Vertical Differentiation and Entry Deterrence: A Reconsideration*

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Abstract

In this work we emphasize why market coverage should be considered endogenous for a correct analysis of entry deterrence in vertical differentiation models and discuss the implications of this endogeneity for that analysis. We consider contexts without quality costs and also contexts with convex fixed quality costs.

JEL classification code: L13, D43

Keywords: Vertical differentiation, market coverage, entry deterrence, quality competition.

January 2005

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1 Introduction

This work presents a new analysis of entry deterrence in vertical differentiation models. We focus on the market coverage configuration issue. We emphasize why market coverage should be considered endogenous for a correct analysis of entry deterrence in those models and discuss the implications of this endogeneity. The approach that we present has been overlooked in the literature as the problem of entry in vertical differentiation contexts when there is endogenous market coverage has not received enough attention. This approach would call for a revision of those analyses that assume a given market coverage configuration.

We consider that there are at most two firms in the market, an established firm, or incumbent, and an entrant that faces an entry cost. The consideration of only one potential entrant suffices to show the implications of endogenous market coverage for a correct analysis of entry deterrence. To allow for further entry of firms, maintaining endogenous market coverage, would complicate the analysis without adding much to our contribution.[1] We can assume, for instance, that there are reasons related to technological accessibility that limit to two firms the maximum number of active firms that may exist in the market.

Previous analyses with more than two firms or more than one potential entrant assume market coverage exogenously given or impose a market configuration at equilibrium (see Hung and Schmitt (1988), Donnenfeld and Weber (1992 and 1995), Constantatos and Perrakis (1997 and 1999) and Scarpa (1998).[2] As a consequence, those analyses do not consider endogenous market coverage and do not allow for an analysis of entry deterrence like the one developed in this work.

We model a three-stage game in our analysis: in the first stage the incumbent chooses production technology with its associated quality level; in the second stage the entrant decides on entry after observing the quality of the established firm and chooses quality in the case of entry; finally in the third stage the firms that are active in the market select prices (simultaneously if

[1] Market coverage generally increases with the number of firms. But, markets are often uncovered. Therefore, this market coverage configuration should be present in most analyses with more than two competing firms.

[2] Some of these papers (Hung and Schmitt (1988) and Constantatos and Perrakis (1997 and 1999)) center in the case of “natural duopoly”, where the market will be covered. See Shaked and Sutton (1983) for an analysis of natural oligopolies.

there is entry). Hence, we recognize that firms can change their prices in a short period of time whereas a change in the technology takes a longer period of time. In this setting it is clear that firms may reduce price competition through quality differentiation.

Products of firms are functionally identical and are sold to a population of consumers differing in their "taste for quality" (or in their incomes). Each consumer prefers a product of higher quality, but consumers differ on the intensity of their preference for quality. The analysis assumes away any asymmetries of information about quality between firms and consumers. In this framework we look for a subgame perfect equilibrium.[3] We consider first a context where there are not quality costs and then the case with convex fixed quality costs.

Many analyses of vertical differentiation consider an uncovered market. This configuration is imposed assuming that some consumers are willing to pay nothing, or almost nothing, for any good whatever quality it may have.[4] This seems very unrealistic in most contexts. Although markets are often not covered, we do not want to consider as potential consumers to these consumers that are willing to pay nothing, or almost nothing, for any of the goods that may be offered in the market. Hence, we assume that all consumers care about quality.

From our results, the observation that the market is not covered, or the knowledge that the market will be uncovered not only when entry is deterred but also when entry is accommodated, does not allow us to assume, in the analysis of entry deterrence, that the market is always not covered. Even if we observe that markets like the one we are interested in are uncovered, it is not correct to assume that the market we study is always not covered. We emphasize in this work that we can never discard the covered market configuration in the analysis of entry deterrence. When the market happens to be not covered, this market coverage configuration is the final result of a process that implies the consideration of covered market structures.

Moreover, not all markets are uncovered. We observe also markets that are covered or almost covered.[5] In general, when entry is accommodated the

[3] Gabszewicz and Thisse (1979) and Shaked and Sutton (1982) settled the basis for many later analyses of quality differentiation.

[4] A complete analysis of entry deterrence in an uncovered market may be found in Lutz (1997).

[5] The markets of several white goods are an example. If we consider that the potential market for a white good is formed by all individuals or families that live in an apartment

market may end up covered or uncovered, depending on the heterogeneity of consumers valuations of quality and on the quality cost function.[6] In some cases product differentiation will be small, price competition intense and the market will end up covered. In other cases products will be highly differentiated and firms will not cover the market as price competition is reduced. To assume that the market is always covered may imply an undesired restriction on the parameters and functions considered.

In this work we notice that when entry is accommodated and the market is covered, the consumer with lowest willingness to pay for quality obtains no surplus. The lower quality firm would find profitable to lower its quality if it leaved positive surplus to this consumer. By doing so, besides any cost savings, the low quality firm allows its rival to set a higher price which eases price competition. This process would continue until the surplus of the consumer with lowest willingness to pay for quality is eliminated.

Instead, we show that in the analysis of entry deterrence we have to consider situations where the consumer with lowest willingness to pay for quality obtains positive surplus. This occurs in the study of the situation where the entrant would enter as the high quality firm. The selection of quality by the incumbent to deter entry would cause small product differentiation and intense price competition if the entrant would enter as the high quality firm. The reasons are that the high quality entrant would not differentiate much its product to save on quality costs or that the interval of technologically feasible qualities above the quality selected by the incumbent is small.[7]

All these market coverage configurations were introduced in Shaked and Sutton (1982). We will say that the market is covered with a corner solution when the low quality firm quotes the price which is just sufficient to cover the market. In this case the consumer with lowest willingness to pay for quality obtains no surplus. We will say that the market is covered with a interior solution when the market is covered and the consumer with lowest

or a house, there are many local markets for white goods in the more developed countries that are close to full market coverage.

[6] However, it is often assumed in the literature that the market is covered or that it is uncovered for exogenous reasons, or some restrictions are imposed on the models that guarantee a specific market configuration at equilibrium.

[7] This may also occur in other situations: for instance, if entry is accommodated when the regulator establishes a minimum quality standard. A small interval of feasible qualities above the minimum quality standard or a fast increase in quality costs above the standard may cause that result.

willingness to pay for quality obtains positive surplus. The third possible market coverage configuration is uncovered market.

To decide on entry deterrence the incumbent considers all feasible market coverage configurations that might result from entry. He takes into account how his decision on quality affects the market configuration that would result from the best response of the entrant. To this end, he notices that the entrant might enter as the high quality firm or as the low quality firm. From the endogeneization of market coverage we obtain that the market configuration that would result if the entrant entered as the low quality firm and the market configuration that would result if entry is accommodated are different to the market configuration that would result if the entrant entered as the high quality firm.

The paper is organized as follows: Section 2 sets the model. Section 3 studies entry deterrence without quality costs, considering that technology only allows for a maximum quality level equal to S. In Section 4 we extend our analysis to situations where there are convex fixed quality costs. The last section summarizes briefly the results. The proofs are included in the Appendix.

2 The model

In this work we consider situations where there are at most two firms in the market, an established firm or incumbent (I), which is active, and an entrant (E) that may decide to operate also in that market. Only one product is allowed for each firm. The entrant faces an entry cost equal to F but the entry cost of the incumbent is sunk when the entry game starts. This asymmetry on entry costs between the incumbent and the entrant does not affect our results, as, to study entry deterrence, we want an incumbent which is always active in the market and, besides, the comparison between the profits of the incumbent when entry is deterred and when entry is accommodated is unaffected by the size of any entry cost the incumbent might have.

Let us model a three-stage game as follows: in the first stage the incumbent chooses his quality level s_I; in the second stage the entrant decides on entry after observing s_I and chooses a quality level s_E in the case of entry; finally in the third stage firms compete simultaneously in prices, p_I and p_E, if firm E decides to enter, or firm I chooses p_I, if firm E does not enter. We assume a framework of perfect and complete information.

Products, that are functionally identical, are sold to a population of consumers differing in their marginal valuation of quality. Consumers may purchase either a single unit of the good from one of the firms or none at all. Consumers' preferences are described as follows: a consumer, identified by j, enjoys (indirect) utility $U(j) = js - p$ when consuming a product of quality s sold at a price p.[8] His utility is zero if he refrains from buying.

The population of consumers is described by the parameter j which is uniformly distributed between v and bv, with v positive and b greater than one. The assumption of v positive is necessary to allow for the possibility of a covered market. If $v = 0$, as in Aoki and Prusa (1996), the market would not be covered as the consumer with marginal valuation of quality equal to v would not buy any good with positive price. We normalize the number of consumers to one and assume $v = 1$ without loss of generality. The parameter b measures the heterogeneity in consumer tastes for quality. We also assume $b > 2$, as we will show that whenever $1 < b \leq 2$ the market is preempted by the high quality firm.

We consider in section 3 a situation where there are not quality costs. The analysis of entry deterrence with convex fixed quality costs is developed in section 4. When there are not quality costs we suppose that technology only allows for a maximum quality level equal to S.[9] Moreover, we assume, without loss of generality, that variable production costs are zero.

When entry occurs and there are two firms in the market, let us use subindexes h and l for the high and low quality firm, respectively, and denote the corresponding demand functions by $D_h(p)$ and $D_l(p)$. Consumer j will be willing to buy the product of firm i, with $i = 1, 2$, only if $\frac{p_i}{s_i} < j$. Moreover, the consumer indifferent between the product of firm 1 and the product of firm 2 has j such that $js_l - p_l = js_h - p_h$, i.e., $j = \frac{p_h - p_l}{s_h - s_l}$ We know that, when

[8] This structure of preferences is very usual in the literature, following Mussa and Rosen (1978). See Peitz (1995) for the construction of a direct utility function that has as its counterpart an indirect utility function as the one used in this paper. Peitz (1995) shows that the underlying preference relation satisfies reflexivity, transitivity, completeness and local nonsatiation. Additive separability is a reasonable assumption as long as the price of the product is such that the consumer expends only a small fraction of his total budget in the product.

[9] This is equivalent to considering that quality costs are zero up to S and infinite above S. We could also consider that technology implies a minimum quality level and that this minimum is below the quality levels relevant for the analyses in this work.

entry occurs and the market is not covered, $D_l(p) + D_h(p) < 1$ and

$$D_l(p_l, p_h) = \frac{1}{b-1}\left(\frac{p_h - p_l}{s_h - s_l} - \frac{p_l}{s_l}\right)$$

$$D_h(p_l, p_h) = \frac{1}{b-1}\left(b - \frac{p_h - p_l}{s_h - s_l}\right),$$

and when the market is covered, $D_l(p) + D_h(p) = 1$ and

$$D_l(p_l, p_h) = \frac{1}{b-1}\left(\frac{p_h - p_l}{s_h - s_l} - 1\right)$$

$$D_h(p_l, p_h) = \frac{1}{b-1}\left(b - \frac{p_h - p_l}{s_h - s_l}\right).$$

When entry does not occur the demand function of the incumbent will be given by

$$D_I(p_I) = \frac{1}{b-1}\left(b - \max\left\{1, \frac{p_I}{s_I}\right\}\right)$$

and the market will not be covered if $\frac{p_I}{s_I} > 1$.

Through all the work we study equilibria in pure strategies. We will proceed by backward induction to look for a subgame perfect equilibrium. To solve the entry game it is important to know when each market configuration would occur in that game.

3 Entry deterrence without quality costs

Let us present first the monopolistic solution. The profit function of a monopolist (incumbent) is:

$$\pi_I = p_I\left[\frac{1}{b-1}\left(b - \frac{p_I}{s_I}\right)\right]$$

with $p_I \geq s_I$. If $p_I = s_I$ the market would be covered and the monopolist profits would be given by $\pi_I = p_I = s_I$. In this case the monopolist would decide $p_I = s_I = S$ and his profits would become $\pi_I = S$.

When $p_I > s_I$ the market is not covered. In this situation the price decision would be:[10]

$$\frac{\partial \pi_I}{\partial p_I} = 0 \Leftrightarrow p_I = \frac{b}{2}s_I$$

[10]Notice that the market is not covered as $b > 2 \Rightarrow \frac{p_I}{s_I} = \frac{b}{2} > 1$.

Hence, to determine the quality selection, we would have:

$$\pi_I = \frac{b^2 s_I}{4(b-1)}$$

and:

$$\frac{\partial \pi_I}{\partial s_I} = \frac{b^2}{4(b-1)} > 0 \Rightarrow s_I = S$$

In the case of uncovered market the monopolist profits would be $\pi_I = \frac{b^2 S}{4(b-1)}$.

The monopolist will not cover the market as for any b such that $b > 2$ his profits are greater if he does not cover the market than if he covers the market.

When the incumbent tries to deter entry he decides $s_I = kS$, with $k < 1$. The incumbent limits his quality, with respect to the monopolist decision, as an entry-deterring device. In this case, the entrant may consider to enter in the market with a quality level smaller than s_I or with a quality level greater than s_I, and we will show that the resultant market coverage configuration if the entrant entered as the low quality firm is different to the market coverage configuration that results if the entrant entered as the high quality firm.

If the incumbent decides $s_I = kS$, and deters entry with this selection of quality, he will set $p_I = \frac{b}{2}kS$ (by the same reasoning used in the case of monopoly). As the profits of firm I, when it is the only firm in the market, increase with the quality level selected, the incumbent will choose, in the case of entry deterrence, the maximum value of k (hence, the maximum value of s_I) that deters entry. The maximum value of k allowing entry deterrence will depend on the entry cost F, on S and on the heterogeneity of consumers tastes b.

We can prove:

Theorem 1: *Entry is deterred in the following situations:*
i) $2 < b \leq 5$ *and*

$$\frac{(2b-1)^2(b-2)^2}{3(b-1)\left[(2b-1)^2(b+1)+3(b-2)^2\right]} \leq \frac{F}{S} \leq \frac{(b-2)^2}{3(b-1)(b+1)}.$$

ii) $5 \leq b \leq 8.6581$ *and*

$$\frac{(2b-1)^2\left(b-2\sqrt{b-1}\right)}{2(b-1)\left[4b^2+\frac{b}{2}-9\sqrt{b-1}+1\right]} \leq \frac{F}{S} \leq \frac{b-2\sqrt{b-1}}{2(b-1)}.$$

iii) $8.6581 \leq b$ *and*

$$\frac{16b^2(2b-1)^2}{48(b-1)\left[16(2b-1)^2+3b^2\right]} \leq \frac{F}{S} \leq \frac{b^2}{48(b-1)}$$

Proof: See the Appendix.

The lower limit of the interval of values of F where entry is deterred increases with b and is continuous in b (in particular, when $b = 5$ and when $b = 8.6581$). In the analysis of entry deterrence in the proof of Theorem 1 we obtain that the resultant market configuration if the entrant would have entered as the low quality firm is different to the resultant market configuration if the entrant would have entered as the high quality firm. Moreover, we show that when entry deterrence is feasible the incumbent always prefers to deter entry than to accommodate entry (hence, entry will occur only when it is not blockaded and, moreover, entry cannot be deterred). Finally, in the proof of Theorem 1 we also obtain the following equilibrium for the case of deterred entry:

Proposition 1: *When entry is deterred the incumbent decides quality* $s_I = kS$ *and price* $p_I = \frac{b}{2}kS$*, with* k *such that:*

i) $k = \frac{48(b-1)F}{b^2 S}$ *when* $b \geq 8.6581$

ii) $k = \frac{2(b-1)F}{\left(b-2\sqrt{b-1}\right)S}$ *when* $8.6581 \geq b \geq 5$

iii) $k = \frac{3(b-1)(b+1)F}{(b-2)^2 S}$ *when* $5 \geq b > 2$

This value of k is continuous in b (in particular, when $b = 8.6581$ and $b = 5$) and decreases with b. As the heterogeneity of consumers tastes increases, entry becomes more profitable and the incumbent must deviate more from the monopolist solution to deter entry. Moreover, as $\frac{p_I}{s_I} = \frac{b}{2} > 1$ when entry is deterred, the market will not be covered in this equilibrium.

The strategy of the proof of Theorem 1 is as follows: We show that in all cases the revenue of the entrant when he enters as the high quality firm (R_E^h) decreases with k while the revenue of the entrant when he enters as the low quality firm (R_E^l) increases with k. Figure 1 represents the situation:

Figure 1 is depicted with b fixed and it is valid for any value of b. We represent R_E^h linear to simplify the presentation, but the functional form of R_E^h varies with the market configuration (that depends on k) and some of

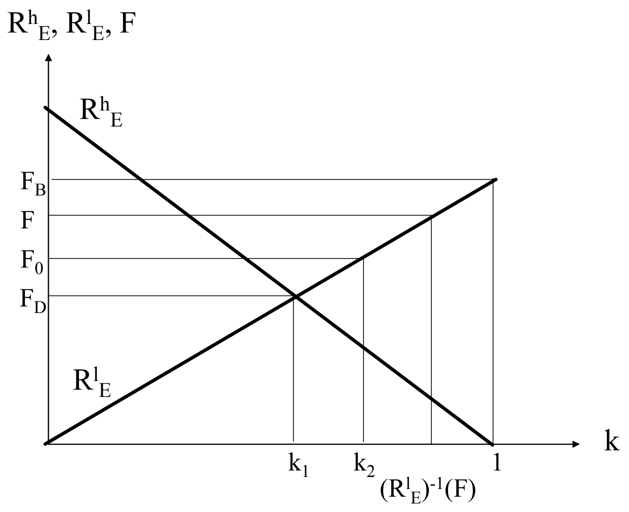

Figure 1:

these functional forms are not linear in k. However, R_E^h is continuous. We measure in the vertical axis R_E^h, R_E^l and F and in the horizontal axis we measure k. When $F \geq F_B$ entry will be blockaded (F_B is the lower limit of the corresponding interval of values of F where entry is blockaded). When entry is blockaded the incumbent chooses $s_I = S$ (that is, $k = 1$) and $R_E^h(1)$ and $R_E^l(1)$ are lower than F.

If $F < F_B$ entry cannot be blockaded. In this situation entry may be deterred if there exists a k such that $R_E^h(k) < F$ and $R_E^l(k) < F$. In Figure 1 entry may be deterred if F is such that $F_D \leq F < F_B$. Suppose that $F = F_0$. For F_0 entry may be deterred with any k such that $k \in [k_1, k_2]$. Among the values of k in this interval we show that, to deter entry, the incumbent prefers $k = k_2$, as the profits of the incumbent when he deters entry increase with s_I (i.e., with k).[11] Hence, when $F_D \leq F < F_B$ the incumbent will select k such that $R_E^l(k) = F$ ($\Leftrightarrow k = (R_E^l)^{-1}(F)$) to deter entry. In Figure 1, F_B, F_D, k_1 and $(R_E^l)^{-1}(F)$ depend on b.

The determination of F_D and $(R_E^l)^{-1}(F)$ requires to study, for each k, the market coverage configuration that would result if the entrant entered

[11] Notice that the incumbent will never select $k < k_1$ to deter entry. From Figure 1 it is clear that if entry may be deterred with a k lower than k_1 there exist values of k greater than k_1 that also deter entry.

as the low quality firm and the market coverage configuration that would result if the entrant entered as the high quality firm. We show in the proof of Theorem 1 that when the incumbent deters entry he selects a value of k such that the market would have been covered with an interior solution if the entrant would have entered as the high quality firm. However, if the entrant would have entered as the low quality firm the resultant market coverage configuration would have been uncovered market or market covered with corner solution, depending on the value of b.

4 Entry deterrence with fixed quality costs

If there are quality costs, i.e., costs that increase with the quality level selected, it is also necessary to endogeneize market coverage in the analysis of entry deterrence. Quality costs may be either fixed, when they do not depend on the production level, or variable, when they do depend on the production level. In the literature there are analyses that consider, in contexts where market coverage is given exogenously, fixed quality costs, as in Aoki and Prusa (1996), Lutz (1997) and Constantatos and Perrakis (1999), and variable quality costs, as in Crampes and Hollander (1995).

We consider that firms face only fixed quality costs. These costs may be considered as the ones required to incorporate (once for all) the technology associated to the corresponding quality level. For instance, the fixed costs of quality may be incurred during the research and development phase of the product. The case of variable quality costs is not considered in this work. Variable quality costs affect the equilibrium of the price subgame, contrary to the case of fixed quality costs, and, therefore, call for a complete new derivation of the results. Moreover, as we need fixed quality costs to have firms committed to a certain quality level during price competition, variable quality costs, if considered, would have to go with fixed quality costs in the analysis.

It is usually assumed that the fixed quality cost function is convex and we incorporate this assumption in our analysis.[12] Let us represent the fixed quality cost function by $c(s)$, with $c'(s) > 0$ and $c''(s) > 0$. The reaction functions corresponding to the quality decisions of firms are presented in the Appendix. With convex fixed quality costs, however, it is not possible

[12] The functional form of the convex fixed quality cost function, together with the rest of parameters, must make possible the existence of an equilibrium with two firms.

to obtain the subgame perfect equilibrium of the entry game in an explicit way when there is endogenous market coverage. Hence, we discuss entry deterrence inthis case using an example. In this example we also obtain that when the incumbent deters entry he selects a value of k such that the market would have been covered with an interior solution if the entrant would have entered as the high quality firm. However, the resultant market configuration if the entrant would have entered as the low quality firm would again depend on b. In this example it is considered that either there is not a maximum quality level or S is never attained in the equilibria considered. We use numerical methods (Scientific WorkPlace) in the calculus.

Example

Consider that $c(s) = \frac{s^2}{7}$, $b = 7$. The monopolist problem with quality costs is

$$\pi_I = p_I \left[\frac{1}{b-1} \left(b - \frac{p_I}{s_I} \right) \right] - \frac{s_I^2}{7}$$

Proceeding as in section 3 we obtain $p_I = \frac{b}{2} s_I$ and $s_I = \frac{7b^2}{8(b-1)}$. Therefore, if entry is not blockaded and entry deterrence is feasible, the quality $k\frac{7b^2}{8(b-1)}$, with $0 < k < 1$, selected by the entrant to deter entry for a given value of b will be the highest quality among those that deter entry for that level of heterogeneity in consumer tastes.

When $b = 7$ we obtain $k_1 = 0.6938$, following the same procedure than in the proof of Theorem 1 (see also Figure 1). If entry may be deterred, the incumbent will select $k \geq 0.6938$ to deter entry. It may be shown that when $k \geq 0.6938$ the market would have been covered with interior solution if the entrant would have entered with high quality, as the configurations of market not covered and market covered with corner solution are not feasible for those values of k if entry occurred in this way (the entrant would obtain negative profits in this latter configurations). However, in the configuration of market covered with interior solution the profits of the high quality entrant would be positive only if $k < 0.71852$ (i.e., $k\frac{7b^2}{8(b-1)} < 5.134$). If $k \geq 0.71852$ entry with high quality would imply losses and, therefore, the entrant would only consider entry with low quality.

It is also obtained that if the entrant would have entered with low quality, when the incumbent can deter entry, the market would have been not covered.[13] Finally, this latter configuration would have also resulted with

[13] Market not covered is the only feasible configuration for this case when $k \geq 0.6938$.

accommodated entry.[14] Hence, it would not be correct either to assume a given market coverage configuration when analysing entry deterrence in this case.

As in the case without quality costs, it is also true that, when entry deterrence is feasible for $b = 7$, the incumbent always prefers to deter entry than to accommodate entry. In this example, if entry is accommodated the market will not be covered and the profits of the incumbent are 5.7046.[15] When entry is deterred the profits of the incumbent are[16]

$$\pi_I^{\det} = \frac{49}{24}(7.1458k) - \frac{(7.1458k)^2}{7}$$

and we obtain that $\pi_I^{\det}$ is greater than 5.7046 for any value of k greater than 0.6938. ■

5 Conclusion

In this paper we have analyzed entry deterrence in a model of vertical differentiation where firms select quality and price, considering that covered or uncovered market are endogenous outcomes of the entry game. As in Shaked and Sutton (1982) there are three different market configurations that may result in the competitive game between an established firm and an entrant: uncovered market, market covered with corner solution and market covered with interior solution.

We have emphasized in this work the relevance of the endogeneity of market coverage for a correct analysis of entry deterrence in vertical differentiation models. We have proved that the market configuration that would result if entry were accommodated and if the entrant would have entered as the low quality firm when the incumbent deters entry are different to the market configuration that would have been obtained if the entrant would have entered as the high quality firm when entry is deterred. Hence, the incumbent must consider all feasible market configurations that may result from entry to decide on entry deterrence.

[14] At the equilibrium with entry (i.e, with F small enough), there may exist a market not covered if $b > 4.698$ and a market covered with corner solution if $2 < b < 4.73$. These parameter regions are obtained as in the proof of Theorem 1.

[15] The qualities selected by the incumbent and the entrant in this case are, respectively, 7.2403 and 1.3789.

[16] The quality selected by a monopolist would be 7.1458 when $b = 7$.

Consider, for instance, that the market would not be covered if entry was accommodated. This may occur as access to technology or entry costs limit the number of active firms in the market and consumers are sufficiently heterogeneous. In this case, to decide on entry deterrence the incumbent must take into account all feasible market configurations, and not only uncovered market. The reason is that the entrant may choose to enter as the high quality firm and the resultant market configuration with that kind of entry may be different from market not covered. Therefore, to assume uncovered market as the exogenously given market configuration in this situation would not be correct for the analysis of entry deterrence. Moreover, if the configuration of market not covered is unfeasible because the heterogeneity in consumers tastes is not large enough, there may still be two feasible market configurations, market covered with corner solution and market covered with interior solution, that the incumbent would have to consider when deciding on entry deterrence.

6 Appendix

6.1 Proof of Theorem 1

Entry is deterred when entry is not blockaded and the profits of the incumbent when entry is deterred are higher than his profits when entry is accommodated. Hence, let us prove first Lemmas 1 and 2 that study the cases of accommodated entry and of blockaded entry, respectively.

Lemma 1: *When entry occurs we have that at the subgame perfect equilibrium:*

i) If $b \geq 8.6581$*, the market will not be covered and:*

-$s_I^{**} = S \qquad p_I^{**} = \frac{2bS}{4S - s_E^{**}}(S - s_E^{**})$

-$s_E^{**} = \frac{4}{7}S \qquad p_E^{**} = \frac{bs_E^{**}}{4S - s_E^{**}}(S - s_E^{**})$

ii) If $8.6581 \geq b > 2$*, the market will be covered with a corner solution and:*

- *when* $8.6581 \geq b \geq 5$*:*

-$s_I^c = S \qquad p_I^c = \frac{s_E^c + b(S - s_E^c)}{2}$

-$s_E^c = \frac{b-1-\sqrt{b-1}}{b-1}S \qquad p_E^c = s_E^c$

- *when* $5 \geq b > 2$*:*

-$s_I^* = S$ $\quad p_I^* = \frac{2b-1}{3}(S - s_E^*)$
-$s_E^* = \frac{b-2}{b+1}S$ $\quad p_E^* = s_E^*$

Proof of Lemma 1: When the entrant enters the market, we can derive first the equilibrium outcomes for the price subgame, using the profit functions and following Wauthy (1996). The parameter regions associated to each market configuration are obtained from the decision of the consumer with the lowest valuation for quality. The market is not covered if this consumer does not buy any of the two products ($v = 1 < \frac{p_1}{s_1}$), it is covered with interior solution if he has a positive surplus when buying the low quality product ($1 > \frac{p_1}{s_1}$) and it is covered with a corner solution when his surplus from buying the low quality product is zero (the rest of situations, where $1 = \frac{p_1}{s_1}$). We use the superscripts **, ^{c} and *, respectively, to denote the equilibrium values at the market configurations of market not covered, market covered with corner solution and market covered with interior solution. At the equilibrium of the price subgame we have:

i) Market not covered whenever $b \geq \frac{4s_h - s_l}{s_h - s_l}$ (or $s_l \leq s_h \frac{b-4}{b-1}$):

$$p_l^{**} = b(s_h - s_l)\frac{s_l}{4s_h - s_l}$$

$$p_h^{**} = b(s_h - s_l)\frac{2s_h}{4s_h - s_l}$$

ii) Market covered with corner solution whenever $\frac{2s_h + s_l}{s_h - s_l} \leq b \leq \frac{4s_h - s_l}{s_h - s_l}$ (or $s_h \frac{b-4}{b-1} \leq s_l \leq s_h \frac{b-2}{b+1}$):

$$p_l^c = s_l$$

$$p_h^c = \frac{s_l + b(s_h - s_l)}{2}$$

iii) Market covered with interior solution whenever $2 \leq b \leq \frac{2s_h + s_l}{s_h - s_l}$ (or $s_l \geq s_h \frac{b-2}{b+1}$):

$$p_l^* = \frac{b-2}{3}(s_h - s_l)$$

$$p_h^* = \frac{2b-1}{3}(s_h - s_l)$$

The market is preempted by firm h, whenever $1 < b \leq 2$.[17] Therefore, to allow for the possibility of entry we focus on situations where there may be two firms in the market and, hence, $b > 2$.

[17] When $b = 2$, the consumer with the lowest valuation of quality is indifferent between buying any of the two goods.

The revenue functions for each market configuration can now be written as follows (the incumbent's profit function, π_I, is identical to the corresponding revenue function, and the entrant's profit function, π_E, is obtained subtracting F from the corresponding revenue function):

i) If the market is not covered:

$$R_h^{**} = \frac{4b^2 s_h^2 (s_h - s_l)}{(b-1)(4s_h - s_l)^2} \tag{1}$$

$$R_l^{**} = \frac{b^2 s_h s_l (s_h - s_l)}{(b-1)(4s_h - s_l)^2} \tag{2}$$

ii) If the market is covered with a corner solution:

$$R_h^c = \frac{[s_l + b(s_h - s_l)]^2}{4(b-1)(s_h - s_l)} \tag{3}$$

$$R_l^c = \frac{s_l\left[(b-2)(s_h - s_l) - s_l\right]}{2(b-1)(s_h - s_l)} \tag{4}$$

iii) If the market is covered with an interior solution:

$$R_h^* = \frac{(2b-1)^2 (s_h - s_l)}{9(b-1)} \tag{5}$$

$$R_l^* = \frac{(b-2)^2 (s_h - s_l)}{9(b-1)} \tag{6}$$

Notice that the revenues of the high quality firm are higher than the revenues of the low quality firm. Observe also that R_l^* is linear and decreasing in s_l and R_h^* is linear and increasing in s_h. Moreover, it is not difficult to show the concavity of R_l^{**} and R_l^c with respect to s_l. Finally, R_h^{**} is concave with respect to s_h but R_h^c is convex with respect to s_h. However, we will show that this convexity will not pose any problems for the analysis in this work.

Let us show now that if the incumbent decides to be the high quality firm he will choose $s_h = S$. From Wauthy (1996) we know that the best reply to quality s_h by a firm deciding on a quality s_l smaller than s_h is:

- $s_l = \frac{4}{7} s_h$ when the market is not covered
- $s_l = \frac{b-1-\sqrt{b-1}}{b-1} s_h$ when the market is covered with corner solution
- $s_l = \frac{b-2}{b+1} s_h$ when the market is covered with interior solution.

As from (1), (3) and (5) we have that R_h increases with s_h in each market configuration, given these best replies by the entrant, the incumbent will set $s_h = S$ if he prefers to be the high quality firm. Although R_h given by (3) is convex, notice that R_h increases with s_h when the market is covered with a corner solution.

Moreover, note that the profits of the incumbent are greater when he is the high quality firm than when he is the low quality firm. Suppose that the incumbent decides quality s' and the entrant enters as the high quality firm (with quality S, by the same argument presented above). When qualities are s' and S a particular market configuration will result. Denoting the best low quality reply to high quality S for this market configuration by $s_l(s_h = S)$, it is, from (1) to (6):

$$R_l(s', S) \leq R_l(s_l(s_h = S), S) < R_h(s_l(s_h = S), S)$$

Thus, the incumbent prefers to be the high quality firm.

Hence, the quality decisions within each market configuration will be:

i) If the market is not covered ($b > 8$):
$s_I^{**} = S$
$s_E^{**} = \frac{4}{7}S$

ii) If the market is covered with a corner solution ($5 \leq b \leq 10$):
$s_I^c = S$
$s_E^c = \frac{b-1-\sqrt{b-1}}{b-1}S$

iii) If the market is covered with an interior solution ($b > 2$):
$s_I^* = S$
$s_E^* = \frac{b-2}{b+1}S$

The parameter regions associated to each market configuration have been derived substituting the corresponding quality equilibrium decisions in the parameter regions derived for the equilibrium outcomes of the price subgame. For instance, from $s_l < s_h \frac{b-4}{b-1}$ and $s_l^{**} = \frac{4}{7}s_h^{**} = \frac{4}{7}S$ we obtain $b > 8$. When $b < 8$, notice that R_l^{**} always increases with s_l and the market will end up covered ($s_l \geq \frac{b-4}{b-1}S$). Furthermore, when $\frac{b-4}{b-1}S \leq s_l \leq \frac{b-2}{b+1}S$, we have that R_l^c increases with s_l if $b < 5$ and that R_l^c decreases with s_l if $b > 10$.

As there are parameter regions where more than one candidate to equilibrium exists, we have to compare the profits of the entrant under each candidate in those regions to obtain the equilibrium selected for each value of b. Let us denote by NC, CC and CI, respectively, the candidates to equilibrium corresponding to the configurations of market not covered, market covered with a corner solution and market covered with an interior solution.

As R_1^* is decreasing in s_1, CI is in the boundary of the region where the market is covered with interior solution (i.e., in the boundary between that region and the region where the market is covered with corner solution). Thus, CI corresponds in fact to a particular corner situation (notice that $p_E^* = \frac{b-2}{3}(s_I^* - s_E^*) = s_E^*$), but it differs, in general, from CC. Therefore, CC will always be preferred by the entrant to CI when the two solutions are defined and differ, i.e., when $5 < b \leq 10$ (if $b = 5$ both solutions coincide as $\frac{b-1-\sqrt{b-1}}{b-1} = \frac{b-2}{b+1}$). When both NC and CC are defined (i.e., when $8 < b \leq 10$), it may be shown that $R_E^{**}(s_E^{**}, S) \gtreqless R_E^c(s_E^c, S) \Leftrightarrow b \gtreqless 8.6581$. Finally, it is easy to check that the profits of the entrant at NC are greater than the profits of this firm at CI when the two solutions are defined, i.e., when $b > 8$. ♦

Since there are neither fixed costs of increasing quality nor greater variable costs to produce a good of higher quality, the incumbent decides $s_I = S$ to reduce price competition. Hence, it is the entrant who determines the equilibrium market configuration. The range of values of F where, for each value of b, there is entry accommodation may be obtained from the analyses of blockaded entry and of deterred entry developed below.

Lemma 2 considers the case of blockaded entry. Entry is blockaded if the incumbent behaves as a monopolist, deciding quality and price as if there was not threat of entry, as the entrant would have negative profits if he entered into the market. The study of blockaded entry is useful for the analysis of entry deterrence not only because we want to focus on situations where entry cannot be blockaded, but also because the decision of the incumbent that deters entry uses as a benchmark the monopolistic quality and price levels corresponding to the case of blockaded entry.

Lemma 2: *When entry is blockaded the incumbent decides $s_I = S$ and $p_I = \frac{b}{2}S$, and the market will not be covered. Moreover, entry is blockaded if:*[18]

i) $b \geq 8.6581$ *and* $\frac{F}{S} \geq \frac{b^2}{48(b-1)}$.

[18] Note that the minimum value of F that permits to block entry increases with b (the heterogeneity of consumers tastes) and with S. Moreover, observe that this lower limit is continuous in b (in particular, when $b = 8.6581$ and when $b = 5$).

ii) $5 \leq b \leq 8.6581$ *and* $\frac{F}{S} \geq \frac{b-2\sqrt{b-1}}{2(b-1)}$.
iii) $2 < b \leq 5$ *and* $\frac{F}{S} \geq \frac{(b-2)^2}{3(b-1)(b+1)}$.

Proof of Lemma 2: When the incumbent selects $s_I = S$ (the quality of monopoly), we know from Lemma 1 that the best the entrant could do is to decide $s_E = \frac{4}{7}S$ if $b \geq 8.6581$ (market not covered), $s_E = \frac{b-1-\sqrt{b-1}}{b-1}S$ if $5 \leq b \leq 8.6581$ (market covered with a corner solution) and $s_E = \frac{b-2}{b+1}S$ if $2 < b \leq 5$ (a different situation of market covered with corner solution). Entry will be blockaded if $\pi_E(s_E, S) \leq 0$. Hence, from (2), (4) and (6) we have that entry is blockaded if:

$$\text{i) } b \geq 8.6581 \text{ and } \frac{F}{S} \geq \frac{b^2}{48(b-1)},$$

$$\text{ii) } 8.6581 \geq b \geq 5 \text{ and } \frac{F}{S} \geq \frac{\left(b-2\sqrt{b-1}\right)}{2(b-1)}, \text{ or}$$

$$\text{iii) } 5 \geq b > 2 \text{ and } \frac{F}{S} \geq \frac{(b-2)^2}{3(b-1)(b+1)}. \ \blacklozenge$$

Proof of Theorem 1: When the incumbent tries to deter entry he decides $s_I = kS$, with $k < 1$. The incumbent limits his quality, with respect to the monopolist decision, as an entry-deterring device. In this case, the entrant may consider to enter in the market with a quality level smaller than s_I. The quality below s_I that is most profitable for the entrant may be derived from the equilibrium in Lemma 1, substituting kS for S. Alternatively, the entrant may consider to enter the market with a quality level greater than s_I. From (1), (3) and (5) it is immediate to see that the quality above s_I most profitable for the entrant would be quality S. In this case, the resultant market configuration would depend on the value of $\frac{s_I}{S} \equiv k$. The market configuration will be market covered with an interior solution if the incumbent decides $k \geq \frac{b-2}{b+1}$, market covered with corner solution if $\frac{b-2}{b+1} \geq k \geq \frac{b-4}{b-1}$ and uncovered market if $\frac{b-4}{b-1} \geq k$.

Let us consider the case $8.6581 \leq b$ in part A of the proof, the case $5 \leq b \leq 8.6581$ in part B and the case $2 < b \leq 5$ in part C. We use Scientific WorkPlace for some calculus.

$A)$ Case $8.6581 \leq b$

The profits of the incumbent when entry is deterred are: $\pi_I^{\det} = \frac{b^2 kS}{4(b-1)}$. When entry is accommodated and $b > 8.6581$, we know from the analysis in section 3 that the profits of the incumbent are: $\pi_I^{ac} = \frac{7b^2 S}{48(b-1)}$. We have $\pi_I^{ac} \leq \pi_I^{\det} \iff k \geq \frac{7}{12}$. If entry deterrence required $k < \frac{7}{12}$ the incumbent would prefer to accommodate entry. Notice that when $b \geq 8.6581$ it is $\frac{b-4}{b-1} > \frac{7}{12}$ and, hence, $\frac{b-2}{b+1} > \frac{7}{12}$.

When $b \geq 8.6581$ the market would be not covered if the entrant entered with quality smaller than s_I and, therefore, the revenue functions would be given by (1) and (2), with $s_h = s_I$ and $s_l = s_E$. To deter entry the incumbent decides $s_I = kS$,with $k < 1$, and the best reply of an entrant that enters with quality smaller than s_I is $s_E = \frac{4}{7} s_I = \frac{4}{7} kS$. Hence, from (2) the entrant would have negative profits and would not enter the market with quality smaller than s_I if:

$$F \geq \frac{b^2 kS}{48(b-1)}$$

However, the entrant may also consider entry with quality (s_E^h) greater than s_I and in this case it would be, from (1), $s_E^h = S$. Let us denote by $R_E^h(k)$ the revenue of the entrant, as a function of k, if he decides to enter with quality S. From (1), (3) and (5) it will be

$$R_E^h(k) = \begin{cases} \frac{4b^2(1-k)S}{(b-1)(4-k)^2} & \text{when } k \leq \frac{b-4}{b-1} \\ \frac{[b(1-k)+k]^2 S}{4(b-1)(1-k)} & \text{when } \frac{b-4}{b-1} \leq k \leq \frac{b-2}{b+1} \\ \frac{(2b-1)^2(1-k)S}{9(b-1)} & \text{when } \frac{b-2}{b+1} \leq k \end{cases}$$

Notice that $R_E^h(k)$ is continuous in k (in particular, when $k = \frac{b-4}{b-1}$ and when $k = \frac{b-2}{b+1}$). Moreover, $R_E^h(0) = \frac{b^2 S}{4(b-1)}$, $R_E^h(1) = 0$ and $\frac{\partial R_E^h(k)}{\partial k} < 0$.[19]

Entry will be deterred if b and F are such that:

i) $\frac{b^2 S}{48(b-1)} \geq F$: entry is not blockaded, and

ii) there exists a k such that

$\rightarrow F \geq \frac{b^2 kS}{48(b-1)}$: entry with quality smaller than s_I is deterred,

$\rightarrow F \geq R_E^h(k)$: entry with quality greater than s_I is deterred.

Hence, to deter entry F must satisfy

[19] Notice that $\frac{\partial \left[\frac{[b(1-k)+k]^2 S}{4(b-1)(1-k)}\right]}{\partial k} < 0$ when $k < \frac{b-2}{b-1}$.

$$\frac{b^2 S}{48(b-1)} \geq F \geq \max\left\{\frac{b^2 kS}{48(b-1)},\, R_E^h(k)\right\}.$$

Observe that with the notation of Figure 1 it is $F_B \equiv \frac{b^2 S}{48(b-1)}$ and $R_E^l \equiv \frac{b^2 kS}{48(b-1)}$. Notice, also, that $\frac{\partial\left[\frac{b^2 kS}{48(b-1)}\right]}{\partial k} > 0$.

Consider that entry is not blockaded. If there exist values of k such that F is greater than $R_E^h(k)$ and $\frac{b^2 kS}{48(b-1)}$ entry will be deterred. Then the incumbent will select $s_I = k^* S$ to deter entry, where k^* is the maximum value of k such that F is greater than $R_E^h(k)$ and $\frac{b^2 kS}{48(b-1)}$. As $R_E^h(k)$ decreases with k and $\frac{b^2 kS}{48(b-1)}$ increases with k, it will always be k^* such that $F = \frac{b^2 k^* S}{48(b-1)} \Leftrightarrow k^* = \frac{48(b-1)F}{b^2 S}$ (in Figure 1 it is $k^* \equiv (R_E^l)^{-1}(F)$).

Let us study the value of k such that $R_E^h(k) = \frac{b^2 kS}{48(b-1)}$ (i.e., let us determine the value of k_1 in Figure 1). If this value of k is greater than $\frac{b-2}{b+1}$ it will be $\frac{b^2 kS}{48(b-1)} = \frac{(2b-1)^2(1-k)S}{9(b-1)} \Leftrightarrow k = \frac{16(2b-1)^2}{16(2b-1)^2+3b^2} < 1$. This is precisely what happens when $8.6581 \leq b$, as then it is $\frac{b-2}{b+1} < \frac{16(2b-1)^2}{16(2b-1)^2+3b^2}$.[20] Hence, when $8.6581 \leq b$ and the incumbent deters entry a market covered with interior solution would have resulted if the entrant entered with quality greater than s_I.

In this case it is then possible to deter entry for a given F when this F satisfies $\frac{b-2}{b+1} \leq \frac{16(2b-1)^2}{16(2b-1)^2+3b^2} \leq k = \frac{48(b-1)F}{b^2 S}$, i.e., if F is such that $F \geq \frac{16(2b-1)^2 b^2 S}{48(b-1)[16(2b-1)^2+3b^2]}$. When $8.6581 \leq b$ entry will be deterred for those values of F such that:

$$\frac{16(2b-1)^2 b^2 S}{48(b-1)\left[16(2b-1)^2+3b^2\right]} \leq F \leq \frac{b^2 S}{48(b-1)}$$

$B)$ Case $5 \leq b \leq 8.6581$

The profits of the incumbent when entry is deterred are: $\pi_I^{\det} = \frac{b^2 kS}{4(b-1)}$. When entry is accommodated and $8.6581 \geq b \geq 5$, we know from the analysis in section 3 that the profits of the incumbent are: $\pi_I^{ac} = \frac{b+2\sqrt{b-1}}{4\sqrt{b-1}} S$. We have $\pi_I^{ac} \leq \pi_I^{\det} \iff k \geq \frac{\sqrt{b-1}\left[b+2\sqrt{b-1}\right]}{b^2}$. If entry deterrence required $k < \frac{\sqrt{b-1}\left[b+2\sqrt{b-1}\right]}{b^2} S$ the incumbent would prefer to accommodate entry.

When $8.6581 \geq b \geq 5$ the market would be covered with a corner solution if the entrant entered with quality smaller than s_I and, therefore, the revenue

[20] This requires $b < 65.02$, which seems realistic.

functions would be given by (3) and (4), with $s_h = s_I$ and $s_l = s_E$. To deter entry the incumbent decides $s_I = kS$, with $k < 1$, and the best reply of an entrant that enters with quality smaller than s_I is $s_E = \left[1 - \frac{1}{\sqrt{b-1}}\right] kS$. Hence, from (4) and using the notation of Figure 1:

$$R_E^l(k) \equiv \frac{b - 2\sqrt{b-1}}{2(b-1)} kS \text{ and } F_B \equiv \frac{b - 2\sqrt{b-1}}{2(b-1)} S$$

However, the entrant may also consider entry in the market with quality (s_E^h) greater than s_I and in this case it would be, from (3), $s_E^h = S$. We will show below that the market would be covered with an interior solution if the entrant entered as the high quality firm. Then,

$$R_E^h(k) = \frac{(2b-1)^2 (1-k)S}{9(b-1)}$$

Notice that

$\rightarrow \frac{\partial\left[\frac{b-2\sqrt{b-1}}{2(b-1)}kS\right]}{\partial k} > 0$. If $k = 0 \Longrightarrow \frac{b-2\sqrt{b-1}}{2(b-1)}kS = 0$. If $k = 1 \Longrightarrow \frac{b-2\sqrt{b-1}}{2(b-1)}kS = \frac{b-2\sqrt{b-1}}{2(b-1)}S > 0$.

$\rightarrow \frac{\partial\left[\frac{(2b-1)^2(1-k)S}{9(b-1)}\right]}{\partial k} < 0$. If $k = 0 \Longrightarrow \frac{(2b-1)^2(1-k)S}{9(b-1)} = \frac{(2b-1)^2 S}{9(b-1)} > 0$. If $k = 1 \Longrightarrow \frac{(2b-1)^2(1-k)S}{9(b-1)} = 0$.

Proceeding as in part A) we obtain (see Figure 1) that $(R_E^l)^{-1}(F) = \frac{2(b-1)F}{\left(b-2\sqrt{b-1}\right)S}$ and $k_1 = \frac{(2b-1)^2}{4b^2+\frac{b}{2}-9\sqrt{b-1}+1} < 1$. Notice that $k_1 = \frac{(2b-1)^2}{4b^2+\frac{b}{2}-9\sqrt{b-1}+1} > \frac{b-2}{b+1}$ when $5 \leq b \leq 8.6581$. Hence, when $5 \leq b \leq 8.6581$ the function $R_E^h(k)$ always cuts the function $R_E^l(k)$ at a value of k such that $k > \frac{b-2}{b+1}$. As a consequence the market would be covered with an interior solution if the entrant entered with high quality. The cases of market covered with corner solution and uncovered market when the entrant enters with quality greater than s_I are not possible. These cases would require that function $R_E^h(k)$, defined in part A of the proof, cuts function $R_E^l(k)$ at a value of k such that $\frac{b-4}{b-1} \leq k \leq \frac{b-2}{b+1}$ in the case of market covered with corner solution and at a value of k such that $k \leq \frac{b-4}{b-1}$ in the case of uncovered market.

Moreover, when $5 \leq b \leq 8.6581$,

$\max\left\{\frac{\sqrt{b-1}\left[b+2\sqrt{b-1}\right]}{b^2}, \frac{(2b-1)^2}{4b^2+\frac{b}{2}-9\sqrt{b-1}+1}\right\} = \frac{(2b-1)^2}{4b^2+\frac{b}{2}-9\sqrt{b-1}+1}$. As $\frac{(2b-1)^2}{4b^2+\frac{b}{2}-9\sqrt{b-1}+1} > \frac{\sqrt{b-1}\left(b+2\sqrt{b-1}\right)}{b^2}$, the incumbent will prefer to deter entry, when entry deterrence is feasible, than to accommodate entry.

Hence, entry deterrence is feasible for a given F when this F satisfies $k_1 \equiv \frac{(2b-1)^2}{4b^2+\frac{b}{2}-9\sqrt{b-1}+1} \leq k = \frac{2(b-1)F}{\left(b-2\sqrt{b-1}\right)S}$, i.e., if F is such that $F \geq \frac{(2b-1)^2\left(b-2\sqrt{b-1}\right)S}{2(b-1)\left[4b^2+\frac{b}{2}-9\sqrt{b-1}+1\right]}$. When $5 \leq b \leq 8.6581$ entry will be deterred for those values of F such that

$$\frac{(2b-1)^2\left(b-2\sqrt{b-1}\right)S}{2(b-1)\left[4b^2+\frac{b}{2}-9\sqrt{b-1}+1\right]} \leq F \leq \frac{b-2\sqrt{b-1}}{2(b-1)}S$$

C) Case $2 < b \leq 5$

The profits of the incumbent when entry is deterred are: $\pi_I^{\det} = \frac{b^2kS}{4(b-1)}$. When entry is accommodated and $5 \geq b > 2$ we know from the analysis in section 3 that the profits of the incumbent are: $\pi_I^{ac} = \frac{(2b-1)^2S}{3(b-1)(b+1)}$. We have $\pi_I^{ac} \leq \pi_I^{\det} \iff k \geq \frac{4(2b-1)^2}{3(b+1)b^2}$. If entry deterrence required $k < \frac{4(2b-1)^2}{3(b+1)b^2}S$ the incumbent would prefer to accommodate entry.

When $5 \geq b > 2$ the market would be covered with interior solution if the entrant entered with quality smaller than s_I and, therefore, the revenue functions would be given by (5) and (6), with $s_h = s_I$ and $s_l = s_E$. To deter entry the incumbent decides $s_I = kS$, with $k < 1$, and the best reply of an entrant that enters with quality smaller than s_I is $s_E = \frac{b-2}{b+1}kS$ (thus, in fact, a different corner solution). Hence, from (6) and using the notation of Figure 1

$$R_E^l(k) \equiv \frac{(b-2)^2kS}{3(b-1)(b+1)} \text{ and } F_B \equiv \frac{(b-2)^2S}{3(b-1)(b+1)}$$

However, the entrant may also consider entry with quality (s_E^h) greater than s_I and in this case it would be, from (5), $s_E^h = S$. As we have seen that entry deterrence by the incumbent requires $k > \frac{b-2}{b+1}$ when $2 < b \leq 5$, if the entrant entered with (high) quality S it would be $\frac{s_I}{S} = k > \frac{b-2}{b+1}$, the market would be covered with an interior solution and:

$$R_E^h = \frac{(2b-1)^2(1-k)S}{9(b-1)}$$

Notice also that $b \in (2,5]$ implies $\frac{4(2b-1)^2}{3(b+1)b^2} > \frac{b-2}{b+1}$. Hence, when the incumbent deters entry he selects k such that the market would be covered with an interior solution if the entrant entered as the high quality firm.

Notice that

$\rightarrow \frac{\partial\left[\frac{(b-2)^2}{3(b-1)(b+1)}kS\right]}{\partial k} > 0$. If $k = 0 \Longrightarrow \frac{(b-2)^2}{3(b-1)(b+1)}kS = 0$. If $k = 1, \Longrightarrow \frac{(b-2)^2}{3(b-1)(b+1)}kS = \frac{(b-2)^2}{3(b-1)(b+1)}S > 0$.

$\rightarrow \frac{\partial\left[\frac{(2b-1)^2(1-k)S}{9(b-1)}\right]}{\partial k} < 0$. If $k = 0 \Longrightarrow \frac{(2b-1)^2(1-k)S}{9(b-1)} = \frac{(2b-1)^2 S}{9(b-1)} > 0$. If $k = 1 \Longrightarrow \frac{(2b-1)^2(1-k)S}{9(b-1)} = 0$.

Proceeding as in part A) we obtain (see Figure 1) that $(R_E^l)^{-1}(F) = \frac{3(b-1)(b+1)F}{(b-2)^2 S}$ and $k_1 = \frac{(2b-1)^2(b+1)}{(2b-1)^2(b+1)+3(b-2)^2}$. Moreover, when $2 < b \leq 5$, $\max\left\{\frac{4(2b-1)^2}{3(b+1)b^2}, \frac{(2b-1)^2(b+1)}{(2b-1)^2(b+1)+3(b-2)^2}\right\} = \frac{(2b-1)^2(b+1)}{(2b-1)^2(b+1)+3(b-2)^2}$, and the incumbent will prefer to deter entry, when entry deterrence is feasible, than to accommodate entry.

Hence, entry deterrence is feasible for a given F when this F satisfies $\frac{(2b-1)^2(b+1)}{(2b-1)^2(b+1)+3(b-2)^2} \leq k = \frac{3(b-1)(b+1)F}{(b-2)^2 S}$, i.e., if F is such that $F \geq \frac{(2b-1)^2(b-2)^2 S}{3(b-1)\left[(2b-1)^2(b+1)+3(b-2)^2\right]}$. When $2 < b \leq 5$ entry will be deterred for those values of F such that

$$\frac{(2b-1)^2(b-2)^2 S}{3(b-1)\left[(2b-1)^2(b+1)+3(b-2)^2\right]} \leq F \leq \frac{(b-2)^2 S}{3(b-1)(b+1)}. \blacksquare$$

6.2 Reaction functions with fixed quality costs

The profit functions of the firms in each market configuration may be obtained substracting fixed quality costs, and the entry cost in the case of the entrant, from the revenue functions presented in the proof of Theorem 1. Let us represent the fixed quality cost function by $c(s)$, with $c'(s) > 0$ and $c''(s) > 0$. The reaction functions corresponding to the quality decisions of firms are:

i) If the market is not covered:

$$\frac{\partial \pi_l^{**}}{\partial s_l} = 0 \Longrightarrow \frac{b^2 s_h^2(4s_h - 7s_l)}{(b-1)(4s_h - s_l)^3} - c'(s_l) = 0 \tag{7}$$

$$\frac{\partial \pi_h^{**}}{\partial s_h} = 0 \Longrightarrow \frac{4b^2 s_h(4s_h^2 - 3s_l s_h + 2s_l^2)}{(b-1)(4s_h - s_l)^3} - c'(s_h) = 0 \tag{8}$$

ii) If the market is covered with corner solution:

$$\frac{\partial \pi_l^c}{\partial s_l} = 0 \Longrightarrow \frac{1}{2} - \frac{s_h^2}{2(b-1)(s_h - s_l)^2} - c'(s_l) = 0 \tag{9}$$

$$\frac{\partial \pi_h^c}{\partial s_h} = 0 \Longrightarrow \frac{b^2}{4(b-1)} - \frac{s_l^2}{4(b-1)(s_h - s_l)^2} - c'(s_h) = 0 \tag{10}$$

iii) If the market is covered with interior solution:

$$\frac{\partial \pi_l^{**}}{\partial s_l} = -\frac{(b-2)^2}{9(b-1)} - c'(s_l) < 0 \tag{11}$$

$$\frac{\partial \pi_h^{**}}{\partial s_h} = 0 \Longrightarrow \frac{(2b-1)^2}{9(b-1)} - c'(s_h) = 0 \tag{12}$$

The equilibrium for each market configuration derives from the corresponding conditions in (7) to (12) (notice that second order conditions are satisfied). Under convex fixed quality costs, the profit function of the low quality firm when the market is not covered and the profit function of the low quality firm when the market is covered with a corner solution are concave with respect to s_l. Moreover, π_l decreases with s_l when the market is covered with an interior solution. Hence, condition (11) implies that s_l will equal the minimum value that permits to attain this market configuration.

The profit function of the high quality firm when the market is not covered and the profit function of the high quality firm when the market is covered with an interior solution are concave with respect to s_h. The feasibility of the configuration of market covered with corner solution requires that the convexity of the cost function compensates the convexity of the revenue function of the high quality firm to obtain a concave profit function of the high quality firm within that configuration. If the profit function of the high quality firm is convex in the configuration of market covered with corner solution, it will never result this configuration at equilibrium.

If there are convex fixed quality costs, the values of b where each market configuration is feasible if the entrant entered as the low quality firm, and the values of b where each market configuration is obtained in that case, depend on the function $c(s)$. Moreover, even for simple fixed quality cost functions, and considering situations where $s_I = S$, it is not possible to obtain neither s_E^{**} nor s_E^c in an explicit way.

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