# Union Formation and Bargaining Rules in the Labor Market*

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**Abstract**

This paper analyzes union formation in a model of bargaining between a firm and several unions. We address two questions: first, the optimal configuration of unions (their number and size) and, second, the impact of the bargaining pattern (simultaneous or sequential). For workers, grouping into several unions works as a price discrimination device which, at the same time, decreases their market power. The analysis shows that optimal union configuration depends on the rules that regulate the bargaining process (monopoly union, Nash bargaining or right to manage).

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# 1. Introduction

The degree of centralization of collective bargaining differs across OECD countries and is one of the most important features of an industrial relations system. In the U.S., Britain, Canada and Japan, among others, unions bargain basically at plant level, at firm-wide level or by industrial sectors. However, in Scandinavian countries, Germany, Austria, France and Italy, collective bargaining is more centralized. Moreover, in the U.S., unlike most continental European countries, trade unions have traditionally been very numerous and have not been linked with political parties. It is worth noting that in recent years the degree of unionization has fallen notably in the U.S. and Britain, while Scandinavian countries have experienced considerable increases in their unionization rates.

The literature on unionized labor markets has developed a wide variety of models of collective bargaining.[1] Recently, several studies have been devoted to explaining how bargaining structures affect wage and employment determination and in some cases the bargaining structure preferred by the participants. These studies have focused mainly on the effect of centralized ( firms bargain with an industry-wide union) versus decentralized bargaining structures ( negotiations take place at firm level with independent unions). Hartog et al. (2002) analyze the firm bargaining regime in the Netherlands, a corporatist country (a bargaining structure in which there is coordination between union federations, employer federations and the national govermment); their results suggest that this feature has small bargaining regime wage effects. In a bargaining model with heterogenous workers and two sectors (union and nonunion), Strand (2003) analyzes why wage dispersion is much greater in countries such as the U.S. and Britain, which have weak and diminishing unions with limited influence, than in Scandinavian countries, where unions are strong and stable. Kiander et al. (2004) study the bargaining regime of 17 OECD countries; their results suggest that the countries where wages are set at firm level use labor taxes less extensively in financing welfare spending than those with centralized

[1] See Oswald (1985) and Farber (1986) for a survey of the most important union models.

bargaining.

The purpose of this paper is to contribute to the understanding of the process of union formation. We analyze workers' incentives to group into teams (unions) to bargain with the firm in different institutional environments. Our focus will be on whether workers are better off when represented by a single union or by several.[2] More precisely, the question will be posed as follows: When a number of workers face a bargaining game with a firm, will they group into a single union, into a few unions, or into many? The answer to this question will depend on the rules of the particular bargaining game determining wage and employment or, in other words, on the institutional environment in the labor market.

We consider, first, a bilateral monopoly in the labor market (*monopoly union model)* in which unions make a "take-it-or-leave-it" offer concerning wages and the firm decides on employment from each union (e.g., Dunlop 1944, and more recently Oswald 1985; Strand 1989; Aronsson et al. 1993; Löfgren 1993). Then we analyze two solutions involving Nash bargaining. In the *Nash bargaining solution* (e.g., MacDonald and Solow, 1981; Barrett and Pattanaik, 1989; Hart and Moutos, 1991), the firm and the union bargain over wages and employment and in the *right to manage model* they bargain only over wages (e.g., Nickell and Andrews, 1983; Oswald, 1985; Manning, 1987; Espinosa and Rhee, 1989; Bughin, 1999; Vannini and Bughin, 2000).[3] First we consider that the firm deals with the different unions sequentially, and then we check how simultaneous negotiations could change the results (e.g., Dobson 1994; Banerji, 2002). For each institutional environment, we derive the equilibrium wage and employment levels for each union and determine the optimal number of unions from the workers' perspective.

Our results indicate that the optimal number of unions depends on the rules of the negotiation process. We show that under the rules of the monopoly union model workers

[2] The formation of groups has been the focus of attention of recent literature in game theory. This analysis has been applied to several economic problems, such as mergers in Cournot markets, partnership formation and international environmental agreements (e.g., Faulí-Oller, 1997, Carraro and Siniscalco, 1998; Espinosa and Inarra, 2001; Espinosa and Macho-Stadler, 2003).

[3] Many empirical studies have tested the appropiate model of trade union behaviour (e.g., Brown and Ashenfelter, 1986; MaCurdy and Pencavel, 1986; De la Rica and Espinosa, 1997).

have no incentive to form several unions: they will all group into a single union (the firm however, would rather have them dispersed into several groups). Nevertheless, when the firm and unions bargain à la Nash to decide on wages and employment, workers are better off if they organize into small unions (and the firm would rather deal with a single union). We also analyze the right to manage model and show that both the firm and the workers would rather have two unions than one.

Labor heterogeneity may be a reason for workers to group into several unions. Byoung Heon Jun (1989) analyzes union formation in a model where workers with different productivity levels may decide to form either a joint union or two separate unions; workers form a joint union when the productivity levels and sizes of the groups are similar. Horn and Wolinsky (1988) show the relationship between the equilibrium pattern of unionization and the degree of substitutability between two types of labor; workers tend to form a single union when the two types are substitutes. In this paper we will assume labor homogeneity to highlight the impact of bargaining rules on the incentives to group into unions. With homogeneous workers our results indicate that dividing into several unions allows workers to price discriminate and extract a higher surplus from the firm, but at the same time it decreases their supply-side market power. We find that in the sequential monopoly union model the market power effect dominates, while in the sequential Nash bargaining model the wage discrimination effect is more important.

Previous literature has developed different models to analyze union configuration. In a model with two groups of workers which are perfect substitutes and where variables are negotiated sequentially, Manning (1987b) obtains that workers would prefer to centralize negotiations (the two unions join forces to bargain with the firm). In a right-to-manage model where bargaining takes place between two firms and a union, Dobson (1994) considers whether the union would prefer to deal with the firms simultaneously or sequentially. Based on Horn and Wolinsky (1988), Naylor (1995) offers an explanation for the empirical observation that firms with several unions and independent bargaining agreements have higher wages than those with a single union.[4]

The rest of the paper is organized as follows. Section 2 presents the model and in Sections 3, 4 and 5 we study the optimal configuration of unions under the rules of the monopoly union model, Nash bargaining and right to manage, respectively. Some extensions of the model are suggested in Section 6. Section 7 concludes.

## 2. The model

In this section we develop a model of union formation in institutional environments where wages and employment are the outcome of sequential bargaining between a firm and several unions.

As in Dobson (1994), there is a rule of order among the unions, and the firm bargains with them sequentially following this rule. Sequential bargaining introduces an asymmetry between unions which are otherwise identical; when bargaining is sequential, agreements reached previously may affect current negotiations. First, we explore the consequences of this asymmetry and in Section 5 we solve the model for simultaneous bargaining.

Unions are assumed to be ex-ante identical, that is, with the same number of members, $m$, and the same objective function; we assume there is a minimum number of members $\underline{m}$. Moreover, we will assume, as in Horn and Wolinsky (1988), that unions do not care about other unions' members, and that they behave independently without cooperating with one another.

### 2.1. Objective function of unions

We assume each union maximizes a representative member's expected utility function.[5] All workers are identical. Denoting by $m$ the size of union membership (number of members), $l$ the union's employment level (number of employed members), $w$ the wage

[4] In recent years several articles have addressed the problem of how to determine and explain union membership (e.g., Moreton, D.R., 1998; Naylor, R. and Cripps, M., 1993; Askildsen, et al. 2002).

[5] An alternative specification could be a Stone-Geary utility function or some other quasi-concave function. See Oswald (1985) for an overview of different union utility functions.

and $S$ the unemploment subsidy (or alternative wage), expected utility for a representative member is:[6]

$$\mu(w,l) = \frac{l}{m}u(w) + \frac{m-l}{m}u(S)$$

We normalize $u(S) = 0$ and assume workers are risk neutral, so that $u(w) = w$. With $T$ unions, the objective function of union $i$ can be expressed as:

$$\mu_i(w_i, l_i) = w_i l_i \qquad i : 1, 2, ..., T \tag{1}$$

where $w_i$ and $l_i$ are the wage and the employment of union $i$.

Our interest in this paper is to determine whether workers would prefer to organize in a single union or in several. Our optimality criterion will be the sum of all workers' utility, although we will also look at the distribution among workers. The expected utility of a worker in union $i$ is $\frac{w_i l_i}{m_i}$. There are $m_i$ members, so the sum of utilities is $w_i l_i$. Adding up the utilities of all workers we get:

$$U_T = \sum_{i=1}^{T} \mu_i = \sum_{i=1}^{T} w_i l_i$$

This expression will be referred to as total union utility. The union configuration maximizing total union utility will be considered optimal from the workers' point of view.

### 2.2. Objective function of firms

We ignore other inputs and consider that the firm maximizes revenue minus labor costs. Thus, when the firm bargains with $T$ unions, its objective is to maximize:

$$\pi_T(w,l) = R(L_T) - \sum_{i=1}^{T} w_i l_i \qquad i : 1, 2, ..., T$$

where $w = (w_0, w_1, ....w_T)$, $l = (l_0, l_1, ....l_T)$, $w_0 \equiv 0$, $l_0 \equiv 0$, $L_T = \sum_{i=1}^{T} l_i$, $R(L_T)$ is the revenue function and the labor demand function is linear.

To simplify the analysis it is assumed that the firm is a price-taker in the product market, selling at a price $p$; $f(L_T) = (zL_T - L_T^2)$, where $z$ is a productivity parameter, is

[6] See Blanchard and Fischer (1989)

the production function. Note that $R(0) = 0$, and that $R(L_T) = pf(L_T)$ is increasing for $L_T < \frac{z}{2}$ , and concave. With this specification, the firm's objective function is given by:

$$\pi_T(w, l) = p\left[zL_T - L_T^2\right] - \sum_{i=1}^{T} w_i l_i \qquad i : 1, 2, ..., T \tag{2}$$

## 3. Sequential monopoly union model

We first study union formation in an environment where a union makes a "take-it-or-leave-it" wage announcement at each stage and the firm decides the employment level for that union. The rules of this wage and employment determination process resemble those of the monopoly union model at each stage of the negotiation process, so we term these rules *sequential monopoly union model.*

We assume that there are $T$ unions, and a rule establishing the order in which unions can make wage offers. Each union $i$ will take into account the result of previous negotiations and its offer $w_i$ will be a function of $(l_0, ...., l_{i-1})$. The timing of the game is as follows. At each stage $i$, the union sets $w_i\,(l_0, ...., l_{i-1})$ and then the firm determines $l_i(w_i, l_0, ..., l_{i-1})$. We solve the game backwards to calculate the subgame perfect equilibrium.

*Stage T:*

At this stage the wage and employment level for union $T$ are decided. Since the firm has already reached agrements with $T-1$ unions, the wages and employment levels agreed upon with other unions will have an influence on the current negotiations.

We compute the firm's best response, $l_T(w_T, l_0, .., l_{T-1})$, to the wage announcement by union $T$. The firm solves:

$$\text{Max}_{l_T \geq 0} \quad \pi_T(w, l)$$

From this problem we obtain the labor demand faced by union $T$:

$$l_T(w_T, l_0, .., l_{T-1}) = \frac{pz - w_T}{2p} - \sum_{j=0}^{T-1} l_j \tag{3}$$

Union $T$ announces $w_T$, taking into account previous negotiations. To determine the optimal wage, $w_T$, union $T$ solves:

$$\begin{array}{ll} \text{Max}_{w_T \geq 0} & \mu_T(w_T, l_T) \\ \text{s.t.} & (3) \end{array}$$

From the first order condition we obtain:

$$w_T(l_0, .., l_{T-1}) = \frac{pz}{2} - p\sum_{j=0}^{T-1} l_j \tag{4}$$

From (3) and (4) we have that the employment level for union $T$ as a function of previous negotiations is given by:

$$l_T(l_0, .., l_{T-1}) = \frac{z}{4} - \frac{1}{2}\sum_{j=0}^{T-1} l_j \tag{5}$$

*Stage i:*

Once wage $w_i$ has been announced, the firm decides $l_i$. Thus, the firm solves the following problem:

$$\begin{array}{lll} \text{Max}_{l_i \geq 0} & \pi_T(w, l) & \\ \text{s.t.} & w_j(l_0,..., l_i) & j : (i+1), (i+2), ..., T \\ & l_j(l_0,..., l_i) & \end{array}$$

From this problem, employment for union $i$ is:

$$l_i(w_i, l_0, ..., l_{i-1}) = \frac{3^{T-i}pz - 4^{T-i}w_i}{3^{T-i}2p} - \sum_{j=0}^{i-1} l_j \tag{6}$$

Union $i$ announces $w_i$. To determine the optimal wage, $w_i$, union $i$ solves:

$$\begin{array}{ll} \text{Max}_{w_i \geq 0} & \mu_i(w_i, l_i) \\ \text{s.t.} & (6) \end{array}$$

From the fist order condition we obtain:

$$w_i(l_0, ..., l_{i-1}) = \frac{3^{T-i}}{4^{T-i}2}pz - \frac{3^{T-i}}{4^{T-i}}p\sum_{j=0}^{i-1} l_j \tag{7}$$

From (6) and (7) we get the employment level of union $i$ as a function of previous negotiations:

$$l_i(l_0, ..., l_{i-1}) = \frac{z}{4} - \frac{1}{2}\sum_{j=0}^{i-1} l_j \tag{8}$$

Table 1 collects the equilibrium results for this negotiation process. There is a relationship between equilibrium levels and the number of unions, $T$, and also between an individual union's utility and the order of that union in the negotiations. We summarize these results as follows.

(a) *Total union utility decreases in the number of unions, $T$.*

(b) *For the firm, equilibrium profits are increasing in $T$.*

(c) *Both efficiency (as measured by total union utility plus profits) and total employment increase with the number of unions, $T$.*

(d) *Utility for union $i$ is negatively related to the total number of unions, $T$, and lower the later it bargains with the firm in the sequential game.*

(e) *Employment level for union $i$ is independent of the total number of unions but lower the later it bargains with the firm in the sequential game.*

(f) *The wage for union $i$ is negatively related both to the number of unions, $T$, and to its index or position in the sequential process, $i$.*

The proof can be found in Appendix A.

As total union utility decreases with the number of unions, $T$, workers have incentives to group into a single union. However, the firm's profits are negatively related to $T$, so the firm would prefer to deal with small unions.[7]

When $T$ increases only the firm benefits from the increase in efficiency. It is also worth noting that the slope of the inverse labor demand function faced by union 1 is lower (in absolute terms) the higher the number of unions, $T$. The intuition behind these results is that as $T$ increases the market gets closer to a competitive labor market (at the limit, market power on the supply side disappears), so efficiency improves, but the side of the market that loses market power is worse off.[8]

Total employment increases with the number of unions, $T$, but each union's employment level is not affected by $T$. When $T$ increases, total employment increases by the same amount as new unions' employment. However, although the employment levels of the previous unions do not change, their wage levels decrease when $T$ increases. In Figure 1 we show how the different variables move with $T$.

On the other hand, one might wonder why the firm hires less workers from those unions (the last ones) that have lower wages. The reason is that employment at the early stages affects the wage at the later ones, so the higher the number of workers hired from the first unions the lower the wages of the last ones.

## 4. Sequential Nash bargaining solution

In this section we study union formation in an environment where the firm and unions bargain over wage and employment.[9] We term the rules of this wage and employment determination process *sequential Nash bargaining solution.*

In this case, at each stage $i$ the firm and union $i$ solve the following problem:

[7] Note that workers are indifferent between one and two unions, but with a small fixed cost of constituting a union we can break this tie.

[8] We can check how as $T$ increases $\theta_T$ is closer to the result that we are going to obtain in the Nash bargaining model, $\theta_T = \frac{1}{4}pz^2$.

[9] See Binmore, Rubinstein and Wolinsky (1986).

$$\begin{array}{lll} \text{Max}_{w_i,\, l_i} & (\mu_i - \mu_i^c)^\gamma (\pi_T - \pi_T^c)^{1-\gamma} & i: 1, 2, ..., T \\ \text{s.t.} & w_j(l_0, ..., l_i) & j: (i+1), (i+2), ..., T \\ & l_j(l_0, ..., l_i) & \end{array}$$

where: $\mu_i$ is the union's objective function as defined in Section 2.1, $\mu_i^c$ is union $i$'s disagreement point, $\pi_T^c$ is the firm's disagreement point, $\gamma$ is unions' bargaining power, and $(1-\gamma)$ is the firm's bargaining power, with $0 < \gamma < 1$.

As unions behave independently we have that $\mu_i^c = 0$ $(i: 1, ...T)$. Moreover, we assume that $\pi_T^c = 0$ $(i: 1, ...T)$, i.e. we assume the firm has to negotiate with all unions.[10] The negotiation process does not proceed to stage $i+1$, and previous agreements are not effective, until an agreement with union $i$ has been reached. Therefore, profits as well as workers' incomes are zero at the disagreement point. This assumption is made to isolate what we call the "price discrimination" effect that appears when there is more than one union. In section 5 we also consider the case in which the firm is not forced to bargain with all unions, and therefore, $\pi_T^c$ is endogenously determined.

**Solving the game**

We solve the game backwards to calculate the subgame perfect equilibrium.

*Stage T:*

At this stage, the firm and union $T$ solve the following problem:

$$\text{Max}_{w_T,\, l_T} \quad (\mu_T)^\gamma (\pi_T)^{1-\gamma} \qquad 0 < \gamma < 1$$

From the first order conditions we obtain the wage and employment level for union $T$ as a function of previous negotiations:

$$l_T = \frac{1}{2} z - \sum_{j=0}^{T-1} l_j \tag{9}$$

[10]See Farber (1986).

$$w_T = \frac{1}{2}\left[\frac{pz^2 - 4\sum_{j=0}^{T-1} w_j l_j}{z - 2\sum_{j=0}^{T-1} l_j}\right]\gamma \tag{10}$$

*Stage $i$:*

At this stage the firm negotiates with union $i$ and both parties simultaneously determine $(w_i, l_i)$, once the firm has reached agreements with unions $1, ...(i-1)$, and therefore, once the wages and employment levels of these unions have already been set. Denote $(w,l)_{i+1}^T$ the equilibrium values in periods $i+1, ...., T$. The firm and union $i$ solve:

$$\begin{array}{lll} \text{Max}_{w_i,\, l_i} & (\mu_i)^\gamma (\pi_T)^{1-\gamma} & \qquad 0<\gamma<1 \\ \text{s.t.} & (w,l)_{i+1}^T & \end{array}$$

From this problem we get:

$$\text{w}_i\text{l}_i = \tfrac{1}{4}\left[pz^2 - 4\sum_{j=0}^{i-1} w_j l_j\right]\gamma \qquad l_i \geq \underline{m} \tag{11}$$

with $i : 1, 2, ...(T-1)$

This outcome implies that there is a continuum of equilibrium solutions at each stage $i$: all those combinations $[w_i,\ l_i]$ that satisfy (11). This is because the firm and union $i$ are indifferent between these combinations. This indifference is obvious in the case of the union. For the firm, it comes from the influence that present negotiations have on later wages and employment. According to (9) and (10), a lower $w_i$ and a higher $l_i$ (which would increase profits at stage $i$ without decreasing union $i's$ utility level) imply a higher $w_T$ and a lower $l_T$, decreasing profits at stage $T$. The result of this trade-off is that the firm is indifferent between combinations $[w_i,\ l_i]$ that satisfy (11). This indifference does not hold at stage $T$ since $[w_T,\ l_T]$ do not affect any future negotiations.

Table 2 shows the equilibrium results for this sequential negotiation process between a firm and several unions. There is a relationship between equilibrium levels and the number of unions, $T$, and also between an individual union's utility and the order of that union in the negotiations, $i$.

Next, we summarize these results.

(a) *Total union utility is increasing in unions' bargaining power,* $\gamma$*, and positively related to the total number of unions,* $T$*.*

(b) *Profits are negatively related to the unions' bargaining power,* $\gamma$*, and to the total number of unions,* $T$*.*

(c) *Neither total welfare nor total employment depends on the total number of unions,* $T$*, or on the unions' bargaining power,* $\gamma$*.*

(d) *Utility for union* $i$ *does not depend on the number of unions but it is negatively related to the order of that union in the sequential bargaining. Moreover, the greater the unions' bargaining power the lower* $\frac{u_i}{u_{i-1}}$*. The effect of* $\gamma$ *on individual union's utility is:*

$$
\begin{array}{llll}
\text{For } i = 1 & \frac{\partial u_i}{\partial \gamma} > 0 & if & 0 < \gamma < 1 \\
 & & & \\
\text{For } i > 1 & \frac{\partial u_i}{\partial \gamma} > 0 & if & 0 < \gamma < \frac{1}{i} \\
 & \frac{\partial u_i}{\partial \gamma} < 0 & if & \frac{1}{i} < \gamma < 1 \\
 & & & \\
 & \frac{\partial^2 u_i}{\partial \gamma^2} < 0 & if & \frac{1}{i} < \gamma < \frac{2}{i} \\
 & \frac{\partial^2 u_i}{\partial \gamma^2} > 0 & if & \frac{2}{i} < \gamma < 1
\end{array}
$$

The proof is given in Appendix B.

Thus, in this case, as total union utility is increasing with the total number of unions, $T$, workers have incentives to group into several unions of the minimum size $\underline{m}$. However, the firm's profits are negatively related to $T$, so the firm would prefer to deal with a single union.

Note that efficiency, as measured by total welfare obtained from sequential negotiations between the firm and unions, is affected neither by the number of unions nor by unions' bargaining power. This result comes from the efficiency of the Nash bargaining solution. The question is how total welfare is shared between the parties. In this case, we can see that the surplus that unions are able to obtain from the firm increases with the number of unions. The intuition behind this result is that when there are several unions they behave as a price discriminating seller. This price discrimination effect drives the main result in this section: it would be optimal for the workers to group themselves into small unions.

The effect of $\gamma$ on an individual union's utility is different for each union $i$. Utility for the first union is always increasing in $\gamma$ while for the rest of the unions it is increasing or independent of the value of $\gamma$. For a given union, an increase in $\gamma$ implies not only an increase in its own bargaining power, but also in other unions' bargaining power, which makes the effect on union $i$'s utility ambiguous. In Figure 2 we show how the different variables move with $T$.

If we compare the results on the optimal configuration of unions that we have obtained in each institutional environment (that is, under the rules of the monopoly union model and under the rules of the Nash bargaining), we see that the optimal number of unions depends on the rules of the negotiation process. This result is collected in the following proposition.

**Proposition 1.** (a) *In the sequential monopoly union model, the optimal union configuration is given by a single union, while in the Nash bargaining model it is given by a number of small unions of size* $\underline{m}$.

(b) *In the sequential monopoly union model, the firm would like to deal with unions of size* $\underline{m}$*, while in the sequential Nash bargaining model, it would like the workers to organize in a single union.*

(c) *In the sequential monopoly union model, both total welfare and total employment are positively related to the number of unions,* $T$*, while in the sequential Nash bargaining model neither of them depends on* $T$*.*

For workers, grouping into several unions works as a price discrimination device which, at the same time, decreases their supply side market power. Proposition 1 states that for the Nash bargaining model the first effect dominates while in the sequential monopoly model the second effect is stronger.

## 5. Sequential Right to Manage Model

In the sequential right to manage model, at each stage $i$ the firm and the union bargain over the wage $w_i$. Then the firm decides on employment from that union $i$. Thus, union $i$ and the firm solve:

$$\begin{array}{lll} \text{Max}_{w_i} & (\mu_i)^\gamma (\pi_T)^{1-\gamma} & \\ \text{s.t.} & l_i^*, & \\ & w_j^*, l_j^* & j = i+1, ...T \end{array}$$

where $l_i^*$, $l_j^*$ and $w_j^*$ denote equilibrium values of the corresponding variables. As in the previous section, we assume that the disagreement point yields zero payoffs.

In contrast to the standard right to manage model, in the sequential version the firm does not decide $l_i$ according to the static labor demand schedule; rather, it takes into account the effect of $l_i$ on all later wages $w_j$ $(j = i+1, ..., T)$ and solves:

$$\begin{array}{lll} \text{Max}_{l_i} & \pi_T(w, l) & \\ \text{s.t.} & w_j^* & j = i+1, ...T \end{array}$$

The sequential right to manage model is difficult to solve and we cannot get explicit solutions for $(w_i, l_i)$ at each stage. Therefore, we solve the case of $T = 2$ and compare the outcome to the equilibrium outcome with only one union.

Table 3 presents the equilibrium results for parameter values $\gamma = 1/2$, $p = 1$, $z = 1$, and two unions.

Table 4 compares the outcome with two unions and a single union and contains the main result of this section. In contrast with the two previous models, here the firm and the workers agree on the best configuration: they would rather have two unions instead of one.

Total welfare is higher when $T = 2$ since employment is higher. This increase in total efficiency is shared by the firm and the unions. The firm benefits from lower wages. Despite the lower wages when $T = 2$, the unions enjoy higher utility due to the higher employment.

## 6. Extensions

In this section we present several extensions; first, we check how simultaneous negotiations can change the above results; and second, we extent the analysis to the Nash bargaining solution with a disagreement point yielding positive profits to the firm.

### 6.1. Simultaneous game

Assume that instead of negotiating with each union in a sequential manner the firm negotiates simultaneously with all unions.

First, we analyze simultaneous negotiations in an environment characterized by the rules of the Nash bargaining.

The timing of the simultaneous Nash bargaining model is given by a single stage at which the problem to be solved by the firm and unions is the following:[11]

$$\text{Max}_{w_i,\, l_i} \quad (\pi_T)^{1-\gamma}(\mu_i)^{\gamma} \qquad \begin{array}{l} i : 1, 2, ..., T \\ 0 < \gamma < 1 \end{array}$$

The results for this case are collected in Table 5. It can be checked that:

(a) *Total union utility increases with the number of unions, $T$.*

(b) *Profits for the firm decrease with the number of unions, $T$.*

(c) *Neither total employment nor total welfare depends on the total number of unions or on the parties' bargaining power.*

[11] Note that we assume the firm has to reach an agreement with all unions for the agreements to be effective.

(d) *Given $T$, all unions obtain the same individual utility level. Moreover, utility for union $i$ is lower the higher the number of unions at the firm, $T$.*

In this case, when the firm negotiates simultaneously with all unions, as in the case of sequential negotiation, total union utility increases in the number of unions, $T$, so workers have incentives to group into several unions of the minimum size. However, as profits decrease in $T$, the firm would prefer workers to group into a single union.

Unlike the result obtained in the sequential Nash bargaining model, all unions obtain the same individual utility level, since there is no asymmetry between unions in this case. Moreover, although total union utility increases in $T$, individual utility is decreasing in $T$.

If the rules of the game were those of the monopoly union model, as in the case of sequential negotiation, the optimal configuration of unions would be given by a single union. In the case of the simultaneous monopoly union model, unions compete à la Bertrand: if a union reduces its wage slightly with regard to the lowest wage announced, it will be the first to attract the labor demand of the firm; unions would go on reducing their wage demand to a wage level $w_R$, which we could identify as the workers' reservation wage or competitive wage level. Thus, the equilibrium wages would be given by:

$$w_i = w_R \qquad \forall\ i : 1, 2, .., T \text{ and } \forall\ T > 1$$

This equilibrium wage, for any number of unions $T > 1$, is always lower than it would be in the case of a single union (the monopoly union model). Thus, the equilibrium outcome in the monopoly union model (one union) always gives workers a higher utility level than the one obtained when there are several unions.

In the case of the simultaneous right to manage model, the firm and each union solve

$$\text{Max}_{w_i} \quad (\pi_T)^{1-\gamma}(\mu_i)^{\gamma}$$

As in the sequential case, the simultaneous right to manage model is difficult to solve and we cannot get explicit solutions for $(w_i, l_i)$ at each stage. Therefore, we solve the cases for $T = 2$ and $T = 3$. The result is shown in Table 6.

Unlike the result obtained in the sequential negotiation, in this case total union utility decreases with the number of unions, $T$, so workers have no incentive to group into several unions. However, as profits increase in $T$, the firm would prefer workers to group into different unions. Total employment and welfare increase with the number of unions. Moreover, when the firm negotiates simultaneously with all unions, each obtains the same individual utility level, since there is no asymmetry between unions in this case.

### 6.2. Disagreement point

In this second extension, we look again at the sequential Nash bargaining model taking into account that a firm may decide not to bargain with a particular union, and then its profits would not be zero but the profits that come from negotiations with the other unions, so:

$$\pi_T^c > 0 \qquad \forall\, T$$

#### 6.2.1. Sequential negotiation

In this case, the firm and union $i$ solve the following problem at each stage $i$ of the sequential negotiation process:

$$\begin{array}{lll} \text{Max}_{w_i,\, l_i} & (\mu_i)^\gamma (\pi_T - \pi^c)^{1-\gamma} & i: 1, 2, ..., T \\ \text{s.t.} & w_j(l_0, ..., l_i) & j: (i+1), (i+2), ..., T \\ & l_j(l_0, ..., l_i) & \end{array}$$

First, we solve the model for any disagreement point, $\pi^c$, and later on we check the case of an endogenous disagreement point.

#### A disagreement point $\pi^c$ independent of the number of unions

The timing of the negotiation process between the firm and unions is the same as in

Section 4. We solve the game backwards to obtain the subgame perfect equilibrium. Table 7 shows the results, which can be summarized as follows:

(a) *The higher the firm's disagreement point the lower both individual union utility and total union utility. This effect reinforces the position of the firm in the negotiation process increasing its profits.*

(b) *Neither total employment nor total welfare depends on the total number of unions; moreover, neither of them depends on unions' bargaining power or the firm's disagreement point.*

(c) *Utility for union* $i$ *is lower the higher the index of the union. The higher the unions' bargaining power the lower* $\frac{u_i}{u_{i-1}}$.

If we compare these results with those obtained in the Nash bargaining model in Section 4, that is, when the firm's disagreement point was zero, we obtain:

$$u_i^d = u_i^{nd} - \pi^c[(1-\gamma)^{i-1}\gamma] \qquad \forall\ i$$

$$U_T^d = U_T^{nd} - \pi^c[1-(1-\gamma)^T] \qquad \forall\ T$$

$$\pi_T^d = \pi_T^{nd} + \pi^c[1-(1-\gamma)^T] \qquad \forall\ T$$

$$\theta_T^d = \theta_T^{nd} \qquad \forall\ T$$

$$L_T^d = L_T^{nd} \qquad \forall\ T$$

where $nd$ denotes the results obtained when $\pi^c = 0$ (Table 2), and $d$ those obtained in this case, with a disagreement point independent of the number of unions.

With a disagreement point $\pi^c$ independent of the number of unions the result is the same as with a zero disagreement point, that is, workers have incentives to group into several unions while the firm would prefer workers to group into a single union.

**An endogenous disagreement point $\pi_T^c$**

Note that the firm's disagreement point is likely to depend on the total number of unions at the firm, $T$. The disagreement point, that is, the profit obtained by the firm if it does not reach an agreement with a union, is the profit obtained by the firm when it negotiates with the rest of the unions, that is:

$$\begin{array}{ll} \pi_T^c = 0 & T = 1 \\ \pi_T^c = \pi_{T-1} & T : 2, 3, ... \end{array}$$

Therefore the firm's disagreement point also depends on both $T$ and $\gamma$. In fact, we have that:

$$\frac{\partial \pi_T^c}{\partial T} = \frac{\partial \pi_{T-1}}{\partial T-1} > 0$$

$$\frac{\partial \pi_T^c}{\partial \gamma} = \frac{\partial \pi_{T-1}}{\partial \gamma} < 0, \text{ for } 0 < \gamma < 1$$

The timing of the negotiation process is the same as in the previous section. Taking into account that we are now dealing with an endogenous disagreement point, $\pi_T^c = \pi_{T-1}$, we solve the game backwards to obtain the subgame perfect equilibrium.

The results derived from the sequential bargaining process as a function of the total number of unions at the firm are shown in Table 8.

Next, we summarize those results.

(a) *Total union utility is lower the higher the total number of unions, $T$, and higher, the higher the unions' bargaining power, $\gamma$.*

(b) *The firm's profits increase with the number of unions, $T$, and decrease with unions' bargaining power, $\gamma$.*

(c) *Utility for union $i$ decreases with the total number of unions and with the union's index. The effect of unions' bargaining power on each union's individual utility level is:*

$$\begin{array}{llll} \textit{For } i = 1 & \frac{\partial u_i}{\partial \gamma} > 0 & if & 0 < \gamma < 1 \\ & & & \\ \textit{For } i > 1 & \frac{\partial u_i}{\partial \gamma} > 0 & if & 0 < \gamma < Z_i \\ & \frac{\partial u_i}{\partial \gamma} < 0 & if & Z_i < \gamma < 1 \end{array}$$

*where $Z_i$ is the positive root, $0 < Z_i < 1$, that solves the equation $\gamma^2 A - \gamma[iB + A] + B = 0$, with $A = \frac{\partial \pi_T^c}{\partial \gamma}$ and $B = (\theta - \pi_T^c)$.*

The proof can be found in Appendix C.

Thus, total union utility is lower the higher the number of unions at the firm, for any value of unions' bargaining power (although the higher $\gamma$, the higher total union utility). This is beacuse the firm's disagreement point is positive and increasing in $T$, so the higher the number of unions the better the position of the firm to carry out negotiation (if it does not reach an agreement with one union, it still has the possibility of reaching agreements with the rest). In contrast to previous cases of Nash bargaining, here workers' market power loss when they break up into several unions dominates the price discrimination effect.

The effect of unions' bargaining power on each union's utility level is different as a function of the index of the union (except the first one, whose utility always increases in $\gamma$). In fact, for each union $i$ there is a value of $\gamma$, $Z_i$, for which its utility is maximized, when $\gamma > Z_i$ utility for union $i$ is negatively related to $\gamma$ and when $\gamma < Z_i$ it is positively related to $\gamma$. The different effect of $\gamma$ on unions' individual utility is due to sequential negotiations. Note also $Z_i$ depends on the number of unions (unlike the result obtained in the Nash bargaining model in Section 4, with zero disagreement point). More precisely, $Z_i$ decreases with the number of unions, $T$ and given $T$, with the index of the union.

If we compare these results with those obtained in Section 4, we can check how in this case the range of values of $\gamma$ for which each union's individual utility increases has broadened (see Figure 3). This is due to a stronger position of the firm in the negotiation process: the first unions that take part in the sequential process cannot extract as much surplus from the firm as they could in the Nash bargaining model with zero disagreement point.

#### 6.2.2. Simultaneous game

The timing of the game is given by a single stage at which the problem to be solved by the firm and unions is the following:

$$\text{Max}_{w_i,\, l_i} \quad (\pi_T - \pi^*)^{1-\gamma}(\mu_i)^\gamma \qquad \begin{array}{l} i:1,2,...,T \\ 0<\gamma<1 \end{array}$$

where $\pi^* = \pi_{T-1}$

The results when $\pi^*$ does not depend on the number of unions, $T$, can be found in Table 10. The conclusions are the same as with a zero disagreement point. Table 9 shows the results with an endogenous disagreement point:

(a) *Total union utility decreases with the number of unions,* $T$.

(b) *Firm's profits increase with the total number of unions,* $T$.

(c) *Neither total employment nor total welfare depends on the total number of unions or on the parties' bargaining power.*

(d) *Given a number of unions at the firm,* $T$*, all unions obtain the same individual utility level. Moreover, individual utility decreases with the number of unions.*

The proof is relegated to Appendix D.

Thus, in this case, as total union utility decreases with the number of unions, $T$, workers have incentives to group into a single union. However, profits increase with $T$, so the firm would prefer workers to group into several unions of the minimum size.

We show the main result of this second extension in the following proposition.

**Proposition 2.** *In an environment characterized by the rules of Nash bargaining with an endogenous disagreement point, the optimal configuration of unions is given by a single union, while the firm would prefer to deal with several unions of the minimum size.*

Making the firm's disagreement point endogenous has such a strong impact on the workers' market power that this effect always dominates the price discrimination effect.

## 7. Concluding Remarks

This paper focuses on the optimal configuration of unions, from the workers' perspective, when bargaining takes place at firm level and there are several unions. We perform this analysis for different institutional environments (rules of the negotiation process), and patterns of bargaining (simultaneous or sequential). It turns out that the rules of the negotiation process make a difference, and whether workers prefer to organize in big or small unions depends on those rules.

In the sequential monopoly union model workers have incentives to constitute a single union to bargain with the firm, while in the Nash bargaining model workers are grouped into small unions. We identify two effects: a wage discrimination effect and a market power effect. Dividing into several unions allows workers to price discriminate and to extract a higher surplus from the firm, but at the same time, it decreases their supply-side market power. We find that in the sequential monopoly union model the market power effect dominates, while in the sequential Nash bargaining model the wage discrimination effect is more important. The terms of the trade-off between the two effects are the same also when negotiations are simultaneous.

In the case of the Nash bargaining model, when we consider an endogenous disagreement point the result changes. In this case, workers would prefer to constitute a single union (with sequential negotiations as well as with simultaneous negotiations).

In this paper we analyze workers' incentives to form big or small unions. A'possible next step is to formalize the question of endogenous union formation, with explicit rules for the process by which workers decide to group into unions. We leave this analysis for further research.

**Appendix A: Sequential Monopoly Union Model**

**(a)** With T unions, total utility can be expressed as:

$$U_T = \sum_{j=1}^{T} \frac{3^{T-j}}{2^{2T+1}} pz^2 \qquad \begin{array}{l} T : 1, 2, ...... \\ j : 1, 2, ..., T \end{array}$$

It can be checked that:

$$\frac{\partial U_T}{\partial T} = \frac{3^T(\ln 3 - 2\ln 2) + 2\ln 2}{4^{T+1}} < 0 \qquad \forall\ T > 1$$

Total union utility with $T$ unions as a function of total union utility with $T-1$ unions is given by:

$$U_T = \frac{1}{4} U_{T-1} + \frac{3^{T-1}}{2^{2T+1}} pz^2$$

**(b)** Firm's profits as a function of $T$ can be written as:

$$\pi_T = \sum_{j=1}^{T} \frac{4^{3-j} 3^{j-1}}{256} pz^2 \qquad \begin{array}{l} T : 1, 2, ...... \\ j : 1, 2, ..., T \end{array}$$

Then:

$$\frac{\partial \pi_T}{\partial T} = -\left(\frac{3}{4}\right)^T \ln\left(\frac{3}{4}\right) \frac{1}{4} pz^2 > 0$$

Profits with $T$ unions as a function of profits with $T-1$ unions are given by:

$$\pi_T = \pi_{T-1} + \frac{4^{3-T} 3^{T-1}}{256} pz^2$$

**(c)** Total welfare can be expressed as:

$$\theta_T = \sum_{j=1}^{T} \frac{4^{3-j} 3}{256} pz^2 \qquad \begin{array}{l} T : 1, 2, ...... \\ \\ j : 1, 2, ..., T \end{array}$$

Thus:

$$\frac{\partial \theta_T}{\partial T} = \left(\frac{1}{4}\right)^{T+1} (\ln 4)\, pz^2 > 0 \qquad \forall\ T$$

Total welfare with $T$ unions as a function of total welfare with $T-1$ unions can be expressed as:

$$\theta_T = \theta_{T-1} + \frac{4^{3-T} 3}{256} pz^2$$

Total employment as a function of the number of unions is given by:

$$L_T = \sum_{j=1}^{T} \frac{2^{j-1}}{2^{T+1}} z \qquad \begin{array}{l} T : 1, 2, ...... \\ j : 1, 2, ..., T \end{array}$$

Hence:

$\frac{\partial L_T}{\partial T} = \frac{\ln 2}{2^{T+1}} > 0$

Total employment with $T$ unions as a function of total employment with $T-1$ unions can be expressed as:

$L_T = \frac{L_{T-1}}{2} + \frac{2^{T-1}}{2^{T+1}} z$

**(d)** The equilibrium outcomes for union $i$ as a function of the levels obtained by the previous union are given by:

$w_i = \frac{2}{3} w_{i-1}$

$l_i = \frac{1}{2} l_{i-1}$

$u_i = \frac{1}{3} u_{i-1}$

Utility for union $i$ as a function of the number of unions can be expressed as:

$u_i = \frac{3^{T-i}}{2^{2T+1}} pz^2 \qquad i : 1, 2, ..., T$

Then:

$\frac{\partial u_i}{\partial T} = \frac{3^{T-i} 2^{2T+1} [\ln 3 - 2 \ln 2]}{(2^{2T+1})^2} pz^2 < 0$

$\frac{\partial u_i}{\partial i} = \frac{-3^{T-i} \ln 3}{2^{2T+1}} pz^2 < 0$

**(e)** The employment level for union $i$ is given by:

$l_i = \frac{2^{T-i}}{2^{T+1}} z \qquad i : 1, 2, ..., T$

and then:

$\frac{\partial l_i}{\partial T} = \frac{\left(2^{T-i} \ln 2\right) 2^{T+1} - \left(2^{T+1} \ln 2\right) 2^{T-i}}{(2^{T+1})^2} z = 0$

$\frac{\partial l_i}{\partial i} = \frac{-2^{T-i} \ln 2}{2^{T+1}} z < 0$

**(f)** The wage for union $i$ as a function of the total number of unions is given by:

$w_i = \frac{3^{T-i} 2^{i-1}}{2^{2T-1}} pz \qquad i : 1, 2, ..., T$

It can be checked that:

$\frac{\partial w_i}{\partial T} = \frac{3^{T-i} 2^{i-1} 2^{T-1} [\ln 3 - 2 \ln 2]}{(2^{2T-1})^2} pz < 0$

$\frac{\partial w_i}{\partial i} = \frac{3^{T-i} 2^{i-1} [\ln 2 - \ln 3]}{2^{2T-1}} pz < 0$

**Appendix B: Sequential Nash bargaining model**

**(a)** Total union utility is given by:

$$U_T = \theta \left[1 - (1-\gamma)^T\right] \qquad \begin{array}{l} T:1,2,... \\ 0<\gamma<1 \end{array}$$

Then:

$$\frac{\partial U_T}{\partial \gamma} = \theta \left[-T(1-\gamma)^{T-1}(-1)\right] > 0$$
$$\frac{\partial U_T}{\partial T} = \theta \left[-(1-\gamma)^T \ln(1-\gamma)\right] > 0$$

**(b)** The firm's profits are given by:

$$\pi_T = \theta(1-\gamma)^T \qquad \begin{array}{l} T:1,2,... \\ 0<\gamma<1 \end{array}$$

Then:

$$\frac{\partial \pi_T}{\partial \gamma} = \theta \left[T(1-\gamma)^{T-1}(-1)\right] < 0$$
$$\frac{\partial \pi_T}{\partial T} = \theta \left[(1-\gamma)^T \ln(1-\gamma)\right] < 0$$

$\pi_T$ as a function of profits with $(T-1)$ unions can be expressed as:

$$\pi_T = \pi_{T-1}(1-\gamma) \qquad \begin{array}{l} T:2,3,... \\ 0<\gamma<1 \end{array}$$

**(c)** Total welfare and total employment as a function of the number of unions are given by:

$$\theta_T = \beta = \tfrac{1}{4}pz^2 \qquad \forall\ T,\ \gamma$$

$$L_T = \tfrac{1}{2}z \qquad \forall\ T,\ \gamma$$

**(d)** Utility for union $i$ as a function of the level obtained by the previous one can be expressed as:

$$u_i = u_{i-1}(1-\gamma) \qquad \begin{array}{l} i:2,...,T \\ 0<\gamma<1 \end{array}$$

Utility for union $i$ as a function of the number of unions is given by:

$$u_i = \tfrac{1}{4}pz^2(1-\gamma)^{i-1}\gamma \qquad \begin{array}{l} i:1,2,...,T \\ 0<\gamma<1 \end{array}$$

Then:

$\frac{\partial u_i}{\partial T} = 0$
$\frac{\partial u_i}{\partial i} = \frac{1}{4} p z^2 (1-\gamma)^{i-1} \gamma \ln(1-\gamma) < 0$

*The effect of* $\gamma$ *on an individual union's utility is:*

| | | | |
|---|---|---|---|
| *For* $i = 1$ | $\frac{\partial u_i}{\partial \gamma} > 0$ | $if$ | $0 < \gamma < 1$ |
| *For* $i > 1$ | $\frac{\partial u_i}{\partial \gamma} > 0$ | $if$ | $0 < \gamma < \frac{1}{i}$ |
| | $\frac{\partial u_i}{\partial \gamma} < 0$ | $if$ | $\frac{1}{i} < \gamma < 1$ |
| | $\frac{\partial^2 u_i}{\partial \gamma^2} < 0$ | $if$ | $\frac{1}{i} < \gamma < \frac{2}{i}$ |
| | $\frac{\partial^2 u_i}{\partial \gamma^2} > 0$ | $if$ | $\frac{2}{i} < \gamma < 1$ |

There is a value of $\gamma$, $\gamma = 1/i$ (which is higher the lower the index of the union) that maximizes utility for union $i$, so when $\gamma < 1/i$, union $i$'s individual utility is positively related to the unions' bargaining power, and when $\gamma > 1/i$, utility for union $i$ is decreasing in $\gamma$, although total union utility is always increasing in $\gamma$. Also, it can be checked that when $\gamma > 2/i$, the negative effect of a rise in $\gamma$ is smaller; the reason is that the higher the unions' bargaining power is, the higher the firm's surplus that the first unions (particularly the first one) can extract, and therefore the lower the firm's surplus to bargain for at the last stages of the sequential bargaining process. Thus, the higher the index of the union the higher the range of values of $\gamma$ for which an increase of $\gamma$ has a negative effect on its utility. In the case of union 2, for example, the optimal situation would be that in which the firm and unions have the same bargaining power. Nevertheless, for the rest of the unions (except the first one), it would be better to have a lower bargaining power than the firm.

**Appendix C: Sequential Nash bargaining solution**

**An endogenous disagreement point** $\pi_T^c$

**(a)** Total union utility is given by:

$$U_T = \theta \prod_{i=1}^{T} \left[1 - (1-\gamma)^i\right] \qquad \begin{array}{l} T : 1, 2, ... \\ 0 < \gamma < 1 \end{array}$$

It can be checked that:

$$\frac{\partial U_T}{\partial T} = \theta \frac{\partial \left( \prod_{i=1}^{T} \left[1-(1-\gamma)^i\right] \right)}{\partial T} < 0$$

$$\frac{\partial U_T}{\partial \gamma} = \sum_{k=1}^{T} \left( \prod_{i=1}^{k-1} [1-(1-\gamma)^i] \right) \left( \prod_{i=k+1}^{T} [1-(1-\gamma)^i] \right) (1-\gamma)^{k-1} k > 0$$

**(b)** As efficiency or total welfare does not depend on the number of unions and it is equal to the sum of total union utility and firm's profits, we can write the firm's profits as:

$$\pi_T = \theta \left[ 1 - \prod_{i=1}^{T} [1-(1-\gamma)^i] \right] \qquad \begin{array}{l} T : 1, 2, ... \\ 0 < \gamma < 1 \end{array}$$

Then:

$$\frac{\partial \pi_T}{\partial T} = -\frac{\partial \prod_{i=1}^{T} \left[1-(1-\gamma)^i\right]}{\partial T} \theta > 0$$

$$\frac{\partial \pi_T}{\partial \gamma} = -\sum_{k=1}^{T} \left( \prod_{i=1}^{k-1} [1-(1-\gamma)^i] \right) \left( \prod_{i=k+1}^{T} [1-(1-\gamma)^i] \right) (1-\gamma)^{k-1} k < 0$$

**(c)** Individual utility for union $i$ is:

$$u_i = \theta(1-\gamma)^{i-1}[1-(1-\gamma)^{T-1}]\gamma^{T-1} \qquad \begin{array}{l} T > 1 \\ 0 < \gamma < 1 \end{array}$$

It can be checked that:

$$\frac{\partial u_i}{\partial i} = \theta(1-\gamma)^{i-1} \ln(1-\gamma)[1-(1-\gamma)^{T-1}]\gamma^{T-1} < 0 \qquad \forall\ \gamma,\ 0 < \gamma < 1$$

$$\frac{\partial u_i}{\partial T} = \left[ \left[ -(1-\gamma)^{T-1} \ln(1-\gamma) \right] \gamma^{T-1} + \left[ 1-(1-\gamma)^{T-1} \right] \gamma^{T-1} \ln \gamma \right] < 0 \qquad \forall\ i$$

Utility for union $i$ as a function of the level obtained by the previous union is given by:

$$u_i = u_{i-1}(1-\gamma) \qquad i: 2, ..., T$$
$$0 < \gamma < 1$$

The effect of unions' bargaining power on each union's individual utility level is:

*For* $i = 1$ $\quad \frac{\partial u_i}{\partial \gamma} > 0 \quad \Rightarrow \quad 0 < \gamma < 1$

*For* $i > 1$ $\quad \frac{\partial u_i}{\partial \gamma} > 0 \quad \Rightarrow \quad 0 < \gamma < Z_i$

$\quad \frac{\partial u_i}{\partial \gamma} < 0 \quad \Rightarrow \quad Z_i < \gamma < 1$

where $Z_i$ is the positive root, $0 < Z_i < 1$, that solves the equation $\gamma^2 A - \gamma[iB + A] + B = 0$, with $A = \frac{\partial \pi_T^c}{\partial \gamma}$ and $B = (\theta - \pi_T^c)$.

We can check that:

$\frac{\partial Z_i}{\partial T} < 0 \qquad i > 1$

$\frac{\partial Z_i}{\partial i} < 0 \qquad i > 1$

With two and three unions at the firm, we obtain the following results:

| | | | $T = 2$ | $T = 3$ |
|---|---|---|---|---|
| $i = 2$ | $\frac{\partial u_i}{\partial \gamma} > 0$ | $\Rightarrow$ | $0 < \gamma < \frac{2}{3}$ | $0 < \gamma < 0,710$ |
| | $\frac{\partial u_i}{\partial \gamma} < 0$ | $\Rightarrow$ | $\frac{2}{3} < \gamma < 1$ | $0,710 < \gamma < 1$ |
| | $\frac{\partial^2 u_i}{\partial \gamma^2} > 0$ | $\Rightarrow$ | $0 < \gamma < \frac{1}{3}$ | $0 < \gamma < 0,44$ |
| | $\frac{\partial^2 u_i}{\partial \gamma^2} < 0$ | $\Rightarrow$ | $\frac{1}{3} < \gamma < 1$ | $0,44 < \gamma < 1$ |
| $i = 3$ | $\frac{\partial u_i}{\partial \gamma} > 0$ | $\Rightarrow$ | - | $0 < \gamma < 0,5657$ |
| | $\frac{\partial u_i}{\partial \gamma} < 0$ | $\Rightarrow$ | - | $0,5657 < \gamma < 1$ |

**Appendix D: Simultaneous Nash bargaining model**

**An endogenous disagreement point** $\pi^*$

**(a)** Total union utility is given by:

$$U_T = \theta \prod_{j=1}^{T} \frac{j\gamma}{(j-1)\gamma+1} \qquad 0 < \gamma < 1$$
$$T : 1, 2, ...$$

Denoting $G_j(\gamma) = \frac{j\gamma}{(j-1)\gamma+1} \qquad j : 1, 2, .., T$

Because of $0 < G_j(\gamma) < 1$, we get that :

$$\frac{\partial U_T}{\partial T} < 0 \qquad \forall\ T, \gamma$$

Total union utility with $T$ unions as a function of total union utility with $(T-1)$ unions is given by:

$$U_T = U_{T-1}\left[\frac{\gamma T}{\gamma(T-1)+1}\right] \qquad 0 < \gamma < 1$$
$$T : 1, 2, ...$$

We can check that total union utility increases with unions' bargaining power:

$$\frac{\partial U_T}{\partial \gamma} = \theta \frac{\partial}{\partial \gamma}[G_1(\gamma)G_2(\gamma)...G_T(\gamma)] = \theta \sum_{i=1}^{T}[G_i^{'}(\gamma) \prod_{\substack{j=1 \\ j\neq i}}^{T} G_j(\gamma)] > 0$$

where: $G_i^{'}(\gamma) = \frac{\partial G_j(\gamma)}{\partial \gamma} = \frac{j}{(\gamma j - \gamma + 1)^2} > 0\ \forall\ j$

**(b)** The firm's profits are given as follows:

$$\pi_T = \theta - U_T = \theta[1 - \prod_{j=1}^{T} \frac{j\gamma}{(j-1)\gamma+1}] \qquad 0 < \gamma < 1$$
$$T : 1, 2, ...$$

Then:

$$\frac{\partial \pi_T}{\partial T} > 0 \qquad \forall\ T, \gamma$$
$$\frac{\partial \pi_T}{\partial \gamma} < 0$$

**(d)** Utility obtained by each union as a function of the number of unions at the firm, is given by:

$$u_i = \frac{1}{4} p z^2 \gamma \prod_{j=1}^{T-1} \frac{j\gamma}{j\gamma+1} \qquad \forall\ i : 1, 2, .., T$$

Denoting $F_j(\gamma) = \frac{j\gamma}{j\gamma+1} \qquad j : 1, 2, .., T-1$

Because of $0 < \gamma < 1$, and therefore $0 < F_j(\gamma) < 1$, we get:

$\frac{\partial u_i}{\partial T} < 0 \qquad \forall\ T,\ i$

It can be checked that, for all $i$, the individual utility level is always increasing on $\gamma$, that is:

$$\frac{\partial u_i}{\partial \gamma} = \theta \prod_{j=1}^{T-1} F_j(\gamma) + \theta\gamma \frac{\partial}{\partial \gamma}[F_1(\gamma)F_2(\gamma)...F_{T-1}(\gamma)] =$$

$$\theta \prod_{j=1}^{T-1} F_j(\gamma) + \theta\gamma \sum_{i=1}^{T-1} [F_i^{'}(\gamma) \prod_{\substack{j=1 \\ j\neq i}}^{T-1} F_j(\gamma)] > 0\ \forall\ i,\ T$$

where: $F_i^{'}(\gamma) = \frac{\partial F_j(\gamma)}{\partial \gamma} = \frac{j}{[j\gamma+1]^2} > 0\ \forall\ j$

Union $i$'s utility with $T$ unions as a function of its utility with $T-1$ is given by:

$$(u_i)_T = (u_i)_{T-1} \left[\frac{\gamma(T-1)}{\gamma(T-1)+1}\right] \qquad 0 < \gamma < 1$$

Thus: $(u_i)_T < (u_i)_{T-1}$

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Table 1

Results for the sequential monopoly union model

| | $T=1$ | $T=2$ | $T=3$ | $T$ |
|---|---|---|---|---|
| $U_T$ | $\frac{1}{8}pz^2$ | $\frac{4}{32}pz^2$ | $\frac{13}{128}pz^2$ | $\sum_{j=1}^{T} \frac{3^{T-j}}{2^{2T+1}}pz^2$ |
| $\pi_T$ | $\frac{1}{16}pz^2$ | $\frac{7}{64}pz^2$ | $\frac{37}{256}pz^2$ | $\sum_{j=1}^{T} \frac{4^{3-j}3^{j-1}}{256}pz^2$ |
| $L_T$ | $\frac{1}{4}z$ | $\frac{3}{8}z$ | $\frac{7}{16}z$ | $\sum_{j=1}^{T} \frac{2^{j-1}}{2^{T+1}}z$ |
| $U_T+\pi_T$ | $\frac{3}{16}pz^2$ | $\frac{15}{64}pz^2$ | $\frac{63}{256}pz^2$ | $\sum_{j=1}^{T} \frac{4^{3-j}3}{256}pz^2$ |
| $w_T$ | $-$ | $-$ | $-$ | $\frac{2^{T-1}}{2^{2T-1}}pz$ |
| $l_T$ | $-$ | $-$ | $-$ | $\frac{1}{2^{T+1}}z$ |
| $u_T$ | $-$ | $-$ | $-$ | $\frac{1}{2^{2T+1}}pz^2$ |

Table 2

Results for the Sequential Nash bargaining model

| | $T=1$ | $T=2$ | $T=3$ | $T$ |
|---|---|---|---|---|
| $U_T$ | $\frac{1}{4}pz^2\gamma$ | $\frac{1}{4}pz^2(2-\gamma)\gamma$ | $\frac{1}{4}pz^2(3-3\gamma+\gamma^2)\gamma$ | $\frac{1}{4}pz^2[1-(1-\gamma)^T]$ |
| $\pi_T$ | $\frac{1}{4}pz^2(1-\gamma)$ | $\frac{1}{4}pz^2(-1+\gamma)^2$ | $-\frac{1}{4}pz^2(-1+\gamma)^3$ | $\frac{1}{4}pz^2(1-\gamma)^T$ |
| $L_T$ | $\frac{1}{2}z$ | $\frac{1}{2}z$ | $\frac{1}{2}z$ | $\frac{1}{2}z$ |
| $\theta_T$ | $\frac{1}{4}pz^2$ | $\frac{1}{4}pz^2$ | $\frac{1}{4}pz^2$ | $\frac{1}{4}pz^2$ |
| $u_1$ | $\frac{1}{4}pz^2\gamma$ | $\frac{1}{4}pz^2\gamma$ | $\frac{1}{4}pz^2\gamma$ | $\frac{1}{4}pz^2\gamma$ |
| $u_2$ | $-$ | $\frac{1}{4}pz^2(1-\gamma)\gamma$ | $\frac{1}{4}pz^2(1-\gamma)\gamma$ | $\frac{1}{4}pz^2(1-\gamma)\gamma$ |
| $u_3$ | $-$ | $-$ | $\frac{1}{4}pz^2(\gamma-1)^2\gamma$ | $\frac{1}{4}pz^2(\gamma-1)^2\gamma$ |
| $u_T$ | $-$ | $-$ | $-$ | $\frac{1}{4}pz^2(1-\gamma)^{T-1}\gamma$ |

Table 3

Sequential Right to Manage with two unions ($\gamma = 1/2, p = z = 1$)

| | Union 1 | Union 2 | Total |
|---|---|---|---|
| Wage | $0,24$ | $0,15$ | $0,222$ |
| Employment | $0,34$ | $0,085$ | $0,425$ |
| Utility | $0,0816$ | $0,01275$ | $0,09435$ |
| Profit | - | - | $0,150025$ |
| Welfare | - | - | $0,244375$ |

Table 4

Sequential Right to Manage Model ($\gamma = 1/2, p = z = 1$)

| | Single union | Two unions |
|---|---|---|
| Weighted average wage $w_m$ | 0,25 | 0,222 |
| Total employment $L_T$ | 0,375 | 0,425 |
| Utility $\mu_T$ | 0,09375 | 0,09435 |
| Profits $\pi_T$ | 0,140625 | 0,150025 |
| Welfare $\theta_T$ | 0,234375 | 0,244375 |

Table 5

Results for the Simultaneous Nash bargaining model

| | $T=1$ | $T=2$ | $T=3$ | $T=4$ | $T$ |
|---|---|---|---|---|---|
| $L_T$ | $\frac{1}{2}z$ | $\frac{1}{2}z$ | $\frac{1}{2}z$ | $\frac{1}{2}z$ | $\frac{1}{2}z$ |
| $\theta_T$ | $\frac{1}{4}pz^2$ | $\frac{1}{4}pz^2$ | $\frac{1}{4}pz^2$ | $\frac{1}{4}pz^2$ | $\frac{1}{4}pz^2$ |
| $\pi_T$ | $\frac{1}{4}(1-\gamma)pz^2$ | $\frac{1}{4}\frac{(1-\gamma)}{\gamma+1}pz^2$ | $\frac{1}{4}\frac{(1-\gamma)}{2\gamma+1}pz^2$ | $\frac{1}{4}\frac{(1-\gamma)}{3\gamma+1}pz^2$ | $\frac{1}{4}\frac{(1-\gamma)}{\gamma(T-1)+1}pz^2$ |
| $U_T$ | $\frac{1}{4}\gamma pz^2$ | $\frac{\gamma}{2(\gamma+1)}pz^2$ | $\frac{3}{4}\frac{\gamma}{2\gamma+1}pz^2$ | $\frac{\gamma}{3\gamma+1}pz^2$ | $\frac{1}{4}\frac{\gamma T}{\gamma(T-1)+1}pz^2$ |
| $u_i$ | $\frac{1}{4}\gamma pz^2$ | $\frac{1}{4}\frac{\gamma}{\gamma+1}pz^2$ | $\frac{1}{4}\frac{\gamma}{2\gamma+1}pz^2$ | $\frac{1}{4}\frac{\gamma}{3\gamma+1}pz^2$ | $\frac{1}{4}\frac{\gamma}{\gamma(T-1)+1}pz^2$ |

Table 6

Results for the simultaneous right to manage model ($\gamma = 1/2, p = z = 1$)

| | $T = 1$ | $T = 2$ | $T = 3$ |
|---|---|---|---|
| Total employment $L_T$ | 0.3750 | 0.3750 | 0.3927 |
| Welfare $\theta_T$ | 0.2343 | 0.2343 | 0.2385 |
| Profits $\pi_T$ | 0.1406 | 0.1406 | 0.1543 |
| Utility $U_T$ | 0.0937 | 0.0937 | 0.0841 |
| Union $i$'s utility $u_i$ | 0.0937 | 0.0468 | 0.028 |
| Union $i$'s employment $l_i$ | 0.3750 | 0.1875 | 0.1309 |
| Union $i$'s wage $w_i$ | 0.25 | 0.25 | 0.2142 |

Table 7

Results for the sequential Nash bargaining model ($\pi^c$ independent of T)

| | $T=1$ | $T=2$ | $T$ |
|---|---|---|---|
| $U_T$ | $\frac{1}{4}\gamma(pz^2-4\pi^c)$ | $-\frac{1}{4}\gamma(\gamma-2)](pz^2-4\pi^c)$ | $\frac{1}{4}[1-(1-\gamma)^T](pz^2-4\pi^c)$ |
| $\pi_T$ | $\frac{1}{4}pz^2(1-\gamma)+\gamma\pi^c$ | $\frac{1}{4}pz^2(-1+\gamma)^2+\gamma(2-\gamma)\pi^c$ | $\frac{1}{4}(1-\gamma)^T(pz^2-4\pi^c)+\pi^c$ |
| $L_T$ | $\frac{1}{2}z$ | $\frac{1}{2}z$ | $\frac{1}{2}z$ |
| $\theta_T$ | $\frac{1}{4}pz^2$ | $\frac{1}{4}pz^2$ | $\frac{1}{4}pz^2$ |
| $u_1$ | $\frac{1}{4}\gamma(pz^2-4\pi^c)$ | $\frac{1}{4}\gamma(pz^2-4\pi^c)$ | $\frac{1}{4}\gamma(pz^2-4\pi^c)$ |
| $u_2$ | $-$ | $-\frac{1}{4}\gamma(-1+\gamma)(pz^2-4\pi^c)$ | $-\frac{1}{4}\gamma(-1+\gamma)(pz^2-4\pi^c)$ |
| $u_T$ | $-$ | $-$ | $\frac{1}{4}\gamma(1-\gamma)^{T-1}(pz^2-4\pi^c)$ |

Table 8

Results for the Nash bargaining model (endogenous disagreement point $\pi_T^c$)

| | $T=1$ | $T=2$ | $T$ |
|---|---|---|---|
| $U_T$ | $\frac{pz^2(1-\gamma)}{4}$ | $\frac{pz^2(1-2\gamma^2+\gamma^3)}{4}$ | $U_T^{nd}[1-(1-\gamma)^{T-1}]\gamma^{T-2}$ |
| $\pi_T$ | $\frac{pz^2(1-\gamma)}{4}$ | $\frac{pz^2(2-\gamma)\gamma^2}{4}$ | $\pi_T^{nd}\left[1-(1-\gamma)^{T-1}\right]\left(\gamma^{T-2}\right)+\frac{pz^2\left[1-\left[1-(1-\gamma)^{T-1}\right]\left(\gamma^{T-2}\right)\right]}{4}$ |
| $L_T$ | $\frac{1}{2}z$ | $\frac{1}{2}z$ | $\frac{1}{2}z$ |
| $\theta_T$ | $\frac{1}{4}pz^2$ | $\frac{1}{4}pz^2$ | $\frac{1}{4}pz^2$ |
| $u_1$ | $\frac{pz^2(1-\gamma)}{4}$ | $\frac{pz^2\gamma^2}{4}$ | $u_1^{nd}[1-(1-\gamma)^{T-1}]\gamma^{T-2}$ |
| $u_2$ | $-$ | $\frac{pz^2(1-\gamma)\gamma^2}{4}$ | $u_2^{nd}[1-(1-\gamma)^{T-1}]\gamma^{T-2}$ |
| $u_T$ | $-$ | $-$ | $u_T^{nd}[1-(1-\gamma)^{T-1}]\gamma^{T-2}$ |

where $nd$ denotes the results obtained when $\pi_T^c=0$ (Table 2).

Table 9

Results for the simultaneous Nash bargaining model

(endogenous disagreement point $\pi_T^c$)

| | $T=1$ | $T=2$ | $T=3$ | $T$ |
|---|---|---|---|---|
| $U_T$ | $\frac{1}{4}pz^2\gamma$ | $\frac{1}{2}pz^2\frac{\gamma^2}{1+\gamma}$ | $\frac{3}{2}pz^2\frac{\gamma^3}{(\gamma+1)(2\gamma+1)}$ | $\frac{1}{4}pz^2\prod_{j=1}^{T}\frac{j\gamma}{(j-1)\gamma+1}$ |
| $\pi_T$ | $\frac{1}{4}pz^2(1-\gamma)$ | $\frac{1}{4}pz^2\frac{(2\gamma+1)(1-\gamma)}{1+\gamma}$ | $\frac{1}{4}pz^2\frac{(1+3\gamma+2\gamma^2-6\gamma^2)}{(\gamma+1)(2\gamma+1)}$ | $\frac{1}{4}pz^2[1-\prod_{j=1}^{T}\frac{j\gamma}{(j-1)\gamma+1}]$ |
| $L_T$ | $\frac{1}{2}z$ | $\frac{1}{2}z$ | $\frac{1}{2}z$ | $\frac{1}{2}z$ |
| $\theta_T$ | $\frac{1}{4}pz^2$ | $\frac{1}{4}pz^2$ | $\frac{1}{4}pz^2$ | $\frac{1}{4}pz^2$ |
| $u_i$ | $\frac{1}{4}pz^2\gamma$ | $\frac{1}{4}pz^2\frac{\gamma^2}{1+\gamma}$ | $\frac{1}{2}pz^2\frac{\gamma^3}{(\gamma+1)(2\gamma+1)}$ | $\frac{1}{4}pz^2\gamma\prod_{j=1}^{T-1}\frac{j\gamma}{j\gamma+1}$ |

where $i:1,2,...,T$.

Table 10

Results for the simultaneous Nash bargaining model

(disagreement point independent of T)

| | $T=1$ | $T=2$ | $T=3$ | $T=4$ | $T$ |
|---|---|---|---|---|---|
| $U_T$ | $\frac{1}{4}\gamma(pz^2-4\pi^c)$ | $\frac{1}{2}\gamma\frac{(pz^2-4\pi^c)}{\gamma+1}$ | $\frac{3}{4}\gamma\frac{(pz^2-4\pi^c)}{2\gamma+1}$ | $\gamma\frac{(pz^2-4\pi^c)}{3\gamma+1}$ | $\frac{1}{4}\gamma\frac{T(pz^2-4\pi^c)}{\gamma(T-1)+1}$ |
| $\pi_T$ | $\frac{4\pi^c\gamma+(1-\gamma)pz^2}{4}$ | $\frac{1}{4}\frac{8\gamma\pi^c+(1-\gamma)pz^2}{\gamma+1}$ | $\frac{1}{4}\frac{12\gamma\pi^c+(1-\gamma)pz^2}{2\gamma+1}$ | $\frac{1}{4}\frac{16\gamma\pi^c+(1-\gamma)pz^2}{3\gamma+1}$ | $\frac{1}{4}\frac{pz^2(1-\gamma)+4\pi^cT\gamma}{\gamma(T-1)+1}$ |
| $L_T$ | $\frac{1}{2}z$ | $\frac{1}{2}z$ | $\frac{1}{2}z$ | $\frac{1}{2}z$ | $\frac{1}{2}z$ |
| $\theta_T$ | $\frac{1}{4}pz^2$ | $\frac{1}{4}pz^2$ | $\frac{1}{4}pz^2$ | $\frac{1}{4}pz^2$ | $\frac{1}{4}pz^2$ |
| $u_i$ | $\frac{1}{4}\gamma(pz^2-4\pi^c)$ | $\frac{1}{4}\gamma\frac{(pz^2-4\pi^c)}{\gamma+1}$ | $\frac{1}{4}\gamma\frac{(pz^2-4\pi^c)}{2\gamma+1}$ | $\frac{1}{4}\gamma\frac{(pz^2-4\pi^c)}{3\gamma+1}$ | $\frac{1}{4}\gamma\frac{(pz^2-4\pi^c)}{\gamma(T-1)+1}$ |

where $i:1,2,..,T$.