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# Shortcuts to adiabaticity in optical waveguides using fast quasiadiabatic dynamics

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**Abstract:** We propose a fast quasiadiabatic approach to the design of optical waveguide devices. This approach distributes the system adiabaticity homogeneously over the device length, thus providing a shortcut to adiabaticity at a shorter device length. A mode sorting asymmetric Y junction is designed by redistributing the adiabaticity of a conventional linearly separating Y junction. Simple procedures for the design of fast quasiadiabatic devices are outlined, and the designed Y junction features large bandwidth at a shorter length than the conventional linearly separating Y junction. The proposed device is verified with beam propagation simulations. A mode conversion efficiency of larger than 99% is observed for the designed Y junction over a 220 nm range.

**OCIS codes:** (000.1600) Classical and quantum physics; (060.1810) Buffers, coupler, routers, switches and multiplexers; (130.3120) Integrated optics devices; (130.2790) Guided waves.

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## 1. Introduction

Since its introduction in 1928 by Born and Fock [1], the adiabatic theorem has been widely applied in theories and experiments in physics. By definition, adiabaticity occurs when the state of a quantum system described by a time-dependent Hamiltonian $H(t)$, initially prepared in an instantaneous eigenstate $|n(0)\rangle$, remains close to the instantaneous eigenstate $|n(t)\rangle$ during its evolution, as long as $H(t)$ changes slowly. The "slowness" is usually measured by the condition

$$\left|\frac{\langle m|\dot{n}\rangle}{\beta_m - \beta_n}\right| \ll 1, \tag{1}$$

where the dot denotes derivative with respect to $t$, and $\beta_{m,n}$ is the eigenvalue associated with eigenstate $|m, n\rangle$. This property allows one to control the evolution of quantum systems by engineering the time evolution of the Hamiltonian. In guided-wave optics, a large class of adiabatic devices operates on the same principle [2–15]. That is, their functionalities rely solely on their geometrical design and the system remains in the same instantaneous eigenmode of the device along the length of the device. By engineering the device geometry along the propagation direction, the field evolution inside can be controlled. So, these devices are sometimes called mode evolution based devices [7, 9]. In general, these adiabatic devices bring with them the benefit of system robustness in terms of broadband operation and good fabrication tolerance, but at the expense of longer device lengths.

In the field of quantum control, shortcuts to adiabaticity (STA) are a class of techniques to speed up slow adiabatic process while keeping or enhancing robustness [16]. By applying STA to optical waveguides, a series of robust devices such as mode converters [17, 18], Y-junctions [19], directional couplers [20–23], mode (de)multiplexers [24], and polarization rotators [25] have been proposed. These approaches, while being robust against particular errors by design, do not guarantee adiabaticity, and a robust STA design against a particular error might be susceptible to other sources of error. Moreover, their implementation might require access to multiple system variables that are not always available. This prompts us to consider a protocol that drives the system as fast as possible while maintaining its adiabaticity as close to the limit as possible using a single control parameter. The fast quasiadiabatic (FAQUAD) approach achieves precisely the stated goal [26], in which the conventional adiabaticity condition Eq. (1) is modified to account for a single control parameter of the system $\gamma$ . By imposing

$$\left|\frac{\langle m|\dot{n}\rangle}{\beta_m - \beta_n}\right| = c \tag{2}$$

in Eq. (1), where $c$ is a small number and using the chain rule $\frac{\partial}{\partial z}|n\rangle = \frac{\partial}{\partial \gamma}\frac{\partial \gamma}{\partial z}|n\rangle$ (we change $t$ to $z$ in subsequent analysis to account for wave propagation in optics), the FAQUAD condition reads

$$\left|\dot{\gamma}\frac{\langle m|\frac{\partial}{\partial \gamma}|n\rangle}{\beta_m - \beta_n}\right| = c, \tag{3}$$

where the dot denotes now derivative with respect to $z$. From Eq. (3), the FAQUAD strategy can be understood as a design protocol to homogeneously distribute adiabaticity during the evolution using a single control parameter $\gamma$.

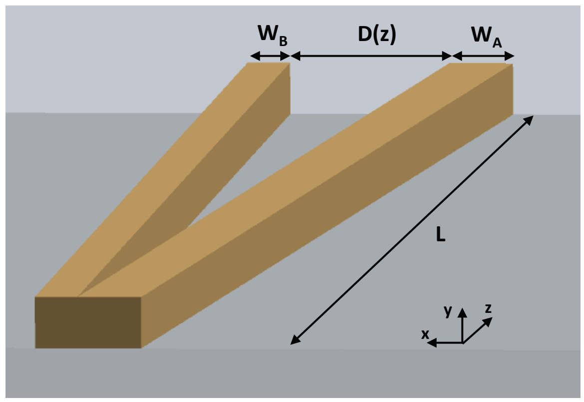


Fig. 1. Schematic of the mode sorting asymmetic Y junction.

## 2. Mode sorting asymmetric Y junctions

In integrated optical waveguides, the asymmetric Y junction can be designed to function as a mode sorter [2,12,13]. It has a two-modes stem and two diverging single-mode arms with different widths. When the fundamental (second) mode of the stem propagates though the junction, it evolves into the fundamental mode of the wider (narrower) output arm, and vice versa. The mode sorting behavior can be attributed to the fact that a mode would evolve into the mode of the output arm with the closest effective index [2]. However, this smooth evolution can only occur when the variation at the junction is slow enough such that the evolution is adiabatic, reducing the coupling between the local eigenmodes (supermodes) of the structure. The adiabatic criterion often leads to a small branching angle between the arms and thus a long device length to achieve the desired arm separation. The challenge in integrated mode sorting Y junction multiplexer/demultiplexer design is thus to reduce the device length while minimizing the crosstalk between the arms. Conventional linearly separating designs optimize the branching angle between the output arms, and a well-known criterion for mode sorting operation of the asymmetric Y junction is given by the mode conversion factor (MCF) as [2]:

$$MCF = \frac{|\beta_A - \beta_B|}{\theta \gamma_{AB}}, \tag{4}$$

where $\theta$ (radians) is the branching angle of the Y junction arms, $\beta_A$ and $\beta_B$ are the propagation constants of the modes supported by single mode arms A and B, and $\gamma_{AB} = 0.5\sqrt{(\beta_A + \beta_B)^2 - (2k_0 n)^2}$ with $n$ the cladding refractive index and $k_0$ the free-space wavenumber. When the MCF is larger (smaller) than 0.43, an asymmetric Y junction acts as a mode sorter (power divider). Previously, a STA based approach was used to design an asymmetric Y junction that is shorter than the linearly separating design [19]. This design requires control of both the waveguide separation and branching arm widths along the device, which might be challenging in terms of fabrication.

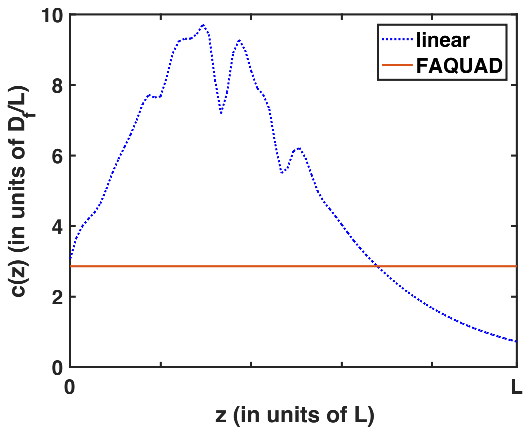


Fig. 2. Device adiabaticities [Eq. (1)] for the linearly separating Y junction (dotted) and the FAQUAD Y junction (solid). This figure also shows the adiabaticity measure in terms of the control parameter [$A(z)$ in Eq. (5)].

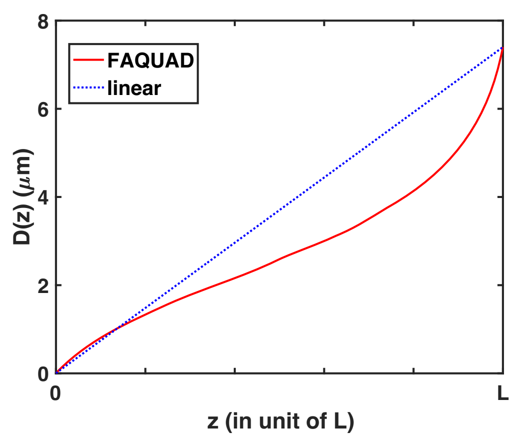


Fig. 3. Waveguide separations $D(z)$ for the linearly separating Y junction (dotted) and the FAQUAD Y junction (solid).

## 3. Fast quasi adiabatic dynamics in optical waveguides

In this paper, we apply the FAQUAD protocol to the design of a mode sorting asymmetric Y junction using only a single control parameter $D(z)$. We discuss systematically the procedure for homogeneously distributing system adiabaticity over the device, which is also applicable to a wide range of adiabatic devices.

We choose a polymer channel waveguide structure for device design. The design parameters are chosen as follows: 3 $\mu$m thick $SiO_2$ ($n$ =1.46) on a Si ($n$ = 3.48) wafer is used for the bottom cladding layer, the core consists of a 2.4 $\mu$m layer of BCB ($n$ = 1.53), and the upper cladding is epoxy ($n$ = 1.50). A schematic of the device is shown in Fig. 1. For the Y junction input and outputs, we choose an input stem waveguide width of 5 $\mu$m supporting two modes, the output single mode waveguides widths are $W_A$=3 $\mu$m and $W_B$=2 $\mu$m, and the total device length is $L$. We target a final waveguide separation $D_f$ of 7.5 $\mu$m so that the coupling between the output branches is negligible. We first calculate the adiabaticity $c_{lin}(z)$ [Eq. (1)] of a linearly separating design $D_{lin}(z) = D_f(z/L)$ by considering the two supermodes $|a\rangle$ and $|b\rangle$ of the structure, and the result is shown in Fig. 2. It is clear that the adiabaticity varies across the device length, and that the coupling between the supermodes is small when $D$ is greater than 7.5 $\mu$m. Using the waveguide separation as the control parameter in Eq. (3), we obtain the functional form of $c_{lin}(z)$

$$c_{lin}(z) = \frac{D_f}{L}\left|\frac{\langle a|\frac{\partial}{\partial D}|b\rangle}{\beta_a - \beta_b}\right| = \frac{D_f}{L}A(z), \tag{5}$$

where $\beta_a$ and $\beta_b$ are the propagation constants corresponding to supermodes $|a\rangle$ and $|b\rangle$, and the function $A(z)$ is a measure of adiabaticity that is strictly related to the differential of $D$ (shown in Fig. 2). From Eq. (3), we can see that in order for the constant adiabaticity condition in Eq. (2) to hold, the waveguide separation should satisfy

$$\left|\frac{dD}{dz}\frac{\langle a|\frac{\partial}{\partial D}|b\rangle}{\beta_a - \beta_b}\right| = c. \tag{6}$$

Using Eq. (5), we obtain

$$\frac{dD}{dz} = \frac{c}{A(z)} = \frac{D_f}{L}\frac{c}{c_{lin}(z)}, \tag{7}$$

where $c = L/[\int_0^L dz/c_{lin}(z)]$ is a constant determined by the following boundary conditions

$$D(0) = 0, \quad D(L) = D_f. \tag{8}$$

Finally, the FAQUAD waveguide separation can be obtained by integrating Eq. (7)

$$D_{FAQUAD}(z) = D_f\frac{\int_0^z (1/c_{lin}(z))dz}{\int_0^L (1/c_{lin}(z))dz}. \tag{9}$$

The steps to design a device using FAQUAD are thus:

- Obtain the functional dependence of device adiabaticity along $z$ for an arbitrary design.
- Calculate the derivative of the single control parameter $\dot{\gamma}$ using the result from step 1.
- Integrate $\dot{\gamma}$ to obtain the evolution of the control parameter $\gamma$.

In the conventional linearly separating design, we can see from Eq. (5) that the branching angle $\theta \approx D_f/L$ needs to be small enough to compensate for the large peak in $A(z)$ to satisfy the overall adiabaticity criteria. On the other hand, for the same $A(z)$, the FAQUAD approach unevenly distributes the separation $D$ along $z$ according to Eq. (7), such that the overall adiabaticity is homogenized. For a given $A(z)$, the FAQUAD approach can thus achieve adiabaticity in a shorter length than the conventional design. Note that it is enough to find the FAQUAD protocol for one particular device length $L$, since the protocol for any other length is simply obtained by scaling [26].

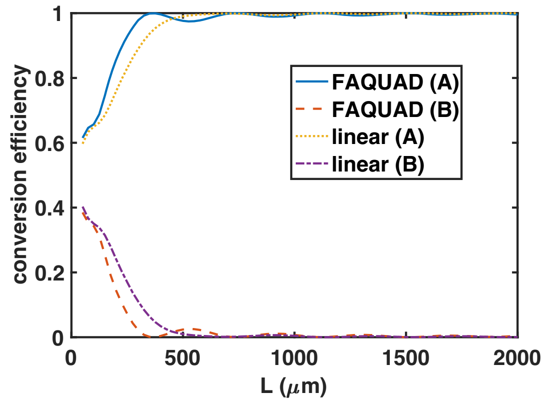


Fig. 4. BPM simulated fractional power (conversion efficiency) in waveguides A and B of the FAQUAD and linearly separating Y junctions using the fundamental mode of the stem waveguide as the input for different device lengths.

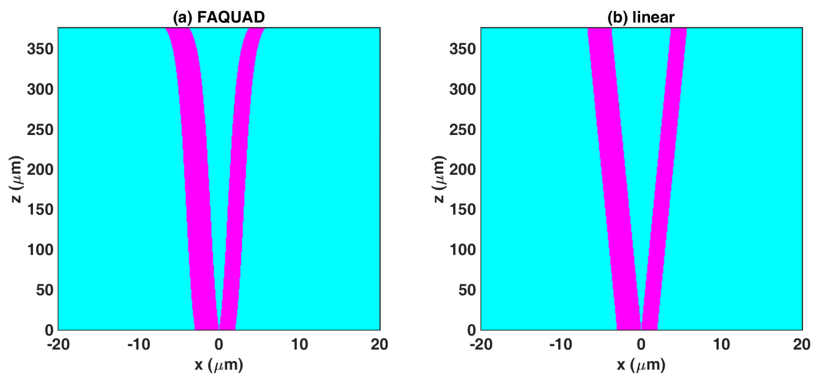


Fig. 5. Geometries of the designed mode sorting asymmetic Y junctions. (a) FAQUAD design (b) linear design.

## 4. Device simulations

Using the steps outlined above, we obtain $D_{FAQUAD}$ from the $c_{lin}$ in Fig. 2, and the result is shown in Fig. 3. The linearly separating design $D_{lin}$ is also shown for reference. In Fig. 2, we show the adiabaticity $c_{FAQUAD}$ calculated for $D_{FAQUAD}$, and it is clear that the adiabaticity is indeed homogenized across the device. We then use beam propagation method (BPM) simulations to verify both $D_{FAQUAD}$ and $D_{lin}$ designs. The device is simulated at 1.55 $\mu$m input wavelength and the TE polarization. In Fig. 4, we show the simulated fractional power (conversion efficiency) in waveguides A and B using the fundamental mode of the stem waveguide as the input at different device lengths for both the FAQUAD and linearly separating designs. As expected, the FAQUAD design provides shortcut at a shorter device length (it achieves 0.9994 efficiency at a length of 375 $\mu$m). On the other hand, the conventional linearly separating design can only achieve greater than 0.999 efficiency with lengths longer than 1100 $\mu$m. The FAQUAD design is nearly three times shorter than the linearly separating design. The oscillatory behavior of the FAQUAD protocol (see Fig. 4) is due to the quantum interference among jumps from A to B at different locations. This is quantitatively explained in [26], where the oscillation period is identified using adiabatic perturbation theory, $T = \frac{2\pi}{\phi_{ab}}$ where $\phi_{ab} = (1/L)\int_0^L \omega_{ba}(z)dz$ and

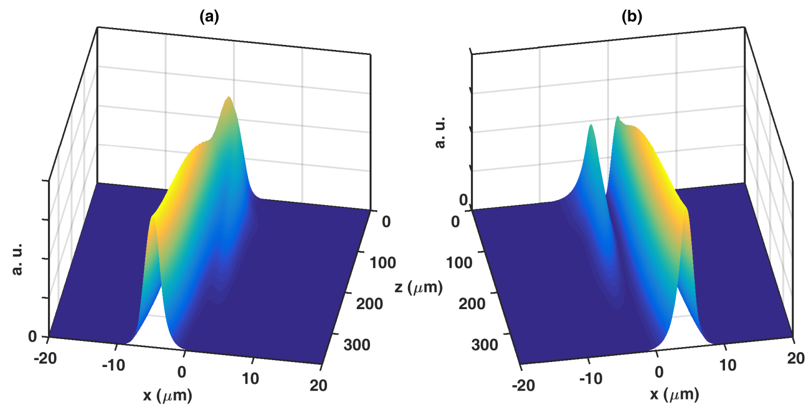


Fig. 6. BPM simulated mode sorting operation of the FAQUAD Y junction. Input (a) fundamental mode (b) second mode.

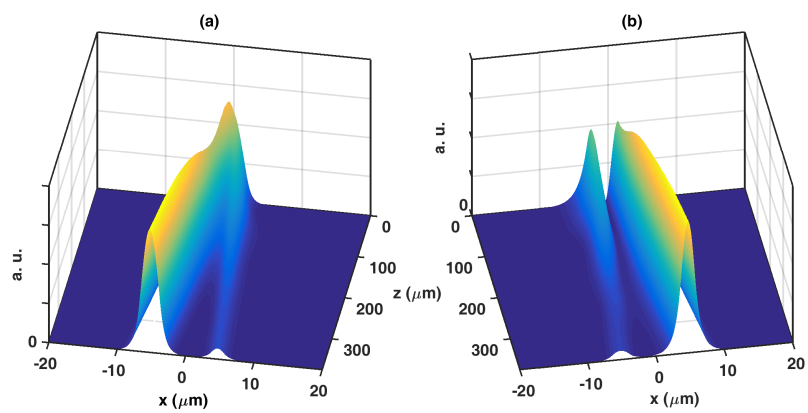


Fig. 7. BPM simulated mode sorting operation of the linearly separating Y junction. Input (a) fundamental mode (b) second mode.

$\omega_{ba}(z) = \beta_b(z) - \beta_a(z)$.

The corresponding FAQUAD Y junction geometry for a $L = 375$ $\mu$m Y junction is shown in Fig. 5(a), and the linearly separating Y junction at the same length is shown in Fig. 5(b). The corresponding BPM simulation results using the first two modes at the input at $z = 0$ are shown in Fig. 6. In Fig. 6(a), the BPM results show that the fundamental mode has evolved to waveguide A at the output, and the evolution of the second mode to waveguide B is shown in Fig. 6(b). For both input modes, there is essentially no cross talk between the output ports for the FAQUAD design. For a comparison, we also show the beam propagation results for the linearly separating Y junction of the same length in Fig. 7. At the outputs, we can clearly see that a substantial amount of light is observed in port B [Fig. 7(a)] and port A [Fig. 7(b)], indicating large cross talk between the ports.

We then look at the device robustness against wavelength variations. Figure 8 shows the simulated wavelength dependence of the conversion efficiency for the linearly separating Y junction and the FAQUAD Y junction both at a $L = 375$ $\mu$m. For the FAQUAD design, it can be seen that for a wide range from 1.4 to 1.68 $\mu$m, the coupling efficiency is larger than 99 %. The good robustness of the device indicates that the adiabaticity of the device is indeed achieved

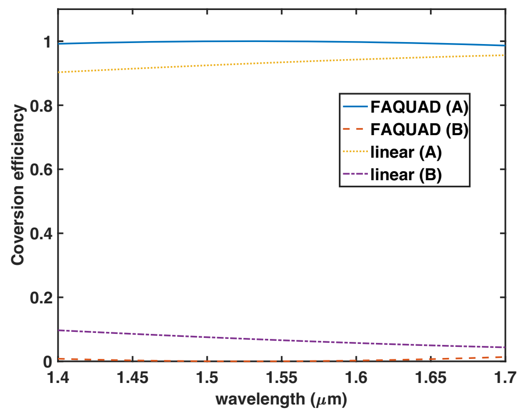


Fig. 8. Simulated wavelength dependence of the conversion efficiencies of the FAQUAD and the linearly separating Y junctions.

with our scheme at a shorter device length than the conventional design.

## 5. Discussions

Next, we discuss the differences between the FAQUAD approach and the other STA techniques in terms of their applications in integrated optics. In particular, we focus on the invariant-based inverse-engineering approach presented in ref. [19]. In the invariant-based design, the system evolution follows the eigenstates of the dynamical invariant. So the system evolution is first designed, and the device parameters are then inversely engineered to achieve mode sorting in a short distance. The nature of the invariant design guarantees that the conversion efficiency is always 100% for any realizable length, while the FAQUAD approach, a quasiadiabatic scheme, does not always guarantee 100% efficiency as discussed above. The invariant-based approach can be made to be robust against particular errors by designing the system evolution, but does not guarantee adiabaticity. Whereas, the FAQUAD approach provides a shortcut to system adiabaticity by homogeneously distributing system adiabaticity over the device length. In the invariant-based design, the device parameters need to be calculated for different device lengths in order to accommodate for changes in system evolution; while, as we discussed in Sec. 3, the FAQUAD design in Fig. 3 is scalable for all device lengths. In terms of device complexity, to obtain the desired dynamical invariant, more variables are required in the invariant-based device design. For the case of mode sorting asymmetric Y junction in ref. [19], the branching waveguide widths $W_A(z)$, $W_B(z)$ and spacing $D(z)$ all need to change with device length, increasing difficulties in fabrication, especially in the case of laser direct-write technique [15]. On the other hand, the FAQUAD approach in this case only requires a single control parameter $D(z)$ while the branch widths are kept as constants, providing advantage in terms of fabrication challenges.

## 6. Conclusion

We have put forward a novel scheme to homogenize the adiabaticity of optical waveguide devices. By evaluating the adiabaticity of a simple linearly separating Y junction, a single control parameter is then used to redistribute the device adiabaticity. The approach provides a shortcut to adiabaticity at a shorter device length. The resulting design shows large bandwidth at a shorter device length as compared to the conventional linearly separating design. The procedures outlined in this paper could also find applications in a wide range of adiabatic waveguide devices.

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