model price dynamics accurately to forecast prices several periods ahead.

Price series for each hour of the day are considered, leading to a multivariate framework. We specify three models: The ARX model; the ARX-J model, which includes jumps; and the ARX-J-CJ model, which includes jumps and cojumps. Cojumps are defined as jumps that occur on the same day. We also transform the price series using several variance stabilizing transformations.

Our results show that using $3\sigma_2$, mlog_1, mlog_2, and T-PIT variance stabilizing transformations provides more accurate forecasts of prices than considering original price data. Furthermore, including jumps and cojumps as covariates further improves price forecasting only for horizons beyond four days.

These conclusions are also of interest to participants in the futures market. Electricity markets around the world are encouraging market agents to participate in futures markets, and the decision to do so is taken after profitability analyses. In particular, Phelix futures are traded on the EEX market. Hence, day-ahead price forecasting helps participants to optimize their bidding strategies for the following days and decide whether to participate in the futures market or not.

Finally, we consider the following lines for future research. First, it should be noted that there is an intraday auction that sets prices every 15 min, and a continuous market with several products (every 15 min, every 30 min and hourly). The incorporation of large-scale intermittent renewable generation and the integration of the European market increase the importance of these markets. Our framework of analysis could also be extended to these markets. However, the different frequencies of price formation would need to be carefully considered in specifying the statistical models. Second, models including other factors that may affect electricity prices, such as load, weather forecasts and reserve margin, could also be considered as more system operators begin disclosing such information. In this case, the day-ahead forecast should be considered to ensure that these factors give the information closest to the time of the forecast. Third, taking into account the integration of the European market, price forecasting could also be assessed for several European electricity markets to check for cojumps between them.

CRediT authorship contribution statement

Aitor Ciarreta: Conceptualization, Methodology, Formal analysis, Writing – review & editing. **Peru Muniain:** Conceptualization, Methodology, Formal analysis, Software, Writing – review & editing. **Ainhoa Zarraga:** Conceptualization, Methodology, Formal analysis, Writing – review & editing.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Appendix. Results of the multivariate DM test for the MAE criterion

See Tables A.1–A.7.

Table A.1
DM using MAE for H1.

Columns: –ARX–

Group	Model	Orig.	$3\sigma_1$	$3\sigma_2$	Logistic$_1$	Logistic$_2$	Asinh$_1$	Asinh$_2$	Mlog$_1$	Mlog$_2$	N-PIT	T-PIT
ARX	$3\sigma_1$	−0.18 (0.43)										
ARX	$3\sigma_2$	−1.16 (0.122)	−1.99 (0.023)									
ARX	Logistic$_1$	1.5 (0.933)	3.88 (>0.999)	4.16 (>0.999)								
ARX	Logistic$_2$	−0.83 (0.205)	−1.47 (0.071)	0.57 (0.715)	−6.55 (<0.001)							
ARX	Asinh$_1$	−0.75 (0.228)	−0.56 (0.289)	0.75 (0.774)	−4.28 (<0.001)	0.57 (0.715)						
ARX	Asinh$_2$	8.81 (>0.999)	6.39 (>0.999)	8.01 (>0.999)	5.13 (>0.999)	7.68 (>0.999)	8.21 (>0.999)					
ARX	Mlog$_1$	−2.96 (0.002)	−1.51 (0.066)	−0.75 (0.227)	−3.92 (<0.001)	−1.16 (0.123)	−2.6 (0.005)	−9.78 (<0.001)				
ARX	Mlog$_2$	−3.34 (<0.001)	−1.31 (0.095)	−0.55 (0.291)	−3.49 (<0.001)	−0.88 (0.189)	−1.8 (0.036)	−9.9 (<0.001)	0.95 (0.829)			
ARX	N-PIT	−1.16 (0.123)	−1.35 (0.089)	−0.01 (0.496)	−4.72 (<0.001)	−0.53 (0.298)	−1.1 (0.135)	−8.1 (<0.001)	0.85 (0.803)	0.58 (0.718)		
ARX	T-PIT	−1.51 (0.065)	−1.21 (0.113)	−0.19 (0.423)	−3.62 (<0.001)	−0.54 (0.294)	−1.05 (0.147)	−8.73 (<0.001)	0.73 (0.767)	0.47 (0.681)	−0.35 (0.361)	
ARX-J	Orig.	2.22 (0.987)	0.5 (0.691)	1.59 (0.944)	−1.2 (0.116)	1.22 (0.888)	1.22 (0.889)	−8.31 (<0.001)	3.68 (>0.999)	4.17 (>0.999)	1.6 (0.945)	2.03 (0.979)
ARX-J	$3\sigma_1$	0.21 (0.583)	3.78 (>0.999)	2.94 (0.998)	−2.93 (0.002)	2.47 (0.993)	1.18 (0.882)	−6.04 (<0.001)	2.01 (0.978)	1.79 (0.964)	2.05 (0.98)	1.78 (0.962)
ARX-J	$3\sigma_2$	−0.76 (0.225)	−1.16 (0.124)	2.62 (0.996)	−3.68 (<0.001)	0.31 (0.622)	−0.15 (0.44)	−7.64 (<0.001)	1.28 (0.899)	1.06 (0.856)	0.72 (0.763)	0.82 (0.794)
ARX-J	Logistic$_1$	1.77 (0.962)	4.7 (>0.999)	4.71 (>0.999)	2.18 (0.986)	7.41 (>0.999)	4.79 (>0.999)	−4.82 (<0.001)	4.29 (>0.999)	3.85 (>0.999)	5.13 (>0.999)	3.97 (>0.999)
ARX-J	Logistic$_2$	−0.28 (0.389)	−0.2 (0.421)	1.84 (0.967)	−5.12 (<0.001)	3.6 (>0.999)	0.75 (0.773)	−7.15 (<0.001)	1.94 (0.974)	1.6 (0.946)	1.64 (0.95)	1.35 (0.911)
ARX-J	Asinh$_1$	−0.15 (0.441)	0.13 (0.55)	1.58 (0.943)	−3.46 (<0.001)	1.84 (0.967)	4.09 (>0.999)	−7.72 (<0.001)	3.72 (>0.999)	2.77 (0.997)	2.25 (0.988)	1.93 (0.973)
ARX-J	Asinh$_2$	9.25 (>0.999)	6.76 (>0.999)	8.45 (>0.999)	5.48 (>0.999)	8.11 (>0.999)	8.66 (>0.999)	2.85 (0.998)	10.25 (>0.999)	10.38 (>0.999)	8.53 (>0.999)	9.18 (>0.999)
ARX-J	Mlog$_1$	−1.84 (0.033)	−0.94 (0.174)	0.09 (0.536)	−3.42 (<0.001)	−0.31 (0.38)	−1.14 (0.126)	−9.22 (<0.001)	3.7 (>0.999)	2.2 (0.986)	0.11 (0.545)	0.34 (0.631)
ARX-J	Mlog$_2$	−2.27 (0.012)	−0.9 (0.185)	0.05 (0.521)	−3.11 (<0.001)	−0.3 (0.382)	−0.92 (0.178)	−9.47 (<0.001)	2.92 (0.998)	2.89 (0.998)	0.06 (0.525)	0.27 (0.605)
ARX-J	N-PIT	−0.78 (0.217)	−0.85 (0.198)	0.64 (0.739)	−4.22 (<0.001)	0.2 (0.581)	−0.3 (0.383)	−7.72 (<0.001)	1.39 (0.917)	1.06 (0.856)	2.49 (0.994)	1.31 (0.905)
ARX-J	T-PIT	−0.98 (0.164)	−0.73 (0.233)	0.51 (0.694)	−3.19 (<0.001)	0.08 (0.532)	−0.31 (0.377)	−8.29 (<0.001)	1.44 (0.925)	1.14 (0.873)	0.97 (0.835)	2.68 (0.996)
ARX-J-CJ	Orig.	2.5 (0.994)	0.54 (0.705)	1.65 (0.95)	−1.17 (0.122)	1.27 (0.897)	1.29 (0.901)	−8.29 (<0.001)	3.8 (>0.999)	4.33 (>0.999)	1.66 (0.951)	2.11 (0.982)
ARX-J-CJ	$3\sigma_1$	0.2 (0.579)	3.73 (>0.999)	2.9 (0.998)	−2.95 (0.002)	2.44 (0.993)	1.17 (0.879)	−6.04 (<0.001)	2 (0.977)	1.78 (0.962)	2.03 (0.979)	1.76 (0.961)
ARX-J-CJ	$3\sigma_2$	−0.76 (0.224)	−1.15 (0.125)	2.69 (0.996)	−3.66 (<0.001)	0.31 (0.621)	−0.15 (0.439)	−7.65 (<0.001)	1.29 (0.901)	1.07 (0.858)	0.72 (0.763)	0.82 (0.794)
ARX-J-CJ	Logistic$_1$	1.76 (0.961)	4.7 (>0.999)	4.69 (>0.999)	2.21 (0.986)	7.19 (>0.999)	4.71 (>0.999)	−4.78 (<0.001)	4.24 (>0.999)	3.81 (>0.999)	5.11 (>0.999)	3.96 (>0.999)
ARX-J-CJ	Logistic$_2$	−0.28 (0.389)	−0.2 (0.419)	1.82 (0.965)	−5.11 (<0.001)	3.46 (>0.999)	0.74 (0.769)	−7.12 (<0.001)	1.92 (0.972)	1.59 (0.944)	1.63 (0.948)	1.34 (0.91)
ARX-J-CJ	Asinh$_1$	−0.1 (0.462)	0.19 (0.575)	1.67 (0.953)	−3.53 (<0.001)	2.04 (0.979)	3.98 (>0.999)	−7.61 (<0.001)	3.58 (>0.999)	2.71 (0.997)	2.42 (0.992)	2.02 (0.978)
ARX-J-CJ	Asinh$_2$	9.18 (>0.999)	6.8 (>0.999)	8.5 (>0.999)	5.51 (>0.999)	8.16 (>0.999)	8.71 (>0.999)	2.2 (0.986)	10.28 (>0.999)	10.4 (>0.999)	8.6 (>0.999)	9.25 (>0.999)
ARX-J-CJ	Mlog$_1$	−1.75 (0.04)	−0.93 (0.177)	0.13 (0.55)	−3.45 (<0.001)	−0.28 (0.391)	−1.11 (0.133)	−9.16 (<0.001)	3.45 (>0.999)	2.1 (0.982)	0.16 (0.562)	0.38 (0.649)
ARX-J-CJ	Mlog$_2$	−1.97 (0.024)	−0.78 (0.216)	0.23 (0.59)	−3.03 (0.001)	−0.13 (0.447)	−0.69 (0.244)	−9.29 (<0.001)	3.56 (>0.999)	3.47 (>0.999)	0.25 (0.6)	0.49 (0.686)
ARX-J-CJ	N-PIT	−0.71 (0.24)	−0.74 (0.23)	0.78 (0.783)	−4.13 (<0.001)	0.37 (0.643)	−0.13 (0.447)	−7.65 (<0.001)	1.5 (0.933)	1.17 (0.879)	3 (0.999)	1.51 (0.934)
ARX-J-CJ	T-PIT	−0.97 (0.166)	−0.75 (0.228)	0.51 (0.693)	−3.23 (<0.001)	0.07 (0.53)	−0.32 (0.374)	−8.23 (<0.001)	1.4 (0.92)	1.11 (0.867)	0.97 (0.834)	2.47 (0.993)

Columns: ARX-J

Group	Model	Orig.	$3\sigma_1$	$3\sigma_2$	Logistic$_1$	Logistic$_2$	Asinh$_1$	Asinh$_2$	Mlog$_1$	Mlog$_2$	N-PIT	T-PIT
ARX-J	$3\sigma_1$	−0.11 (0.456)										
ARX-J	$3\sigma_2$	−1.19 (0.116)	−2.29 (0.011)									
ARX-J	Logistic$_1$	1.48 (0.931)	3.82 (>0.999)	4.28 (>0.999)								
ARX-J	Logistic$_2$	−0.66 (0.253)	−1.38 (0.084)	1.03 (0.847)	−6.59 (<0.001)							
ARX-J	Asinh$_1$	−0.62 (0.268)	−0.53 (0.297)	0.98 (0.836)	−4.15 (<0.001)	0.44 (0.669)						
ARX-J	Asinh$_2$	8.79 (>0.999)	6.42 (>0.999)	8.1 (>0.999)	5.2 (>0.999)	7.62 (>0.999)	8.22 (>0.999)					
ARX-J	Mlog$_1$	−2.63 (0.004)	−1.49 (0.068)	−0.5 (0.308)	−3.87 (<0.001)	−1.21 (0.114)	−2.62 (0.004)	−9.78 (<0.001)				
ARX-J	Mlog$_2$	−3.28 (<0.001)	−1.41 (0.08)	−0.5 (0.308)	−3.51 (<0.001)	−1.09 (0.139)	−2.02 (0.022)	−10.03 (<0.001)	−0.15 (0.439)			
ARX-J	N-PIT	−1.22 (0.112)	−1.61 (0.053)	−0.08 (0.47)	−4.78 (<0.001)	−1.01 (0.157)	−1.48 (0.069)	−8.18 (<0.001)	0.51 (0.696)	0.47 (0.682)		
ARX-J	T-PIT	−1.5 (0.067)	−1.33 (0.091)	−0.14 (0.444)	−3.6 (<0.001)	−0.78 (0.219)	−1.23 (0.109)	−8.79 (<0.001)	0.47 (0.681)	0.46 (0.679)	−0.14 (0.445)	
ARX-J-CJ	Orig.	0.63 (0.735)	0.15 (0.558)	1.24 (0.893)	−1.45 (0.073)	0.71 (0.761)	0.68 (0.751)	−8.77 (<0.001)	2.73 (0.997)	3.42 (>0.999)	1.27 (0.898)	1.56 (0.941)
ARX-J-CJ	$3\sigma_1$	−0.12 (0.453)	−0.42 (0.336)	2.25 (0.988)	−3.84 (<0.001)	1.34 (0.911)	0.52 (0.697)	−6.42 (<0.001)	1.47 (0.93)	1.39 (0.918)	1.59 (0.944)	1.32 (0.906)
ARX-J-CJ	$3\sigma_2$	−1.2 (0.115)	−2.27 (0.012)	−0.01 (0.497)	−4.26 (<0.001)	−1.02 (0.155)	−0.98 (0.163)	−8.11 (<0.001)	0.5 (0.693)	0.5 (0.693)	0.07 (0.53)	0.14 (0.556)
ARX-J-CJ	Logistic$_1$	1.47 (0.93)	3.82 (>0.999)	4.26 (>0.999)	0.13 (0.551)	6.39 (>0.999)	4.07 (>0.999)	−5.16 (<0.001)	3.82 (>0.999)	3.48 (>0.999)	4.76 (>0.999)	3.59 (>0.999)
ARX-J-CJ	Logistic$_2$	−0.66 (0.254)	−1.39 (0.083)	1.02 (0.845)	−6.57 (<0.001)	−0.05 (0.479)	−0.44 (0.331)	−7.58 (<0.001)	1.19 (0.883)	1.07 (0.859)	1 (0.842)	0.77 (0.78)
ARX-J-CJ	Asinh$_1$	−0.55 (0.29)	−0.49 (0.313)	1.07 (0.858)	−4.24 (<0.001)	0.57 (0.717)	0.76 (0.775)	−8.11 (<0.001)	2.56 (0.995)	2.01 (0.978)	1.65 (0.95)	1.32 (0.907)
ARX-J-CJ	Asinh$_2$	8.73 (>0.999)	6.45 (>0.999)	8.16 (>0.999)	5.22 (>0.999)	7.68 (>0.999)	8.27 (>0.999)	−0.97 (0.165)	9.82 (>0.999)	10.05 (>0.999)	8.25 (>0.999)	8.86 (>0.999)
ARX-J-CJ	Mlog$_1$	−2.52 (0.006)	−1.49 (0.068)	−0.48 (0.315)	−3.9 (<0.001)	−1.2 (0.114)	−2.64 (0.004)	−9.72 (<0.001)	0.42 (0.662)	0.27 (0.606)	−0.49 (0.313)	−0.44 (0.33)
ARX-J-CJ	Mlog$_2$	−2.96 (0.002)	−1.3 (0.096)	−0.34 (0.368)	−3.44 (<0.001)	−0.94 (0.173)	−1.82 (0.034)	−9.85 (<0.001)	0.71 (0.76)	2.43 (0.992)	−0.3 (0.383)	−0.26 (0.396)
ARX-J-CJ	N-PIT	−1.14 (0.126)	−1.51 (0.066)	0.07 (0.528)	−4.7 (<0.001)	−0.85 (0.198)	−1.31 (0.095)	−8.11 (<0.001)	0.64 (0.74)	0.59 (0.721)	1.82 (0.965)	0.38 (0.648)
ARX-J-CJ	T-PIT	−1.49 (0.069)	−1.36 (0.087)	−0.15 (0.439)	−3.64 (<0.001)	−0.79 (0.214)	−1.24 (0.107)	−8.72 (<0.001)	0.46 (0.676)	0.45 (0.673)	−0.16 (0.436)	−0.09 (0.466)

Columns: –ARX-J-CJ–

Group	Model	Orig.	$3\sigma_1$	$3\sigma_2$	Logistic$_1$	Logistic$_2$	Asinh$_1$	Asinh$_2$	Mlog$_1$	Mlog$_2$	N-PIT
ARX-J-CJ	$3\sigma_1$	−0.16 (0.436)									
ARX-J-CJ	$3\sigma_2$	−1.25 (0.105)	−2.23 (0.013)								
ARX-J-CJ	Logistic$_1$	1.44 (0.926)	3.84 (>0.999)	4.23 (>0.999)							
ARX-J-CJ	Logistic$_2$	−0.71 (0.239)	−1.35 (0.088)	1.01 (0.843)	−6.37 (<0.001)						
ARX-J-CJ	Asinh$_1$	−0.61 (0.271)	−0.47 (0.319)	1.07 (0.858)	−4.16 (<0.001)	0.57 (0.717)					
ARX-J-CJ	Asinh$_2$	8.71 (>0.999)	6.45 (>0.999)	8.17 (>0.999)	5.18 (>0.999)	7.64 (>0.999)	8.17 (>0.999)				
ARX-J-CJ	Mlog$_1$	−2.62 (0.004)	−1.47 (0.07)	−0.48 (0.315)	−3.86 (<0.001)	−1.19 (0.117)	−2.6 (0.005)	−9.76 (<0.001)			
ARX-J-CJ	Mlog$_2$	−3.09 (<0.001)	−1.29 (0.098)	−0.34 (0.368)	−3.4 (<0.001)	−0.93 (0.176)	−1.82 (0.034)	−9.87 (<0.001)	0.51 (0.695)		
ARX-J-CJ	N-PIT	−1.19 (0.117)	−1.49 (0.068)	0.07 (0.528)	−4.68 (<0.001)	−0.85 (0.199)	−1.47 (0.071)	−8.18 (<0.001)	0.62 (0.734)	0.42 (0.661)	
ARX-J-CJ	T-PIT	−1.55 (0.06)	−1.34 (0.09)	−0.15 (0.439)	−3.63 (<0.001)	−0.79 (0.215)	−1.34 (0.091)	−8.79 (<0.001)	0.43 (0.665)	0.25 (0.599)	−0.41 (0.341)

Multivariate DM test statistic using MAE criterion for forecast horizon 1. P-values in parentheses. A p-value lower than 0.10 indicates that the forecasts of the model of the row are better than those of the model of the column at the 10% significance level. A heat map is used to indicate lower (green) and greater (red) p-values.

Table A.2
DM using MAE for H2.

		–ARX–										
		Orig.	$3\sigma_1$	$3\sigma_2$	Logistic$_1$	Logistic$_2$	Asinh$_1$	Asinh$_2$	Mlog$_1$	Mlog$_2$	N-PIT	T-PIT
ARX	$3\sigma_1$	−2.95 (0.002)										
	$3\sigma_2$	−3.55 (<0.001)	0.63 (0.735)									
	Logistic$_1$	−0.8 (0.213)	4.01 (>0.999)	2.35 (0.99)								
	Logistic$_2$	−2.57 (0.005)	1.65 (0.95)	0.44 (0.669)	−4.44 (<0.001)							
	Asinh$_1$	−1.82 (0.035)	2.56 (0.995)	1.8 (0.964)	−1.82 (0.035)	2.82 (0.998)						
	Asinh$_2$	−0.25 (0.4)	2 (0.977)	2.12 (0.983)	0.43 (0.666)	1.66 (0.952)	1.03 (0.849)					
	Mlog$_1$	−3.73 (<0.001)	1.14 (0.873)	0.92 (0.82)	−1.97 (0.025)	0.45 (0.674)	−1.82 (0.034)	−1.76 (0.04)				
	Mlog$_2$	−4.15 (<0.001)	1.28 (0.899)	1.25 (0.895)	−1.58 (0.058)	0.64 (0.74)	−1.12 (0.132)	−1.7 (0.045)	0.99 (0.839)			
	N-PIT	−3.55 (<0.001)	−0.71 (0.24)	−1.1 (0.135)	−4.44 (<0.001)	−1.98 (0.024)	−4.54 (<0.001)	−2.32 (0.01)	−2.23 (0.013)	−2.15 (0.016)		
	T-PIT	−4.32 (<0.001)	−0.82 (0.205)	−1.48 (0.07)	−3.2 (<0.001)	−1.57 (0.058)	−3.17 (<0.001)	−2.66 (0.004)	−2.47 (0.007)	−2.59 (0.005)	−0.54 (0.295)	
ARX-J	Orig.	−0.64 (0.262)	2.83 (0.998)	3.36 (>0.999)	0.66 (0.746)	2.44 (0.993)	1.66 (0.952)	0.1 (0.539)	3.5 (>0.999)	3.82 (>0.999)	3.46 (>0.999)	4.22 (>0.999)
	$3\sigma_1$	−2.54 (0.006)	2.22 (0.987)	0.27 (0.608)	−3.29 (<0.001)	−0.26 (0.396)	−1.82 (0.035)	−1.71 (0.044)	−0.48 (0.314)	−0.66 (0.254)	1.46 (0.928)	1.38 (0.917)
	$3\sigma_2$	−3.4 (<0.001)	0.61 (0.73)	0.05 (0.52)	−2.36 (0.009)	−0.41 (0.342)	−1.8 (0.036)	−2.1 (0.018)	−0.86 (0.196)	−1.16 (0.122)	1.13 (0.87)	1.43 (0.924)
	Logistic$_1$	−0.04 (0.484)	5.35 (>0.999)	3.35 (>0.999)	5.87 (>0.999)	6.09 (>0.999)	3.59 (>0.999)	0.15 (0.558)	3.06 (0.999)	2.59 (0.995)	5.55 (>0.999)	4.09 (>0.999)
	Logistic$_2$	−2.02 (0.022)	3.06 (0.999)	1.41 (0.92)	−3 (0.001)	3.37 (>0.999)	−1.09 (0.139)	−1.27 (0.102)	0.56 (0.711)	0.26 (0.601)	3 (0.999)	2.28 (0.989)
	Asinh$_1$	−0.93 (0.177)	3.97 (>0.999)	2.96 (0.998)	0.19 (0.574)	5.03 (>0.999)	5.01 (>0.999)	−0.43 (0.333)	3.48 (>0.999)	2.55 (0.995)	5.99 (>0.999)	4.21 (>0.999)
	Asinh$_2$	0.34 (0.632)	2.57 (0.995)	2.78 (0.997)	0.88 (0.811)	2.23 (0.987)	1.58 (0.943)	3.1 (>0.999)	2.4 (0.992)	2.36 (0.991)	2.92 (0.998)	3.31 (>0.999)
	Mlog$_1$	−2.61 (0.005)	2.1 (0.982)	2.06 (0.98)	−1.09 (0.138)	1.68 (0.953)	−0.11 (0.455)	−1.2 (0.115)	3.27 (>0.999)	2.05 (0.98)	3.44 (>0.999)	3.5 (>0.999)
	Mlog$_2$	−3.49 (<0.001)	1.51 (0.935)	1.51 (0.935)	−1.33 (0.091)	0.96 (0.831)	−0.7 (0.242)	−1.51 (0.065)	1.48 (0.93)	0.96 (0.832)	2.43 (0.992)	2.77 (0.997)
	N-PIT	−2.78 (0.003)	0.69 (0.753)	0.13 (0.551)	−3.36 (<0.001)	−0.33 (0.372)	−2.7 (0.003)	−1.78 (0.038)	−0.72 (0.234)	−0.86 (0.194)	4.08 (>0.999)	2.18 (0.985)
	T-PIT	−3.97 (<0.001)	−0.44 (0.328)	−0.99 (0.162)	−2.95 (0.002)	−1.19 (0.116)	−2.76 (0.003)	−2.45 (0.007)	−1.93 (0.027)	−2.07 (0.019)	0.17 (0.569)	1.14 (0.873)
ARX-J-CJ	Orig.	−0.64 (0.261)	2.83 (0.998)	3.35 (>0.999)	0.66 (0.745)	2.43 (0.993)	1.66 (0.951)	0.09 (0.537)	3.49 (>0.999)	3.81 (>0.999)	3.46 (>0.999)	4.22 (>0.999)
	$3\sigma_1$	−2.46 (0.007)	2.63 (0.996)	0.44 (0.671)	−3.15 (<0.001)	−0.01 (0.497)	−1.67 (0.047)	−1.65 (0.05)	−0.36 (0.36)	−0.54 (0.294)	1.61 (0.946)	1.5 (0.933)
	$3\sigma_2$	−3.38 (<0.001)	0.65 (0.743)	0.13 (0.552)	−2.34 (0.01)	−0.37 (0.357)	−1.76 (0.039)	−2.08 (0.019)	−0.82 (0.207)	−1.12 (0.131)	1.16 (0.877)	1.46 (0.928)
	Logistic$_1$	−0.01 (0.497)	5.39 (>0.999)	3.39 (>0.999)	5.83 (>0.999)	6.12 (>0.999)	3.66 (>0.999)	0.17 (0.568)	3.11 (>0.999)	2.63 (0.996)	5.58 (>0.999)	4.12 (>0.999)
	Logistic$_2$	−1.94 (0.026)	3.31 (>0.999)	1.56 (0.94)	−2.81 (0.002)	3.89 (>0.999)	−0.86 (0.194)	−1.21 (0.112)	0.71 (0.761)	0.39 (0.652)	3.19 (>0.999)	2.4 (0.992)
	Asinh$_1$	−0.88 (0.19)	4.03 (>0.999)	3.01 (0.999)	0.3 (0.619)	5.14 (>0.999)	5.15 (>0.999)	−0.4 (0.345)	3.55 (>0.999)	2.61 (0.996)	6.02 (>0.999)	4.24 (>0.999)
	Asinh$_2$	0.41 (0.659)	2.65 (0.996)	2.87 (0.998)	0.94 (0.827)	2.31 (0.989)	1.65 (0.951)	3.36 (>0.999)	2.49 (0.994)	2.45 (0.993)	3 (0.999)	3.39 (>0.999)
	Mlog$_1$	−2.76 (0.003)	1.89 (0.97)	1.78 (0.962)	−1.3 (0.097)	1.42 (0.922)	−0.49 (0.313)	−1.31 (0.095)	2.57 (0.995)	1.42 (0.922)	3.17 (>0.999)	3.21 (>0.999)
	Mlog$_2$	−3.21 (<0.001)	1.71 (0.957)	1.75 (0.96)	−1.18 (0.119)	1.19 (0.884)	−0.43 (0.333)	−1.38 (0.084)	2.06 (0.98)	1.57 (0.942)	2.69 (0.996)	3.01 (0.999)
	N-PIT	−2.7 (0.003)	0.83 (0.797)	0.25 (0.597)	−3.3 (<0.001)	−0.17 (0.434)	−2.54 (0.006)	−1.72 (0.042)	−0.58 (0.28)	−0.74 (0.23)	4.42 (>0.999)	2.31 (0.99)
	T-PIT	−3.91 (<0.001)	−0.43 (0.334)	−0.97 (0.167)	−2.97 (0.001)	−1.19 (0.117)	−2.77 (0.003)	−2.43 (0.008)	−1.9 (0.029)	−2.04 (0.021)	0.22 (0.586)	1.16 (0.877)

		ARX-J										
		Orig.	$3\sigma_1$	$3\sigma_2$	Logistic$_1$	Logistic$_2$	Asinh$_1$	Asinh$_2$	Mlog$_1$	Mlog$_2$	N-PIT	T-PIT
ARX-J	$3\sigma_1$	−2.46 (0.007)										
	$3\sigma_2$	−3.41 (<0.001)	−0.26 (0.396)									
	Logistic$_1$	0.1 (0.542)	4.89 (>0.999)	3.44 (>0.999)								
	Logistic$_2$	−1.91 (0.028)	2.09 (0.982)	1.47 (0.929)	−5.25 (<0.001)							
	Asinh$_1$	−0.77 (0.221)	3.4 (>0.999)	3.08 (0.999)	−2.08 (0.019)	3.71 (>0.999)						
	Asinh$_2$	0.5 (0.692)	2.29 (0.989)	2.8 (0.997)	0.28 (0.609)	1.83 (0.967)	0.95 (0.83)					
	Mlog$_1$	−2.54 (0.006)	1.52 (0.936)	2.17 (0.985)	−2.3 (0.011)	0.67 (0.749)	−2.13 (0.017)	−1.84 (0.033)				
	Mlog$_2$	−3.53 (<0.001)	0.96 (0.83)	1.61 (0.946)	−2.4 (0.008)	0.07 (0.527)	−2.27 (0.011)	−2.19 (0.014)	−1.76 (0.039)			
	N-PIT	−2.72 (0.003)	−0.11 (0.456)	0.11 (0.546)	−4.82 (<0.001)	−1.59 (0.056)	−4.78 (<0.001)	−2.37 (0.009)	−2.08 (0.019)	−1.22 (0.111)		
	T-PIT	−4.01 (<0.001)	−1.06 (0.145)	−1.05 (0.148)	−3.92 (<0.001)	−1.99 (0.023)	−3.96 (<0.001)	−3.11 (<0.001)	−3.24 (<0.001)	−2.46 (0.007)	−1.73 (0.042)	
ARX-J-CJ	Orig.	−0.04 (0.486)	2.46 (0.993)	3.39 (>0.999)	−0.11 (0.457)	1.91 (0.972)	0.76 (0.777)	−0.5 (0.308)	2.52 (0.994)	3.52 (>0.999)	2.71 (0.997)	4 (>0.999)
	$3\sigma_1$	−2.39 (0.008)	1.44 (0.925)	0.45 (0.672)	−4.77 (<0.001)	−1.81 (0.035)	−3.26 (<0.001)	−2.23 (0.013)	−1.39 (0.082)	−0.84 (0.201)	0.28 (0.61)	1.18 (0.881)
	$3\sigma_2$	−3.38 (<0.001)	−0.22 (0.412)	0.27 (0.606)	−3.42 (<0.001)	−1.43 (0.077)	−3.04 (0.001)	−2.77 (0.003)	−2.13 (0.017)	−1.57 (0.059)	−0.08 (0.468)	1.08 (0.861)
	Logistic$_1$	0.14 (0.555)	4.94 (>0.999)	3.48 (>0.999)	0.72 (0.766)	5.32 (>0.999)	2.18 (0.985)	−0.25 (0.402)	2.35 (0.991)	2.44 (0.993)	4.87 (>0.999)	3.95 (>0.999)
	Logistic$_2$	−1.83 (0.034)	2.37 (0.991)	1.63 (0.948)	−5.11 (<0.001)	1.46 (0.927)	−3.49 (<0.001)	−1.77 (0.038)	−0.51 (0.304)	0.07 (0.527)	1.8 (0.964)	2.12 (0.983)
	Asinh$_1$	−0.72 (0.236)	3.47 (>0.999)	3.13 (>0.999)	−1.98 (0.024)	3.82 (>0.999)	0.74 (0.77)	−0.92 (0.179)	2.23 (0.987)	2.35 (0.991)	4.86 (>0.999)	4.01 (>0.999)
	Asinh$_2$	0.58 (0.718)	2.37 (0.991)	2.89 (0.998)	0.33 (0.629)	1.91 (0.972)	1.02 (0.846)	0.86 (0.806)	1.92 (0.973)	2.28 (0.989)	2.45 (0.993)	3.2 (>0.999)
	Mlog$_1$	−2.71 (0.003)	1.3 (0.903)	1.87 (0.97)	−2.52 (0.006)	0.4 (0.655)	−2.56 (0.005)	−1.95 (0.025)	−1.84 (0.033)	0.87 (0.807)	1.8 (0.964)	2.93 (0.998)
	Mlog$_2$	−3.24 (<0.001)	1.16 (0.877)	1.87 (0.969)	−2.26 (0.012)	0.3 (0.619)	−2.04 (0.021)	−2.05 (0.02)	−1.01 (0.156)	1.86 (0.969)	1.48 (0.93)	2.73 (0.997)
	N-PIT	−2.64 (0.004)	0.03 (0.513)	0.24 (0.594)	−4.78 (<0.001)	−1.45 (0.073)	−4.63 (<0.001)	−2.31 (0.01)	−1.92 (0.028)	−1.09 (0.137)	1.12 (0.868)	1.88 (0.97)
	T-PIT	−3.96 (<0.001)	−1.05 (0.146)	−1.03 (0.152)	−3.94 (<0.001)	−2 (0.023)	−3.99 (<0.001)	−3.09 (<0.001)	−3.23 (<0.001)	−2.44 (0.007)	−1.75 (0.04)	0.18 (0.571)

		–ARX-J-CJ–									
		Orig.	$3\sigma_1$	$3\sigma_2$	Logistic$_1$	Logistic$_2$	Asinh$_1$	Asinh$_2$	Mlog$_1$	Mlog$_2$	N-PIT
ARX-J-CJ	$3\sigma_1$	−2.39 (0.009)									
	$3\sigma_2$	−3.37 (<0.001)	−0.4 (0.343)								
	Logistic$_1$	0.14 (0.556)	4.82 (>0.999)	3.46 (>0.999)							
	Logistic$_2$	−1.83 (0.034)	2.1 (0.982)	1.58 (0.943)	−5.18 (<0.001)						
	Asinh$_1$	−0.71 (0.238)	3.32 (>0.999)	3.1 (>0.999)	−2.08 (0.019)	3.61 (>0.999)					
	Asinh$_2$	0.57 (0.717)	2.31 (0.989)	2.86 (0.998)	0.3 (0.619)	1.84 (0.967)	0.99 (0.838)				
	Mlog$_1$	−2.69 (0.004)	1.17 (0.879)	1.84 (0.967)	−2.57 (0.005)	0.24 (0.594)	−2.67 (0.004)	−2.04 (0.021)			
	Mlog$_2$	−3.23 (<0.001)	1.04 (0.851)	1.83 (0.967)	−2.3 (0.011)	0.16 (0.565)	−2.12 (0.017)	−2.13 (0.016)	−0.14 (0.445)		
	N-PIT	−2.63 (0.004)	−0.14 (0.445)	0.2 (0.581)	−4.83 (<0.001)	−1.66 (0.048)	−4.72 (<0.001)	−2.39 (0.008)	−1.65 (0.05)	−1.35 (0.089)	
	T-PIT	−3.95 (<0.001)	−1.18 (0.119)	−1.07 (0.143)	−3.98 (<0.001)	−2.13 (0.017)	−4.04 (<0.001)	−3.18 (<0.001)	−2.93 (0.002)	−2.71 (0.003)	−1.91 (0.028)

Multivariate DM test statistic using MAE criterion for forecast horizon 2. P-values in parentheses. A *p*-value lower than 0.10 indicates that the forecasts of the model of the row are better than those of the model of the column at the 10% significance level. A heat map is used to indicate lower (green) and greater (red) p-values.

Table A.3
DM using MAE for H3.

		–ARX– Orig.	–ARX– $3\sigma_1$	–ARX– $3\sigma_2$	–ARX– Logistic$_1$	–ARX– Logistic$_2$	–ARX– Asinh$_1$	–ARX– Asinh$_2$	–ARX– Mlog$_1$	–ARX– Mlog$_2$	–ARX– N-PIT	–ARX– T-PIT
ARX	$3\sigma_1$	−4.02 (<0.001)										
ARX	$3\sigma_2$	−4.31 (<0.001)	1.81 (0.965)									
ARX	Logistic$_1$	−1.72 (0.042)	3.45 (>0.999)	1.43 (0.924)								
ARX	Logistic$_2$	−3.34 (<0.001)	2.37 (0.991)	−0.11 (0.457)	−3.26 (<0.001)							
ARX	Asinh$_1$	−2.52 (0.006)	3.07 (0.999)	1.34 (0.91)	−0.84 (0.2)	2.92 (0.998)						
ARX	Asinh$_2$	−1.85 (0.032)	1.44 (0.925)	1.05 (0.852)	0 (0.5)	0.89 (0.814)	0.22 (0.586)					
ARX	Mlog$_1$	−4.24 (<0.001)	2.4 (0.992)	1.28 (0.901)	−0.87 (0.192)	1.37 (0.915)	−0.77 (0.221)	−0.5 (0.309)				
ARX	Mlog$_2$	−4.93 (<0.001)	2.27 (0.988)	1.44 (0.925)	−0.73 (0.234)	1.23 (0.891)	−0.54 (0.296)	−0.5 (0.308)	0.17 (0.568)			
ARX	N-PIT	−4 (<0.001)	−0.03 (0.489)	−1.25 (0.106)	−3.64 (<0.001)	−1.64 (0.05)	−4 (<0.001)	−1.41 (0.08)	−2.66 (0.004)	−2.27 (0.011)		
ARX	T-PIT	−4.81 (<0.001)	−0.42 (0.336)	−1.75 (0.04)	−2.64 (0.004)	−1.45 (0.073)	−2.89 (0.002)	−1.74 (0.041)	−2.92 (0.002)	−2.74 (0.003)	−0.69 (0.246)	
ARX-J	Orig.	0.17 (0.568)	4.16 (>0.999)	4.35 (>0.999)	1.81 (0.965)	3.45 (>0.999)	2.61 (0.995)	1.87 (0.969)	4.2 (>0.999)	4.75 (>0.999)	4.14 (>0.999)	4.91 (>0.999)
ARX-J	$3\sigma_1$	−3.58 (<0.001)	1.84 (0.967)	−0.74 (0.229)	−2.75 (0.003)	−0.89 (0.187)	−2.29 (0.011)	−1.14 (0.128)	−1.62 (0.053)	−1.55 (0.061)	0.84 (0.798)	1.04 (0.85)
ARX-J	$3\sigma_2$	−4.28 (<0.001)	1.42 (0.922)	−0.32 (0.374)	−1.51 (0.065)	−0.05 (0.48)	−1.43 (0.076)	−1.1 (0.135)	−1.37 (0.085)	−1.5 (0.066)	1.08 (0.86)	1.52 (0.936)
ARX-J	Logistic$_1$	−0.88 (0.19)	4.75 (>0.999)	2.53 (0.994)	5.47 (>0.999)	5.09 (>0.999)	3.09 (0.999)	0.61 (0.729)	2.2 (0.986)	1.92 (0.972)	4.88 (>0.999)	3.62 (>0.999)
ARX-J	Logistic$_2$	−2.7 (0.003)	3.61 (>0.999)	0.98 (0.837)	−1.68 (0.046)	3.26 (>0.999)	−0.96 (0.169)	−0.46 (0.322)	−0.08 (0.467)	−0.12 (0.453)	2.78 (0.997)	2.24 (0.988)
ARX-J	Asinh$_1$	−1.44 (0.076)	4.65 (>0.999)	2.7 (0.997)	1.85 (0.968)	5.36 (>0.999)	4.92 (>0.999)	0.47 (0.68)	2.87 (0.998)	2.28 (0.989)	5.54 (>0.999)	4.07 (>0.999)
ARX-J	Asinh$_2$	−1.02 (0.153)	2.29 (0.989)	2.01 (0.978)	0.63 (0.736)	1.69 (0.954)	0.94 (0.827)	3.44 (>0.999)	1.37 (0.914)	1.42 (0.922)	2.21 (0.987)	2.63 (0.996)
ARX-J	Mlog$_1$	−3.3 (<0.001)	3.08 (0.999)	2.11 (0.983)	−0.17 (0.431)	2.29 (0.989)	0.45 (0.673)	−0.1 (0.461)	2.3 (0.989)	1.78 (0.962)	3.42 (>0.999)	3.55 (>0.999)
ARX-J	Mlog$_2$	−4.15 (<0.001)	2.44 (0.993)	1.66 (0.951)	−0.45 (0.325)	1.54 (0.938)	−0.1 (0.458)	−0.31 (0.379)	0.91 (0.82)	0.9 (0.815)	2.52 (0.994)	2.9 (0.998)
ARX-J	N-PIT	−3.28 (<0.001)	1.43 (0.924)	−0.07 (0.472)	−2.49 (0.006)	0.02 (0.508)	−2.37 (0.009)	−0.87 (0.193)	−1.22 (0.111)	−1.07 (0.143)	3.56 (>0.999)	2.18 (0.985)
ARX-J	T-PIT	−4.65 (<0.001)	−0.24 (0.403)	−1.48 (0.069)	−2.58 (0.005)	−1.3 (0.097)	−2.78 (0.003)	−1.61 (0.053)	−2.7 (0.004)	−2.52 (0.006)	−0.36 (0.36)	0.42 (0.663)
ARX-J-CJ	Orig.	0.11 (0.546)	4.18 (>0.999)	4.38 (>0.999)	1.82 (0.965)	3.48 (>0.999)	2.62 (0.996)	1.84 (0.967)	4.25 (>0.999)	4.8 (>0.999)	4.17 (>0.999)	4.94 (>0.999)
ARX-J-CJ	$3\sigma_1$	−3.42 (<0.001)	2.61 (0.995)	−0.38 (0.351)	−2.47 (0.007)	−0.38 (0.351)	−1.98 (0.024)	−1.01 (0.156)	−1.33 (0.092)	−1.27 (0.101)	1.18 (0.881)	1.31 (0.904)
ARX-J-CJ	$3\sigma_2$	−4.23 (<0.001)	1.58 (0.943)	−0.12 (0.452)	−1.48 (0.069)	0.05 (0.518)	−1.38 (0.083)	−1.05 (0.148)	−1.29 (0.099)	−1.41 (0.079)	1.18 (0.882)	1.62 (0.948)
ARX-J-CJ	Logistic$_1$	−0.9 (0.184)	4.69 (>0.999)	2.5 (0.994)	5.22 (>0.999)	5 (>0.999)	3.02 (0.999)	0.59 (0.723)	2.16 (0.985)	1.88 (0.97)	4.82 (>0.999)	3.58 (>0.999)
ARX-J-CJ	Logistic$_2$	−2.7 (0.003)	3.48 (>0.999)	0.93 (0.823)	−1.75 (0.04)	3.04 (0.999)	−1.01 (0.156)	−0.47 (0.318)	−0.12 (0.451)	−0.15 (0.439)	2.7 (0.997)	2.19 (0.986)
ARX-J-CJ	Asinh$_1$	−1.42 (0.077)	4.61 (>0.999)	2.69 (0.996)	1.86 (0.969)	5.31 (>0.999)	4.85 (>0.999)	0.47 (0.682)	2.87 (0.998)	2.28 (0.989)	5.52 (>0.999)	4.06 (>0.999)
ARX-J-CJ	Asinh$_2$	−1.03 (0.151)	2.3 (0.989)	2.02 (0.979)	0.63 (0.737)	1.7 (0.955)	0.95 (0.829)	3.34 (>0.999)	1.38 (0.916)	1.43 (0.924)	2.22 (0.987)	2.64 (0.996)
ARX-J-CJ	Mlog$_1$	−3.28 (<0.001)	3.05 (0.999)	2.05 (0.98)	−0.19 (0.423)	2.28 (0.989)	0.43 (0.668)	−0.11 (0.457)	2.23 (0.987)	1.67 (0.953)	3.39 (>0.999)	3.47 (>0.999)
ARX-J-CJ	Mlog$_2$	−3.76 (<0.001)	2.75 (0.997)	2.01 (0.978)	−0.22 (0.414)	1.9 (0.971)	0.27 (0.607)	−0.14 (0.444)	1.69 (0.955)	1.66 (0.952)	2.89 (0.998)	3.23 (>0.999)
ARX-J-CJ	N-PIT	−3.23 (<0.001)	1.5 (0.933)	−0.02 (0.492)	−2.49 (0.006)	0.1 (0.54)	−2.32 (0.01)	−0.84 (0.201)	−1.15 (0.124)	−1.01 (0.156)	3.65 (>0.999)	2.21 (0.986)
ARX-J-CJ	T-PIT	−4.77 (<0.001)	−0.5 (0.307)	−1.75 (0.04)	−2.8 (0.003)	−1.56 (0.059)	−3.05 (0.001)	−1.72 (0.043)	−2.97 (0.001)	−2.77 (0.003)	−0.78 (0.218)	−0.21 (0.417)

		ARX-J Orig.	ARX-J $3\sigma_1$	ARX-J $3\sigma_2$	ARX-J Logistic$_1$	ARX-J Logistic$_2$	ARX-J Asinh$_1$	ARX-J Asinh$_2$	ARX-J Mlog$_1$	ARX-J Mlog$_2$	ARX-J N-PIT	ARX-J T-PIT
ARX-J	$3\sigma_1$	−3.83 (<0.001)										
ARX-J	$3\sigma_2$	−4.65 (<0.001)	0.6 (0.727)									
ARX-J	Logistic$_1$	−0.96 (0.17)	4.34 (>0.999)	2.66 (0.996)								
ARX-J	Logistic$_2$	−2.87 (0.002)	2.89 (0.998)	1.17 (0.878)	−4.17 (<0.001)							
ARX-J	Asinh$_1$	−1.54 (0.061)	4.19 (>0.999)	2.89 (0.998)	−0.9 (0.184)	4.19 (>0.999)						
ARX-J	Asinh$_2$	−1.08 (0.14)	1.99 (0.977)	2.11 (0.983)	−0.02 (0.491)	1.24 (0.893)	0.21 (0.583)					
ARX-J	Mlog$_1$	−3.58 (<0.001)	2.53 (0.994)	2.45 (0.993)	−1.6 (0.055)	1.12 (0.868)	−1.94 (0.026)	−0.95 (0.17)				
ARX-J	Mlog$_2$	−4.58 (<0.001)	1.91 (0.972)	2.03 (0.979)	−1.69 (0.045)	0.51 (0.695)	−1.98 (0.024)	−1.21 (0.112)	−1.25 (0.106)			
ARX-J	N-PIT	−3.47 (<0.001)	0.65 (0.743)	0.06 (0.523)	−4.24 (<0.001)	−1.44 (0.075)	−4.84 (<0.001)	−1.65 (0.049)	−2.24 (0.012)	−1.45 (0.073)		
ARX-J	T-PIT	−4.96 (<0.001)	−0.91 (0.181)	−1.38 (0.084)	−3.66 (<0.001)	−2.2 (0.014)	−4.17 (<0.001)	−2.51 (0.006)	−3.64 (<0.001)	−2.92 (0.002)	−2.15 (0.016)	
ARX-J-CJ	Orig.	−0.13 (0.447)	3.85 (>0.999)	4.68 (>0.999)	0.95 (0.829)	2.88 (0.998)	1.54 (0.938)	1.06 (0.855)	3.62 (>0.999)	4.63 (>0.999)	3.49 (>0.999)	4.99 (>0.999)
ARX-J-CJ	$3\sigma_1$	−3.67 (<0.001)	2.28 (0.989)	−0.22 (0.412)	−4.1 (<0.001)	−2.36 (0.009)	−3.91 (<0.001)	−1.86 (0.031)	−2.24 (0.013)	−1.64 (0.05)	−0.29 (0.385)	1.19 (0.883)
ARX-J-CJ	$3\sigma_2$	−4.6 (<0.001)	0.74 (0.769)	0.61 (0.73)	−2.65 (0.004)	−1.09 (0.137)	−2.87 (0.002)	−2.04 (0.021)	−2.38 (0.009)	−1.93 (0.027)	0.03 (0.51)	1.49 (0.931)
ARX-J-CJ	Logistic$_1$	−0.98 (0.164)	4.28 (>0.999)	2.63 (0.996)	−0.42 (0.338)	4.09 (>0.999)	0.83 (0.796)	0.01 (0.503)	1.56 (0.94)	1.66 (0.952)	4.19 (>0.999)	3.62 (>0.999)
ARX-J-CJ	Logistic$_2$	−2.88 (0.002)	2.76 (0.997)	1.11 (0.866)	−4.29 (<0.001)	−0.33 (0.37)	−4.31 (<0.001)	−1.25 (0.106)	−1.15 (0.125)	−0.54 (0.294)	1.37 (0.915)	2.15 (0.984)
ARX-J-CJ	Asinh$_1$	−1.53 (0.063)	4.17 (>0.999)	2.88 (0.998)	−0.88 (0.189)	4.19 (>0.999)	0.1 (0.54)	−0.2 (0.419)	1.96 (0.975)	1.98 (0.976)	4.84 (>0.999)	4.17 (>0.999)
ARX-J-CJ	Asinh$_2$	−1.09 (0.138)	2 (0.977)	2.12 (0.983)	−0.02 (0.491)	1.25 (0.894)	0.21 (0.584)	0.01 (0.504)	0.96 (0.832)	1.22 (0.89)	1.66 (0.952)	2.52 (0.994)
ARX-J-CJ	Mlog$_1$	−3.56 (<0.001)	2.5 (0.994)	2.38 (0.991)	−1.64 (0.051)	1.1 (0.864)	−2.02 (0.022)	−0.95 (0.171)	−0.12 (0.452)	1.11 (0.867)	2.22 (0.987)	3.57 (>0.999)
ARX-J-CJ	Mlog$_2$	−4.16 (<0.001)	2.24 (0.988)	2.41 (0.992)	−1.49 (0.068)	0.87 (0.809)	−1.67 (0.047)	−1.02 (0.153)	−0.27 (0.394)	2.1 (0.982)	1.82 (0.966)	3.28 (>0.999)
ARX-J-CJ	N-PIT	−3.42 (<0.001)	0.73 (0.766)	0.11 (0.544)	−4.27 (<0.001)	−1.38 (0.085)	−4.83 (<0.001)	−1.62 (0.053)	−2.18 (0.015)	−1.39 (0.082)	0.43 (0.666)	2.18 (0.985)
ARX-J-CJ	T-PIT	−5.1 (<0.001)	−1.19 (0.116)	−1.66 (0.048)	−3.89 (<0.001)	−2.48 (0.007)	−4.43 (<0.001)	−2.62 (0.004)	−3.94 (<0.001)	−3.19 (<0.001)	−2.6 (0.005)	−1.59 (0.056)

		–ARX-J-CJ– Orig.	–ARX-J-CJ– $3\sigma_1$	–ARX-J-CJ– $3\sigma_2$	–ARX-J-CJ– Logistic$_1$	–ARX-J-CJ– Logistic$_2$	–ARX-J-CJ– Asinh$_1$	–ARX-J-CJ– Asinh$_2$	–ARX-J-CJ– Mlog$_1$	–ARX-J-CJ– Mlog$_2$	–ARX-J-CJ– N-PIT
ARX-J-CJ	$3\sigma_1$	−3.68 (<0.001)									
ARX-J-CJ	$3\sigma_2$	−4.63 (<0.001)	0.35 (0.637)								
ARX-J-CJ	Logistic$_1$	−0.97 (0.166)	4.04 (>0.999)	2.61 (0.996)							
ARX-J-CJ	Logistic$_2$	−2.89 (0.002)	2.26 (0.988)	1.04 (0.85)	−4.22 (<0.001)						
ARX-J-CJ	Asinh$_1$	−1.53 (0.063)	3.89 (>0.999)	2.86 (0.998)	−0.81 (0.209)	4.3 (>0.999)					
ARX-J-CJ	Asinh$_2$	−1.06 (0.144)	1.87 (0.969)	2.05 (0.98)	−0.01 (0.498)	1.26 (0.896)	0.21 (0.582)				
ARX-J-CJ	Mlog$_1$	−3.59 (<0.001)	2.21 (0.986)	2.31 (0.99)	−1.6 (0.055)	1.13 (0.87)	−2.04 (0.021)	−0.96 (0.169)			
ARX-J-CJ	Mlog$_2$	−4.23 (<0.001)	1.97 (0.976)	2.32 (0.99)	−1.45 (0.073)	0.9 (0.817)	−1.68 (0.047)	−1.03 (0.151)	−0.19 (0.423)		
ARX-J-CJ	N-PIT	−3.45 (<0.001)	0.37 (0.643)	0.03 (0.512)	−4.22 (<0.001)	−1.31 (0.095)	−4.85 (<0.001)	−1.63 (0.052)	−2.15 (0.016)	−1.76 (0.039)	
ARX-J-CJ	T-PIT	−5.13 (<0.001)	−1.48 (0.069)	−1.77 (0.038)	−3.85 (<0.001)	−2.42 (0.008)	−4.43 (<0.001)	−2.63 (0.004)	−3.85 (<0.001)	−3.55 (<0.001)	−2.64 (0.004)

Multivariate DM test statistic using MAE criterion for forecast horizon 3. P-values in parentheses. A p-value lower than 0.10 indicates that the forecasts of the model of the row are better than those of the model of the column at the 10% significance level. A heat map is used to indicate lower (green) and greater (red) p-values.

Table A.4
DM using MAE for H4.

–ARX– (columns)

Group	Model	Orig.	$3\sigma_1$	$3\sigma_2$	Logistic$_1$	Logistic$_2$	Asinh$_1$	Asinh$_2$	Mlog$_1$	Mlog$_2$	N-PIT	T-PIT
ARX	$3\sigma_1$	−4.45 (<0.001)										
ARX	$3\sigma_2$	−4.62 (<0.001)	2.28 (0.989)									
ARX	Logistic$_1$	−2.05 (0.02)	3.19 (>0.999)	1.03 (0.848)								
ARX	Logistic$_2$	−3.69 (<0.001)	2.46 (0.993)	−0.44 (0.328)	−2.82 (0.002)							
ARX	Asinh$_1$	−2.69 (0.004)	3.44 (>0.999)	1.21 (0.887)	−0.21 (0.417)	3.4 (>0.999)						
ARX	Asinh$_2$	−2.23 (0.013)	1.38 (0.916)	0.77 (0.779)	−0.02 (0.492)	0.78 (0.783)	0.03 (0.513)					
ARX	Mlog$_1$	−4.49 (<0.001)	2.8 (0.997)	1.19 (0.883)	−0.5 (0.309)	1.69 (0.954)	−0.7 (0.242)	−0.26 (0.396)				
ARX	Mlog$_2$	−5.15 (<0.001)	2.79 (0.997)	1.61 (0.946)	−0.28 (0.388)	1.66 (0.952)	−0.3 (0.384)	−0.17 (0.431)	0.76 (0.776)			
ARX	N-PIT	−4.05 (<0.001)	0.54 (0.705)	−1.13 (0.128)	−2.94 (0.002)	−1.12 (0.132)	−3.73 (<0.001)	−1.12 (0.131)	−2.44 (0.007)	−2.25 (0.012)		
ARX	T-PIT	−4.79 (<0.001)	0.07 (0.529)	−1.61 (0.054)	−2.16 (0.015)	−1.07 (0.143)	−2.68 (0.004)	−1.4 (0.081)	−2.68 (0.004)	−2.7 (0.003)	−0.59 (0.278)	
ARX-J	Orig.	−0.15 (0.439)	4.49 (>0.999)	4.51 (>0.999)	2.06 (0.98)	3.7 (>0.999)	2.66 (0.996)	2.13 (0.983)	4.24 (>0.999)	4.74 (>0.999)	4.08 (>0.999)	4.77 (>0.999)
ARX-J	$3\sigma_1$	−4.03 (<0.001)	1.71 (0.956)	−1.24 (0.107)	−2.53 (0.006)	−1.05 (0.148)	−2.63 (0.004)	−1.09 (0.137)	−2 (0.023)	−2.07 (0.019)	0.25 (0.6)	0.54 (0.707)
ARX-J	$3\sigma_2$	−4.78 (<0.001)	1.56 (0.94)	−0.72 (0.237)	−1.2 (0.115)	0.08 (0.532)	−1.41 (0.079)	−0.92 (0.178)	−1.45 (0.074)	−1.86 (0.031)	0.79 (0.784)	1.18 (0.881)
ARX-J	Logistic$_1$	−1.21 (0.114)	4.53 (>0.999)	2.13 (0.983)	5.6 (>0.999)	4.74 (>0.999)	2.6 (0.995)	0.63 (0.735)	1.85 (0.968)	1.49 (0.931)	4.31 (>0.999)	3.19 (>0.999)
ARX-J	Logistic$_2$	−3.05 (0.001)	3.55 (>0.999)	0.61 (0.729)	−1.32 (0.094)	2.94 (0.998)	−1.31 (0.096)	−0.37 (0.358)	−0.38 (0.353)	−0.55 (0.293)	2.22 (0.987)	1.86 (0.968)
ARX-J	Asinh$_1$	−1.87 (0.031)	4.47 (>0.999)	2.19 (0.986)	1.73 (0.959)	4.98 (>0.999)	3.56 (>0.999)	0.47 (0.679)	2.22 (0.987)	1.57 (0.942)	4.69 (>0.999)	3.48 (>0.999)
ARX-J	Asinh$_2$	−1.44 (0.074)	2.22 (0.987)	1.72 (0.958)	0.59 (0.723)	1.56 (0.94)	0.72 (0.765)	3.31 (>0.999)	1.09 (0.863)	1.05 (0.854)	1.89 (0.971)	2.24 (0.988)
ARX-J	Mlog$_1$	−4.11 (<0.001)	2.81 (0.998)	1.33 (0.908)	−0.31 (0.377)	1.86 (0.969)	−0.35 (0.365)	−0.15 (0.439)	0.62 (0.732)	0.03 (0.51)	2.5 (0.994)	2.66 (0.996)
ARX-J	Mlog$_2$	−5.05 (<0.001)	2.24 (0.987)	0.96 (0.832)	−0.51 (0.306)	1.21 (0.886)	−0.62 (0.267)	−0.32 (0.373)	−0.25 (0.403)	−0.81 (0.21)	1.81 (0.965)	2.16 (0.985)
ARX-J	N-PIT	−3.18 (<0.001)	2.34 (0.99)	0.26 (0.602)	−1.46 (0.072)	0.96 (0.832)	−1.71 (0.043)	−0.48 (0.315)	−0.71 (0.239)	−0.8 (0.213)	4.06 (>0.999)	2.41 (0.992)
ARX-J	T-PIT	−4.71 (<0.001)	0.12 (0.546)	−1.5 (0.067)	−2.22 (0.013)	−1.06 (0.145)	−2.72 (0.003)	−1.34 (0.09)	−2.62 (0.004)	−2.62 (0.004)	−0.48 (0.314)	0.1 (0.539)
ARX-J-CJ	Orig.	−0.01 (0.495)	4.66 (>0.999)	4.69 (>0.999)	2.16 (0.985)	3.87 (>0.999)	2.79 (0.997)	2.17 (0.985)	4.48 (>0.999)	5.02 (>0.999)	4.24 (>0.999)	4.93 (>0.999)
ARX-J-CJ	$3\sigma_1$	−3.91 (<0.001)	2.35 (0.991)	−0.96 (0.169)	−2.33 (0.01)	−0.66 (0.253)	−2.41 (0.008)	−0.99 (0.162)	−1.78 (0.038)	−1.85 (0.032)	0.53 (0.704)	0.77 (0.779)
ARX-J-CJ	$3\sigma_2$	−4.69 (<0.001)	1.74 (0.959)	−0.47 (0.319)	−1.14 (0.126)	0.2 (0.58)	−1.34 (0.09)	−0.86 (0.196)	−1.34 (0.09)	−1.74 (0.041)	0.9 (0.816)	1.3 (0.903)
ARX-J-CJ	Logistic$_1$	−1.26 (0.103)	4.42 (>0.999)	2.07 (0.981)	5.16 (>0.999)	4.63 (>0.999)	2.48 (0.993)	0.59 (0.722)	1.78 (0.962)	1.42 (0.922)	4.2 (>0.999)	3.12 (>0.999)
ARX-J-CJ	Logistic$_2$	−3 (0.001)	3.68 (>0.999)	0.68 (0.753)	−1.24 (0.108)	3.24 (>0.999)	−1.21 (0.113)	−0.33 (0.37)	−0.29 (0.387)	−0.47 (0.321)	2.33 (0.99)	1.92 (0.973)
ARX-J-CJ	Asinh$_1$	−1.81 (0.035)	4.58 (>0.999)	2.27 (0.988)	1.89 (0.971)	5.22 (>0.999)	3.85 (>0.999)	0.5 (0.692)	2.32 (0.99)	1.66 (0.951)	4.82 (>0.999)	3.56 (>0.999)
ARX-J-CJ	Asinh$_2$	−1.4 (0.081)	2.26 (0.988)	1.77 (0.962)	0.62 (0.733)	1.6 (0.945)	0.76 (0.776)	3.39 (>0.999)	1.13 (0.871)	1.09 (0.863)	1.93 (0.973)	2.28 (0.989)
ARX-J-CJ	Mlog$_1$	−3.96 (<0.001)	2.93 (0.998)	1.45 (0.926)	−0.21 (0.418)	2.03 (0.979)	−0.17 (0.434)	−0.09 (0.464)	0.99 (0.839)	0.32 (0.626)	2.63 (0.996)	2.75 (0.997)
ARX-J-CJ	Mlog$_2$	−4.4 (<0.001)	2.84 (0.998)	1.68 (0.953)	−0.09 (0.465)	1.87 (0.969)	0.01 (0.505)	−0.03 (0.487)	1.07 (0.858)	0.67 (0.748)	2.41 (0.992)	2.74 (0.997)
ARX-J-CJ	N-PIT	−3.06 (0.001)	2.55 (0.995)	0.42 (0.662)	−1.28 (0.101)	1.23 (0.89)	−1.45 (0.073)	−0.4 (0.343)	−0.5 (0.309)	−0.62 (0.269)	4.51 (>0.999)	2.6 (0.995)
ARX-J-CJ	T-PIT	−4.87 (<0.001)	−0.16 (0.436)	−1.77 (0.038)	−2.45 (0.007)	−1.33 (0.091)	−3.03 (0.001)	−1.44 (0.075)	−2.94 (0.002)	−2.9 (0.002)	−0.93 (0.175)	−0.51 (0.304)

ARX-J (columns)

Group	Model	Orig.	$3\sigma_1$	$3\sigma_2$	Logistic$_1$	Logistic$_2$	Asinh$_1$	Asinh$_2$	Mlog$_1$	Mlog$_2$	N-PIT	T-PIT
ARX-J	$3\sigma_1$	−4.17 (<0.001)										
ARX-J	$3\sigma_2$	−5.04 (<0.001)	0.86 (0.806)									
ARX-J	Logistic$_1$	−1.21 (0.114)	4.12 (>0.999)	2.31 (0.99)								
ARX-J	Logistic$_2$	−3.12 (<0.001)	2.87 (0.998)	0.93 (0.823)	−3.76 (<0.001)							
ARX-J	Asinh$_1$	−1.87 (0.031)	3.99 (>0.999)	2.42 (0.992)	−0.97 (0.166)	3.71 (>0.999)						
ARX-J	Asinh$_2$	−1.38 (0.084)	1.94 (0.974)	1.91 (0.972)	−0.06 (0.476)	1.13 (0.87)	0.18 (0.573)					
ARX-J	Mlog$_1$	−4.14 (<0.001)	2.26 (0.988)	1.76 (0.961)	−1.73 (0.042)	0.68 (0.752)	−2.06 (0.02)	−0.98 (0.165)				
ARX-J	Mlog$_2$	−5.15 (<0.001)	1.72 (0.957)	1.44 (0.925)	−1.72 (0.042)	0.21 (0.584)	−1.95 (0.026)	−1.2 (0.115)	−0.91 (0.18)			
ARX-J	N-PIT	−3.25 (<0.001)	1.65 (0.951)	0.53 (0.703)	−3.36 (<0.001)	−0.43 (0.333)	−3.55 (<0.001)	−1.22 (0.111)	−0.98 (0.163)	−0.48 (0.316)		
ARX-J	T-PIT	−4.85 (<0.001)	−0.54 (0.295)	−1.19 (0.117)	−3.37 (<0.001)	−1.97 (0.024)	−3.75 (<0.001)	−2.2 (0.014)	−2.87 (0.002)	−2.26 (0.012)	−2.75 (0.003)	
ARX-J-CJ	Orig.	0.3 (0.617)	4.34 (>0.999)	5.23 (>0.999)	1.28 (0.899)	3.26 (>0.999)	1.97 (0.976)	1.42 (0.922)	4.4 (>0.999)	5.49 (>0.999)	3.39 (>0.999)	5.01 (>0.999)
ARX-J-CJ	$3\sigma_1$	−4.04 (<0.001)	1.73 (0.958)	−0.57 (0.283)	−3.94 (<0.001)	−2.48 (0.007)	−3.78 (<0.001)	−1.83 (0.034)	−2.03 (0.021)	−1.51 (0.066)	−1.37 (0.085)	0.78 (0.783)
ARX-J-CJ	$3\sigma_2$	−4.95 (<0.001)	1.03 (0.85)	0.69 (0.756)	−2.27 (0.011)	−0.83 (0.203)	−2.37 (0.009)	−1.83 (0.034)	−1.66 (0.048)	−1.31 (0.094)	−0.44 (0.329)	1.32 (0.906)
ARX-J-CJ	Logistic$_1$	−1.27 (0.103)	4.01 (>0.999)	2.25 (0.988)	−0.97 (0.166)	3.65 (>0.999)	0.8 (0.789)	0.02 (0.507)	1.65 (0.951)	1.66 (0.951)	3.24 (>0.999)	3.29 (>0.999)
ARX-J-CJ	Logistic$_2$	−3.06 (0.001)	3 (0.999)	0.99 (0.84)	−3.73 (<0.001)	0.59 (0.722)	−3.65 (<0.001)	−1.09 (0.138)	−0.58 (0.28)	−0.13 (0.447)	0.55 (0.708)	2.04 (0.979)
ARX-J-CJ	Asinh$_1$	−1.81 (0.035)	4.12 (>0.999)	2.5 (0.994)	−0.85 (0.197)	3.95 (>0.999)	0.52 (0.698)	−0.15 (0.44)	2.19 (0.986)	2.03 (0.979)	3.71 (>0.999)	3.82 (>0.999)
ARX-J-CJ	Asinh$_2$	−1.34 (0.089)	1.99 (0.977)	1.96 (0.975)	−0.03 (0.488)	1.17 (0.878)	0.22 (0.586)	0.37 (0.643)	1.02 (0.846)	1.24 (0.893)	1.26 (0.896)	2.24 (0.987)
ARX-J-CJ	Mlog$_1$	−3.97 (<0.001)	2.39 (0.991)	1.87 (0.969)	−1.65 (0.05)	0.85 (0.801)	−1.95 (0.026)	−0.9 (0.185)	0.73 (0.768)	1.19 (0.882)	1.14 (0.874)	2.96 (0.998)
ARX-J-CJ	Mlog$_2$	−4.47 (<0.001)	2.33 (0.99)	2.18 (0.985)	−1.34 (0.09)	0.86 (0.804)	−1.38 (0.084)	−0.87 (0.191)	0.76 (0.776)	3.37 (>0.999)	1.08 (0.86)	2.88 (0.998)
ARX-J-CJ	N-PIT	−3.12 (<0.001)	1.87 (0.97)	0.69 (0.754)	−3.24 (<0.001)	−0.17 (0.432)	−3.35 (<0.001)	−1.13 (0.129)	−0.76 (0.222)	−0.3 (0.381)	1.27 (0.898)	2.96 (0.998)
ARX-J-CJ	T-PIT	−5.02 (<0.001)	−0.83 (0.202)	−1.46 (0.072)	−3.61 (<0.001)	−2.26 (0.012)	−4.04 (<0.001)	−2.3 (0.011)	−3.19 (<0.001)	−2.55 (0.005)	−3.25 (<0.001)	−1.39 (0.082)

–ARX-J-CJ– (columns)

Group	Model	Orig.	$3\sigma_1$	$3\sigma_2$	Logistic$_1$	Logistic$_2$	Asinh$_1$	Asinh$_2$	Mlog$_1$	Mlog$_2$	N-PIT
ARX-J-CJ	$3\sigma_1$	−4.21 (<0.001)									
ARX-J-CJ	$3\sigma_2$	−5.15 (<0.001)	0.74 (0.77)								
ARX-J-CJ	Logistic$_1$	−1.34 (0.091)	3.83 (>0.999)	2.21 (0.986)							
ARX-J-CJ	Logistic$_2$	−3.2 (<0.001)	2.63 (0.996)	0.9 (0.816)	−3.61 (<0.001)						
ARX-J-CJ	Asinh$_1$	−1.91 (0.028)	3.91 (>0.999)	2.45 (0.993)	−0.68 (0.248)	3.89 (>0.999)					
ARX-J-CJ	Asinh$_2$	−1.38 (0.083)	1.87 (0.969)	1.88 (0.97)	0.01 (0.504)	1.13 (0.87)	0.18 (0.573)				
ARX-J-CJ	Mlog$_1$	−4.22 (<0.001)	2.16 (0.985)	1.77 (0.962)	−1.57 (0.058)	0.75 (0.773)	−2.08 (0.019)	−0.94 (0.174)			
ARX-J-CJ	Mlog$_2$	−4.78 (<0.001)	2.12 (0.983)	2.07 (0.981)	−1.27 (0.102)	0.77 (0.78)	−1.46 (0.071)	−0.92 (0.18)	0.35 (0.637)		
ARX-J-CJ	N-PIT	−3.27 (<0.001)	1.59 (0.945)	0.6 (0.725)	−3.11 (<0.001)	−0.28 (0.388)	−3.52 (<0.001)	−1.17 (0.121)	−0.92 (0.178)	−0.9 (0.185)	
ARX-J-CJ	T-PIT	−5.2 (<0.001)	−1.08 (0.14)	−1.6 (0.055)	−3.53 (<0.001)	−2.33 (0.01)	−4.13 (<0.001)	−2.34 (0.01)	−3.27 (<0.001)	−3.18 (<0.001)	−3.47 (<0.001)

Multivariate DM test statistic using MAE criterion for forecast horizon 4. P-values in parentheses. A *p*-value lower than 0.10 indicates that the forecasts of the model of the row are better than those of the model of the column at the 10% significance level. A heat map is used to indicate lower (green) and greater (red) p-values.

Table A.5

DM using MAE for H5.

–ARX– (columns)

		Orig.	$3\sigma_1$	$3\sigma_2$	Logistic$_1$	Logistic$_2$	Asinh$_1$	Asinh$_2$	Mlog$_1$	Mlog$_2$	N-PIT	T-PIT
ARX	$3\sigma_1$	−4.28 (<0.001)										
ARX	$3\sigma_2$	−4.49 (<0.001)	2.03 (0.979)									
ARX	Logistic$_1$	−2.21 (0.014)	2.33 (0.99)	0.61 (0.729)								
ARX	Logistic$_2$	−3.59 (<0.001)	1.9 (0.972)	−0.52 (0.302)	−1.96 (0.025)							
ARX	Asinh$_1$	−2.67 (0.004)	2.98 (0.999)	1.01 (0.845)	0.4 (0.655)	3.08 (0.999)						
ARX	Asinh$_2$	−1.61 (0.053)	1.9 (0.972)	1.48 (0.93)	0.65 (0.741)	1.37 (0.915)	0.62 (0.734)					
ARX	Mlog$_1$	−4.32 (<0.001)	2.69 (0.996)	1.25 (0.894)	0.05 (0.519)	1.82 (0.965)	−0.27 (0.392)	−0.85 (0.198)				
ARX	Mlog$_2$	−4.91 (<0.001)	2.71 (0.997)	1.71 (0.956)	0.22 (0.586)	1.81 (0.965)	0.07 (0.527)	−0.78 (0.219)	0.89 (0.812)			
ARX	N-PIT	−3.82 (<0.001)	0.48 (0.686)	−0.97 (0.166)	−2.13 (0.017)	−0.8 (0.211)	−3.18 (<0.001)	−1.59 (0.056)	−2.28 (0.011)	−2.13 (0.017)		
ARX	T-PIT	−4.39 (<0.001)	0.42 (0.663)	−1.06 (0.144)	−1.37 (0.085)	−0.53 (0.298)	−2.04 (0.021)	−1.76 (0.039)	−2.23 (0.013)	−2.29 (0.011)	0.04 (0.515)	
ARX-J	Orig.	−2.37 (0.009)	3.63 (>0.999)	3.52 (>0.999)	1.64 (0.95)	2.91 (0.998)	1.92 (0.973)	0.89 (0.814)	3.05 (0.999)	3.33 (>0.999)	3.16 (>0.999)	3.6 (>0.999)
ARX-J	$3\sigma_1$	−4.09 (<0.001)	0.43 (0.668)	−1.6 (0.055)	−2.12 (0.017)	−1.36 (0.087)	−2.68 (0.004)	−1.82 (0.035)	−2.36 (0.009)	−2.42 (0.008)	−0.26 (0.398)	−0.24 (0.406)
ARX-J	$3\sigma_2$	−5.46 (<0.001)	−0.16 (0.437)	−3.42 (<0.001)	−1.55 (0.06)	−1.08 (0.139)	−2.16 (0.015)	−2.31 (0.01)	−2.8 (0.003)	−3.35 (<0.001)	−0.43 (0.334)	−0.5 (0.309)
ARX-J	Logistic$_1$	−1.53 (0.063)	3.51 (>0.999)	1.54 (0.938)	4.71 (>0.999)	3.66 (>0.999)	1.71 (0.956)	−0.12 (0.45)	1.08 (0.86)	0.77 (0.779)	3.38 (>0.999)	2.29 (0.989)
ARX-J	Logistic$_2$	−3.19 (<0.001)	2.56 (0.995)	0.14 (0.557)	−1.06 (0.144)	1.88 (0.97)	−1.72 (0.043)	−1.1 (0.136)	−0.98 (0.163)	−1.11 (0.134)	1.48 (0.931)	1.03 (0.849)
ARX-J	Asinh$_1$	−2.04 (0.021)	3.85 (>0.999)	1.8 (0.964)	1.85 (0.967)	4.41 (>0.999)	2.87 (0.998)	−0.22 (0.414)	1.51 (0.935)	0.95 (0.828)	4.04 (>0.999)	2.73 (0.997)
ARX-J	Asinh$_2$	−1.05 (0.148)	2.48 (0.994)	2.16 (0.985)	1.07 (0.857)	1.91 (0.972)	1.1 (0.864)	2.35 (0.991)	1.43 (0.923)	1.4 (0.919)	2.1 (0.982)	2.33 (0.99)
ARX-J	Mlog$_1$	−4.27 (<0.001)	2.32 (0.99)	0.96 (0.832)	−0.07 (0.472)	1.52 (0.936)	−0.45 (0.326)	−0.9 (0.183)	−0.43 (0.335)	−0.93 (0.175)	1.95 (0.974)	1.88 (0.97)
ARX-J	Mlog$_2$	−5.28 (<0.001)	1.81 (0.965)	0.6 (0.726)	−0.28 (0.388)	0.95 (0.828)	−0.68 (0.248)	−1.13 (0.128)	−1.02 (0.153)	−1.83 (0.033)	1.37 (0.915)	1.45 (0.927)
ARX-J	N-PIT	−3 (0.001)	2.09 (0.982)	0.28 (0.611)	−0.71 (0.239)	1.08 (0.861)	−1.35 (0.089)	−0.98 (0.164)	−0.73 (0.233)	−0.84 (0.201)	3.47 (>0.999)	1.72 (0.958)
ARX-J	T-PIT	−4.51 (<0.001)	0.06 (0.523)	−1.35 (0.088)	−1.67 (0.047)	−0.87 (0.192)	−2.43 (0.008)	−1.88 (0.03)	−2.59 (0.005)	−2.59 (0.005)	−0.5 (0.308)	−0.79 (0.215)
ARX-J-CJ	Orig.	−1.95 (0.026)	3.89 (>0.999)	3.82 (>0.999)	1.81 (0.965)	3.16 (>0.999)	2.15 (0.984)	1.01 (0.843)	3.46 (>0.999)	3.79 (>0.999)	3.39 (>0.999)	3.85 (>0.999)
ARX-J-CJ	$3\sigma_1$	−3.97 (<0.001)	1.14 (0.873)	−1.28 (0.1)	−1.88 (0.03)	−0.94 (0.175)	−2.41 (0.008)	−1.71 (0.044)	−2.11 (0.018)	−2.18 (0.015)	0.05 (0.521)	0.02 (0.508)
ARX-J-CJ	$3\sigma_2$	−5.31 (<0.001)	0.16 (0.563)	−2.94 (0.002)	−1.42 (0.078)	−0.85 (0.197)	−2.02 (0.022)	−2.16 (0.016)	−2.61 (0.005)	−3.13 (<0.001)	−0.21 (0.416)	−0.26 (0.399)
ARX-J-CJ	Logistic$_1$	−1.56 (0.059)	3.45 (>0.999)	1.51 (0.934)	4.45 (>0.999)	3.6 (>0.999)	1.64 (0.95)	−0.14 (0.444)	1.05 (0.853)	0.74 (0.77)	3.31 (>0.999)	2.25 (0.988)
ARX-J-CJ	Logistic$_2$	−3.08 (0.001)	2.89 (0.998)	0.35 (0.637)	−0.8 (0.213)	2.49 (0.994)	−1.39 (0.082)	−1.01 (0.155)	−0.75 (0.226)	−0.91 (0.182)	1.73 (0.958)	1.21 (0.886)
ARX-J-CJ	Asinh$_1$	−1.98 (0.024)	3.89 (>0.999)	1.86 (0.968)	1.99 (0.977)	4.52 (>0.999)	3.08 (0.999)	−0.18 (0.427)	1.6 (0.945)	1.02 (0.847)	4.07 (>0.999)	2.78 (0.997)
ARX-J-CJ	Asinh$_2$	−1.09 (0.138)	2.45 (0.993)	2.12 (0.983)	1.04 (0.852)	1.88 (0.97)	1.07 (0.858)	2.09 (0.982)	1.39 (0.918)	1.36 (0.913)	2.07 (0.981)	2.29 (0.989)
ARX-J-CJ	Mlog$_1$	−4.26 (<0.001)	2.23 (0.987)	0.86 (0.804)	−0.13 (0.449)	1.45 (0.926)	−0.56 (0.287)	−0.92 (0.178)	−0.61 (0.269)	−1.04 (0.148)	1.87 (0.969)	1.77 (0.961)
ARX-J-CJ	Mlog$_2$	−4.72 (<0.001)	2.28 (0.989)	1.13 (0.87)	0.01 (0.504)	1.44 (0.925)	−0.25 (0.4)	−0.9 (0.185)	−0.1 (0.461)	−0.74 (0.229)	1.81 (0.965)	1.87 (0.97)
ARX-J-CJ	N-PIT	−2.93 (0.002)	2.21 (0.987)	0.36 (0.64)	−0.62 (0.267)	1.23 (0.89)	−1.23 (0.11)	−0.93 (0.175)	−0.62 (0.267)	−0.75 (0.228)	3.62 (>0.999)	1.8 (0.964)
ARX-J-CJ	T-PIT	−4.59 (<0.001)	−0.09 (0.465)	−1.5 (0.067)	−1.8 (0.036)	−1.02 (0.154)	−2.61 (0.004)	−1.93 (0.027)	−2.76 (0.003)	−2.74 (0.003)	−0.74 (0.231)	−1.08 (0.14)

ARX-J (columns)

		Orig.	$3\sigma_1$	$3\sigma_2$	Logistic$_1$	Logistic$_2$	Asinh$_1$	Asinh$_2$	Mlog$_1$	Mlog$_2$	N-PIT	T-PIT
ARX-J	$3\sigma_1$	−3.53 (<0.001)										
ARX-J	$3\sigma_2$	−4.93 (<0.001)	−0.37 (0.357)									
ARX-J	Logistic$_1$	−0.96 (0.17)	3.46 (>0.999)	2.45 (0.993)								
ARX-J	Logistic$_2$	−2.56 (0.005)	2.55 (0.995)	1.69 (0.954)	−3.09 (<0.001)							
ARX-J	Asinh$_1$	−1.3 (0.096)	3.79 (>0.999)	2.93 (0.998)	−0.31 (0.377)	3.77 (>0.999)						
ARX-J	Asinh$_2$	−0.31 (0.378)	2.43 (0.992)	3.1 (>0.999)	0.53 (0.702)	1.65 (0.95)	0.68 (0.752)					
ARX-J	Mlog$_1$	−3.24 (<0.001)	2.2 (0.986)	2.72 (0.997)	−1.22 (0.112)	0.84 (0.8)	−1.75 (0.04)	−1.51 (0.066)				
ARX-J	Mlog$_2$	−4.21 (<0.001)	1.71 (0.956)	2.56 (0.995)	−1.27 (0.102)	0.36 (0.642)	−1.68 (0.047)	−1.79 (0.037)	−0.89 (0.186)			
ARX-J	N-PIT	−2.36 (0.009)	2 (0.977)	1.58 (0.943)	−2.41 (0.008)	0.27 (0.606)	−2.94 (0.002)	−1.48 (0.069)	−0.57 (0.285)	−0.16 (0.437)		
ARX-J	T-PIT	−3.84 (<0.001)	−0.11 (0.455)	0.18 (0.573)	−2.68 (0.004)	−1.45 (0.074)	−3.28 (<0.001)	−2.47 (0.007)	−2.42 (0.008)	−1.87 (0.031)	−2.51 (0.006)	
ARX-J-CJ	Orig.	0.79 (0.786)	3.78 (>0.999)	5.26 (>0.999)	1.1 (0.865)	2.8 (0.997)	1.5 (0.934)	0.44 (0.669)	3.67 (>0.999)	4.74 (>0.999)	2.58 (0.995)	4.11 (>0.999)
ARX-J-CJ	$3\sigma_1$	−3.39 (<0.001)	1.78 (0.963)	0.69 (0.753)	−3.23 (<0.001)	−2.08 (0.019)	−3.54 (<0.001)	−2.32 (0.01)	−1.94 (0.026)	−1.46 (0.072)	−1.68 (0.047)	0.39 (0.652)
ARX-J-CJ	$3\sigma_2$	−4.75 (<0.001)	−0.04 (0.485)	1.48 (0.931)	−2.34 (0.01)	−1.47 (0.071)	−2.81 (0.003)	−2.92 (0.002)	−2.52 (0.006)	−2.3 (0.011)	−1.4 (0.081)	0.08 (0.53)
ARX-J-CJ	Logistic$_1$	−0.98 (0.163)	3.41 (>0.999)	2.43 (0.992)	−0.42 (0.338)	3.06 (0.999)	0.24 (0.594)	−0.55 (0.291)	1.18 (0.882)	1.24 (0.892)	2.34 (0.99)	2.63 (0.996)
ARX-J-CJ	Logistic$_2$	−2.44 (0.007)	2.89 (0.998)	1.88 (0.97)	−2.85 (0.002)	1.59 (0.944)	−3.42 (<0.001)	−1.56 (0.06)	−0.6 (0.274)	−0.16 (0.436)	0.02 (0.507)	1.63 (0.949)
ARX-J-CJ	Asinh$_1$	−1.25 (0.106)	3.86 (>0.999)	2.99 (0.999)	−0.18 (0.429)	3.89 (>0.999)	0.51 (0.694)	−0.64 (0.26)	1.88 (0.97)	1.75 (0.96)	3.05 (0.999)	3.32 (>0.999)
ARX-J-CJ	Asinh$_2$	−0.36 (0.361)	2.4 (0.992)	3.06 (0.999)	0.5 (0.693)	1.62 (0.947)	0.65 (0.742)	−0.37 (0.357)	1.47 (0.93)	1.75 (0.96)	1.45 (0.927)	2.44 (0.993)
ARX-J-CJ	Mlog$_1$	−3.22 (<0.001)	2.12 (0.983)	2.57 (0.995)	−1.3 (0.096)	0.76 (0.776)	−1.93 (0.027)	−1.51 (0.065)	−0.39 (0.347)	0.62 (0.734)	0.49 (0.688)	2.29 (0.989)
ARX-J-CJ	Mlog$_2$	−3.62 (<0.001)	2.18 (0.986)	3.04 (0.999)	−1.01 (0.156)	0.84 (0.799)	−1.31 (0.095)	−1.52 (0.064)	0.3 (0.619)	2.31 (0.99)	0.58 (0.72)	2.33 (0.99)
ARX-J-CJ	N-PIT	−2.29 (0.011)	2.12 (0.983)	1.64 (0.949)	−2.38 (0.009)	0.4 (0.657)	−2.84 (0.002)	−1.43 (0.076)	−0.46 (0.323)	−0.07 (0.471)	0.57 (0.715)	2.56 (0.995)
ARX-J-CJ	T-PIT	−3.94 (<0.001)	−0.27 (0.395)	0.05 (0.521)	−2.82 (0.002)	−1.61 (0.054)	−3.46 (<0.001)	−2.53 (0.006)	−2.61 (0.005)	−2.03 (0.021)	−2.78 (0.003)	−0.68 (0.247)

–ARX-J-CJ– (columns)

		Orig.	$3\sigma_1$	$3\sigma_2$	Logistic$_1$	Logistic$_2$	Asinh$_1$	Asinh$_2$	Mlog$_1$	Mlog$_2$	N-PIT
ARX-J-CJ	$3\sigma_1$	−3.64 (<0.001)									
ARX-J-CJ	$3\sigma_2$	−5.1 (<0.001)	−0.37 (0.356)								
ARX-J-CJ	Logistic$_1$	−1.13 (0.129)	3.18 (>0.999)	2.31 (0.99)							
ARX-J-CJ	Logistic$_2$	−2.68 (0.004)	2.45 (0.993)	1.67 (0.953)	−2.8 (0.003)						
ARX-J-CJ	Asinh$_1$	−1.45 (0.074)	3.6 (>0.999)	2.87 (0.998)	−0.1 (0.46)	3.55 (>0.999)					
ARX-J-CJ	Asinh$_2$	−0.48 (0.315)	2.29 (0.989)	2.88 (0.998)	0.52 (0.7)	1.53 (0.937)	0.61 (0.731)				
ARX-J-CJ	Mlog$_1$	−3.64 (<0.001)	1.86 (0.969)	2.36 (0.991)	−1.27 (0.102)	0.52 (0.697)	−2.05 (0.02)	−1.48 (0.069)			
ARX-J-CJ	Mlog$_2$	−4.13 (<0.001)	1.94 (0.974)	2.82 (0.998)	−0.98 (0.164)	0.63 (0.735)	−1.39 (0.082)	−1.49 (0.068)	0.47 (0.68)		
ARX-J-CJ	N-PIT	−2.51 (0.006)	1.8 (0.964)	1.46 (0.928)	−2.31 (0.011)	0.12 (0.546)	−2.97 (0.001)	−1.41 (0.08)	−0.38 (0.352)	−0.49 (0.313)	
ARX-J-CJ	T-PIT	−4.21 (<0.001)	−0.55 (0.292)	−0.22 (0.414)	−2.78 (0.003)	−1.8 (0.036)	−3.51 (<0.001)	−2.5 (0.006)	−2.46 (0.007)	−2.49 (0.006)	−2.82 (0.002)

Multivariate DM test statistic using MAE criterion for forecast horizon 5. P-values in parentheses. A *p*-value lower than 0.10 indicates that the forecasts of the model of the row are better than those of the model of the column at the 10% significance level. A heat map is used to indicate lower (green) and greater (red) p-values.

Table A.6
DM using MAE for H6.

ARX rows and columns:

		Orig.	$3\sigma_1$	$3\sigma_2$	Logistic$_1$	Logistic$_2$	Asinh$_1$	Asinh$_2$	Mlog$_1$	Mlog$_2$	N-PIT	T-PIT
ARX	$3\sigma_1$	−4.38 (<0.001)										
ARX	$3\sigma_2$	−5.03 (<0.001)	1.32 (0.906)									
ARX	Logistic$_1$	−2.28 (0.011)	2 (0.977)	0.71 (0.76)								
ARX	Logistic$_2$	−3.74 (<0.001)	1.5 (0.934)	−0.2 (0.422)	−1.77 (0.039)							
ARX	Asinh$_1$	−2.76 (0.003)	2.57 (0.995)	1.15 (0.876)	0.44 (0.67)	2.81 (0.998)						
ARX	Asinh$_2$	−1.71 (0.043)	1.82 (0.965)	1.65 (0.95)	0.65 (0.742)	1.36 (0.912)	0.62 (0.731)					
ARX	Mlog$_1$	−4.58 (<0.001)	2.29 (0.989)	1.46 (0.928)	0.08 (0.532)	1.68 (0.954)	−0.26 (0.398)	−0.84 (0.2)				
ARX	Mlog$_2$	−5.4 (<0.001)	2.28 (0.989)	1.83 (0.966)	0.19 (0.574)	1.62 (0.948)	−0.01 (0.495)	−0.81 (0.208)	0.59 (0.723)			
ARX	N-PIT	−3.66 (<0.001)	0.89 (0.814)	−0.23 (0.408)	−1.44 (0.075)	−0.1 (0.458)	−2.33 (0.01)	−1.35 (0.088)	−1.63 (0.052)	−1.51 (0.066)		
ARX	T-PIT	−4.32 (<0.001)	0.58 (0.718)	−0.42 (0.337)	−1.04 (0.149)	−0.21 (0.416)	−1.63 (0.052)	−1.59 (0.056)	−1.77 (0.038)	−1.78 (0.037)	−0.26 (0.399)	
ARX-J	Orig.	−3.04 (0.001)	3.52 (>0.999)	3.76 (>0.999)	1.55 (0.939)	2.85 (0.998)	1.78 (0.963)	0.75 (0.775)	2.92 (0.998)	3.3 (>0.999)	2.81 (0.998)	3.32 (>0.999)
ARX-J	$3\sigma_1$	−4.49 (<0.001)	−1.03 (0.151)	−1.63 (0.051)	−2.32 (0.01)	−1.94 (0.026)	−2.89 (0.002)	−1.97 (0.024)	−2.55 (0.005)	−2.53 (0.006)	−1.3 (0.097)	−0.92 (0.177)
ARX-J	$3\sigma_2$	−6.34 (<0.001)	−1.35 (0.088)	−4.76 (<0.001)	−1.96 (0.025)	−1.89 (0.03)	−2.68 (0.004)	−2.82 (0.002)	−3.48 (<0.001)	−3.98 (<0.001)	−1.57 (0.058)	−1.62 (0.053)
ARX-J	Logistic$_1$	−1.73 (0.042)	2.91 (0.998)	1.41 (0.92)	3.77 (>0.999)	3.03 (0.999)	1.16 (0.877)	−0.23 (0.408)	0.78 (0.781)	0.57 (0.714)	2.44 (0.993)	1.75 (0.96)
ARX-J	Logistic$_2$	−3.55 (<0.001)	1.54 (0.939)	−0.01 (0.497)	−1.46 (0.072)	0.53 (0.701)	−2.2 (0.014)	−1.26 (0.104)	−1.35 (0.089)	−1.35 (0.089)	0.31 (0.621)	0.35 (0.637)
ARX-J	Asinh$_1$	−2.39 (0.008)	3.04 (0.999)	1.57 (0.941)	1.21 (0.887)	3.5 (>0.999)	1.67 (0.953)	−0.38 (0.351)	0.93 (0.823)	0.56 (0.713)	2.81 (0.997)	2 (0.977)
ARX-J	Asinh$_2$	−1.33 (0.091)	2.17 (0.985)	2.08 (0.981)	0.91 (0.819)	1.69 (0.954)	0.91 (0.818)	1.57 (0.941)	1.2 (0.886)	1.2 (0.886)	1.66 (0.951)	1.93 (0.973)
ARX-J	Mlog$_1$	−4.86 (<0.001)	1.58 (0.943)	0.71 (0.762)	−0.34 (0.367)	0.95 (0.828)	−0.96 (0.169)	−1.1 (0.136)	−1.51 (0.066)	−1.63 (0.051)	0.93 (0.825)	1.05 (0.853)
ARX-J	Mlog$_2$	−6.18 (<0.001)	1.04 (0.851)	0.21 (0.582)	−0.6 (0.274)	0.35 (0.636)	−1.21 (0.114)	−1.42 (0.078)	−2.16 (0.015)	−2.99 (0.001)	0.38 (0.649)	0.55 (0.709)
ARX-J	N-PIT	−2.92 (0.002)	2.17 (0.985)	0.76 (0.777)	−0.23 (0.409)	1.47 (0.929)	−0.76 (0.223)	−0.83 (0.203)	−0.33 (0.37)	−0.43 (0.333)	2.97 (0.999)	1.65 (0.95)
ARX-J	T-PIT	−4.2 (<0.001)	0.61 (0.729)	−0.37 (0.355)	−1.08 (0.14)	−0.19 (0.423)	−1.7 (0.044)	−1.53 (0.063)	−1.74 (0.041)	−1.73 (0.042)	−0.21 (0.418)	0.07 (0.527)
ARX-J-CJ	Orig.	−2.49 (0.006)	3.85 (>0.999)	4.15 (>0.999)	1.76 (0.961)	3.18 (>0.999)	2.07 (0.981)	0.91 (0.818)	3.44 (>0.999)	3.91 (>0.999)	3.11 (>0.999)	3.64 (>0.999)
ARX-J-CJ	$3\sigma_1$	−4.33 (<0.001)	−0.28 (0.391)	−1.3 (0.098)	−2.07 (0.019)	−1.49 (0.068)	−2.61 (0.005)	−1.84 (0.033)	−2.28 (0.011)	−2.27 (0.012)	−0.97 (0.166)	−0.65 (0.259)
ARX-J-CJ	$3\sigma_2$	−6.16 (<0.001)	−1.06 (0.146)	−4.25 (<0.001)	−1.83 (0.034)	−1.66 (0.048)	−2.53 (0.006)	−2.63 (0.004)	−3.28 (<0.001)	−3.75 (<0.001)	−1.36 (0.087)	−1.37 (0.086)
ARX-J-CJ	Logistic$_1$	−1.75 (0.04)	2.87 (0.998)	1.39 (0.917)	3.58 (>0.999)	2.99 (0.999)	1.11 (0.867)	−0.25 (0.403)	0.75 (0.773)	0.54 (0.706)	2.38 (0.991)	1.72 (0.957)
ARX-J-CJ	Logistic$_2$	−3.57 (<0.001)	1.48 (0.93)	−0.04 (0.483)	−1.53 (0.063)	0.43 (0.665)	−2.29 (0.011)	−1.27 (0.102)	−1.39 (0.083)	−1.38 (0.084)	0.27 (0.605)	0.32 (0.625)
ARX-J-CJ	Asinh$_1$	−2.29 (0.011)	3.11 (>0.999)	1.66 (0.952)	1.41 (0.921)	3.63 (>0.999)	2.02 (0.978)	−0.33 (0.372)	1.09 (0.862)	0.7 (0.757)	2.89 (0.998)	2.09 (0.982)
ARX-J-CJ	Asinh$_2$	−1.3 (0.097)	2.16 (0.984)	2.06 (0.98)	0.92 (0.82)	1.68 (0.954)	0.92 (0.82)	1.47 (0.929)	1.21 (0.886)	1.2 (0.886)	1.66 (0.951)	1.92 (0.973)
ARX-J-CJ	Mlog$_1$	−4.66 (<0.001)	1.7 (0.956)	0.86 (0.804)	−0.22 (0.411)	1.12 (0.868)	−0.77 (0.221)	−1.01 (0.157)	−1.06 (0.145)	−1.23 (0.109)	1.08 (0.859)	1.17 (0.879)
ARX-J-CJ	Mlog$_2$	−5.58 (<0.001)	1.44 (0.925)	0.7 (0.759)	−0.32 (0.374)	0.79 (0.786)	−0.8 (0.211)	−1.17 (0.121)	−1.28 (0.101)	−1.88 (0.03)	0.78 (0.783)	0.95 (0.829)
ARX-J-CJ	N-PIT	−2.82 (0.002)	2.3 (0.989)	0.86 (0.805)	−0.08 (0.467)	1.65 (0.95)	−0.57 (0.284)	−0.77 (0.221)	−0.19 (0.426)	−0.31 (0.378)	3.23 (>0.999)	1.77 (0.962)
ARX-J-CJ	T-PIT	−4.4 (<0.001)	0.26 (0.603)	−0.7 (0.243)	−1.35 (0.089)	−0.52 (0.3)	−2.06 (0.019)	−1.67 (0.047)	−2.12 (0.017)	−2.06 (0.02)	−0.79 (0.215)	−0.67 (0.253)

ARX-J columns:

		Orig.	$3\sigma_1$	$3\sigma_2$	Logistic$_1$	Logistic$_2$	Asinh$_1$	Asinh$_2$	Mlog$_1$	Mlog$_2$	N-PIT	T-PIT
ARX-J	$3\sigma_1$	−3.78 (<0.001)										
ARX-J	$3\sigma_2$	−5.73 (<0.001)	−0.97 (0.166)									
ARX-J	Logistic$_1$	−0.98 (0.163)	3.34 (>0.999)	2.64 (0.996)								
ARX-J	Logistic$_2$	−2.73 (0.003)	2.4 (0.992)	2.08 (0.981)	−3.01 (0.001)							
ARX-J	Asinh$_1$	−1.44 (0.076)	3.54 (>0.999)	3.12 (>0.999)	−0.46 (0.324)	3.47 (>0.999)						
ARX-J	Asinh$_2$	−0.37 (0.356)	2.36 (0.991)	3.34 (>0.999)	0.49 (0.687)	1.61 (0.946)	0.68 (0.752)					
ARX-J	Mlog$_1$	−3.54 (<0.001)	2 (0.977)	2.99 (0.999)	−1.21 (0.113)	0.76 (0.776)	−1.68 (0.046)	−1.48 (0.069)				
ARX-J	Mlog$_2$	−4.84 (<0.001)	1.43 (0.923)	2.74 (0.997)	−1.35 (0.088)	0.18 (0.572)	−1.76 (0.039)	−1.84 (0.033)	−1.29 (0.098)			
ARX-J	N-PIT	−2.08 (0.019)	2.75 (0.997)	2.5 (0.994)	−1.56 (0.059)	1.28 (0.899)	−1.68 (0.047)	−1.14 (0.126)	0.28 (0.611)	0.66 (0.744)		
ARX-J	T-PIT	−3.35 (<0.001)	1.03 (0.848)	1.69 (0.955)	−1.88 (0.03)	−0.36 (0.359)	−2.2 (0.014)	−1.89 (0.029)	−1.14 (0.128)	−0.56 (0.288)	−1.94 (0.026)	
ARX-J-CJ	Orig.	0.99 (0.84)	4.12 (>0.999)	6.13 (>0.999)	1.17 (0.88)	3.05 (0.999)	1.71 (0.956)	0.53 (0.704)	4.16 (>0.999)	5.61 (>0.999)	2.36 (0.991)	3.68 (>0.999)
ARX-J-CJ	$3\sigma_1$	−3.59 (<0.001)	1.86 (0.968)	1.31 (0.904)	−3.11 (<0.001)	−1.91 (0.028)	−3.27 (<0.001)	−2.23 (0.013)	−1.72 (0.042)	−1.16 (0.123)	−2.42 (0.008)	−0.73 (0.232)
ARX-J-CJ	$3\sigma_2$	−5.52 (<0.001)	−0.64 (0.262)	1.67 (0.952)	−2.52 (0.006)	−1.85 (0.032)	−2.98 (0.001)	−3.14 (<0.001)	−2.76 (0.003)	−2.45 (0.007)	−2.32 (0.01)	−1.44 (0.075)
ARX-J-CJ	Logistic$_1$	−1 (0.158)	3.3 (>0.999)	2.62 (0.996)	−0.33 (0.372)	2.98 (0.999)	0.4 (0.656)	−0.5 (0.307)	1.19 (0.882)	1.33 (0.908)	1.5 (0.933)	1.84 (0.967)
ARX-J-CJ	Logistic$_2$	−2.75 (0.003)	2.3 (0.989)	2.03 (0.979)	−3.11 (<0.001)	−0.24 (0.405)	−3.59 (<0.001)	−1.62 (0.053)	−0.8 (0.212)	−0.21 (0.415)	−1.33 (0.092)	0.33 (0.628)
ARX-J-CJ	Asinh$_1$	−1.34 (0.09)	3.63 (>0.999)	3.2 (>0.999)	−0.24 (0.405)	3.62 (>0.999)	0.83 (0.797)	−0.62 (0.267)	1.88 (0.97)	1.89 (0.971)	1.87 (0.969)	2.29 (0.989)
ARX-J-CJ	Asinh$_2$	−0.35 (0.363)	2.35 (0.991)	3.31 (>0.999)	0.5 (0.69)	1.61 (0.946)	0.69 (0.754)	0.11 (0.545)	1.48 (0.931)	1.84 (0.967)	1.15 (0.875)	1.89 (0.97)
ARX-J-CJ	Mlog$_1$	−3.31 (<0.001)	2.12 (0.983)	3.04 (0.999)	−1.1 (0.135)	0.93 (0.824)	−1.53 (0.064)	−1.39 (0.083)	0.83 (0.796)	1.55 (0.939)	−0.1 (0.458)	1.26 (0.897)
ARX-J-CJ	Mlog$_2$	−4.16 (<0.001)	1.84 (0.967)	3.14 (0.999)	−1.09 (0.137)	0.63 (0.734)	−1.38 (0.084)	−1.57 (0.058)	−0.09 (0.466)	2.27 (0.988)	−0.26 (0.397)	0.99 (0.84)
ARX-J-CJ	N-PIT	−1.98 (0.024)	2.89 (0.998)	2.55 (0.995)	−1.45 (0.073)	1.46 (0.928)	−1.48 (0.069)	−1.08 (0.14)	0.42 (0.661)	0.76 (0.775)	0.76 (0.778)	2.06 (0.981)
ARX-J-CJ	T-PIT	−3.58 (<0.001)	0.68 (0.752)	1.37 (0.915)	−2.15 (0.016)	−0.69 (0.245)	−2.55 (0.005)	−2.04 (0.021)	−1.52 (0.065)	−0.9 (0.185)	−2.53 (0.006)	−1.67 (0.047)

ARX-J-CJ columns:

		Orig.	$3\sigma_1$	$3\sigma_2$	Logistic$_1$	Logistic$_2$	Asinh$_1$	Asinh$_2$	Mlog$_1$	Mlog$_2$	N-PIT
ARX-J-CJ	$3\sigma_1$	−3.93 (<0.001)									
ARX-J-CJ	$3\sigma_2$	−5.95 (<0.001)	−0.99 (0.161)								
ARX-J-CJ	Logistic$_1$	−1.2 (0.116)	3.07 (0.999)	2.5 (0.994)							
ARX-J-CJ	Logistic$_2$	−3.08 (0.001)	1.84 (0.967)	1.8 (0.964)	−3.08 (0.001)						
ARX-J-CJ	Asinh$_1$	−1.61 (0.054)	3.36 (>0.999)	3.06 (0.999)	−0.18 (0.428)	3.75 (>0.999)					
ARX-J-CJ	Asinh$_2$	−0.51 (0.304)	2.22 (0.987)	3.11 (>0.999)	0.51 (0.696)	1.62 (0.947)	0.63 (0.736)				
ARX-J-CJ	Mlog$_1$	−3.9 (<0.001)	1.85 (0.968)	2.82 (0.998)	−1.08 (0.14)	0.98 (0.835)	−1.72 (0.043)	−1.39 (0.083)			
ARX-J-CJ	Mlog$_2$	−4.91 (<0.001)	1.57 (0.942)	2.89 (0.998)	−1.07 (0.142)	0.66 (0.745)	−1.52 (0.064)	−1.57 (0.058)	−0.5 (0.307)		
ARX-J-CJ	N-PIT	−2.26 (0.012)	2.56 (0.995)	2.38 (0.991)	−1.39 (0.082)	1.52 (0.935)	−1.69 (0.045)	−1.09 (0.139)	0.24 (0.596)	0.37 (0.646)	
ARX-J-CJ	T-PIT	−3.93 (<0.001)	0.38 (0.647)	1.11 (0.867)	−2.11 (0.017)	−0.66 (0.255)	−2.65 (0.004)	−2.03 (0.021)	−1.63 (0.052)	−1.33 (0.092)	−2.63 (0.004)

Multivariate DM test statistic using MAE criterion for forecast horizon 6. P-values in parentheses. A p-value lower than 0.10 indicates that the forecasts of the model of the row are better than those of the model of the column at the 10% significance level. A heat map is used to indicate lower (green) and greater (red) p-values.

Table A.7
DM using MAE for H7.

–ARX– (columns)

		Orig.	$3\sigma_1$	$3\sigma_2$	Logistic$_1$	Logistic$_2$	Asinh$_1$	Asinh$_2$	Mlog$_1$	Mlog$_2$	N-PIT	T-PIT
ARX	$3\sigma_1$	−4.3 (<0.001)										
ARX	$3\sigma_2$	−4.93 (<0.001)	1.11 (0.867)									
ARX	Logistic$_1$	−2.09 (0.018)	2.34 (0.99)	1.06 (0.855)								
ARX	Logistic$_2$	−3.56 (<0.001)	1.74 (0.959)	0.13 (0.553)	−2.14 (0.016)							
ARX	Asinh$_1$	−2.58 (0.005)	2.76 (0.997)	1.38 (0.917)	0.09 (0.534)	2.95 (0.998)						
ARX	Asinh$_2$	−1.72 (0.043)	1.9 (0.972)	1.83 (0.966)	0.55 (0.707)	1.35 (0.912)	0.59 (0.722)					
ARX	Mlog$_1$	−4.14 (<0.001)	2.74 (0.997)	2 (0.977)	0.01 (0.504)	1.96 (0.975)	−0.05 (0.478)	−0.74 (0.229)				
ARX	Mlog$_2$	−4.92 (<0.001)	2.62 (0.996)	2.35 (0.991)	0.09 (0.535)	1.77 (0.961)	0.08 (0.531)	−0.74 (0.229)	0.39 (0.652)			
ARX	N-PIT	−3.56 (<0.001)	1.11 (0.867)	0.03 (0.512)	−1.77 (0.038)	−0.13 (0.448)	−2.38 (0.009)	−1.33 (0.091)	−1.88 (0.03)	−1.66 (0.048)		
ARX	T-PIT	−4.11 (<0.001)	0.98 (0.837)	0.09 (0.537)	−1.09 (0.138)	−0.03 (0.487)	−1.43 (0.076)	−1.48 (0.069)	−1.75 (0.04)	−1.73 (0.042)	0.09 (0.537)	
ARX-J	Orig.	−2.83 (0.002)	3.45 (>0.999)	3.75 (>0.999)	1.4 (0.919)	2.73 (0.997)	1.71 (0.956)	0.79 (0.785)	2.67 (0.996)	3.07 (0.999)	2.75 (0.997)	3.14 (>0.999)
ARX-J	$3\sigma_1$	−4.24 (<0.001)	−0.35 (0.362)	−1.14 (0.128)	−2.46 (0.007)	−1.77 (0.038)	−2.85 (0.002)	−1.94 (0.026)	−2.71 (0.003)	−2.6 (0.005)	−1.22 (0.112)	−1.06 (0.145)
ARX-J	$3\sigma_2$	−5.76 (<0.001)	−1.02 (0.155)	−3.52 (<0.001)	−2.04 (0.02)	−1.73 (0.042)	−2.56 (0.005)	−2.72 (0.003)	−3.45 (<0.001)	−3.85 (<0.001)	−1.45 (0.073)	−1.69 (0.046)
ARX-J	Logistic$_1$	−1.58 (0.058)	3.07 (0.999)	1.67 (0.952)	3.5 (>0.999)	3.19 (>0.999)	1.46 (0.928)	−0.16 (0.438)	0.8 (0.788)	0.62 (0.731)	2.65 (0.996)	1.73 (0.958)
ARX-J	Logistic$_2$	−3.35 (<0.001)	1.68 (0.953)	0.29 (0.613)	−1.88 (0.03)	0.53 (0.7)	−2.37 (0.009)	−1.24 (0.107)	−1.57 (0.058)	−1.46 (0.072)	0.34 (0.632)	0.18 (0.571)
ARX-J	Asinh$_1$	−2.19 (0.014)	3.19 (>0.999)	1.77 (0.962)	0.98 (0.838)	3.61 (>0.999)	1.7 (0.956)	−0.35 (0.365)	0.74 (0.772)	0.48 (0.684)	2.86 (0.998)	1.81 (0.965)
ARX-J	Asinh$_2$	−1.44 (0.075)	2.19 (0.986)	2.18 (0.985)	0.74 (0.771)	1.62 (0.947)	0.81 (0.791)	1.11 (0.866)	1.01 (0.845)	1.04 (0.85)	1.57 (0.942)	1.74 (0.959)
ARX-J	Mlog$_1$	−4.19 (<0.001)	2.19 (0.986)	1.44 (0.925)	−0.28 (0.389)	1.45 (0.926)	−0.53 (0.298)	−0.91 (0.181)	−1.01 (0.156)	−1.06 (0.145)	1.38 (0.916)	1.23 (0.891)
ARX-J	Mlog$_2$	−5.33 (<0.001)	1.63 (0.949)	1.05 (0.853)	−0.54 (0.296)	0.78 (0.781)	−0.84 (0.202)	−1.22 (0.112)	−1.72 (0.043)	−2.3 (0.011)	0.78 (0.784)	0.76 (0.778)
ARX-J	N-PIT	−2.51 (0.006)	2.96 (0.998)	1.44 (0.925)	0.07 (0.53)	2.2 (0.986)	0.01 (0.506)	−0.59 (0.277)	0.05 (0.522)	−0.06 (0.476)	4.19 (>0.999)	1.97 (0.976)
ARX-J	T-PIT	−3.75 (<0.001)	1.39 (0.918)	0.47 (0.681)	−0.88 (0.19)	0.34 (0.632)	−1.17 (0.121)	−1.25 (0.105)	−1.33 (0.091)	−1.32 (0.093)	0.71 (0.761)	0.84 (0.799)
ARX-J-CJ	Orig.	−1.82 (0.035)	4.01 (>0.999)	4.45 (>0.999)	1.75 (0.96)	3.26 (>0.999)	2.18 (0.985)	1.12 (0.869)	3.5 (>0.999)	4.08 (>0.999)	3.24 (>0.999)	3.7 (>0.999)
ARX-J-CJ	$3\sigma_1$	−4.09 (<0.001)	0.39 (0.652)	−0.79 (0.215)	−2.22 (0.013)	−1.32 (0.094)	−2.56 (0.005)	−1.8 (0.036)	−2.43 (0.008)	−2.34 (0.01)	−0.86 (0.195)	−0.76 (0.224)
ARX-J-CJ	$3\sigma_2$	−5.57 (<0.001)	−0.69 (0.244)	−3.03 (0.001)	−1.9 (0.028)	−1.5 (0.067)	−2.4 (0.008)	−2.54 (0.006)	−3.24 (<0.001)	−3.62 (<0.001)	−1.23 (0.11)	−1.43 (0.077)
ARX-J-CJ	Logistic$_1$	−1.56 (0.059)	3.05 (0.999)	1.67 (0.952)	3.41 (>0.999)	3.19 (>0.999)	1.46 (0.928)	−0.15 (0.44)	0.8 (0.789)	0.62 (0.732)	2.61 (0.995)	1.71 (0.957)
ARX-J-CJ	Logistic$_2$	−3.32 (<0.001)	1.72 (0.957)	0.31 (0.624)	−1.86 (0.031)	0.62 (0.733)	−2.31 (0.01)	−1.22 (0.11)	−1.52 (0.065)	−1.41 (0.079)	0.38 (0.646)	0.2 (0.581)
ARX-J-CJ	Asinh$_1$	−2.05 (0.02)	3.35 (>0.999)	1.93 (0.973)	1.35 (0.911)	3.89 (>0.999)	2.39 (0.992)	−0.26 (0.399)	1.01 (0.843)	0.69 (0.755)	3.09 (0.999)	1.98 (0.976)
ARX-J-CJ	Asinh$_2$	−1.41 (0.08)	2.19 (0.986)	2.18 (0.985)	0.75 (0.775)	1.62 (0.948)	0.82 (0.795)	1.14 (0.872)	1.03 (0.848)	1.05 (0.853)	1.58 (0.943)	1.74 (0.959)
ARX-J-CJ	Mlog$_1$	−3.9 (<0.001)	2.45 (0.993)	1.69 (0.955)	−0.06 (0.475)	1.78 (0.962)	−0.17 (0.431)	−0.76 (0.222)	−0.24 (0.405)	−0.43 (0.333)	1.66 (0.952)	1.48 (0.93)
ARX-J-CJ	Mlog$_2$	−4.6 (<0.001)	2.19 (0.986)	1.67 (0.953)	−0.16 (0.438)	1.4 (0.919)	−0.29 (0.386)	−0.89 (0.186)	−0.55 (0.291)	−0.89 (0.186)	1.33 (0.908)	1.3 (0.903)
ARX-J-CJ	N-PIT	−2.36 (0.009)	3.19 (>0.999)	1.6 (0.945)	0.35 (0.637)	2.52 (0.994)	0.36 (0.641)	−0.5 (0.31)	0.29 (0.613)	0.13 (0.552)	4.56 (>0.999)	2.16 (0.985)
ARX-J-CJ	T-PIT	−3.94 (<0.001)	1.03 (0.848)	0.13 (0.552)	−1.15 (0.126)	0.01 (0.502)	−1.51 (0.065)	−1.4 (0.081)	−1.7 (0.044)	−1.65 (0.05)	0.17 (0.566)	0.09 (0.536)

ARX-J (columns)

		Orig.	$3\sigma_1$	$3\sigma_2$	Logistic$_1$	Logistic$_2$	Asinh$_1$	Asinh$_2$	Mlog$_1$	Mlog$_2$	N-PIT	T-PIT
ARX-J	$3\sigma_1$	−3.55 (<0.001)										
ARX-J	$3\sigma_2$	−5.15 (<0.001)	−0.92 (0.178)									
ARX-J	Logistic$_1$	−0.88 (0.189)	3.35 (>0.999)	2.65 (0.996)								
ARX-J	Logistic$_2$	−2.61 (0.005)	2.16 (0.985)	1.89 (0.97)	−3.3 (<0.001)							
ARX-J	Asinh$_1$	−1.35 (0.089)	3.51 (>0.999)	2.99 (0.999)	−0.71 (0.239)	3.76 (>0.999)						
ARX-J	Asinh$_2$	−0.52 (0.302)	2.27 (0.988)	3.18 (>0.999)	0.35 (0.635)	1.52 (0.936)	0.57 (0.715)					
ARX-J	Mlog$_1$	−3.06 (0.001)	2.42 (0.992)	3.2 (>0.999)	−1.12 (0.132)	1.25 (0.895)	−1.28 (0.1)	−1.21 (0.114)				
ARX-J	Mlog$_2$	−4.23 (<0.001)	1.83 (0.967)	3.1 (>0.999)	−1.25 (0.106)	0.6 (0.727)	−1.4 (0.081)	−1.55 (0.061)	−1.18 (0.118)			
ARX-J	N-PIT	−1.71 (0.044)	3.36 (>0.999)	2.81 (0.998)	−1.19 (0.116)	2.11 (0.982)	−0.93 (0.176)	−0.83 (0.204)	0.48 (0.684)	0.79 (0.786)		
ARX-J	T-PIT	−2.93 (0.002)	1.59 (0.944)	2.15 (0.984)	−1.62 (0.053)	0.17 (0.569)	−1.69 (0.046)	−1.54 (0.062)	−0.96 (0.17)	−0.43 (0.334)	−1.86 (0.031)	
ARX-J-CJ	Orig.	1.87 (0.969)	4.11 (>0.999)	5.85 (>0.999)	1.21 (0.887)	3.11 (>0.999)	1.79 (0.963)	0.86 (0.806)	3.93 (>0.999)	5.33 (>0.999)	2.16 (0.985)	3.47 (>0.999)
ARX-J-CJ	$3\sigma_1$	−3.38 (<0.001)	1.98 (0.976)	1.29 (0.901)	−3.13 (<0.001)	−1.69 (0.045)	−3.24 (<0.001)	−2.13 (0.017)	−2.13 (0.017)	−1.55 (0.061)	−3.02 (0.001)	−1.27 (0.102)
ARX-J-CJ	$3\sigma_2$	−4.91 (<0.001)	−0.56 (0.286)	1.63 (0.949)	−2.52 (0.006)	−1.66 (0.049)	−2.84 (0.002)	−2.98 (0.001)	−2.97 (0.002)	−2.79 (0.003)	−2.63 (0.004)	−1.89 (0.03)
ARX-J-CJ	Logistic$_1$	−0.87 (0.191)	3.33 (>0.999)	2.65 (0.996)	0.1 (0.541)	3.3 (>0.999)	0.72 (0.764)	−0.34 (0.366)	1.12 (0.869)	1.25 (0.894)	1.18 (0.881)	1.6 (0.945)
ARX-J-CJ	Logistic$_2$	−2.58 (0.005)	2.2 (0.986)	1.9 (0.972)	−3.3 (<0.001)	0.24 (0.594)	−3.7 (<0.001)	−1.51 (0.066)	−1.2 (0.115)	−0.56 (0.286)	−2.06 (0.02)	−0.14 (0.443)
ARX-J-CJ	Asinh$_1$	−1.21 (0.114)	3.7 (>0.999)	3.13 (>0.999)	−0.34 (0.369)	4.07 (>0.999)	1.35 (0.911)	−0.47 (0.318)	1.57 (0.941)	1.61 (0.946)	1.29 (0.902)	1.87 (0.969)
ARX-J-CJ	Asinh$_2$	−0.49 (0.311)	2.27 (0.988)	3.16 (>0.999)	0.36 (0.64)	1.54 (0.938)	0.58 (0.72)	0.18 (0.571)	1.22 (0.889)	1.56 (0.941)	0.84 (0.799)	1.54 (0.938)
ARX-J-CJ	Mlog$_1$	−2.75 (0.003)	2.68 (0.996)	3.39 (>0.999)	−0.91 (0.181)	1.58 (0.944)	−0.95 (0.172)	−1.05 (0.146)	1.4 (0.92)	1.76 (0.961)	−0.16 (0.437)	1.23 (0.892)
ARX-J-CJ	Mlog$_2$	−3.41 (<0.001)	2.4 (0.992)	3.62 (>0.999)	−0.9 (0.184)	1.22 (0.888)	−0.89 (0.186)	−1.21 (0.114)	0.41 (0.659)	2.78 (0.997)	−0.27 (0.392)	1.01 (0.844)
ARX-J-CJ	N-PIT	−1.56 (0.06)	3.6 (>0.999)	2.93 (0.998)	−0.98 (0.164)	2.44 (0.993)	−0.59 (0.277)	−0.72 (0.234)	0.71 (0.761)	0.97 (0.835)	1.39 (0.918)	2.08 (0.981)
ARX-J-CJ	T-PIT	−3.15 (<0.001)	1.22 (0.889)	1.8 (0.964)	−1.89 (0.029)	−0.16 (0.436)	−2.02 (0.022)	−1.68 (0.046)	−1.33 (0.091)	−0.78 (0.219)	−2.44 (0.007)	−1.67 (0.047)

–ARX-J-CJ– (columns)

		Orig.	$3\sigma_1$	$3\sigma_2$	Logistic$_1$	Logistic$_2$	Asinh$_1$	Asinh$_2$	Mlog$_1$	Mlog$_2$	N-PIT
ARX-J-CJ	$3\sigma_1$	−3.94 (<0.001)									
ARX-J-CJ	$3\sigma_2$	−5.65 (<0.001)	−0.95 (0.171)								
ARX-J-CJ	Logistic$_1$	−1.2 (0.115)	3.12 (>0.999)	2.52 (0.994)							
ARX-J-CJ	Logistic$_2$	−3.08 (0.001)	1.75 (0.96)	1.68 (0.953)	−3.31 (<0.001)						
ARX-J-CJ	Asinh$_1$	−1.64 (0.051)	3.43 (>0.999)	2.98 (0.999)	−0.35 (0.362)	4.01 (>0.999)					
ARX-J-CJ	Asinh$_2$	−0.83 (0.202)	2.13 (0.983)	2.97 (0.999)	0.35 (0.638)	1.52 (0.935)	0.49 (0.687)				
ARX-J-CJ	Mlog$_1$	−3.56 (<0.001)	2.39 (0.992)	3.16 (>0.999)	−0.92 (0.18)	1.53 (0.937)	−1.24 (0.107)	−1.07 (0.143)			
ARX-J-CJ	Mlog$_2$	−4.46 (<0.001)	2.13 (0.983)	3.35 (>0.999)	−0.9 (0.184)	1.17 (0.879)	−1.12 (0.132)	−1.22 (0.111)	−0.41 (0.342)		
ARX-J-CJ	N-PIT	−2 (0.023)	3.27 (>0.999)	2.76 (0.997)	−0.97 (0.166)	2.39 (0.992)	−0.97 (0.167)	−0.74 (0.231)	0.4 (0.655)	0.47 (0.682)	
ARX-J-CJ	T-PIT	−3.71 (<0.001)	0.89 (0.814)	1.53 (0.937)	−1.87 (0.031)	−0.19 (0.424)	−2.21 (0.014)	−1.68 (0.046)	−1.6 (0.055)	−1.35 (0.088)	−2.65 (0.004)

Multivariate DM test statistic using MAE criterion for forecast horizon 7. P-values in parentheses. A p-value lower than 0.10 indicates that the forecasts of the model of the row are better than those of the model of the column at the 10% significance level. A heat map is used to indicate lower (green) and greater (red) p-values.

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