Profitable Strategic Delegation with Conjectural Variations[1]

Aitor Ciarreta

University of the Basque Country UPV/EHU
Fundamentos del Análisis Económico II.
Avda. Lehendakari Aguirre 83, 48015
Bilbao (Spain).
E-mail address: aitor.ciarreta@ehu.eus

Javier García-Enríquez (corresponding author)

University of the Basque Country UPV/EHU
Dpto. Economía Aplicada III (Econometría y Estadística).
Avda. Lehendakari Aguirre 83, 48015
Bilbao (Spain).
E-mail address: javier.garcia@ehu.eus

[1] Aitor Ciarreta and Javier García-Enríquez gratefully acknowledge funding from the Spanish Ministry of the Economy and Competitiveness and from ERDF via research grants ECO2013-40935-P, ECO2015-64467-R and ECO2016-76884-P and from the Basque Government through Grupo Consolidado IT383-13.

Profitable Strategic Delegation with Conjectural Variations

Current Version: March 2017

Abstract

Firms delegate strategic decisions to managers because they find it profitable to do so. In the product market, when agents make conjectures about the reaction of their rivals to marginal changes in their own strategies, the set of equilibriums can be enlarged with respect to the case of no conjectures. This paper takes a duopolistic linear market parameterization where firms selling differentiated products can delegate either price or output decisions to managers. We show that it is a dominant strategy for firms to delegate no matter whether firms are Cournot or Bertrand competitors, although the equilibrium is not necessarily efficient. Futhermore, in equilibrium Cournot competition is more profitable for firms than Bertrand competition. Finally, requiring consistency in conjectures yields the same outcome no matter what type of strategic interaction and managerial choice there is on the part of firms.

JEL classification: D21; D43

Keywords: Strategic delegation; conjectural variations; duopoly; Cournot; Bertrand.

1 Introduction

In duopolistic markets firms' organizational choice and the nature of product market competition are two structural features that determine profitability and social welfare. Moreover, conjectural variations (CVs, hereafter), which refer to beliefs of each firm about the conduct of the others in the product market, are an additional element that is usually neglected in these studies. This paper sheds light on this issue and addresses the problem of how the set of equilibriums changes when CVs are considered.

The choice of quantity or price as a strategic variable determines market outcomes. Strategic interaction between two firms that compete by setting quantities is different from interaction between two firms who compete by setting prices. Singh and Vives (1984) and Cheng (1985) show that Cournot competition always yields higher prices and lower welfare than Bertrand competition when goods are substitutes. Moreover, firms' profits are higher under Cournot competition. However, if goods are complements Bertrand competition is more profitable. Häckner (2000) shows that these results are sensitive to the duopoly assumption. In a general n firm case, where firms are sorted by quality of supply in the utility function, it is not evident which type of competition is more efficient.

Strategic choices can be made directly by an entrepreneur to maximize profits or can be delegated to a manager who is offered an incentive contract to achieve the same target. However, separation of ownership and control in corporations raises the issue of under what conditions it is profitable to delegate strategic decisions to managers. In this line, a paper by Fershtman and Judd (1987) (F&J hereafter) analyzes the case of linear incentive contracts between profits and sales. Under Cournot competition with homogeneous products owners find it optimal to offer a linear combination that depends on the cost asymmetry between them, whilst under Bertrand competition with differentiated products owners find it optimal to encourage managers to be more concerned with profits.[1] There is a large body of literature following F&J's paper that analyzes delegation using a strategic approach for different incentive contracts (Sklivas, 1987, Fumas, 1992, Miller and Pazgal, 2002, Jansen et al., 2007, Manasakis et al., 2010, and Mizuno, 2013, are just few of them). In this paper, we focus on an F&J type contract.[2]

Regardless of whether owners or managers take price or quantity decisions, agents can make CVs on how their rival will respond to their own actions on the market competition stage. Thus,.Cournot (1838) in his pioneer work studied competition in markets for homogeneous goods with two competitors, imposing the behavioral hypothesis that firm i determines its production level taking as given the production level of its rival j. Hence, Cournot duopolists are said to have zero CVs: each one conjectures that the competitor will not react to a change in its own production level. Early critiques of Cournot's zero conjectures approach can be found in Bowley (1924), who introduced CV equilibriums into Cournot's duopoly: firm i conjectures about how the rival j will change its actions (price, quantity or any other decision variable) with respect to potential adjustments in the first firm's actions. The consistency restriction, proposed by Harrod (1939) and Leontief (1936), imposes the requirement that firm i's conjecture must actually be correct, that is, conjectural best responses and conjectured reactions coincide. This yields the concept of consistent CV equilibriums.[3] Bresnahan (1981),

[1] F&J do not analyze Cournot competition with differentiated products but, as expected, there are no further insights when quantity competition with differentiated products is analyzed.

[2] Whereas the theoretical literature about strategic delegation is extense, the empirical studies are rather scarce. Among them we can find Vroom and Gimeno (2007), which supports price-leadership on the hotel industry with strategic delegation; or Bloomfield (2017), which supports theoretical results by F&J in a sample of large corporations.

[3] But one may rightly ask why the conjectural variation should be a constant. It seems that the conjectures made by firms should take into account and depend on the characteristics of the situation, including the production levels of the two firms, q_i and q_j. A priori many functional forms could be specified, but the linear form is the easiest to analyze. Boyer and Moreaux (1983) propose: $\frac{dq_i}{dq_j} = \alpha_j + \beta_j q_j + \gamma_j q_i$. However, we restrict ourselves to the constant case to shed light on the effects of

working on consistent CVs, shows how there may indeed be no equilibrium and proposes a more general concept in which a multiplicity of equilibriums may arise. Laitner (1980) rationalizes conjectures within the context of perfect information, but he also shows how the set of "rational conjectural equilibriums" may still be large and may not substantially restrict the larger set of conjectural equilibriums.

The case of the CV under price competition has received less attention than the one under quantity competition. Within the context of product differentiation, as considered here, it seems reasonable to analyze CVs under price competition because, as Stigler (1940) emphasizes, "*in the case of duopoly with differentiated products, the possibility of price competition becomes slightly more realistic*". The main argument is that prices are more likely to be observed by firms and it is easier to change in response to the other firm's prices. In this line, Liang (1989) estimates price CVs to measure the degree of price competition in a product differentiated oligopoly.

In general, computing CV equilibriums means solving systems of differential equations. In a duopoly, Bresnahan (1981) finds that there are no analytic solutions whilst Olsder (1981) finds a multiplicity of (numerical) solutions. In situations of complete information and common knowledge of rationality, players should consider only strategies that remain after the iterated elimination of dominated strategies. Often, only Nash equilibriums remain. However, this does not solve the problem of dynamic inconsistency. This problem is usually solved by means of Harrod's conditions (Harrod, 1939), which do not restrict the wide diversity of a priori possible equilibriums. As a result, a growing literature has developed to analyze the implications for duopoly theory of requiring that the conjectures held by the firms be consistent or rational.[4]

CVs have been incorporated into the analysis of duopoly theory, although they are few studies when delegation is under study. Among these are Hwang and Mai (1995) and Vetter (2016) for the homogenous product case.[5] This paper provides new insights into the analysis of the relationship between managerial organization, CV market competition and efficiency by taking a strategic delegation perspective. We show under a linear parameterization that no matter what type of strategic interaction exists, it is a dominant strategy for both firms to delegate. However, we do not always have a Prisoner's Dilemma as in F&J, because for some values of the parameters the result is actually

making conjectures.

[4]See Boyer and Moreaux (1983) and more recently Giocoli (2005) for an insightful study on consistency of conjectures.

[5]In particular, Hwang and Mai (1995) analyze the case of CVs for owners and managers; whilst Vetter (2016) considers evolutionary stable conjetures.

profit-maximizing for firms. Furthermore, when consistent CVs are considered as defined by Harrod (1939) then profits are the same no matter what organizational choice or type of strategic interaction is considered. Thus, being managerial or self-managed under either Cournot or Bertrand does not make any difference. This is a remarkable result to add to the existing literature on the topic. Therefore, the equivalence of Cournot and Bertrand competition outcomes can be found in a different setting. Note that this is not always the case. For example, Pal (2015) shows that, under the presence of network externalities, managerial delegation does not yield to the equivalence of Bertrand and Cournot equilibria.

The rest of the paper is organized as follows. Section 2 develops the linear demand duopoly model with product differentiation. Sections 3 and 4 solve the model under Cournot and Bertrand competition, respectively. We discuss implications for competition without imposing restrictions on conjectures and also when consistency is required. In Section 5, we compare price and quantity competition, and the implications for optimal management design. Final comments are summarized in Section 6. All the proofs are relegated to an online appendix.

2 The model

Consider two products, i and j, that differ in a number of characteristics that firms have already chosen, thus the degree of product differentiation is predetermined. Assume a representative consumer with a taste for variety who derives utility from the consumption of both goods according to a quadratic specification as in Dixit (1979) and Singh and Vives (1984).

$$U\left(q_i, q_j\right) = w + a_i q_i + a_j q_j - \frac{1}{2}\left(b_i q_i^2 + 2d q_i q_j + b_j q_j^2\right),$$

where w is a numeraire good, and q_i, q_j are the consumption of good i,j, respectively. For the sake of simplicity assume $a_i = a_j = 1$ and $b_i = b_j = 1$. The predetermined degree of product substitutability is summarized in parameter d such that $d \in (0,1)$. This consumer has a budget constraint where income, Y, is spent on goods i and j and the numeraire, $Y = p_i q_i + p_j q_j + w$. The consumer maximizes net utility by choosing (q_i, q_j) which yields the inverse demand functions $p_i = 1 - q_i - dq_j$ and $p_j = 1 - q_j - dq_i$, respectively, where the corresponding demands functions are $q_i = \frac{1}{1+d} - \frac{1}{1-d^2}p_i + \frac{d}{1-d^2}p_j$ and $q_j = \frac{1}{1+d} - \frac{1}{1-d^2}p_j + \frac{d}{1-d^2}p_i$, respectively.

Firms are symmetric, each producing one good with unit cost c. Firms can be either

Cournot competitors (referred to as C : the quantity game) or Bertrand competitors (B : the price game). Furthermore, firms can choose between two different organizational modes:

Managerial: We refer to a firm where the owner, who seeks to maximize expected profits, delegates strategic price or quantity decisions to a manager in exchange for a payment. The contract signed by the manager, O_i, is a linear combination of profits, Π_i, and sales, R_i: $O_i^{\alpha} = \alpha_i \Pi_i + (1 - \alpha_i) R_i$ in the case of Cournot, and $O_i^{\beta} = \beta_i \Pi_i + (1 - \beta_i) R_i$ in the case of Bertrand, where $\alpha_i, \beta_i \in \mathbb{R}$.[6]

Self-managed: The owner takes strategic variable decisions himself to maximize profits: Π_i in case of Cournot and Π_i in case of Bertrand.

Players make CVs in the competition game. Under quantity competition denote by λ_i the CV of firm i on j's response to a quantity change in i, $\lambda_i = dq_j/dq_i$, where $\lambda_i \in [-1, 1]$; whilst under price competition μ_i is the CV of firm i on j's response to a price change of i, $\mu_i = dp_j/dp_i$, where $\mu_i \in [-1, 1]$. Positive conjectures are associated with more collusive behavior, with $\lambda_i = 1$ and $\mu_i = 1$ being the most restrictive case (each firm believes that the rival will exactly imitate any change in price or quantity, which leads to firms behaving as a monopolist), whilst negative conjectures are associated with more competitive behavior, with $\lambda_i = -1$ and $\mu_i = -1$ being the most efficient case (each firm believes that the rival will exactly offset any change in price or quantity, which leads to firms behaving as perfect competitors). We impose ex-ante symmetric conjectures to alleviate the burden of carrying many parameters through the analysis. Although evidence or uncertain data can yield differences in beliefs of firms that result in different priors, our model does not look more deeply into these sources of asymmetric behavior.

There are four possible strategies that firms can follow for each type of market interaction:

Strategy I: Firms simultaneously choose not to delegate, (ND,ND). In this case $\alpha_i = \alpha_j = 1$ ($\beta_i = \beta_j = 1$ in the price game) and pure profit maximization is encouraged. Denote as $\Pi^{C,I}$ ($\Pi^{B,I}$ in the price game) the equilibrium profits for each firm (equal by symmetry).

[6]Note that weights in the optimal contract are not necessarily between 0 and 1. For instante, if $\alpha_i > 1$ (or $\beta_i > 1$) the owner encorauges a manager to be over concerned on profits. This optimal contract prevents the manager from being too agressive in the market.

Strategy II: Managerial organization is chosen by both firms, (D,D), and a two-stage game follows. In stage 1 owners simultaneously choose incentive parameters to maximize profits. Then, in stage 2 competition follows and managers compete either in quantities or in prices to maximize payoffs, O_i^C or O_i^B, respectively. Denote as $\Pi^{C,II}$ ($\Pi^{B,II}$ in the price game) the equilibrium profits for each firm (equal by symmetry).

Strategy III: Firm i chooses managerial organization and firm j does not, (D,ND), so in stage 1 the owner of i chooses the incentive contract and in stage 2 competition follows. Denote as $\Pi_i^{C,III}$ and $\Pi_j^{C,III}$ ($\Pi_i^{B,III}$ and $\Pi_j^{B,III}$ in the price game) the equilibrium profits for firms i and j, respectively.

Strategy IV: This is symmetric to (D,ND). Denote as $\Pi_i^{C,IV}$ and $\Pi_j^{C,IV}$ ($\Pi_i^{B,IV}$ and $\Pi_j^{B,IV}$ in the price game) the equilibrium profits for firms i and j, respectively.

Hence, the normal form of the strategic delegation game with CVs is

		j: ND	j: D
i	ND	Π^I, Π^I	Π_i^{IV}, Π_j^{IV}
	D	Π_i^{III}, Π_j^{III}	Π^{II}, Π^{II}

where by symmetry of the firms it holds that $\Pi_i^{III} = \Pi_j^{IV}$ and $\Pi_j^{III} = \Pi_i^{IV}$. Note that the superscripts have been removed, so it can represent either game. We look for the sub-game perfect CV Nash equilibriums and consistent sub-game perfect CV Nash equilibriums, the latter to restrict the set of solutions that fulfill the consistency requirement of conjectures as defined by Harrod (1939).

3 Game under Cournot competition

I: Neither firm delegates

Assume that shareholders do not delegate production decisions to managers. This is the familiar case of Cournot competition with product differentiation and CVs (see for example Pfaffermayr, 1999, and Boone, 2008). Therefore, the best response function for firm i is

$$q_i^I(q_j) = \frac{1 - c - dq_j}{2 + d\lambda}.$$

The reaction function of firm i is downward sloping, that is, $\partial q_i^I / \partial q_j < 0$ for every λ and d. Quantity decisions are regarded as strategic substitutes according to Bulow et al. (1985). Hence, the symmetric Cournot-Nash equilibrium quantities and profits are

$$q^{C,I} = \frac{1-c}{2+d+d\lambda},$$
$$\Pi^{C,I} = \frac{(1+d\lambda)(1-c)^2}{(2+d+d\lambda)^2}.$$

Note that, as expected, output is decreasing when owners make more cooperative conjectures on their rival's reaction to quantity change. Profits are increasing in λ since making more cooperative conjectures favours higher prices and lower production levels for each firm such that, for every d, the solution tends to monopolistic behavior.

II: Both firms delegate

Assume that both firms simultaneously and independently hire managers to make quantity decisions and offer contracts O_i^α and O_j^α, respectively, in stage 1. The game is solved backwards. Therefore, in stage 2 the manager of firm i chooses q_i to maximize payoff, that is, $\partial O_i^\alpha / \partial q_i = 0$, given α_i. Denote as $q_i^{II}(q_j)$ the firm i's corresponding best response function, which takes the form

$$q_i^{II}(q_j) = \frac{1-\alpha_i c - d q_j}{2+d\lambda}. \tag{1}$$

Note that, as in the no delegation case, $\partial q_i^{II} / \partial q_j < 0$, that is, quantities are strategic substitutes. Delegation and conjectures have an impact on the optimal response of managers. The choice of α_i in stage 1 has a direct effect on output compared to the case when $\alpha_i = 1$. If $\alpha_i < 1$ (> 1) then the reaction function shifts outwards (inwards), since the manager views $\alpha_i c$ as the true marginal cost. Conjectures rotate the slope: if $\lambda > 0$ (< 0) the slope is in absolute value flatter (steeper) than in the zero pure Cournot conjectures. Therefore, we explore how both parameters interact in equilibrium.

The Nash equilibrium strategies of the production stage as a function of the incentive parameters and conjectures are,

$$q_i^{II}(\alpha_i, \alpha_j) = \frac{2-d+d\lambda-(2+d\lambda)c\alpha_i + cd\alpha_j}{(2-d+d\lambda)(2+d+d\lambda)}. \tag{2}$$

It holds that $\partial q_i^{II}(\alpha_i, \alpha_j) / \partial \alpha_i < 0$, that is, the incentive contract giving more weight to profits than to sales forces managers to adopt a less aggressive behavior in the product market. However, $\partial q_i^{II}(\alpha_i, \alpha_j) / \partial \alpha_j > 0$, that is, the optimal response to less aggressive behavior by the rival triggers less restrictive behavior by the firm. In any event, it holds that $\left|\partial q_i^{II}(\alpha_i,\alpha_j)/\partial\alpha_i\right| > \left|\partial q_i^{II}(\alpha_i,\alpha_j)/\partial\alpha_j\right|$, which means that own effects are larger than cross effects.

In stage 1 owners simultaneously choose the optimal contract that maximizes profits (Π_i^{II}) subject to the equilibrium strategies (**??**) of the managers and given conjectures,

$$\max_{\alpha_i} \left(1 - c - q_i^{II}(\alpha_i,\alpha_j) - dq_j^{II}(\alpha_i,\alpha_j)\right) q_i^{II}(\alpha_i,\alpha_j).$$

Lemma 1 *The first order condition of profit maximization, $\partial\Pi_i^{II}/\partial\alpha_i = 0$, yields the system of linear best reaction functions $\alpha_i(\alpha_j) = k_0 - k_1\alpha_j$, where k_0 and k_1 depend on the conjectures made by managers in stage 2 and the structural parameters c and d. Moreover, necessary and sufficient condition for the existence of a unique and stable equilibirum is $|k_1| < 1$.*

The way in which these parameters interact largely affects the optimal contract that shareholders offer to their managers to maximize profits. As shown in the proof, given c, the (λ, d) space is divided into two regions such that each one determines whether delegation strategies are substitutes (Region I) or complements (Region II).[7] Given the restrictions of the parameters of the model in Figure 1, the curve represents combinations of λ and d such that the slopes of the response functions $\alpha_i(\alpha_j)$ are zero ($k_1 = 0$).

[7] The decisions of the players are called strategic complements if they mutually reinforce one another, and they are called strategic substitutes if they mutually offset one another. This means that, for firm i, when α_j increases (i.e. the owner in firm j is more concerned about profits) α_i increases (decreases) if they are strategic complements (substitutes).

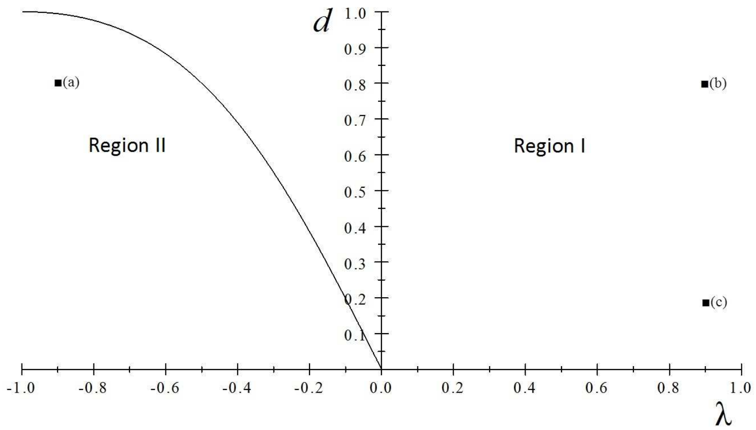

Figure 1: Combinations of λ and d such that $k_1 = 0$ and resulting regions.

We illustrate incentive response functions $\alpha_i\,(\alpha_j)$ of Lemma 1 in Figures 2(a), 2(b) and 2(c) for different values of the parameters and taking $c = 0.2$.

For any value of d, when there are positive conjectures strategic delegation best response functions are negative sloping. In particular, there are two cases depending on the interaction between the degree of product substitutability and conjecture: either $\alpha_i, \alpha_j < 0$ (Figure 2(b)) or $\alpha_i, \alpha_j > 0$ (Figure 2(c)). However, note that in both cases the best response for firm i to a larger α_j is a smaller α_i, that is, owners respond to more competitive behavior of the rival with more restrictive incentive to keep margins high. This is not the case for negative conjectures where, for any pair (λ, d) in Region II, delegation strategies are positive sloping and it induces less aggressive behavior on the part of the managers (Figure 2(a)).

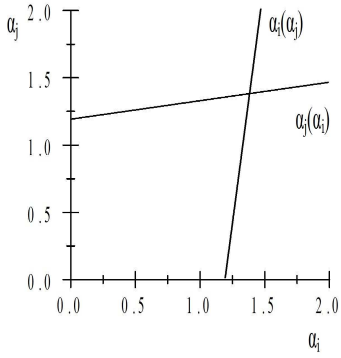

Figure 2(a): $\lambda = -0.9, d = 0.8$

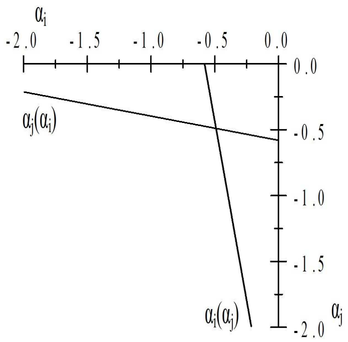

Figure 2(b): $\lambda = 0.9, d = 0.8$

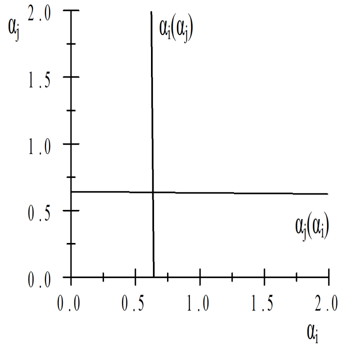

Figure 2(c): $\lambda = 0.9, d = 0.2$

We look for the symmetric equilibrium. Theorem 1 summarizes the results of the quantity game with delegation.

Theorem 1 *Under Cournot competition, if c and d are known in stage 1 by owners, and agents make CVs in the production stage, then for every $\lambda \in [-1, 1]$ the unique symmetric Nash equilibrium of the game with delegation is to offer a linear combination of profits and sales with weight*

$$\alpha^{II} = 1 - \frac{d\left(2\lambda + d\left(1 + \lambda^2\right)\right)(1-c)}{(4 + d(2-d) + d(2+d)\lambda)c}, \tag{3}$$

where $\alpha^{II} \in \left[1 - \frac{d(1-c)}{2c}, 1 + \frac{d(1-d)(1-c)}{(2-d^2)c}\right]$. As a result, the equilibrium quantity and profits for each firm are

$$q^{C,II} = \frac{(2 + d\lambda)(1-c)}{4 + d(2-d) + d(2+d)\lambda},$$

$$\Pi^{C,II} = \frac{(2 + d\lambda)\left(2 + d\lambda - d^2\right)(1-c)^2}{(4 + d(2-d) + d(2+d)\lambda)^2}.$$

Theorem 1 reveals an interesting forecast on optimal contract design. When managers make more cooperative conjectures, owners offer a contract where more weight is put on sales. Even though managers perceive a lower unit cost than under pure profit maximization (that is, when $0 < \alpha < 1$), the effect on output choice is decreasing with conjectures. By contrast, when conjectures are perfectly competitive ($\lambda = -1$) $\alpha^{II} > 1$.

This means owners have to design a contract that forces managers to be more aware of profit maximization. As a result, market outcome moves away from perfect competition. In this case, owners design an optimal contract that incentivates managers to behave as if the marginal cost was higher than the real one, and encourages less aggressive sales behavior than would be the case in the absence of delegation.

In Figure 2(a) $\alpha^{II} = 1.38$, in Figure 2(b) $\alpha^{II} = -0.49$ and in Figure 2(c) $\alpha^{II} = 0.64$. Therefore, in equilibrium α^{II} can be even negative, which means that managers are encouraged to be over-concerned with sales. This is the optimal contract when products are closer substitutes and conjectures are cooperative.

At this point it is illustrative to show what the optimal α^{II} looks like in the case where Cournot-type of conjectures are simultaneously held by both owners. It can be shown that $\alpha^{II}(\lambda = 0) = 1 - \frac{(1-c)d^2}{c(4+2d-d^2)}$, thus taking $d = 1$ we are in exactly the same context as in F&J and equation $(7a)$ is obtained for the symmetric firm case. Finally, if $d = 0$ then $\alpha^{II} = 1$, owners should just offer a contract that encourages pure profit maximization.

III/IV: One firm delegates

In this case there is an asymmetry in the timing of the game that is generated by each firm having a different organizational choice. This results in a sequential game where firm i chooses managerial organization in stage 1 whilst firm j does not. In linear demand and linear cost models without conjectures, under Cournot competition there is a strategic advantage in being the first mover. In this case, the potential advantage comes from the anticipation that owners have in designing optimal contracts provided their rival does not go managerial. Then, given that $\alpha_j = 1$, the optimal $\alpha_i \in \mathbb{R}$ is obtained from the solution to the system of reaction functions,

$$q_i^{III}(q_j) = \frac{1 - \alpha_i c - dq_j}{2 + d\lambda},$$
$$q_j^{III}(q_i) = \frac{1 - c - dq_i}{2 + d\lambda}.$$

Thus, in stage 2 the Nash equilibrium depends on α_i:

$$q_i^{III}(\alpha_i) = \frac{2 - d + d\lambda + cd - (2 + d\lambda)c\alpha_i}{(2 - d + d\lambda)(2 + d + d\lambda)},$$
$$q_j^{III}(\alpha_i) = \frac{2 - d + d\lambda - c(2 + d\lambda) + cd\alpha_i}{(2 + d + d\lambda)(2 - d + d\lambda)}.$$

Both firms' output choices depend on α_i. Particularly, $\partial q_i^{III}(\alpha_i)/\partial \alpha_i < 0$ and $\partial q_j^{III}(\alpha_i)/\partial \alpha_i > 0$. Compared to the case where there is no delegation, choosing $\alpha_i < 1$

in equilibrium causes firm j to decrease its output and firm i to expand its output. This result is rational given the strategic substitutability of quantities no matter what the degree of product heterogeneity is. Proposition 1 summarizes the results of the game.

Proposition 1 *Under Cournot competition, if c and d are known in stage 1 by all agents, firm i does delegate output decisions in stage 2, and firm j does not go managerial, then for every $\lambda \in [-1, 1]$ the unique Nash equilibrium of the game is*

$$\alpha^{III} = \frac{\begin{array}{c}\left((d\lambda)^3 + (6-d)(d\lambda)^2 + (12 - d(2+d))\,d\lambda + \left(8 - d^2(2+d)\right)\right)c \\ -d\left(d + 2\lambda + d\lambda^2\right)(2 - d + d\lambda)\end{array}}{2(2+d\lambda)(2+d\lambda-d^2)\,c},$$

$$q_i^{C,III} = \frac{(2-d+d\lambda)(1-c)}{2(2+d\lambda-d^2)},$$

$$q_j^{C,III} = \frac{\left(4 - 2d(1-\lambda) - d^2(1+\lambda)\right)(1-c)}{2(2+d\lambda)(2+d\lambda-d^2)}$$

$$\Pi_i^{C,III} = \frac{(2-d+d\lambda)^2}{4(2+d\lambda-d^2)(2+d\lambda)}(1-c)^2,$$

$$\Pi_j^{C,III} = \frac{(1+d\lambda)\left(4-2d(1-\lambda)-d^2(1+\lambda)\right)^2}{4(2+d\lambda-d^2)^2(2+d\lambda)^2}(1-c)^2$$

where $\alpha^{III} \in \left[1 - \frac{2d(1-c)}{(2-d)(2+d)c}, 1 + \frac{2d(1-d)(1-c)}{(4-d^2)c}\right]$.

We illustrate the forecasts in Proposition 1 for the same three cases as above. In (a) $\alpha^{III} = 1.33$, in (b) $\alpha^{III} = -0.76$ and in (c) $\alpha^{III} = 0.63$. Note that in all three cases $\alpha^{III} < \alpha^{II}$.

Comparison of results:

First, in Proposition 2 we compare the optimal contracts for the cases of both firms delegating with those in the case when only one firm delegates.

Proposition 2 *If only one firm delegates then more aggressive sales behavior is encouraged than when both firms delegate, that is, $\alpha^{II} - \alpha^{III} > 0$.*

Therefore, no matter whether delegation decisions are strategic substitutes or strategic complements, the optimal contract always encourages agents to be more aware of profits than sales when both firms delegate output decisions. The normal form of the game is represented below.

		j ND	j D
i	ND	$\frac{1+d\lambda}{(2+d+d\lambda)^2}, \frac{1+d\lambda}{(2+d+d\lambda)^2}$	$\frac{(1+d\lambda)\left(4-2d(1-\lambda)-d^2(1+\lambda)\right)^2}{4(2+d\lambda-d^2)^2(2+d\lambda)^2}, \frac{(2-d+d\lambda)^2}{4(2+d\lambda-d^2)(2+d\lambda)}$
	D	$\frac{(2-d+d\lambda)^2}{4(2+d\lambda-d^2)(2+d\lambda)}, \frac{(1+d\lambda)\left(4-2d(1-\lambda)-d^2(1+\lambda)\right)^2}{4(2+d\lambda-d^2)^2(2+d\lambda)^2}$	$\frac{(2+d\lambda)\left(2+d\lambda-d^2\right)}{(4+d(2-d)+d(2+d)\lambda)^2}, \frac{(2+d\lambda)\left(2+d\lambda-d^2\right)}{(4+d(2-d)+d(2+d)\lambda)^2}$

All equilibrium profits are scaled by $(1-c)^2$.

Proposition 3 characterizes the Nash equilibrium of the game. It generalizes the F&J homogenous product case to allow product differentiation and conjectures in the production stage. We show that the equilibrium strategies do not depend on whether variables α_i and α_j are strategic substitutes or strategic complements, although the efficiency of the solution does.

Proposition 3 *In the strategic delegation game with CVs, a unique Nash equilibrium in dominant strategies does exist, in which both firms decide to delegate. Moreover, this equilibrium is Pareto optimal only if α_i and α_j are strategic complements.*

The model predicts the existence of a unique Nash equilibrium in dominant strategies where both firms delegate. However, the game is not necessarily a Prisoner's Dilemma because a Pareto efficient outcome is achieved when delegation strategies are complements.

Similar results have been obtained in the literature under different frameworks. For example, Bhattacharjee and Pal (2013) find that, in the presence of strong network externalities, firms obtain higher equilibrium profits under strategic delegation than under no-delegation, being this equilibrium Pareto optimal. Fanti and Meccheri (2017) show that, in the presence of asymmetric and convex costs, managerial delegation is a Nash equilibrium, but if costs are different enough, in equilibrium, the more efficient firm obtains a higher profit than under no delegation. This implies that, in that case, the managerial delegation game does not represent a Prisoners' Dilemma.

Concentrating on consistent CV equilibriums, the condition that the slope of the best response function has to be equal to the conjecture is the same no matter whether firms delegate or not, and it is given by, $\frac{-d}{2+d\lambda} = \lambda$. Thus, for every $d > 0$, solving in terms of λ implies a unique equilibrium of conjectures as a function of the degree of product differentiation: $\lambda^* = \frac{-1+\sqrt{1-d^2}}{d} \in (-1, 0)$, which is outside Cournot's zero-conjectures. The consistent CV, λ^*, implies that the equilibrium profits for each firm and each pair of strategies are

$$\Pi_*^C = \frac{\sqrt{1-d^2}}{\left(1+d+\sqrt{1-d^2}\right)^2}(1-c)^2.$$

Therefore, when only consistent CVs are considered delegating gives no strategic advantage over self-management since all the equilibrium outcomes are the same. This result summarizes some predictions in the relevant literature. If $d = 1$ (perfect substitutes) the result obtained is that of Perry (1982), who shows that when the number of firms is fixed and the marginal costs are constant, competitive behavior is the unique consistent equilibrium in CV, that is, $\Pi_*^C = 0$. Finally, if $d = 0$ the solution is restricted to the equitable cartel case, that is, $\Pi_*^C = \frac{(1-c)^2}{2}$.

4 Game under Bertrand competition

I: Neither firm delegates

First consider the case in which owners do not delegate price decisions to managers. This is the case of Bertrand competition with differentiated products and CVs (See Pfaffermayr, 1999). Hence, the reaction function for firm i can be expressed as

$$p_i^I(p_j) = \frac{1-d+c(1-d\mu)+dp_j}{2-d\mu}.$$

The reaction function of firm i is upward sloping, that is, $\partial p_i^I/\partial p_j > 0$ for every μ and d. Thus, price decisions are regarded as strategic complements according to Bulow et al. (1985) regardless of the conjectures. Since we are looking for the symmetric Bertrand equilibrium, we have that

$$p^{B,I} = \frac{1-d+(1-d\mu)c}{2-d-d\mu}.$$

Note that, as expected, $\partial p^{B,I}/\partial\mu > 0$, that is, in the symmetric equilibrium prices are increasing when the owner makes more cooperative conjectures as to the rival's reaction to a price change. Finally, equilibrium profits are

$$\Pi^{B,I} = \frac{(1-d)(1-d\mu)(1-c)^2}{(1+d)(2-d-d\mu)^2},$$

which are also increasing with conjectures, $\partial\Pi^{B,I}/\partial\mu > 0$.

II: Both firms delegate

Assume that both firms simultaneously and independently hire managers to take price decisions and offer contracts O_i^β and O_j^β, respectively in stage 1. The game is solved backwards. Therefore, in stage 2 under strategic delegation, managers simultaneously and independently choose p_i to maximize the payoff function O_i^β offered by owners, given β_i:

$$\max_{p_i} (p_i - \beta_i c) \left(\frac{1}{1+d} - \frac{1}{1-d^2} p_i + \frac{d}{1-d^2} p_j \right),$$

where β_i is the marginal cost scale parameter that owners instruct managers to consider when taking price decisions. Delegation also plays the same role as in the Cournot game. Managers view $\beta_i c$ as the true marginal cost when taking price decisions. Then, from the first order condition of profit maximization, $\partial O_i^\beta / \partial p_i = 0$, we obtain the reaction function for each firm as,

$$p_i^{II}(p_j) = \frac{1 - d + c(1 - d\mu)\beta_i + dp_j}{2 - d\mu}. \tag{4}$$

Note that compared to the case of pure profit maximization ($\beta_i = 1$) delegation shifts the best response function outwards (inwards) as long as $\beta_i > 1$ (< 1). Prices are strategic complements, $\partial p_i^{II}(p_j) / \partial p_j > 0$ no matter what conjectures firms make. The choice of managerial organization of the firm has a direct effect on the strategic behavior of the firms; $\partial p_i^{II}(p_j) / \partial \beta_i > 0$, thus choosing a larger β_i favors more cooperative behavior for every μ. Therefore, the Nash equilibrium strategies of the price stage-game can be obtained as a function of the payoff incentive parameters and conjectures

$$p_i^{II}(\beta_i, \beta_j) = \frac{(1-d)(2+d-d\mu) + (1-d\mu)((2-d\mu)\beta_i + d\beta_j)c}{(2+d-d\mu)(2-d-d\mu)}. \tag{5}$$

Note that both $\partial p_i^{II}(\beta_i, \beta_j) / \partial \beta_i > 0$ and $\partial p_i^{II}(\beta_i, \beta_j) / \partial \beta_j > 0$, that is, strategic delegation pushes prices up in equilibrium no matter what conjectures firms make. Owners giving more weight to profits encourage a less aggressive pricing policy. Note also that $\left|\partial p_i^{II}(\beta_i, \beta_j) / \partial \beta_i\right| > \left|\partial p_i^{II}(\beta_i, \beta_j) / \partial \beta_j\right|$, which means that own effects are larger than cross effects.

In stage 1 owners simultaneously choose the optimal contract that maximizes profits (Π_i^{II}) subject to the equilibrium strategies (**??**) of the managers and given conjectures,

$$\max_{\beta_i} \left(p_i^{II}(\beta_i, \beta_j) - c\right) \left(\frac{1}{1+d} - \frac{1}{1-d^2} p_i^{II}(\beta_i, \beta_j) + \frac{d}{1-d^2} p_j^{II}(\beta_i, \beta_j) \right),$$

Lemma 2 *The first order condition of profit maximization, $\partial \Pi_i^{II} / \partial \beta_i = 0$, yields the system of linear best reaction functions $\beta_i(\beta_j) = m_0 + m_1 \beta_j$, where m_0 and m_1 depend* *on the conjectures made by managers in stage 2 and the structural parameters c and d. Moreover, necessary and sufficient condition for the existence of a unique and stable equilibirum is* $|m_1| < 1$.

How these parameters interact substantially affects the optimal contract that shareholders offer to their managers to maximize profits. As shown in the proof, given c the (β, d) space is divided into two regions such that each one determines whether delegation strategies are substitutes (Region II) or complements (Region I). Given the restrictions of the parameters of the model in Figure 3, the curve represents combinations of μ and d such that the slopes of the response functions $\beta_i\,(\beta_j)$ are zero ($m_1 = 0$).

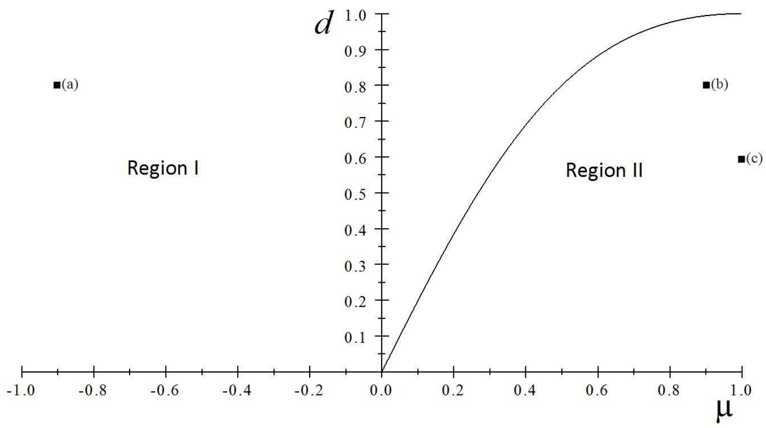

Figure 3: Combinations of μ and d such that $m_1 = 0$ and resulting regions.

We illustrate the incentive response functions $\beta_i\,(\beta_j)$ of Lemma 2 in Figures 4(a), 4(b) and 4(c) for different values of the parameters and taking $c = 0.2$.

For any value of d, when there are negative conjectures, strategic delegation best response functions are positive sloping (Figure 4(a)). This is not the case for positive conjectures where, for any pair (μ, d) in Region II, delegation strategies are negative sloping and it induces more aggressive behavior on the part of the managers. In particular, there are two cases depending on the interaction between the degree of product substitutability and conjecture: either $\beta_i, \beta_j > 0$ (Figure 4(b)) or $\beta_i, \beta_j < 0$ (Figure 4(c)). However, note that in both cases the best response for firm i to a larger β_j is a smaller β_i, that is, owners respond to more competitive behavior of the rival with more restrictive incentive to keep margins high.

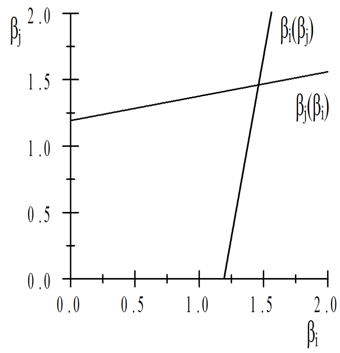

Figure 4(a): $\mu = -0.9, d = 0.8$

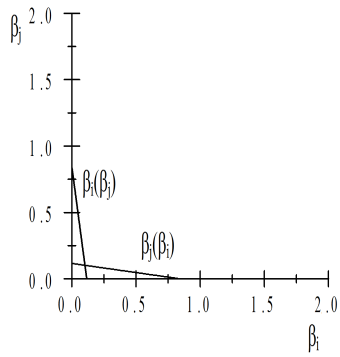

Figure 4(b): $\mu = 0.9, d = 0.8$

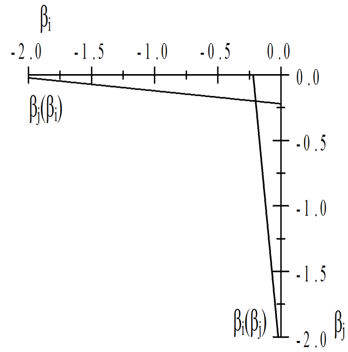

Figure 4(c): $\mu = 1, d = 0.6$

Theorem 2 summarizes the results of the price game with delegation.

Theorem 2 *Under Bertrand competition, if* c *and* d *are known in stage* 1 *by owners, and agents make CVs in the production stage, then for every* $\mu \in [-1, 1]$ *the unique symmetric Nash equilibrium of the game with delegation is to offer a linear combination of profits and sales with weight*

$$\beta^{II} = \frac{d\,(1-d)\left(d - 2\mu + d\mu^2\right) + (2 - d - d\mu)\left(2 - d\mu - d^2\right)c}{c\,(1 - d\mu)\,(4 - 2d\,(1+\mu) - d^2\,(1-\mu))},$$

where $\beta^{II} \in \left[1 - \frac{d(1-c)}{2c}, 1 + \frac{d(1-d)(1-c)}{c(2-d^2)}\right]$. *As a result*

$$p^{B,II} = \frac{(1-d)\,(2 - d\mu) + \left(2 - d^2 - d\mu\right)c}{4 - 2d\,(1+\mu) - d^2\,(1-\mu)},$$

$$\Pi^{B,II} = \frac{(1-d)\,(2-d\mu)\left(2 - d\mu - d^2\right)(1-c)^2}{(1+d)\,(4 - 2d\,(1+\mu) - d^2\,(1-\mu))^2}.$$

In Figure 4(a) $\beta^{II} = 1.46$, in Figure 4(b) $\beta^{II} = 0.10$ and in Figure 4(c) $\beta^{II} = -0.20$. Therefore, in equilibrium β^{II} can be even negative, which means that managers are encouraged to be over-concerned with sales. This is the optimal contract when products are closer substitutes and conjectures are cooperative.

In the extreme cases, when goods are independent ($d = 0$), then $\beta^{II} = 1$, so owners should optimally encourage pure profit maximization; but there is no discussion when products are perfect substitutes because the model reduces to perfect competition no matter what conjectures are formed. If products are imperfect substitutes then the lower bound is still positive but less than one. If $\mu = 0$ then F&J equation (24) is obtained.

III/IV: One firm delegates

In this case there is an asymmetry that arises from firms having different organizational choices. Assume that firm i chooses managerial organization whilst firm j does not. Hence, the system of reaction functions is

$$p_i^{III}(p_j) = \frac{1 - d + c(1 - d\mu)\beta_i + dp_j}{2 - d\mu},$$
$$p_j^{III}(p_i) = \frac{1 - d + c(1 - d\mu) + dp_i}{2 - d\mu}.$$

This is a sequential price game where firm i chooses managerial organization in stage 1 whilst firm j does not. In linear demand and linear cost models without conjectures, price competition results in a second-mover strategic advantage. In this case, the potential advantage comes from the possibility that firm i has of anticipating prices chosen in the second stage knowing firm's j organizational choice. Then, given that $\beta_j = 1$, the optimal $\beta_i \in \mathbb{R}$ is obtained from the two-stage game. In stage 2, the Nash equilibrium depends on β_i,

$$p_i^{III}(\beta_i) = \frac{(1-d)(2+d-d\mu) + (1-d\mu)dc + (1-d\mu)(2-d\mu)c\beta_i}{(2+d-d\mu)(2-d-d\mu)}, \tag{7a}$$
$$p_j^{III}(\beta_i) = \frac{(1-d)(2+d-d\mu) + (1-d\mu)(2-d\mu)c + (1-d\mu)dc\beta_i}{(2+d-d\mu)(2-d-d\mu)}. \tag{7b}$$

β_i affects prices in the same way: $\partial p_i^{III}(\beta_i)/\partial\beta_i > 0$ and $\partial p_j^{III}(\beta_i)/\partial\beta_i > 0$, that is, contracts that give more incentives to profits than sales encourage less aggressive price behavior. Furthermore, the difference in prices $p_i^{III}(\beta_i) - p_j^{III}(\beta_i) = \frac{c(1-d\mu)(\beta_i-1)}{(2+d-d\mu)}$ depends on the contract choice; if $\beta_i > 1$ then $p_i^{III}(\beta_i) > p_j^{III}(\beta_i)$, otherwise $p_i^{III}(\beta_i) < p_j^{III}(\beta_i)$. Owner i in stage 1 maximizes profits subject to the Nash equilibrium prices obtained in stage 2. Proposition 4 summarizes the main properties of the equilibrium.

Proposition 4 *Under Bertrand competition, if c and d are known in stage* 1 *by all agents, firm i delegates price decisions in stage* 2*, and firm j does not do so, then for* *every $\mu \in [-1,1]$ the unique Nash equilibrium of the game is*

$$\beta^{III} = \frac{\begin{array}{c}\left((1+d)\,d^3\mu^3 - \left(6+3d-d^2\right)d^2\mu^2 + \left(12+2d+d^3-5d^2\right)d\mu - \left(8-6d^2+d^3+d^4\right)\right)c \\ -d\,(1-d)\,(2+d-d\mu)\left(d-2\mu+d\mu^2\right)\end{array}}{-2c(2-d\mu)(1-d\mu)(2-d\mu-d^2)},$$

$$p_i^{B,III} = \frac{(1+d)(2-d-d\mu)c+(1-d)(2+d-d\mu)}{2(2-d\mu-d^2)},$$

$$p_j^{B,III} = \frac{(1-d)\left(4+2d(1-\mu)-d^2(1+\mu)\right)+\left(4+2d(1-3\mu)-d^2\left(1+\mu-2\mu^2\right)-d^3(1-\mu)\right)c}{2(2-d\mu)(2-d\mu-d^2)},$$

$$\Pi_i^{B,III} = \frac{(1-d)(2+d-d\mu)^2(1-c)^2}{4(1+d)(2-d\mu-d^2)(2-d\mu)},$$

$$\Pi_j^{B,III} = \frac{(1-d)(1-d\mu)\left(4+2d(1-\mu)-d^2(1+\mu)\right)^2(1-c)^2}{4(1+d)(2-d\mu)^2(2-d\mu-d^2)^2},$$

where $\beta^{III}(-1) > \beta^{III}(1)$.

Note that unlike the Cournot case, there is no monotonicity in $\partial\beta^{III}/\partial\mu$. We illustrate the predictions in Proposition 4 for the same three cases as above. In (a) $\beta^{III} = 1.38$, in (b) $\beta^{III} = -0.02$, and in (c) $\beta^{III} = -0.31$. Note that in all three cases $\beta^{II} > \beta^{III}$.

Comparison of results:

Proposition 5 enables the optimal delegation contracts to be compared when both firms delegate with respect to the case of the sequential type one firm delegation game.

Proposition 5 *If only one firm delegates then more aggressive sales behavior is encouraged than when both firms delegate, that is, $\beta^{II} - \beta^{III} > 0$.*

The normal form of the game with profit pairs for each possible strategy taken simultaneously by both firms is reported below.

j

		ND	D
i	ND	$\frac{1-d\mu}{(2-d-d\mu)^2}, \frac{1-d\mu}{(2-d-d\mu)^2}$	$\frac{(1-d\mu)\left(4+2d(1-\mu)-d^2(1+\mu)\right)^2}{4(2-d\mu)^2(2-d\mu-d^2)^2}, \frac{(2+d-d\mu)^2}{4(2-d\mu-d^2)(2-d\mu)}$
	D	$\frac{(2+d-d\mu)^2}{4(2-d\mu-d^2)(2-d\mu)}, \frac{(1-d\mu)\left(4+2d(1-\mu)-d^2(1+\mu)\right)^2}{4(2-d\mu)^2(2-d\mu-d^2)^2}$	$\frac{(2-d\mu)\left(2-d\mu-d^2\right)}{(4-2d(1+\mu)-d^2(1-\mu))^2}, \frac{(2-d\mu)\left(2-d\mu-d^2\right)}{(4-2d(1+\mu)-d^2(1-\mu))^2}$

All equilibrium profits are scaled by $\frac{(1-d)(1-c)^2}{(1+d)}$.

Proposition 6 summarizes the Nash equilibrium of the price game.

Proposition 6 *In the strategic delegation game with CVs, a unique Nash equilibrium in dominant strategies does exist, in which both firms decide to delegate. Moreover, this equilibrium is Pareto optimal only if β_i and β_j are strategic complements.*

In equilibrium, both owners delegate price decisions to managers, and design contracts which are a combination of profits and sales, and the result can be Pareto efficient or not depending on how the incentives interact. This result is in the line of the Cournot equilibrium discussed in Section 3.

Concentrating on consistent CV equilibriums, the condition that the slope of the best response function has to be equal to the conjecture is $\frac{d}{2-d\mu} = \mu$, no matter what strategy firms follow. Thus, for every $d > 0$ it implies a unique equilibrium of conjectures as a function of the degree of product differentiation, $\mu^* = \frac{1-\sqrt{1-d^2}}{d} \in (-1, 0)$, outside Bertrand's zero-conjectures. This μ^* is the same no matter what strategy firms follow, and the equilibrium profits for each firm are,

$$\Pi_*^B = \frac{\sqrt{1-d^2}}{\left(1+d+\sqrt{1-d^2}\right)^2} (1-c)^2 .$$

Therefore, being self-managed and being managerial yield the same profits. If $d = 1$ then $\Pi_*^B = 0$, which is the Bertrand equilibrium with homogeneous products, and if $d = 0$ then $\Pi_*^B = (1-c)^2 /2$, which are the monopoly profits.

5 Profitable delegation

In the previous two sections we have shown that the nature of market interaction plays a key role in the incentives to delegate managerial decisions. It remains to be shown under what type of strategic interaction and values of the parameters delegation is more profitable. We consider the different payoffs when both firms play delegation under each type of strategic interaction. Table 1 simulates values where, without loss of generality, $c = 0.2$ and for few values of d and conjectures.

Cheng (1985) shows that when goods are substitutes Cournot equilibrium prices (quantities) are higher than Bertrand equilibrium prices (quantities) and, as a result, a quantity strategy dominates a price strategy. As shown in Table 1, this result also holds when firms can delegate strategic decisions to managers and there are CV in the market

Table 1: Π^C (left) and Π^B (right) with delegation

$\lambda, \mu \setminus d$	0.2		0.5		0.8	
-1	0.131945	0.131945	0.097959	0.097959	0.058131	0.058131
-0.8	0.131952	0.131950	0.098304	0.098089	0.062443	0.058751
-0.6	0.131958	0.131954	0.098600	0.098229	0.065471	0.059465
-0.4	0.131964	0.131959	0.098857	0.098382	0.067704	0.060296
-0.2	0.131970	0.131965	0.099081	0.098549	0.069413	0.061274
0	0.131975	0.131970	0.099280	0.098733	0.070760	0.062443
0.2	0.131981	0.131975	0.099456	0.098935	0.071846	0.063862
0.4	0.131986	0.131981	0.099614	0.099159	0.072740	0.065621
0.6	0.131991	0.131987	0.099755	0.099408	0.073488	0.067852
0.8	0.131995	0.131993	0.099884	0.099686	0.074122	0.070760
1	0.132000	0.132000	0.100000	0.100000	0.074667	0.074667

competition stage. However, if the analysis is restricted to consistent conjectures, then Cournot and Bertrand profits are the same, $\Pi^B_* = \Pi^C_*$ for any $d \in (0,1)$. This result is in clear contrast with those in Singh and Vives (1984) and in Cheng (1985). This is because the unique consistent CV is more competitive under Cournot than under Bertrand. Figure 5 plots λ^* and μ^* as a function of d.

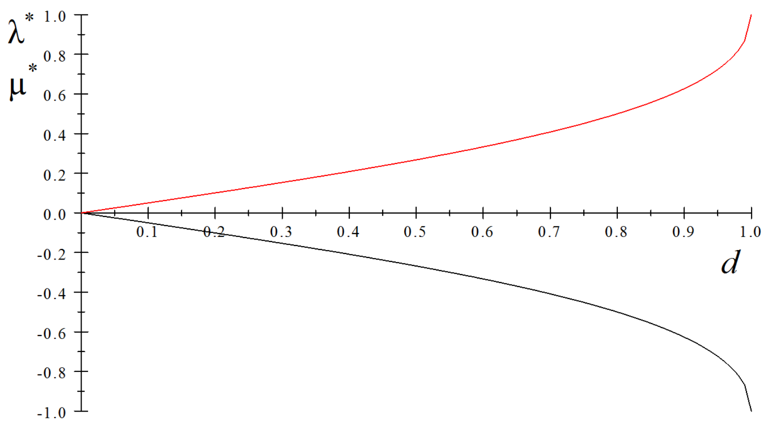

Figure 5: λ^* and μ^* as a function of d.

The consistent CV is negative (positive) under Cournot (Bertrand) competition for every $d \in (0,1)$.

Therefore, if owners delegate either quantity or price decisions to managers, requiring consistency in conjectures implies no strategic advantage no matter what type of competition is considered. This result is important because under not very stringent assumptions on market structure, firms' behavior, and conjectures, we reach the same conclusions as in Kreps and Scheinkman (1983) and Buccirossi (2001).

6 Conclusions

Delegation of strategic decisions such as price and quantity is common in large corporations. The limited information on market conditions held by shareholders prevents them from running firms themselves. Choosing the profit-maximizing contract depends on the nature of competition in the product market. This paper extends the literature on this topic by allowing decision makers to make CVs on how their rival reacts to a change in their own output or price. Firms can be either managerial or self-managed. Delegation is analyzed based on an F&J (1987) type contract.

We prove that regardless of the type of strategic interaction in the product market, there is a dominant strategy: to delegate. However, it is not always the result of a Prisoner's Dilemma because there are values of the parameters for which the profits of the firms in the subgame perfect Nash equilibrium are efficient. Therefore, an efficient solution can be achieved.

There is widespread criticism of how the set of equilibriums increases when conjectures are incorporated in the best response of agents to their rival's behavior. However, a new prediction useful for market organization arises when consistency in conjectures is required because there is no strategic advantage in going managerial or being self-managed, or competing in prices or quantities. Profits are the same in equilibrium under all possible scenarios.

This study can be extended in several directions. First, alternative incentive contracts can be explored, however we are actually concerned with the simplest formulation and try to reach conclusions on how firms' organization is affected by allowing conjectures in the product market. Second, the assumption of equal conjectures is not a restrictive assumption when information is symmetric and complete. If firms have access to the same information set then priors are likely to be the same. However, we can assume different specifications at the cost of more cumbersome algebra. Third, in this paper we restrict conjectures to the product market competition stage of the game, but needless to say conjectures can also be analyzed in the strategic delegation stage. Fourth, generalizations beyond linear specifications are interesting to analyze how sensitive results are to different demand and cost specifications. This issue remains to be explored.

References

[1] Bhattacharjee, T. and Pal, R. (2013). "Network externalities and strategic managerial delegation in Cournot duopoly: is there a prisoners' dilemma?", *Review of Network Economics,* 12, pp. 343–353.

[2] Bloomfield, M. (2017). "Product Market Competition, Disclosure and Strategic Delegation: Evidence from Revenue-Based Compensation", available at SSRN: https//ssrn.com/abstract=2862069

[3] Boone, J. (2008). "Competition: Theoretical parameterizations and empirical measures", *Journal of Institutional and Theoretical Economics,* 164, pp. 587-611.

[4] Bowley, A.L. (1924). "The Mathematical Groundwork of Economics". Oxford: Oxford University Press.

[5] Boyer, M. and Moreaux, M. (1983). "Consistent Versus Non-Consistent Conjectures in Duopoly Theory: Some Examples", *The Journal of Industrial Economics*, 32(1), pp. 97-110.

[6] Bresnahan, T. F. (1981). "Duopoly Models with Consistent Conjectures", *American Economic Review*,71, pp. 934-945.

[7] Buccirossi, P. (2001). "Price Competition and a Probabilistic Demand Yield the Cournot Outcome, the Bertrand Outcome and Much More", Working Paper no. 41, University of Rome La Sapienza, available at http://dep.eco.uniroma1.it/.

[8] Bulow, J., Geanakoplos, J. and Klemperer, P. (1985). "Multimarket oligopoly: strategic substitutes and strategic complements", *Journal of Political Economy,* 93, pp. 488-511.

[9] Cheng, L. (1985). "Comparing Bertrand and Cournot Equilibria: A Geometric Approach", *The RAND Journal of Economics*, 16(1), pp. 146-152.

[10] Cournot, A. (1838). "Recherches sur les principes mathématique de la théorie des richesses". Paris: Hachette.

[11] Dixit, A. (1979). "A Model of Duopoly Suggesting a Theory of Entry Barriers", *The Bell Journal of Economics,* 10, pp. 20-32.

[12] Fanti, L. and Meccheri, N. (2017). "A note on managerial delegation with asymmetric and convex costs", *Managerial and Decision Economics*, 38, pp. 279-284.

[13] Fershtman, C. and Judd, K. (1987). "Equilibrium incentives in oligopoly", *American Economic Review*,77, pp. 927-940.

[14] Fumas, V.S. (1992). "Relative Performance Evaluation of Management: The Effects of Industrial Competition and Risk Sharing", *International Journal of Industrial Organization,* 10, pp. 473-489.

[15] Giocoli, N. (2005). "The escape from conjectural variations: the consistency condition in duopoly theory from Bowley to Fellner", *Cambridge Journal of Economics*, 29(4), pp. 601-618.

[16] Häckner, J. (2000). "A Note on Price and Quantity Competition in Differentiated Oligopolies", *Journal of Economic Theory,* 93(2), pp. 233-239.

[17] Harrod, R.F. (1939). "An Essay in Dynamic Theory", *The Economic Journal*, 49, pp. 14-33.

[18] Hwang, H. and Mai, C.C. (1995). "Strategic management under duopoly", *Managerial and Decision Economics*, 16, pp. 239-247.

[19] Jansen, T., van Lier A. and van Witteloostuijn, A. (2009). "On the impact of managerial bonus systems on firm profit and market competition: the cases of pure profit, sales, market share and relative profits compared", *Managerial and Decision Economics*, 30, pp. 141–153.

[20] Kreps, D. and Scheinkmann, J. (1983). "Quantity precommitment and Bertrand competition yield Cournot outcomes", *Bell Journal of Economics*, 14, pp. 326–337.

[21] Laitner, J. (1980). ""Rational" Duopoly Equilibrium", *Quarterly Journal of Economics*, 95, pp. 641-662.

[22] Leontief, W. (1936). "Stackelberg on Monopolistic Competition", *The Journal of Political Economy*, 44(4), pp. 554-559.

[23] Liang, J.N. (1989). "Price Reaction Functions and Conjectural Variations", *Review of Industrial Organization*, 4(2), pp. 31-58.

[24] Manasakis, V., Mitrokostas, E. and Petrakis, E. (2010). "Endogenous Managerial Incentive Contracts in a Differentiated Duopoly, With and Without Commitment", *Managerial and Decision Economics*, 31, pp. 531–543.

[25] Miller, N. and Pazgal, A. (2002). "Relative Performance as a Strategic Commitment Mechanism", *Managerial and Decision Economics* 23, pp. 51-68.

[26] Mizuno, K. (2013). "Managerial incentives and endogenous coalition formation with externalities", *Mathematical Social Sciences*, 66, pp. 33-43.

[27] Olsder, G.J. (1981). "A Critical Analysis of a New Equilibrium Concept", Memorandum Nr. 329, Dept. Applied Maths., Twente University of Technology, The Netherlands.

[28] Pal, R. (2015). "Cournot vs. Bertrand under relative performance delegation: Implications of positive and negative network externalities", *Mathematical Social Sciences*, 75, pp. 94-101.

[29] Perry, M.K. (1982). "Oligopoly and Consistent Conjectural Variations", *The Bell Journal of Economics*, 13(1), pp. 197-205.

[30] Pfaffermayr, M. (1999). "Conjectural-Variation Models and Supergames with Price Competition in a Differentiated Product Oligopoly", *Journal of Economics*, 70(3), pp. 309-326

[31] Singh, N. and Vives, X. (1984). "Price and Quantity Competition in a Differentiated Duopoly", *The Rand Journal of Economics*, 15(4), pp. 546-554.

[32] Sklivas, S.D. (1987). "The strategic choice of managerial incentives", *Rand Journal of Economics*, 18 (3), pp 452-458.

[33] Stigler, G.J. (1940). "Notes on the Theory of Duopoly", *Journal of Political Economy*, 48(4), pp. 521-541.

[34] Vetter, H. (2016). "Delegation and incentives under evolutionary conjectures", *Managerial and Decision Economics*, DOI: 10.1002/mde.2816.

[35] Vroom, G. and Gimeno, J. (2007). "Ownership form, Managerial Incentives, and the Intensity of Rivalry", *Academy of Management Journal*, 50(4), pp. 901 – 922.