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# UNEMPLOYMENT RATES AND POPULATION CHANGES IN SPAIN

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This paper discusses the long run effect of changes in the age distribution of Spanish population on the unemployment rate, disaggregated by sex and age segments in the light of cointegration theory given the non stationarity of the series. Four main results are obtained. First, empirical analysis does not provide a clear scheme concerning the long run relationships between population variables and the specific unemployment rates for different groups. Second, as a first approximation one can detect the existence of, at least, one long run equilibrium relationship in all sex-age groups, except for the ones including the middle aged unemployed female workers and oldest unemployed female workers. Third, a more thorough analysis enables us to justify the existence of such long run relationships for the youngest male workers and middle aged male workers. One cannot argue, however, a joint evolution of population variables and the unemployment rates associated with female workers and to the oldest male workers. Fourth, the short run dynamics of unemployment rates for the youngest (male and female workers) and the middle age male workers is affected by the transitory deviations from these long run relationships. Hence, from an applied economics point of view, the result stresses the likely failure of employment policies which do not take into account the heterogeneous composition of unemployed workers. It stresses the need to design particular policies for specific groups of workers depending on their age and sex characteristics.

JEL classification codes: E24, J19, J64
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## I. Introduction

This paper analyzes the effects of population age structure on unemployment rates disaggregated by age and sex in Spain between the third quarter of 1976 and the fourth quarter of 1998.[1]

Two seemingly unrelated issues such as the unemployment rate and its high degree of persistence in developed countries and the influence of demographic factors on economic activity have received considerable attention in the economic literature. Concerning the former, the focus generally centers on the aggregate unemployment rate, because references dealing with disaggregate unemployment rates by sex and age are scarce. With respect to the latter, there is also an extensive literature, largely motivated by the increasing aging of the population in western economies.[2] For instance, one can find numerous references dealing with the influence that the population structure exerts on education, pension and health expenditures, public expenditures in general and even public revenues.

However, the existing literature on these two issues (unemployment and demographics) considered together is much less extensive, though there are some well-known references along this line. For instance, Welch (1979), Berger (1985) and Fair and Dominguez (1991) have studied the effects of the US age structure of population on the labor market; Zimmermann (1992) and Schmidt (1993) focus on the effects of changes in the age composition on the unemployment rates for specific age segments and sex in Germany.

Focusing on the Spanish economy, Bover and Arellano (1995) discuss the main factors leading to an increase in women's participation rate in Spain during the 1980's; Castillo and Jimeno (1996) find that changes in the active population do not contribute significantly to explain the Spanish

[1] The Spanish National Statistical Institute changed the methodology and the definition of the labor variables in 1999, thus precluding us from using a larger sample size with homogeneous time series.

[2] The population pyramid is changing its structure dramatically. There are two main reasons in the Spanish case. First, a remarkable fall in birth rates since the 1960s: from 21.98 births per thousand in 1964 to 9.37 per thousand in 1997. And, second, the progressive increase in the life expectancy of individuals (see INE, 2000 and Fernández Cordón, 1998).

unemployment rate. There are, however, no empirical works on population aging as a relevant factor to explain the high unemployment rates among some groups of population in Spain and, in particular, among the youngest.[3] Limiting its scope to a specific group, Ahn et al. (2000) may be considered an exception, suggesting the existence of a positive relationship between the relative size of young to old population and their associated unemployment rates.

The present article follows this line of research. Specifically, our purpose is to analyze the influence that changes in the age distribution of population may exert upon unemployment rates, disaggregated by age and sex. In particular, if workers with different ages are not perfect substitutes, a crowding-out effect among the cohorts shows up, thereby larger cohorts are affected by higher unemployment rates (see Ahn et al., 2000). Thus, the largest young population during the sample period (the baby boomers of the 1960s) will exhibit the highest unemployment rate. So, we analyze the possible cointegrating relationships between two demographic variables and unemployment rates. The article may be considered, therefore, as an extension of Zimmermann (1992) and Schmidt (1993).

We obtain four main results. First, the sample evidence does not give precise information about the existence of long-run relationships between population variables and unemployment rates, disaggregated by sex and age. Second, in a preliminary estimation (where we include all the variables that our theoretical model suggests relevant and where we run the corresponding cointegration tests) the analysis suggests the presence of such relationships in all age segments, except for middle aged female workers and oldest female workers. Third, after running a more refined estimation (where we exclude non significant series) a joint long-run evolution of demographic variables and unemployment rates can be justified only for the youngest male workers and for middle aged male workers. Fourth, the short run dynamics of the unemployment rates of the youngest workers (male and female) and the male workers in the middle aged segment is affected by transitory deviations from these long run relationships.

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[3] Unemployment rates in Spain are among the highest in OECD countries, especially for younger (male and female) workers (see OCDE, 1994 and Ahn *et al*., 2000).

Our main conclusion is, therefore, that changes in the age structure of Spanish population do play an important role in the unemployment rates of, at least, some age segments in the case of male workers. In particular, the youngest unemployed male workers and middle aged unemployed male workers seem to be affected by changes in the demographic composition of the labor force. From an applied economics point of view, the result stresses the likely failure of employment policies which do not take into account the heterogeneous composition of unemployed workers. Thus, the natural recommendation that follows for policy makers is the need to design particular policies for specific groups of workers depending on their age and sex characteristics.

The rest of the paper is organized as follows. In Section II a theoretical model is presented to formalize the effects of changes in the age structure of population on disaggregate unemployment rates, the variables in the paper are defined and the analysis of stationarity is executed. In Section III the study of the cointegrating relationships is carried out. Moreover, we run tests concerning cointegrating vectors and weighting matrix. Section IV concludes and draws the main conclusions.

## II. Theoretical Model and Data

### A. The Model

In this subsection, a simple model is presented to formalize the effects of changes in the age structure of population upon unemployment rates. It is a career phase model in essence, and it thus captures the fact that every worker goes through successive career or professional stages during his/her working life (see Welch, 1979 and Schmidt, 1993).[4]

The basic idea behind career phase models is that, at any moment in the individual's working life, the worker goes through a transition process between two phases. In our case, the phase is characterized by the age segment to which the individual belongs. Assuming that these phases are associated with

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[4] This approach is quite similar to that of the life cycle model, because the stage or phase the worker is in is determined by his/her age.

different degrees of expertise in the labor market (the older the worker, the greater his/her expertise, for instance), one can interpret the transition process in terms of acquiring knowledge to increase human capital stock. In particular, we consider three stages in the worker's life: young workers, who we denote by $j = 1$; middle aged workers, denoted by $j = 2$; and elder workers, denoted by $j = 3$.[5] That makes a total of six groups of workers: three age segments for both sexes, the latter denoted by $i = m$ for men, and $i = w$ for women.

Stated briefly, the model assumes that when the worker starts the $j$-th stage (at the age of $x_j$), he/she leaves the previous stage behind, $j - 1$ (except, of course, when $j$ is the first one), and he/she also starts the transition to the next stage, $j + 1$ (except, of course, when $j$ is the last one). More precisely, when in $j$-th stage, the worker devotes a (decreasing with age $x$) share $p_j^i(x)$ of his/her time to this stage; the rest of the time, in an (increasing) share $1 - p_j^i(x)$, is devoted to accumulating human capital to access the following stage, $j + 1$. Formally, $p_j^i(x)$ is such that $dp_j^i(x)/dx < 0$, for $x \in [x_j, x_{j+1}]$, $p_j^i(x_j) = 1$, and $p_j^i(x_{j+1}) = 0$.

Thus, the number of workers offering labor services in $group_j^i$ (i.e., their labor supply) is obtained as:

$$A_j^i = \int_{x_{j-1}}^{x_j} [1 - p_{j-1}^i(x)] a^i(x) dx + \int_{x_j}^{x_{j+1}} p_j^i(x) a^i(x) dx, \tag{1}$$

for $j = 1, 2, 3$, $i = m, w$, where $a^i(x)$ stands for active population of age $x$ and sex $i$.[6] Notice that both individuals in the $j$-th age segment and individuals in

[5] We are certainly aware that, first, these age segments are not necessarily homogeneous and, second, that alternative partitions might also be considered (see, e.g., Ahn *et al*., 2000). One has to set a limit to identify some reasonable groups, however: by specifying these three age segments the model captures those workers who have recently entered the labor market; those who have attained a certain level of qualification; and those with the highest level of expertise in the labor market. Schmidt (1993) considers seven age segments and notes that this excess disaggregation, along with the small sample size, might explain the rejection of the existence of long run relationships between the size of the cohorts and the unemployment rates for some of those age segments.

[6] By construction, for stage $j = 1$, the first term on the right hand side of (1) must be identically zero. That is also the case for the second term in the last stage ($j = 3$ in our case).

the previous age segment *j*-1 offer their labor services to the *j*-th segment. The former do it in a share $p_j^i(x)$, whereas the latter in a $1 - p_{j-1}^i(x)$ share.

Production is represented by an aggregate production function $Y = f(N, Z)$. $N$ denotes the total amount of labor, assumed to be a function of all groups of workers in the economy, i.e., $N = g(N_1^m, N_1^w, N_2^m, N_2^w, N_3^m, N_3^w)$, with $N_j^i$ standing for the number of workers of sex *i* in the *j*-th age segment. Different groups of workers are allowed to exhibit different productivities; otherwise, *g(.)* would be the sum of all the $N_j^i$'s. $Z$ denotes all other production factors, fixed in the short run and representing the state of the economy.

Suppose that for given wages for the three age segments $s_j$ $(j = 1, 2, 3)$ (to be determined later on), and for a given $Z$, firms choose the labor inputs corresponding to the six worker groups, $N_j^i$, so that they solve the following problem:

$$\max_{\{N_j^i\}} f(N, Z) - \sum_{j=1}^{3} s_j (N_j^m + N_j^w) \tag{2}$$

$$\text{s.t. } N = g(N_1^m, N_1^w, N_2^m, N_2^w, N_3^m, N_3^w).$$

We are implicitly allowing for different wages for different age segments (see, for instance, Welch, 1979 and Berger, 1985), although wages are assumed to be the same for both sexes.[7] Nothing precludes the model from allowing for sex discrimination, but we do not include that possibility to keep the exposition as simple as possible.

From that maximization problem, we obtain a system of first order necessary conditions from which labor demand functions for each group are derived:

$$N_j^{i*} = h_j^i(s_1, s_2, s_3, Z), \quad j = 1, 2, 3, \ i = m, w, \tag{3}$$

which depend on wages for all age segments and on economic activity.

Having obtained the number of employed workers and the number of

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[7] Welch (1979) and Berger (1985) find significant effects of the age structure of population upon wages, and show that elderly workers (with a larger expertise) earn higher wages than the younger workers who have entered the labor market recently.

individuals offering their labor services, we can derive the unemployment rates associated with each group:

$$U_j^i = \frac{A_j^i - N_j^i}{A_j^i}, \quad j = 1, 2, 3, \quad i = m, w. \tag{4}$$

Therefore, the $U_j^i$'$s$ are determined by the wages associated with all age segments, the distribution of the active population (by age and sex), and the level of economic activity, $U_j^i = f_j^i[s_1, s_2, s_3, a^i(x), Z]$.

To close the model, suppose finally that there is a union that, representing the whole labor force in the economy, sets the wages for all workers. The union is assumed to have preferences defined over wages and employment for all groups and represented by a utility function *V*. This way, the setting of the wage by the union will require that *V* be maximized subject to firms' labor demand for every group:

$$\max_{\{w_j\}} V(s_1, N_1^m, N_1^w, s_2, N_2^m, N_2^w, s_3, N_3^m, N_3^w) \tag{5}$$

$$\text{s.t. } N_j^{i*} = h_j^i(s_1, s_2, s_3, Z), \quad j = 1, 2, 3, \quad i = m, w.$$

The first order necessary conditions for that problem are given by:

$$\frac{\partial V}{\partial s_j} + \sum_{k=1}^{3} \sum_{i=m,w} \frac{\partial V}{\partial N_k^i} \frac{\partial N_k^i}{\partial s_j} = 0, \quad j = 1, 2, 3. \tag{6}$$

Finally, from (6) we obtain optimal wages as functions of economic activity alone, $s_j^* = s_j(Z)$, $j$ = 1, 2, 3. The same result can be also found in Schmidt (1993).

In sum, the specific unemployment rate for each group is determined by economic activity level (that, at the same time, determines the wages) and the age structure of the active population $(U_j^i = \ell_j^i[a^i(x), Z])$. In our case, we use the *GDP* growth rate (which we denote by *Y*) as a proxy for *Z*.[8] We follow

[8] Alternatively, one could use the aggregate unemployment rate (Schmidt, 1993). Both variables are related, however. The relation, of course, is given by Okun's law.

Zimmermann (1992) when specifying the age structure and approximate it by means of two variables: the *relative* mean age and the relative size of “mature” population (which we denote by $E_i$ and $T_i$, respectively, and define more precisely in the next subsection). An alternative specification would consist in describing this age structure by means of the cohort sizes. This way, one would take into account both the age and the population size (see Schmidt, 1993).[9] Thus, the unemployment rate for every sex and age segment is characterized by:

$$U_j^i = \ell^i{}_j(E_i, T_i, Y), \quad j = 1,2,3, \quad i = m, w. \tag{7}$$

Now, our concern is to confirm whether such rates are really determined by their fundamentals, at least in the long run, for the Spanish economy.[10]

## B. Definition of Variables

The data sources used are the *Active Population Survey* and the *Quarterly National Accounts*, both collected by the National Statistical Institute (INE, 2000). The data are quarterly and the sample size extends from 1976:3 to 1998:4 (*i.e.*, from the 3rd quarter of 1976 to the 4th quarter of 1998). The variables are defined below:

- Unemployment rates, $U_j^i$, previously defined in (4). $j$ = 1 refers to workers between 16 and 29 (both included), $j$ = 2 refers to workers from 30 to 44 (both included), and $j$ = 3 refers to workers of 45 and over; $i$ = $m$ refers to male and $i$ = $w$ refers to female workers.
- Relative size of mature population, $T_i$. People between 40 and 65 (both inclusive) are considered mature, where 65 is the legal retirement age. $T_i$ is defined as the ratio of the size of mature population to the size of population

[9] The author does not assure, however, that cohort sizes are an accurate measure to explain the unemployment rate specific to each age segment.

[10] Before leaving the subsection, two assumptions implicit in the model should be noted. First, we ignore the presence of migratory flows and therefore assume that migration has not been a determining factor for the Spanish age structure of population. Second, the transition process between the phases in the individual’s career is exogenous and, therefore, cohort size independent.

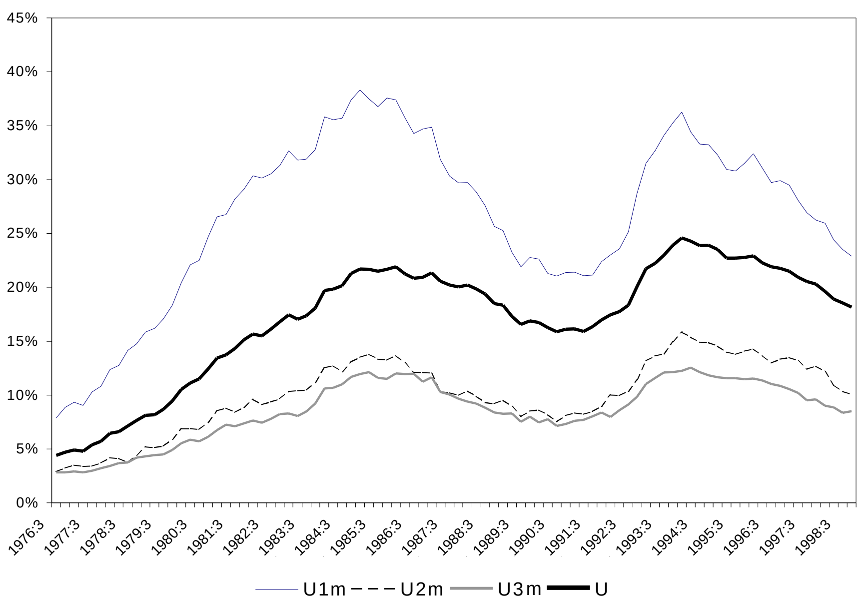
between 16 and 39. The rest of the population is left out due to its negligible participation in the labor force.

- Relative mean age of mature population, $E_i$. It is defined as the ratio of the mean age among mature population to the mean age among population between 16 and 39.

- Growth rate of GDP, *Y*. The quarterly growth rate of real GDP, base 1986. Including this variable allows us to capture the influence of the business cycle on unemployment rates.

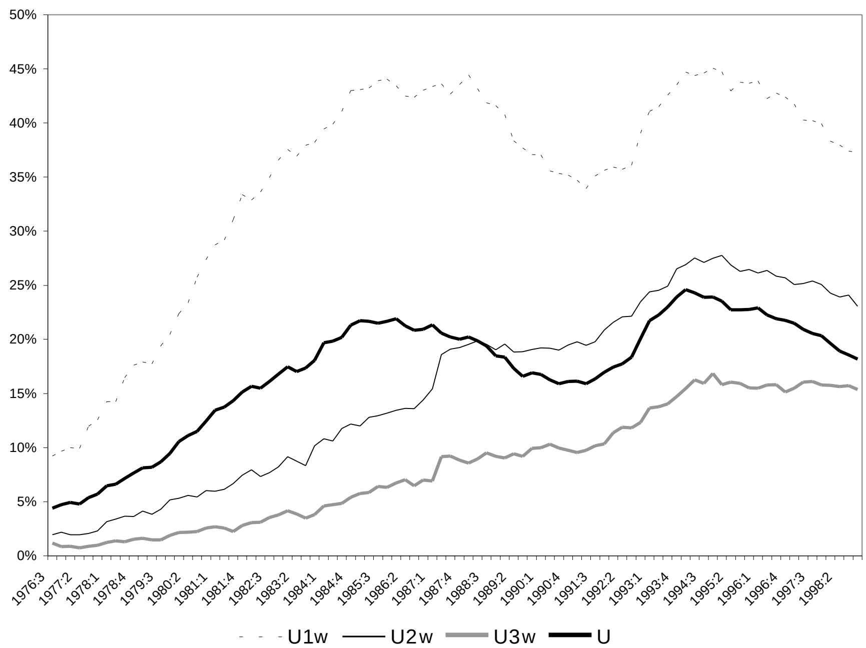
Disaggregate unemployment rates by sex and age, along with the aggregate unemployment rate *U*, are plotted in Figure 1 (for male workers) and in Figure 2 (for female workers).

**Figure 1: Unemployment Rates (Male Workers)**




It is also noticeable that an analysis of unemployment series shows, first, higher rates for female workers than for male workers and, second, higher rates for the youngest than for the oldest (for both male and female workers). We believe that these differences motivate the need to decompose the aggregate unemployment rate by age and sex.

**Figure 2: Unemployment Rates (Female Workers)**

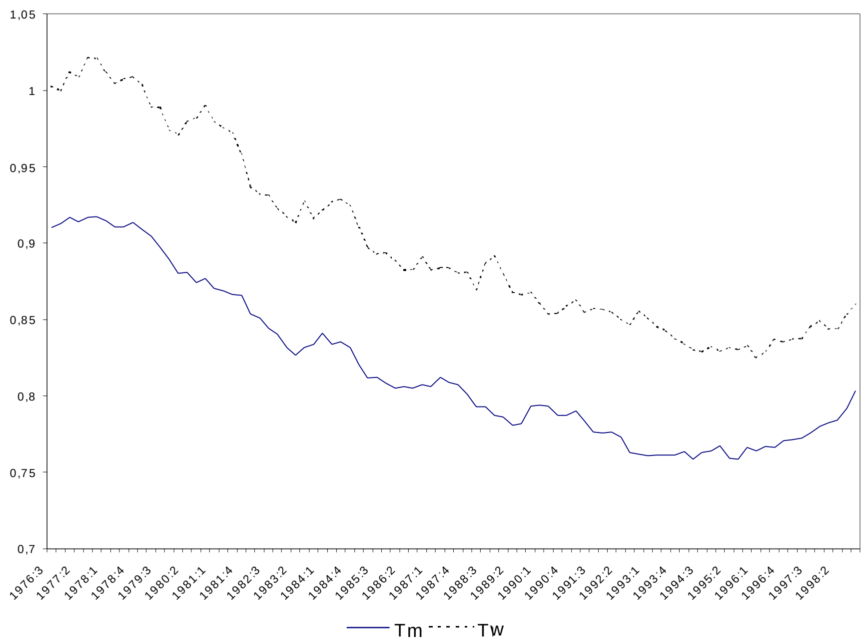



Visual inspection of the series suggests that while all are countercyclical, some are more pronounced (the aggregate rate, the male rates and the rate of the youngest female workers). In particular, the unemployment rates of the older than 30 female workers ($U_2^w$ and $U_3^w$) seem to show different patterns from the other series. This will be confirmed in Section III since the results that we obtain for these two groups will be different from the results for the rest of groups.

Finally, concerning the population variables, the pattern in the series ($E_i$, $T_i$) (the relative mean age and the relative size of mature population, respectively) is similar in both sexes (see Figures 3 and 4). For instance, $T_i$ shows, in general, a decreasing trend until the mid-90s, reflecting the pressure exerted by those born during the baby boom. From about 1994, however, $T_i$ suffers a change in its behavior, showing smooth stability or even a slight increase. The smaller number of births since the late 70s begins to exert its effect on the youth population as this generation reaches working age.

Likewise, one can infer a smooth negative relationship between the relative size of mature population and the unemployment rates for the period 1976-

**Figure 3: Size of Population**



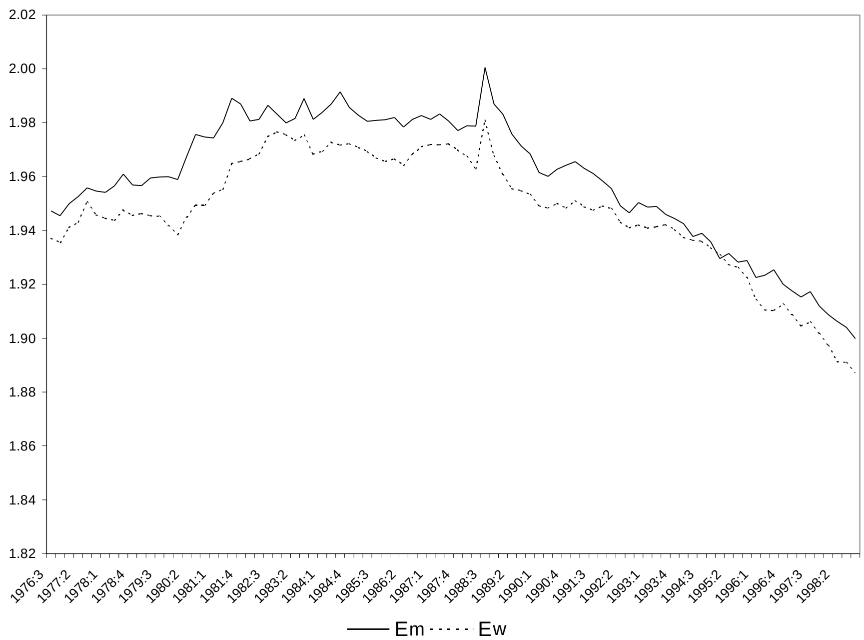

1998. The unemployment rates for each specific age segment, in general, tend to increase during the first half of the sample (from 1976 to about 1986) and to decrease from the mid-90s. The opposed pattern is shown in the relative size of mature population, both male and female: a decrease at first and a change later in its behavior during the second half of the 90s. Therefore, one can expect a negative sign for the effect of $T_i$ on $U_j^i$.

Finally, it should be noticed that none of the series seems to have a constant mean. Therefore, they are expected to be non-stationary or generated by a stochastic process with (at least) one unit root. If the series turned out to be non-stationary, then traditional estimation methods might not be appropriate. In that case, cointegration theory will provide the right methodology. This is the issue dealt with in the two following subsection.

## C. Stationarity and Cointegration Analysis

After running several statistical procedures (more precisely, the Dickey

**Figure 4: Age of Population**




and Fuller test, the Phillips and Perron test, and the Kwiatkowski, Phillips, Schmidt and Shin test), our expectations are confirmed.[11] Nevertheless, some discrepancy in the results does not allow us to conclude about the order of integration of all variables. There exists evidence of stationarity of the first differences of the demographic variables (*E*, *T*), the GDP growth rate (*Y*), the unemployment rates of elderly workers regardless of their sex $(U_3^m$ and $U_3^w)$, and middle aged female workers $(U_2^w)$. This implies that all relevant information about the present and the future evolution of the series is summarized in their past behavior.

The results are not that conclusive, however, for the unemployment rates of workers in the first age segment, both male and female $(U_1^m$ and $U_1^w)$, and male workers in the second age segment $(U_2^m)$.[12] In any case, and as we

[11] The detailed results of the tests are not shown here for the sake of space saving, but can be obtained from the authors upon request.

[12] Zimmermann (1992) found that, regardless of age and sex of individuals, German unemployment rates had a unique unit root. Likewise, he showed the non stationarity of

mentioned above, given that the series are *not* stationary, even though we have not found out with certainty whether they are I(1) or I(2), traditional estimation methods may not be appropriate.

Cointegration theory provides the appropriate framework for analysis because it facilitates the estimation of the long run parameters in an equation with non-stationary series. Likewise, the short run relationships can be studied by means of the error correction model. In other words, it also allows us to merge short run dynamics with the long run equilibrium.

Let us consider $U_j^i$ the variable to be explained and $E_i$, $T_i$ and $Y$ the explanatory variables ($i = m, w$ and $j = 1, 2, 3$). Assuming a labor market where workers with different ages are not perfect substitutes, one may conclude the existence of a crowding-out effect among the cohorts.[13] In other words: larger cohorts are affected by higher unemployment rates (see Ahn et al., 2000). So, for example, the largest youth population during the sample period (due to the baby boom of the 1960s) will exhibit the highest unemployment rate. One could expect, therefore, a negative relationship between the relative size of mature population and the unemployment rate.[14]

We test the presence of long run equilibrium relationships among the variables using two alternative approaches: the one proposed by Engle and Granger (1987) and the one proposed by Johansen (1988). We use these two different methods because upon following the first one some counter-intuitive results show up.

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the demographic variables, the evidence being much clearer for the relative mean age than for the relative mean size of the population.

[13] This assumption follows the lines of the vintage human capital literature. Individuals of different cohorts may exhibit different levels of human capital for two reasons: first, their accumulated on-the-job expertise is different; and, second, individuals born at different points in time enjoy different levels of aggregate knowledge that affects their own private human capital production as an externality.

[14] This relationship is positive in Zimmermann (1992), because the definition of the demographic variables is exactly the contrary: young relative to mature population. Schmidt (1993) and Ahn et al. (2000) have also found that increases in the cohort size are associated with increases in the unemployment rates. We shall endeavor to confirm this hypothesis using Spanish economy data for 1976:3-1998:4.

## III. Results

### A. Long-run Relationship by Sex-Age Groups: Engle and Granger Cointegration

Concerning Engle and Granger's procedure, and as in Zimmermann (1992) and Schmidt (1993), we estimate a regression model, for each specific group, among the levels of the variables:

$$U^{i}_{jt} = b_0 + b_1 E_{it} + b_2 T_{it} + b_3 Y_t + \varsigma_t, \tag{8}$$

for $i = m, w$, $j = 1, 2, 3$ and $t = 1, 2, ..., T$. The OLS estimator is adequate for this purpose, as it is super-consistent, but the common inference procedures are not appropriate because their probability distribution is not a normal distribution. The results (for both male and female workers) are shown in Table 1.

The results obtained are contrary to expectations: the presence of a unit root in the residuals is not rejected in any of the models. Consequently, there is no long run equilibrium relationship between the unemployment rate, the variables representing the age structure of population and the GDP growth rate. In other words, the evidence of a long run joint evolution among the implicated variables cannot be assured. This conclusion can be applied to all the groups studied, regardless of their sex and age.[15]

These results can be interpreted in several different ways. First, we have considered that all series are I(1) and the existence of cointegration is conditioned to stationary residuals. This could be different if some series were I(2), as might be the case with $U^m_1$, $U^w_1$ and $U^m_2$. If so, we would be including variables with different orders of integration and we would be trying

[15] With a view to analyzing the robustness of these results, we considered some alternative specifications. In particular, we estimated some models which include only one variable representing the demographic structure: either the age or the population size. As a third alternative, we have added a trend to the equation (8). (See Schmidt, 1993 and Ahn et al., 2000). The same conclusion is obtained in all cases: it seems that these series, therefore, do not follow a common pattern in the long run.

**Table 1. Cointegration Test: Engle and Granger's Approach. Male and Female Workers**

| Regressor | Variable to be explained | | | | | |
|---|---|---|---|---|---|---|
| | Male workers | | | Female workers | | |
| | $U_1^m$ | $U_2^m$ | $U_3^m$ | $U_1^w$ | $U_2^w$ | $U_3^w$ |
| Constant | -1.562 | 0.350 | 0.009 | -1.496 | 2.509 | 2.263 |
| | (-3.53) | (2.10) | (0.07) | (-3.91) | (14.74) | (22.45) |
| $E_m / E_w$ | -1.450 | 0.117 | 0.248 | 1.713 | -0.622 | -0.783 |
| | (6.12) | (1.30) | (3.53) | (8.30) | (-6.78) | (-14.41) |
| $T_m / T_w$ | -1.212 | -0.574 | -0.002 | -1.649 | -1.267 | -1.267 |
| | (-10.58) | (-13.29) | (-1.91) | (-21.16) | (-36.55) | (-36.55) |
| $Y$ | -0.009 | -0.004 | -0.002 | -0.002 | -0.0004 | -0.0004 |
| | (-2.82) | (-2.88) | (-1.91) | (-0.74) | (-0.32) | (-0.58) |
| $ADF_{res}$ | -2.767 | -2.852 | -2.546 | -1.601 | -3.821 | -4.064$^{***}$ |

Note: $ADF_{res}$ is the statistic to test the null hypothesis of a unit root in the residuals (critical values in Engle and Yoo, 1987). $^{***}$ represents rejecting $H_0$ at the 10% level.

to explain an I(2) variable by using I(1) variables. Second, some omitted variable in the regression might affect the equilibrium, so that the error term would make the residual non-stationary. Third, another justification may be found in some variables of the model, particularly those which vary slowly. Variability of data referring to population variables is very low in small samples, which implies a lack of precision in the estimation.

## B. Long-run Relationship by Sex-Age Groups: Johansen Cointegration

As for Johansen's approach (see Johansen, 1988, 1991, and Johansen and Juselius, 1990, 1992), let $\mathbf{X}_t$ be a vector, $\mathbf{X}'_t = \left[U_{jt}^i \; E_{it} \; T_{it} \; Y_t\right]$, whose dimension, $k$, is given by the number of series in the model (in our case,

$k$ = 4). We assume that $\mathbf{X}_t$ admits a representation of error correction model (ECM):

$$\Delta \mathbf{X}_t = \Pi \mathbf{X}_{t-1} + \sum_{\iota=1}^{p-1} \Gamma_\iota \Delta \mathbf{X}_{t-\iota} + \mu + \eta_t, \tag{9}$$

where $\mu$ is a vector of constants and $\eta_t$ a ($k$-dimensional) independent stochastic process, normally distributed, with zero mean and covariance matrix $\Omega$. $\Pi$, a $k \times k$ matrix, is the long run impact matrix, and it is this matrix that provides information about the existence of cointegrating relationships among the series in $\mathbf{X}_t$. If $\Pi$ has reduced rank equal to $r$ ($r < k$), we can conclude that there exist $r$ long run equilibrium relationships, and $\Pi$ can be written as $\Pi = \alpha \beta'$. Then, $\beta$ is a $k \times r$ matrix that contains the $r$ cointegrating vectors and $\boldsymbol{\alpha}$, with a similar order, includes the short run adjustment coefficients of each of the variables to the long run equilibrium.[16]

The determination of the possible existence (and, where appropriate, the number) of some long run equilibrium relationships among the series in $\mathbf{X}_t$ is made using the *trace*, *Tr*, and the maximal eigenvalue statistics, $\lambda_{\max}$. We carried out the tests assuming first that the model contains a deterministic linear trend ($Tr$, $\lambda_{\max}$) and, later, assuming that this trend is not present ($Tr^*$, $\lambda^*_{\max}$).[17] Following Johansen (1991), we use a statistic defined as $Tr^*(r)$ -$Tr(r)$ to test for the null hypothesis of absence of a linear trend.[18] The results are shown in Tables 2 and 3.

Consider first male workers (Table 2). We began by analyzing the youngest unemployed male workers $U_1^m$, $E_m$, $T_m$ and $Y$, assuming that all variables are $I(1)$.

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[16] We have checked that the assumptions in the model are satisfied: first, independence across time of the error term and, second, normality of the errors. The latter is not essential because the maximum likelihood method is robust to the absence of this property (see Gonzalo, 1994).

[17] Critical values for their asymptotic distributions can be found in Johansen and Juselius (1990).

[18] To find the number of long run relationships among the variables, we also follow a graphical analysis of the possible cointegrating relationships. The representations are not included here for reasons of space.

**Table 2. Cointegration Test (Male Workers): Johansen's Approach**

**Presence of a Linear Trend (Unemployment Rate: Levels)**

| | $U_1^m(p^*=6)$ | | $U_2^m(p^*=6)$ | | $U_3^m(p^*=6)$ | |
|---|---|---|---|---|---|---|
| $H_0$ | $T_r$ | $\lambda_{\max}$ | $T_r$ | $\lambda_{\max}$ | $T_r$ | $\lambda_{\max}$ |
| $r = 0$ | 77.56 $^{*}$ | 35.19 $^{*}$ | 73.72 $^{*}$ | 32.86 $^{*}$ | 71.88 $^{*}$ | 35.07 $^{*}$ |
| $r \leq 1$ | 42.37 $^{*}$ | 21.52 $^{**}$ | 40.86 $^{*}$ | 23.71 $^{**}$ | 36.81 $^{*}$ | 20.48 $^{***}$ |
| $r \leq 2$ | 20.84 $^{*}$ | 13.07 $^{***}$ | 17.15 $^{**}$ | 10.97 | 16.32 $^{**}$ | 11.07 |
| $r \leq 3$ | 7.77 $^{*}$ | 7.77 $^{*}$ | 6.17 $^{**}$ | 6.17 $^{**}$ | 5.25 $^{**}$ | 5.25 $^{**}$ |

**Absence of a Linear Trend (Unemployment Rate: Levels)**

| | $U_1^m(p^*=6)$ | | $U_2^m(p^*=6)$ | | $U_3^m(p^*=6)$ | |
|---|---|---|---|---|---|---|
| $H_0^*$ | $Tr^*$ | $\lambda_{\max}^*$ | $Tr^*$ | $\lambda_{\max}^*$ | $Tr^*$ | $\lambda_{\max}^*$ |
| $r = 0$ | 77.84 $^{*}$ | 29.66 $^{**}$ | 75.84 $^{*}$ | 31.76 $^{**}$ | 72.21 $^{*}$ | 30.89 $^{**}$ |
| $r \leq 1$ | 48.18 $^{*}$ | 22.81 $^{**}$ | 44.07 $^{*}$ | 21.24 $^{***}$ | 41.32 $^{*}$ | 21.04 $^{***}$ |
| $r \leq 2$ | 25.36 $^{*}$ | 17.18 $^{**}$ | 22.83 $^{**}$ | 15.56 $^{***}$ | 20.27 $^{**}$ | 13.21 |
| $r \leq 3$ | 8.18 $^{***}$ | 8.18 $^{***}$ | 7.27 | 7.27 | 7.06 | 7.06 |

**Presence of a Linear Trend (Unemployment Rate: First Differences)**

| | $\Delta U_1^m(p^*=6)$ | | $\Delta U_2^m(p^*=5)$ | |
|---|---|---|---|---|
| $H_0$ | $T_r$ | $\lambda_{\max}$ | $T_r$ | $\lambda_{\max}$ |
| $r = 0$ | 83.95 $^{*}$ | 47.15 $^{*}$ | 77.52 $^{*}$ | 39.82 $^{*}$ |
| $r \leq 1$ | 36.81 $^{*}$ | 23.28 $^{**}$ | 37.70 $^{*}$ | 24.95 $^{**}$ |
| $r \leq 2$ | 13.52 $^{***}$ | 9.43 | 12.75 | 9.39 |
| $r \leq 3$ | 4.09 $^{**}$ | 4.09 $^{**}$ | 3.36 $^{***}$ | 3.36 $^{***}$ |

**Table 2. (Continued) Cointegration Test (Male Workers): Johansen's Approach**

**Absence of a Linear Trend (Unemployment Rate: First Differences)**

| | $\Delta U_1^m (p^* = 6)$ | | $\Delta U_2^m (p^* = 5)$ | |
|---|---|---|---|---|
| $H_0^*$ | $Tr^*$ | $\lambda^*_{max}$ | $Tr^*$ | $\lambda^*_{max}$ |
| $r = 0$ | 94.69 $^{*}$ | 47.39 $^{*}$ | 83.26 $^{*}$ | 40.00 $^{*}$ |
| $r \leq 1$ | 47.29 $^{*}$ | 26.49 $^{*}$ | 43.26 $^{*}$ | 25.07 $^{**}$ |
| $r \leq 2$ | 20.81 $^{**}$ | 15.49 $^{***}$ | 18.19 $^{***}$ | 12.07 |
| $r \leq 3$ | 5.31 | 5.31 | 6.12 | 6.12 |

Note: In the first row both the unemployment rate (levels or first differences) and the number of lags, $p^* = p - 1$, included in the corresponding *ECM* are shown. $^{*}$, $^{**}$ and $^{***}$ denote the rejection of $H_0$ at the 1%, 5% and 10% level, respectively. *Tr* and $\lambda_{max}$ ($Tr^*$ and $\lambda^*_{max}$) are the trace and maximal eigenvalue statistics in a model with deterministic linear trend (without deterministic linear trend). *r* denotes the number of cointegrating vectors.

The existence of four cointegrating relationships is not rejected, thus reflecting the stationarity of the four variables.[19] This result differs from the one obtained in the previous section. There, we showed that $U_1^m$ contained at least one unit root. This may be the problem: $U_1^m$ has been included in the model as a stationary variable in first differences, but this might not be the case. Therefore, this result gives rise to a second analysis in which we consider that $U_1^m \sim I(2)$. In this case, three of the four statistics ($T_r$, $\lambda_{max}$ and $\lambda^*_{max}$) do not reject the existence of two possible cointegrating relationships at the 5% level. A different conclusion is derived from the $Tr^*$ statistic, because $H_0^* : r \leq 2$ is rejected at that level. This rejection is, nevertheless, marginal, as the values of the statistics are almost equal to the critical values tabulated (20.81 vs. 20.17, at the 5% significance level). So, we consider the existence of two long run relationships

[19] The exception arises from the analysis of $\lambda_{max}$ for the null of $H_0 : r \leq 2$ which is not rejected at the 5% level, although it is at the 10% level. The same occurs when we analyze $H_0^* : r \leq 3$ in a model without a linear trend.

**Table 3. Cointegration Test (Female Workers): Johansen's Approach**

**Presence of a Linear Trend**

| | $U_1^w(p^*=6)$ | | $U_2^w(p^*=3)$ | | $U_3^w(p^*=4)$ | |
|---|---|---|---|---|---|---|
| $H_0$ | $T_r$ | $\lambda_{\max}$ | $T_r$ | $\lambda_{\max}$ | $T_r$ | $\lambda_{\max}$ |
| $r = 0$ | 60.32 * | 34.83 * | 43.15 | 18.12 | 41.02 | 17.89 |
| $r \leq 1$ | 25.49 | 15.39 | 25.03 | 13.71 | 23.13 | 12.11 |
| $r \leq 2$ | 10.10 | 8.14 | 11.31 | 7.87 | 11.02 | 8.40 |
| $r \leq 3$ | 1.96 | 1.96 | 3.44 *** | 3.44 *** | 2.62 | 2.62 |

**Absence of a Linear Trend**

| | $U_1^w(p^*=6)$ | | $U_2^w(p^*=3)$ | | $U_3^w(p^*=4)$ | |
|---|---|---|---|---|---|---|
| $H_0^*$ | $Tr^*$ | $\lambda_{\max}^*$ | $Tr^*$ | $\lambda_{\max}^*$ | $Tr^*$ | $\lambda_{\max}^*$ |
| $r = 0$ | 74.90 * | 38.50 * | 57.31 ** | 28.29 ** | 52.12 *** | 21.71 |
| $r \leq 1$ | 36.39 ** | 15.69 | 29.02 | 14.04 | 30.40 | 16.36 |
| $r \leq 2$ | 20.70 ** | 12.33 | 14.97 | 9.72 | 14.04 | 9.85 |
| $r \leq 3$ | 8.37 *** | 8.37 *** | 5.25 | 5.25 | 4.19 | 4.19 |

Note: In the first row the unemployment rate and the number of lags, $p^* = p - 1$, included in the corresponding *ECM* are shown. $^*$, $^{**}$ and $^{***}$ denote the rejection of $H_0$ at the 1%, 5% and 10% level, respectively. $T_r$ and $\lambda_{\max}$ ($Tr^*$ and $\lambda_{\max}^*$) are the trace and maximal eigenvalue statistics in a model with deterministic linear trend (without deterministic linear trend). $r$ denotes the number of cointegrating vectors.

in a model without a linear trend.[20] When the unemployed male workers in the middle aged $U_2^m$ are included in $\mathbf{X}_t$, we obtain the same result. Finally, we

[20] With regard to the trend, some procedures can be used to detect its presence. First, by testing $H_0 : r \leq 2$ and $H_0^* : r \leq 2$. Neither of the two hypotheses is rejected, which does not imply the presence of a linear trend. Second, $Tr^* - Tr = 7.29 > \chi^2_{(2,0.05)}$, thereby rejecting the null hypothesis of absence of a linear trend. Given that both results are contradictory, the final decision was based on the graphical analysis of the residuals: the model without a linear trend provides some more satisfactory graphic representations in stationarity terms.

consider unemployed male workers over 44, $U_3^m$. There exists ambiguity when we choose the number of cointegrating vectors, because both testing procedures produce different results. The final conclusion has been based on the maximal eigenvalue statistics.[21] As a consequence, we conclude with the existence of only one long run relationship in a model without a linear trend.

Considering now female workers (Table 3). Starting by young unemployed female workers $U_1^w$, here we obtain the same result we did for $U_3^m$; the presence of only one long run equilibrium relationship between the series in $\mathbf{X}_t$ is not rejected.[22] When the model includes $U_2^w$, we can infer that there is no long run relationship among the middle aged unemployed female workers and the rest of the series in $\mathbf{X}_t$. The values of the statistics to test for $H_0 : r = 0$ allow us to reject this hypothesis in a model without a linear trend. However, these are close to the critical values tabulated. That is, this hypothesis would not be rejected at a significance level only slightly lower than 5%. To finish, we consider the model that includes the unemployment rate of the eldest female workers $U_3^w$, obtaining a similar result: there is no long run relationship between $U_3^w$, $E_w$, $T_w$ and $Y$.

In summary, the disparity in the results is worthy of note. The use of Engle and Granger's (1987) approach gives unexpected results. We find no relationship among the unemployment rates and the variables representing the evolution of the population pyramid, regardless of sex and age of unemployed, except when the first differences in $U_1^m$ are included.[23]

[21] The graphic representation of the cointegrating residuals is illustrative in this case: a stationary behavior can be seen in those obtained from a model without a linear trend, being the opposed when the trend is not present.

[22] There exists some ambiguity about whether the analysis must be made in a model with or without a linear trend. A visual inspection of the cointegrating residuals would support this last result. The tests (on $\boldsymbol{\beta}$ and $\boldsymbol{\alpha}$) that we are going to implement later have been carried out for both cases, obtaining robust results under both specifications.

[23] As for coherence, we make new regressions using Engle and Granger' approach. The new feature appears in the dependent variables we use. These will be the first differences of the unemployment rate and not their levels. In particular, we analyze two cases: the one including $\Delta U_1^m$ and the one including $\Delta U_2^m$. The results, with regard to the existence of equilibrium relationships, are similar to the ones obtained with Johansen's procedure in the first case ($ADF_{res}$ = *-4.866*) but differ in the second one ($ADF_{res}$ = -3.926). That is, when we use Engle and Granger's approach for $\Delta U_1^m$ and $\Delta U_2^m$, only one long run relationship among the demographic variables, the *GDP* growth rate and the first differences

Following Johansen's procedure, however, the conclusions are modified in some cases. We do not detect long run equilibrium relationships among the population variables and the unemployment rates of the middle aged (from 30 to 44) female workers and oldest (45 and over) female workers, but we do detect equilibrium relationships for the remaining sex-age groups.

To sum up, there is at least one cointegrating relationship between the series. This is the case for male workers (regardless the age), and for the youngest female workers. For these groups, the unemployment rate, the relative mean age and relative size of mature population, and the GDP growth rate evolve jointly thus keeping at least one long run equilibrium relationship. There exists, therefore, a common evolution between these series, thus showing that population aging has significant long run effects upon some specific Spanish unemployment rates.

Zimmermann (1992) and Schmidt (1993) obtain similar results. None of them finds a clear long run relationship between the age distribution of German population and unemployment rates for specific age and sex.

For Spain, Ahn et al. (2000) do not obtain conclusive results either. More precisely, they find that the influence of the size of the youth population upon the youth unemployment rate differs depending on the specific population group considered and on the demographic variables included.

### C. Further Testing on Long-run Relationship by Sex-Age Groups

We are going to analyze some characteristics relative to the cointegrating vectors, $\beta_t$ ($\ell = 1, 2, ..., r$), and the weighting matrix, $\boldsymbol{\alpha}$, only for those groups for which we have not rejected the existence of, at least, one equilibrium relationship: namely, male unemployed in the three age segments and youngest female unemployed (see Johansen and Juselius, 1990, 1992). Two aims are pursued.

Firstly, we test whether all series ($U^i_j$, $E_i$, $T_i$ and $Y$) are part of the cointegrating vector(s) or if some of them should be excluded. In the former

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in the unemployment rates of the youngest male workers (between 16 to 29) is detected, rejecting this relationship if the male workers in the higher age segment are analyzed (from 30 to 44).

case, all the series play the same role in the long run relationships. In the latter case, the long run behavior of the system would not depend on the excluded variable(s). To ascertain the cointegrating relationship between $U_j^i$, $E_i$ and $T_i$ none of them should be excluded. Although some of these series cannot affect the long run behavior, its short run dynamics could be affected by deviations from this long run equilibrium.

Johansen and Juselius (1990, 1992) propose a test for this purpose. The results, for all the variables in $\mathbf{X}_t$ are presented in Table 4.

**Table 4. "Exclusion" Test**

| | Age | $D_j^m$ | $E_i$ | $T_i$ | $Y$ |
|---|---|---|---|---|---|
| | 16-29 | 30.76 * | 20.35 * | 2.68 | 29.86 * |
| Male workers | 30-44 | 13.97 * | 20.63 * | 10.27 * | 17.96 * |
| | ≥45 | 0.009 | 9.80 * | 0.12 | 9.40 * |
| Female workers | 16-29 | 1.03 | 13.39 * | 0.11 | 19.41 * |

Note: $D_j^m = U_j^m$ if $U_j^m$ is $I(1)$, and $D_j^m = \Delta U_j^m$ if $U_j^m$ is $I(2)$. * denotes the rejection of $H_0$ of exclusion of the variable at the 1% significance level.

The hypothesis of exclusion of the $T_i$ variable is not rejected in three out of four models. Therefore, the relative size of the mature population does not seem to influence the long run behavior of the system. Thus, the effect of changes in the demographic structure is captured by the relative mean age of mature population, $E_i$.

However, in the fourth model, the one that includes $\Delta U_2^m$, the incidence of changes in population is shown by both demographic variables, $E_m$ and $T_m$. In this case, we reject the exclusion of all the variables in the cointegrating vectors, so that all of them affect the long run behavior of the system.

There are two models (those containing oldest unemployed male workers and youngest unemployed female workers) in which both $T_i$ and the unemployment rates must be excluded. This implies that the cointegrating vectors are integrated only by the GDP growth rate and by the relative mean age of mature population (*i.e.*, $Y$ and $E_i$). A long run relationship between these variables might have a reasonable justification from an economic point of view to the extent that the population aging might really affect the GDP.

To sum up, we cannot justify, in general, the existence of a long run equilibrium relationship between the demographic variables (the relative size and the relative mean age of mature population), the variable that represents the economic situation (the growth rate of GDP) and the unemployment rates, disaggregated by sex and age. The exception is given when we analyze the youngest male workers and the middle aged male workers. In these groups, one can actually find a long run joint evolution between the unemployment rates and the population variables.

## D. Short-run Dynamics

Secondly, a cointegrating vector can be interpreted as an equilibrium relationship from which variables can deviate only temporarily. The elements in the $\alpha$ matrix represent the speed of response of these variables in the short run to transitory deviations from the long run relationships. Thus, it is worth wondering whether all the variables in the system react to such deviations or, on the contrary, this adjustment is produced only by some of them. A weak exogeneity test would give an answer to this question.

We must be aware that the study of short run adjustments of some variables (such as $E_i$ and $T_i$) might not be appropriate here, because the changes in the age composition of population are not expected to be instantaneous. As a result, when we implement the tests, we are implicitly assuming that the remaining variables make the corresponding short run changes (in this case, there is only one variable left, GDP growth rate).

The procedure is as follows. If $\alpha_{k\ell} = 0$, where $k = U, E, T, Y$ and $\ell = 1, 2, ..., r$ (where $r$ denotes the maximum number of long run equilibrium relationships or cointegrating vectors), this would be interpreted as the absence of adjustment of the $k$ variable in the presence of transitory deviations from the equilibrium relationships. In other words, $k$ would be a weakly exogenous variable (see Johansen and Juselius, 1990, 1992). The values of the test statistics are shown in Table 5.

One does not find homogeneous results for all groups. The systems including the youngest male (from 16 to 29) unemployed workers and the

**Table 5. Weak Exogeneity Test**

| | Age | $D_j^m$ | $E_i$ | $T_i$ | $Y$ |
|---|---|---|---|---|---|
| | 16-29 | 15.46 $^{*}$ | 6.30 $^{**}$ | 11.98 $^{*}$ | 8.26 $^{**}$ |
| Male workers | 30-44 | 7.36 $^{**}$ | 7.01 $^{**}$ | 15.26 $^{*}$ | 7.33 $^{**}$ |
| | ≥45 | 1.20 | 0.39 | 9.30 $^{*}$ | 1.12 |
| Female workers | 16-29 | 14.84 $^{*}$ | 1.16 | 4.29 $^{**}$ | 0.02 |

Note: $D_j^m = U_j^m$ if $U_j^m$ is $I(1)$, and $D_j^m = \Delta U_j^m$ if $U_j^m$ is $I(2)$. $^{*}$ and $^{**}$ denote the rejection of $H_0 : \alpha_{k\ell} = 0$ at the 1% and 5% significance level, respectively.

middle aged (from 30 to 44) unemployed male workers reject the weak exogeneity of all variables. That is, those unemployment rates, the population variables and the GDP growth rate do get adjusted in the short run to reach the long run equilibrium relationship among them.

Looking at the system that includes the oldest (45 and over) unemployed male workers, only the relative size of mature population is adjusted to achieve this relationship. $T_i$, therefore, does not affect the long run behavior of the system (see Table 4), but it does have influence on its short run path.

Finally, we consider the unemployed female workers with ages between 16 and 29. The weak exogeneity hypothesis is not rejected for the relative mean age of mature population and for the GDP growth rate. Such variables, therefore, are not affected by the cointegrating relationships in their short run evolution. Their unemployment rate and the relative size of female mature population will be adjusted to achieve such relationships, in which the other two variables ($E_w$ and $Y$) will take part.

In short, these results do not confirm the starting hypothesis about the weak exogeneity of the demographic variables. Once again, the reduced informative contents may explain this result.

One might be tempted to follow an impulse response analysis to obtain additional information about these relationships between the variables. This procedure may be appropriate in other contexts but not here. Our purpose in this paper is to analyze the population incidence on unemployment rates, disaggregated by sex and age segment: an analysis that considers shocks in the demographic structure is of reduced relevance because the changes in the Spanish population pyramid, as in most countries, are produced very slowly.

Once the characteristics of the cointegrating vectors and the weighting matrix are known, we analyze the structural relationships which can be inferred from their estimations. $\boldsymbol{\beta}'\mathbf{X}_t = 0$ represents the equilibrium relationships between the variables in $\mathbf{X}_t$, and the elements of $\boldsymbol{\alpha}$ represent the adjustment of these variables towards the equilibrium. Appendix shows the estimations of $\boldsymbol{\beta}$ and $\boldsymbol{\alpha}$ for the models where long run relationships have been found.

There are certain limitations to interpreting the meaning of the estimated long run relationships among the series. When there is more than one relationship, we cannot be sure about which one should be picked up. Moreover, even though the cointegrating vector is unique, we would not have enough information to identify the long run equilibrium relationships. Why? The matrix $\Pi = \boldsymbol{\alpha}\,\boldsymbol{\beta}'$ that appears in the estimated error correction model is identifiable, but $\boldsymbol{\beta}$ and $\boldsymbol{\alpha}$ matrices are *not*. Therefore, the cointegrating vectors cannot be interpreted directly as long run equilibria, because one would be ignoring the dynamic relationships between the variables of the short run system.

Nevertheless, it is possible to prove the existence of the cointegrating relationships. Thus, we can confirm the presence of relationships between the demographic trends and the youngest unemployed male workers and middle aged unemployed male workers. Moreover, although the interpretation of these relationships by using the coefficients of the normalized equation is not satisfactory, we can infer, at least, whether this is positive or negative. So, we can write $\boldsymbol{\beta}'\,\mathbf{X}_t = 0$ as[24]

$$\beta_\ell^U U_{jt}^i + \beta_\ell^E E_{it} + \beta_\ell^T T_{it} + \beta_\ell^Y Y_t + \beta_{0,\ell} = 0, \quad \ell = 1, 2, \ldots, r, \tag{10}$$

and we can normalize the cointegrating relationships for a specific unemployment rate (the variable of our interest). We only consider the two models in which long run equilibrium relationships have been found, *i.e.*, those including the youngest male workers and middle aged male workers. (Remember that two cointegrating vectors have been found in both cases.)

[24] We consider that the error correction model does not include the vector of constants, so that the dimension of $\boldsymbol{\beta}$ increases in one unit.

The equations referring to the youngest unemployed male workers (from 16 to 29) do not provide clear information about the effect that demographic structure can exert on the unemployment rate: the sign associated with the population variable $E_{mt}$ is positive in one case and negative in other.

When we analyze the middle aged unemployed male workers, the sign associated with $T_{mt}$ is not clear, although the sign of $E_{mt}$ is positive in both cointegrating vectors. Thus, there is a positive relationship between the age and the change in the unemployment rate, while the relationship between the evolution of GDP and the unemployment rate is, as expected, negative.

In short, we cannot confirm our initial hypothesis about the existence of a negative relationship between the relative size of mature population and the unemployment rates, disaggregated by sex and age. However, the results show that increases in the relative mean age of mature population increase the middle aged unemployment rate of male workers. Zimmermann (1992) offers a similar conclusion.

## IV. Conclusions

In this paper, we have analyzed the long run incidence of changes in the age structure of the population upon unemployment rates, disaggregated by age and sex in Spain during the period 1976:3-1998:4. The age structure has been approximated by the relative mean age of mature population and the relative size of the group of mature population with respect to the group of rest of the working age population. Unemployment rates have been defined considering three age segments: the first between 16 and 29 years old; the second between 30 and 44; and the third of 45 and over.

We have obtained four main results. First, the estimations obtained in the empirical analysis do not provide a clear scheme about the relationships between the evolution of the age structure of population and the unemployment rates. Second, in a first approximation one can detect the existence of, at least, one long run equilibrium relationship between the unemployment rates, the relative mean age of mature population, the relative size of mature population and the GDP growth rate in all cases, except for middle aged female workers and oldest female workers. Third, a more thorough analysis of the significance degree of the variables in such equilibrium relationships suggests that unemployment rates for the eldest male workers and the youngest female workers can be excluded from their corresponding error correction models. The same happens with the relative size of mature population in all cases except for middle aged male workers. As a result of the tests, therefore, one can only justify the existence of long run relationships between the demographic variables and the unemployment rates of the youngest male workers and middle aged male workers. One cannot argue, however, joint evolution for population variables and the unemployment rates of the youngest female workers and of the oldest male workers. Fourth, the short run dynamics of the unemployment rates of the youngest male workers and middle aged male workers and that of the youngest female workers are affected by transitory deviations from these long run relationships.

Zimmermann (1992) and Schmidt (1993)'s papers on the German case are no more conclusive. Their estimations show that there exists a long run relationship between the population variables and the specific unemployment rates for only a few groups. Something similar occurs in Ahn et al. (2000) when they study the incidence of Spanish youth population on the unemployment rates of young workers (both male and female).

One cannot confirm, therefore, that changes in the age structure of Spanish population play an important role in the unemployment rates of all age segments. Only the youngest unemployed male workers and middle aged unemployed male workers seem to be affected by changes in the demographic composition of the labor force.

The effect of population variables on the number of unemployed workers may depend on activity rates: low activity rates may cause low incidence of population variables on unemployment rates. This reasoning may be valid to argue the absence of response of the unemployment rates for 30-plus female workers and 44-plus male workers: the activity rates were 28.5% and 19.3%, respectively, in 1976:3, and 63.1% and 18.8% in 1998:4. In the same periods, the activity rates for the oldest male workers were 67.5% and 44.9%. They are all very low compared to those of middle aged male workers: 97.5% and 95.2%.

All results must be interpreted cautiously. The reduced number of observations and, in consequence, the reduced informative contents of the sample limit the interpretation of the estimations in a way.

An extension of the model that allows for adjustments in wages and for a more important role in the different educational levels could open another line of research.

## Appendix. Maximum Likelihood Estimations

The maximum likelihood estimations of the cointegrating vectors, $\boldsymbol{\beta}$, and the weighting matrix, $\boldsymbol{\alpha}$, are shown below.

- Unemployed male workers aged between 16 and 29:

$$\boldsymbol{\beta}^{*} = \begin{bmatrix} 576.31 & 58.62 \\ -53.16 & 52.75 \\ 2.96 & -0.32 \\ 95.90 & -101.67 \end{bmatrix}, \quad \boldsymbol{\alpha} = \begin{bmatrix} -0.00284 & 0.00146 \\ 0.00105 & -0.0000543 \\ 0.000557 & 0.00177 \\ -0.06 & -0.05 \end{bmatrix}$$

- Unemployed male workers aged between 30 and 44:

$$\boldsymbol{\beta} = \begin{bmatrix} -259.27 & -731.47 \\ 51.57 & 36.90 \\ 9.93 & -24.80 \\ -1.04 & -1.52 \\ -106.15 & -47.90 \end{bmatrix}, \quad \boldsymbol{\alpha} = \begin{bmatrix} 0.000551 & 0.00127 \\ 0.000899 & 0.000661 \\ -0.00177 & 0.000341 \\ 0.06 & 0.06 \end{bmatrix}$$

Note: $\boldsymbol{\beta}^{*}$ denotes $\boldsymbol{\beta}$ once $T_{mt}$ variable has been eliminated ($T_{mt}$ has been eliminated as the test of exclusion for this variable indicates (see Table 4 in the main text).

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