Electronic modeling of a PEMFC with logarithmic amplifiers

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Abstract

The main purpose of this project has been to simulate the behavior of a PEM-type fuel cell working in a stationary regimen using an equivalent circuit (EC). The EC designed with *Multisim* circuit design software reproduces the three characteristics sections of a polarization curve for a proton exchange membrane fuel cell (PEMFC). The main characteristic of this EC is that it offers the possibility to adapt the power range of the fuel cell in the simulated electronic model. To do so, a transconductance is used to allow adjusting the load current range. The EC allows fitting the simulated results to any commercial PEM fuel cell polarization curves and power ranges, adjusting parameters such as the charge current I_{FC} and the reversible voltage of the PEM fuel cell E_{rev}, and then setting the resistor values in the losses blocks and in the amplifiers.

Validation of the EC has been performed by simply adjusting the empirical data obtained with an Electrochem commercial 25 cm^2 active area PEMFC to the different analog blocks of the EC. Adjustments were carried out by using Mathcad 14 calculation algorithms.

Keywords: Fuel cell; PEMFC; Polarization characteristics; Dynamic equivalent circuit model; Multisim; Mathcad

Introduction

Polymer Electrolyte Fuel Cells (PEMFCs) have been the subject of numerous research projects for the last twenty decades due to its technological advantages; low operational temperature, easy and safe operational modes and a wide scope of application in power systems. Currently, a large majority of the models proposed for PEMFCs consist of mathematical equations which are not very useful in simulating systems with power converters [1–3]. The fuel cell polarization curve can be easily implemented in many circuit simulation softwares, by using Matlab-Simulink [4–10], Mathcad [11], FC power system simulator (FC-SIM) [12]and PSPICE [13],the lack of simplicity, due to the implementation of thermodynamic and energy conservation equations, makes it necessary to develop a more simple and versatile model that easily simulate PEMFC working at different powers.

The empirical curve that allows studying the performance of any PEMFC working in stationary state is known as the polarization curve [14–18,[19] [20]; [21] [22]; [23] [24]; [25],26,27]. In this work a simple and novel equivalent circuit model is shown to accurately simulate the entire polarization curve of a PEMFC.

From an electrochemical point of view, different types of fuel cell (DMFC, PEMFC, PAFC, MCFC and SOFC) generate in their performance a representative polarization curve V–I which follows logarithmic expressions when representing the activation and mass transport losses. In this work a simulation model with logarithmic electronic components that generate the same response V–I of a PEM type fuel cell has been developed, in which the simulated curves fit the experimental curves of a trial. Moreover, by using the representative expression of each type of fuel cell and the logarithmic electronic components, other simulation models could be implemented. If the components of the simulation model are implemented in the physical domain of electronic hardware, the same electrical response V–I should be obtained within the power range in which all electronic elements of the model are established.

A fuel cell is a complex system, involving different physical phenomena: thermal, electrochemical, electrical, etc. Several auxiliary components are required to operate correctly, e.i. the hydrogen and oxygen system supplies, the cooling system and humidification system. The lack of guaranty in the product, its price, hydrogen costs and operational lifetime of PEM fuel cell system are limiting factors when used as a laboratory element. To overcome such limiting factors, replacement of PEM fuel cell system by a hardware emulator capable of accurately copying their behavior, could be carried out. In relation to this, the simple electronic circuit model shown in this work could be useful for a designer in power electronics.

The reason for implementing the function of the polarization curve with electronic elements such as differential amplifiers, logarithmic amplifiers, resistors, current sources, is that all these electronic elements used in the simulation are transferable to hardware element by element, to reproduce the same electrical response as that obtained in a polarization curve of the PEMFC. Each element of each electronic component model exactly represents the function of the polarization curve. The same function can be implemented in the Matlab/Simulink environment, which must then be converted and compiled into a C-program, which is finally programmed into a dSPACE and/or DSP controller. To electrically reproduce that polarization curve programmed in C-program with hardware elements, the controller takes control of Duty Cycle of an IGBT transistor, PWM control to regulate the output voltage and current of a DC/DC Buck converter type, as can be seen in some of the PEMFC emulators referenced in this paper [28–32]. In such emulators, solutions can not be identified through the hardware elements used e.i. the different components of the polarization curve, as this is only programmed and is not related as an equivalent circuit itself.

Besides all these variety in emulators one thing in common is that they use large amount of hardware and complex software when simulating the behavior of a PEM fuel cell such both in their waveforms as in the electrical parameters; voltage, current and power. On the contrary, the simple electronic circuit model shown in this work uses fewer components to simulate the behavior of the fuel cell in their waveforms. A transconductance is proposed in order to simulate a charge demanding current from the PEMFC. Also, to adapt the power range of the fuel cell in the simulated electronic model, a current source controlled by a voltage source is used.

To implement the proposed electronic model circuit and to emulate the electric parameters of the PEMFC, the following hardware components would be needed:

Power Supply HM8040-3 HAMEG (356 €), Programmable Power Supply AIM-TTI QPX1200SP (1590 €), Solderless Breadboard 3M (150 €) AMP LOG101 (35 €), OP AMP UA741CPE4 (3 €), some resistors and capacitors. Total cost = 2131 €.

Thus, in the proposed equivalent electronic model such behaviors are emulated by logarithmic amplifiers LOG100 [33] and LOG101 [34]. These devices allow the modeling of the activation and mass transport losses region of a PEMFC when simulating its behavior starting from a characteristic current load, I_{FC}. Moreover, resistive load have been used to electronically simulate PEMFC ohmic losses. Thus, the electrochemical processes that govern a PEMFC behavior can be related with the representative equation of each electronic device used in the EC.

The design of the EC was developed using a Multisim Power Pro Edition/Full Edition de National Instruments (version 10.0.144) environment. Experimental data, were obtained from commercial Electrochem 25 cm^2 active area PEMFC, (Electrochem, Ref: EFC25-02SP) using the 850e station from Scribner Associates Fuel Cell Test System and a 885 Fuel Cell Potentiostat, Fig. 1, which will be used to validate the EC model. The conditions used are: oxygen flow of 0.5 l/min and atmospheric pressure of reagents. Mathcad's 14 algorithms were used in order to adjust the parameters of the electronic model developed to the empirical data obtained.

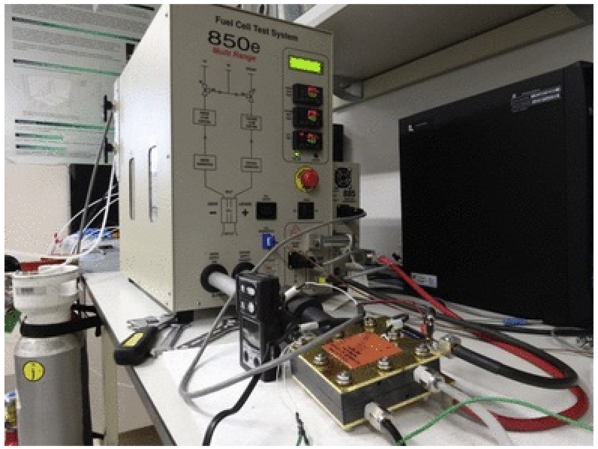

Fig. 1 Setup for the controlled PEM fuel cell test.

Electrochemical approach

Two commonly used modeling approaches can be found in literature concerning the simulation of PEMFCs performance: (a) mechanistic approaches and (b) empirically-based approaches. The former describes almost all the electrochemical and physicochemical aspects involved in a PEMFC performance [6–9]. On the other hand, the latter can predict the polarization curve of a PEMFC at different operating conditions, at least when reproduced in a small operating range, by only consideration three different polarizations in the representative fuel cell equation [14].

$$V_{FC} = E_{rev} - \eta_{act} - \eta_{ohmic} - \eta_l \quad (1)$$

where V_{FC} and E_{rev} are the output voltage and the reversible voltage of the fuel cell, respectively. The reversible voltage is calculated using [10]:

$$E_{rev} = E^o + \frac{RT}{2F}\ln\left(\frac{P_{H_2}\cdot P_{O_2}^{\frac{1}{2}}}{P_{H_2O}}\right) \quad (2)$$

Where R is the universal gas constant, T is the fuel cell temperature, F is the Faraday's constant and P_i the partial pressures of reactant and product, assuming an ideal gas behavior. Concerning the polarization processes, the activation losses η_{act} are caused by the sluggishness of the reaction on the surface of the electrodes. The rate of these electrochemical reactions can be expressed by the Tafel equation [35] as follows:

$$\eta_{act} = \frac{2.3RT}{\alpha nF}\ln\left(\frac{I_{FC}}{I_O}\right) \quad (3)$$

Where IFC is the output current of the fuel cell, α is the transfer coefficient, n is the number of electrons involved in the reaction and I_0 is the exchange current. The ohmic overvoltage can be expressed as:

$$\eta_{ohmic} = I_{FC}R^{\text{int}} \quad (4)$$

Where, R^{int}, is defined as the sum of the electric and proton transfer resistances. The concentration losses η_l represents the exhaustion of the reactants at the surface of the electrodes as the fuel is consumed. Output voltages of PEMFCs can be modified when fed with a mixed flow of gases (e.g., 0_2, N_2, and vapor H_20 in the cathode or little purified H_2 in the anode) and consumption of any of the reactants at the electrode surfaces takes place, due to concentration gradients and reduction of the partial pressure processes. Defining a limiting current, I_L, as the current achieved when the reactant is consumed at a rate equal to the maximum supply flow, and making the assumption that the partial pressure falls linearly down to zero when the fuel cell current I_{FC} is increased from zero to this limiting current, then the expression for the overpotential due to concentration losses is given by Ref. [10]:

$$\eta_l = -\frac{RT}{nF}\ln\left(1 - \frac{I_{FC}}{I_l}\right) \quad (5)$$

Other irreversible process to take into account concerning the output voltage drop in the low current region is the internal current losses, I_n. Such currents are related to the quality of the polymer membranes and the active area of the fuel cell membrane and, which may result in electron crossing from the anode to the cathode. Therefore, in order to model such currents, it is convenient to include their values to the total fuel cell current as ($I_{FC} + I_n$). A precise approximation of the value of internal current densities is ±2 mA cm^{-2} [10].

Thus, considering all the potential losses, the final expression for a PEMFC voltage can be represented as:

$$V_{FC} = E_{rev} - A\ln\left(\frac{I_{FC}}{I_O}\right) - R^{\text{int}}I_{FC} + B\ln\left(1 - \frac{I_{FC}}{I_l}\right) \quad (6)$$

Where, A, is the activation polarization constant and B the concentration polarization constant. These constant values are obtained by means of experimental processes.

Equivalent circuit model

The empirical model of the PEMFC, Eq. (6) can be linked to an electronic circuit and thus, emulate the internal electrochemical behavior of the PEMFC with analog signals obtaining its representative output voltage and current signals. To carry out such an issue, logarithmic amplifiers with active components have been used, as well as operational amplifiers following a differential configuration, summing amplifier, inverting and non-inverting amplifiers, as well as voltage followers.

Commercial logarithmic amplifiers

In order to electronically reproduce the two logarithmic sections of an operational PEMFC i.e. activation polarization and mass polarization regions, logarithmic amplifiers in its commercial integrated circuit version from Texas Instrument LOG 100 or LOG101 Fig. 2, have been used.

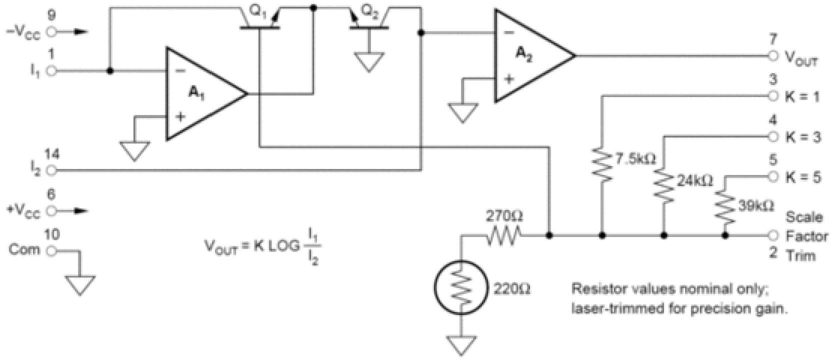

Fig. 2 Integrated circuit LOG100 (by courtesy of Texas Instrument).

The ideal transfer function of this integrated circuit is:

$$V_{OUT} = k\log\frac{I_1}{I_2} \tag{7}$$

Where k is the scale factor with units of volts/decade, I_1 is the numerator input current and I_2 the denominator input current. Fig. 3(a) shows both, the operating current ranges of the logarithmic amplifier, and the transfer function with varying k and I_1. In order to implement the natural logarithms of Eq. (6), the logarithmic relation of the transfer function of the integral circuit LOG100 must be adjust in its k constant to a value of 2.3.

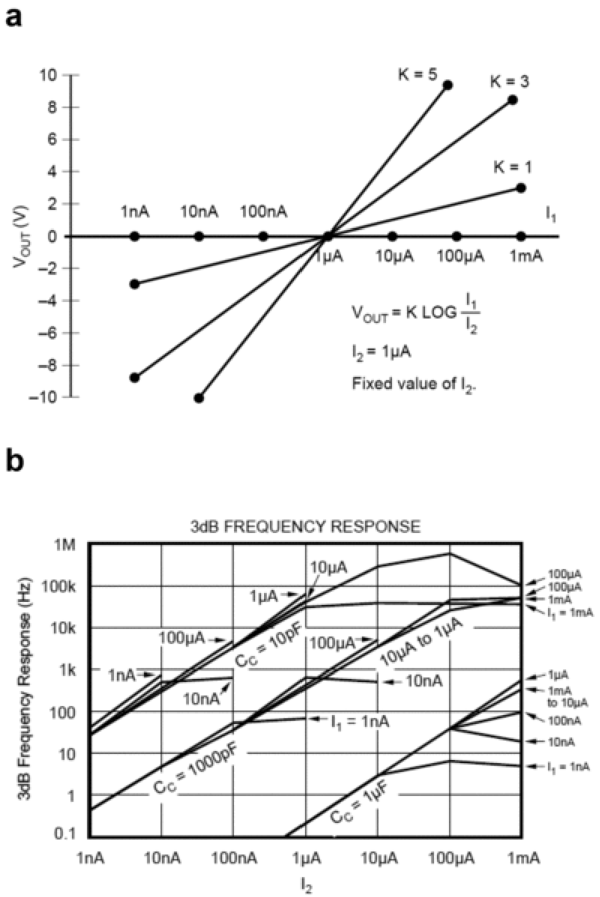

Fig. 3 (a) Dependence of the transfer function of LOG100 amplifier with K and I_1 parameters (by courtesy of Texas Instrument). (b) Frequency compensation and current relationship as a function of the capacitor used C_c (by courtesy of Texas Instrument).

As the commercial values fixed for k are 1, 2 or 5, a new resistor (10.3 kΩ) has been externally inserted between pins 3 and 7 of the LOG100 in order to obtain a constant value for k of 2.3, and thus, to ensure the natural logarithm correlation. Moreover, the use of a capacitor, C_c, between pins 7 and 14 has been used in order to frequency compensation. The size of the capacitor is a function of the input currents. Thus, the smallest value of the capacitor is determined by the maximum value of I_2 and the minimum value of I_1. Larger values of C_c will make the LOG100 more stable, but will reduce the frequency response. In this study a typical value of 150 μF has been used. Fig. 3(b) shows the frequency response and the current relationship (I_1, I_2) dependence on the capacitor size used for frequency compensation.

Base model of the logarithmic amplifier

As it was mention before, the value of k may be changed by increasing or decreasing the voltage divider resistor normally connected to the output. Among all the possible circuit configurations, connection of pin 7, corresponding to V_{out}, to any of the 3, 4, or 5 pins, allows achieving values of scale factor of k = 1, k = 3 or k = 5. The values of k are of key importance since they establish the quantity (volts/decades) in which the V_{out} will vary. Fig. 4 shows the simplified model, representative of any of the three different configurations.

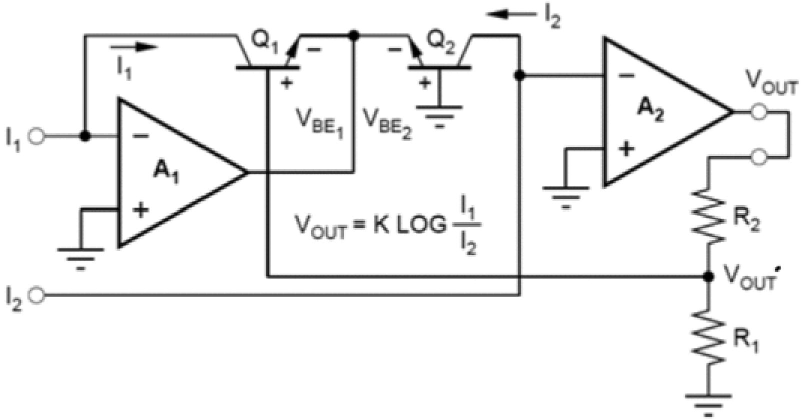

Fig. 4 Simplified model of the integrator LOG100 (by courtesy of Texas Instrument).

The ideal transfer function of the logarithmic integrated circuit can be obtained through the voltage relationships between the components of the base model.

Thus, the theory of operation of the amplifier relies on the relationship between the base-emitter voltages of the transistors Q_1 and Q_2. Moreover, at the base of transistor Q_1 the voltage value, $V_{OUT'}$, is generated as a result of summing the base-emitter voltages (V_{BEi}). This $V_{OUT'}$ is expressed as:

$$V_{OUT'} = V_{BE_1} - V_{BE_2} \quad (8)$$

The base-emitter voltage of a bipolar transistor is:

$$V_{BE} = V_T \ln\frac{I_C}{I_S} \quad (9)$$

Where V_T the temperature dependence of voltage, I_C is the collector current and I_s, the reverse saturation current. V_T is related with the absolute temperature, as shown in Eq. 10

$$V_T = \frac{KT}{q} \quad (10)$$

Where K is the Boltzman's constant and q, is the electron charge.

Substituting Eq. (10) into Eq. (9) yields

$$V_{OUT'} = V_{T_1}\ln\frac{I_1}{I_{S_1}} - V_{T_2}\ln\frac{I_2}{I_{S_2}} \quad (11)$$

In case the transistors are isothermal ($V_{T1} = V_{T2}$) and matched, i.e. both have the same reverse saturation current (SC), which occurs when they are integrated on the same chip, then Eq. (11) can be written as:

$$V_{OUT'} = V_T \ln\frac{I_1}{I_2} \quad (12)$$

Or, in order to reproduced the ideal transfer function, V_{OUT}

$$V_{OUT'} = nV_T \log\frac{I_1}{I_2} \quad (13)$$

where n values is 2.3

On the other hand, by taken into account the divider resistor between V_{OUT} and $V_{OUT'}$, in which resistors R_1 and R_2 are involved, the scale factor can be fitted as:

$$V_{OUT} = V_{OUT'}\frac{(R_1 + R_2)}{R_1} = \frac{(R_1 + R_2)}{R_1} nV_T \log\frac{I_1}{I_2} \quad (14)$$

$$V_{OUT} = k\log\frac{I_1}{I_2} \quad (15)$$

Base equivalent circuit (BEC)

To obtain the different voltage losses values which appear plotted for a conventional PEMFC polarization curve, a charge increasing the current demand from the PEMFC across the current working range is needed. To simulate such charge, a pulse source that works producing a periodic voltage ramp-up, and whose rise time is equal to the signal period, is proposed. Moreover, the pulsed value is equal to the I_{FC} charge current working range. Thus, the pulse source attacks a voltage-controlled current source, whose output current depends on the voltage applied at the input terminals. Both, the input voltage (ΔV_{in}) and output current (ΔI_{out}) are related by a parameter called transconductance (G), which is the ratio of the output current to the input voltage (in direct current circuits), eq. (16). It is measured in mhos (also known as Siemens) S (1 SIEMENS = 1 A per volt), and can have any value from mmhos to kmhos.

$$g_m = \frac{\Delta I_{out}}{\Delta V_{in}} \quad (16)$$

Working at different power values can be achieved by changing both, the pulsed value in the pulse source and the PEMFC's E_{rev} value. Moreover, values of V_{sample} can be obtained for a particular I_{FC} value when using different resistor values, R_{sample}, considered as resistor samples. By means of amplifying V_{sample} values, V_{in} values can be obtained. Thus, all the voltage losses are obtained by applying V_{in} voltage to the different resistive, activation and concentration voltage losses blocks. After summing all the voltage losses obtained according to I_{FC} values, a subtraction is done between the reversible voltage of the PEMFC, E_{rev}, and the sum of all voltage losses. Thereby, this method allows getting all the real voltage values of the PEMFC, V_{FC}, in each section of a PEMFC polarization curve. Fig. 5, shows the structure of the BEC.

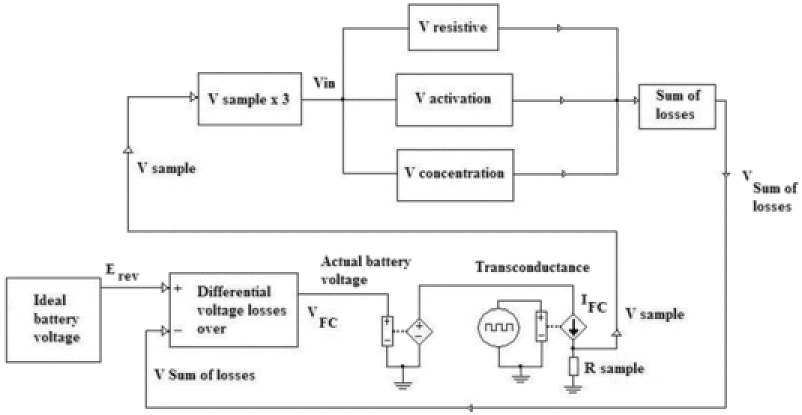

Fig. 5 Base model of the PEMFC equivalent circuit.

Full equivalent circuit

In this section, the different elements making up the total equivalent circuit are discussed. These elements or subcircuits are shown in Fig. 6, and relate to: the summing voltage element, the amplifier V_{sample}, the resistive losses, the activation losses, the concentration losses, and the differential subcircuits.

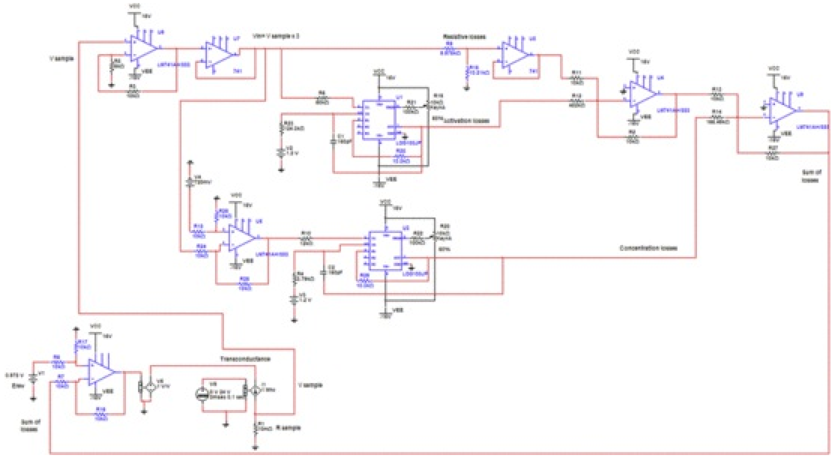

Fig. 6 Full equivalent circuit of the PEMFC equivalent circuit.

Amplifier subcircuit V_{sample}

Values of V_{in}, subsequently used by different analog blocks to calculate the total polarization losses, are obtained by amplifying V_{sample} voltage using a non-inverting amplifier with a gain factor of 3. In order to eliminate voltage losses in V_{in} values, this subcircuit has a voltage follower (unit buffer amplifier) that matches impedance values, Fig. 7(a).

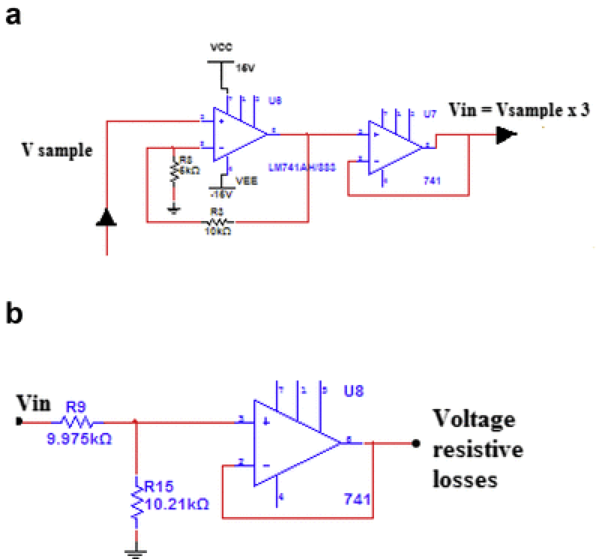

Fig. 7 (a) Amplifier subcircuit, V_{Sample}. (b) Ohmic losses subcircuit.

Resistive losses subcircuit

The first analog block simulating the polarization losses is the resistive losses subcircuit, Fig. 7(b). As can be observed, the voltage values corresponding to resistive losses are obtained by means of a voltage divider formed by R15 and R9. In order to match the impedance values a voltage follower (unit buffer amplifier) is placed before the summing losses subcircuit, Fig. 9.

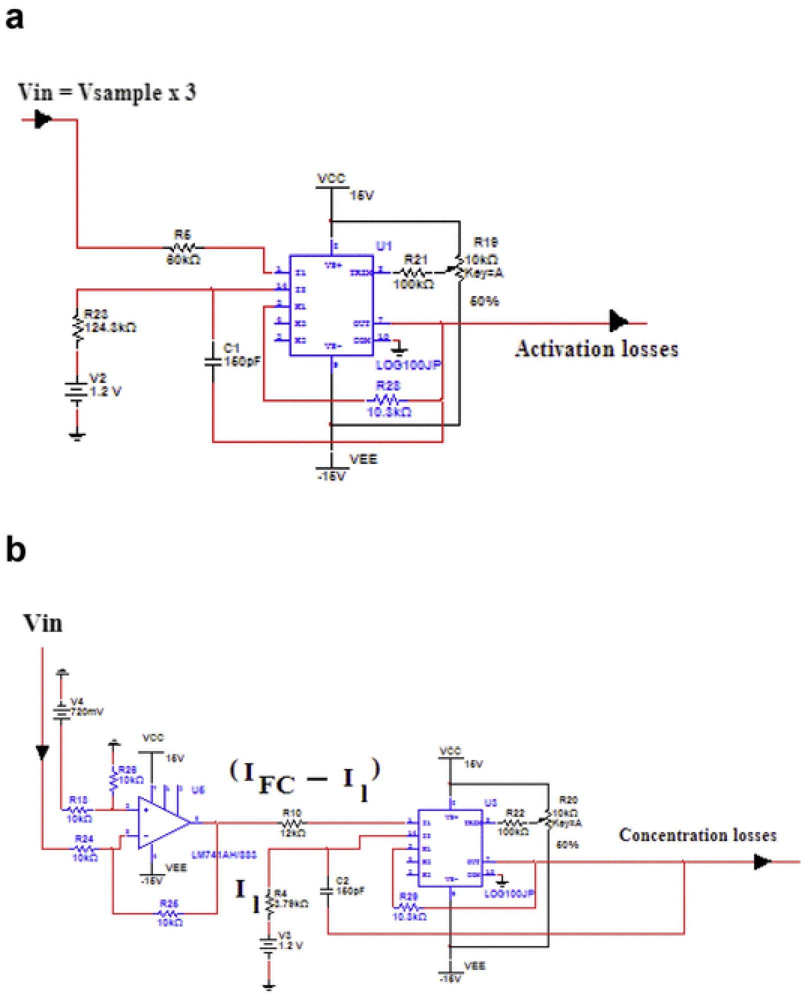

Fig. 8 (a) Activation losses subcircuit. (b) Concentration losses subcircuit.

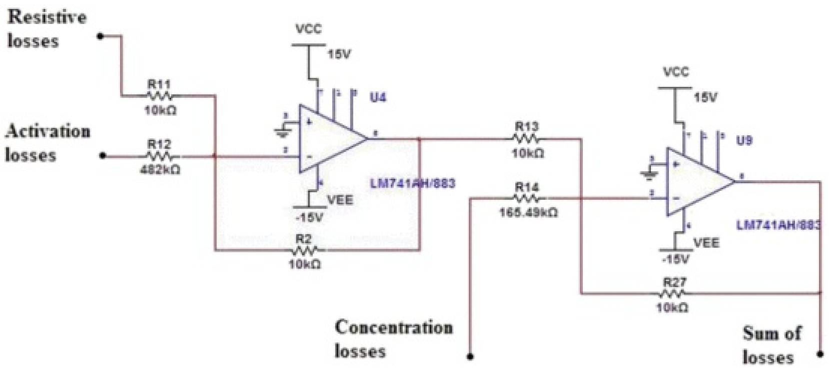

Fig. 9 Summing losses subcircuit.

Activation losses subcircuit

The second analog block, the activation losses subcircuit, is shown in Fig. 8 (a). Input current values, I_1 and I_2, can be obtained when V_{in} values and a fixed voltage value of 1.2 V, and are both divided by resistor R5 and, R23 respectively. These constitute the input currents to the first block.

Thus, I_1 and I_2 will represent to I_{FC} and I_0 in Eq. (17), respectively, allowing emulating the PEMFC activation losses.

$$\eta_a = A\ln\left(\frac{I_{FC}}{I_O}\right) \tag{17}$$

Concentration losses subcircuit

The third analog block is that which reproduces the concentration polarization, Fig. 8 (b). In a first step and starting from V_{in} values and reference voltage values of 1.2 V and 720 mV, the expression (V_{in}-720 mV) is calculated. Dividing such difference by resistor R4, the input current, I_1, is obtained. On the other hand, dividing the reference voltage value (1.2 V) by resistor R10, the input current, I_2, is obtained. Both, I_1 and I_2, are input current values used for the activation losses subcircuit.

It must be taken into account that in this subcircuit model I_1 and I_2 represent the numerator ($I_L - I_{FC}$) and denominator I_L in Eq. (18), respectively.

$$\eta_l = -B\ln\left(1-\frac{I_{FC}}{I_l}\right) \tag{18}$$

Summing of voltage losses

All the previous polarization losses will be summed weighting each voltage contribution by means of fully adjustable amplification factors, as expressed by Eqs. 19, and 20. The subcircuit developed for the summing of the voltage losses is represented in Fig. 9.

$$V_{lossess} = V_{resistive}\cdot\frac{R_2}{R_{11}} - V_{activation}\cdot\frac{R_2}{R_{12}} + V_{concentration}\cdot\frac{R_{27}}{R_{14}} \tag{19}$$

$$V_{losses} = V_{resistive}\cdot\frac{10k\Omega}{10k\Omega} - V_{activation}\cdot\frac{10k\Omega}{482k\Omega} + V_{concentration}\cdot\frac{10k\Omega}{165k\Omega} \tag{20}$$

PEMFC differential voltage subcircuit and PEMFC working current adjusting subcircuit

Fig. 10 shows a block consisting of two analog subcircuits. The first subcircuit includes a differential amplifier which is used to measure voltage differences between the voltage value of the ideal battery, E_{rev}, and the sum of all voltage losses of the PEMFC. This first subcircuit controls a voltage-controlled source eq. (21), which gain value E is 1, and which gives V_{FC} voltage values [36,37].

$$\mathrm{E} = \frac{\mathrm{V_{OUT}}}{\mathrm{V_{IN}}}; 1 = \frac{\mathrm{V_{FC}}}{(\mathrm{E_{rev}} - \sum \mathrm{V_{losses}})} \tag{21}$$

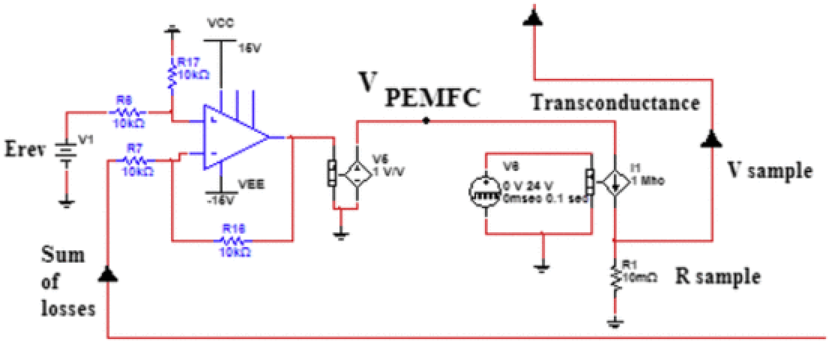

Fig. 10 Differential subcircuit: battery and loss control *vs* load current.

The function of the second subcircuit is to adjust the PEMFC working current. To adapt the power range of the fuel cell in the simulated electronic model, a transconductance is proposed. This would simulate a charge demanding current from the PEMFC, and would adapt the power range of the fuel cell in the simulated electronic model. Thus, it is a current source controlled by a voltage source. By applying a ramp voltage source, the controlled current source generates a current ramp enabling adjustment of the load current range (power level). To implement this transconductance in the physical domain and to get a range of adjustable current, programmable current source was used. As previously mention in Section 3.3, a pulse source that works producing a periodic voltage ramp-up, and whose rise time is equal to the signal period, is proposed in order to simulate a charge demanding current from the PEMFC. Thereby, changing the nature of the voltage pulse allows working at different powers, changing I_{FC} working range, and the E_{rev} value.

Finally, values of V_{sample} can be obtained for a particular I_{FC} value when using different resistor values, R_{sample}, considered as resistor samples. This sample voltage value, after being amplified, see Fig. 7(a), will be converted into V_{in}, which correspond to the voltage values that are used to carry out the subsequent analogical operation for the different polarization losses measurements.

The classic polarization curve of a PEMFC and the current load evolution *versus* time has been simulated using the complete equivalent circuit. Fig. 11, depicts the polarization curves for experimental and simulated values of the PEMFC tests developed at 40 °C and 70 °C. In this case to carry out this simulation, resistive values of the equivalent circuit are previously obtained by performing the adjustments of the equivalent circuit response to the commercial Electrochem PEMFC polarization curves at 40 °C and 70 °C using Mathcad 14, see Section 4.

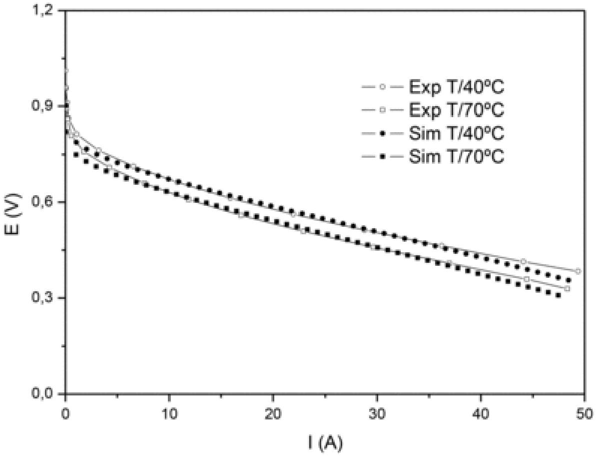

Fig. 11 Experimental and simulated polarization curves at 40 °C and 70 °C. 25 cm^2 active area PEMFC oxygen flow 0.5 l/min at atmospheric pressure of reagents.

Adjustment of the equivalent circuit response to the polarization curve of a commercial PEMFC

Mathematical models of the analog blocks

In this section, mathematical models of the different analog blocks, which make up the equivalent circuit, as well as the steps followed to get all the parameter values, are shown.

The sample voltage, V_{sample} can be obtained by multiplying a fixed range of current load value (from 0 A to 48 A), by a low sample resistor value, R_{sample}, of 10 mΩ.

$$V_{sample} = R_{sample} \cdot I_{FC} = 0.01 \cdot I_{FC} \quad (22)$$

For the different I_{FC} values, V_{sample} varied between 0 and 480 mV. Therefore, to work in a higher voltage range V_{sample} has been amplified using a non-inverting amplifier by a factor of 3, Fig. 7(a), with the range of output voltage values, V_{in} between 0 V and 1.44 V.

$$V_{in} = V_{sample} \cdot 3 = 0.01 \cdot I_{FC} \cdot 3 = 0.03 \cdot I_{FC} \quad (23)$$

This wider voltage range for V_{in} has allowed obtaining the voltage loss for each section of the polarization curve, as a function of the load current.

Using a resistive divider formed by R15 and R9, Fig. 7(b), the voltage losses corresponding to the ohmic resistance of the battery, $V_{resistive}$, have been obtained. Eq. (24) shows the mathematical expression of this element.

$$V_{resistive} = V_{in} \frac{R15}{(R15 + R9)} \quad (24)$$

So as to get the activation losses of the PEMFC as a function of current load, I_{FC}, V_{in} has been applied to the numerator of the integrated circuit LOG100JP (see Fig. 8(a)). Thus, the current input used in the numerator I_1, is a function of the resistor R5 and the range voltage of V_{in}. On the other hand, the current of denominator I_2, is a function of resistor R23 and the reference voltage value, 1.2 V. Therefore, the logarithmic activation losses for the first section of the PEMFC polarization curve are defined in function of I_{FC} as:

$$V_{activation} = 2.3\cdot\log\left(\frac{\frac{V_{in}}{R5}}{\frac{1.2}{R23}}\right) \quad (25)$$

Regarding the mass losses section of the polarization curve, a differential voltage has been used in the numerator of the second integrated circuit LOG100JP, which value correspond to the difference between the maximum limit voltage value of V_{in} (720 $0\cdot10^{-3}$ V) and V_{in}. See Fig. 8(b). Moreover, the current input value of the numerator for the second integrated circuit LOG100JP is a function of the resistor R10 and the differential voltage of ($720\cdot10^{-3}$–V_{in}). The denominator current, I_2 is a function of the resistor R4 and the reference voltage value 1.2 V. Thus, logarithmic concentration losses for the PEMFC as a function of I_{FC} can be expressed as:

$$V_{concentration} = 2.3\cdot\log\left[\frac{\frac{\left(720\cdot10^{-3}-V_{in}\right)}{R10}}{\frac{1.2}{R4}}\right] \quad (26)$$

Fig. 9 represents the sum of the total voltage losses using summing amplifiers.

With regard to the ohmic losses, a gain value of 1 has been applied. This value corresponds to the same value obtained for the resistor relationship that the resistive losses produce when summing the total losses. Moreover, when adjusting the transfer function of the equivalent circuit to the Electrochem curve using Mathcad software, resistive values for R11 and R13 are obtained.

$$\frac{R_2}{R_{11}}\cdot\frac{R_{27}}{R_{13}} = \frac{10k\Omega}{10k\Omega}\cdot\frac{10k\Omega}{10k\Omega} = 1 \quad (27)$$

Therefore, a gain value of 0.034965 has been applied to the activation losses the same value obtained for the resistor relationship that the activation losses produce when summing the total losses. When adjusting the transfer function of the equivalent circuit to the Electrochem curve using Mathcad software, resistive values for R12 and R13 can be obtained.

$$\frac{R_2}{R_{12}}\cdot\frac{R_{27}}{R_{13}} = \frac{10k\Omega}{286k\Omega}\cdot\frac{10k\Omega}{10k\Omega} = 0.034965 \quad (28)$$

Regarding the gain for the mass polarization, a value of 2.577 has been used. This value corresponded to the same value obtained for the resistor relationship when summing the total losses. So, when adjusting the transfer function of the equivalent circuit by using Mathcad to the Electrochem curve, the resistive values obtained corresponded to R27 and R14.

$$\frac{R_{27}}{R_{14}} = \frac{10k\Omega}{3.88k\Omega} = 2.577 \quad (29)$$

Once calculated all the resistive relationships, summing of the total voltage losses of the equivalent circuit is expressed as shown in Eq. (30).

$$V_{losses} = V_{resistive} - V_{activation}\cdot\frac{R_2}{R_{12}} + V_{concentration}\cdot\frac{R_{27}}{R_{14}} \quad (30)$$

Summing of the total voltage losses by means of summing amplifiers was previously shown in Fig. 9.

The real equivalent circuit voltage, taking into account such losses, see Fig. 10, is expressed as.

$$V_{PEMFC} = E_{rev} - V_{losses} \quad (31)$$

Mathcad adjustment of the equivalent circuit response to the Electrochem PEMFC polarization curve

$$\mathrm{f}(\mathrm{I}_{FC}, \mathrm{R}_i) = \mathrm{E}_{rev} - \frac{(3\cdot0.01\cdot R15)}{(R15+R9)} - \frac{10}{R12}\cdot2.3\cdot\log\left(\frac{\frac{3\cdot0.01\cdot\mathrm{I}_{FC}}{60}}{\frac{1.2}{R23}}\right) + \frac{10}{R14}\cdot2.3\cdot\log\left[\frac{\frac{\left(720\cdot10^{-3}-3\cdot0.01\cdot\mathrm{I}_{FC}\right)}{12}}{\frac{1.2}{R4}}\right] \quad (32)$$

The mathematical expression representing the equivalent circuit performance designed to simulate an operational PEMFC curve is:

As can be seen, this function, $f(I_{FC}, R_i)$, is I_{FC} and R_i values dependent. The latter values make it possible not only the adjustment of the different losses values, but also to set the gain values to be applied to each analog block within the total sum of the circuit losses.

As mentioned before, validation of $f(I_{FC}, R_i)$, was conducted using the experimental values obtained from a commercial 25 cm^2 active area PEMFC (Electrochem, Ref: EFC25-02SP) at 40 °C and 75 °C and an oxygen flow of 0.5 l/min at atmospheric pressure of reagents. Fig. 12, shows the experimental and simulation values for the polarization and power curves at 40 °C and 70 °C.

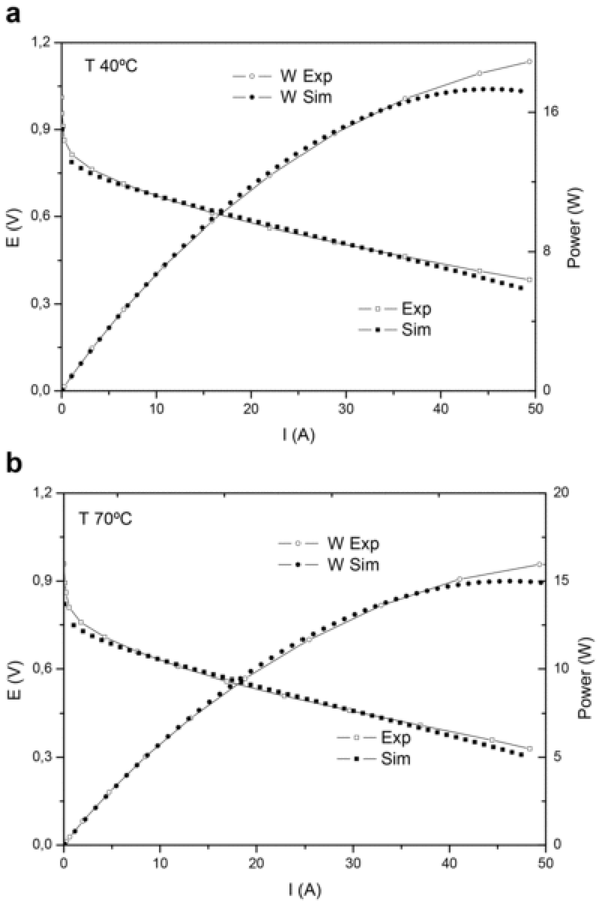

Fig. 12 (a) Experimental and simulated polarization and power curves at 40 °C. 25 cm^2 active area PEMFC oxygen flow 0.5 l/min at atmospheric pressure of reagents. (b) Experimental and simulated polarization and power curves at 70 °C. 25 cm^2 active area PEMFC oxygen flow 0.5 l/min at atmospheric pressure of reagents.

Table 1 shows the initialization and set values obtained for different resistive values of the equivalent circuit. Resistors values (R_i) obtained are within a valid range, which allows the operation of the equivalent circuit without exceeding its limit values. As can be seen in Figs. 11 and 12, these resistive values provided good fit between the equivalent circuit response and the polarization curve corresponding to the performance of the commercial PEMFC.

Table 1 Setting values obtained for different resistive values of the equivalent circuit.

Resistances to adjust in the EC	Setting values obtained at 40 °C (kΩ)	Setting values obtained at 70 °C (kΩ)
R15	109	237
R9	96.33	162.9

R12	307.2	320.9
R14	93.9	99
R23	100	138.9
R4	1.366	1.007

Conclusion

In this work a simulation model with logarithmic electronic components which generated the same V–I response of a PEM type fuel cell has been developed. Moreover, the simulated curves fit the experimental curves of a trial. Also, the versatility of this model allows, by using the representative parameters of each type of fuel cell and by using accurate logarithmic electronic components, to simulate any other PEM fuel cell performance.

To adapt the power range of the fuel cell in the simulated electronic model, a transconductance is proposed. This enables to simulate a charge demanding current from the PEMFC, and to adapt the power range of the fuel cell in the simulated electronic model; it is a current source controlled by a voltage source.

Besides all the variety in emulators most of them use large amount of hardware and complex software when simulating the behavior of a PEM fuel cell in their waveforms as in the electrical parameters; voltage, current and power. On the contrary, the simple electronic circuit model shown in this work uses fewer components to simulate such behavior, thus lowering the manufacturing costs as well as the data processing time.

The equivalent EC performance has been validated by fitting this model polarization curve to the experimental data, obtained from commercial Electrochem 25 cm^2 active area PEMFC, by optimizing six resistive parameters.

Uncited references

[19] [20]; [21] [22]; [23] [24]; [25].

Acknowledgments

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Highlights

- An electronic equivalent circuit reproduces the stationary operation of a PEMFC.
- The EC performance has been validated by fitting this model to experimental data.
- This EC offers the possibility to adapt the power range of the fuel cell.

Queries and Answers

Query: Please check the Equations. (14), (25), (26) and correct if necessary.
Answer: The equations are correct.

Query: As Refs. [28] and [29] were identical, the latter has been removed from the reference list and subsequent references have been renumbered.
Answer: That's right.

Query: Uncited references: This section comprises references that occur in the reference list but not in the body of the text. Please position each reference in the text or, alternatively, delete it. Any reference not dealt with will be retained in this section. Thank you.
Answer: The references have been corrected.

Query: Please confirm that given names and surnames have been identified correctly.
Answer: That's right.

Query: The supplied figures 6, 7, 8, 9 and 10 are in poor quality. Please provide the better quality figures.
Answer: The quality and resolution of Figures 6, 7, 8, 9 and 10 have been upgraded and are individually attached to the query number five.