# DISCLOSURE OF R&D KNOWLEDGE IN A MIXED DUOPOLY

by

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In this paper we analyze whether firms want to disclose their R&D knowledge for free in a mixed duopoly. We extend the paper by Poyago-Theotoky (1999) on strategic R&D with endogenous spillovers by considering a mixed duopoly. We find that when spillovers of information are treated as endogenous the public firm fully discloses its information so long as R&D levels are chosen non-cooperatively. The private firm does not disclose its information. This result holds if the private firm is foreign-owned. We have extended the main model to consider the threat of entry faced by an incumbent public firm and whether it is privatised or not. We find that the public firm is not privatised and there is entry if the fixed entry cost is low enough.

**Key words:** R&D Duopoly · Mixed Duopoly · Non-Tournament Model · Endogenous Spillovers · Privatization

**JEL Classification** O30 · L13 · L32.

*We thank two referees for helpful comments and suggestions. Financial support from Ministerio de Ciencia y Tecnología (ECO2015-66803-P) and Grupos de Investigación UPV/EHU (GIU17/051) is gratefully acknowledged.

## 1 INTRODUCTION

In many industries public and private firms compete in the product market. This is especially important in Europe and Asia, for instance, in the markets for cars, ships and steel manufactures (see De Fraja (2009), Chang (2007)). An important factor in market competition between public and private firms is their production technology.[1] Thus, public and private firms invest in R&D to improve their technologies and thereby reduce their costs of production and obtain a competitive advantage over rivals. This issue has been widely analysed in the literature on R&D.

Some of the relevant studies have analyzed R&D investments with exogenous technological spillovers by private firms (see d'Aspremont and Jacquemin (1988), Kamien *et al*. (1992), Poyago-Theotoky (1995), Kawasaki *et al.* (2014)). This analysis has also considered exogenous spillovers when public and private firms compete in product markets (see Heywood and Ye (2009), Gil-Moltó *et al*. (2011) Kesavayutha and Zikos (2013), Haruna and Goel (2017)).[2]

The above papers consider R&D investments in the context of a non-tournament R&D competition model (as opposed to a patent race) in which there are technology spillovers, assuming that spillovers are exogenously given. These papers have been extended to study endogenous spillovers in a non-tournament R&D private duopoly. Poyago-Theotoky (1999) analyzes whether private firms have an incentive to disclose part or all of the information that they produce in their R&D activities. She shows that private firms never disclose any of their information when they choose their R&D non-cooperatively. However, under cooperative R&D firms share their information fully. Tesoriere (2008) extends the above analysis by considering collusion by firms. He shows that when the market is large enough, the optimal collusion arrangement requires perfect knowledge disclosure between firms.

[1] This issue has been analyzed when firms have both constant marginal production costs and convex production costs (see Bárcena-Ruiz (2012), Matsumura and Shimizu (2010), Bárcena-Ruiz and Garzón (2005)).

[2] Some studies have analyzed R&D investments in mixed markets assuming patent races, where a new product or a new process is introduced (see Delbono and Denicolo (1993), Poyago-Theotoky (1998), Ishibashi and Matsumura (2006)). Related analyses consider cost-reducing incentives in a mixed duopoly market (see Lin and Ogawa (2005), Matsumura and Matsushima (2004), Nishimori and Ogawa (2002)).

The above two papers consider endogenous spillovers in a non-tournament R&D private duopoly. As far as we know, this issue has not been studied in a context of mixed markets. This analysis is relevant because in many industries firms increasingly share know-how and resources. It is increasingly common to observe that firms give their patents away for free. Through this behaviour, firms seem to act against the original concept of the patent: To establish temporary monopolies to collect the returns on their R&D investment. Given the increasing trend of disclosing R&D knowledge by firms and given that public and private firms compete in the product market, it seems important to analyze why public and private firms may disclose their knowledge.[3] Thus, in this paper we analyze the incentives that public and private firms have to disclose R&D knowledge that reduces their costs of production.

Ishibashi and Matsumura (2006) discuss one example that is useful in motivating the analysis undertaken in this paper: Competition on the 'genome project' between Celera Genomics and public institutions. They point out that "the purpose of the international genome project was not to make current profits, but to make the information public and to improve worldwide welfare in the future by the open use of this information. On the other hand, the purpose of Celera Genomics was to make profits". Moreover, they point out that "If a private firm monopolises such very important knowledge, it must become a serious obstacle to the future progress of human science and bio-technology and for the development of important industries such as the medical and agricultural industries, which could result in large losses of worldwide welfare".[4] A related example is the case of DARPA, the research arm of the U.S. Department of Defense. It has made an effort to shift toward open-source machine learning technologies. It has developed a catalog of state-of-the-art machine learning, visualization and other technologies that anyone can download, use and modify to

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[3]Ziegler *et al*. (2014) argue that a growing number of firms give away their patents for free. They discuss different motives for open intellectual property strategies such as economic, technological, and social reasons. They point out that speeding up the innovation process and profiting from network effects are important motives.

[4]Ishibashi and Matsumura (2006) discuss other examples of research projects by Japanese universities. Bozeman (2000) reviews literature on technology transfer, focusing mainly on literature analyzing technology transfer from universities and government laboratories. Similarly, Aarhus University, a Danish public university, publishes all its results on the innovative Open Science platform for free (scitech.au.dk).

build custom AI tools.[5] A further example is the case of the big tech company Lenovo which has given away free patents to startups (Business Telegraph, May 23, 2019). Around 68 percent of the shares in Lenovo are owned by the Chinese government.

The above examples illustrate two situations which it would be interesting to analyze: First, whether firms in a mixed duopoly disclose their R&D knowledge; and second, the threat of entry faced by a monopolist incumbent firm that invests in R&D, and whether or not the incumbent firm is privatised if it is publicly-owned.

We start by analyzing the first issue. We assume a mixed duopoly in which the private and public firms chose R&D levels non-cooperatively to reduce their marginal costs of production. After investing in R&D, firms decide simultaneously whether or not to disclose their R&D knowledge. Thus, the cost of production of each firm can be reduced in two ways: First, directly by R&D expenditure by the firm itself; and secondly indirectly via the R&D expenditure of the rival firm if it discloses its knowledge. We find that the public firm fully discloses its information to reduce the marginal cost of the private firm, which increases the output of the industry, and with it the consumer surplus and social welfare. The production cost of the private firm is reduced in two ways: First, R&D levels are strategic complements,[6] so the R&D level chosen by the private firm increases with that of the public firm. Second, the public firm fully discloses its knowledge for free. However, by disclosing a positive amount of R&D knowledge the private firm lowers the marginal cost of the public firm, which decreases its market share, so the private firm does not disclose information. We also find that the public firm fully discloses its knowledge to the private firm even if the private firm is foreign-owned.

The second issue considered in the paper is the threat of entry faced by an incumbent firm, and whether or not it is privatised if it is publicly-owned.[7]We extend the main model by considering that there is an incumbent public firm and a private firm that has to pay a fixed cost to enter the market. This issue is especially relevant when sectors where R&D

[5]See http://www.iflscience.com/technology/why-big-tech-companies-are-open-sourcing-their-ai-systems/.

[6]It should be noted that in a private duopoly R&D levels are strategic substitutes so investment by one firm decreases with that of the other.

[7]This issue is related to the literature that analyzes free entry by private firms into mixed markets (see Anderson et al. (1997), Matsumura and Kanda (2005), Ino and Matsumura (2010)), and to the literature that analyzes privatization of public firms in mixed markets (see Matsumura and Shimizu (2010), Bárcena-Ruiz and Garzón (2017, 2018), Sato and Matsumura (2019)).

knowledge is important are considered. We find that the public firm is not privatised and, in that case, the private firm enters the market if the fixed entry cost is low enough; if that cost is high enough there is a public monopoly. This result holds if the entrant is foreign-owned. We also find that the value of the fixed cost such that there is no entry is greater if the entrant is foreign-owned than if it is domestic-owned, so there is less entry in the first case.

The rest of the paper is organised as follows. Section 2 presents the model. Sections 3, 4 and 5 analyze a mixed duopoly with a domestic private firm, a private duopoly with domestic firms, and a mixed duopoly with a foreign private firm, respectively. Section 6 compares the results obtained in the three cases. Section 7 analyzes privatization and free entry, and Section 8 concludes.

## 2 THE MODEL

We consider an economy that comprises two firms producing a homogeneous good, denoted by 0 and 1, respectively. Firm 0 is public while firm 1 is private. As usual, the public firm maximises social welfare while the private firm maximises its own profit. The demand function is given by $p = a - q_0 - q_1$.

Firms invest in R&D which reduces their costs of production, so we are considering process innovation. The costs of production of each firm can be reduced in two ways: directly by R&D expenditure carried out by the firm itself, and indirectly via the R&D expenditure of the other firm (through spillovers). Following Gil-Moltó *et al.* (2011), we assume that if firms $i$ and $j$ choose $x_i$ and $x_j$ R&D levels, resulting from expenditure in R&D, the cost function of firm $i$ is given by $C(q_i, x_i, x_j) = (c - x_i - \beta_j x_j)q_i + q_i^2$, where $\beta_j \in [0, 1]$ is the amount of information that firm $j$ discloses, $i \neq j$; $i, j = 0, 1$.[8] Thus, $\beta_j = 0$ means that firm $j$ does not transfer its technology and $\beta_j = 1$ means that firm $j$ transfers its technology for free.[9]

[8] Increasing marginal cost of production is an assumption widely used in the literature on mixed oligopoly to avoid a trivial solution (see, for example, De Fraja and Delbono (1989), Bárcena-Ruiz and Garzón (2005)). If the marginal cost of production is assumed to be constant, the public firm will produce a quantity such that the market price equals its marginal cost, so the private firm is driven out of the market.

[9] If firms produce goods that are independent in demand but share a common R&D base, it would probably not be harmful to disclose their results. However, when they produce a homogeneous good they compete in the product market, so disclosing information would reduce their market share and, thus, their profits.

The cost of R&D is assumed to be quadratic, reflecting diminishing returns on R&D expenditures (see d'Aspremont and Jacquemin (1988)). Therefore, the expenditure on R&D of firm $i$ is given by $I(x_i) = \gamma x_i^2$, $i$=0, 1, where parameter $\gamma$ measures the efficiency of technology, so a low value of this parameter indicates greater efficiency in R&D technology.

The profit function of firm $i$ is given by:

$$\pi_i = p\, q_i - (c - x_i - \beta_j x_j)\, q_i - q_i^2 - \gamma x_i^2,\ i \neq j;\ i, j = 0, 1. \tag{1}$$

We assume that $\gamma \geq 1$ to ensure that the profit of the firms is positive in all cases. This assumption also ensures that the output and R&D levels of the firms are positive in all cases.

The producer surplus comprises the profits obtained by both firms: $PS = \pi_0 + \pi_1$. As usual, the consumer surplus is given by $CS = (q_0 + q_1)^2/2$. The social welfare function comprises the consumer surplus, $CS$, and the producer surplus, $PS$:

$$W = CS + PS = \pi_0 + \pi_1 + (q_0 + q_1)^2/2\ . \tag{2}$$

The timing of the game is as follows. In the first stage firms simultaneously decide their R&D levels non-cooperatively. After observing the decision taken in the first stage, in the second stage firms decide how much of the knowledge created in the first stage to disclose (i.e. firms $i$ and $j$ choose $\beta_i$ and $\beta_j$, respectively). Finally, in the third stage firms make quantity decisions.[10] We solve backwards to get a subgame perfect equilibrium.

First we consider a mixed duopoly with a domestic private firm, then we extend the model to consider a private duopoly and the case of a mixed duopoly with a foreign private firm. Finally, we compare the three cases.

## 3 MIXED DUOPOLY

We denote this case by superscript $M$. In the third stage firm 1 chooses the output level, $q_1$, that maximises its profit, given by (1) for $i$ = 1. Firm 0 chooses the output level, $q_0$, that

[10] Spillovers take the form of information-sharing, so firms first decide their cost-reducing R&D expenditures and then decide how much of the knowledge created in the first stage to make public.

maximises social welfare, given by (2). Solving these problems, the following first order conditions emerge:

$$\frac{\partial W}{\partial q_0} = a - c - 3q_0 - q_1 + x_0 + x_1\beta_1 = 0,$$
$$\frac{\partial \pi_1}{\partial q_1} = a - c - q_0 - 4q_1 + x_1 + x_0\beta_0 = 0.\text{[11]} \tag{3}$$

From (3), the outputs of the firms are obtained as a function of R&D levels and the disclosure of information by firms:

$$q_0 = (3a - 3c + 4x_0 - x_1 - x_0\beta_0 + 4x_1\beta_1)/11,$$
$$q_1 = (2a - 2c + 3x_1 - x_0 - x_1\beta_1 + 3x_0\beta_0)/11. \tag{4}$$

In the second stage firm *i* decides how much of the knowledge created in the first stage to disclose. Firm 1 chooses the disclosure level, $\beta_1$, that maximises its profit, given by (1) for $i = 1$, where $q_0$ and $q_1$ are given by (4). Firm 0 chooses the disclosure level, $\beta_0$, that maximises social welfare, given by (2). The first order conditions of these problems are the following:

$$\frac{\partial W}{\partial \beta_0} = 14\, x_0(2a - 2c + 3x_1 - x_0 - x_1\beta_1 + 3x_0\beta_0)/121 = 14\, x_0(q_1)/11,$$
$$\frac{\partial \pi_1}{\partial \beta_1} = -4\, x_1(2a - 2c + 3x_1 - x_0 - x_1\beta_1 + 3x_0\beta_0)/121 = -4\, x_1(q_1)/11. \tag{5}$$

The two firms produce a positive output, so $q_0$ and $q_1$ are positive. From (5) it emerges that $\partial W/\partial \beta_0 > 0$ and $\partial \pi_1/\partial \beta_1 < 0$ since $q_1 > 0$. Thus, the following result is obtained.

[11] The second order conditions are satisfied since $\partial^2 W/\partial (q_0)^2 = -3 < 0$ and $\partial^2 \pi_1/\partial (q_1)^2 = -4 < 0$.

*Proposition 1*: In the mixed duopoly, in equilibrium, the public firm fully discloses its knowledge and the private firm does not disclose its information, so $\beta_0^M$ = 1 and $\beta_1^M$= 0.

Proposition 1 shows that $\beta_0^M$ = 1 and $\beta_1^M$= 0, so the public firm fully discloses its knowledge created in the first stage, while the private firm does not disclose its knowledge. By disclosing information, the private firm affects the marginal cost of the public firm in two ways: First, it reduces the intercept of the marginal cost of production, $c - x_0 - \beta_1 x_1$. Second, the marginal cost of the public firm increases since it gains market share. The first effect dominates so the marginal cost of production of the public firm decreases with $\beta_1$. Thus, the private firm does not disclose information since its objective function is its own profit and by disclosing information it loses market share and profit at the expense of the public firm. By disclosing information the public firm reduces the marginal cost of production of the private firm, which raises the output of the industry, and thus the consumer surplus and social welfare. Therefore, as the objective function of the public firm is social welfare, it fully discloses its R&D knowledge. In this way the public firm tries to correct the underproduction arising from imperfect competition in the product market. By disclosing its information, the public firm achieves a reduction in the cost of production of the private firm which increases market competition, so the market failure due to imperfect competition is mitigated.

In the first stage of the game, firm 1 chooses the R&D level, $x_1$, that maximises its profit given by (1) for *i*=1. Firm 0 chooses the R&D level, $x_0$, that maximises social welfare given by (2). From the first order conditions of these problems, the following reaction functions in R&D levels emerge:[12]

$$x_0 = \frac{59(a-c)+28x_1}{242\gamma-59},\ x_1 = \frac{12(a-c+x_0)}{121\gamma-18}. \tag{6}$$

From (6) it results that $\partial x_0/\partial x_1 = 28/(242\gamma - 59) > 0$ and $\partial x_1/\partial x_0 = 12/(121\gamma - 18) > 0$, so R&D level are strategic complements. This means that the R&D level chosen by one firm increases with that chosen by the other. If the private firm increases its R&D

[12] The second order conditions are satisfied since $\partial^2 W/\partial(x_0)^2 = (59 - 242\gamma)/121 < 0$ and $\partial^2 \pi_1/\partial(x_1)^2 = 2(18 - 121\gamma)/121 < 0$.

level, its marginal cost of production decreases, which raises its output level and the output of the industry. However, from the point of view of the public firm, whose objective function is social welfare, the increase in the output of industry is not enough because the private firm does not disclose information ($\beta_1 = 0$). Thus, if the private firm increases its R&D level, $x_1$, the public firm reacts by also increasing its R&D level, $x_0$, to reduce the marginal cost of production of both firms (since $\beta_0 = 1$). This raises the output of the industry, the consumer surplus and social welfare.

When the public firm increases its R&D level, its marginal cost of production decreases, which raises its output. The output of the private firm also increases, since the public firm fully discloses its information. However, the increase is smaller than that of the public firm, so the private firm reacts by increasing its R&D level in order to increase its market share and profits.

It should be noted that the R&D level chosen by the public firm increases more than that of the private firm with the level of its rival. This is because the objective function of the public firm is social welfare, so it cares about the output of industry, while the objective function of the private firm is its own profit. As R&D levels are strategic complements, the private firm is encouraged to increase its R&D level due to R&D level of the public firm. However, by fully disclosing its information the public firm encourages the private firm to reduce its R&D level in order to reduce the total cost of investment in R&D.

From (1) to (6) and Proposition 1 the following emerges:

TABLE 1

RESULTS OBTAINED UNDER A MIXED DUOPOLY

| | Public firm | Private firm |
|---|---|---|
| R&D | $x_0^M = \frac{(a-c)(59\gamma-6)}{6-95\gamma+242\gamma^2}$ | $x_1^M = \frac{24(a-c)\gamma}{6-95\gamma+242\gamma^2}$ |
| Output | $q_0^M = \frac{6(a-c)\gamma(11\gamma-2)}{6-95\gamma+242\gamma^2}$ | $q_1^M = \frac{44(a-c)\gamma^2}{6-95\gamma+242\gamma^2}$ |
| Profits | $\pi_0^M = \frac{(a-c)^2\gamma(852\gamma-5065\gamma^2+4356\gamma^3-36)}{(6-95\gamma+242\gamma^2)^2}$ | $\pi_1^M = \frac{32(a-c)^2\gamma^3(121\gamma-18)}{(6-95\gamma+242\gamma^2)^2}$ |

| | |
|---|---|
| Output of industry | $Q^M = q_0^M + q_1^M = \frac{2(a-c)\gamma(55\gamma-6)}{6-95\gamma+242\gamma^2}$ |
| Welfare | $W^M = \frac{(a-c)^2\gamma(924\gamma-6961\gamma^2+14278\gamma^3-36)}{(6-95\gamma+242\gamma^2)^2}$ |

It is easy to see that $x_0^M > x_1^M$ since $\gamma \geq 1$. The public firm invests more in R&D to increase the output of the industry since its objective function is social welfare so it cares about the consumer surplus, while the objective function of the private firm is its own profit. The public firm has a higher marginal cost than the private firm, but because it cares about the consumer surplus it produces more: $q_0^M > q_1^M$. Finally $\pi_0^M > \pi_1^M$ if and only if $\gamma > 9.0818$. Therefore, the advantage in costs of the private firm (since $\beta_0^M = 1$ and $\beta_1^M = 0$) means that it earns more profits except when the inefficiency of R&D is great enough (i.e. if $\gamma > 9.0818$).

## 4 PRIVATE DUOPOLY

We now consider the case in which both firms are private, denoted by superscript *P*. In the third stage firm *i* chooses the output level, $q_i$, that maximises its profit, given by (1). Solving this problem, the following first order condition emerges:

$$\frac{\partial \pi_i}{\partial q_i} = a - c - 4q_i - q_j + x_i + x_j\beta_j = 0,\ i \neq j;\ i, j=0, 1. \tag{7}$$

[13]

From (7), the output of the firms is obtained as a function of R&D levels and the disclosure of information by firms:

$$q_i = (3a - 3c + 4x_i - x_j - x_i\beta_i + 4x_j\beta_j)/15,\ i \neq j;\ i, j=0, 1. \tag{8}$$

In the second stage firm *i* chooses the disclosure level, $\beta_i$, that maximises its profit given by (1). The first order conditions of this problem are the following:

$$\frac{\partial \pi_i}{\partial \beta_i} = -4\,x_i(3a - 3c + 4x_i - x_j - x_i\beta_i + 4x_j\beta_j)/225 = -4\,x_i(q_i)/15. \tag{9}$$

[13] The second order condition is satisfied since $\partial^2\pi_i/\partial(q_i)^2 = -4 < 0$, *i* =0, 1.

As $q_i$ > 0, it emerges from (9) that $\partial\pi_i/\partial\beta_i < 0$, $i$=0, 1. Thus, the following result is obtained.

*Proposition 2*: In the private duopoly, in equilibrium, firms do not disclose their knowledge, so $\beta_i^P = 0$, $i$ = 0, 1.

Proposition 2 shows that $\beta_i^P = 0$ so private firm *i* does not disclose its R&D knowledge. This leads to the same result as in Poyago-Theotoky (1999). By disclosing a positive amount of R&D knowledge, private firm *i* lowers the marginal cost of production of private firm *j*, which decreases its market share and profits. As a result, private firm *i* does not disclose information.

In the first stage of the game, firm *i* chooses the R&D level, $x_i$, that maximises its profit. From the first order condition of this problem, the following reaction functions in R&D levels emerge: [14]

$$x_i = \frac{8(3(a-c)-x_j)}{225\gamma-32},\ i \neq j;\ i, j=0, 1. \tag{10}$$

From (10) it results that $\partial x_i/\partial x_j = -8/(225\gamma - 32) < 0$, so R&D levels are strategic substitutes, unlike the mixed duopoly cases. This means that the R&D level of one firm decreases with that of the other. If private firm 0 increases its R&D level the marginal cost of production of private firm 1 decreases, but by less than in the mixed duopoly because in that case firm 0 is public and fully discloses its information while in the private duopoly firms do not disclose information. Thus, if a private firm increases its R&D level, $x_i$, private firm *j* reduces $x_j$ to reduce the cost of R&D and raise its profits. From (1), (2), (8), (10) and Proposition 2, the following emerges:

TABLE 2

RESULTS OBTAINED UNDER A PRIVATE DUOPOLY

[14] The second order condition is satisfied since $\partial^2\pi_i/\partial(x_i)^2 = 2(32 - 225\gamma)/225 < 0$, $i$ =0, 1.

| | Private firms |
|---|---|
| R&D | $x_1^P = x_2^P = \frac{8(a-c)}{75\gamma-8}$ |
| Output | $q_1^P = q_2^P = \frac{15(a-c)\gamma}{75\gamma-8}$ |
| Profits | $\pi_1^P = \pi_2^P = \frac{2(a-c)^2\gamma(225\gamma-32)}{(75\gamma-8)^2}$ |
| Output of industry | $Q^P = q_1^P + q_2^P = \frac{30(a-c)\gamma}{75\gamma-8}$ |
| Welfare | $W^P = \frac{2(a-c)^2\gamma(675\gamma-64)}{(75\gamma-8)^2}$ |

## 5 Mixed duopoly with a foreign-owned private firm

We now consider a mixed duopoly in which private firm 1 is foreign-owned. This case is denoted by superscript *F*. Domestic social welfare is now defined as:

$$W = CS + \pi_0. \tag{11}$$

Solving this case (see Appendix) the following result is obtained.

*Proposition 3*: In the mixed duopoly with a foreign private firm, in equilibrium, the public firm fully discloses its knowledge while the private firm does not, so $\beta_0^F = 1$ and $\beta_1^F = 0$.

As when the private firm is domestic-owned, the public firm fully discloses its R&D knowledge created in the first stage since this reduces the marginal cost of production of the private firm, which increases the output of the industry and, thus, the consumer surplus and social welfare. It also increases the profit of the foreign-owned firm, but this is not included in social welfare. The increase in the consumer surplus leads the public firm to fully disclose its knowledge to the foreign private firm. As in the other cases, the private firm does not disclose its knowledge.

## 6 Comparison of results

From the results in the above sections, the following result is obtained.

*Proposition 4*: In equilibrium, (i) $\beta_0^M = \beta_0^F = 1$ and $\beta_1^M = \beta_1^F = \beta_0^P = \beta_1^P = 0$; (ii) $x_0^M > x_0^F > x_0^P$, $x_1^M > x_1^P > x_1^F$ if $\gamma$<4.0901 and $x_1^P > x_1^M > x_1^F$ if $\gamma$>4.0901; (iii) $Q^F > Q^M > Q^P$; (iv) $\pi_1^M > \pi_1^F > \pi_1^P$ if and only if 1<$\gamma$<1.2437, $\pi_1^M > \pi_1^P > \pi_1^F$ if and only if 1.2437<$\gamma$<3.1313 and $\pi_1^P > \pi_1^M > \pi_1^F$ if and only if $\gamma$>3.1313; and (v) $W^M > W^P > W^F$.

From Propositions 1 to 3 it emerges that the disclosure of information chosen by the firms in the different cases considered is the following: $\beta_0^M = \beta_0^F = 1$ and $\beta_1^M = \beta_1^F = \beta_0^P = \beta_1^P = 0$. Therefore, the public firm always fully discloses its information independently of whether the private firm is domestic or foreign-owned, which means that the public firm fully discloses its knowledge even to a foreign private firm. This is because the public firm maximises social welfare, so it cares about the total output of the industry (and the consumer surplus) and not only about profits. Private firms care about their own profits, so they do not disclose information in any of the cases since this would reduce the marginal cost of production of the rival firm.

We find that the nationality of the owner of the private firm affects the R&D level of the public firm: $x_0^M > x_0^F > x_0^P$. Therefore, public firm 0 invests more in R&D if the private firm is domestic-owned than if it is foreign-owned. By investing more in R&D, the public firm reduces the marginal cost of production of the private firm (since $\beta_0 = 1$), which increases its profits. The objective function of the public firm is social welfare, which includes the profit of the private firm if it is domestic-owned, so the R&D level of the public firm is greater in that case. Moreover, firm 0 invests more in R&D if it is public than if it is private, since in the former case its objective function is social welfare and it cares about the consumer surplus. By investing more in R&D, the public firm lowers the marginal cost of production of both firms, which raises the output of the industry, the consumer surplus and social welfare. A privatised firm invests less in R&D since its objective function is now its own profits.

We also find that the investment decision of the private firm depends on whether it is domestic or foreign-owned (due to the behaviour of the public firm) and on the efficiency of the R&D technology as measured by parameter $\gamma$. If the efficiency of the R&D technology is high (i.e. when $\gamma < 4.0901$) we find that $x_1^M > x_1^P > x_1^F$; if the efficiency of the R&D technology is low (i.e. when $\gamma > 4.0901$) we find that $x_1^P > x_1^M > x_1^F$.[15]

In the mixed duopoly, firm 1 invests less in R&D for all $\gamma$ if it is foreign-owned (i.e. $x_1^M > x_1^F$). When firm 0 is public, R&D levels are strategic complements so the R&D level of firm 1 increases with that of firm 0. As firm 0 chooses a higher R&D level when it is publicly-owned and firm 1 is domestic-owned ($x_0^M > x_0^P$), the R&D level of firm 1 is greater when it is domestic-owned if the efficiency of the R&D technology is high (i.e. $x_1^M > x_1^P$). However, if the efficiency of the R&D technology is low, the high cost of R&D means that the R&D level of firm 1 is greater in the private duopoly (i.e. $x_1^P > x_1^M$).

The fact that the public firm fully discloses its information ensures that the output of the industry is greater in the mixed duopoly than in the private duopoly, and in the former case is greater if the private firm is domestic than if it is foreign-owned ($Q^F > Q^M > Q^P$). This means that social welfare is greater in the mixed duopoly than in the private duopoly when the private firm is domestic-owned ($W^M > W^P$). However, social welfare is lower in the mixed duopoly with a foreign private firm than in the private duopoly ($W^F < W^P$), becouse in the former case domestic social welfare only comprises the profit of firm 0 while in the second case it also comprises the profit of private firm 1.

In the mixed duopoly the profit of the private firm is greater if it is domestic-owned ($\pi_1^M > \pi_1^F$). In the mixed duopoly the public firm discloses its information regardless of the ownership of the private firm. However, the R&D levels chosen by the two firms in the mixed duopoly are higher if the private firm is domestic-owned ($x_0^M > x_0^F$ and $x_1^M > x_1^F$). Thus, in the mixed duopoly a domestic private firm has an advantage in costs over a foreign private firm, so it obtains greater profits.

In the mixed duopoly, regardless of the ownership of the private firm, the profit of the private firm is greater than in the private duopoly if and only if the efficiency of R&D is great enough ($\pi_1^M > \pi_1^P$ if and only if $\gamma < 3.1313$, and $\pi_1^F > \pi_1^P$ if and only if $\gamma < 1.2437$). This is because the public firm fully discloses its knowledge and chooses a greater level of R&D than in the private duopoly ($x_0^M > x_0^F > x_0^P$). The advantage in costs of the private firm in the mixed duopoly means that it earns more profits except when the efficiency of the R&D is low enough. This comparison is meaningful when firm 1 is considered as a potential

[15] It can be shown that $x_1^P = x_1^M$ for $\gamma = 4.0901$.

entrant firm that has to pay a fixed cost to enter the market. This case is analyzed in the next section.

## 7 Privatization and free entry

We now extend the game to consider that the private firm has to pay a fixed cost *f* to enter the market. We also analyze whether or not the government privatises the incumbent public firm. To do this we consider a five-stage game with the following timing: In the first stage the government decides whether or not the public firm is privatised. In the second stage the potential entrant determines whether or not to enter the market by incurring a fixed cost *f*. In the third stage firms simultaneously decide their R&D levels non-cooperatively; after observing this decision, in the fourth stage firms decide how much of the knowledge created in the first stage to disclose. Finally, in the fifth stage firms make quantity decisions.

We first analyze the game considering that the potential entrant is a domestic private firm. When the public firm is not privatised the results of the last three stages are the same as those obtained in Section 3, when it is privatised they are the same as Section 4. From Tables 1 and 2, taking into account that the entrant incurs a fixed cost *f*, the profits of the entrant firm in the mixed and private duopolies, respectively, are:

$$\pi_1^M = \frac{32(a-c)^2\gamma^3(121\gamma-18)}{(6-95\gamma+242\gamma^2)^2} - f = f^M - f,\ \pi_1^P = \frac{2(a-c)^2\gamma(225\gamma-32)}{(75\gamma-8)^2} - f = f^P - f.$$

In the second stage, the private firm enters the market if it obtains positive profits. There is entry under a mixed (private) duopoly if $f<f^M$ ($f<f^P$), so there is a public (private) monopoly if $f \geq f^M$ ($f \geq f^P$). This leads to the following result.

*Proposition 5*: In equilibrium, if the public firm is not privatised there is entry if $f<f^M$, so there is a public monopoly if $f \geq f^M$. If the public firm is privatised there is entry if $f<f^P$, so there is a private monopoly if $f \geq f^P$.

Proposition 5 shows that regardless of whether the public firm is privatised or not there is no entry if the fixed entry cost is high enough, so the market structure is a monopoly by the incumbent firm. It is easy to see that $f^M < f^P$ if and only if $\gamma$>3.1313. This is because market competition is stronger under a mixed duopoly than under a private duopoly, so the private firm obtains lower profits (without taking the fixed costs into account) in the former case. The results obtained in the public and private monopoly cases are relegated to the Appendix.

To solve the first stage of the game it is necessary to compare the social welfare obtained in the different cases. Solving the first stage of the game (see Appendix) gives he following result.

*Proposition 6*: In equilibrium, the public firm is never privatised, and there is entry if and only if $f < f^M$.

*Proof:* See Appendix.

In case of entry there is a duopoly in the product market, so market competition is weak. As a result, the public firm is never privatised since it would additionally decrease market competition, thus reducing social welfare. Therefore the incumbent is always a public firm. In that case there is entry only when $f < f^M$.

We also extend the game to consider that the potential entrant is foreign-owned. The results of the last three stages obtained in the mixed and private duopolies when the private firm is foreign-owned are shown in the Appendix. From Tables 2 and A1, taking into account that the entrant incurs a fixed cost $f$, the profits of the foreign entrant firm in the mixed and private duopolies, respectively, are:

$$\pi_1^F = \frac{144(a-c)^2\gamma^3(8\gamma-1)}{(3-44\gamma+144\gamma^2)^2} - f = f^F - f,\ \pi_1^P = \frac{2(a-c)^2\gamma(225\gamma-32)}{(75\gamma-8)^2} - f = f^P - f.$$

In the second stage, there is entry under a mixed (private) duopoly with a foreign private firm if $f<f^F$ ($f<f^P$), so there is a public (private) monopoly if $f \geq f^F$ ($f \geq f^P$). In the first stage we compare the social welfare obtained in the different cases and find the following.

*Proposition 7*: In equilibrium, when the potential entrant is foreign-owned, the public firm is never privatised, and there is entry if and only if $f<f^F$. Moreover, $f^F < f^M$.

*Proof:* See Appendix.

The explanation of the first result obtained in Proposition 7 is omitted because it is similar to that obtained in Proposition 6. The public firm produces more if the private firm is foreign-owned than if is domestic-owned. As a result, the private firm obtains greater profits in the second case, which explains why $f^F < f^M$. This means that there is less entry if the entrant is foreign-owned than if it is domestic-owned.

## 8 CONCLUSIONS

R&D investments in the context of a non-tournament R&D competition model with exogenous technological spillovers are an issue extensively studied in theoretical literature, especially with the assumption of imperfect market competition. The analysis conducted here assumes private and mixed oligopolies. One strand of these studies analyzes endogenous spillovers in a non-tournament R&D private duopoly. It is shown that a private firm never discloses any of its information when choosing its R&D non-cooperatively. As far as we know, this issue has not been studied under the assumption of a mixed duopoly, which is the main objective of this paper.

We find that in a mixed duopoly the public firm fully discloses its information when it chooses R&D investments levels non-cooperatively. It thus reduces the marginal cost of the private firm, which increases the output of the industry, the consumer surplus and social welfare. This result holds if the private firm is foreign-owned, so the public firm fully discloses its knowledge to a foreign-private firm. However, private firms do not disclose their knowledge.

We extend the previous analysis to consider the threat of entry faced by an incumbent public firm and whether or not it is privatised. We find that the public firm is not privatised, and in this case the private firm enters the market if the fixed cost of entry is low enough; if that cost is high enough there is a public monopoly. The same result is obtained if the entrant is foreign-owned. We also find that there is less entry if the entrant is foreign-owned than if it is domestic-owned.

## APPENDIX

*Mixed duopoly with a foreign-owned private firm*

In the third stage firm 1 chooses $q_1$, which maximises its profit, given by (1) for $i$=1, and firm 0 chooses $q_0$, which maximises social welfare, given by (11). Solving these problems reveals that: $q_0 = (a - c + x_0 + x_1\beta_1)/3$ and $q_1 = (2a - 2c + 3x_1 - x_0 - x_1\beta_1 + 3x_0\beta_0)/12$. In the second stage firm 1 chooses $\beta_1$, which maximises its profit and firm 0 chooses $\beta_0$, which maximises social welfare, given by (11). Solving reveals that $\partial W/\partial\beta_0 = x_0(q_1)/16>0$ and $\partial\pi_1/\partial\beta_1 = -x_1(q_1)/3<0$ since $q_1 > 0$, so $\beta_0^F=1$ and $\beta_1^F=0$. In the first stage, firm 1 chooses $x_1$, which maximises its profit and firm 0 chooses $x_0$, which maximises social welfare, given by (11). Solving these problems gives the following:

TABLE A1

RESULTS UNDER A MIXED DUOPOLY WITH A FOREIGN PRIVATE FIRM

| | Public firm | Private firm |
|---|---|---|
| R&D | $x_0^F = \frac{(a-c)(26\gamma-3)}{3-44\gamma+144\gamma^2}$ | $x_1^F = \frac{12(a-c)\gamma}{3-44\gamma+144\gamma^2}$ |
| Output | $q_0^F = \frac{6(a-c)\gamma(8\gamma-1)}{3-44\gamma+144\gamma^2}$ | $q_1^F = \frac{24(a-c)\gamma^2}{3-44\gamma+144\gamma^2}$ |
| Profits | $\pi_0^F = \frac{(a-c)^2\gamma(192\gamma-1108\gamma^2+1152\gamma^3-9)}{(3-44\gamma+144\gamma^2)^2}$ | $\pi_1^F = \frac{144(a-c)^2\gamma^3(8\gamma-1)}{(3-44\gamma+144\gamma^2)^2}$ |
| Output of industry | $Q^F = q_0^F + q_1^F = \frac{6(a-c)\gamma(12\gamma-1)}{3-44\gamma+144\gamma^2}$ | |
| Welfare | $W^F = \frac{(a-c)^2\gamma(210\gamma-1540\gamma^2+3744\gamma^3-9)}{(3-44\gamma+144\gamma^2)^2}$ | |

As $\gamma \geq 1$, it is easy to see that $x_0^F > x_1^F$, $q_0^F > q_1^F$ and $\pi_0^F < \pi_1^F$.

*Private duopoly with a foreign private firm*

This case is denoted by superscript *PF*. Given that both firms are private, the same result is obtained as in Section 4. Only the level of social welfare is different, since now firm 1 is foreign-owned so social welfare comprises only the profits of firm 0 and the consumer surplus. Thus, the following result is obtained:

$$W^{PF} = \frac{4(a-c)^2\gamma(225\gamma-16)}{(8-75\gamma)^2}.$$

*Monopoly*

The case of a public monopoly is denoted by superscript *MN*. Firm 1 does not enter the market, so $q_1 = 0$, $x_1 = 0$ and $\beta_0 = \beta_1 = 0$. Solving Section 3 with these assumptions gives the expressions for R&D, output, profits and welfare (see Table A2).

The case of a private monopoly is denoted by superscript *PN*. Firm 1 does not enter the market so $q_1 = 0$, $x_1 = 0$ and $\beta_0 = \beta_1 = 0$. Solving Section 4 with these assumptions gives the expressions for R&D, output, profits and welfare (see Table A2).

TABLE A2

RESULTS UNDER A MONOPOLY

| | Public monopoly | Private monopoly |
|---|---|---|
| R&D | $x_0^{MN} = \frac{a-c}{6\gamma-1}$ | $x_0^{PN} = \frac{a-c}{8\gamma-1}$ |
| Output | $q_0^{MN} = \frac{2(a-c)\gamma}{6\gamma-1}$ | $q_0^{PN} = \frac{2(a-c)\gamma}{8\gamma-1}$ |
| Profits | $\pi_0^{MN} = \frac{(a-c)^2\gamma(4\gamma-1)}{(6\gamma-1)^2}$ | $\pi_0^{PN} = \frac{(a-c)^2\gamma}{8\gamma-1}$ |
| Welfare | $W^{MN} = \frac{(a-c)^2\gamma}{6\gamma-1}$ | $W^{PN} = \frac{(a-c)^2\gamma(10\gamma-1)}{(8\gamma-1)^2}$ |

*Proof of Proposition* 6

From Tables 1, 2 and A2, taking into account the fixed entry cost, the following emerges:

TABLE A3

WELFARE IN EACH CASE WHEN THE POTENTIAL ENTRANT IS DOMESTIC-OWNED

| | Duopoly | Monopoly |
|---|---|---|
| Public | $W^M = \frac{(a-c)^2\gamma(924\gamma-6961\gamma^2+14278\gamma^3-36)}{(6-95\gamma+242\gamma^2)^2} - f$ | $W^{MN} = \frac{(a-c)^2\gamma}{6\gamma-1}$ |
| Private | $W^P = \frac{2(a-c)^2\gamma(675\gamma-64)}{(75\gamma-8)^2} - f$ | $W^{PN} = \frac{(a-c)^2\gamma(10\gamma-1)}{(8\gamma-1)^2}$ |

As $f^M$<$f^P$ if and only if $\gamma$>3.1313, when $\gamma$<3.1313, $W^M$ must be compared with $W^P$ for $f$<$f^P$, $W^M$ with $W^{PN}$ for $f^P$<$f$<$f^M$, and $W^{MN}$ with $W^{PN}$ for $f$>$f^M$. When $\gamma$>3.1313, $W^M$ must be compared with $W^P$ for $f$<$f^M$, $W^{MN}$ with $W^P$ for $f^M$<$f$<$f^P$, and $W^{MN}$ with $W^{PN}$ for $f$>$f^P$. It can be shown that in all cases social welfare is greater if the public firm is not privatised.

*Proof of Proposition* 7

From Tables A1, A2 and the value of $W^{PF}$ obtained in the private duopoly with a foreign firm, taking into account the fixed entry cost *f*, the following emerges:

TABLE A4

WELFARE IN EACH CASE WHEN THE POTENTIAL ENTRANT IS FOREIGN-OWNED

| | Duopoly | Monopoly |
|---|---|---|
| Public | $W^F = \frac{(a-c)^2\gamma(210\gamma-1540\gamma^2+3744\gamma^3-9)}{(3-44\gamma+144\gamma^2)^2} - f$ | $W^{MN} = \frac{(a-c)^2\gamma}{6\gamma-1}$ |
| Private | $W^{PF} = \frac{4(a-c)^2\gamma(-16+225\gamma)}{(8-75\gamma)^2} - f$ | $W^{PN} = \frac{(a-c)^2\gamma(10\gamma-1)}{(8\gamma-1)^2}$ |

It is easy to see that $f^F < f^P$ if and only if $\gamma$>1.2437. Thus, when $\gamma$<1.2437, $W^F$ must be compared with $W^{PF}$ for $f$<$f^P$, $W^{PN}$ with $W^F$ for $f^P$<$f$<$f^F$, and $W^{MN}$ with $W^{PN}$ for $f$>$f^F$. When $\gamma$>1.2437, $W^F$ must be compared with $W^{PF}$ for $f$<$f^F$, $W^{MN}$ with $W^{PF}$ for $f^F$<$f$<$f^P$, and $W^{MN}$ with $W^{PN}$ for $f$>$f^P$. It can be shown that in all cases social welfare is greater if the public firm is not privatised.

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