Analytical and numerical approach of an End Notched Flexure test configuration with an inserted roller for promoting mixed mode I/II.

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Abstract

A test configuration for studying mixed-mode is analyzed analytically and numerically. It is based on the End Notched Flexure test, inserting a roller in the cracked part in order to promote mixed mode I/II. The analytical approach includes the calculation of the force exerted by the roller, the midpoint displacement, the compliance of the test, and the relative displacement of both arms of the crack. Moreover, the energy release rate is determined based on the complementary strain energy. With respect to numerical analysis, the two-step extension procedure is used for determining energy release rates in mode I and mode II. Comparison between analytical and numerical results has been carried out in order to check the suitability of the test method.

Keywords: Composites, Delamination, Fracture mechanics, Finite element analysis, Mixed mode fracture.

1 Introduction

Interlaminar fracture is one of the most common failure modes in composite materials. As delamination in composites is often a mixed-mode fracture, it is important that the composite toughness be measured at different mode mixtures [1-4]. Using fracture mechanics to characterize the onset and growth of delamination has become a generally accepted practice. According to Griffith-Irwin linear elastic fracture mechanics, the crack initiation and propagation is governed by the critical strain energy release rate G [5,6] .

A standardized method for characterization of the mixed-mode I–II fracture toughness was introduced in 2001 [6], named mixed mode bending test (MMB). The fixture proposed in the standard ASTM D6671 [6] is a modified version of that originally proposed by Crews and Reeder [3,7] . Chen et al. [8] carried out a modification on the MMB test rig for making it easier to calculate G, avoiding to introduce the weight of the lever. Blanco et al. [9] proposed a solution for determining the distance of the lever arm in the MMB test with better accuracy. Tenchev and Falzon [10], presented an analytical solution for the MMB problem, when the crack has propagated beyond the middle of the beam.

For the MMB test the ratio between mode I and mode II, named mixed-mode ratio, remains constant. In the mixed-mode end load split test (MMELS) the mode mix depends on the crack extension. This test is a modified version of the end load split test (ELS), where the interlaminar crack in a beam-type specimen is forced to propagate under mixed-mode and has been analysed by different researchers. The studies of Hashemi et al. [11] and Kinloch et al. [4] are based on the beam theory formulated by Williams [12]. They proposed a series of analytical expressions for the characterization of the test. Blanco et al. [13] demonstrated that beam theory is capable of modelling MMELS test when the crack is centred in the thickness.

Other test and specimen configurations have been proposed in order to analyse the mixed mode I/II fracture. The asymmetric double-cantilever beam (ADCB), where the crack plane is out of the laminate midplane, generating a mixed mode load state at the crack tip. Mangalgiri et al. [14] were the first to apply the ADCB test. The European Structural Integrity Society (ESIS) TC4 group studied it in the 1990´s. [15]. Ducept et al. [16] carried out experiments on ADCB glass fibre reinforced epoxy composite samples and compared results with analytical and numerical ones. Bennati et al. [17,18] developed an enhanced beam theory model for the ADCB test based on the experimental work developed by Ducept et al. ADCB configuration has been analyzed by other authors [19-21]

Szekrényes proposed three different specimens for analyzing mixed mode I/II: Two prestressed specimens, one based on the end-notched flexure (ENF) specimen [22], and other based on the ELS specimen [23]. The main characteristic of these methods is that the mode I is provided by the insertion of a steel roller at the delamination plane. The third proposal, developed by Szekrényes and Uj [24], was the over-leg bending specimen, which is a modification of the single-leg bending -specimen, where the load is introduced eccentrically.

Kolluri et al. [25] introduced a new miniature setup capable of applying a mixed mode bending load to a bilayer delamination sample with a pre-crack. In this set up a microscope is necessary to observe the delamination.

Bonhomme et al. [26] proposed the two-step extension procedure (TSEP) as an alternative to the virtual crack closure technique (VCCT), which allows to determine *G*. Mollón et al. [27], compared experimental results of *G* obtained by ADCB and reduced by an analytical method based on a modified beam theory with numerical results obtained by the TSEP. They found a good agreement among all the studied methods.

The aim of this study is to present a novel analytical model for a mixed-mode I/II interlaminar fracture test configuration for unidirectional composites. It is similar to ENF, but the specimen has an inserted roller for promoting mixed mode. This configuration was proposed by Szekrényes [22] for the particular position in which the roller is above the support. In the present work, the configuration is generalized for any position of the roller. Numerical analysis is also carried out by FEM applying the TSEP.

Nomenclature

a, a_I, Δa	Crack length, corrected crack length and crack increment, respectively
c_0, c_i	Distances from the support to the position of the roller when it is at the outer side and at the inner side of the support, respectively
b, $2h$	Width and thickness of the specimen, respectively
d_{interf}	Distance from the support to the edge of the specimen, when there is interference
A_1, A_2, A_0	Surface areas of the upper and lower arms in the cracked zone, and of the whole section in the uncracked zone, respectively
C	Compliance of the test
E_f	Flexural modulus
E_L, E_T, G_{LT}, ν_{LT}	Longitudinal, transverse, in-plane shear elastic moduli and Poisson ratio, respectively
F_{x1i}, F_{y1i}, u_{1i}, v_{1i}	Forces at the crack tip and horizontal and vertical displacements of the released nodes, respectively
G_I, G_{II}, G	Mode I, mode II and total energy release rates, respectively
I_1, I_2, I_0	Second moments of area with respect to the middle plane of the upper and lower arms in the cracked zone, and of the whole section in the uncracked zone, respectively
M_1, M_2, Q_1, Q_2	Bending moments and shear forces in the cracked zone
M_I, M_{II}, Q_I, Q_{II}	Bending moments and shear forces in the crack tip due to mode I, and mode II, respectively
L	Half span of the test
P	Applied load
R	Roller radius
W, U, U^*	Work done by external forces, strain energy and complementary strain energy, respectively
Y	Force exerted by the roller
δ	Displacement of the load application point of the specimen
δ_{rel}	Relative displacement between the upper and lower arms in the cracked zone
P_0, δ_0	Initial load and initial displacement, respectively
ADCB	Asymmetric double-cantilever beam
ELS	End load split test
MMELS	Mixed-mode end load split test

2 Description of the test

The test configuration is based on the ENF test. In order to get mixed mode, a roller is introduced between the two surfaces of the crack: The mode II is provided by the external load, and the mode I is obtained by the opening of the crack due to the insertion of the roller as shown in Fig. 1.

This configuration has already been used by Szekrényes [22] in the particular case where the roller is directly above the support. In the present analysis the configuration of the test is generalized. The roller position can be located at the outer side or at the inner side of the support, as shown in Fig. 1.

In the analytic approach it is assumed the hypothesis of small bending rotations. Thus, the horizontal components of forces that the supports and the roller exert on the specimen are neglected. Otherwise, even in the case of small bending rotations, the span reduction due to the variation in the contact point between the specimen and support rollers is not negligible depending on the radius of the rollers [28] It is expected a similar behaviour for the inserted roller. Nevertheless, it has been seen that those effects are not important in ENF tests if the roller radii is 2.5 mm or lesser [29]. As the radii of the inserted rollers used in the present work are lesser than that value, the effects concerning span variations are not considered. Furthermore, the effect of friction between rollers and specimen is neither considered.

3 Redundant Force and Compliance

3.1 Roller positioned at the outer side

3.1.1 Force exerted by the roller

Fig. 2 shows the test configuration and dimensions when the roller is located at the outer side of the support.

The roller is replaced by the forces that generate the displacement. Being Y that force, and $P/2$ the reaction force, bending moments and shear forces in the crack zone are shown in Fig. 3.

The upper arm is named 1 and the lower arm is named 2. The internal forces and moments in the cracked zone are:

$$\begin{array}{ll}
0 < x < c_o & \\
Q_1 = Y & M_1 = Yx \\
Q_2 = -Y & M_2 = -Yx \\
& \\
c_o < x < a & \\
Q_1 = Y & M_1 = Yx \\
Q_2 = \left(\dfrac{P}{2} - Y\right) & M_2 = \left(\dfrac{P}{2} - Y\right)x - \dfrac{P}{2}c_o
\end{array} \tag{1}$$

Bending moments and shear forces in the zone without crack are shown in Fig. 4.

Applying equilibrium equations it results:

$$a < x < c_0 + L$$
$$Q = \frac{P}{2} \qquad M = \frac{P}{2}\left(x - c_0\right) \tag{2}$$

Bending moments and shear forces in the right half of the specimen are:

$$0 < x' < L$$
$$Q = -\frac{P}{2} \qquad M = \frac{P}{2}x' \tag{3}$$

The force exerted by the roller can be determined applying the Castigliano-Engesser theorem [30], which in the case of shear and bending is given by:

$$\frac{\partial U^*}{\partial F_i} = \int_L \frac{M}{E_f I}\frac{\partial M}{\partial F_i}dx + \int_L \frac{6Q}{5G_{LT}A}\frac{\partial Q}{\partial F_i}dx = \delta_i \tag{4}$$

Being U^* the complementary strain energy; F_i an independent applied force; and δ_i the displacement of the application point of F_i in the direction of F_i.

The relative displacement associated to the forces exerted by the roller is $2R$, R being the roller radius. Thus, the derivative of the complementary strain energy U^* with respect to the redundant unknown Y is:

$$\frac{\partial U^*}{\partial Y} = 2R \tag{5}$$

Bending moments and shear forces depend on Y only in the cracked zone. Replacing M, Q and their derivatives with respect to Y from Eq. (1) in Eq. (4) and applying Eq. (5) the redundant force is:

$$Y = \frac{2R + P\left[\frac{(a-c_o)^2(2a+c_o)}{E_f bh^3} + \frac{3(a-c_o)}{5G_{LT}bh}\right]}{\frac{8a^3}{E_f bh^3} + \frac{12a}{5G_{LT}bh}} \tag{6}$$

Where a is the crack length; E_f the flexural modulus; G_{LT} is the in-plane shear modulus which is equal to the out-of-plane shear modulus $G_{LT'}$ assuming that the material is transversely isotropic; R is the radius of the roller, b the width of the specimen; and $2h$ the total thickness of the specimen. Considering only bending effects the force Y is:

$$Y = \frac{R}{4}\frac{E_f bh^3}{a^3} + \frac{P(a-c_o)^2(2a+c_o)}{8a^3} \tag{7}$$

According to Eq. (7) the force Y is always positive, for any position of the roller.

3.1.2 *Displacement of the load application point*

The displacement of the load application point is determined by the unit load method and the Engesser-Castigliano´s theorem. Since Y has been considered the redundant unknown, the equivalent basic system is the original one without the roller. Moreover, when a unit load is applied in the middle of the specimen, the upper arm is an unloaded cantilever. Then, it is not considered in Fig. 5.

The derivatives obtained in the different parts of Fig. 5 are written in lowercase letters, being:

$$\begin{aligned}
&c_o < x < a \\
&q_2 = \frac{1}{2} \qquad m_2 = \frac{1}{2}\left(x - c_o\right) \\
&a < x < c_o + L \\
&q = \frac{1}{2} \qquad m = \frac{1}{2}\left(x - c_o\right) \\
&0 < x' < L \\
&q = -\frac{1}{2} \qquad m = \frac{1}{2}x'
\end{aligned} \tag{8}$$

Hence, the displacement of the load application point is given by

$$\delta = \int_0^a \frac{M_1 m_1}{E_f I_1} dx + \int_0^a \frac{6Q_1 q_1}{5G_{LT} A_1} dx + \int_a^{c_o+L} \frac{Mm}{E_f I_0} dx + \int_a^{c_o+L} \frac{6Qq}{5G_{LT} A_0} dx + \int_a^{L} \frac{Mm}{E_f I_0} dx' + \int_a^{L} \frac{6Qq}{5G_{LT} A_0} dx' \tag{9}$$

Substituting the values from Eq.(1), Eq.(2), Eq.(3) and Eq. (8) in Eq.(9), the displacement is:

$$\delta = \frac{1}{8E_f bh^3}\left[P\left(7(a-c_o)^3 + 2L^3\right) + 8Y(3a^2 c_o - 2a^3 - c_o{}^3)\right] + \frac{3}{5G_{LT} bh}\left[\frac{P}{4}\left(a - c_o + 2L\right) - Y(a - c_o)\right] \tag{10}$$

Taking into account only the term that corresponds to bending effects, and replacing the value of Y given in Eq.(7) in Eq.(10), the displacement can be expressed as:

$$\delta = \frac{P}{8E_f bh^3}\left[\frac{\left(3a^3 + c_o{}^3 + 3ac^2\right)\left(a - c_o\right)^3}{a^3} + 2L^3\right] - \frac{R}{4}\frac{(a-c_o)^2(2a+c_o)}{a^3} \tag{11}$$

According to Eq. (11), after inserting the roller and without applying load, the displacement of the load application point is given by the second term of the right hand, which is negative. Then, as the unit load has been applied downwards, the initial displacement is upwards.

3.1.3 *Relative Displacement between the crack-arm ends*

When the roller is inserted, the two arms of the crack are separated. The relative displacement between the crack-arm ends has been determined in order to analyse possible interference. Two opposite unit loads are applied on the basic system corresponding to the specimen, as shown in Fig. 6

The derivatives obtained in the cracked zone of Fig. 6 are:

$$\begin{aligned}
&0 < x < a \\
&q_1 = 1 \qquad m_1 = x + d \\
&q_2 = -1 \qquad m_2 = -\left(x + d\right)
\end{aligned} \tag{12}$$

Substituting the forces and moments from Eq.(1) and Eq.(12) in the Engesser-Castigliano theorem of Eq. (4), the relative displacement between the two tips of the crack arms is given by:

$$\delta_{rel} = -P\left[\frac{(a-c_o)^2(2a+c_o+3d)}{E_f bh^3} + \frac{3(a-c_o)}{5G_{LT} bh}\right] + Y\left[\frac{4a^2(2a+3d)}{E_f bh^3} + \frac{12a}{5G_{LT} bh}\right] \tag{13}$$

Taking into account only terms that corresponds to bending effects and replacing the value of Y given in Eq. (7) in Eq. (13), the relative displacement is:

$$\delta_{rel} = P\left[\frac{3(a-c_o)^2 c_o d}{2aE_f bh^3}\right] + R\left(\frac{2a+3d}{a}\right) \tag{14}$$

When the relative displacement is positive the two arms get separated. When it is equal to zero or negative, there is interference. According to Eq. (14) the relative displacement between the two tips of the crack arms is always positive and there is not interference in any case.

3.1.4 Compliance

As it can be seen in Eq.(10), there is an initial negative displacement when P=0. Therefore, when the displacement is null, there is a positive load.

The initial conditions when shear effects are not considered can be calculated replacing P=0 and δ=0 in Eq.(11):

$$\begin{aligned} \delta_0 &= -\frac{R}{4}\frac{(a-c_o)^2(2a+c_o)}{a^3} \\ P_0 &= -\frac{R}{4}\frac{(a-c_o)^2(2a+c_o)}{a^3} 8E_f bh^3 \left[\frac{(a-c_o)^3(3a^3+3ac_o{}^2+c_o{}^3)}{a^3} + 2L^3\right]^{-1} \end{aligned} \tag{15}$$

The expression of the compliance of the applied load-displacement is:

$$C = \frac{\delta}{P-P_0} = \frac{\delta_0}{P_0} \tag{16}$$

Replacing the values of Eq.(15) in Eq.(16), compliance can be expressed as:

$$C = \frac{1}{8E_f bh^3}\left[\frac{(a-c_o)^3(3a^3+3ac_o{}^2+c_o{}^3)}{a^3} + 2L^3\right] \tag{17}$$

According to Eq.(17) the compliance does not depend on the radius of the roller.

3.2 Roller positioned at the inner side

3.2.1 Force exerted by the roller

In this case the position of the roller is defined by c_i, as shown in

Fig. 7.

Moments and shear forces in the cracked zone are:

$$
\begin{aligned}
&0 < x < c_i \\
&Q_1 = 0 \qquad M_1 = 0 \\
&Q_2 = \frac{P}{2} \qquad M_2 = \frac{P}{2}x \\
&c_i < x < a \\
&Q_1 = Y \qquad M_1 = Y(x - c_i) \\
&Q_2 = \left(\frac{P}{2} - Y\right) \qquad M_2 = \frac{P}{2}x - Y(x - c_i)
\end{aligned}
\tag{18}
$$

Moments and shear forces in the zone without crack and in the other half of the specimen are:

$$
\begin{aligned}
&a < x < L \\
&Q = \frac{P}{2} \qquad M = \frac{P}{2}x \\
&0 < x' < L \\
&Q = -\frac{P}{2} \qquad M = \frac{P}{2}x'
\end{aligned}
\tag{19}
$$

Following a similar procedure to the previous section, the force between the two arms is given by:

$$
Y = \frac{2R + \dfrac{P(a-c_i)^2(2a+c_i)}{E_f bh^3} + \dfrac{3P(a-c_i)}{5G_{LT}bh}}{\dfrac{8(a-c_i)^3}{E_f bh^3} + \dfrac{12(a-c_i)}{5G_{LT}bh}}
\tag{20}
$$

Considering only bending effects, the Y force is:

$$
Y = \frac{R}{4}\frac{E_f bh^3}{(a-c_i)^3} + \frac{P(2a+c_i)}{8(a-c_i)}
\tag{21}
$$

The Y force is always positive, for any position of the roller, since $a>c_i$ as shown in

Fig. 7.

3.2.2 *Displacement of the load application point*

In spite of the basic system is the same as in the previous case, the derivatives of forces and moments are determined in different intervals, being:

$$\begin{aligned}
&0 < x < a \\
&q_2 = \frac{1}{2} \qquad m_2 = \frac{1}{2}x \\
&a < x < L \\
&q = \frac{1}{2} \qquad m = \frac{1}{2}x \\
&0 < x' < L \\
&q = -\frac{1}{2} \qquad m = \frac{1}{2}x'
\end{aligned} \tag{22}$$

Replacing the moments and forces from Eq.(18), (19) and (22) in Eq (4), the displacement of the load application point can be expressed as:

$$\delta = \frac{1}{8E_f bh^3}\left[P\left(7a^3 + 2L^3\right) - 8Y(a-c_i)^2(2a+c_i)\right] + \frac{1}{20G_{LT}bh}\left[P(3a+6L) - 12Y(a-c_i)\right] \tag{23}$$

Taking into account only bending effects, substituting the value of the force Y (Eq.(21)), the displacements is:

$$\delta = \frac{P}{8E_f bh^3}\left(3a^3 + 2L^3 + c_i^{\,3} + 3ac_i^{\,2}\right) - \frac{R}{4}\frac{(2a+c_i)}{(a-c_i)} \tag{24}$$

According to Eq.(24) the initial displacement is upwards.

3.2.3 *Relative Displacement of the crack-arm ends*

The basic system is the same as in the previous case, shown in Fig. 6. Thus, the derivatives in the cracked zone are the ones of Eq.(22). Replacing the forces and moments from Eq. (18) and Eq.(12) in Eq.(4), the relative displacement between the crack-arm ends can be expressed as:

$$\delta_{rel} = -P\left[\frac{a^2(2a+3d)}{E_f bh^3} + \frac{3a}{5G_{LT}bh}\right] + Y\left[\frac{4(a-c_i)^2(2a+c_i+3d)}{E_f bh^3} + \frac{12(a-c_i)}{5G_{LT}bh}\right] \tag{25}$$

Taking into account only bending effects and replacing the value of the force Y given in Eq.(21) in Eq.(25) the relative displacement is:

$$\delta_{rel} = -P\left[\frac{c_i\left[3a(c_i+d) + c_i(c_i+3d)\right]}{2E_f bh^3}\right] + R\left(\frac{2a+c_i+3d}{a-c_i}\right) \tag{26}$$

In this case it is possible to calculate interference conditions imposing $\delta_{rel} = 0$ in Eq.(26). Hence, a critical value of the distance from the support to the specimen end, named d in Fig. 6, is obtained:

$$d_{interf} = \frac{c_i^{\,2}(-a+c_i)(3a+c_i)P + 2b(2a+c_i)E_f h^3 R}{3(a-c_i)c_i(a+c_i)P - 6E_f bh^3 R} \tag{27}$$

If $d<d_{\text{interf}}$ there is not interference and the proposed analysis is valid. If $d>d_{\text{interf}}$ there is contact between the crack-arm ends. In this case, the analysis is not valid.

3.2.4 *Compliance*

The initial displacement δ_0 and initial load P_0 are obtained from Eq.(24), imposing P=0 and δ=0 respectively:

$$\begin{aligned} \delta_0 &= -\frac{R}{4}\frac{(2a+c_i)}{(a-c_i)} \\ P_0 &= \frac{2RE_f bh^3(2a+c_i)}{(a-c_i)(3a^3+c_i{}^3+2L^3+3ac_i{}^2)} \end{aligned} \tag{28}$$

Replacing the values of Eq.(28) in Eq.(16), compliance can be expressed as:

$$C = \frac{3a^3+2L^3+c_i{}^3+3ac_i{}^2}{8E_f bh^3} \tag{29}$$

In this case, as in the previous one, the compliance is independent on the radius of the roller.

3.3 Roller positioned above the support

Regarding the particular case in which the roller is directly positioned above the support, that is c_o=c_i=0 the forces obtained from Eq. (6) and Eq. (20) are the same being:

$$Y = \frac{P}{4} + \frac{R/2}{\left(\dfrac{2a^3}{E_f bh^3}+\dfrac{3a}{5G_{LT}bh}\right)} = \frac{P}{4}+\frac{R}{4}\frac{E_f bh^3}{a^3}\left[1+\frac{3}{10}\left(\frac{h}{a}\right)^2\frac{E_f}{G_{LT}}\right]^{-1} \tag{30}$$

If R=0, the test is an ENF test and Y=P/4 is obtained.

For the displacement of the load application point, when c_o=c_i=0, Eq. (10) and Eq.(23) lead to:

$$\delta = \frac{1}{8E_f bh^3}\left[P\left(7a^3+2L^3\right)-16Ya^3\right]+\frac{1}{20G_{LT}bh}\left[P(3a+6L)-12Ya\right] \tag{31}$$

Replacing the value of Y (Eq.(30)) in the former equation, the displacement can be expressed as:

$$\delta = P\left[\left(\frac{3a^3+2L^3}{8E_f bh^3}\right)+\left(\frac{3L}{10G_{LT}bh}\right)\right]-\frac{R}{2} \tag{32}$$

If R=0, the displacement is the same as in the ENF test.

For the relative displacement between the two crack-arm ends, when c_o=c_i=0, Eq. (13) and Eq.(25) lead to:

$$\delta_{rel}(c_i=c_o=0) = -P\left[\frac{a^2(2a+3d)}{8E_f bh^3}+\frac{3a}{5G_{LT}bh}\right]+Y\left[\frac{4a^2(2a+3d)}{E_f bh^3}+\frac{12a}{5G_{LT}bh}\right] \tag{33}$$

Replacing c_o=c_i=0, in Eq.(14) and Eq.(26) that include only bending effects, the relative displacement is:

$$\delta_{rel} = R\left(\frac{2a+3d}{a}\right) \tag{34}$$

According to Eq.(34) the relative displacement is always positive. That means that for any d distance, the two tips remain separated.

The initial displacement and the initial load are obtained from Eq.(32), being:

$$\begin{aligned} \delta_0 &= -\frac{R}{2} \\ P_0 &= \frac{R}{2}\left(\frac{3a^3+2L^3}{8E_f bh^3} + \frac{3L}{10G_{LT}bh}\right)^{-1} \end{aligned} \tag{35}$$

Replacing values from Eq.(35) in Eq.(16) the compliance is:

$$C = \frac{3a^3+2L^3}{8E_f bh^3} + \frac{3L}{10G_{LT}bh} \tag{36}$$

According to Eq. (36), the compliance is the same as in the ENF test and it does not depend on the roller radius. The crack length, based on the compliance, can be expressed as:

$$a = \left(\frac{8E_f bh^3}{3}C - \frac{2}{3}L^3 - \frac{4E_f h^2 L)}{5G_{LT}}\right)^{1/3} \tag{37}$$

Arrese et al. [29] developed a new method for determining the crack length at each point of the test based on the experimental compliance in ENF tests. Hence, the same method can be used when the effect of bending rotations at supports is negligible, since the compliance for ENF test and for the present test configuration are the same. Therefore, there is no need to monitor the crack length using any optical method, which is one of the most important drawbacks for any test that includes mode II. This problem was addressed by Szekrényes for the present configuration [22].

4 Energy Release Rate

4.1 Energy Balance Criterion

According to Griffith [31] and Irwin [32], the energy balance in a small crack advance can be expressed as:

$$dW = dU + Gbda \tag{38}$$

Where dW is the work done by external forces; dU is the change in strain energy; G is the energy needed for the crack advance per unit area; b is the width of the crack; and da is the differential crack advance. The differential work done by the external forces F_i in their respective displacements δ_i, assuming the repeated index convention is $dW = F_i d\delta_i$. Then, Eq. (38) results in:

$$dU = F_i d\delta_i - Gbda \tag{39}$$

In spite of the crack advance is an irreversible process, it is assumed that in an elemental variation the strain energy is an exact differential. According to Eq.(39) the state variables are δ_i and a. Then it results that:

$$dU = \frac{\partial U}{\partial \delta_i} d\delta_i + \frac{\partial U}{\partial a} da \quad (40)$$

Identifying first summands of Eq.(39) and Eq.(40), it results that $\partial U / \partial \delta_i = F_i$, which is the first theorem of Castigliano. Identifying the second summands it results:

$$G = -\frac{1}{b}\left(\frac{\partial U}{\partial a}\right)_{\delta = cte} \quad (41)$$

Furthermore the complementary strain energy U^* is defined as:

$$U^* = F_i \delta_i - U \quad (42)$$

Differentiating Eq.(42), and replacing in the energy balance equation of Eq.(38) it results:

$$dU^* = \delta_i dF_i + Gbda \quad (43)$$

In this case the state variables are F_i and a, thus:

$$dU^* = \frac{\partial U^*}{\partial F_i} dF_i + \frac{\partial U^*}{\partial a} da \quad (44)$$

Identifying the first summands of Eq.(43) and Eq.(44) it results the theorem of Engesser-Castigliano given in Eq. (4). Identifying the second summands it results:

$$G = \frac{1}{b}\left(\frac{\partial U^*}{\partial a}\right)_{F_i = cte} \quad (45)$$

The above explanations are carried out because in this case there are two forces that carry out work, Y and P. The work done by Y is related to the finite displacement imposed by the roller. The work done by P is the usual one, due to the application of the load. According to Eq. (45) the complementary strain energy must be used for determining G if forces are used as state variables. In a linear elastic system, the complementary strain energy is the same that the strain energy, being:

$$U^*(F_i, a) = \frac{1}{2}\int_l \frac{M^2}{E_f I} dx + \frac{1}{2}\int_l \frac{6Q^2}{5G_{LT}A} dx \quad (46)$$

In order to obtain the derivative with respect to a, as the integral limits depend on that parameter, Leibniz's integral rule is appropriate. Given a function $f = f(x,a)$ this rule states that [33]:

$$\frac{\partial}{\partial z}\int_{a(z)}^{b(z)} f(x,z)dx = \int_{a(z)}^{b(z)} \frac{\partial f}{\partial z} dx + f(b(z),z)\frac{\partial b}{\partial z} - f(a(z),z)\frac{\partial a}{\partial z} \quad (47)$$

As loads remain constant, in the derivation process it is assumed that $U^* = U^*\left(x,a\right)$. Applying Eq. (47) to Eq. (46) it results:

$$\frac{\partial U^*}{\partial a} = \int_{u(a)}^{v(a)} \frac{M}{E_f I}\frac{\partial M}{\partial a} dx + \int_{u(a)}^{v(a)} \frac{6Q}{5G_{LT}A}\frac{\partial Q}{\partial a} dx + U^*(v(a),a)\frac{\partial v}{\partial a} - U^*(u(a),a)\frac{\partial u}{\partial a} \quad (48)$$

As M and Q do not depend on a, the first and second terms of the right hand of Eq. (48) are 0.

It is worth pointing out that in the present case the approach based on the derivative of the compliance is not suitable, as the work is not carried out by a unique force, but by two.

4.2 Roller positioned at the outer side

Determining U^* from Eq. (46) after replacing the moments and shear forces from Eq.(1), Eq.(2) and Eq.(3), the energy release rate is determined replacing Eq. (48) in Eq. (45), being:

$$G=\frac{3\left[4Ya-P(a-c_o)\right]^2}{4E_f b^2h^3}+\frac{3(P-4Y)^2}{40G_{LT}b^2h}+\frac{9P^2(a-c_o)^2}{16E_f b^2h^3} \tag{49}$$

When R=0 and c_o=0 the result corresponds to ENF configuration. Then, the partition of the energy release rate, due to each fracture mode, can be expressed as follows:

$$\begin{aligned} G_I &= \frac{3\left[4Ya-P(a-c_o)\right]^2}{4E_f b^2h^3}+\frac{3(P-4Y)^2}{40G_{LT}b^2h} \\ G_{II} &= \frac{9P^2(a-c_o)^2}{16E_f b^2h^3} \end{aligned} \tag{50}$$

Replacing the value of Y given in Eq.(6) in Eq.(50), and taking into account only bending effects, G_I and G_{II} are given by:

$$\begin{aligned} G_I &= \frac{3R^2Eh^3}{4a^4}-\frac{3PRc_o(a^2-c_o{}^2)}{4a^4b}+\frac{3P^2c_o{}^2(a^2-c_o{}^2)^2}{16a^4E_f b^2h^3} \\ G_{II} &= \frac{9P^2(a-c_o)^2}{16E_f b^2h^3} \end{aligned} \tag{51}$$

It is worth pointing out that when the roller is positioned at the outer side, the value (a-c_o) is the usual crack length measured in the ENF test configuration, according to Fig. 2.

4.3 Roller positioned at the inner side

Following the same procedure as in the previous section, determining U^* from Eq. (46) after replacing the moments and shear forces from Eq.(18) and Eq.(19), the energy release rate is determined replacing Eq. (48) in Eq. (45), being:

$$G=\frac{3\left[4Y(a-c_i)-Pa\right]^2}{4E_f b^2h^3}+\frac{3(P-4Y)^2}{40G_{LT}b^2h}+\frac{9P^2a^2}{16E_f b^2h^3} \tag{52}$$

By similar reasoning to the previous case, the partition of the energy release rate due to each fracture mode can be expressed as:

$$\begin{aligned} G_I &= \frac{3\left[4Y(a-c_i)-Pa\right]^2}{4E_f b^2h^3}+\frac{3(P-4Y)^2}{40G_{LT}b^2h} \\ G_{II} &= \frac{9P^2a^2}{16E_f b^2h^3} \end{aligned} \tag{53}$$

Replacing the value of Y of Eq.(20) in Eq.(53), and taking into account only bending effects, G_I and G_{II} are given by:

$$G_I = \frac{3R^2E_fh^3}{4(a-c_i)^4} + \frac{3PRc_i}{4b(a-c_i)^2} + \frac{3P^2c_i^{\,2}}{16E_fb^2h^3}$$
$$G_{II} = \frac{9P^2a^2}{16E_fb^2h^3} \tag{54}$$

4.4 Roller positioned above the support

Regarding the particular case in which the roller is positioned above the support, the energy release rate and the contribution of each mode can be determined, replacing c_i=0 or c_o=0 in Eq. (50) and Eq.(52) , respectively. The results obtained from both equations are the same. When shear effects are not considered, G_I and G_{II} can be obtained from Eq. (51) and Eq. (54) being:

$$G_I = \frac{3R^2E_fh^3}{4a^4}$$
$$G_{II} = \frac{9P^2a^2}{16E_fb^2h^3} \tag{55}$$

Results of Eq.(55) are the same as those obtained by Szekrényes [22]. In the following paragraphs it is shown that mode I in this particular configuration is associated to the work done for introducing the roller. Before starting the external load application, the roller is inserted between the two surfaces of the crack. A force appears between the two arms Y for enforcing the displacement $2R$. The initial complementary strain energy can be expressed as follows:

$$U = U^* = 2\left(\frac{1}{2}Y_0R\right) \tag{56}$$

where the value of the force Y when P=0 is called Y_0. Replacing the value of Y_0 from Eq. (30) in Eq.(56), considering only bending effects and then calculating the energy release rate according to Eq.(45) it results:

$$G_{Initial} = \frac{3R^2E_fh^3}{4a^4} \tag{57}$$

The result obtained in Eq. (57) is the same result obtained in Eq.(55) for G_I.

4.5 William's partition approach

According to Williams [12], it is possible to determine the energy release rate G of a delaminated specimen based on the applied moments at the end of a crack. The mode partitioning consists of separating the total G, into the opening component G_I and the shear component G_{II}, being $G = G_I + G_{II}$.

Considering the moment case shown in Fig. 8, the opening mode only requires moments in opposite senses. Thus $-M_1$ is applied in the upper arm and M_1 in the lower arm. Pure mode II is obtained when the curvature of the two arms is the same.

The relation between moments for each fracture mode and moments at the crack tip, and between shear forces for each fracture mode and shear forces at the crack tip are:

$$M_I = \frac{M_2 - M_1}{2}$$
$$M_{II} = \frac{M_2 + M_1}{2}$$
$$Q_I = \frac{Q_2 - Q_1}{2}$$
$$Q_{II} = \frac{Q_2 + Q_1}{2} \tag{58}$$

The mode partition, being b the section width and $2h$ the total thickness, can be expressed as follows:

$$G_{I_M} = \frac{12M_I^2}{E_f b^2 h^3}$$
$$G_{I_Q} = \frac{6Q_I^2}{5G_{LT} b^2 h} \tag{59}$$
$$G_{II} = \frac{9M_{II}^2}{E_f b^2 h^3}$$

When the roller is at the outer side, the moments and forces at the crack tip are shown in Fig. 9:

The bending moments and shear forces are then:

$$\begin{aligned} Q_1 &= Y & M_1 &= -Ya \\ Q_2 &= \frac{P}{2} - Y & M_2 &= Ya - \frac{P}{2}(a - c_o) \end{aligned} \tag{60}$$

Replacing the values from Eq.(60) in Eq.(59), the energy release rate can be expressed as:

$$G = \frac{3\left[4Ya - P(a - c_o)\right]^2}{4E_f b^2 h^3} + \frac{3(P - 4Y)^2}{40G_{LT} b^2 h} + \frac{9P^2(a - c_o)^2}{16E_f b^2 h^3} \tag{61}$$

The result obtained in Eq. (61) is the same as that obtained in Eq.(49).

When the roller is positioned at the inner side, the moments and shear forces at the crack tip are:

$$\begin{aligned} Q_1 &= Y & M_1 &= -Y(a - c_i) \\ Q_2 &= \frac{P}{2} - Y & M_2 &= Y(a - c_i) - \frac{P}{2}a \end{aligned} \tag{62}$$

Replacing the values from Eq.(62) in Eq. (59), the energy release rate can be expressed as follows

$$G = \frac{3\left[4Y(a - c_i) - Pa\right]^2}{4E_f b^2 h^3} + \frac{3(P - 4Y)^2}{40G_{LT} b^2 h} + \frac{9P^2 a^2}{16E_f b^2 h^3} \tag{63}$$

The result obtained in Eq. (63) is the same as that obtained in Eq.(52). Therefore, the values of G obtained in the present study agree with those obtained by William's partition method.

5 Analysis of mode mixity

For a given specimen geometry shown in Table. 1 and a given material, there are two parameters that can be varied to achieve a wide range of mode mixity: the roller radius and the roller position.

The material used to perform analytical and numerical calculations has been the HexPly AS4/8552 RC34 from Hexcel Composites, which properties are given in Table 2. This laminate incorporates the Hexcel 8552 modified epoxy resin in order to improve toughness properties.

In spite of the tensile, compressive and flexure modulus are different in a unidirectional composite [34], the three modulus have been considered as equal as a first approach. Then, the tensile modulus given in Table 2 has been used in analytic and numeric calculations.

The critical values of P and δ, corresponding to crack initiation, named P_c and δ_c respectively, have been determined using the values of G_{Ic} and G_{IIc} of Table 2 according to the linear failure criterion given by [35]:

$$\frac{G_I}{G_{Ic}} + \frac{G_{II}}{G_{IIc}} = 1 \tag{64}$$

The criterion of Eq. (64) has been selected as a simple way for determining the values P_c and δ_c that correspond to crack initiation. Furthermore, in order to take into account the effect of rotation of the crack arms at the crack tip when G_I is determined, the crack length must be corrected in the analytic approach. According to Williams, the effect of the end rotation could be modelled by increasing the real length by an amount Δa given by [36]:

$$\frac{\Delta a}{h} = \sqrt{\frac{E_L}{11G_{LT}}\left[3-2\left(\frac{\Gamma}{1+\Gamma}\right)^2\right]} \qquad \Gamma = 1.18\frac{\sqrt{E_L E_T}}{G_{LT}} \tag{65}$$

This correction is related to the stiffness of the specimen along the thickness, which is not included in beam theory. Then, the mode I Double Cantilever Beam test is modelled as an elastic foundation in the middle plane of the specimen for obtaining Eq. (65). This correction is not applied to numerical analysis, as the stiffness along the thickness is incorporated in the model. The correction of the crack length in mode I is included in the standard ASTM D5528 [37]. With respect to G_{II}, the sliding crack advance mode associated to it has no relation with the stiffness along the thickness and any correction factor with respect to the crack length is not included in the recently published standard ASTM D7905 [38]. Thus, the crack length used in the analytic approach for G_I has been corrected weighting the contribution of mode I according to:

$$a_I = a + \frac{G_I}{G}\Delta a \tag{66}$$

The values of G_I and G_{II} have been obtained substituting P_c, the mechanical properties of Table 2 and the geometric parameters of Table. 1 in Eq.(6) and Eq.(20) for obtaining the Y force for the outer and inner cases, respectively. Then, Eq. (50) and in Eq.(53) are used for determining G_I and G_{II} in the outer and inner cases, respectively. As mentioned before, the corrected crack length a_I from Eq.(66) is applied to the determination of G_I.

Fig. 10 shows the influence of the variation of the radius of the roller for three different locations of it. Three curves giving the mode ratio corresponding to each location are shown. The position defined by c=0 means that the roller is positioned above the support. The position defined by c=10 mm means that the roller is at 10mm from the support at the inner side. Finally the position defined by c=-10 mm, means that the roller is at 10mm at the outer side.

As it can be seen in Fig. 10, when the roller radius increases, the mode ratio (G_{II} /G) decreases. When the roller is positioned at 10 mm at the inner side, varying the radius from 0 mm up to 0.8 mm all the mode ratios can be achieved.

Fig. 11 shows the influence of the variation of the location of the roller for three different radii. Three curves giving the mode ratio corresponding to each radius are shown.

For a given radius, the mode ratio decreases when the roller moves towards the crack tip, as a greater opening mode is promoted. When the roller radius is 1 mm, varying its position from 14 mm towards the specimen end, up to 8 mm towards the crack tip, all the mode ratios can be achieved.

Besides c and R parameters, varying the initial crack length, the mode ratio could also be varied. Moreover, taking into account other specimen geometries, the mode ratio also changes. It can be concluded that all mixed modes can be achieved with variations in the test configuration.

6 Numerical Approach (FEM)

An Ansys package was used to perform the numerical calculations. In order to calculate G_I and G_{II} energy release rates the TSEP or Two-Step Crack Closure Technique was followed. In this method, the crack path is modelled using pairs of coincident nodes coupled together as shown in Fig. 12 .a. In the first step when the imposed load or displacement reaches a critical value (P_c, δ_c) the forces at the crack tip (1-1') are calculated as shown in Fig. 12.b. The imposed displacement in the sample (δ_c) is then held in a second step and the coupled degrees of freedom (DOF) of the nodes at the crack tip 1-1' are released, as shown in Fig. 12.c. Displacements are then calculated in this second step.

The energy release rate is then calculated by means of the loads obtained in the first step and the displacements from the second step calculated in the same node pair 1-1'using the following expressions:

$$\begin{aligned} G_I &= \frac{1}{2b\Delta a}\sum_{i=1}^{n} F_{y1i}(v_{1i} - v_{1'i}) \\ G_{II} &= \frac{1}{2b\Delta a}\sum_{i=1}^{n} F_{x1i}(u_{1i} - u_{1'i}) \end{aligned} \tag{67}$$

where:

− F_{x1i}, F_{y1i}: forces at the crack tip (nodes 1-1'), being $F_{x1i}= F_{x1i'}$ and $F_{y1i}= F_{y1i'}$
− u_{1i}, v_{1i}: horizontal and vertical displacements of the released nodes (nodes 1-1')
− Δa: crack increment

The suffix i takes into account the extension to a 3D system, where i nodes are placed along the crack front.

The element used to model the test was the PLANE 42 with the plain strain option. The element size was set to 0.1 mm.

7 Comparison between analytical and numerical results

Fig. 13 shows the sample geometry and test configuration used in calculations and the deformed shape obtained in the FE analysis.

With the aim of comparing analytic and FEM results, rollers of 0.5 mm and 0.7 mm radii were placed above the support and at 10 mm from the support at the inner side. Other roller of 1.5 mm was placed to 10 mm from the support at the outer side. The crack length was a = 40 mm in all cases, as shown in Fig. 13.

Table 3 shows the values of P_c, Y_c and δ_c obtained analytically for the different cases. For the numerical calculations, the imposed displacement in the sample was the value of δ_c. Without any load application, when the roller is moved towards the crack tip, the mode I contribution increases. Therefore, the load and displacement needed to achieve the linear failure criterion are lower. Table 3 includes also the values of P_c and Y_c obtained numerically and the relative errors of the analytic values with respect to numerical ones. The results obtained for P_c and Y_c by the analytic approach and those obtained by FEM agree, being the relative errors lower than 3% and 5%, respectively.

The analytical values of G_I and G_{II} have been obtained as explained in section 5. The results for the energy release rate due to each mode are listed in Table. 4. When the roller is placed at the outer side, the mode I contribution is very low. Thus, only one case of R = 1.5 mm has been analyzed. In the case of G_I the lowest relative error occurs when the mode I is more dominant (case 4). It is probably related to the fact that the relative error of Y_c is also minimum in that case. In the case of G_{II} errors are near 1% in all cases.

8 Conclusions

A test configuration for obtaining mixed mode I/II interlaminar fracture in unidirectional composites has been proposed. It is a generalization of a previous proposal based on the ENF configuration with an inserted roller. Internal forces, mid point displacement, compliance and energy release rate have been determined analytically. Furthermore, it has been shown that all mixed modes can be achieved with simple variations concerning the radius or the position of the insterted roller.

The analytic values have been compared with those obtained by FEM. The Two Step Extension Procedure has been used for obtaining the energy release rate by FEM. The values obtained by the new analytic approach agree reasonably with those obtained numerically.

After having checked the analytic approach with FEM, and as different mode mixtures can be easily obtained, experiments can be carry out in order to determine interlaminar failure of composites under mixed mode I/II with the proposed procedure.

Due to the difficulty of determining optically the crack length in ENF tests, the proposed configuration can be used for opening the crack with the roller at the outer side of the crack tip. In that case, the mode I contribution is very small but it is expected that the crack tip advance will be monitored more easily than in the case of ENF test.

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