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# Angle of Loll Calculation By Cubic Spline

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ABSTRACT

Several years ago the Basque Government supported the programming of the software ARKITSAS in order to provide all existing vessels with a specific software to calculate stability, cargo and longitudinal strength data. The aim of this article is to present the part of that research concerning the definition of the static stability curve by cubic spline in its initial end when the metacentric height is negative. Taking into account that the slope at initial end is known, the precision of the results for low heeling angles may be improved and, in this way, the accuracy in the calculation of loll should be enhanced. This method of calculation is compared to other traditional methods used for wall-sided ships by the application to three different ships.

## 1. Introduction

The static stability curve represents the values of GZ arms for the different heeling angles. However, the stability booklet usually provides GZ arms for every ten or fifteen degrees of heeling angles. This means that the rest of righting arms have to be obtained by drawing the static transversal stability curve passing through the known data. In this way, the global cubic splines seems to be one of the most suitable methods to define the static stability curve since the same have to pass through some control points. The local splines pass also through the control points, although its degree of smoothness is lower than the global splines. On the other hand, the B-splines would not be suitable because the curve would not pass through the control points. Consequently, the GZ arms obtained by global cubic spline interpolation appear to be appropriate enough.

The ends of spline curve are usually free under this interpolation method. However, the slope of the static stability curve for the heel of 0° (when the vessel is up righted) is known specifically, it is equal to the metacentric height divided by the value of one radian in degrees. Therefore, it is possible to fix the initial end of the static stability curve to make the GZ arms within the initial stability more precise and, taking into account that the angle of loll is usually small; its calculation would be suitable by means of non-free end cubic spline method.

When the ship is 'wall-sided' the approximate formula in (1) may be enough to calculate the angle of loll (Barrass and Derret, 2012). However, this method of calculation would not be suitable for those ships that have very fine bowlines and stern contours. Even in the case of box shaped ships the angle of loll calculated by this formula is not as accurate as that obtained graphically from the static stability curve. For this reason, the method presented in this paper takes advantage of the graphic definition of the static stability curve by cubic spline to obtain the angle of loll by the cross of this curve and the abscissa axis.

$$\tan\varphi_{loll} = \sqrt{\frac{2 \cdot GM}{BM}} \qquad (1)$$

## 2. Static stability curve definition by cubic spline

The static stability curve defined by global cubic splines is made up of different portions of curves connected in control points called knots which coincide with the known data. Therefore the curve is a piece-wise function defined by multiple subfunction in the form of the equation in (2), $n$ being the total data provided in the cross stability curves.

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$$GZ_i(\varphi) = A_i + B_i \cdot (\varphi - \varphi_i) + C_i \cdot (\varphi - \varphi_i)^2 + D_i \cdot (\varphi - \varphi_i)^3 \quad i = 0, n \qquad (2)$$

Obviously, the adjoining subfunctions coincide in value at knots and, moreover, the slope and the curvature at knots is the same for the adjoining subfunctions, which makes the whole curve smooth. Therefore, the following conditions have to be fulfilled (Borse, 1991):

1st condition: the value of a cubic subfunction at initial knot is known.

$$GZ_i(\varphi_i) = A_i \qquad (3)$$

2nd condition: the value of two different adjoining subfunctions at common knot is the same.

$$GZ_i(\varphi_{i+1}) = GZ_{i+1}(\varphi_{i+1}) \qquad (4)$$

3rd condition: the slope of two different adjoining subfunctions at common knot is the same, which means that the derivative of both subfunctions is the same as well at common knot.

$$GZ_i'(\varphi_{i+1}) = GZ_{i+1}'(\varphi_{i+1}) \qquad (5)$$

4th condition: the curvature of two different adjoining subfunctions at common knot is the same, which means that the second derivative of both subfunctions is the same as well.

$$GZ_i''(\varphi_{i+1}) = GZ_{i+1}''(\varphi_{i+1}) \qquad (6)$$

The definition of a curve by global cubic splines is related to the task carried out in the past at the shipyards by the draughtsmen who used a flexible strip of metal for drawing curve lines. The strip was fixed at the points or nodes through which the curve had to pass and the ends of the strip were left free. Bearing this in mind, the ends of a curve defined by global cubic spline may be left free if the curvature is nil at the ends. Thus, another additional condition may be added for the ends of the curve:

$$GZ_0''(\varphi_0) = 0 \qquad GZ_n''(\varphi_n) = 0 \qquad (7)$$

Nevertheless, as has been mentioned before, the end of the first curve segment is fixed in the case of the static stability curve, since the slope at the end is known (Rawson, 2001).

$$GZ_0'(\varphi_0) = \frac{GM}{57{,}3} \qquad (8)$$

All these conditions will let us calculate the coefficients of the different cubic subfunctions. Thus, the coefficients $C_i$ can be obtained either by substitution, if the GZ data are equidistant, or by Gauss-Siedel. As the cross stability data may not be equidistant, the method of Gauss-Siedel seems to be the most adequate. Therefore, the coefficients $C_i$ are obtained by the equation in (9).

$$C \cdot a = R \qquad (9)$$

where:

$$C^t = [C_1 \quad C_2 \quad \dots \quad C_{n-2}]$$

$$R^t = [R_1 \quad R_2 \quad \dots \quad R_{n-1}]$$

$$a = \begin{bmatrix} a_{1,1} & a_{1,2} & 0 & 0 & \cdots & 0 & 0 \\ a_{2,1} & a_{2,2} & a_{2,3} & 0 & \cdots & 0 & 0 \\ 0 & a_{3,2} & a_{3,3} & a_{3,4} & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \cdots & 0 & 0 \\ 0 & 0 & 0 & 0 & \cdots & a_{n-1,n-2} & a_{n-1,n-1} \end{bmatrix}$$

$$a_{i,i-1} = \varphi_i - \varphi_{i-1}$$

$$a_{i,i} = 2 \cdot (\varphi_{i+1} - \varphi_{i-1})$$

$$a_{i,i+1} = \varphi_{i+1} - \varphi_i$$

$$R_i = \frac{3 \cdot (A_{i+1} - A_i)}{(\varphi_{i+1} - \varphi_i)} - \frac{3 \cdot (A_i - A_{i-1})}{(\varphi_i - \varphi_{i-1})}$$

To solve the equation 9, it is necessary to give an initial value to the coefficients $C_i$ and to apply an iterative process until convergence criteria is reached or a maximum of iterations is exceeded, depending on the degree of accuracy. In first instance the coefficients $C_i$ at the ends of the curve ($C_0$ and $C_n$) would be equal to zero by application of the formulae in (7).

The coefficient $B_i$ is calculated by the formula in (16), which is obtained from the first four conditions.

$$B_i = \frac{A_{i+1} - A_i}{\varphi_{i+1} - \varphi_i} - \frac{(\varphi_{i+1} - \varphi_i) \cdot (2 \cdot C_i + C_{i+1})}{3} \qquad (10)$$

The coefficients $D_i$ are obtained by the equation in (11) that it is reached from the fourth condition.

$$D_i = \frac{C_{i+1} - C_i}{3 \cdot (\varphi_{i+1} - \varphi_i)} \qquad (11)$$

Once, all the coefficients have been obtained, the initial end of the curve has to be fixed by giving new values to $B_0$ and $C_0$. The coefficient $B_0$ will be equal to the slope at the first end and $C_0$ will be obtained from the equations in (10) and (11).

$$B_0 = \frac{GM}{57{,}3} \qquad (12)$$

$$C_0 = \frac{3}{2 \cdot \varphi_1} \cdot \left[ \frac{A_1 - A_0}{\varphi_1} - \frac{GM}{57.3} \right] - 0.5 \cdot C_1 \qquad (13)$$

If these new coefficients $B_0$ and $C_0$ are applied to the first four conditions, the values of

$A_{1,1}$ and $R_1$ will also vary.

$$A_{1,1} = 2 \cdot \varphi_2 - 0.5 \quad (14)$$

$$R_1 = \frac{3 \cdot (A_2 - A_1)}{(\varphi_2 - \varphi_1)} - \frac{3 \cdot (A_1 - A_0)}{\varphi_1} + \frac{3}{2 \cdot \varphi_1} \cdot \left[ \frac{GM}{57.3} - \frac{A_1 - A_0}{\varphi_1} \right] \quad (15)$$

Taking into account the new values obtained from the formulae (9) to (15), the equation in (9) is applied again to calculate the values of $C_i$ and the rest of coefficients. Thus, the first end of the static stability curve will be fixed.

## 3. Angle of loll determination by spline

As the angle of loll is usually small, it may be considered included within the first segment of the spline curve or, what is the same, within the first subfunction. Given that the GZ is equal to zero when the ship is up righted [$GZ_0(0^o)=0$] and when the ship is heeled the angle of loll [$GZ_0(\phi_{loll})=0$], the equation in (2) is applied to obtain the angle of loll as follows:

$$A_0 + B_0 \cdot \varphi + C_0 \cdot \varphi^2 + D_0 \cdot \varphi^3 = 0$$

As $GZ_0(0^o)=A_0=0$, then:

$$B_0 \cdot \varphi + C_0 \cdot \varphi^2 + D_0 \cdot \varphi^3 = 0$$

$$\varphi \cdot (B_0 + C_0 \cdot \varphi + D_0 \cdot \varphi^2) = 0$$

Bearing in mind that $\phi_0=0^o$, the angle of loll is obtained from the second grade equation:

$$B_0 + C_0 \cdot \varphi_{loll} + D_0 \cdot \varphi_{loll}^2 = 0 \quad (16)$$

The coefficients $B_0$, $C_0$ and $D_0$ are obtained from the equations in (11), (12) and (13).

In the improbable case that the angle of loll was located in the second subfunction, it will have to be calculated from the equation in (17).

$$A_1 + B_1 \cdot \varphi_{loll} + C_1 \cdot \varphi_{loll}^2 + D_1 \cdot \varphi_{loll}^3 = 0 \quad (17)$$

The coefficients $A_1$, $B_1$, $C_1$ and $D_1$ would be obtained from the equations in (3), (10), (9) and (11).

## 4. Practical application

In this chapter the angle of loll is calculated by means of the spline method for three different types of vessels. The values so obtained will be compared to those calculated by the approximate formula in (1). Obviously, the ships are unstable in all cases to get a negative GM. On the other hand, the static stability curve is drawn by cubic splines either fixing the ends or leaving them free.

FIRST CASE.- 16,600 dwt bulk-carrier (wall-sided).

**Table 1**: 16,600 dwt bulk-carrier hydrostatic data

| | |
|---|---|
| Draft | 10 meters |
| TKM (vertical distance from keel to transversal metacentre) | 9.707 meters |
| GM (metacentric height) | 0.043 (-) |
| VCB (vertical center of buoyance) | 5,305 meters |
| BM (metacentric radious) | 4.402 meters |

**Source:** Authors.

**Table 2:** GZ arms for 16,600 dwt bulk-carrier, draft 10 meters and GM 0.043(-) meters.

| $\phi$ | 0° | 10° | 20 | 30° | 40° | 50° | 60° | 75° |
|---|---|---|---|---|---|---|---|---|
| GZ | 0.000 | 0.003 | 0.028 | 0.089 | 0.262 | 0.263 | 0.020 | -0.574 |

**Source:** Authors.

Taking into account the hydrostatic data and the GZ arms from the table 1 and 2, respectively, the coefficients are obtained for the first spline subfunction.

$B_0$ = -7.5044·$10^{-4}$

$C_0$ = 6.7556·$10^{-5}$

$D_0$ = 3.7487·$10^{-6}$

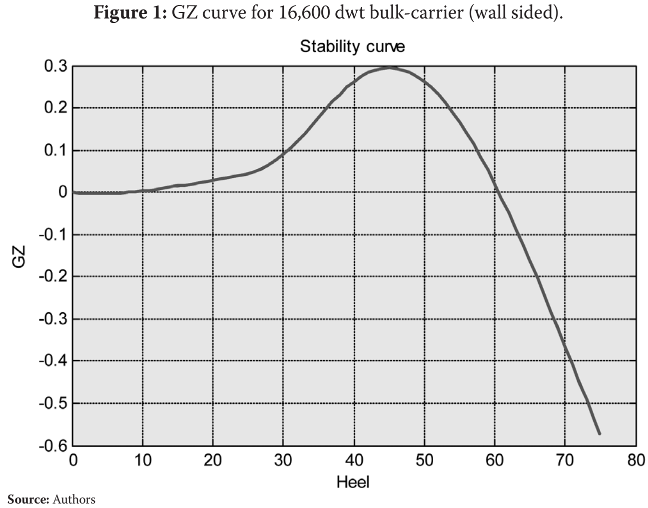
The angle of loll is thus calculated from the equation in (16).

$\phi_{loll}$ = 7.76°

On the other hand, the angle of loll is also obtained from the formula in (1).

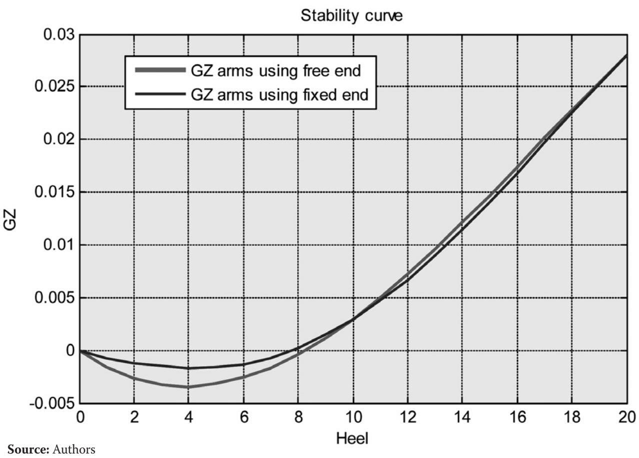
$\phi_{loll}$ = 7.96°

**Figure 1:** GZ curve for 16,600 dwt bulk-carrier (wall sided).




**Source:** Authors

**Figure 2:** Small angles' detail of the GZ curve for 16,600 dwt bulk-carrier (wall sided).




**Source:** Authors

SECOND CASE.- 150,000 dwt tanker (wall-sided)

**Table 3:** 150,000 dwt tanker hydrostatic data.

| | |
|---|---|
| Draft | 15.9 meters |
| TKM (vertical distance from keel to transversal metacentre) | 20.08 meters |
| GM (metacentric height) | 0.02 (-) |
| VCB (vertical center of buoyance) | 8.25 meters |
| BM (metacentric radious) | 11.83 meters |

**Source:** Authors.

**Table 4:** GZ arms for 16,600 dwt bulk-carrier, draft 15.9 meters and GM 0.02(-) meters.

| $\phi$ | 0° | 10° | 20° | 30° | 40° | 50° | 60° | 70° | 80° | 90° |
|---|---|---|---|---|---|---|---|---|---|---|
| GZ | 0 | 0.023 | 0.055 | -0.55 | -1.52 | -2.805 | -4.239 | -5.664 | -6.979 | -8.108 |

**Source:** Authors.

In case that the hydrostatic data and the GZ arms are those showed in table 3 and 4, respectively, the coefficients of first spline subfunction are expressed below.

$B_0 = -3.4904 \cdot 10^{-4}$

$C_0 = -2.4409 \cdot 10^{-4}$

$D_0 = 5.0899 \cdot 10^{-5}$

The angle of loll is thus calculated from the equation in (16).

$\phi_{loll} = 5.95°$

On the other hand, the angle of loll obtained from the formula in (1).

$\phi_{loll} = 3.33°$

**Table 5:** Sailing yacht hydrostatic data.

| | |
|---|---|
| Draft | 2.71 meters |
| TKM (vertical distance from keel to transversal metacentre) | 3.35 meters |
| GM (metacentric height) | 0.05 m. (-) |
| VCB (vertical center of buoyance) | 1.94 meters |
| BM (metacentric radious) | 1.41 meters |

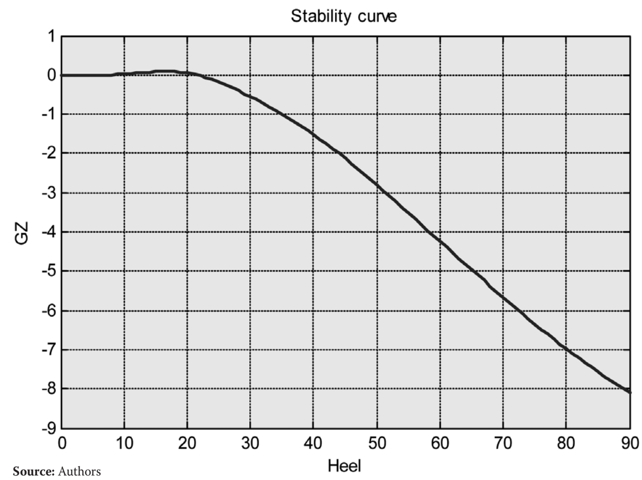
**Source:** Authors.

**Table 6:** GZ arms for sailing yacht, draft 2.71 meters and GM 0.05(-) meters.

| $\phi$ | 0° | 10° | 20° | 30° | 40° | 50° | 60° |
|---|---|---|---|---|---|---|---|
| GZ | 0 | 0.0096 | 0.032 | 0.02 | -0.015 | -0.1246 | -0.244 |

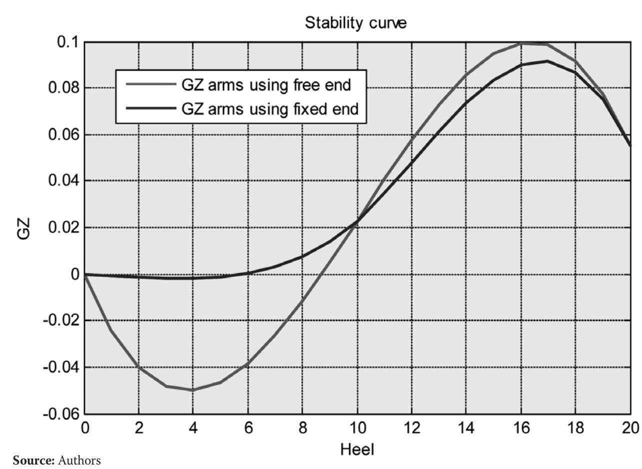
**Source:** Authors.

**Figure 3:** GZ curve for 150,000 dwt tanker (wall-sided).




**Source:** Authors

**Figure 4:** Small angles' detail of the GZ curve for 150,000 dwt tanker (wall-sided).




**Source:** Authors

THIRD CASE.- Sailing yacht (round-shaped)

The coefficients obtained from the hydrostatic data and the GZ arms from the table 5 and 6, respectively, are shown below:

$B_0 = -8.7260 \cdot 10^{-4}$

$C_0 = 1.9003 \cdot 10^{-4}$

$D_0 = -6.7682 \cdot 10^{-7}$

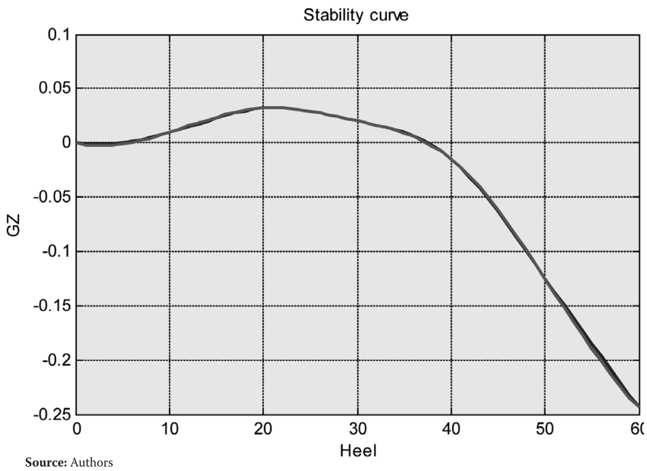
The angle of loll obtained from the equation in (16).

$\phi_{loll} = 4.67°$

On the other hand, the angle of loll obtained from the formula in (1).

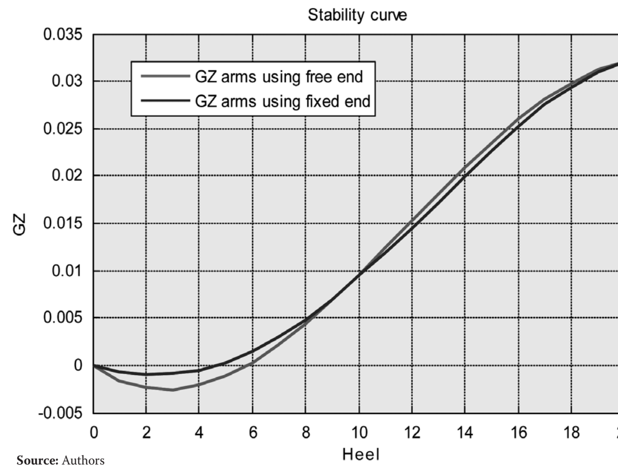
$\phi_{loll} = 14.9°$

**Figure 5:** GZ curve for sailing yacht (round-shaped).




**Source:** Authors

**Figure 6:** Small angles' detail of the GZ curve for sailing yacht (round-shaped).




**Source:** Authors

## 5. Conclusions

The results obtained in the practical application show that the approximate formula is not accurate enough for round-shaped ships, although it may be useful for wall-sided ships. The results of the applications of both methods in the 16,600 dwt bulk-carrier In the case of the 150,000 dwt tanker, there is a difference of almost three degrees between both methods of calculation, which seems to be excessive for a box-shaped ship with small block coefficient; nevertheless, it must be taken into account that the ship's conditions to reach a negative metacentric height have been forced too much. This is due to the fact that the shape and hydrostatic particulars of that type of ship provide her with excessive stability. Therefore the height of the centre of gravity estimated for the tanker in the practical case is hypothetical and out of the possible stability criteria in the construction of this type of ships (Riola and Pérez, 2009). In the case of the sailing yacht, the difference of the results between both methods is bigger than ten degrees, which means that the approximate formula is not valid for non wall-sided ships.

The global cubic splines let us obtain graphically and analytically the angle of loll in an accurate way. However, it is essential to fix the end where the slope of the static stability curve is known. Otherwise, there may be an error of more than three degrees. Therefore, it is advisable to fix the end when a math program such as Matlab or Mathematica is used to define the static stability curve.

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