Privatisation and Vertical Integration
under a Mixed Duopoly

JUAN CARLOS BÁRCENA-RUIZ

Universidad del País Vasco UPV/EHU

and

MARÍA BEGOÑA GARZÓN

Universidad del País Vasco UPV/EHU

Abstract. This paper analyses the privatisation of public firms when private firms may be vertically integrated with their suppliers. We consider a mixed duopoly with a vertically integrated public firm. The private firm bargains the price of the input with its supplier if they are not vertically integrated. We find that for a given bargaining power of the private firm, it vertically integrates with its supplier if goods are weak substitutes. We also find that there is less vertical integration in the mixed duopoly than in the private duopoly. Finally, in general, the public firm is privatised when goods are close substitutes and the bargaining power of the private firm is low enough.

Key words: privatisation, vertical integration, mixed duopoly
JEL classification: L22, L33, L40

Corresponding author: María Begoña Garzón, Departamento de Fundamentos del Análisis Económico I, Universidad del País Vasco. Avenida Lehendakari Aguirre, 83; 48015 Bilbao, Spain. Phone: (34) 94 601 38 15; Fax: (34) 94 601 38 91; e-mail: mariabegona.garzon@ehu.es

1. Introduction

The literature analysing the vertical boundaries of firms is very extensive. Lafontaine and Slade (2007) point out that understanding the boundaries of firms and the choice between interacting in a firm and in a market is one of the most important issues in economics.[1] This issue has been studied from the assumption of competition between private firms, but few papers have analysed it for public firms. This latter issue is the main objective of the paper.

Strategic factors explaining vertical integration have been studied extensively in the relevant literature. In this regard, Bonnano and Vickers (1988) assume a duopoly model with price competition and analyse whether a manufacturing firm prefers to sell its product through an independent retailer (vertical separation) or directly to consumers (vertical integration). They show that vertical separation can be used as a collusion device. Jullien and Rey (2007) study collusion between upstream firms under vertical restrictions on retail prices. Nocke and White (2007) consider a repeated game and analyse the impact of vertical mergers on the ability of upstream firms to collude. Arya et al. (2008) study the production of key inputs outsourced to a vertically integrated retail competitor with upstream market power. Miklós-Thal et al. (2011) analyse the competitive effects of various contractual provisions under the assumption that rival retailers make offers to a single manufacturer.

The said papers analyse vertical integration while assuming that firms are private. However, in many industries in today's economy there is interaction between private and public firms, e.g. in the market for cars, health care, ships and steel manufacture (see De Fraja, 2009). This is especially significant in Europe and Asia, where governments still own a significant percentage of the firms in the different sectors of industry (see Parker and Saal, 2003; Chang, 2007). The privatization of public firms is a relevant issue in the analysis of mixed markets. European countries have privatised

---

[1] Lafontaine and Slade (2007) review the findings of empirical studies that examine two main questions: first, the types of transaction that are best brought within the firm; and second, the consequences of vertical integration for economics outcomes such us prices, quantities and profits. For other surveys on vertical integration see Whinston (2003) and Lafontaine and Slade (2001).

some of their state corporations in recent years (see, Megginson and Netter, 2001; Parker and Saal, 2003). Thus, one of the issues analysed by the literature on mixed oligopoly is the decision by governments whether to privatise a single public firm (see, for example, De Fraja and Delbono, 1989, 1990; Corneo and Jeanne, 1994).[2]

As far as we know, few papers have analysed privatisation and vertical integration under mixed markets. Bose and Gupta (2013) analyse the decision whether to privatise a public bilateral monopoly from a welfare perspective. They analyse downstream and upstream privatisation under two scenarios: a bilateral monopoly with an upstream private firm and a downstream public firm, and a bilateral monopoly with an upstream public firm and a downstream private firm. In case of privatisation the downstream and upstream firms are owned by different owners, so they do not analyse vertical integration. Glaeser and Scheinkman (1996) consider privatisation depending on demand uncertainties in a single industry with three sectors: an upstream firm, a downstream firm and a retailer. They do not analyze whether firms are vertically integrated or not. Wen and Yuan (2010) study the privatisation of a vertically regulated public utility from a public finance perspective. The government compares the net benefits between operating the industry as an integrated public utility or divesting the assets into the private sector. Matsumura and Matsushima (2012) consider two downstream firms and two input suppliers. Each input supplier is owned by a different government, and each government decides whether to privatise it or not.

The literature that analyses the privatisation of public firms usually assumes that firms do not buy inputs from providers.[3] However, in practice some firms buy inputs from providers while others are vertically integrated. Moreover, the literature on

---

[2] These papers have been extended to consider, among other factors, partial privatisation of public firms (Matsumura, 1998), strategic privatisation assuming international trade (Bárcena-Ruiz and Garzón, 2005*a*, 2005*b*), sequential privatisation of public firms (Matsumura and Shimizu 2010), privatisation when the public firm is as efficient as private firms (Bárcena-Ruiz, 2012), short-run and long-run relationships of trade and privatisation policies (Cato and Matsumura, 2015), privatisation of state holding corporations (Bárcena-Ruiz and Garzón, 2017; Dong et al., 2018), and privatisation under an interdependent payoff structure (Matsumura and Okamura, 2015).

[3] A related issue is the analysis of mixed markets when firms hire labour and bargain with a unionised labor force (see Willner, 1999; Ishida and Matsushima, 2008; Bárcena-Ruiz and Garzón, 2009).

privatisation and vertical integration considers that there is no competition in the product market, while in practice firms compete with each other. In order to fill this gap in the literature this paper analyses the decision whether to privatize a vertically integrated public firm. This firm competes with a private firm that must decide whether to vertically integrate or not.

An example that helps to motivate the paper corresponds to the shipbuilding sector. In this sector, vertically integrated public shipyards compete with private ones, some of which subcontract the construction of part of the ships. For example, Zamakona Yards is a Spanish private firm dedicated to the construction of ships of up to 110 meters long, of different classes (see www.zamakonayards.com). This firm works closely with leading engineering design and development companies. The Spanish public firm Izar is a vertical integrated firm that constructs civil ships. This firm has been privatised recently (see www.sepi.es). Another example corresponds to the delivery of letters and packages. After the liberalisation of this market in 2011 public firms began to compete with private firms in the European Union.[4] Correos is a Spanish public firm that is in charge of receiving the letters and packages of the clients and later deliver them, so it is vertically integrated. However, private firms usually subcontract the final delivery of the packages and letters, so they are not vertically integrated. A similar situation exists in the United Kingdom, country that privatised the public firm Royal Mail in 2013.

Taking into account the above example, we consider an industry made up of a public firm and a private firm (a mixed duopoly). Firms produce heterogeneous goods, compete on prices and buy inputs from suppliers. The public firm is vertically integrated, while the private firm has to decide whether to integrate with its provider or not. If firms are not integrated they bargain the price of the input. We consider the Nash

---

4 The European Parliament approved the Third Postal Directive, setting December 31st 2010 as the deadline for the abolition of postal monopolies and the full liberalisation of postal services in the E.U.

bargaining solution where the parties hold bargaining power.[5] We also analyse the decision by the government of whether to privatise the public firm or not. In case of privatisation the privatised firm remains vertically integrated.[6]

We find that under both mixed and private duopolies the decision of the private firm and its supplier on whether to integrate or not depends on the parameters of the model. The parameters involved are the degree to which goods are substitutes and the bargaining power of the private firm. If the degree to which goods are substitutes increases, market competition becomes stronger. This favours non integration because if they integrate market competition is still stronger so total profits are lower. If the bargaining power of the private firm decreases, the price of the input rises and market competition becomes weaker, which favours integration. These effects go in opposite directions so the result depends on which of them dominates. When goods are weak substitutes the second effect dominates, so the private firm and its supplier vertically integrate. When goods are close substitutes the first effect dominates, so the private firm and its supplier remain as independent firms. We also find that there is less vertical integration in the mixed duopoly. This is because the public firm is vertically integrated and takes the consumer surplus into account when deciding its price. Thus, market competition is stronger in the mixed duopoly than in the private duopoly, which favours non integration.

Finally, we study whether the government privatises the public firm or not. We find that, in general, the public firm is privatised if goods are close substitutes and the bargaining power of the private firm is low enough. In that case the private firm and its supplier remain as independent firms only if the degree to which goods are substitutes is

---

[5] In many industries the price of the input between firms and providers are set through bargaining. For example, Gal-Or (1997) considers a model where health insurers and providers (hospitals) bargain over how much the insurer pays to the hospital to provide health care for its clients. Chipty and Snyder (1999) point out that in cable-television large, horizontally-integrated cable operators are able to bargain for lower prices in their negotiations with suppliers of programme services.

[6] It can be shown that vertical separation of the privatized firm reduces welfare. Thus, the government has an incentive to prohibit vertical separation of the privatized firm. To simplify the presentation of the results we do not therefore consider this last case.

high enough. The results obtained in this paper may help to understand, for example, why some European countries have privatised part of their Social Security Systems. In Europe the Social Security System acts as an insurer and provider of health care and is thus, a vertically integrated public firm that competes with private insurers and hospitals, which may or may not be vertically integrated.

The rest of the paper is organised as follows. Section 2 presents the model. Sections 3 and 4 analyse the decision by a private firm and its supplier of whether or not to vertically integrate in a mixed duopoly and a private duopoly, respectively. Section 5 analyses the decision by the government on whether to privatise the public firm, and Section 6 concludes.

2. Model

We consider two firms that produce a differentiated good: one firm is public and the other is private, denoted by 0 and 1, respectively. On the consumption side, there is a continuum of consumers of the same type. The representative consumer's utility function is given by:

$$U(q_0, q_1) = a(q_0 + q_1) - \frac{1}{2} \left((q_0)^2 + 2bq_0q_1 + (q_1)^2 \right), \ 1 > b \geq 0,[7]$$

where parameter b measures the degree to which goods are substitutes. The representative consumer maximises $U(q_0, q_1) - p_0q_0 - p_1q_1$, where q_i is the amount of the good i and p_i is its price ($i = 0, 1$). Thus, demand functions are given by:

$$q_i = \frac{a(1-b) - p_i + bp_j}{1 - b^2}, \ i \neq j; \ i, j = 0, 1. \tag{1}$$

As firms produce heterogeneous goods, the consumer surplus is given by:

---

[7] We consider a simplified version of the model used by Vives (1984). Following Vives (1984), we assume that $b<1$ to ensure that the function $U(q_0, q_1)$ is strictly concave.

$$CS = U(q_0, q_1) - p_0 q_0 - p_1 q_1 = \frac{2a(1-b)(a-p_0-p_1) + (p_0)^2 - 2bp_0p_1 + (p_1)^2}{2(1-b^2)}. \quad (2)$$

It takes one unit of input to produce each unit of output at both firms. We normalise the marginal cost of producing the input to zero. The public firm is vertically integrated with its supplier, so the price of the input is zero. The private firm outsources the production of the input to a private supplier and pays a unit input price, w. Therefore, the profit function of firm i is given by:

$$\pi_i = (p_i - w) \, q_i, \; i = 0, 1, \quad (3)$$

where q_i is given by (1), with $w=0$ for the profit function of the public firm.

We consider that the private firm and its supplier bargain over the price of the input. The private firm seeks to maximise its profits, π_1, while the supplier seeks to maximise its incomes, wq_1. The outcome of the bargaining on the input price is modelled in terms of the two-person Nash bargaining solution in which the bargaining power of the private firm is measured by α, and that of the supplier by $1-\alpha$.

The producer surplus comprises the profits obtained by both firms plus the revenues obtained by the private supplier of the input. Thus, the producer surplus is given by $PS = \pi_0 + \pi_1 + wq_1$. As usual, the social welfare function comprises the consumer surplus, given by (2), and the producer surplus. Specifically, this function is expressed as:

$$W = CS + PS. \quad (4)$$

This paper sets out to analyse whether the government privatises the public firm or not when the private firm and its supplier decide whether to become a vertically integrated firm or not. We propose a four-stage game with the following timing. In the first stage the government decides whether to privatise the public firm or not. In the second stage the private firm and its supplier decide whether to become a vertically integrated firm or not. If they decide to become a vertically integrated firm then the input price is zero. If they decide not to then in the third stage the input price is bargained. Finally, in the

fourth stage, firms make price decisions simultaneously.[8] The equilibrium concept used is the subgame perfect equilibrium solved by backward induction.

In the first stage the government decides whether to privatise the public firm or not, so there are two subgames to be analysed: first, the government does not privatise the public firm, so there is a mixed duopoly in the product market (denoted by the superscript *M*); and second, the government privatises the public firm, so there is a private duopoly in the product market (denoted by the superscript *P*). We now analyse the first subgame.

3. Mixed duopoly

In this case a public firm competes with a private firm in the product market. Given that in the second stage the private firm and its supplier decide whether to be vertically integrated or not, there are two subgames to be analysed. In the first one the private firm and its supplier are not vertically integrated (denoted by the superscript *N*). In the second they are vertically integrated (denoted by the superscript *I*). We now analyse the first subgame.

3.1. The private firm and its supplier are not vertically integrated

Given the price of the input bargained in the third stage of the game, in the fourth stage the private firm chooses the price, p_1, that maximises its profit given by (3) for $i=1$. The public firm chooses the price, p_0, that maximises social welfare given by (4). From the first order conditions of these problems we obtain the reaction functions of the firms in prices:

$$p_1 = \frac{a(1-b)+w+bp_0}{2}, \; p_0 = bp_1. \tag{5}$$

---

[8] A rationalisation for using Bertrand competition in a mixed duopoly can be found in Matsumura and Ogawa (2012).

As usual, given that goods are substitutes, the reaction functions of firms in prices are upward sloping. Thus, if one firm increases its price its rival reacts by doing likewise. From (5) we obtain the following:

$$p_1 = \frac{a(1-b)+w}{2-b^2}, p_0 = \frac{b(a(1-b)+w)}{2-b^2}. \tag{6}$$

An increase in the price of the input raises the price of the output set by the private firm $(\partial p_1/\partial w = 1/(2-b^2) > 0)$. The reaction functions of firms in prices are upward sloping, so the price of the output chosen by the public firm also increases although to a lesser extent since the public firm takes consumer surplus into account when choosing the price of the output $(\partial p_0/\partial w = b/(2-b^2) > 0)$. Therefore, an increase in the price of the input raises output prices and reduces market competition.

By substituting (6) in (1) and (3) we obtain the output of firm 1 and its profit, respectively, as a function of w:

$$q_1 = \frac{a-w(1+b)}{(1+b)(2-b^2)}, \ \pi_1 = \frac{(1-b)(a-w(1+b))^2}{(1+b)(2-b^2)^2}. \tag{7}$$

It is easy to see from (7) that the output of firm 1 and its profit decrease with w. In the third stage the price of the input is negotiated. The result of the negotiation between the supplier of the input and the private firm is given by the solution to the following problem:

$$w = \underset{w}{argmax} \ (\pi_1)^\alpha (wq_1)^{1-\alpha}, 0 < \alpha < 1, \tag{8}$$

where π_1 and q_1 are given by (7). Parameter α measures the bargaining power of the private firm and (1- α) measures the bargaining power of the supplier. The greater the bargaining power of the private firm is, the lower the price of the input resulting from the bargaining process is. Solving this problem gives the following result.

Lemma 1. *Under a mixed duopoly, when the private firm and its supplier are not vertically integrated, in equilibrium:*

$$p_1^{NM} = \frac{a(3-2b^2-\alpha)}{2(1+b)(2-b^2)}, \quad p_0^{NM} = \frac{ab(3-2b^2-\alpha)}{2(1+b)(2-b^2)}, \quad q_1^{NM} = \frac{a(1+\alpha)}{2(1+b)(2-b^2)}, \quad q_0^{NM} = \frac{a}{1+b},$$

$$w^{NM} = \frac{a(1-\alpha)}{2+2b}, \quad \pi_1^{NM} = \frac{a^2(1-b)(1+\alpha)^2}{4(1+b)(2-b^2)^2}, \quad \pi_0^{NM} = \frac{a^2b(3-2b^2-\alpha)}{2(1+b)^2(2-b^2)},$$

$$PS^{NM} = \frac{a^2(1+4b-2b^3+\alpha)(3-2b^2-\alpha)}{4(1+b)^2(2-b^2)^2}, \quad CS^{NM} = \frac{a^2(17-16b^2+4b^4+8b(1+\alpha)-4b^3(1+\alpha)+\alpha(2+\alpha))}{8(1+b)^2(2-b^2)^2},$$

$$W^{NM} = \frac{a^2(23+4b(8-b(5+b(8-b-2b^2)))+6\alpha-4b^2\alpha-\alpha^2)}{8(1+b)^2(2-b^2)^2}.$$

The higher cost of the private firm implies, from (6), that it sets a higher price than the public firm. As the public firm takes the consumer surplus into account and has lower costs, since it is vertically integrated, it produces more.

3.2. The private firm and its supplier are vertically integrated

In the fourth stage of the game, the private firm chooses the price, p_1, that maximises its profit given by (3) for $i=1$, where $w=0$ since it is vertically integrated. The public firm chooses the price, p_0, that maximises social welfare given by (4). Solving these problems simultaneously we obtain expression (6) for $w=0$. From expressions (1) to (4) we obtain the following result.

Lemma 2. *Under a mixed duopoly, when the private firm and its supplier are vertically integrated, in equilibrium:*

$$p_1^{IM} = \frac{a(1-b)}{2-b^2}, \quad p_0^{IM} = \frac{a(1-b)b}{2-b^2}, \quad q_1^{IM} = \frac{a}{(1+b)(2-b^2)}, \quad q_0^{IM} = \frac{a}{1+b}, \quad \pi_1^{IM} = \frac{a^2(1-b)}{(1+b)(2-b^2)^2},$$

$$\pi_0^{IM} = \frac{a^2(1-b)b}{(1+b)(2-b^2)}, \quad PS^{IM} = \frac{a^2(1-2b^2+b^3)}{(2-b^2)^2}, \quad CS^{IM} = \frac{a^2(5-b-3b^2-b^3)}{2(1+b)(2-b^2)^2},$$

$$W^{IM} = \frac{a^2(7+b-7b^2-b^3+2b^4)}{2(1+b)(2-b^2)^2}.$$

Although both firms have the same costs, the public firm takes the consumer surplus into account so it sets a lower price and produces more.

From Lemmas 1 and 2 we obtain that $W^{IM} > W^{NM}$, so social welfare is greater in the mixed duopoly when the private firm vertically integrates. In this case, the private firm produces more than when it is not vertically integrated with its supplier since the price of the input is lower. Thus, the consumer surplus is greater in case of integration, which implies also a greater social welfare.

The second stage of this subgame remains to be solved.

3.3. Decision on integration

To analyse the decision of the private firm and its supplier on whether to vertically integrate or not their total profits under integration, π_1^{IM}, must be compared with those obtained if they are independent firms, $\pi_1^{NM} + w^{NM} q_1^{NM}$. They integrate if $\pi_1^{IM} = p_1^{IM} q_1^{IM} > \pi_1^{NM} + w^{NM} q_1^{NM} = p_1^{NM} q_1^{NM}$. Let $\alpha^M = 1 - 2b^2$ denote the level of parameter α such that $p_1^{IM} q_1^{IM} = p_1^{NM} q_1^{NM}$, where $\alpha^M = 0$ for $b = 1/\sqrt{2} \approx 0.7071$. From Lemmas 1 and 2 the following result, shown in Figure 1, is obtained.

Proposition 1. *Under a mixed duopoly, in equilibrium, the private firm and its supplier vertically integrate if α< α^M . If α≥ max{ α^M , 0} they remain as independent firms.*

This Proposition and Figure 1 show that for a given bargaining power of the private firm it vertically integrates with its supplier if goods are weak substitutes. To explain the result obtained in Proposition 1 the total profits obtained by the two firms under vertical integration, $p_1^{IM} q_1^{IM}$, must be compared with those obtained when they are independent firms, $p_1^{NM} q_1^{NM}$. It is easy to see that $p_1^{NM} > p_1^{IM}$ and $q_1^{NM} < q_1^{IM}$.[9]

[INSERT FIGURE 1 ABOUT HERE]

---

[9] For the public firm $p_0^{NM} > p_0^{IM}$ and $q_0^{NM} = q_0^{IM}$. The output of the public firm is the same in both cases since the production cost of firm 1 under non integration is the income of its supplier, so they cancel out in social welfare.

We consider first that $b=0$, which means that goods are independent in demand, so firm 1 is a monopolist in its market. Thus, firm 1 and its supplier vertically integrate and share the monopolistic profits between them.

Next we consider the opposite case: goods are close substitutes ($b>0.7071$). In this case firm 1 and its supplier prefer to be independent firms. Although firm 1 produces more under vertical integration, the lower price means lower total profits for firm 1 and its supplier. This is because when goods are close substitutes competition in the product market is stronger under vertical integration. Therefore, stronger market competition favours non integration.

Finally, we provide an intuition into the result obtained for intermediate values of parameter b ($0.7071>b>0$). As b increases, market competition becomes stronger which favours non integration. On the other hand, as parameter α decreases the bargaining power of the supplier increases, which raises the price of the input, thus reducing market competition. This counteracts the effect of the increase in parameter b. As a result, the value of parameter α such that the private firm and its supplier are indifferent whether to vertically integrate or not (see Figure 1) decreases as b increases: $\partial \alpha^M / \partial b < 0$. Therefore, if $\alpha < \alpha^M$ the private firm and its supplier vertically integrate, and if $\alpha \geq \max \{ \alpha^M, 0 \}$ they remain as independent firms.

To solve the first stage of the game, the subgame in which the government privatises the public firm remains to be analysed.

4. Private duopoly

We assume that if the vertically integrated public firm is privatised the government sells it to a single private investor, so firm 0 is an integrated private firm. In the second stage, private firm 1 and its supplier decide whether to become vertically integrated or not, so there are two subgames to be analysed. In the first one they decide to remain as

independent firms and in the second they decide to vertically integrate. We now analyse the first subgame.

4.1. Firm 1 *and its supplier are not vertically integrated*

In this case, the government sells the public firm to a single private investor, so there are two private firms competing in the product market but only firm 0 is vertically integrated. In the fourth stage of the game private firm i chooses the price, p_i, that maximises its profit given by (3), $i = 0, 1$, with $w=0$ for firm 0. Solving these problems simultaneously we obtain the following:

$$p_1 = \frac{a(2-b-b^2)+2w}{4-b^2} \ , \ p_0 = \frac{a(2-b-b^2)+bw}{4-b^2}. \tag{9}$$

By substituting (9) in (1) and (3) we obtain the output of firm 1 and its profit, respectively, as a function of w:

$$q_1 = \frac{a(2-b-b^2)-(2-b^2)w}{(4-b^2)(1-b^2)} \ , \ \pi_1 = \frac{(a(2-b-b^2)-(2-b^2)w)^2}{(4-b^2)^2(1-b^2)}. \tag{10}$$

In the third stage, the price of the input is bargained. The result of the bargaining between private firm 1 and its supplier is given by the solution to problem (8), where π_1 and q_1 are given by (10). Solving this problem we obtain the following result.

Lemma 3. *Under a private duopoly, when only one firm is vertically integrated, in equilibrium:*

$$p_1^{NM} = \frac{a(1-b)(3-b^2-\alpha)}{(2-b)(2-b^2)} \ , \ p_0^{NM} = \frac{a(1-b)(4-2b^2+b(1-\alpha))}{2(2-b)(2-b^2)},$$

$$q_1^{NP} = \frac{a(1+\alpha)}{(2-b)(2-b^2)}, \ q_0^{NM} = \frac{a(4+b-2b^2-b\alpha)}{2(2-b)(1+b)(2-b^2)},$$

$$w^{NP} = \frac{a(2-b-b^2)(1-\alpha)}{2(2-b^2)}, \ \pi_1^{NP} = \frac{a^2(1-b)(1+\alpha)^2}{4(2-b)^2(1+b)}, \ \pi_0^{NP} = \frac{a^2(1-b)(4+b(1-2b-\alpha))^2}{4(2-b)^2(1+b)(2-b^2)^2},$$

$$PS^{NP} = \frac{a^2(1-b)(28+8b(1-\alpha)-4b^3(1-\alpha)+8\alpha-4\alpha^2+2b^4(3+\alpha)-b^2(25+10\alpha-3\alpha^2))}{4(2-b)^2(1+b)(2-b^2)^2},$$

$$CS^{NP} = \frac{a^2(20+b(4-b(19+b(1-4b)))+8\alpha-2b^2(5+b(1-2b))\alpha+(1-b)(4-3b^2)\alpha^2)}{8(2-b)^2(1+b)(2-b^2)^2},$$

13

$$W^{NP} = \frac{a^2(76+24b^4+4(6-\alpha)\alpha-4b(9-\alpha)(1+\alpha)-4b^5(3+\alpha)+b^3(41+(26-3\alpha)\alpha)-b^2(85+\alpha(14-3\alpha)))}{8(2-b)^2(1+b)(2-b)^2}.$$

Firm 0, the integrated private firm, has a lower cost than firm 1 so it sets a lower price and produces more.

4.2. *Firm 1 and its supplier are vertically integrated*

In the fourth stage of the game, private firm i chooses the price, p_i, that maximises its profit given by (3), $i = 0, 1$, with $w=0$. Solving these problems simultaneously we obtain expression (9) for $w=0$. From (1) to (4) we obtain the following result.

Lemma 4. *Under a private duopoly, when both firms are vertically integrated, in equilibrium:*

$$p_i^{IP} = \frac{a(1-b)}{2-b}, \quad q_i^{IP} = \frac{a}{2+b-b^2}, \quad \pi_i^{IP} = \frac{a^2(1-b)}{(2-b)^2(1+b)},$$

$$PS^{IP} = \frac{2a^2(1-b)}{(2-b)^2(1+b)}, \quad CS^{IP} = \frac{a^2}{(2-b)^2(1+b)}, \quad W^{IP} = \frac{a^2(3-2b)}{(2-b)^2(1+b)}, \quad i = 0, 1.$$

Both firms have the same costs, so they set the same price, produce the same amount of output and obtain the same profits.

The second stage of this subgame remains to be solved.

4.3. *Decision on integration*

In order to analyse the decision of private firm 1 and its supplier on whether to vertically integrate or not the total profits obtained under vertical integration, π_1^{IP}, must be compared with those obtained when firms remain independent, $\pi_1^{NP} + w^{NP}q_1^{NP}$. They vertically integrate if $\pi_1^{IP} > \pi_1^{NP} + w^{NP}q_1^{NP}$. Let $\alpha^P = 1 - b^2$ denote the value of parameter α such that $\pi_1^{IP} = \pi_1^{NP} + w^{NP}q_1^{NP}$. From Lemmas 3 and 4 the following result (shown in Figure 2) is obtained.

Proposition 2. *Under a private duopoly, in equilibrium, private firm 1 and its supplier vertically integrate if $\alpha < \alpha^P$. If $\alpha \geq \alpha^P$ they remain as independent firms.*

The explanation of the result obtained in Proposition 2 is similar to that of Proposition 1. In both cases, when $b=0$ private firm 1 and its supplier vertically integrate. If b is high enough (which now means that b tends to 1), firm 1 and its supplier prefer to remain as independent firms. Finally, α^P decreases as b increases ($\partial \alpha^P / \partial b < 0$).

4.4. Comparison of the results obtained under mixed and private duopolies

From Propositions 1 and 2 the following result is obtained.

Proposition 3. *In equilibrium, $\alpha^P > \alpha^M$, so there is less vertical integration in the mixed duopoly than in the private duopoly.*

As the public firm is vertically integrated and takes the consumer surplus into account, it sets a lower price than private firms. Thus, market competition is stronger in the mixed duopoly than in the private duopoly. As seen in Proposition 1, stronger market competition favors non integration. This explains why $\alpha^P > \alpha^M$, so there is less vertical integration in the mixed duopoly. This result is illustrated in Figure 2, where there is vertical integration under a mixed (private) duopoly if $\alpha < \alpha^M$ ($\alpha < \alpha^P$), i.e. to the left of the dotted (continuous) curve.

[INSERT FIGURE 2 AROUND HERE]

Next we solve the first stage of the game.

5. Privatisation decision

15

In the first stage of the game the government decides whether to privatise the public firm or not. Next we compare the results shown in Lemmas 1 to 4 to solve this stage of the game. We show in the Appendix that $W^{IM} > W^{IP}$, $W^{NM} > W^{NP}$ if and only if $\alpha > \alpha^{W1}$, and $W^{NM} > W^{IP}$ if and only if $\alpha > \alpha^{W2}$, with $\alpha^{W2} > \alpha^{W1}$.[10] We identify the following zones depending on the values of parameters α and b. Zone I contains the parameter values such that $\alpha \leq \alpha^{M}$. Zone II contains the parameters values such that $\alpha^{M} < \alpha \leq min\{$ α^{W2}, $\alpha^{P}\}$. Zone III contains the parameters values such that $\alpha^{P} < \alpha \leq \alpha^{W1}$. Zone IV contains the parameters values such that $\alpha > max\{\alpha^{P}, \alpha^{W1}\}$. Finally, the remaining parameters values are included in Zone V. These five zones are shown in Figure 3, which illustrates the following result.

Proposition 4. *In equilibrium, the government privatises the public firm in Zones II and III but not in Zones I, IV and V. The private firm and its supplier vertically integrate only in Zones I and II.*[11]

[INSERT FIGURE 3 AROUND HERE]

Propositions 1 and 2 show that in Zone I the private firm and its supplier vertically integrate in both the mixed and private duopolies (since $\alpha < \alpha^{M} < \alpha^{P}$). In this zone the government does not privatise the public firm. This is because the price set by the public firm is lower than that set by the private firms (since it takes the consumer surplus) into account and the marginal cost of the public firm is no greater than that of the private firms. Thus, in the mixed duopoly market competition is stronger than in the private duopoly. As the consumer surplus increases while the producer surplus decreases with output level, we obtain that the consumer surplus is greater ($CS^{IM} > CS^{IP}$) and the producer surplus is lower in the mixed duopoly ($PS^{IM} < PS^{IP}$). The effect of the consumer surplus on social welfare dominates, so social welfare is greater

---

[10] These critical values of parameter α are relegated to the Appendix.

[11] It can be shown that if firms compete on quantities, the government does not privatise the public firm and the private firm and its supplier vertically integrate for all parameters values.

in the mixed duopoly, which means that the government does not privatise the public firm.

When $max\{\alpha^M, 0\} \leq \alpha < \alpha^P$, i.e. in Zones II and V, Propositions 1 and 2 show that the private firm and its supplier do not integrate in the mixed duopoly but do integrate in the private duopoly. Proposition 3 shows that the government does not privatise the public firm in Zone V but does so in Zone II. This is due to the effect that parameters b and α have on social welfare. First, as b increases the consumer surplus increases more in the private duopoly than in the mixed duopoly, because as b increases market competition becomes greater and the public firm takes this into account and reduces its output. This favours the privatisation of the public firm. Second, as α increases the bargaining power of the private firm increases, which reduces the price of the input, thus increasing market competition in the mixed duopoly. Thus, as α increases social welfare in the mixed duopoly rises. However, social welfare does not vary with α in the private duopoly since the two firms are integrated. This favours the non privatisation of the public firm. As a result, in Zone *V*, where α is high enough and b is low enough, the government does not privatise the public firm. However in Zone II, where α is low enough and b is high enough, the government does privatise the public firm.

Propositions 1 and 2 show that if $\alpha \geq \alpha^P$, i.e. in Zones III and IV, the private firm and its supplier remain as independent firms in both the mixed and private duopolies. Thus, parameter α affects social welfare in both cases. In Zone IV the government does not privatise the public firm but in Zone III it does. There are two factors that explain this result. First, as discussed above, an increase in parameter b favours the privatisation of the public firm. Second, an increase in parameter α increases market competition. However, market competition increases more in the mixed duopoly than in the private duopoly since the public firm takes consumer surplus into account when deciding its price. Thus, as α increases both the consumer surplus and social welfare increase more in the mixed duopoly than in the private duopoly, which favours the non privatization of the public firm. As a result, in Zone IV, where parameter α is high enough, the government does not privatise the public firm. However in Zone III, where parameter b is high enough, the government does privatise the public firm.

6. Conclusion

Empirical evidence shows that in the real world some firms buy inputs from providers while others are vertically integrated. However, the literature that analyses the privatisation of public firms usually assumes that firms do not buy inputs from providers. Moreover, the literature that analyses privatisation and vertical integration does not consider competition in the product market by firms. Seeking to fill this gap in the literature, this paper analyses the decision whether to privatize a vertically integrated public firm that competes with a private firm, which must decide whether to vertically integrate with its supplier or not.

We find that under both mixed and private duopolies, the decision of the private firm and its supplier on whether to vertically integrate or not depends on the parameters of the model: the degree to which goods are substitutes and the bargaining power of the private firm. We also find that there is less vertical integration in the mixed duopoly than in the private duopoly. The public firm is vertically integrated and its objective function takes the consumer surplus into account, so market competition is stronger in the mixed than in the private duopoly, and stronger competition favours non integration. Finally we study whether the government privatises the public firm or not. We find that, in general, the public firm is privatised if goods are close substitutes and the bargaining power of the private firm is low enough. In that case, the private firm and its supplier integrate only when the bargaining power of the private firm is sufficiently low for a given degree of substitutability between goods.

Although the timing considered in the paper is natural, the opposite timing (the private firm chooses its vertical structure, and then the government chooses whether to privatise) might also be plausible. This is a relevant issue since, as Lee et al. (2018) show, the timing of privatisation may affect the equilibrium outcomes. We leave this issue for future research.

Appendix

Proof of Proposition 3

A comparison of the social welfare obtained in the different cases shows the following:

i) $W^{IM} - W^{IP} = a^2(1-b)^3(4+4b-b^2-2b^3)/(2(2-b)^2(1+b)(2-b^2)^2) > 0.$

ii) $W^{NM} - W^{NP} = a^2(16+8b^7-4b^5(7-\alpha)-4b^6(4-\alpha)+4b^3(7+\alpha)-4b(1+4\alpha-\alpha^2) -$

$$4b^2(16-9\alpha+2\alpha^2)+b^4(59-30\alpha+3\alpha^2))/(8(2-b)^2(1+b)^2(2-b^2)^2).$$

The above expression equals zero for $\alpha = \alpha^{W1} = (8b-18b^2-2b^3+15b^4-2b^5-2b^6 +$

$2\sqrt{b(2-3b+b^2)^2(-4+b+12b^2+3b^3-8b^4-3b^5+2b^6+b^7)})/(4b-8b^2+3b^4) < 1.$ We obtain that $W^{NM} > W^{NP}$ if $\alpha > \alpha^{W1}$ and $W^{NM} < W^{NP}$ if $\alpha < \alpha^{W1}$.

iii) $W^{NM} - W^{IP} = a^2(-4+4b-25b^2+16b^3+36b^4-24b^5-12b^6+8b^7+24\alpha-24b\alpha -$

$$10b^2\alpha+16b^3\alpha-4b^4\alpha-4\alpha^2+4b\alpha^2-b^2\alpha^2)/(8(-2+b)^2(1+b)^2(-2+b^2)^2).$$

The above expression equals zero for $\alpha = \alpha^{W2} = (12-12b-5b^2+8b^3-2b^4 -$

$2\sqrt{2}\sqrt{(1+b)(6b-3b^3+b^4-4)^2})/(2-b)^2 < 1.$ We obtain that $W^{NM} > W^{IP}$ if $\alpha > \alpha^{W2}$ and $W^{NM} < W^{IP}$ if $\alpha < \alpha^{W2}$.

References

Arya, A., Mittendorf, B., Sappington, D., 2008. Outsourcing, vertical integration, and price vs. quantity competition. International Journal of Industrial Organization 26, 1-16.

Bárcena-Ruiz, J.C., 2012. Privatization when the public firm is as efficient as private firms. Economic Modelling 29, 1019-1023.

Bárcena-Ruiz, J.C., Garzón, M.B., 2005*a*. Economic integration and privatisation under diseconomies of scale. European Journal of Political Economy 21, 247-267.

Bárcena-Ruiz, J.C., Garzón, M.B., 2005*b*. International trade and strategic privatization. Review of Development Economics 9, 502-513.

Bárcena-Ruiz, J.C., Garzón, M.B., 2009. Relocation and public ownership of firms. Journal of the Japanese and International Economies 23, 71-85.

Bárcena-Ruiz, J.C., Garzón, M.B., 2017. Privatization of state holding corporations. Journal of Economics 120 (2), 171-188.

Bonnano, G., Vickers, J., 1988. Vertical separation. The Journal of Industrial Economics 3, 257- 265.

Bose, A., Gupta, B., 2013. Mixed markets in bilateral monopoly. Journal of Economics 110, 141-164.

Cato, S., Matsumura, T., 2015. Optimal privatisation and trade policies with endogenous market structure. The Economic Record 91 (294), 309-323.

Chang, H.J., 2007. State-owned enterprise reform. National development strategies. Policy notes. United Nations. Department for Economics and Social Affairs.

Chipty, T., Snyder, C.M., 1999. The role of firm size in bilateral bargaining: A study of the cable television industry. The Review of Economics and Statistics 81 (2), 326–340.

Corneo, G., Jeanne, O., 1994. Oligopole mixte dans un marché commun. Annales d'Economie et de Statistique 33, 73-90.

De Fraja, G., Delbono, F., 1989. Alternatives strategies of a public enterprise in oligopoly. Oxford Economic Papers 41, 302-311.

De Fraja G., Delbono, F., 1990. Game theoretic models of mixed oligopoly. Journal of Economic Surveys 4, 1-17.

De Fraja, G., 2009. Mixed oligopoly: old and new. Working Paper No. 09/20. University of Leicester.

Dong, Q., Bárcena-Ruiz, J.C., Garzón, M.B., 2018. Partial privatization of state holding corporations. The Manchester School 86 (1), 119-138.

Gal-Or, E., 1997. Exclusionary equilibria in health-care markets. Journal of Economics and Management Strategy 6 (1), 5-43.

Glaeser, E.L., Scheinkman, J.A., 1996. The transition to free markets: where to begin privatization. Journal of Comparative Economics 22 (1), 23-42.

Ishida, J., Matsushima, N., 2008. Should civil servants be restricted in wage bargaining? A mixed-duopoly approach. Journal of Public Economics 93 (3-4), 634–646.

Jullien, B., Rey, P., 2007. Resale price maintenance and collusion. RAND Journal of Economics 38 (4), 983-1001.

Lafontaine, F., Slade, M., 2001. Incentive contracting and the Franchise decision. Game Theory and Business Applications. Chatterjee, K., Samuelson, W. (eds.) Kluwer Academic Press, 133-188.

Lafontaine, F., Slade, M., 2007. Vertical integration and firm boundaries: The Evidence. Journal of Economic Literature 45 (3), 629-685.

Lee, S-H., Matsumura, T., Sato, S., 2018. An analysis of entry-then-privatization model: welfare and policy implications. Journal of Economic 123 (1), 14-88.

Matsumura, T., 1998. Partial privatization in mixed duopoly. Journal of Public Economics 70, 473-483.

Matsumura, T., Ogawa, A., 2012. Price versus quantity in a Mixed Duopoly. Economic Letters 116 (2), 174-177.

Matsumura, T., Okamura, M., 2015. Competition and privatization policies revisited: the payoff interdependence approach. Journal of Economics 116, 137-150.

Matsumura, T., Matsushima, N., 2012. Airport privatization and international competition. The Japanese Economic Review 63 (4), 431-450.

Matsumura, T., Shimizu, D., 2010. Privatization waves. The Manchester School 78, 609-625.

Megginson, W.L., Netter, J.M., 2001. From state to market: A survey of empirical studies on privatization. Journal of Economic Literature 39, 321-389.

Miklós-Thal, J., Rey, P., Vergé, T., 2011. Buyer power and intrabrand Coordination. Journal of the European Economic Association 9 (4), 721-741.

Nocke, V., White, L., 2007. Do vertical mergers facilitate upstream Collusion? American Economic Review 97, 1321-1339.

Parker, D., Saal, D. eds., 2003. International Handbook on Privatization. Edward Elgar Publishing, Cheltenham, UK.

Vives, X., 1984. Duopoly information equilibrium: Cournot and Bertrand. Journal of Economic Theory 34, 71-94.

Wen, J-F., Yuan, L., 2010. Optimal privatization of vertical public utilities. Canadian Journal of Economics 43 (3), 816-831.

Whinston, M.D., 2003. On the transaction costs determinants of vertical integration. Journal of Law, Economics and Organization 19, 1-23.

Willner, J., 1999. Policy objectives and performance in a mixed market with bargaining. International Journal of Industrial Organization 17 (1), 137-145.
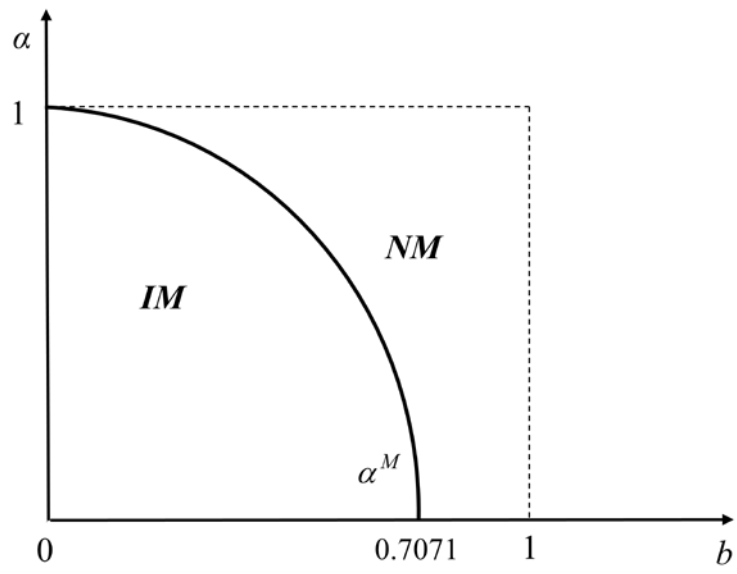
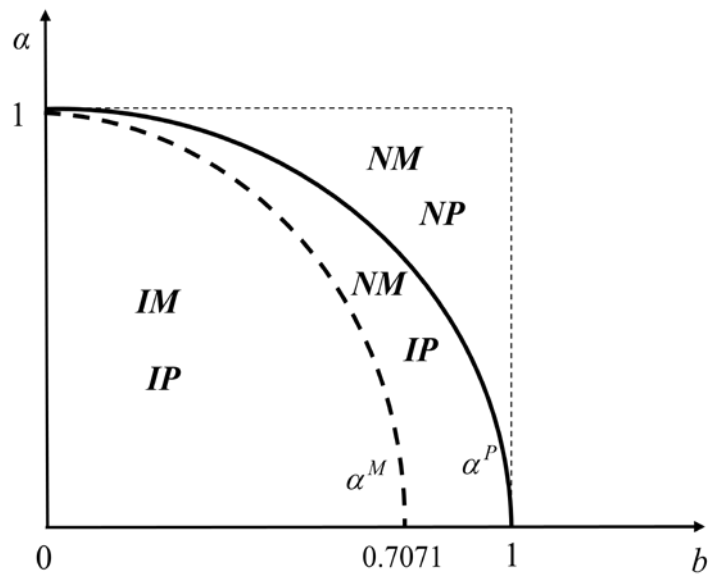**Fig. 1.** Decision on integration under a mixed duopoly.

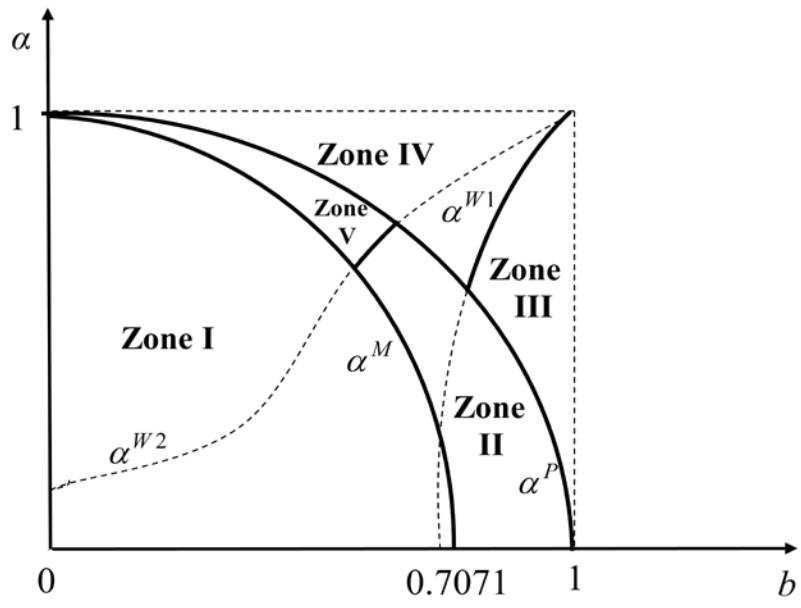**Fig. 2.** Decision on integration under a private duopoly and a mixed duopoly.

Fig. 3. Decision on privatization.