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WELFARE EFFECTS OF THIRD-DEGREE PRICE DISCRIMINATION: IPPOLITO MEETS SCHMALENSEE AND VARIAN

by

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Welfare Effects of Third-Degree Price Discrimination: Ippolito Meets Schmalensee and Varian

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Abstract

Based on a pioneering work by Ippolito (1980) we construct a simple model which allows the welfare effects of third-degree price discrimination to be well understood and explained. The decomposition of the change in welfare into a misallocation effect and an output effect has advantages over the well-established analysis by Schmalensee (1981) and Varian (1985). In particular, our approach provides a graphic analysis which clarifies the welfare analysis of third-degree price discrimination. (*JEL* D42, L12, L13)

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I. INTRODUCTION

Price discrimination under imperfect competition is an important area of economic research,[1] and third-degree price discrimination, the most prevalent form of price discrimination, is a major item in any standard treatment of monopoly theory covered in intermediate and advanced microeconomics courses (see, for instance, Pindyck and Rubinfeld, 2008, or Varian, 1992, 2006). Under third-degree price discrimination the seller can charge different prices to consumers belonging to different groups or submarkets. For example, the seller may charge different prices to customers who are separated geografically (the home and the export markets) or that are differentiated by age (senior citizen's discounts), occupation (student discounts), time of purchases (initial equipment and replacement purchases), or by enduse (milk for liquid consumption or for further processing). Moving from non-discrimination to discrimination raises the firm's profits, harms consumers in markets where the prices increase and benefits the consumers who face lower prices. Consequently, the overall effect on welfare is undetermined.

Understanding the conditions under which the change in social welfare can be signed has concerned economists at least from the earlier work by Pigou (1920) and Robinson (1933). A move from uniform pricing to third-degree price discrimination generates, as will be shown below, two effects:[2] first, price discrimination causes a misallocation of goods from high to low value users (that is, output is not efficiently distributed to the highest-value end); second, price discrimination affects total output. Therefore, since price discrimination is viewed as an inefficient way of distributing a given quantity of output between different consumers or submarkets, a necessary condition for price discrimination to increase social welfare is that it should increase total output.[3] In consequence, in order for price discrimination to increase welfare a positive output effect must offset the negative effect of distributional inefficiency.

Schmalensee (1981)'s direct approach to the welfare effect and Varian's celebrated bounds on social welfare (1985, 1989,1992) have dominated both

[1] See Stole (2007), Armstrong (2008) and Liu and Serfes (2010) for excellent theoretical surveys. See also McAfee (2008) for a modern view of price discrimination and for antitrust implications.

[2] McAfee (2008) provides a nice explanation of these effects.

[3] See, for example, Robinson (1933), Schmalensee (1981), Varian (1985), Schwartz (1990) and more recently Bertoletti (2004). However, when marginal costs varies across markets that result does not maintain (see, Bertoletti, 2009).

the research and the teaching of the welfare effects of third-degree price discrimination for the last twenty-five years. Our analysis, inspired by the pioneering work by Ippolito (1980) and its generalization to n markets by Aguirre (2008), offers some advantages over Schmalensee's and Varian's. Firstly, it focuses directly on the change in welfare (instead of on indirect Lagrangian techniques or on exogenous bounds) and allows the output effect (that is, the social valuation of the change in total output) to be distinguished neatly from the misallocation effect. In addition we show how it is possible to prove the theorem that "a necessary condition for third-degree price discrimination to increase social welfare is that total output increases" by using this decomposition. Our approach to the welfare effects of discrimination is also more intuitive and can be illustrated graphically.

II. **ANALYSIS**

Consider a monopolist selling a good in two perfectly separated markets. The demand function in market i $(i = 1, 2)$ is given by $D_i(p_i)$, where p_i is the price charged in that market and the inverse demand function is $p_i(q_i)$, where q_i is the quantity sold. Unit cost, c, is assumed to be constant. The price elasticity in market i is given by $\varepsilon_i(p_i) = -[D_i^{'}(p_i)p_i]/D_i(p_i)$. The profit function in market i, $\pi_i(p_i) = (p_i - c)D_i(p_i)$, is assumed to be strictly concave, $\pi_i^{''} < 0$.

Profit Maximization

Under simple monopoly pricing, profits are maximized by charging all consumers a common price p^0 such that:

$$\sum_{i=1}^{2} D_i(\, p^0) + (\, p^0 - c) \sum_{i=1}^{2} D_i^{'}(\, p^0) = 0. \tag{1}$$

Therefore, under uniform pricing, the optimal policy is given by $(p^0 - c)/p^0 = 1/\varepsilon(p^0)$, where p^0 denotes the uniform price and $\varepsilon(p^0)$ is the elasticity of the aggregate demand at p^0. If we let $D(p) = \sum_{i=1}^{2} D_i(p)$ denote the aggregate demand, then this elasticity is simply the weighted average elasticity: $\varepsilon(p^0) = \sum_{i=1}^{2} \alpha_i(p^0)\varepsilon_i(p^0)$, where the elasticity of market i is weighted by the "share" of that market at the optimal uniform price, $\alpha_i(p^0) = D_i(p^0)/\sum_{i=1}^{2} D_i(p^0)$. Let q_i^0 denote the quantity sold in market i, $q_i^0 = D_i(p^0)$ $(i = 1, 2)$, and q^0

denote the total output, $q^0 = \sum_{i=1}^2 D_i(p^0)$, under uniform pricing which can be expressed as follows:

$$q^0 = \sum_{i=1}^{2} D_i(p^0) = -\sum_{i=1}^{2}(p^0 - c)D_i^{'}(p^0). \tag{2}$$

It is illustrative to evaluate the marginal profit in market i $(i = 1, 2)$ at the optimal uniform pricing:

$$\pi_i^{'}(p^0) = D_i(p^0) + (p^0 - c)D_i^{'}(p^0) = p^0 D_i^{'}(p^0)\left[\frac{D_i(p^0)}{p^0 D_i^{'}(p^0)} + \frac{(p^0 - c)}{p^0}\right]. \tag{3}$$

Therefore, the marginal profit becomes:

$$\pi_i^{'}(p^0) = p^0 D_i^{'}(p^0)\left[-\frac{1}{\varepsilon_i(p^0)} + \frac{1}{\varepsilon(p^0)}\right]. \tag{4}$$

The monopolist is willing to increase (decrease) the price in market i if the elasticity in that market, $\varepsilon_i(p^0)$, is lower (higher) than the elasticity of the aggregate demand, $\varepsilon(p^0)$.
Condition (4) can be written, equivalently, as

$$\pi_i^{'}(p^0) = p^0 D_i^{'}(p^0)\left[-\frac{1}{\varepsilon_i(p^0)} + \frac{D_i(p^0) + D_j(p^0)}{D_i(p^0)\varepsilon_i(p^0) + D_j(p^0)\varepsilon_j(p^0)}\right],$$

which leads to

$$\pi_i^{'}(p^0) = \frac{D_i(p^0)D_j(p^0)}{[D_i(p^0)\varepsilon_i(p^0) + D_j(p^0)\varepsilon_j(p^0)]}\left[\varepsilon_j(p^0) - \varepsilon_i(p^0)\right], \tag{5}$$

where $i, j = 1, 2, j \neq i$. Note that $\pi_i^{'}(p^0) > 0$ iff $\varepsilon_j(p^0) > \varepsilon_i(p^0)$, $i, j = 1, 2, j \neq i$. Therefore, if possible the monopolist would want to increase the price in the market with lower elasticity of demand and to reduce the price in the market with higher elasticity of demand.

Under price discrimination, the optimal policy for the monopolist is given by:

$$D_i(p_i^d) + (p_i^d - c)D_i^{'}(p_i^d) = 0, \;\; i = 1, 2, \tag{6}$$

where p_i^d denotes the optimal price in market i (and profit functions are assumed to be strictly concave in the relevant range). Under price discrimination, the optimal policy for the monopolist can be expressed as $(p_i^d - c)/p_i^d = 1/\varepsilon_i(p_i^d)$, $i = 1, 2$, where p_i^d denotes the optimal price in market i, and $\varepsilon_i(p_i^d) = -[D_i^{'}(p_i^d)p_i^d]/D_i(p_i^d)$ is the price-elasticity in market i. That is, the Lerner index in each market is inversely proportional to its elasticity of demand and the monopolist therefore sets a higher price in the market with the lower elasticity of demand. The quantity sold in market i is $q_i^d = D_i(p_i^d)$, $i = 1, 2$. The total output under price discrimination is $q^d = \sum_{i=1}^{2} q_i^d = \sum_{i=1}^{2} D_i(p_i^d)$. Given the first order conditions in (6), total output can be expressed as:

$$q^d = \sum_{i=1}^{2} D_i(p_i^d) = -\sum_{i=1}^{2}(p_i^d - c)D_i^{'}(p_i^d). \tag{7}$$

The change in the quantity sold in market i is given by $\Delta q_i = q_i^d - q_i^0$, $i = 1, 2$. We assume with no loss of generality that market 1 is the market with the lower elasticity of demand (the strong market) and market 2 the market with the higher elasticity (the weak market) . We have implicitly assumed that both markets are served under both price regimes so, given the strict concavity of the profit functions, then $p_1^d > p^0 > p_2^d$.[4] Therefore, price discrimination decreases the output in market 1 and increases output in market 2: $\Delta q_1 < 0$ and $\Delta q_2 > 0$. The effect of third-degree price discrimination on social welfare depends crucially on the change in the total output given by $\Delta q = q^d - q^0 = \Delta q_1 + \Delta q_2$. We next show that the demand curvature plays a relevant role in determining the effect on total output.

The Change in Total Output and Demand Curvature

Given conditions (2) and (7), the change in total output, $\Delta q = q^d - q^0$, is given by:

$$\Delta q = q^d - q^0 = -\sum_{i=1}^{2}(p_i^d - c)D_i^{'}(p_i^d) + \sum_{i=1}^{2}(p^0 - c)D_i^{'}(p^0). \tag{8}$$

[4]See in Nahata et al. (1990) the analysis when profit functions are not strictly concave.

We can write condition (8) as:

$$\Delta q=-\sum_{i=1}^{2}\left\{\int_{p^0}^{p_i^d} d\left[(p_i-c)D_i^{'}(p_i)\right]\right\}. \tag{9}$$

Therefore, we get:

$$\begin{aligned}\Delta q &= -\sum_{i=1}^{2}\left\{\int_{p^0}^{p_i^d}\left[D_i^{'}(p_i)+(p_i-c)D_i^{''}(p_i)\right]dp_i\right\},\\ &= -\sum_{i=1}^{2}\left\{\Delta q_i+\int_{p^0}^{p_i^d}(p_i-c)D_i^{''}(p_i)dp_i\right\},\\ &= -\frac{1}{2}\sum_{i=1}^{2}\left\{\int_{p^0}^{p_i^d}(p_i-c)D_i^{''}(p_i)dp_i\right\}.\end{aligned} \tag{10}$$

From (10) we obtain that the effect of third-degree price discrimination on total output depends on the demand curvature in each market.[5] If the demand in the lower elasticity market is strictly concave (strictly convex) and the demand in the higher elasticity market is strictly convex (strictly concave) then total output increases (decreases) with price discrimination. When all demands are linear output remains unchanged.

Welfare Effects

A move from uniform pricing to price discrimination generates a welfare change of:

$$\Delta W=\int_{q_1^0}^{q_1^d}\left[p_1(q)-c\right]dq+\int_{q_2^0}^{q_2^d}\left[p_2(q)-c\right]dq, \tag{11}$$

that is, the change in welfare is the sum across markets of the cumulative difference between price and marginal cost for each market between the output

[5] See, for example, Robinson (1933), Schmalensee (1981), Shih, Mai and Liu (1988), Cheung and Wang (1994), Cowan (2007) and Aguirre, Cowan and Vickers (2010).

under single pricing and the output under price discrimination.[6] As output decreases in the market with lower elasticity of demand and increases in the market with higher elasticity of demand, the first term in (11) is the welfare loss in market 1, whereas the second term is the welfare gain in market 2.[7] Figure 1 illustrates how the welfare effect of third-degree price discrimination is measured as the addition of the (negative) change in total surplus in market 1 and the (positive) change in total surplus in market 2.[8]

[6] We consider the case of quasilinear-utility function with an aggregate utility function of the form $\sum_{i=1}^{2}[u_i(q_i)+y_i]$, where q_i is consumption in submarket i and y_i is the amount to be spent on other consumption goods, $i = 1, 2$. It is assumed that $u_i^{'} > 0$ and $u_i^{''} < 0$, $i = 1, 2$.

[7] The overall effect on welfare may be positive or negative. See Aguirre, Cowan and Vickers (2010) for sufficient conditions based on the shape of the demand and inverse demand functions to determine the sign of the welfare effect.

[8] Ippolito (1980) and more recently Cowan (2011) analyze the effect of third-degree price discrimination on consumer surplus and find reasonable settings where the effect is positive. In a related paper Leeson and Sobel (2008) consider costly price discrimination. Note that if consumer surplus increases then social welfare would increase even though price discrimination costs offset the private incentive to price discriminate.

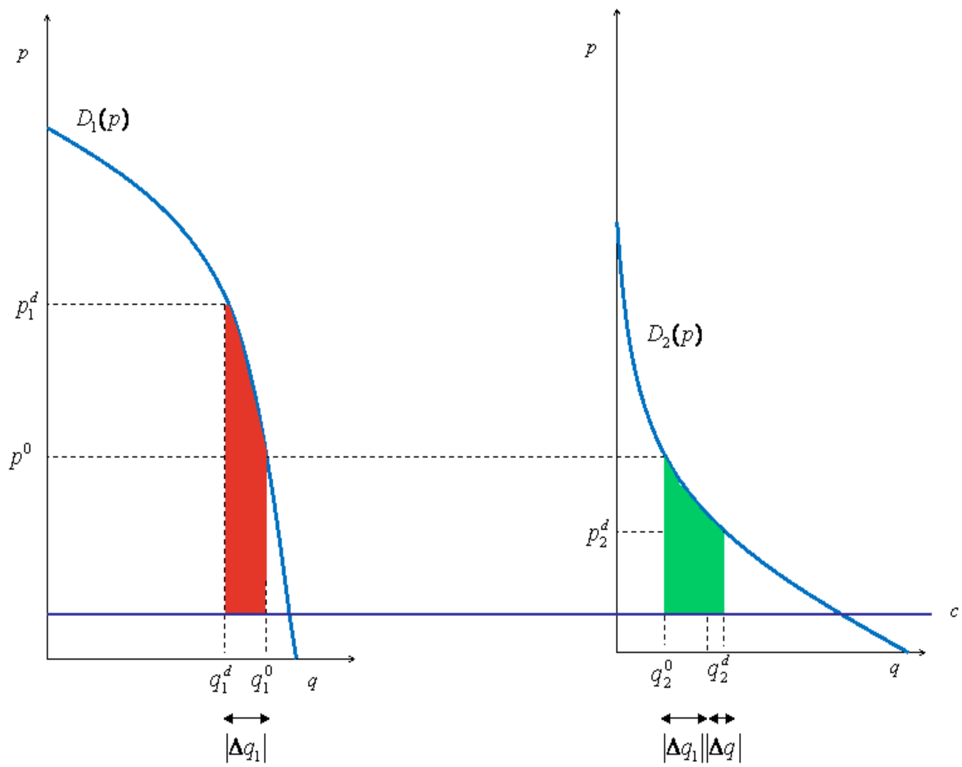

Figure 1. Welfare effects of third-degree price discrimination.

We want to break down the effect on social welfare into two effects: a misallocation effect, which can be interpreted as the welfare loss due to the transfer of q units of production from market 1 (the market with lower elasticity) to the market 2 (the market with higher elasticity), and an output effect, which can be interpreted as the effect of the change in total output on social welfare. Obviously, the effect of total output on social welfare crucially depends on whether third-degree price discrimination increases total output or not. Then we decompose the change in welfare into the two effects for the case where price discrimination increases total output.[9]

We assume that $\Delta q \geq 0$ and since the change in total output is given by

[9] The Appendix considers the case where price discrimination reduces total output.

$\Delta q = \Delta q_1 + \Delta q_2$, the change in output in market 2 is $\Delta q_2 = \Delta q - \Delta q_1 = \Delta q + |\Delta q_1| \geq 0$. We can express the change in welfare as:

$$\begin{aligned}
\Delta W &= \int_{q_1^0}^{q_1^d} p_1(q)dq + \int_{q_2^0}^{q_2^0 - \Delta q_1} p_2(q)dq \\
&\quad + \int_{q_2^0 - \Delta q_1}^{q_2^d} \left[p_2(q) - c\right] dq, \qquad (12)
\end{aligned}$$

Given that $q_i^d = q_i^0 + \Delta q_i$, $i = 1, 2$ we have:

$$\begin{aligned}
\Delta W &= \int_{q_1^0}^{q_1^0 + \Delta q_1} p_1(q)dq + \int_{q_2^0}^{q_2^0 - \Delta q_1} p_2(q)dq \\
&\quad + \int_{q_2^0 - \Delta q_1}^{q_2^d} \left[p_2(q) - c\right] dq, \qquad (13)
\end{aligned}$$

which under quasilinear utility $p_i(q_i) = u_i^{'}(q_i)$, $i = 1, 2$, becomes:

$$\begin{aligned}
\Delta W &= \int_{q_1^0}^{q_1^0 + \Delta q_1} u_1^{'}(q)dq + \int_{q_2^0}^{q_2^0 - \Delta q_1} u_2^{'}(q)dq \\
&\quad + \int_{q_2^0 - \Delta q_1}^{q_2^d} \left[u_2^{'}(q) - c\right] dq, \qquad (14)
\end{aligned}$$

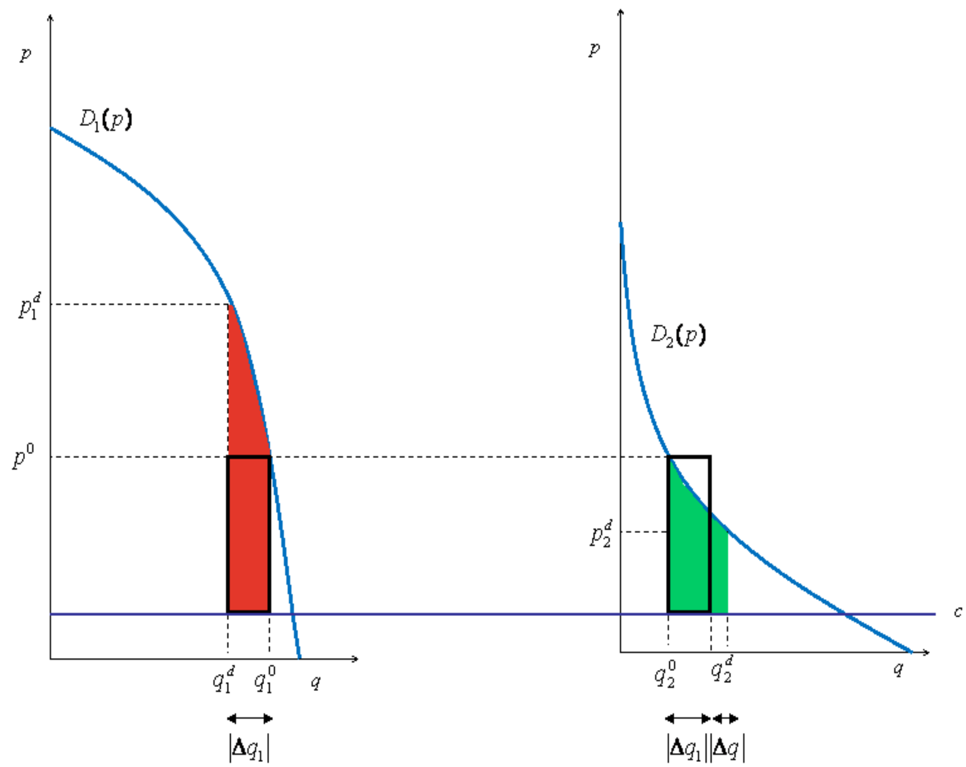

Figure 2. The addition and subtraction of $(p^0 - c)|\Delta q_1|$.

Taking into account that the optimal uniform price satisfies $p^0 = u_1'(q_1^0) = u_2'(q_2^0)$ and by adding and subtracting $(p^0 - c)\Delta q_1$, see Figure 2, we can express the change in welfare as:

$$\Delta W = ME + OE, \tag{15}$$

where the misallocation effect, ME, and the output effect, OE, when total output increases $\Delta q \geq 0$ are given by:

$$ME = \int_{q_1^0}^{q_1^0+\Delta q_1} [u_1'(q) - u_1'(q_1^0)]dq + \int_{q_2^0}^{q_2^0-\Delta q_1} [u_2'(q) - u_2'(q_2^0)]dq, \tag{16}$$

$$OE = \int_{q_2^0 - \Delta q_1}^{q_2^d} \left[u_2'(q) - c \right] dq. \tag{17}$$

The misallocation effect (16) can be written as $ME = -[u_1(q_1^0) - u_1(q_1^0 - |\Delta q_1|)] + [u_2(q_2^0 + |\Delta q_1|) - u_2(q_2^0)]$ and may therefore be interpreted as the welfare loss due to the transfer of $|\Delta q_1|$ units of production from market 1 to market 2. The output effect (17), OE, can be interpreted as the effect of additional output on social welfare. It is positive because the social valuation of the increase in output exceeds the marginal social cost. Figure 3 illustrates the output effect (the green area) and the misallocation effect (the red area).

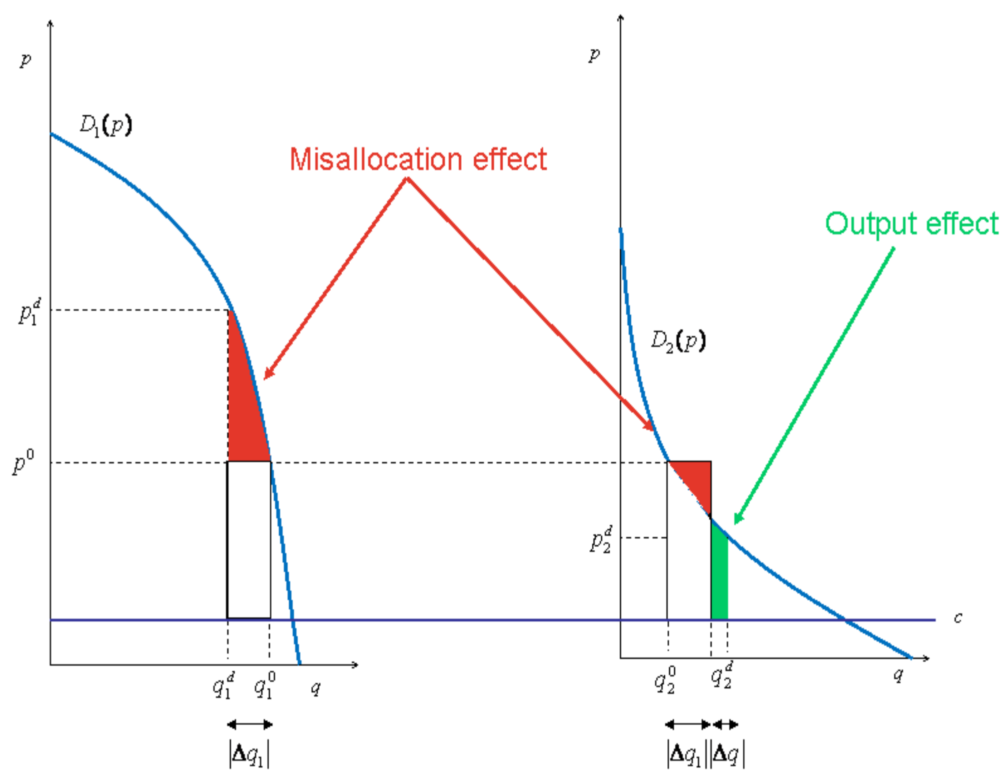

Figure 3. Output and Misallocation Effects.

Some important lessons can be drawn from the above analysis:
(i) *An increase in total output is a necessary* (but of course not sufficient) *condition for third-degree price discrimination to increase social welfare.* This conclusion is not based on exogenous bounds. Since the misallocation effect, (16), is always non-positive then a positive output effect (based on an increase in output) is needed to increase social welfare. In fact, that argument represents an earlier, easier and more intuitive demonstration of the theorem that an increase in output is a necessary condition for discrimination raises social welfare. Under linear demand, given that total output remains constant, social welfare is reduced by price discrimination.
(ii) *Market Opening.* In the above analysis we assume that both markets are served under both price regimes. We now analyze the case in which third-degree price discrimination serves to open markets; that is, we assume that market 2 is only served under third-degree price discrimination. Note that in this case $p_1^d = p^0 > p_2^d$ and therefore $\Delta q_1 = 0$ and $\Delta q_2 = q_2^d > 0$. Therefore, in this case price discrimination not only increases social welfare but also implies a Pareto improvement. Notice that the misallocation effect would be zero and the output effect would obviously be positive given that total output increases.
(iii) The use of linear demands is not appropiate for illustrating the welfare effects of third-degree price discrimination. Non-specialized readers might reach the conclusion that the only way for third-degree price discrimination to increase welfare is by opening markets. However, the change in welfare depends on two effects: a misallocation effect and an output effect. It is easy to construct examples where price discrimination increases social welfare but both markets are served.

Aguirre, Cowan and Vickers (2010) find sufficient conditions, based on the curvatures of direct and inverse demand functions for third-degree price discrimination to increase (or decrease) social welfare. Their main results are that welfare is higher with discrimination when inverse demand in the weak market is more convex than that in the strong market and the price difference with discrimination is small, and discrimination reduces welfare when the direct demand function is more convex in the high-price market.

Cowan (2011) shows that aggregate consumer surplus is higher with discrimination if the ratio of pass-through to the price elasticity (at the uniform price) is the same or larger in the weak market.[10] As an application he shows

[10] Pass-through is extensively analyzed by Weyl and Fabinger (2011) and shown to be a

that discrimination always increases surplus for *logit demand functions* whose pass-through rates exceed 0.5 (so demand is convex). Note that an increase in the consumer surplus ensures an increase in social welfare given that price discrimination increases profits (at least for a monopolist). Therefore, with this demand family results are just contrary to those under linear demand: the output effect always dominates the misallocation effect for logit demand functions (with pass-through rates exceeding 0.5).

The constant elasticity demand family is very appropriate for illustrating the tradeoff between the two effects given that as total output increases with discrimination (see, Ippolito, 1980, Aguirre, 2006, and Aguirre, Cowan and Vickers, 2010) output effect is positive. If both the share of the strong market under uniform pricing and the difference between demand elasticities are big enough then the output effect dominates to the misallocation effect (see, Aguirre, 2011).

(iv) Third-degree price discrimination is a topic covered by any microeconomics text book. However, there is a gap in the literature with respect to an appropriate graphical analysis of the effects on social welfare.[11] The above analysis fills this gap and provides a graphic treatment that is accessible for most readers and highlights the welfare effects of third-degree price discrimination.

We next compare our analysis with Schmalensee's and Varian's.

(v) Schmalensee (1981) in his graphical analysis decomposed social welfare into two effects:[12]

unifying concept which generalizes and simplifies the analysis of many industrial organization models.

[11] Graphical presentations of third-degree price discrimination typically focus on the comparison of the corresponding profit maximization problems. See for example Round and McIver (2006) and Weber and Pasche (2008) for recent analysis.

[12] In order to facilitate comparison we have rewritten his graphical analysis in terms of our notation.

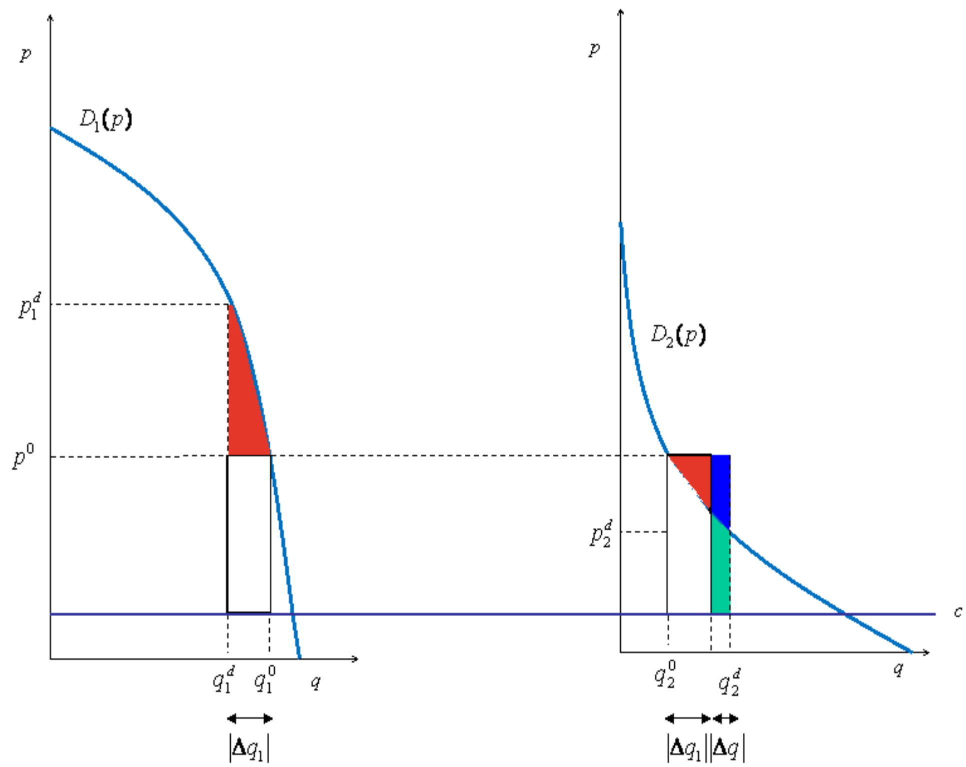

Figure 4. Schmalensee (1981)'s Output and Misallocation Effects.

$$
\begin{aligned}
\Delta W &= \int_{q_1^0}^{q_1^0+\Delta q_1} [u_1'(q) - u_1'(q_1^0)]dq + \int_{q_2^0}^{q_2^0+\Delta q_2} [u_2'(q) - u_2'(q_2^0)]dq \\
&\quad + \int_{q_2^0-\Delta q_1}^{q_2^d} \left[u_2'(q_2^0) - c\right] dq, \qquad (18)
\end{aligned}
$$

where the third term on the right hand in (18) may be equivalently written as $(p^0 - c)\Delta Q$. Schmalensee (1981) named the first two terms the (negative) distribution effect and the last term the output effect. In contrast, in our paper the output effect is the social valuation of an increase in output (that is the valuation of the consumers in the most elastic market). Figure 4 shows how in the Schmalensee's approach the distribution effect and the output effect are overstated and overlapped. His ouput effect (the green area plus the blue area), $(p^0 - c)\Delta Q$, exaggerates the social valuation of the increase in total output. It is more reasonable to define the output effect as the valuation of the additional output by the elastic market consumers (that is, those consumers enjoying the increase in output); i.e. the green area. On the other hand, the Schmalensee's distribution effect overestimates the negative effect of distributional inefficiency: the red areas plus the (negative) blue area. Our approach presents two advantages: first, it allows to interpret the misallocation effect as the welfare loss due to the transfer of $|\Delta q_1|$ units of production from consumers in market 1 to consumers in market 2 and, second, it identifies the output effect by stating the social valuation of an increase in output as the increase in total surplus of consumer in market 2, the most elastic demand market.

(vi) Varian (1985) obtained upper and lower bounds on the change in welfare when moving from uniform pricing to third-degree price discrimination. By using the property of concavity of the utility function the bounds on welfare change are:

$$(p^0 - c)\Delta Q \geq \Delta W \geq (p_1^d - c)\Delta q_1 + (p_2^d - c)\Delta q_2. \tag{19}$$

or in terms of marginal willingness to pay:

$$\int_{q_2^0-\Delta q_1}^{q_2^d} \left[u_2^{'}(q_2^0) - c\right] dq \geq \Delta W \geq \int_{q_1^0}^{q_1^d} \left[u_1^{'}(q_1^d) - c\right] dq + \int_{q_2^0}^{q_2^d} \left[u_2^{'}(q_2^d) - c\right] dq. \tag{20}$$

The upper bound provides a necessary condition for price discrimination to increase social welfare (that is, an increase in output) and the lower bound a sufficient condition. Our approach presents some advantages over Varian's. One crucial advantage relates to the graphical analysis: his graphic treatment goes not very further from the one market case as it appears in Varian (1992)'s text book. On the other hand, the bounds are not very informative. Consider for example the two cases used by Varian (1992) to illustrate the bounds: (i) linear demands and (ii) market opening. In both cases, our approach allows to compute exactly the welfare change. (i) Under linear demands (20) becomes:

$$0 > \Delta W > \int_{q_1^0}^{q_1^d} \left[u_1^{'}(q_1^d) - u_2^{'}(q_2^d) \right] dq_1. \tag{21}$$

and however our analysis states, from (15), (16) and (17), that:

$$\Delta W = ME = \int_{q_1^0}^{q_1^0+\Delta q_1} [u_1^{'}(q) - u_1^{'}(q_1^0)]dq + \int_{q_2^0}^{q_2^0-\Delta q_1} [u_2^{'}(q) - u_2^{'}(q_2^0)]dq < 0. \tag{22}$$

On the other hand, when uniform pricing serves to open markets ($p_1^d = p^0 > p_2^d$, $\Delta q_1 = 0$ and $\Delta q_2 = q_2^d > 0$) the bounds on social welfare are:

$$\int_{q_2^0-\Delta q_1}^{q_2^d} \left[u_2^{'}(q_2^0) - c \right] dq \geq \Delta W \geq \int_{q_2^0}^{q_2^d} \left[u_2^{'}(q_2^d) - c \right] dq. \tag{23}$$

while the change in social welfare is given by:

$$\Delta W = OE = \int_{q_2^0-\Delta q_1}^{q_2^d} \left[u_2^{'}(q) - c \right] dq. \tag{24}$$

III. CONCLUDING REMARKS

Based on a pioneering paper by Ippolito (1980) we construct a simple model which allows the welfare effects of third-degree price discrimination to be well understood and explained. The decomposition of the change in welfare into a misallocation effect and an output effect has advantages over the well-established analyses by Schmalensee (1981) and Varian (1985). In particular, our approach provides a graphic analysis which clarifies the welfare analysis of third-degree price discrimination.

APPENDIX

Here we decompose the change in welfare into two effects for cases where third-degree price discrimination does not increase total output. When total output does not increase $\Delta q \leq 0$ it is more illustrative to express the change in welfare as:

$$\begin{aligned} \Delta W &= \int_{q_1^0}^{q_1^0-\Delta q_2} p_1(q)dq + \int_{q_2^0}^{q_2^d} p_2(q)dq \\ &\quad + \int_{q_1^0-\Delta q_2}^{q_1^d} [p_1(q) - c]dq, \end{aligned} \tag{25}$$

and

$$\begin{aligned} \Delta W &= \int_{q_1^0}^{q_1^0-\Delta q_2} p_1(q)dq + \int_{q_2^0}^{q_2^0+\Delta q_2} p_2(q)dq \\ &\quad + \int_{q_1^0-\Delta q_2}^{q_1^d} [p_1(q) - c]dq, \end{aligned} \tag{26}$$

and

$$\begin{aligned} \Delta W &= \int_{q_1^0}^{q_1^0-\Delta q_2} u_1^{'}(q) + \int_{q_2^0}^{q_2^0+\Delta q_2} u_2^{'}(q)dq \\ &\quad + \int_{q_1^0-\Delta q_2}^{q_1^d} [u_1^{'}(q) - c]dq. \end{aligned} \tag{27}$$

By adding and subtracting $(p^0 - c)\Delta q_2$ the misallocation and the output effects can be expressed as follows:

$$ME = \int_{q_1^0}^{q_1^0-\Delta q_2} [u_1^{'}(q) - u_1^{'}(q_1^0)]dq + \int_{q_2^0}^{q_2^0+\Delta q_2} [u_2^{'}(q) - u_2^{'}(q_2^0)]dq, \tag{28}$$

$$OE = \int_{q_1^0 - \Delta q_2}^{q_1^d} [u_1^{'}(q) - c] dq. \tag{29}$$

The misallocation effect can be written as $ME = -[u_1(q_1^0) - u_1(q_1^0 - \Delta q_2)] + [u_2(q_2^0 + \Delta q_2) - u_2(q_2^0)]$ and may therefore be interpreted as the welfare loss due to the transfer of Δq_2 units of production from market 1 to market 2. The output effect, OE, can be interpreted as the effect of the reduction in output on social welfare. It is negative because the social valuation of the increase in output exceeds the marginal social cost.

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