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**INTERNAL ORGANIZATION OF
FIRMS AND CARTEL FORMATION**

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Internal Organization of Firms and Cartel Formation

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Abstract

We study the endogenous formation of cartels in two contexts. First, we consider internal-external stability based models which, due to firms' free-riding incentives, lead to the formation of very small stable cartels (if any). Second, we introduce the dynamic aspect of coalition formation. That is, when considering a cartel we take into account also any other cartel that can be reached through a succession of moves. We apply notions such as the generalized stable sets and the absorbing sets solutions which predict that collusion of the whole industry can occur with some regularity. Then we apply the two approaches to a Cournot game, and study the influence that the internal organization of firms has on the size of the cartels that form by means of a comparison between a situation where ownership and management are not separated and one in which they are.

JEL Classification: L22, C70, C72.

Key words: Cartels, stability, absorbing sets solution, strategic delegation.

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1 Introduction

This work deals with the study of endogenous formation of cartels. D’Aspremont *et al.*’s [1983] concept of a stable cartel is the first contribution in this area: a stable cartel is one where for all firms in the cartel there is no incentive to defect from the cartel (internal stability), and for all firms outside the cartel there is no incentive to join the cartel (external stability). While the definition of stability due to d’Aspremont *et al.* [1983] excludes the possibility of coalitional deviations, *i.e.* only deviations by a single firm are considered, Thoron [1998] develops a model of coalition formation in oligopolies which incorporates the possibility of coalitional deviations. Applying the concepts of a strong Nash equilibrium and a coalition-proof Nash equilibrium to the coalition formation model, the concepts of a strong stable cartel and a coalition-proof stable cartel are defined. The formation of a cartel can be interpreted as a public good which induces positive externalities on firms outside the cartel. Then, due to the free-riding incentive that those positive externalities create, collusive agreements are highly unstable. Consequently, the stability concepts mentioned above predict stable cartels (if any) formed by a small number of firms.

In this work we study the formation of cartels in two different contexts. First, we analyze internal-external stability based models. It is found that, under very general assumptions,¹ strong stable cartels may not exist, while stable and coalition-proof stable cartels do exist, but they are formed by a small number of firms. Second, we introduce the dynamic aspect of coalition formation. In this part, we allow for coalitional deviations of firms, and assume that certain groups of firms are able to force a transition from a given cartel to a new one. We assume that firms show myopic behavior, *i.e.* they do not take into account the consequences of a move. In this context, a natural solution concept would be that of a strong stable cartel. However, since internal-external stability based models assume that cartels only form if there is some guarantee of long-term stability, due to the free-riding incentive there exist many situations for which no strong stable cartels can form. This leads us to analyze what may happen in situations where no inherently stable agreement exists. We study how firms in those cases may join or leave a cartel, whenever the new situation provides larger profits for the firms that induce the change. We describe how transitions between cartels may take place, and take up the question of how firms react if a firm or a group of firms deviate. In this way, we allow firms to move successively in discrete steps unless they converge upon cartels which are stable. This process of formation and destruction of cartels is modeled by means of an abstract game (von Neumann and Morgenstern [1944]). Then, we use solution concepts for abstract games such as the notions of a generalized stable set (van Deemen [1991]) and an elementary dynamic solution (Shenoy [1979]). In particular, we

¹Assumption **A1**: the profit of an independent firm increases as the number of firms in the cartel increases. Assumption **A2**: given any non-degenerated cartel, an independent firm receives a greater profit than a firm in the cartel. Assumption **A3**: a cartel formed by a single firm is equivalent to the inexistence of a cartel, and all firms receive the same profits in these two situations.

interpret an elementary dynamic solution (or absorbing set) as a set of cartels to which the transitions may become restricted in the long run. In this way, it is possible to predict which cartels might be observed with some regularity, and which cartels will not be observed.²

Our results show that a unique absorbing set exists, and provide a characterization of that set. Two different situations may arise. First, we find conditions where there is a unique cartel-size (number of firms in a cartel), say k^* , from which no profitable move exists. Moreover, we prove that, starting at any cartel, after a finite number of moves, firms will form a cartel of size k^* . And from then on, no cartel of different size can form. Since the unique cartel-size from which no profitable move exists is k^* , this is the unique size of a strongly stable cartel, the unique cartel-size in the unique elementary dynamic solution, and also the unique cartel-size in the unique generalized stable set. Second, we find conditions where there is a set of cartel-sizes, say \mathcal{A} , such that no profitable move from one cartel in \mathcal{A} to a cartel not in \mathcal{A} exists. Moreover, we prove that, starting at any cartel, after a finite number of moves, firms will form a cartel-size in \mathcal{A} . And from then on, no cartel outside \mathcal{A} can form. Since firms join and leave cartels endlessly, no strongly stable cartel exists. However, the absorbing sets solution selects a unique group of cartels to which the process of formation and destruction of cartels is restricted in the long run.

In the rest of the study we apply the two approaches of cartel formation mentioned before to a Cournot game. We further study the influence of the internal organization of firms on the size of the cartels that form in the context of an oligopolist industry. The literature that analyzes the formation of cartels in oligopoly markets usually treats firms as economic agents with the sole objective of profit maximization. However, for modern corporations, ownership and management are often separated. Fershtman and Judd [1987], Sklivas [1987] and Vickers [1985], examine the incentive contracts that owners of competing firms give their managers and how these incentive contracts affect the oligopoly outcome. These works show that profit-maximizing owners will turn their managers away from strict profit maximization for strategic reasons. We study the influence that the internal organization of firms has on the size of the cartels that form by means of a comparison between a situation where ownership and management are not separated and a situation in which they are separated. For the latter we assume, as in Fershtman and Judd [1987], that firms' managers will be given an incentive to maximize an objective function consisting of a linear combination of profits and sales revenue. Under quantity competition, this leads to lower prices and higher quantities than strict profit maximization (see, for example, Fershtman and Judd [1987]). Firms may form cartels in order to reduce market competition, which in turn leads to larger profits. We show that the free-riding incentives decrease and the incentives to form cartels increase

²Bloch [1996] proposes an alternative dynamic approach in which the formation of a cartel depends on the sequentiality of the coalition formation process. Espinosa and Inarra [2000] propose a model which allows the formation of multiple cartels, and use the notion of von Neumann and Morgenstern stable sets.

when firms' owners delegate production decisions to managers.³

We show that strategic delegation reinforces the incentives to form cartels in both the static and the dynamic contexts, that influence being stronger in the dynamic context. At this point we restrict the analysis to a linear Cournot model as the simplest way of illustrating our results. However, this analysis can be extended to symmetric oligopoly models with arbitrary cost and demand functions. In fact, we only require these assumptions on firms' profits: *i*) the profit of an independent firm increases as the cartel-size increases, and *ii*) given any non-degenerated cartel, an independent firm receives a greater profit than a firm in the cartel (a cartel formed by a single firm is equivalent to the inexistence of a cartel, and all firms receive the same profits in these two situations). Under strategic delegation, managers produce a larger quantity than a profit-maximizer firm, and firms' profits are hence lower than in the strict profit maximization case. Under Cournot competition the cartel and each of the independent firms produce the same quantity. Consequently, the loss of profits induced by strategic delegation is smaller for the firms in the cartel, since those firms share the losses while independent firms do not. As a result, the free-riding incentive becomes lower and the incentive to form cartels becomes higher when firms' owners delegate production decisions to managers. Moreover, the unique absorbing set under strategic delegation can be obtained by deleting a number of smallest cartels from the absorbing set obtained under strict profit maximization. Then, the set of cartels to which the moves will be restricted in the long run contains larger cartels under strategic delegation than under strict profit maximization.

The rest of the paper is organized as follows. Section 2 analyzes the formation of cartels, applying the notions of stability, strong stability and coalition-proof stability, and provides a characterization of stable and coalition-proof stable cartels. In Section 3 we introduce equilibrium concepts for abstract games, and prove existence and uniqueness of an absorbing set. Section 4 describes the general features of a Cournot oligopoly model with a homogeneous product when firms' owners delegate production decisions to managers, and shows how strategic delegation affects firms' decisions to form cartels. Finally, Section 5 contains some conclusions as well as pointers for further research.

2 Stable, strongly stable, coalition-proof stable cartels

Situations where all decisions are made simultaneously can be modeled as *non-cooperative games in strategic form*. A game in strategic form specifies the possible strategies of every player in the game and the outcomes associated with all possible strategy profiles that can be chosen by the players. Formally:

³González-Maestre and López-Cuñat [2001] study the interactions between mergers and strategic delegation for a linear Cournot oligopoly model, and apply the concept of subgame perfect equilibrium. Faulí-Oller and Motta [1996] analyze a model with three initial firms that can merge, where only the owner of one firm delegates production decisions to a manager.

A game in strategic form is a tuple $\Gamma = (N; (S_i)_{i \in N}; (\pi_i)_{i \in N})$, where $N = \{1, \dots, n\}$ denotes the player set, S_i denotes the strategy space of player i , and π_i denotes the payoff function of player i , which assigns to every $s \in S = \prod_{j \in N} S_j$ a payoff $\pi_i(s) \in \mathbb{R}$.

According to this definition, every player $i \in N$ chooses a strategy $s_i \in S_i$, and all players choose these strategies simultaneously and independently. Every player then receives a payoff that in general will depend not only on his own choice but also on the choices of all other players. If strategy profile $s \in (s_j)_{j \in N}$ is chosen, then player i receives $\pi_i(s)$. The standard equilibrium concept for games in strategic form is the concept of *Nash equilibrium* (Nash [1950]), which we define in the following.

Consider a game in strategic form $\Gamma = (N; (S_i)_{i \in N}; (\pi_i)_{i \in N})$. Then the strategy profile $s^* \in S$ is a Nash equilibrium of Γ if for all $i \in N$ and all $s_i \in S_i$, $\pi_i(s^*) \geq \pi(s_i, s_{-i}^*)$ where $(s_i, s_{-i}^*) = (s_1^*, \dots, s_{i-1}^*, s_i, s_{i+1}^*, \dots, s_n^*)$.

According to this definition, a strategy profile is a Nash equilibrium if no player has an incentive to unilaterally deviate from it, *i.e.* no player can unilaterally improve his payoff. However, even if a Nash equilibrium is played, a coordinated deviation by some players might improve the payoff of all deviating players. The *strong Nash equilibrium* concept (Aumann [1959]) considers this possibility, and restricts the set of equilibria compared to the set of Nash equilibria.

Consider a game in strategic form $\Gamma = (N; (S_i)_{i \in N}; (\pi_i)_{i \in N})$. Let $S_T = \prod_{i \in T} S_i$ and let $s_{N \setminus T}$ be the vector of strategies of firms not in T . Then the strategy profile $s^* \in S$ is a strong Nash equilibrium of Γ if there is no coalition $T \subseteq N$ and strategy profile $s_T \in S_T$ such that $\pi_i(s_T, s_{N \setminus T}^*) \geq \pi_i(s^*)$ for all $i \in T$, with the inequality being strict for at least one $i \in T$.

This notion of a strong stable strategy profile is very restrictive, since existence is not ensured. Bernheim *et al.* [1987] propose the notion of *coalition-proof Nash equilibrium*, which also takes into account coalitional deviations but considers a deviation of a coalition only if it is not threatened by further deviations by subcoalitions. These further deviations are considered only if they are not threatened to even further deviation of subsubcoalitions, etcetera. The equilibrium is defined recursively.

Consider a game in strategic form $\Gamma = (N; (S_i)_{i \in N}; (\pi_i)_{i \in N})$. For every $T \subset N$ and $s_{N \setminus T}^* \in S_{N \setminus T}$, let $\Gamma(s_{N \setminus T}^*)$ be the game induced on the players of coalition T by the strategies $s_{N \setminus T}^*$. That is, $\Gamma(s_{N \setminus T}^*) = (T; (S_i)_{i \in T}; (\pi_i^*)_{i \in T})$, where $\pi_i^* : S_T \rightarrow \mathbb{R}$ is given by $\pi_i^*(s_T) = \pi_i(s_T, s_{N \setminus T}^*)$ for all $s_T \in S_T$ and all $i \in T$. Coalition-proof Nash equilibria are defined inductively. In a one-player game with player set $N = \{i\}$, $s_i^* \in S = S_i$ is a coalition-proof Nash equilibrium of $\Gamma = (\{i\}; S_i; \pi_i)$ if s_i^* maximizes π_i over S_i . Now let Γ be a game with $n > 1$ players. Assume that coalition-proof Nash equilibria have been defined for games with less than n players. Then a strategy profile $s^* \in S_N$ is called *self-enforcing* if for all $T \subset N$, s_T^* is a coalition-proof Nash equilibrium of $\Gamma(s_{N \setminus T}^*)$. Now, the strategy profile s^* is a coalition-proof Nash equilibrium of Γ if s^* is self-enforcing and there is no other self-enforcing strategy profile

$s \in S_N$ with $\pi_i(s) > \pi_i(s^*)$ for all $i \in N$.

This solution concept leads to a set of strategy profiles which are self-enforcing (*i.e.* not based on the players' commitment to cooperation), which includes the set of strong Nash equilibria.

2.1 Symmetric cartel games

We restrict our study to non-cooperative games in strategic form which have the following characteristics. The player set N is formed by n identical firms (*i.e.* they all have the same set of actions available). Only one coalition (cartel) K can be formed at a time. Each firm must then make a choice between entering the cartel (action C) or remaining independent (action I). Therefore, the strategy space of firm i takes a binary form: $S_i = \{C, I\}$. An oligopoly game between the cartel and the independent firms determines the payoffs. As firms are *ex ante* identical we assume that profits inside the cartel are shared equally.⁴ A game that takes this particular form will be called a *symmetric cartel game*.

For a symmetric cartel game, a pure strategy-profile is completely determined by the subset K of firms that choose action C (*i.e.* the firms that form the cartel). Then, we say that K is a stable cartel if the strategy-profile associated with K is a Nash-equilibrium, K is a strongly stable cartel if the strategy-profile associated with K is a strong Nash-equilibrium, and K is a coalition-proof stable cartel if the strategy-profile associated with K is a coalition-proof Nash-equilibrium.

Alternatively, stability, strong stability and coalition-proof stability of cartels can be formulated in terms of internal-external stability. Let $\pi^C(k)$ be the profit of a firm that belongs to a cartel of size k . And let $\pi^I(k)$ be the profit of an independent firm when a cartel of size k forms.⁵ Assume:

A1 $\pi^I(k) > \pi^I(k - 1)$ for all $k \geq 2$.

A2 $\pi^I(k) > \pi^C(k)$ for all $k \geq 2$.

A3 $\pi^C(1) = \pi^I(1) = \pi^I(0)$.

Assumption **A1** says that the profit of an independent firm increases as the cartel-size increases. Assumption **A2** says that, given any non-degenerated cartel, an independent firm receives a greater profit than a firm in the cartel. And Assumption **A3** says that a cartel of size $k = 1$ is equivalent to the inexistence of a cartel, and that all firms receive the same profits in these two situations.⁶ Then,

⁴This simplifying assumption is quite common in the literature. Suppose for example that firms inside a cartel bargain 'a la Nash', since firms are symmetric the distribution of gains will be symmetric.

⁵Throughout the paper we will identify a cartel with a capital letter and the size of the cartel (*i.e.* the number of firms, since firms are identical) with its corresponding small letter. Due to symmetry of firms, all that matters is the specification of a *cartel-size* k .

⁶These are very general assumptions, which most oligopolist models (such as Cournot, Bertrand and price-leadership) fulfill.

Lemma 1 (Thoron [1998] Theorem 1) *A cartel-size k is stable if and only if:*

- i) $\pi^C(k) \geq \pi^I(k-1)$ and*
- ii) $\pi^I(k) \geq \pi^C(k+1)$.*

That is, a stable cartel satisfies a property of internal stability: for all firms in the cartel there is no incentive to defect unilaterally from the cartel, and a property of external stability: for all firms outside the cartel there is no incentive to join the cartel unilaterally.

Lemma 2 (Thoron [1998] Theorems 2 and 3) *A cartel-size k is strongly stable if and only if:*

- i) $\pi^C(k) \geq \pi^I(k-l)$ for all $l \in \{1, \dots, k\}$ and*
- ii) $\pi^I(k) > \pi^C(k+l)$ for all $l \in \{1, \dots, n-k\}$.*

That is, a strongly stable cartel satisfies a property of strong internal stability: for all possible coalitions of firms in the cartel there is no incentive to jointly defect from the cartel, and a property of strong external stability: for all possible coalitions of firms outside the cartel there is no incentive to jointly add to the cartel.

Following Thoron [1998], we say that a coalitional deviation is *beneficial* if either *i)* the new situation provides larger profits for all firms that induce the change, or *ii)* the profits for the firms that induce the change are the same as in the previous situation, but the profits of either those who remain in the cartel or those who remain independent increase. We allow a coalitional deviation only if it is beneficial. Then,

Lemma 3 *A cartel-size k is coalition-proof stable if and only if:*

- i) for all $l \in \{1, \dots, k\}$ such that $\pi^C(k) < \pi^I(k-l)$, there is $m \in \{1, \dots, l-1\}$ such that $\pi^C(k-m) \geq \pi^I(k-l)$ and*
- ii) for all $l \in \{1, \dots, n-k\}$ such that $\pi^I(k) \leq \pi^C(k+l)$, there is $m \in \{1, \dots, l-1\}$ such that $\pi^I(k+m) > \pi^C(k+l)$.*

Proof. Consider a cartel of size k . Suppose first that $\pi^C(k) \geq \pi^I(k-l)$ for all $l \in \{1, \dots, k\}$ and $\pi^I(k) > \pi^C(k+l)$ for all $l \in \{1, \dots, n-k\}$. From Assumption **A2** we have that $\pi^I(k-l) > \pi^C(k-l)$ for all $l \in \{1, \dots, k\}$. Therefore, $\pi^C(k) = \pi^I(k-l) \Rightarrow \pi^C(k) > \pi^C(k-l)$. Then, if $\pi^C(k) = \pi^I(k-l)$ the deviation of l firms that leave the cartel is not beneficial. As a result, if $\pi^C(k) \geq \pi^I(k-l)$ for all $l \in \{1, \dots, k\}$ and $\pi^I(k) > \pi^C(k+l)$ for all $l \in \{1, \dots, n-k\}$ there is no coalitional deviation from k which is beneficial, and k is thus the size of a coalition-proof stable cartel.

Suppose now that $\pi^C(k) < \pi^I(k-l)$ for some $l \in \{1, \dots, k\}$. Then, a coalition L of l firms finds it profitable to defect from the cartel. As the firms in L become independent, then the only possibility for further deviation by a subcoalition of L is that firms in such a subcoalition rejoin the cartel. From Assumption

A2 we have that $\pi^I(k-m) > \pi^C(k-m)$ for all $m \in \{1, \dots, l-1\}$. Therefore, $\pi^C(k-m) = \pi^I(k-l) \Rightarrow \pi^I(k-m) > \pi^I(k-l)$. As a result, a further deviation by a subcoalition of L which is beneficial exists if and only if $\pi^C(k-m) \geq \pi^I(k-l)$ for some $m \in \{1, \dots, l-1\}$.

Suppose now that $\pi^I(k) \leq \pi^C(k+l)$ for some $l \in \{1, \dots, n-k\}$. Then, a coalition L of l firms finds it profitable to join the cartel. Then, the only possibility for further deviation by a subcoalition of L is that firms in that subcoalition leave the cartel and become independent again. From Assumption **A2** we have that $\pi^I(k+m) > \pi^C(k+m)$ for all $m \in \{1, \dots, l-1\}$. Therefore, $\pi^I(k+m) = \pi^C(k+l) \Rightarrow \pi^C(k+l) > \pi^C(k+m)$. As a result, further deviation by a subcoalition of L which is beneficial exists if and only if $\pi^I(k+m) > \pi^C(k+l)$ for some $m \in \{1, \dots, l-1\}$. ■

The first condition in Lemma 3 says that whenever a coalition of firms finds it profitable to defect from the cartel, then a subcoalition of that coalition of firms finds it profitable to rejoin the cartel. The second condition says that whenever a coalition of firms finds it profitable to join the cartel, then a subcoalition of that coalition of firms finds it profitable subsequently to defect from the cartel and become independent again.

D'Aspremont *et al.* [1983] prove the existence of stable cartels. Thoron [1998] proves that under assumptions **A1** to **A3** there is a unique coalition-proof stable cartel, which is the greatest stable cartel. We now provide a characterization of stable and coalition-proof stable cartels.

Define k^* as the maximum size of a cartel for which any firm that unilaterally leaves the cartel receives a strictly smaller profit. Formally,

$$k^* := \max \{k \mid \pi^C(k) \geq \pi^I(k-1)\}.$$

From Assumption **A3** we have that $\pi^C(1) = \pi^I(0)$. Consequently, k^* is well-defined and $k^* \geq 1$. Then,

Proposition 1 *The cartel of size k^* is a stable cartel.*

Proof. We have $\pi^C(k^*) \geq \pi^I(k^*-1)$ and $\pi^C(k^*+1) < \pi^I(k^*)$ from definition of k^* . From Assumption **A2** we have that $\pi^I(k^*-1) > \pi^C(k^*-1)$. Therefore, $\pi^C(k^*) = \pi^I(k^*-1) \Rightarrow \pi^C(k^*) > \pi^C(k^*-1)$. And then, if $\pi^C(k^*) = \pi^I(k^*-1)$ the unilateral deviation of one firm that leaves the cartel is not beneficial. Conditions *i*) and *ii*) in Lemma 1 are then satisfied, and k^* is the size of a stable cartel. ■

Proposition 2 *The cartel of size k^* is the unique coalition-proof stable cartel.*

Proof. We first prove that k^* is the size of a coalition-proof stable cartel. We have $\pi^C(k^*) \geq \pi^I(k^*-1)$ by definition of k^* , and $\pi^I(k^*-1) \geq \pi^I(k^*-l)$ for all $l \in \{1, \dots, k^*\}$ by Assumption **A1**. Therefore, $\pi^C(k^*) \geq \pi^I(k^*-l)$ for all $l \in \{1, \dots, k^*\}$ and condition *i*) in Lemma 3 is then satisfied. Suppose now that $\pi^I(k^*) < \pi^C(k^*+l)$ for some $l \in \{1, \dots, n-k^*\}$, then $\pi^C(k^*+l) < \pi^I(k^*+l-1)$

by definition of k^* , and condition *ii*) in Lemma 3 is then satisfied. Consequently, k^* is the size of a coalition-proof stable cartel.

We next prove that $k \neq k^*$ cannot be the size of a coalition-proof stable cartel. If $k > k^*$, then $\pi^C(k) < \pi^I(k-1)$ by definition of k^* . Therefore, condition *i*) in Lemma 3 is not satisfied. Consider now $k < k^*$. We have $\pi^I(k) \leq \pi^I(k^*)$ by Assumption **A1**, and $\pi^I(k^*-1) \leq \pi^C(k^*)$ by definition of k^* . Let $l = k^* - k$, then for all $m < l$ we have $\pi^I(k+m) \leq \pi^I(k+l-1) = \pi^I(k^*-1) \leq \pi^C(k^*) = \pi^C(k+l)$. Therefore, condition *ii*) in Lemma 3 is not satisfied, and k^* is the unique size of a coalition-proof stable cartel. ■

While the existence of a strong stable cartel cannot be guaranteed, at least one stable cartel exists under Assumption **A2** on the payoff function, which says that an independent firm receives a greater payoff than a firm in the cartel. The result of existence and uniqueness of a coalition-proof stable cartel is also independent of how firms compete in the market. In fact, we only need Assumption **A1** on the payoff function, which says that the profit to an independent firm increases as the cartel-size increases. **A1** and **A2** are very general assumptions since they only require that independent firms benefit from the lower market competition induced by larger cartels. However, as we will see in Section 4, the sizes of the stable and coalition-proof stable cartels are often very small or even not different to cartel-size $k = 1$. We next study the dynamic aspect of cartel formation, which will provide us with a set of cartels that are likely to form at some time even if they are not stable.

3 Abstract games

The notion of an *abstract game* (von Neumann and Morgenstern [1944]) provides us with a general framework for studying the formation of cartels.

An abstract game is a pair (\mathcal{S}, R) where \mathcal{S} is a set of alternatives and R is a binary relation defined on that set. For $a, b \in \mathcal{S}$, if aRb we will say that *a dominates b*. A *path* from a to b in \mathcal{S} is a sequence of alternatives $a = a_0, a_1, a_2, \dots, a_m = b \in \mathcal{S}$ such that $a_{i-1}Ra_i$ for all $i \in \{1, \dots, m\}$.

There are many situations for which no inherently stable cartel (as defined in Section 2) exists. In those cases, it may be interesting to analyze how firms may form a cartel and how firms may leave a cartel, whenever the new situation provides larger (or at least no smaller) profits for the firms that induce the change. An abstract game in which the set of alternatives represents the set of all possible cartels, and the binary relation represents the possible transitions between cartels may be a useful instrument for analyzing how firms may react when a firm or a group of firms deviate.

Von Neumann and Morgenstern [1944] suggested the notion of a *stable set* as a solution concept for abstract games. A stable set satisfies two conditions: *i*) neither of any two alternatives in a stable set dominates the other (internal stability), and *ii*) any alternative outside a stable set is dominated by an alternative in the set (external stability). A drawback of the notion of a stable set

is that existence is not ensured. As a response, van Deemen [1991] introduces the concept of a generalized stable set and proves existence.

Let R^T be the *transitive closure* of a binary relation R (i.e. aR^Tb means that there is a path from a to b).

Definition 1 Let (\mathcal{S}, R) be an abstract game. A nonempty set $\mathcal{G} \subseteq \mathcal{S}$ is called a *generalized stable set* if and only if:

- i) for all $a, b \in \mathcal{G}$ ($a \neq b$): not aR^Tb and
- ii) for all $b \in \mathcal{S} \setminus \mathcal{G}$ there is $a \in \mathcal{G}$ such that aR^Tb .

The first condition (called internal stability of domination) says that neither of any two alternatives in a generalized stable set dominates the other, not even through a path. The second condition (called external stability of domination) says that any alternative outside a generalized stable set is dominated by an alternative in the set, either directly or through a path.

Shenoy [1979] introduces the concept of an *elementary dynamic solution* (which we call *absorbing set*) for abstract games. We call the *absorbing sets solution* for an abstract game to the collection of all its absorbing sets.⁷

Definition 2 Let (\mathcal{S}, R) be an abstract game. A nonempty set $\mathcal{A} \subseteq \mathcal{S}$ is called an *absorbing set* if and only if:

- i) for all $a, b \in \mathcal{A}$ ($a \neq b$): aR^Tb and
- ii) there is no $b \in \mathcal{S} \setminus \mathcal{A}$ and $a \in \mathcal{A}$ such that bR^Ta .

The first condition says that any two alternatives in an absorbing set dominate each other, if not directly then through a path. This implies that whenever the negotiation process leads to an alternative that belongs to an absorbing set, it may shift to any other alternative that belongs to the same absorbing set in a finite number of steps. The second condition says that no alternative outside the absorbing set dominates an alternative in the set, not even through a path. This implies that once the negotiation process leads to an alternative that belongs to an absorbing set, from this time on, any alternative that does not belong to this absorbing set is impossible to reach (even if it belongs to another absorbing set). This solution concept captures the dynamic aspect of negotiations. That is, when considering an alternative in \mathcal{S} , we consider also any alternative in \mathcal{S} that can be reached through a succession of moves.

A relationship between generalized stable sets and absorbing sets can be established.

Proposition 3 (Inarra, Kuipers, Olaizola [2004a] Theorem 5) Let $\{\mathcal{A}_1, \dots, \mathcal{A}_k\}$ be the absorbing sets solution for the abstract game (\mathcal{S}, R) . Then, the set \mathcal{G} is generalized stable if and only if $|\mathcal{G} \cap \mathcal{A}_i| = 1$ for $i = 1, \dots, k$.

⁷Each absorbing set coincides with the *elementary dynamic solution* introduced by Shenoy [1979]. The union of all elementary dynamic solutions is called *dynamic solution*. This solution was previously defined by Kalai *et al.* [1976] under the name of the *admissible set*. Schwartz [1974] also introduces an equivalent solution, called *top cycle solution*.

As the set of alternatives \mathcal{S} (the set of all possible cartels that can be formed) is finite, the game (\mathcal{S}, R) has at least one absorbing set (see Theorem 1 in Kalai and Schmeidler [1977]). We will prove that under assumptions **A1** to **A3** on the payoff function a unique absorbing set exists. When each of the absorbing sets contains a single element, the collection of all absorbing sets coincides with the (unique) generalized stable set. In this case, we will show that every element in the generalized stable set is strongly stable, and moreover one of them is certain to be reached after a finite number of moves.

To define an abstract game, a set of alternatives \mathcal{S} and a binary relation R on that set are needed. In our context, all possible cartels constitute the set of alternatives. We now propose a binary relation which we view as appropriate to describe the arguments that firms construct in the process of cartel formation.

Let P and Q be two cartels and let M be a *minimum set of firms* necessary to force the move from P to Q . Note that this minimum set may not be unique, a coalition M is a minimum set of firms necessary to force the move from P to Q if one of the following two conditions holds: *i*) $Q \subseteq P$ and $M = P \setminus Q$, *ii*) $M = Q$.⁸ We say that the move from P to Q is profitable via M (denoted as $P \xrightarrow{M} Q$) if no firm in M loses with this move and at least one firm in M strictly gains. However, not all profitable moves seem equally reasonable. We assume that firms perform some type of filtering on the possible moves so that only some profitable moves (called robust moves) remain.⁹ Suppose that the firms in M are considering the move from P to Q , then they are likely to take other possible moves from P into consideration as well before making a decision. We next formalize this idea.

Let $\pi_i(S)$ be firm i 's profit when cartel S forms. Consider the profitable move $P \xrightarrow{M} Q$. We say that the move from P to Q' via M' is a *counter-proposal* for $P \xrightarrow{M} Q$ if the following three conditions hold: *i*) the move from P to Q' is profitable via M' , *ii*) $M \cap M' \neq \emptyset$, and *iii*) $\pi_i(Q') \geq \pi_i(Q)$ for all $i \in M \cap M'$ with at least one strict inequality. Then,

Definition 3 We say that $P \xrightarrow{M} Q$ is a robust move, and denote $P \xrightarrow{M}_r Q$, if there is no counter-proposal for $P \xrightarrow{M} Q$.

From a given cartel we can expect a different one to emerge via a deviating group of firms, which profit (or do not lose) from this move. The information provided by the profit function thus suggests a process of transitions from one cartel to another which we identify with robust moves. Then aRb is equivalent to the existence of M such that the move from b to a via M is robust. In this way, it is possible to predict what cartels might be observed with some regularity in the long run, and what cartels will not be observed. Consequently,

⁸Note that condition *ii*) implies that a cartel forms if and only if all potential members agree to form the cartel. By contrast, in Section 2 we consider games in which nonmembers can join the existing cartel without permission of the previous members.

⁹A discussion about filtering processes can be found in Inarra, Kuipers and Olaizola [2004b].

we can interpret an absorbing set as a set of cartels to which the transitions may become restricted in the long run.¹⁰

We next characterize the generalized stable sets and the absorbing sets solution under assumptions **A1** to **A3** on the profit function. These are very general assumptions, which most oligopolist models (such as Cournot, Bertrand and price-leadership) fulfill. These assumptions reflect two important aspects of oligopolist markets. First, if additional firms join a cartel, firms in the cartel reduce their output in order to internalize the positive externalities of output reduction, and market price rises. Consequently, firms in the cartel incur the cost of output reduction while firms outside the cartel profit from the higher price. Second, independent firms and the cartel produce the same quantity in equilibrium. Since inside the cartel the division of profits is equitable, then an independent firm receives a higher profit than a firm in the cartel.

Define \hat{k} as the cartel-size for which the payoff to firms inside the cartel is maximal. If there are different cartel-sizes for which this payoff is maximal, then choose the smallest of them. Formally,

$$\hat{k} := \min\{k \mid \pi^C(k) \geq \pi^C(l) \text{ for all } l \in N\}.$$

Further, define \bar{k} as the minimum number of firms needed for a cartel such that an independent firm receives a profit at least as large as the profit of firms inside a cartel of size \hat{k} . Formally,

$$\bar{k} := \min\{k \mid \pi^I(k) > \pi^C(\hat{k})\}.$$

Observe that $\pi^I(k) > \pi^C(\hat{k})$ for all $k \geq \bar{k}$, and $\pi^I(\bar{k} - 1) \leq \pi^C(\hat{k})$ and $\pi^I(k) < \pi^C(\hat{k})$ for all $k < \bar{k} - 1$, since the function $\pi^I(\cdot)$ is strictly decreasing by assumption **A1**. Also observe that $\bar{k} \leq \hat{k}$, since $\pi^I(\hat{k}) > \pi^C(\hat{k})$ by assumption **A2**. Recall from Section 2 that the unique size of a coalition-proof stable cartel is

$$k^* := \max\{k \mid \pi^C(k) \geq \pi^I(k - 1)\}.$$

Since $\pi^I(k - 1) > \pi^C(\hat{k}) \geq \pi^C(k)$ for all $k > \bar{k}$, it follows directly from the definition of k^* that $k^* \leq \bar{k}$.

Proposition 4 *If $k^* < \bar{k}$, then no strongly stable cartel exists and the unique absorbing set is $\mathcal{A} = \{\{K\} \mid \bar{k} - 1 \leq k \leq \hat{k}\}$. If $k^* = \bar{k}$, then the unique size of a strongly stable cartel is $k = \bar{k} = k^*$, and each elementary dynamic solution consists of one strongly stable cartel, i.e. the unique absorbing set is $\mathcal{A} = \{\{K\} \mid k = \bar{k}\}$.*

¹⁰Inarra, Kuipers and Olaizola [2004b] also model the formation of cartels in a Cournot oligopoly model by means of an abstract game. Their analysis is more general on the one hand, since they allow for the existence of more than one cartel at a time (the set of alternatives \mathcal{S} being the collection of all partitions of N). But on the other hand, they restrict to a specific profit function.

Proof. *i)* Consider first the case $k^* < \bar{k}$. Let K be a cartel of size $k \geq \bar{k}$ and let $i \in K$. Then $K \xrightarrow{i} K \setminus \{i\}$. This follows from the fact that $k \geq \bar{k} > k^*$, hence $\pi^I(k-1) > \pi^C(k)$ by definition of k^* . We even have $K \xrightarrow{i}_r K \setminus \{i\}$. To see this, first note that a counter-proposal, if it exists, cannot be to a cartel of size k or $k-1$. Consider a move to a cartel of size $m > k$. Then at least one independent firm must enter the cartel. Such a firm loses, since $\pi^I(k) > \pi^I(\bar{k}) > \pi^C(\hat{k}) \geq \pi^C(m)$. Hence, the move is not profitable and cannot be a counter-proposal for $K \xrightarrow{i} K \setminus \{i\}$. Now, consider a move to a cartel of size $m < k-1$. Then, no matter whether firm i will be independent or in the cartel, it will receive less than $\pi^I(k-1)$, since $\pi^C(m) < \pi^I(m) < \pi^I(k-1)$. Therefore, such a move cannot be a counter-proposal for $K \xrightarrow{i} K \setminus \{i\}$ either.

Let K be a cartel of size $k < \bar{k}$. Then let \hat{K} denote a cartel of size \hat{k} , such that K is a proper subset of \hat{K} (this choice is possible, since $\hat{k} \geq \bar{k} > k$). We have $K \xrightarrow{\hat{K}} \hat{K}$. This follows since $\pi^I(k) \leq \pi^C(\hat{k})$ and $\pi^C(k) \leq \pi^C(\hat{k})$, which shows that neither the independent firms that join the cartel nor the firms that were already in the cartel lose. Moreover, at least one of the two groups strictly gain, since $\pi^I(k) = \pi^C(\hat{k})$ and $\pi^C(k) = \pi^C(\hat{k})$ leads to the contradiction $\pi^C(k) < \pi^I(k) = \pi^C(\hat{k}) = \pi^C(k)$, where the strict inequality is by Assumption **A2**. We even have $K \xrightarrow{\hat{K}}_r \hat{K}$. To see this, note that for any move to another cartel, say of size m , and such that an independent firm w.r.t. \hat{K} joins the cartel, will receive a payoff that is strictly less than $\pi^I(\hat{k})$, since $\pi^C(m) \leq \pi^C(\hat{k}) < \pi^I(\hat{k})$. Also, for any move to a cartel, not \hat{K} , such that all independent firms w.r.t. \hat{K} stay independent, the firms that stay inside the cartel will receive a payoff strictly less than $\pi^C(\hat{k})$, since then $m < \hat{k}$, hence $\pi^C(m) < \pi^C(\hat{k})$ by definition of \hat{k} . Therefore, no counter-proposal exists for the move $K \xrightarrow{\hat{K}} \hat{K}$.

We have proved that a profitable (even robust) move exists at every cartel. This shows that no stable cartel exists. We have also proved that for every cartel, a sequence of robust moves exists leading to cartels of size $\bar{k}-1, \dots, \hat{k}$, and in fact *any* cartel of such size can be reached. This shows that there is a unique elementary dynamic solution, which contains $\{K \mid \bar{k}-1 \leq k \leq \hat{k}\}$. To prove that the unique elementary dynamic solution is equal to this set, we still need to prove that no robust move from a cartel in this set to a cartel outside this set exists. To see that this is true, first consider a cartel K of size k with $\bar{k} \leq k \leq \hat{k}$. We will show that in this case, the moves $K \xrightarrow{i} K \setminus \{i\}$ for $i \in K$ are the only robust moves. To this end, consider a profitable move, say $K \xrightarrow{M} K'$, not of the type $K \xrightarrow{i} K \setminus \{i\}$. Then for the size of K' , say k' , we have $k' > k$ or $k' < k-1$. We have demonstrated already that a move from a cartel of size k with $k \geq \bar{k}$ to a cartel of size $k' > k$ is not profitable, so in fact we must have $k' < k-1$. Then a firm $i \in K$ exists that is an independent firm w.r.t. K' , hence $i \in M$. Therefore, $K \xrightarrow{i} K \setminus \{i\}$ is a counter-proposal for $K \xrightarrow{M} K'$, since $\pi^I(k-1) > \pi^I(k')$. Now consider a cartel of size $\bar{k}-1$. We will show that in

this case, the moves of the type $K \xrightarrow{\hat{K}} \hat{K}$ (where K is a proper subset of \hat{K} and \hat{K} is of size \hat{k}), are the only robust moves. To this end, consider a profitable move, say $K \xrightarrow{M} K'$, for which the size of K' is not in the range from $\bar{k} - 1$ to \hat{k} . Let k' denote the size of K' . If $k' < \bar{k} - 1$, then at least one of the independent firms w.r.t. K' is a mover and this firm, say i , receives $\pi^I(k') < \pi^C(\hat{k})$, where the inequality follows by definition of \hat{k} . Now construct \hat{K} of size \hat{k} with $i \in \hat{K}$ and $K \subseteq \hat{K}$. Then we see that $K \xrightarrow{\hat{K}} \hat{K}$ is a counter-proposal for $K \xrightarrow{M} K'$. If $k' > \hat{k}$, then $K \xrightarrow{N} \hat{K}$ is a counter-proposal (where \hat{K} is a proper subset of K' of size \hat{k}), since $\pi^C(\hat{k}) \geq \pi^C(k')$ (firms that stay in the cartel do not lose compared to K'), $\pi^I(\hat{k}) > \pi^C(\bar{k}) \geq \pi^C(k')$ (firms that *become* independent strictly gain), and $\pi^I(\hat{k}) > \pi^I(k')$ (firms that *stay* independent strictly gain).

ii) Consider now the case $k^* = \bar{k}$. For a cartel K of size $k > k^* = \bar{k}$ and $i \in K$, we have that $K \xrightarrow{i} K \setminus \{i\}$ is a robust move (the proof of this is given in *i*). For a cartel K of size $k < \bar{k}$, the move $K \xrightarrow{\hat{K}} \hat{K}$ is robust, where K is a proper subset of \hat{K} and \hat{K} is of size \hat{k} (the proof of this is also given in *i*). We have thus proved that for an arbitrary cartel a sequence of robust moves exists leading to a cartel of size $k^* = \bar{k}$.

We will now show that no profitable move exists at a cartel of size \bar{k} . First consider a move to a cartel of size m with $m < \bar{k}$. Then at least one independent firm in the smaller cartel is a mover, and its payoff is $\pi^I(m) \leq \pi(k^* - 1) \leq \pi^C(k^*)$, hence this firm does not gain. For the possible movers that stay inside the cartel, we have a payoff of $\pi^C(m) < \pi^I(m) \leq \pi^C(k^*)$, so these firms even lose. Therefore, the move is not profitable.

Now consider a move to a cartel of size m with $m > \bar{k}$. Then at least one independent firm w.r.t. the cartel of size \bar{k} is inside the cartel of size m . This firm loses. Then the independent firm(s) that enter the cartel lose, since $\pi^I(\bar{k}) > \pi^C(\hat{k}) \geq \pi^C(m)$. Hence, such a move is not profitable either.

We see that no profitable move exists at a cartel of size \bar{k} . It follows that each cartel of size $\bar{k} = k^*$ is an elementary dynamic solution. These are also the only elementary dynamic solutions, since robust moves at all other cartels exist. It follows that the absorbing set equals $\{\{K\} \mid k = \bar{k}\}$. From the fact that no profitable move at a cartel of size \bar{k} exists, it also follows that these cartels are strongly stable. Since a strongly stable cartel is also coalition-proof stable, and since the unique size of a coalition-proof stable cartel is $k^* = \bar{k}$, it follows that $k^* = \bar{k}$ is the unique size of a strongly stable cartel. ■

From Propositions 3 and 4 we have the following straightforward result.

Corollary 1 *If $k^* < \bar{k}$, then each cartel in $\mathcal{A} = \{\{K\} \mid \bar{k} - 1 \leq k \leq \hat{k}\}$ is generalized stable. If $k^* = \bar{k}$, then the unique size of a generalized stable cartel is $\bar{k} = k^*$.*

Then, if $k^* < \bar{k}$, after a finite number of moves, firms will form a cartel with at least $\bar{k} - 1$ and no more than \hat{k} firms. From this time on, any cartel of size

smaller than $\bar{k} - 1$ or larger than \hat{k} can no longer form, while there is a positive probability of forming any cartel of size $\bar{k} - 1 \leq k \leq \hat{k}$. Since firms join and leave cartels endlessly, no strongly stable cartel exists. However, the absorbing sets solution selects a group of cartels to which the process of formation and destruction of cartels is restricted in the long run. On the other hand, if $k^* = \bar{k}$, after a finite number of moves, firms will form a cartel of size $k = \bar{k} = k^*$. From this time on, no cartel of different size can form. Since the unique cartel-size from which no profitable move exists is $k = \bar{k} = k^*$, it is the unique strongly stable cartel-size, the unique cartel-size in the unique elementary dynamic solution, and also the unique cartel-size in the unique generalized stable set.

In the rest of the paper we apply the two cartel formation approaches analyzed before to a Cournot game. We also study the influence that the internal organization of firms has on the size of the cartels that form in the context of an oligopolist industry by comparing a situation where ownership and management are not separated with one where they are separated.

4 Firms' internal organization and cartel-size

We analyze a linear Cournot model as the simplest way of illustrating our results. However, this analysis can be extended to symmetric oligopoly models with arbitrary cost and demand functions. We only require assumptions **A1** to **A3** on firms' profits.

4.1 A Cournot oligopoly model

Consider a single industry consisting of n firms that produce a homogeneous good. Assume linear demand function $p = 1 - Q$, with p being the price of the good and Q being the total quantity produced in the industry. Assume constant marginal cost of production c , $0 < c < 1$, equal for all firms. Firms compete in the market by setting quantities simultaneously.

In order to analyze whether strategic delegation of production decisions to managers affects the formation of cartels, we first introduce the case in which firms' owners do not delegate production decisions as a benchmark, we then turn to the strategic delegation case.

4.1.1 Profit-maximizer firms

We model the formation of cartels as a non-cooperative game. Assume that firms play a two stage game. In the first stage firms' owners decide whether to join a cartel or not, then at this stage firms' actions have a binary form: to cooperate (action C) or not cooperate (action I). The cartel will be formed by the firms that choose action C , while firms that choose I remain independent. Therefore, if k firms decide to form a cartel, $n - k$ firms remain independent. In the second stage, firms' owners take production decisions, firms in the cartel playing as a single player against firms outside the cartel. Firms in the cartel *jointly*

decide their quantities taking into account the behavior of the independent firms. Independent firms maximize their individual profits taking into account the behavior of the cartel. Firms in the cartel and independent firms choose their quantities simultaneously. Inside the cartel the profits are shared equally.

The aggregate coalitional profit of a cartel K is:

$$\sum_{i \in K} (1 - Q) q_i - c q_i.$$

Since firms in a cartel join forces to make production decisions, the problem of firms in K is to choose the *per member* quantity q_i for all $i \in K$ which maximizes the coalitional profit, taking the quantities of the independent firms as given. We can consider an independent firm as a cartel formed by a single firm. The maximization problem an independent firm faces is then a particular case of the problem solved by coalition K .

Let $q_{nd}^C(k)$ be the quantity produced by a firm that belongs to a cartel of size k . And let $q_{nd}^I(k)$ be the quantity produced by an independent firm when a cartel of size k forms. Through the work, subindex nd will denote the case where owners do not delegate production decisions to managers, *i.e.* the strict profit maximization case. The first order conditions provide,

$$q_{nd}^I(k) = \frac{1 - c}{n - k + 2} \quad \text{and} \quad q_{nd}^C(k) = \frac{1}{k} q_{nd}^I(k).$$

Let $p_{nd}(k)$ be the market price when a cartel of size k forms. Let $\pi_{nd}^C(k)$ be the profit of a firm that belongs to a cartel of size k . And let $\pi_{nd}^I(k)$ be the profit of an independent firm when a cartel of size k forms. Substituting the optimal quantities we obtain the associated equilibrium price and profits. Then,

$$p_{nd}(k) = \frac{1 + c(n - k + 1)}{n - k + 2},$$

$$\pi_{nd}^I(k) = \frac{(1 - c)^2}{(n - k + 2)^2} \quad \text{and} \quad \pi_{nd}^C(k) = \frac{1}{k} \pi_{nd}^I(k).$$

Note that the only Pareto optimal coalition is the cartel formed by all firms. Note also the positive externality induced by collusion: firms in the cartel reduce their production, rising market price, and as a result firms outside the cartel receive larger profits (Assumption **A2**). On the other hand, independent firms receive greater profits when larger cartels form (Assumption **A1**). This leads to a free-riding incentive, since firms in the cartel incur the cost of output reduction while firms outside the cartel profit from the higher price.

4.1.2 Strategic delegation

Assume now that each firm's owner delegates production decisions to a manager in order to improve his strategic position in the market. As in Fershtman and Judd [1987], we assume that owners offer 'take it or leave it' linear incentive

schemes to risk-neutral managers. The manager of firm i receives a payoff: $\beta_i + B_i O_i$, where β_i and B_i are constant, $B_i > 0$, and O_i is a linear combination of profits and sales revenue. Formally, firm i 's manager will be given an incentive to maximize:

$$O_i = \alpha_i (p - c) q_i + (1 - \alpha_i) p q_i,$$

where q_i is the quantity produced by firm i . And α_i is the incentive parameter chosen by firm i 's owner, we make no restrictions on α_i . O_i can be rewritten as:

$$O_i = (p - c\alpha_i) q_i.$$

As this last equation shows, firm i 's manager considers $c\alpha_i$ as the marginal cost of production when taking production decisions. In this way, firm i 's owner can make his manager more (less) aggressive, *i.e.* he can make his manager produce a higher (lower) output level than a profit-maximizer firm by choosing an incentive parameter such that the marginal cost of production considered by the manager is lower (higher) than that considered by a profit-maximizer firm.

Assume that firms play a three stage game: an intermediate stage to the two stage game described for the strict profit maximization case is added. In the first stage, firms' owners decide whether to join the cartel or not. In the second stage, firms' owners decide what incentives are given to their managers. And in the third stage, firms' managers take production decisions, with firms in the cartel playing as a single player against firms outside the cartel, which behave as independent firms.¹¹

Once a cartel forms, the manager of the cartel is given to maximize a linear combination of cartel sales and cartel profits. Let $\alpha^C(k)$ be the incentive parameter chosen by the owner of a cartel of size k . Then, the objective function $O(k)$ that the manager of a cartel formed by k firms has to maximize can be written as:

$$O(k) = (1 - Q) \sum_{i \in K} q_i - c\alpha^C(k) \sum_{i \in K} q_i.$$

The cartel's manager and the independent firms' managers are assumed to choose their quantities simultaneously. Let $\alpha^C(k)$ and $\alpha_i^I(k)$ be the incentive parameters chosen by the owner of a cartel of size k and the owner of independent firm i when a cartel of size k forms, respectively. Let $q_{id}^C(k)$ be the quantity produced by a firm that belongs to a cartel of size k . And let $q_{id}^I(k)$ be the quantity produced by an independent firm when a cartel of size k forms.

¹¹Once a cartel forms, we assume that the managers of the firms that join the cartel are all given the same function to maximize: a linear combination of cartel sales (sum of sales of all firms in the cartel) and cartel profits (sum of profits of all firms in the cartel). In order to simplify the exposition of the work, we can then substitute in our discussion the collection of managers of a cartel by a single manager, who competes in the product market against the managers of the independent firms. We can also substitute the collection of owners of a cartel by a single owner, since every owner of a firm in the cartel assumes as his objective the maximization of the aggregate cartel profit.

Throughout this study, subindex d denotes the case where owners delegate production decisions to managers. The first order conditions provide,

$$\begin{aligned} q_{id}^I(k) &= \frac{1 - (n - k + 1) c \alpha^C(k) + c \sum_{i \notin K} \alpha_i^I(k)}{n - k + 2} \quad \text{for all } i \notin K, \\ q_{id}^C(k) &= \frac{1 + (n - k + 1) c \alpha^C(k) - c \sum_{i \notin K} \alpha_i^I(k)}{n - k + 2} \quad \text{for all } i \notin K. \end{aligned}$$

Now firms' owners simultaneously choose the incentive parameters that are given to managers in order to maximize their profits. Then,

$$\alpha^C(k) = \alpha_i^I(k) = 1 - \frac{(1 - c)(n - k)}{c(2 + (n - k + 2)(n - k))} \quad \text{for all } i \notin K.$$

And therefore,¹²

$$q_d^I(k) = \frac{(1 - c)(n - k + 1)}{2 + (n - k + 2)(n - k)} \quad \text{and} \quad q_d^C(k) = \frac{1}{k} q_d^I(k).$$

Let $p_d(k)$ be the market price when a cartel of size k forms. Let $\pi_d^C(k)$ be the profit of a firm that belongs to a cartel of size k . And let $\pi_d^I(k)$ be the profit of an independent firm when a cartel of size k forms. Substituting the optimal quantities we obtain the associated equilibrium price and profits. Then,

$$\begin{aligned} p_d(k) &= \frac{1 + c(n - k + 1)^2}{2 + (n - k + 2)(n - k)}, \\ \pi_d^I(k) &= \frac{(1 - c)^2 (n - k + 1)}{(2 + (n - k + 2)(n - k))^2} \quad \text{and} \quad \pi_d^C(k) = \frac{1}{k} \pi_d^I(k). \end{aligned}$$

As in the strict profit maximization case, firms' profits fulfill assumptions **A1** to **A3**. Note that all $\alpha^C(k)$ and $\alpha_i^I(k)$ are less than unity (with the exception of the monopoly case, in which they are equal to unity).¹³ Consequently, all managers consider a lower marginal cost of production than in the strict profit maximization case. As a result, all managers behave more aggressively (*i.e.* produce a larger quantity) than a profit-maximizer firm, and all firms' profits are hence lower than in the strict profit maximization case. We next analyze how this fact affects to the formation of cartels by using the two different approaches introduced in Sections 2 and 3.

We first characterize the stable cartels for the linear Cournot oligopoly model under both strategic delegation and strict profit maximization.

¹²Since in equilibrium $q_{id}^C(k) = q_{jd}^C(k)$ for all $i, j \in K$, and $q_{id}^I(k) = q_{jd}^I(k)$ for all $i, j \notin K$, from now on the subscript i will be ignored.

¹³The case $\alpha^C(k) = \alpha_i^I(k) = 1$ is equivalent to the strict profit maximization case.

4.2 Cartel stability

When firms' owners do not delegate production decisions to managers the following result is obtained.

Proposition 5 *If $n = 2$, then $k = 2$ is the only stable, the only strongly stable and the only coalition-proof stable cartel-size. If $n \geq 3$, then $k = 1$ is the only stable and the only coalition-proof stable cartel-size, and there is no strongly stable cartel.*¹⁴

Proof. Straightforward computations show that if $n = 2$ we have $\pi_{nd}^C(2) > \pi_{nd}^I(1)$, and if $n \geq 3$ we have $\pi_{nd}^I(k-1) > \pi_{nd}^C(k)$ for all k . Then, for $n = 2$ we have $k_{nd}^* = 2$, and for $n \geq 3$ we have $k_{nd}^* = 1$. As a result, the unique stable cartel according to Lemma 1, and also the unique coalition-proof stable cartel according to Proposition 2 is the one formed by the two firms if $n = 2$, and the degenerate one formed by a single firm if $n \geq 3$. On the other hand, for $n = 2$ we have $\bar{k}_{nd} = k_{nd}^*$, and for $n \geq 3$ we have $\bar{k}_{nd} > k_{nd}^*$. Then, from Proposition 5 we have that the cartel formed by the two firms is the only strongly stable one for $n = 2$, and from Proposition 4 we have that there is no strongly stable cartel for $n \geq 3$. ■

We now extend the analysis by allowing firms' owners to delegate production decisions to managers. The following result is obtained.

Proposition 6 *If $n \leq 3$, then $k = n$ is the only stable, the only strongly stable and the only coalition-proof stable cartel-size. If $n = 4$, then $k = 2$ is the only stable and the only coalition-proof stable cartel-size, and there is no strongly stable cartel. If $n \geq 5$, then $k = 1$ is the only stable and the only coalition-proof stable cartel-size, and there is no strongly stable cartel.*

Proof. Straightforward computations show that if $n \leq 3$ we have $\pi_d^C(k) > \pi_d^I(k-1)$ for all k , if $n = 4$ we have $\pi_d^I(k-1) > \pi_d^C(k)$ if and only if $k \geq 3$, and if $n \geq 5$ we have $\pi_d^I(k-1) > \pi_d^C(k)$ for all k . Then, for $n = 2$ we have $k_d^* = 2$, for $n = 3$ $k_d^* = 3$, $n = 4$ $k_d^* = 2$, and for $n \geq 5$ we have $k_d^* = 1$. Then, from Lemma 1 we have that the unique stable cartel (and according to Proposition 2 the unique coalition-proof stable cartel) is the cartel formed by all firms if $n \leq 3$, the one formed by two firms if $n = 4$, and the degenerate one formed by a single firm if $n \geq 5$. On the other hand, for $n = 2$ and $n = 3$ we have $\bar{k}_d = k_d^*$, and for $n \geq 4$ we have $\bar{k}_d > k_d^*$. Then, from Proposition 5 we have that the cartel formed by all firms is the only strongly stable one for $n \leq 3$, and from Proposition 4 we have that there is no strongly stable cartel for $n \geq 4$. ■

These results on cartel stability illustrate the free-riding incentives induced by the formation of cartels, which for the Cournot oligopoly model proves to be so strong that no cartel is formed (except when there is a very low number of firms in the industry). From Propositions 5 and 6 we obtain the following straightforward result.

¹⁴This is a result obtained by Thoron [1998].

Proposition 7 *If $n = 2$ or $n \geq 5$, the results on cartel stability are equal under strategic delegation and strict profit maximization. If $n = 3$ or $n = 4$, larger stable cartels can be sustained under strategic delegation.*

As noted before, firms in the cartel reduce their production, raising the market price, and firms outside the cartel benefit from the increased price and receive larger profits. Moreover, firms belonging to a larger cartel obtain lower profits. These facts explain the high instability of cartels. However, we find that under strategic delegation, firms' free-riding incentives are weakened, and the size of the unique stable cartel, the size of the unique strongly stable cartel and the size of the unique coalition-proof stable cartel (if any) are not smaller than under strict profit maximization. In fact, in the triopoly and tetrapoly cases strictly larger cartels are formed when owners delegate production decisions to managers. This fact may be relevant, since we can find many examples of industries in which very few firms are present in the market.

The analysis of the dynamic aspect of coalition formation will provide us with more insights about the influence that the internal organization of firms has on cartel formation.

4.3 Absorbing sets solution

We have defined \hat{k} as the cartel-size for which the payoff to a firm inside the cartel is maximum. Straightforward computations show that $\hat{k} = n$ for the linear Cournot model under both the strategic delegation and the strict profit maximization cases. Then, \bar{k} is defined as the minimum number of firms needed to form a cartel such that every independent firm receives a profit at least as large as the profit it obtains if the whole industry colludes.

When firms' owners do not delegate production decisions we obtain:

$$\bar{k}_{nd} = \left\lceil 2 + n - 2n^{\frac{1}{2}} \right\rceil.$$

And when they do delegate production decisions:

$$\bar{k}_d = \left\lceil 1 + n - \frac{1}{6^{\frac{1}{2}}} Y - \left(\frac{1}{Y} 6^{\frac{1}{2}} n - \frac{1}{6} Y^2 - 1 \right)^{\frac{1}{2}} \right\rceil,$$

where $Y = (Z + \frac{1}{2}4 - 2)^{\frac{1}{2}}$ and $Z = (27n^2 - 8 + 3n(81n^2 - 48)^{\frac{1}{2}})^{\frac{1}{3}}$. Then,

Proposition 8 *If $n = 2$, $\mathcal{A}_{nd} = \mathcal{A}_d = \{K \mid k = 2\}$. If $n = 3$, $\mathcal{A}_{nd} = \{K \mid k \geq 1\}$ and $\mathcal{A}_d = \{K \mid k = 3\}$. If $n \geq 4$, $\mathcal{A}_{nd} = \{\{K\} \mid k \geq \bar{k}_{nd} - 1\}$ and $\mathcal{A}_d = \{\{K\} \mid k \geq \bar{k}_d - 1\}$.*

Proof. Straightforward computations show that for $n = 2$ we have $\bar{k}_{nd} = k_{nd}^* = \bar{k}_d = k_d^* = 2$, for $n = 3$ we have $\bar{k}_{nd} = 2 > k_{nd}^* = 1$ and $\bar{k}_d = k_d^* = 3$, and for $n \geq 4$ we have $\bar{k}_{nd} > k_{nd}^*$ and $\bar{k}_d > k_d^*$. Then, according to Proposition 4 the result holds. ■

For $n = 2$, under both strict profit maximization and strategic delegation, the collusion of the whole industry will take place at some time, and from then on the situation will remain unchanged since it is the unique cartel in the unique absorbing set, and therefore no move from it can be robust. For $n = 3$, under strict profit maximization firms will endlessly form and dissolve cartels, and no cartel can be excluded from this transition process since at any cartel a robust move can be found. However, under strategic delegation the collusion of the whole industry will take place at some time, and from then on the situation will remain unchanged. For $n \geq 4$, independently of the starting point, after a finite number of moves firms will form a cartel with at least $\bar{k} - 1$ firms, under both strict profit maximization and strategic delegation. From then on, the formation and destruction of cartels will be restricted to cartels with at least $\bar{k} - 1$ firms. Moreover, for $n \geq 4$, the cartel formed by n firms belongs to the unique absorbing set, and thus the collusion of the whole industry will occur with some regularity in time.

We now compare the results obtained under strict profit maximization and strategic delegation. Straightforward computations show that $\bar{k}_d > \bar{k}_p$. From Proposition 8 we then have:

Proposition 9 *If $n = 2$, $\mathcal{A}_{nd} = \mathcal{A}_d$. If $n \geq 3$, $\mathcal{A}_d = \mathcal{A}_{nd} \setminus \{K \mid \bar{k}_{nd} \leq k < \bar{k}_d\}$.*

Consequently, the unique absorbing set under strategic delegation can be obtained by deleting a number of the smallest cartels from the absorbing set obtained under strict profit maximization. Then, the set of cartels to which the moves will be restricted in the long run contains larger cartels under strategic delegation than under strict profit maximization. As we have seen, firms' profits are lower under strategic delegation than under strict profit maximization. As the cartel and each of the independent firms produce the same quantity, then the loss of profit induced by strategic delegation is smaller for the firms in the cartel, since those firms share losses while independent firms do not. As a result, the free-riding incentive becomes lower and the incentive to form cartels becomes higher when firms' owners delegate production decisions to managers. However, as the results obtained by applying the notions of stability, strong stability and coalition-proof Nash stability show, although the free-riding incentive is weakened under strategic delegation it remains strong enough to prevent the stability of cartels. The reason for the formation of larger cartels lies then in firms' higher incentive to form cartels under strategic delegation. As n is the cartel-size for which the payoff to firms inside the cartel is maximal and a single firm always finds it profitable to leave a cartel, then given any cartel there are two possible moves to another cartel: either all firms together induce the collusion of the whole industry or some firms leave the cartel and become independent. As the minimum number of firms needed to form a cartel such that every independent firm receives a profit at least as large as the profit it obtains in the monopoly case is larger under strategic delegation than under strict profit maximization, the free-riding process 'stops earlier' under strategic delegation.

5 Conclusions

We restrict our study to oligopolist industries where firms' profits are characterized by assumptions **A1** to **A3**. These are very general assumptions, which most oligopolist models (such as Cournot, Bertrand and price-leadership) fulfill. These assumptions reflect two important aspects of oligopolist markets. First, if additional firms join a cartel, firms in the cartel reduce their output in order to internalize the positive externalities of output reduction, and the market price rises. Consequently, firms in the cartel incur the cost of output reduction while firms outside the cartel profit from the higher price. Second, independent firms and the cartel produce the same quantity in equilibrium. Since the division of profits inside the cartel is equitable, an independent firm therefore receives a higher profit than a firm in the cartel. Within this framework, we study the formation of cartels in two different contexts. First, we analyze internal-external stability based models which, due to firms' free-riding incentives, lead to the inexistence of stable cartels (except when there is a very low number of firms in the industry). Second, to analyze what may happen in those cases where no inherently stable cartel exists, we introduce the dynamic aspect of cartel formation. We prove that there is a unique absorbing set, and provide a characterization. The absorbing sets solution selects a group of cartels to which the process of formation and destruction of cartels is restricted in the long run even if no cartel in such a group is stable.

We also study the influence that delegation of production decisions has on the size of the cartels that form. At this moment, we restrict the analysis to a linear Cournot model as a simplest way of illustrating our results. However, this analysis could be extended to symmetric oligopoly models with arbitrary cost and demand functions. We only require assumptions **A1** to **A3** on firms' profits. We show that strategic delegation reinforces the incentives to form cartels in both the static and the dynamic contexts, with this influence being stronger in the dynamic context.

Our model assumes firms' identical cost structure and homogeneous goods. Possible extensions could consider the case of heterogeneous goods as well as the case in which firms have access to different technologies, since it would be interesting to analyze which firms are most likely to join in a cartel in those cases.

Another extension could take into account the fact that while collusive agreements create positive externalities on firms outside the cartel, cost-reducing alliances induce negative externalities on other firms. An analysis of this case could lead to very different conclusions.

6 References

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