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Assessment of the operational flexibility of virtual power plants to facilitate the integration of distributed energy resources and decision-making under uncertainty

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ABSTRACT

Distributed energy resources (DERs) are elements that actively participate in the supply of renewable energy and contribute to the decarbonization of the power system. However, they lack two factors necessary to take advantage of their operational flexibility: observability and controllability. In this sense, Virtual Power Plants (VPPs) are a feasible alternative to provide the necessary requirements for the optimal management of a set of distributed units. Therefore, knowledge of the technical and energy characteristics of each unit that makes up the VPP is a necessary condition for the effective integration of DERs into the power system. This paper proposes a methodology to graphically represent, quantify and exploit the aggregate operational flexibility of a set of units. The proposed methodology is based on five metrics related to active and reactive power, which serve as a tool to facilitate the VPP Operator's decision-making under uncertainty. Consequently, achieving the coordinated operation of several distributed units makes it possible to achieve common objectives. For instance, frequency and voltage regulation, compliance with a planned power curve, or dealing with the variability of renewable energies. The proposal is applied to a theoretical case study and through real operational tests between a hydroelectric unit and a photovoltaic plant. Finally, it is shown that the results obtained are a useful tool in real-time.

1. Introduction

1.1. Justification and scope of research

Operational flexibility can be defined as the ability of the system to provide a power balance between supply and demand, and also to maintain service continuity in the event of contingencies [1,2]. Operational flexibility has commonly been provided by large power plants, geographically distant from the end consumer. Thus, operational decision-making considers two uncertainties: demand variability and energy price. However, the expansion of generation through DERs and renewable energies introduces variable and random characteristics of the energy resource in the system. These characteristics make generation availability an additional uncertainty for the Transmission system operator (TSO).

All these factors have led to a review of the concept of operational flexibility [3] and organizations such as the International Energy Agency (IEA) have emphasized the importance of proposing new assessment methodologies [1]. Currently, it could be defined as the ability of the system to cope with the uncertainty of generation and demand, maintaining reliability at a reasonable cost [4,5]. These concepts derive the interest in the study of operational flexibility in four research areas [6]: First: Analysis of the different resources, such as generation, storage, and demand management technologies. Second: Technical assessment of the operational flexibility of a system. Third: Analysis and design of new electricity markets for the supply of ancillary services and, finally: Regulatory aspects for exploiting the operational flexibility services of the systems. This work is focused on the second area of research.

From a technical point of view, the management of operational flexibility in a system requires two very important factors: observability and controllability. "Observability" refers to the power system operator's ability to monitor and obtain detailed information about the behavior and status of the power units in real-time. "Controllability"

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Nomenclature

CCP	Common connection point
DER	Distributed energy resource
DSO	Distribution system operator
TSO	Transmission system operator
VPP	Virtual power plant

refers to the power system operator's ability to effectively control and adjust the power generation units in real-time. These factors are fundamental requirements to ensure the stability, reliability, and efficiency of power system operation. Additionally, these terms are closely related to "dispatchability," which refers to the ability of a power generation unit to be controlled and scheduled to meet the specific needs of the system in terms of power production and operation. Therefore, the capacity of a unit to modulate power dispatch over time depends on the system operator's ability to manage and control each technology and thus take advantage of the generator's flexibility [7]. This means, the technical capability of an operator to modulate the power flow at specific grid nodes and maintain the balance between supply and demand [8]. According to this intrinsic dependence on observability and controllability, there are three categories to classify operational flexibility resources in the system: *A. Actual flexibility resources:* Considers the units that the TSO uses in real-time for power modulation. It includes traditional units that have observability and controllability, such as hydroelectric power plants or thermal plants. *B. Operational flexibility reserves:* This is a part of the real flexibility resources. However, due to technical or economic considerations, are held in reserve to be used in the future or as backup against contingencies. *C. Potential operational flexibility resources:* Comprises those units that participate in the operation of the system but lack observability or controllability. Consequently, the resource exists, but an appropriate mechanism is needed to provide services of flexibility. This group includes the DERs and the scope of this research.

Therefore, the power system requires a mechanism that facilitates the integration of DERs and allows the necessary management to take advantage of potential flexibility resources. In this sense, the scientific literature proposes the concept of virtual power plants [9–11]. VPP aggregates and manages multiple DERs, giving the System Operator observability and control of a set of distributed elements [12]. In this way, through the VPP, the exploitation of the operational flexibility of the distributed units is facilitated. In addition, the VPP allows coordination of the operation of each element according to the availability of the energy resource, the technical availability, the associated operating costs, and the system requirements [13]. Similarly, the VPP is capable of managing different technologies located in various geographical areas [14] and contributes to the complementarity between units, covering the energy resource deficit of one technology with another DER. This management mechanism makes it possible to counteract uncertainty and optimize the operational flexibility of a set of distributed elements [15]. In this context, the VPP Operator becomes an integrating agent between a group of DERs and the TSO or Distribution System Operator (DSO) with the responsibility to dispatch DERs at the lowest cost and within the safety limits of the system. The coordination between power system participants (VPP Operator, TSO, and DSO) is a fundamental aspect of DERs integration. The paper [15] provides details on both the architectures and coordination schemes identified for this purpose. Finally, Fig. 1 shows the concept of VPP as the virtual interconnection of several distributed units through communication protocols (controllability) and their dispatch coordination with the system operator (observability).

1.2. Previous studies

Knowledge of the operational flexibility of large power plants is a necessary condition to guarantee the operation and safety of the system. However, with the expansion of generation through renewable energies and DERs, there is a set of potential operational flexibility resources that

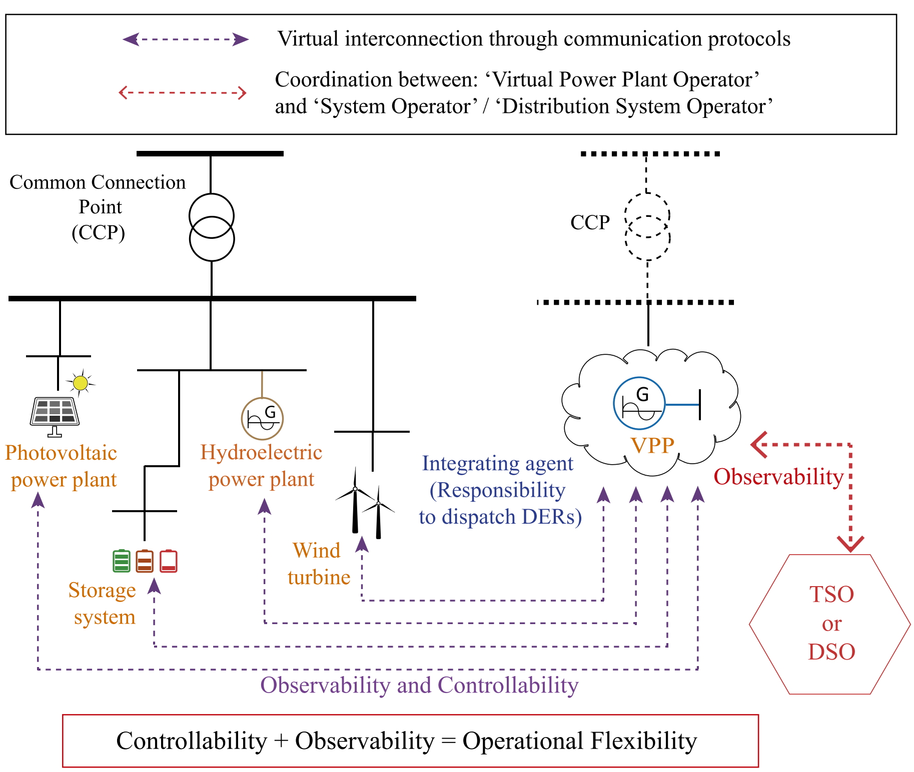

Fig. 1. Operational flexibility of a set of DERs through a VPP.

are not being exploited. Indeed, the knowledge and assessment of the operational flexibility in distributed units is also a necessary condition for the integration of DERs in the power system [16]. Flexibility is often expressed in terms of active power because it is an essential requirement for the operation of the power system; however, considering active power as the only metric to assess operational flexibility could be insufficient. For this reason, new alternatives have been proposed for its analysis. The Insufficient Ramping Resource Expectation (IRRE) metric is proposed in [17] to measure power system flexibility for use in long-term planning. To assess the operational flexibility in the system and allow the integration of renewable energies, in [18] three measurement metrics are proposed (active power, energy reserve, and a power ramp) for a more accurate and complete representation of operational flexibility. This assessment methodology was used in [8] to obtain a flexibility volume and plot the three flexibility metrics on a three-dimensional Cartesian plane. The graphical tool obtained allows an understanding of the variability of technical parameters over time and facilitates decision-making during the dispatch of units. These works are focused on the management of large generation plants that have observability and control capacity. However, in distribution grids, generation management is not common, so their technical flexibility cannot be exploited. Therefore, the VPP concept plays an important role in increasing the operational flexibility of DERs.

The operational flexibility of VPPs is usually estimated and represented in terms of active and reactive power using aggregation techniques, but they do not consider ramp capacity or energy reserves. According to the literature, there are two fundamental methodologies to estimate this capability diagram: Monte Carlo estimation [19–21] and optimization methods [22–24]. The Monte Carlo estimation describes the feasible operating area of the VPP and calculates the time required to modulate power from one operating point to another [19]. These procedures are very useful for assessing operational flexibility; however, a large number of scenarios are required to obtain reliable results. This means very long calculation times. On the other hand, optimization methods involve the modeling of the AC network, considering the system constraints in each time-step. The results obtained depend on the complexity and size of the modeled system, the more complex the network, the greater the computational effort. In addition, long computation times are required, and the optimization results could lose precision due to the quadratic equations inherent in the mathematical model.

In summary, the literature review indicates a limited number of studies dedicated to the assessment of the operational flexibility of VPPs, including metrics encompassing capacity, energy reserves, and power ramp. While existing research on operational flexibility presents promising results, their practical application may be impeded by time constraints, particularly in ancillary services or real-time markets. Furthermore, there is a dearth of studies examining the utilization of ramp and reactive power in DERs. Similarly, to the best of the authors' knowledge, no studies have been conducted to evaluate the reactive power flexibility in VPPs. All of these identified gaps are considered essential for the technical integration into the electric power system.

In conclusion, to optimize the hourly power offer, provide the ancillary services, and manage its reserves, the VPP Operator requires knowledge of the operational flexibility of the set of DERs. This means making decisions in near real-time and under conditions of uncertainty. In this sense, there is a need for a methodology that assesses the stochastic behavior of a set of DERs and obtains immediate results on the operational flexibility of the units. Therefore, obtaining a strategy to assess operational flexibility in a simple and fast way, facilitates decision-making about the dispatch of the VPP in real-time. In addition, it minimizes uncertainties and takes advantage of the energy complementarity between the different technologies.

1.3. Novelty and contributions

The hypothesis of this research raises the following fact: Knowledge of the technical and energy availability of DERs and the possibility of managing these resources would increase the operational flexibility of the power system and facilitate decision-making under uncertain conditions during the operation of a VPP.

This paper has integrated the concepts of operational flexibility in power systems, and the fundamental concepts of VPPs to present a methodological proposal that facilitates decision-making during the management and dispatch of the active and reactive power of a set of DERs associated with a VPP. This work presents the following contributions:

- A methodological proposal is presented to assess and graphically represent the operational flexibility of a VPP with several common connection points to the distribution network.
- This methodological proposal makes it possible to obtain information on the available operational flexibility of a set of DERs.
- The results of this work have presented an alternative solution to the gaps identified in the literature. In addition, different applications to exploit the proposed methodology have been described, with the potential to shape future research and strengthen the alternatives for the integration of DERs.

The rest of the paper is organized as follows. Section 2 presents an analysis of the theoretical concepts to assess and represent the operational flexibility of the power system. Section 3 proposes a methodology to assess and graphically represent the aggregate operational flexibility of a VPP, which serves as a tool to facilitate decision-making during the dispatch of DERs. Section 4 presents the results of a theoretical case study and through actual operational tests between a hydroelectric unit and a photovoltaic plant. Section 5 presents the discussion of results. Section 6 contains the conclusions of this work.

2. Assessment of operational flexibility in the power system

The complexity of the operation and management of a power system is defined by its stochastic characteristics over time and by the level of uncertainty of its variables. Operational flexibility is the ability of the system to respond to these stochastic variations and cope with uncertainty in real-time. Therefore, the operational flexibility of a unit always depends on the time scale [25], for example, speed of response to frequency and voltage variations, power ramp rate, limits to modulate dispatch and, energy reserves for future use. In any case, the time factor is a necessary condition for assessing the operational flexibility of the system.

From a technical perspective, operational flexibility needs can be summarized in four categories [26]:

- Active power modulation requirement: It is responsible for the real-time balance between generation and demand. It maintains frequency stability and its time scale varies from fractions of a second to several minutes.
- Reactive power modulation requirement: It is necessary to maintain voltage levels within predefined limits in a specific node of the network. Therefore, it is a requirement of local or regional flexibility and its time scale varies from seconds to several minutes.
- Energy requirements: Establishes the balance between generation and demand in medium and long-term dispatch schedules. Its time scale varies from hours to months.
- Transfer capacity: It is the flexibility of the system to transfer the scheduled dispatch from one unit to another. This strategy is required in cases of energy deficit due to forecasting errors, technical failures, or economic conditions. Its time scale varies from minutes to hours. The transfer capacity can be used by the VPP Operator to take

advantage of energy complementarity between different DERs and meet a scheduled dispatch curve.

For analysis purposes, the operational flexibility of a DER should be characterized and categorized by appropriate metrics related to active and reactive power. However, each of these parameters has intrinsic characteristics related to the time scale. From these particularities, five metrics are proposed to assess operational flexibility in a more precise way.

In terms of active power, three specific metrics are used to assess operational flexibility [2,8,18,27,28]:

- Range of active power capacity (π), measured in [MW]. This metric could take negative values if storage units, such as batteries, are involved.
- Modulation speed or active power ramp rate (ρ), measured in [MW/min]. The slope of this ramp can be upward ($+$) or downward ($-$), depending on the modulation requirement.
- Energy supply (ε), measured in [MW·h]. This metric evaluates how long the unit could operate at constant power for a defined time. This metric could take negative values if they are storage units, such as batteries.

On the other hand, reactive power modulation requirements are of great importance at a local scale, where voltage control is required at specific nodes of the network [27]. For these cases, two new metrics is proposed:

- Range of reactive power capacity (φ), measured in [MVAr]. This metric could take positive or negative values, depending on the technology of each unit.
- Modulation speed or reactive power ramp rate (ψ), measured in [MVAr/min]. The slope of this ramp can be upward ($+$) or downward ($-$), depending on the modulation requirement.

As mentioned by Ulbig et al. in [8], an important feature is that the ramp, power, and energy metrics are directly linked through integration and differentiation operations in the time domain: energy is the integral of power, which in turn is the integral of the power ramp rate:

$$Energy \rightleftarrows^{d/dt}_{\int dt} Power \rightleftarrows^{d/dt}_{\int dt} Ramp - Rate\varepsilon = \left(\frac{1}{2}\rho\cdot t^2\right) \rightleftarrows^{d/dt}_{\int dt} \pi = (\rho\cdot t) \rightleftarrows^{d/dt}_{\int dt} \rho\varphi = (\psi\cdot t) \rightleftarrows^{d/dt}_{\int dt} \psi \quad (1)$$

The five metrics analyzed are closely related. On the one hand, the temporal co-dependency relationship according to Eq. (1), and on the other hand, the technical relationship between the active power and the reactive power of each DER, according to its exclusive capacity curve.

The graphical representation of the operational flexibility metrics is shown in Fig. 2.

The five metrics considered ($\pi,\rho,\varepsilon,\varphi,\psi$), allow for the comprehensive assessment of the operational flexibility of the unit and the power system. Through these metrics, the operator can assess compliance with various flexibility services. For example, monitoring of a scheduled dispatch curve, regulation of frequency and voltage, and energy reserves. In addition, knowledge of these metrics allows dispatch to be optimized through the power transfer capacity between various units.

On the other hand, the metrics established to evaluate the operational flexibility of a generation unit are estimated through energy resource forecasts. In addition, it is necessary to know the technical limitations of the unit or system constraints. The estimated values can be plotted on two or three-dimensional Cartesian planes to delimit flexibility planes and volumes. The vertices or extreme points of each figure represent the maximum available values of ($\pi^{\pm},\rho^{\pm},\varepsilon^{\pm},\varphi^{\pm},\psi^{\pm}$), as shown in Fig. 3.

3. Methodology to assess the operational flexibility of a virtual power plant

3.1. Methodological proposal for the assessment and of operational flexibility of VPPs

DERs are active elements in distribution networks that represent a permanent challenge during the management and operation of the power system. DERs can be of any technology, although they are generally related to medium and small-capacity renewable energies and storage systems. It is usual that, due to their variable and random characteristics and their reduced capacity compared to the entire power system, they are unobservable by the TSO [29]. Therefore, there are limitations to participating in the electricity markets individually and dynamically. One strategy for integrating DERs into the electrical system is through VPPs, which is based on the ’cloud’ concept to aggregate and control a set of DERs as a single virtual element associated with the power system [15]. A fundamental aspect of the VPP Operator is the ability to communicate with each distributed element. The VPP Operator has real-time access to the operation of each DER, thus having the capacity to control and manage its operation. On the other hand, the VPP plays the role of integrating agents with the TSO, providing the observability of DERs to participate dynamically in the electricity market. Such markets include the day-ahead (DAM), real-time (RTM), ancillary service (ASM), and futures (FM) markets.

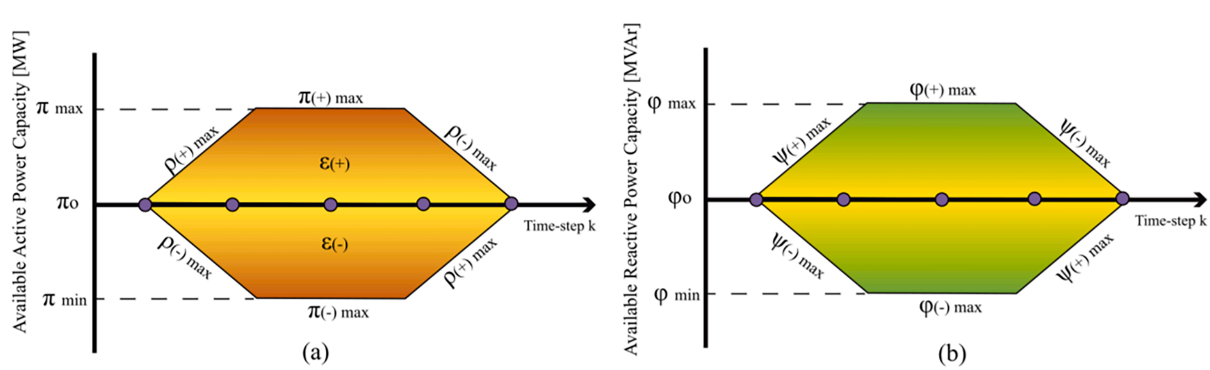

Fig. 2. Operational flexibility metrics: (a) Active power: (π,ρ,ε), (b) Reactive power: (φ,ψ).

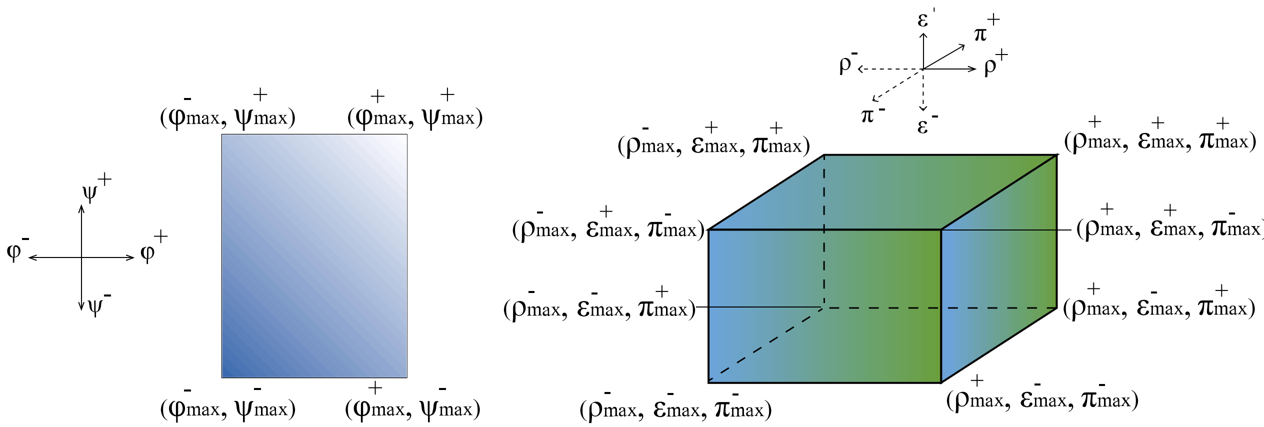

Fig. 3. Graphical representation of the plane and volume of operational flexibility of a generation unit.

The evaluation of the operational flexibility of the VPP consists of calculating the total aggregation of a set of units and comparing it with the system requirements at a specific instant of time. The proposed methodology is based on four steps:

i. Step one. Analysis of flexibility for each unit: Technical and energy analysis of each DER and obtainment of a probabilistic valorization of occurrence.
ii. Step two. Analysis of aggregated flexibility: Estimation of the *available operational flexibility* of the set of DERs. Grouping the

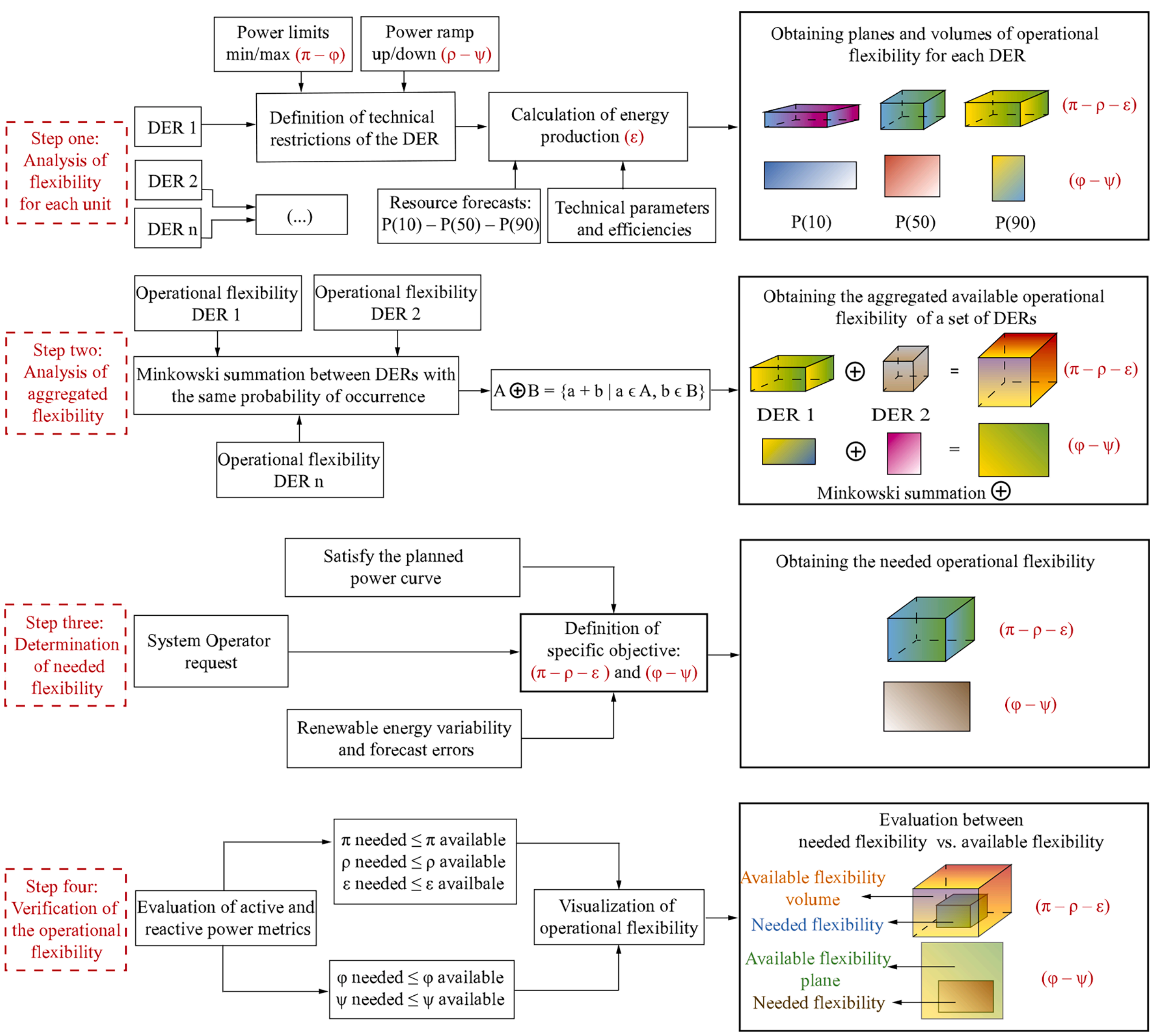

Fig. 4. Scheme of the methodology to assess and graphically represent the operational flexibility.

results with the same probability of occurrence and calculation of the aggregate operational flexibility of the set.

iii. Step three. Determination of needed flexibility: Evaluate the specific objective required by the system and estimate the *needed operational flexibility* to achieve it.

iv. Step four. Verification of the operational flexibility: Evaluate and compare the available operational flexibility with the needed operational flexibility by the system.

Through the results obtained in the proposed methodology, the VPP Operator acquires an effective tool to facilitate decision-making during the dispatch and management of the set of DERs. The general scheme of this methodology is shown in Fig. 4, while the technical details of the calculation are explained in the following section.

3.2. Assessment of the operational flexibility of a VPP

Flexibility qualities of each DER can be aggregated and combined among different technologies to counteract their limitations and achieve common objectives. For example, compliance with the dispatch of a programmed power curve, supply of ancillary services, or optimizing the technical and economic benefits of the whole set of units. In effect, the VPP intends to provide an additional service that the DERs could not individually provide. The four steps of the methodological proposal for assessing operational flexibility in a VPP are described below.

i) Step one. Analysis of flexibility for each unit:

The metrics proposed to evaluate the flexibility of each unit (π,ρ,ε,φ, ψ), are used to identify operational flexibility planes and volumes for each DER. This requires knowledge of the technical parameters and specific constraints of each technology. In addition, the Common Connection Points (CCPs) of the VPP must be very well-defined to efficiently manage the available resources, above all, management related to reactive power (φ,ψ).

Because the metrics considered for the evaluation of the available flexibility are based on the forecasts of the energy resource, the flexibility planes and volumes associated with a probability of occurrence can be calculated. For example, the river flow of a hydroelectric power plant can be estimated with a probability of certainty of 10, 50, or 90 % (or any other percentage). Therefore, the volume of operational flexibility of this unit is also probabilistic. This facilitates the VPP Operator to make decisions during dispatch scheduling. In other words, planes, and volumes with a certain probability of occurrence could be obtained, thus reducing the uncertainty on the generation side.

ii) Step two. Analysis of aggregated flexibility:

The operational flexibility of a set of DERs can be obtained by means of the combination of the operational flexibility of individual DERs, through a mathematical procedure known as 'Minkowski Summation'. This procedure states that given two sets of cartesian coordinates: A y B, their Minkowski summation, denoted by $A \oplus B$, is their point-wise sum [30], according to the following equation:

$$A \oplus B = \{a + b | a \in A; b \in B\} \tag{2}$$

To illustrate it, a two-dimension example is given as shown in Fig. 5. Consider the Minkowski Sum of the following two polygons:

Through this mathematical tool, the aggregate flexibility of a set of DERs can be calculated. Fig. 6 shows these concepts applied to a VPP with two CCPs.

The proposal schematized in Fig. 6 allows for evaluating the operational flexibility of a VPP with several CCPs, through the volumes of flexibility ($\pi^{\pm}$, $\rho^{\pm}$, $\varepsilon^{\pm}$). In addition, if a similar procedure is applied, planes of flexibility related to reactive power would be obtained ($\varphi^{\pm}$, $\psi^{\pm}$). In this way, the VPP Operator acquires information on the five technical parameters available in each section of the system, and it determines the available operational flexibility in the VPP. For example, for frequency control, the aggregated volume of flexibility by all the units associated with distribution networks 1 and 2 can be used. Whereas, for voltage control at CCP 1, only the plane of operational flexibility available with the associated DERs in distribution network 1, would be evaluated.

iii) Step three. Determination of needed flexibility:

The requirements of π,ρ,ε,φ, and ψ by the TSO for each time-period are found. This means, the request for active and reactive power modulation, to contribute to system balancing through frequency regulation or voltage regulation at a specific node of the network. On the other hand, the VPP Operator can also define the necessary flexibility requirements, for example, by evaluating the parameters needed to ensure compliance with a scheduled power curve. In this way, the VPP Operator can manage its operation by power transfer between units to cope with the variability of renewable energies or reduce impacts due to forecast errors. Consequently, the same process used to represent the available flexibility graphically can be used, this time, to describe the operational flexibility needed graphically.

iv) Step four. Verification of the operational flexibility:

The stochastic behavior of renewable energies and the dynamic interaction between DERs and the power system are important features during VPPs operation. Therefore, the planes and volumes of operational flexibility are also variable over time. This implies that operational flexibility must be constantly evaluated for each time-step to facilitate decision-making and guarantee the achievement of objectives. To evaluate the needed flexibility and the available flexibility of the VPP, the mathematical conditions that always must be met are:

$$\pi_{Needed} \leq \pi_{Available} \tag{3}$$

$$\rho_{Needed} \leq \rho_{Available} \tag{4}$$

$$\varepsilon_{Needed} \leq \varepsilon_{Available} \tag{5}$$

$$\varphi_{Needed} \leq \varphi_{Available} \tag{6}$$

$$\psi_{Needed} \leq \psi_{Available} \tag{7}$$

Finally, the VPP Operator proceeds to compare graphically both the plane ($\varphi^{\pm}$,$\psi^{\pm}$) and the volume ($\pi^{\pm}$,$\rho^{\pm}$,$\varepsilon^{\pm}$) of available flexibility, with the plane and volume of needed flexibility, as shown in Fig. 7.

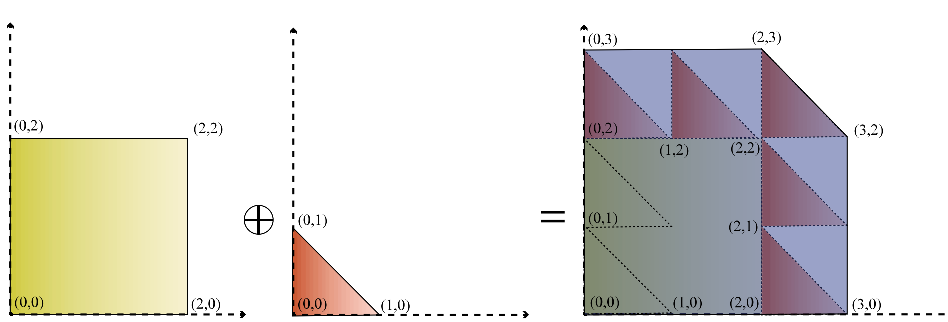

Fig. 5. Illustration of Minkowski Summation.

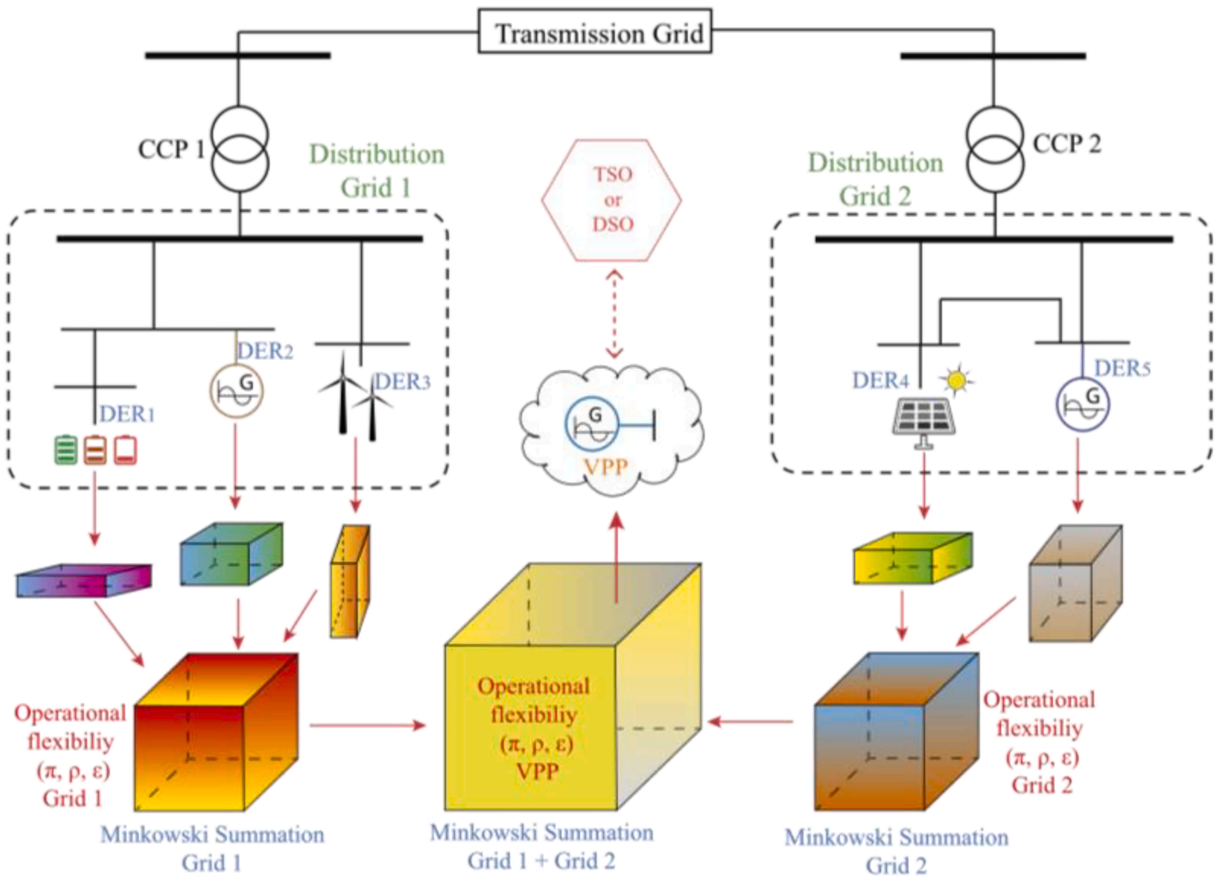

Fig. 6. Aggregated operational flexibility through a VPP.

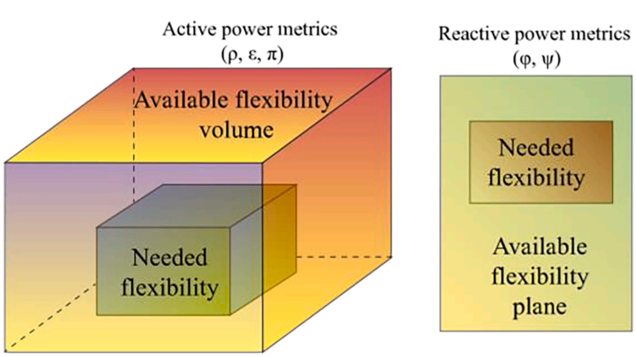

Fig. 7. Assessment between flexibility needed vs flexibility available.

Through this visual tool, the VPP Operator evaluates compliance with the requirements solicited by the TSO. This means that the VPP has sufficient resources only if its plane and volume of available operational flexibility fully enclose the area and volume of needed flexibility. If any side of the needed flexibility figures is out of phase or protrudes from the figures of available flexibility, the requirements expressed in any of Eqs. (3)–(7) would not be met. In this case, the VPP Operator should decide to reschedule the dispatch of its units or simply set limits for the supply of its services.

In conclusion, the results obtained can be calculated almost immediately, providing the VPP Operator with a real-time tool and simple interpretation that facilitates decision-making during service offerings, and in dispatch programming of the DERs that make up the VPP.

4. Application of the proposal to assess the flexibility of a VPP

To apply the methodology proposed in Section 3 of this paper, two types of analysis have been developed. First, a theoretical case study is presented to evaluate the operational flexibility of a VPP with two CCPs and four technologies. Second, practical tests were performed on real-generation units. Operational testing allowed evaluation of the available flexibility of $\pi^{\pm}, \rho^{\pm}, \varepsilon^{\pm}$. In addition, this analysis verified the power transfer capability between DERs and active power modulation to compensate for the production variability of a PV plant.

4.1. Application of the methodology in a theoretical case study

The methodology proposed in this paper is applicable in systems with a large number of DERs associated with the VPP. In fact, the results obtained serve as preliminary information during the dispatch optimization calculations of a set of units. However, for illustrative purposes of the methodology, this theoretical case study evaluates the operational flexibility of the VPP shown in Fig. 8.

VPP of the study case is made up as follows:

- Distribution grid 1 (CCP1): A subsystem with 7 MW of installed power (5 MW photovoltaic and 2 MW storage).
- Distribution grid 2 (CCP2): A subsystem with 13 MW of installed power (10 MW hydro and 3 MW wind).

In the initial period 't0' the VPP supplies the system with the following power values:

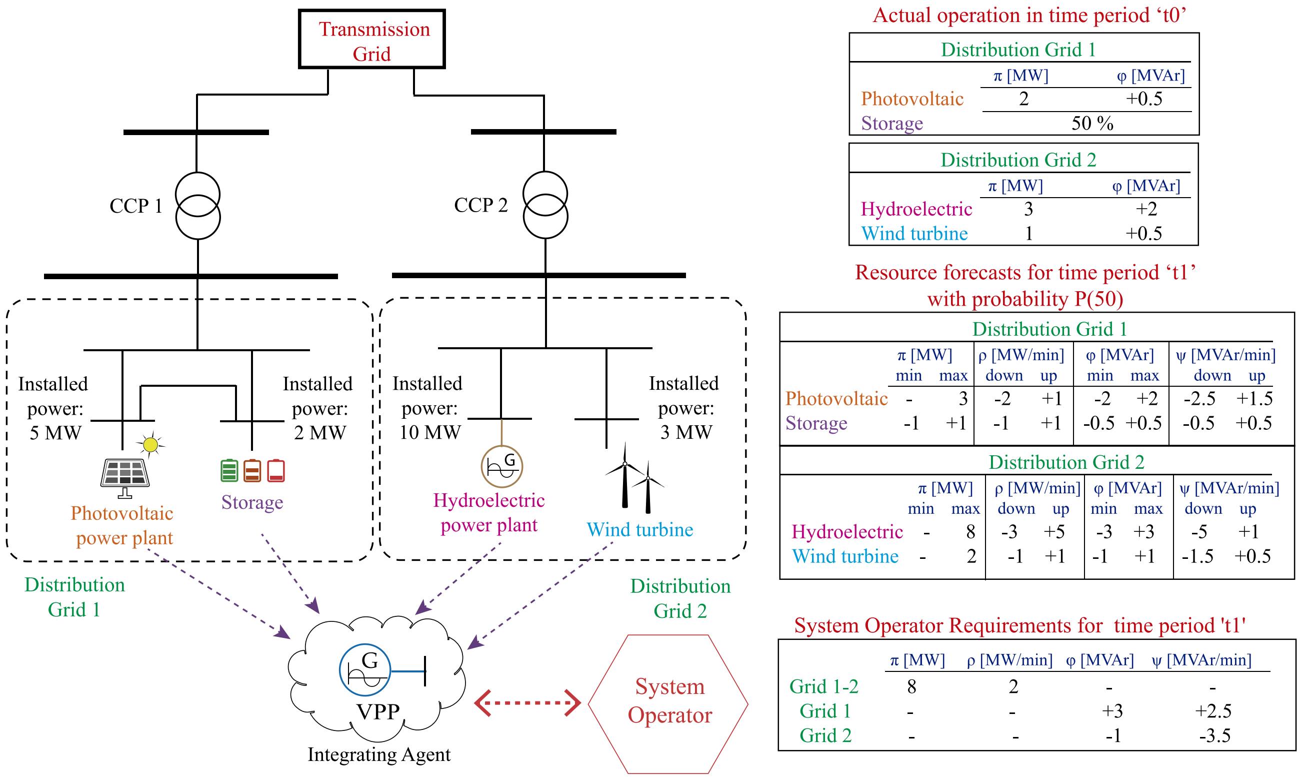

Fig. 8. Theoretical case study to apply the proposed methodology.

- Distribution grid 1 (CCP1): Supplies power of 2 MW and + 0.5 MVAr (photovoltaic power plant). While the storage system maintains a 50 % load.
- Distribution grid 2 (CCP2): Supplies power of 4 MW and + 2.5 MVAr (Hydroelectric: 3 MW, +2 MVAr and Wind turbine: 1 MW, +0.5 MVAr).

The evaluation of the operational flexibility of the VPP for the next period 't1', follows the methodology outlined in Section 3.

i) Step one. Analysis of flexibility for each unit:

The VPP Operator forecasts the available resources in period 't1'. It verifies the technical constraints of each unit and its capability diagrams. Finally, it estimates the individual operational flexibility of each DER, according to the probability of occurrence and their respective reactive power capacities as would correspond to their specific capability diagrams (active vs reactive power) (Table 1):

For the practical purposes of this case study, the development of the methodology is continued with the active power data corresponding to P (50):

- Photovoltaic: Higher solar radiation is forecast, with a supplied possibility of 3 MW and ±2 MVAr. Therefore, with respect to 't0', it can increase its production by an additional +1 MW and +1.5 MVAr or reduce its production to 0 MW and −2 MVAr, at modulation rates of 1 MW/min and 1.5 MVAr/min (upward) and 2 MW/min and 2.5 MVAr/min (downward).

Table 1
Example of occurrence probabilities for the period 't1'.

Unit	P(10)	P(50)	P(90)
Photovoltaic	4 MW|±2.2 MVAr	3 MW|±2 MVAr	2 MW|±1.8 MVAr
Hydroelectric	10 MW|±3.5 MVAr	8 MW|±3 MVAr	7 MW|±2.8 MVAr
Wind	3 MW|±1.3 MVAr	2 MW|±1 MVAr	1 MW|±0.5 MVAr

- Storage system: At 50 % charge, this DER can supply or demand the system ± 1 MW and ±0.5 MVAr. The power ramps are ±0.5 MW/min and ±0.5 MVAr/min.
- Hydroelectric: Higher hydrological flow is forecast, with a supplied possibility of 8 MW and ±3 MVAr. Therefore, with respect to 't0', it can increase its production by an additional +5 MW and +1 MVAr or reduce its production to 0 MW and −3 MVAr, at modulation rates of 5 MW/min and 1 MVAr/min (upward) and 3 MW/min and 5 MVAr/min (downward).
- Wind turbine: Higher wind speed is forecast, with a supplied possibility of 2 MW and ±1 MVAr. Therefore, with respect to 't0', it can increase its output by an additional +1 MW and +0.5 MVAr or reduce its production to 0 MW and −1 MVAr, at modulation rates of 1 MW/min and 0.5 MVAr/min (upward) and 1 MW/min and 1.5 MVAr/min (downward).

ii) Step two. Analysis of aggregated flexibility:

The volumes and planes of the aggregated operational flexibility of the set of units are calculated and shown in Fig. 9. The aggregate operational flexibility graphs are constructed through the Minkowski sum and correspond to a probability of occurrence P(50). The VPP Operator can calculate the aggregate flexibility for other probabilities of occurrence, e.g., P(10) or P(90).

iii) Step three. Determination of needed flexibility:

According to the information shown in Fig. 8, for period 't1' the TSO requests active and reactive power modulation, according to the following parameters: π: 8 MW, ρ: 2 MW/min, φ_{Grid1}:3 MVAr, ψ_{Grid1}: 2.5 MVAr/min, φ_{Grid2}: −1 MVAr, and ψ_{Grid2}: −3.5 MVAr/min. Therefore, the objective to be met is established and the flexibility required by the system is determined.

iv) Step four. Verification of the operational flexibility:

The available operational flexibility is evaluated and compared with the required operational flexibility by the system. It is verified that the five needed flexibility parameters are less than the available flexibility parameters (Eqs. (3)–(7)). The results are shown in Fig. 10.

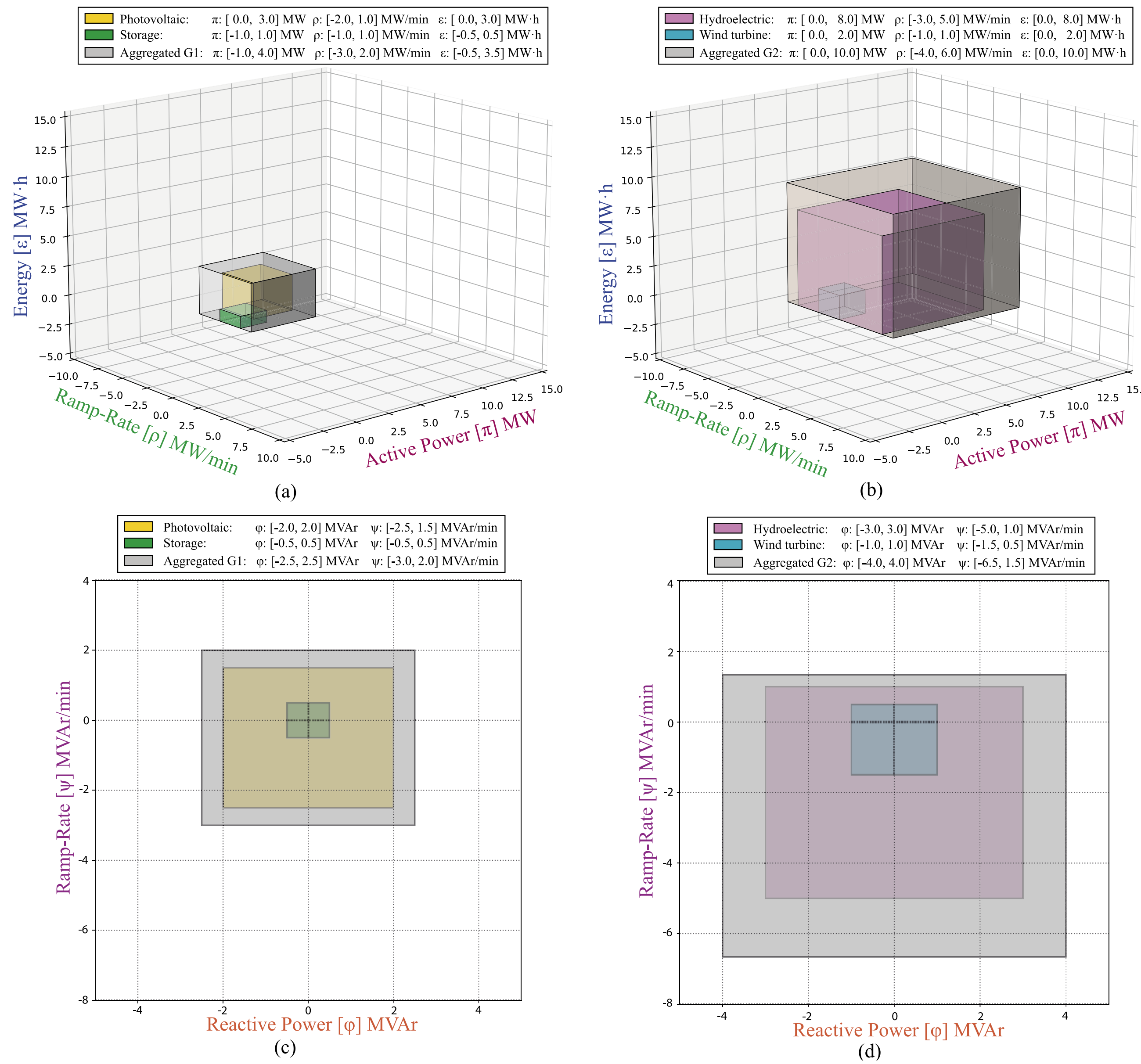

Fig. 9. (a) Volume of aggregated flexibility in grid 1, (b) Volume of aggregated flexibility in grid 2, (c) Plane of aggregated flexibility in grid 1, (d) Plane of aggregated flexibility in grid 2.

The results presented in Fig. 10 (b) demonstrate the feasibility of the VPP to meet the requirements of π, ρ, ε, solicited by the TSO. In addition, the VPP Operator could make new service offers in 't1', as it still has enough flexibility to operate in a higher active power range (P(50) of probability). However, with respect to the reactive power requirements shown in Fig. 10 (c) and (d), the non-compliance with the conditions requested for grid 1 is verified. In this case, the VPP Operator shall communicate to the TSO the lack of available flexibility for 't1'.

Finally, the VPP Operator can evaluate the available operational flexibility with other probabilities of occurrence, e.g., P(10) and P(90), and compare with the operational flexibility required by the system. In this way, decision-making is facilitated to accept or reject risks in service offerings.

4.2. Application of the methodology in actual tests

ELECAUSTRO is a public power generation company in Ecuador with experience in the management and operation of DERs, mainly hydroelectric technology [31]. Through the Department of Planning and Technical Studies, operational tests were conducted to assess the flexibility of a 4 MW hydroelectric unit (H1) with a reservoir, interconnected at 69 kV. The flexibility estimates $(\pi^{+}, \rho^{\pm}, \varepsilon^{+})$ were used to check the active power modulation capability and to compensate for actual production variations of a 1 MW PV plant (PV1), interconnected at 22 kV. The hydroelectric unit and the photovoltaic plant are not interconnected with each other, so they are considered two independent CCPs. The single-line diagram of the system is shown in Fig. 11:

The main objective of this case study is to demonstrate the coordinated operation capability of H1 and PV1 units to meet a requested dispatch curve. The graphical representation of operational flexibility is intended to illustrate the periods of time when the Hydro-Photovoltaic group has the capacity to fulfill a technical request and the periods when energy resources are insufficient. In this case study, the assessment of operational flexibility was conducted in four one-hour time periods.

From a technical perspective, the hydroelectric unit represents the firm power of this system, while the photovoltaic unit operates with its

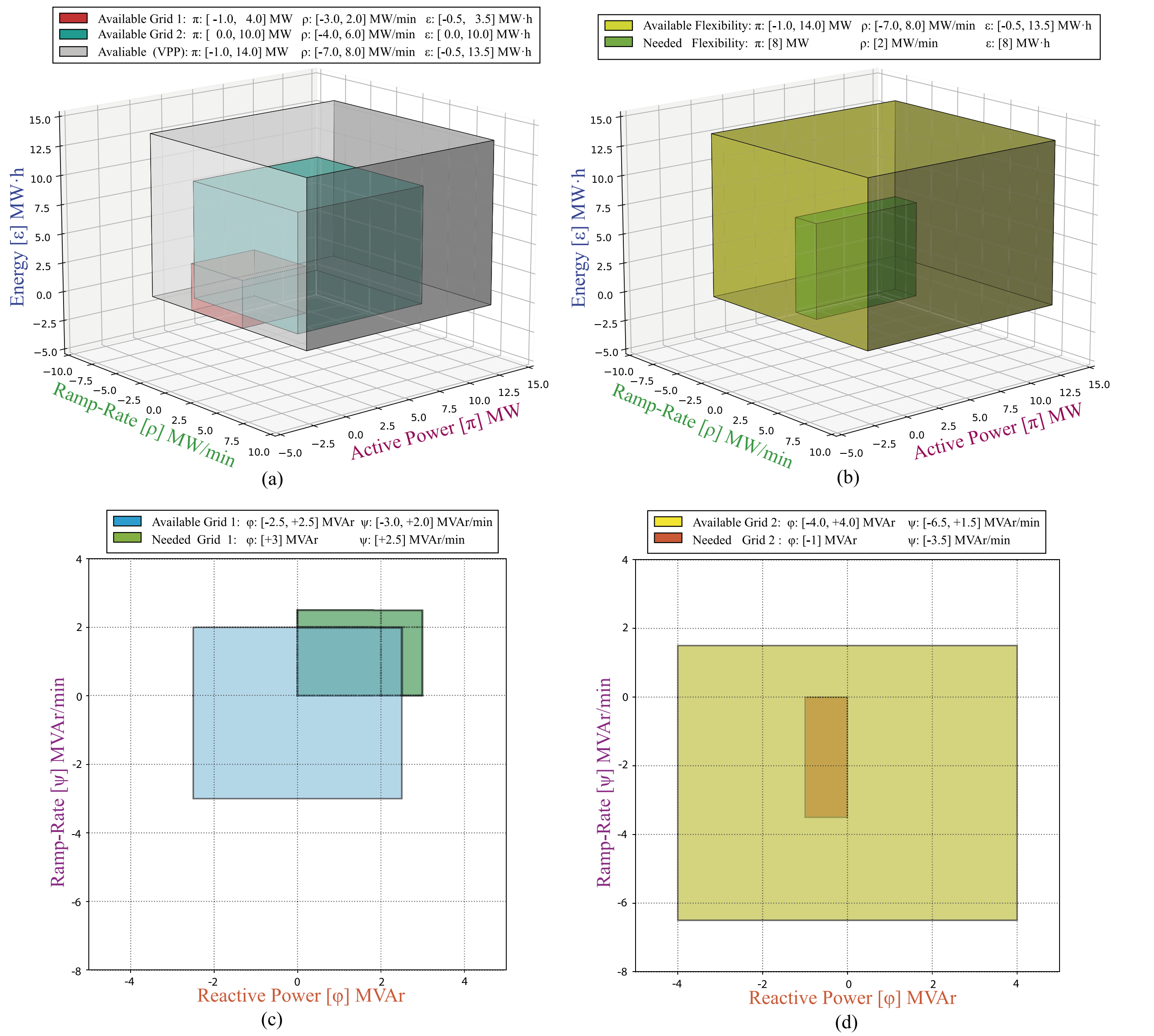

Fig. 10. (a) Volume of aggregated flexibility in the VPP, (b) Assessment of the operational flexibility (π, ρ, ε) in all system, (c) Assessment of the operational flexibility (φ, ψ) in grid 1, (d) Assessment of the operational flexibility (φ, ψ) in grid 2.

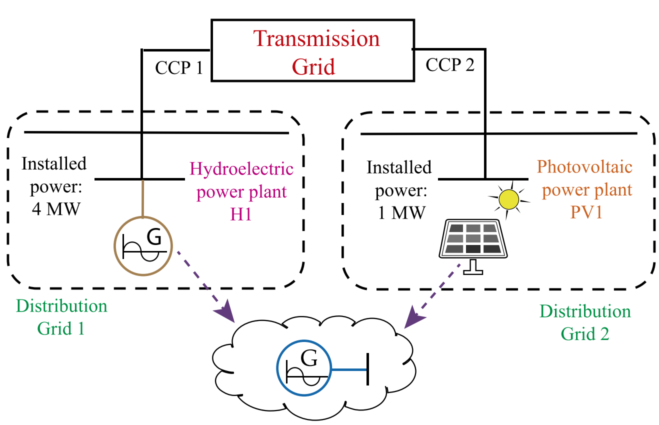

Fig. 11. Real case study to apply the proposed methodology.

variable and random characteristics. Therefore, the capacity of the H1 reservoir was utilized to offset the production variability of PV1.

i) Step one. Analysis of flexibility for each unit:

First, it was verified that there are no restrictions in the system or hydrological flow limitations. Consequently, the hydroelectric unit has the capacity to modulate its dispatch without technical or energy limitations (maximum flexibility of π^{+}, ε^{+}). Regarding upward and downward ramp-rate (ρ), these characteristics are shown in Fig. 12 (a).

According to operating tests, if the unit is at minimum synchronous power (0.5 MW), it will reach its rated power in 25 s. In the opposite direction, the stop of the operating unit requires 17 s. Thus, this unit has an average modulation ramp-rate of 0.14 MW/s (upward) and 0.24 MW/s (downward). This is equivalent to a ramp of 8.4 MW/min and 14.4 MW/min upward and downward, respectively. On the other hand, if the unit is out of sync, additional time is required to activate its auxiliary services and to open valves. Still, in these circumstances this unit reaches its nominal power in 180 s, meaning a maximum average of 1.3 MW/min. Regarding PV1, the energy forecasts estimate a nominal production of 1 MW, with

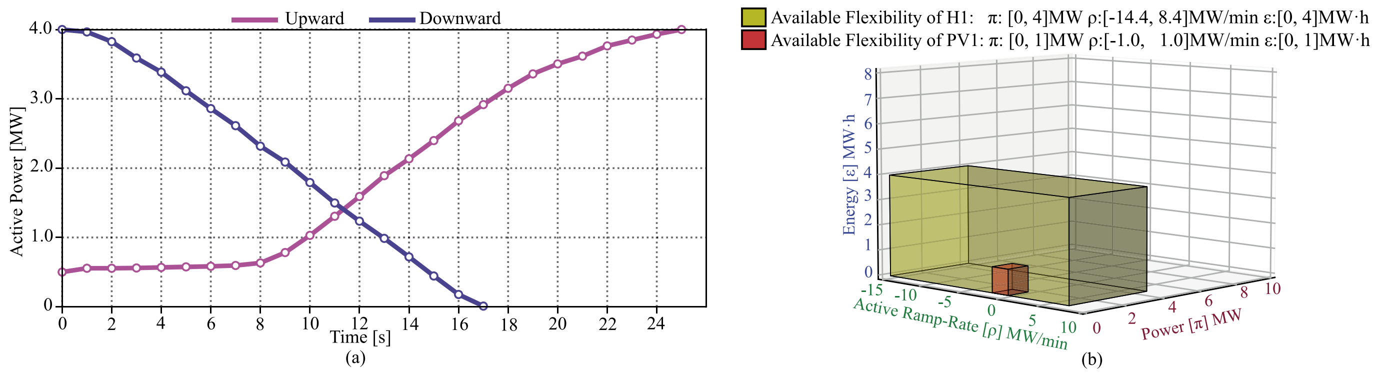

Fig. 12. (a) Upward and downward ramp-rate (ρ) of the 4 MW hydroelectric unit, (b) Available flexibility volume of H1 and PV1.

probability P(50). Since the ramp values of PV1 are not available, for practical purposes of this case study it is 1 MW/min. The available flexibility of H1 and PV1 are shown in Fig. 12 (b).

ii) Step two. Analysis of aggregated flexibility:

According to the results obtained in step 1, we proceed to calculate the Minkowski summation between H1 and PV1, obtaining the following values of aggregate operational flexibility: $\pi : [0, 5]$ MW; $\rho[-15.4, 9.4]$ MW/min, and $\varepsilon : [0, 5]$ MW·h.

iii) Step three. Determination of needed flexibility:

The analysis of this case study contemplates a limited timeline of four hours of operation. The objective is to supply 4 MW during time periods 't1', 't2' and 't3', while for period 't4' the supply of 6 MW is requested. Clearly, during the fourth time period, requests for active power and energy parameters exceeded the technical capacity of the hydro-photovoltaic group. The purpose of this request is to exceed the available flexibility.

For the corresponding analysis, the required operational flexibility is determined by the power fluctuation of the PV plant and the established dispatch curve.

The operational management of this set of units was defined through the following coordination scheme:

- In periods of time when there are records of photovoltaic production, hydroelectric production decreases proportionally. In this way, the unused water from the reservoir is considered storage of the solar energy produced by PV1.
- Conversely, in periods of time when PV production records decrease due to lack of radiation, the hydroelectric unit compensates for the deficit of the solar resource by increasing production.

iv) Step four. Verification of the operational flexibility:

According to step three, the requested dispatch curve during periods t1, t2, and t3 is 4 MW, while in t4, the requested power increases to 6 MW. Due to the robust technical and energetic characteristics of the hydro unit, H1 has sufficient capacity to operate in coordination with PV1 (Fig. 12 (b)). During the first three time periods, the variability of the solar resource is adequately compensated by H1. However, in the last period of time, the technical capacity of the hydro-photovoltaic group is not sufficient to meet the requested dispatch curve. Fig. 13 shows the operating curves of the H1 and PV1 units and the tracking of the requested power curve for each time step.

During the coordinated operation, it is observed that the aggregate operational flexibility in t1, t2, and t3 is greater than the flexibility needed by the system (Fig. 14 (a)). However, during period t4, a deficit in flexibility with respect to π and, ε is evident (Fig. 14 (b)). For this step-time, the VPP Operator should manage an additional DER to meet the system requirements, otherwise, the VPP would not have the capacity to meet the needed technical parameters, as shown in Fig. 13.

As shown in Fig. 14 (a), the most robust available flexibility parameter is the power ramp. The hydro-photovoltaic group has sufficient capacity to modulate its operation in short periods of time. However, the power and energy parameters have a very limited range of flexibility compared to the requested dispatch curve. This limitation is evident in Fig. 14 (b) when the flexibility resources prove insufficient for the system's requirements. In this analysis, the assessment of operational flexibility highlights a deficit in two technical parameters (π, ε). The flexibility graph suggests to the VPP operator the necessity of dispatching additional energy units to compensate for the deficit between available and required resources. In instances where no additional units are available, as in this case study, the VPP Operator must communicate the resource shortage and initiate a dispatch replanning process.

Assessing the available flexibility provides the VPP operator with

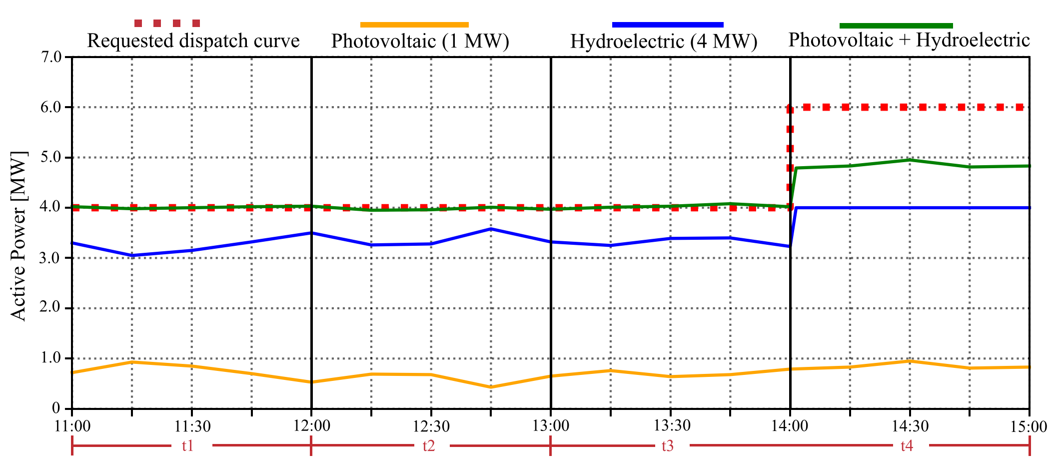

Fig. 13. Coordinated operation between a hydro unit and a PV plant.

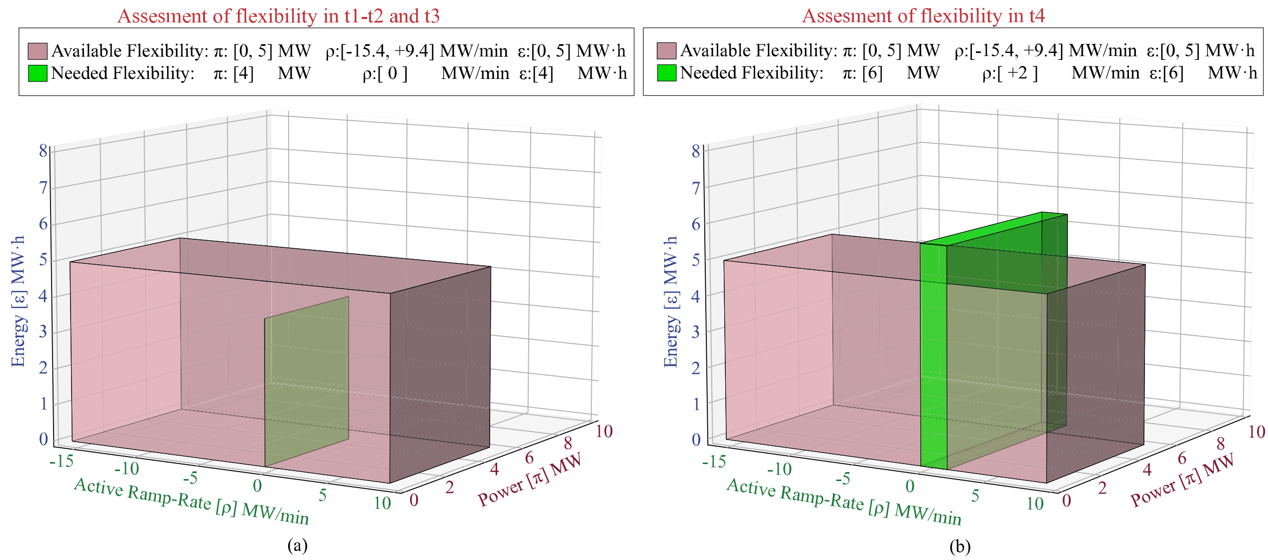

Fig. 14. Assessment of available flexibility vs Needed flexibility: (a) t1, t2 and t3, (b) t4.

crucial information for forecasting and managing resources in each subsequent time period, thereby enabling effective management in the face of uncertainty.

This case study demonstrates the potential of the proposed methodology for a VPP composed of multiple distributed units. With this methodology, the assessment of operative flexibility suggests the VPP operator with the ability to leverage capacity transfer between different technologies, exploit energy complementarity, or opt for storage usage.

5. Discussion

The research presented in this paper aims to contribute to integrating renewable energies into the power system. Since these are stochastic generation units, the results obtained do not guarantee the actual operation of the VPP. However, the proposed methodology provides a tool that facilitates the decision-making of the VPP Operator.

Renewable energies' forecasts are probabilistic, so considering the probability of a certain generation level is necessary during the Operator's analysis of the VPP. Planes and volumes of flexibility are not unique or constant over time and may differ in dimensions from one probability percentage to another. This is evident in the practical case study, where PV1 PV generation predicted a nominal output of 1 MW for the four time periods, with a probability of occurrence P(50). However, the actual operating results showed power fluctuations that had to be compensated by the hydroelectric unit. If H1 did not have sufficient flow, it was possible that the dispatch curve would not be covered in t1, t2, or t3.

Therefore, weather forecasts remain relevant, and the assessment of available flexibility vs required flexibility is also probabilistic. In this sense, the VPP Operator should start the loop again of the methodology proposed and evaluate the available flexibility with other probabilities of occurrence before making dispatch decisions for each DER or before offering new services to TSO. The tool presented in this article becomes a guide for the VPP operator while offering services to the system and for the energy management of a set of DERs. The usefulness of this tool was tested in the theoretical and practical case studies.

The advantages of this methodological proposal concerning the proposals identified in the study of the art are the following. First, it applies to the modern concept of VPPs, which include two or more CCPs. Second, the evaluation of operational flexibility is not limited to active power parameters but also allows the evaluation of reactive power for voltage control at specific points in the network (multiple CCPs). Third, the results are obtained immediately and do not require great computational effort (time ranges from milliseconds to a few seconds), depending on the amount of DER of the VPP. Finally, this paper allowed the development of actual operation tests to validate this proposal.

It is essential to mention that this tool becomes more relevant for VPPs with many units DERs. For instance, when evaluating a set of hundreds of units, the volumes, and planes of the aggregated flexibility of this entire set provide valuable information for the VPP Operator. In addition, the available flexibility ranges are obtained immediately without requiring complex mathematical calculations or excessive simulations as required by the Monte Carlo methodology.

6. Conclusions

The results obtained in this paper allowed the hypothesis proposed at the beginning of the research to be tested and led to the following conclusions:

- The graphical representation of the operational flexibility quickly indicates the fulfillment or deficit of the technical requirements of the system provided by the VPP.
- The proposed tool for the graphical representation of operational flexibility in a VPP facilitates the operator's decision-making during the operation and dispatch of units in real-time.
- Knowing the operational flexibility of the set of DERs facilitates the transfer capacity between units of different technologies.
- The coordinated operation of a set of DERs increases the operational flexibility of the system and reduces the uncertainty of unmanageable renewable energies.
- Knowing and managing the operational flexibility of a set of DERs allow the integration of renewable energies into the power system.
- Knowing and managing the operational flexibility of a set of DERs allow participation in the ancillary services market, maximizing the technical benefits of the system.

The integral knowledge of the technical parameters, and the control of the flexible characteristics of the DERs, allows the VPP to behave similarly to a conventional plant. In this way, the VPP acquires the capacity to offer ancillary services, power reserve, and operational flexibility. Of course, all this is possible only if the energy resource is available and if there is the capacity to control each DER. Consequently, the technical services provided by VPP encompass all four categories of operational flexibility: active power modulation, reactive power modulation, DER dispatch scheduling capability, and inter-unit operation transfer capability.

Once this operational flexibility is assessed, the VPP Operator can effectively coordinate its operations with the TSO or DSO to optimize the utilization of energy resources. Understanding the available operational flexibility also facilitates decision-making, enables participation in various electricity markets, efficient planning of unit dispatch, prompt response to the system's technical requirements or could be used to verify the availability of flexibility for the VPP to operate within the limits established by dynamic operating envelope. Therefore, assessing the operational flexibility of a VPP is crucial for the technical integration and efficient operation of DERs.

The benefits provided by VPP when there are several CCPs are emphasized: On the one hand, it allows the VPP Operator and the TSO to manage energy resources in a bottom-up approach. For example, if it is required to control the voltage levels at a network node, the VPP has control over the set of DERs that interconnect to a specific CCP. In this way, the VPP Operator can manage the operation in specific sectors of the grid. Another common case is participation in frequency regulation. In these circumstances, the VPP Operator may decide to dispatch all available DERs, only limited by the technical constraints of the network. This results in an increase in the flexibility of the power system with units that, individually, have no management capacity.

According to the concept of VPPs, the elements that make it up do not have geographical restrictions. This means the capacity acquired by the VPP Operator to evaluate and take advantage of the energy complementarity between different technologies. However, a robust communication and control system is essential for the VPP to ensure optimal management in real-time. Consequently, it is concluded that operational flexibility management is necessary for the effective integration of DERs into the power system. While VPP is the necessary means to grant observability and control of these units.

On the other hand, the operational flexibility available in a VPP is closely related to the dispatchability of DERs because it provides valuable technical information such as capacity, energy, and power ramp. This dispatchability involves important economic considerations, as the management and operation of the VPP must be performed at the lowest possible cost. To this end, the VPP must be evaluated through a mathematical optimization model. This optimization model must consider as input information the technical parameters of each DER and its flexibility availability, as well as the technical constraints and operating costs of each unit. In summary, evaluating operational flexibility is a preliminary requirement that serves as input information in the mathematical model of economic optimization of the VPP. This topic is beyond the scope of this paper; however, it has been identified as a line of future research.

The methodology for assessing the operational flexibility of a VPP is an essential part of a broader integral methodology that describes the operation of a VPP. Knowledge of operational flexibility alone does not provide information about the dispatchability of each DER. It does, however, provide valuable information that facilitates decision-making for the operation of the VPP.

CRediT authorship contribution statement

Juan C. Sarmiento-Vintimilla: Conceptualization, Methodology, Software, Validation, Formal analysis, Investigation, Data curation, Writing – original draft, Writing – review & editing, Visualization, Project administration. **D. Marene Larruskain:** Conceptualization, Methodology, Resources, Writing – original draft, Writing – review & editing, Visualization, Supervision. **Esther Torres:** Conceptualization, Methodology, Resources, Writing – original draft, Writing – review & editing, Visualization, Supervision. **Oihane Abarrategi:** Conceptualization, Methodology, Writing – review & editing, Visualization.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

No data was used for the research described in the article.

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