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# Two-Part Tariff Licensing Mechanisms

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## Abstract

Most of the patent licensing agreements that are observed include royalties, in particular per-unit or ad valorem royalties. This paper shows that in a differentiated duopoly that competes á la Cournot the optimal contract for an internal patentee *always* includes a positive royalty. Moreover, we show that the patentee would prefer to use ad valorem royalties rather than per-unit royalties when goods are complements or when they are substitutes and the degree of differentiation is sufficiently low. The reason is that by including an ad valorem royalty in the licensing contract the patentee can commit strategically to be more (less) aggressive when goods are complements (substitutes) since his licensing revenues become increasing with the price of output of his rival. As a result, licensing may hurt consumers although it always increases social welfare.

*Keywords:* Patent Licensing, Royalty, Cournot Duopoly, Product Differentiation.

*JEL classification:* D45.

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# 1 Introduction

OECD studies has been gathering evidence about the expansion of the volume and value of patent licensing over recent years, relating this phenomenon to broad changes in the modes of innovation, globalisation and strengthened market competition (OECD 2009, and other references therein).

The owner of a patent charges some payments for using the new technology to the authorised firms obtaining in this way a return on his investment in research and development. The profits of the patentee will depend on the structure adopted by the licensing agreement. In this regard, some econometric papers have studied which industry and firm characteristics may explain differences in licensing contracts. Vishwasrao (2007) finds that licensing contracts are more likely to use royalties when sales are relatively high, whereas volatile sales and greater profitability favor fixed fee contracts.

The theoretical literature on patent licensing, initiated by Arrow (1962), has analysed the performance of fixed fees and royalties payments as instruments for the licensing of patented cost-reducing innovations. Initially, contributions focused on innovations provided by firms outside the industry where this innovation will apply. Kamien and Tauman (1984, 1986), Katz and Shapiro (1986) and Kamien et al (1992) considering an oligopoly industry showed the superiority, from the point of view of the patentee, of upfront fee mechanisms over a per-unit royalty mechanism for the case of homogeneous goods under Cournot or Bertrand competition. Later, studies that consider a patent holder which is himself a producer within the industry obtain that the patentee may prefer royalty contracts rather than fixed-fee contracts under both type of competitions (see for example Wang (1998), Wang and Yang (1999) and Kamien and Tauman (2002)).<sup>1</sup>

Empirical evidence about licensing contracts reveals that in actual practices the innovator is often one of the incumbent firms in the industry, that most of the contracts include a positive royalty and that combinations of upfront fees and royalties are commonly observed in practice (see Macho-Stadler et al. (1996), Bousquet et al.

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<sup>1</sup>For the case of an external patentee the reasons that may explain that superiority of a per-unit royalty mechanism include uncertainty (e.g., Jensen and Thursby, 2001 and other references therein), product differentiation (Muto, 1993), strategic delegation (Saracho, 2002), the degree of competitive behavior in the product market (Saracho, 2005) or the restriction that the number of licenses must be an integer (Sen, 2005).

(1998) and another references therein).

Recently, theoretical literature has analysed the use of two-part tariff contracts, consisting in a fixed fee plus a per-unit royalty in different contexts. Mukherjee and Balasubramanian (2001) and Faulí-Oller and Sandonís (2002) consider technology transfer in horizontally differentiated Cournot and Bertrand duopolies with substitute goods and show that the optimal contract always includes a positive royalty. Sen and Tauman (2007) show for both, an internal and an external patentee in an homogeneous good Cournot oligopoly, that the optimal per-unit royalty plus fixed fee contract involves a positive royalty for relatively significant innovations.

Theoretical literature however, has not paid so much attention to other evidence reported by empirical data in licensing contracts: as the widespread use of ad valorem royalties. For example, the study of french data by Bousquet et al. (1998) shows that 96% of royalties in licensing contracts are ad valorem royalties.<sup>2</sup>

Some studies have included this type of payments in the theoretical analysis and as a result some interesting aspects have been added to this issue. Hernández-Murillo and Llobet (2006) study the optimal licensing agreement between an external patentholder of a cost reducing innovation and firms that have heterogeneous uses for the technology in a monopolistic competition framework. Considering a payment corresponding to a share of gross revenues of the licensee together with a flat fee, they obtain that firms with a higher valuation for the innovation choose a contract in which they pay a higher flat fee and retain a higher share of revenues. Erutku and Richelle (2007) include royalties that may depend on firm's output and on total output of the industry in a context of an external patentee licensing the innovation to a Cournot oligopoly. They show that a licensor specifying both a fixed fee and a royalty can obtain revenues equal to the profit that a monopolist endowed with the innovation could make on the market. San Martín and Saracho (2010) focusing on an incumbent innovator show that in the classic homogeneous good Cournot duopoly an internal patentee will always prefer the ad valorem royalty to the classic per-unit royalty. In fact the optimal two part tariff licensing, per-unit royalty plus fixed fee or ad valorem royalty plus fixed fee, implies a pure ad valorem contract (a fee equal to zero and a positive ad valorem royalty). They justify the presence of ad valorem royalties by appealing to their influence in the strategic behavior of the patentee.

In this paper, we study the optimal two-part tariff contract for an insider firm

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<sup>2</sup>Sakakibara (2010) carries out a regression analysis to examine the determinants of the price of patent licensing with 661 patent licensing contracts in Japan. Using the royalty rate, as a percentage of sale, and the lump-sum payment as proxies for the price, the article suggests that the royalty rate represents patent licensing price better than the lump-sum payment.

that has a patent on a process innovation in a differentiated Cournot duopoly. In this way, we are able to analyse how the use of ad valorem or per-unit royalties in licensing contracts, may differ depending on both the kind of goods produced by the industry (substitutes or complementary goods) and the degree of product differentiation between goods.<sup>3</sup>

In the analysis we also aim to analyze the effects of the chosen payment scheme may have on consumers and social welfare. As it is known, from a social perspective, licensing can generate both, positive and negative effects. An important positive aspect is that licensing is a way to diffuse new technologies. However, we also know that licensing could be used as a collusive device between the firms in such a way that reduces consumer surplus and social welfare. It happens, for example, in Faulí-Oller and Sandoñis (2002), where authors characterize situations where licensing a cost reducing innovation to a rival firm using two-part tariff contracts (fixed fee plus per-unit royalty) reduces social welfare. Also, in San Martín and Saracho (2010) it is shown that although social welfare is greater with licensing than without it consumer surplus is lower.

Summarising, the analysis in this paper allows us to justify the presence of ad valorem royalties in licensing contracts based on the kind of goods produced in the duopoly industry and on the degree of differentiation between goods. In particular, for non drastic innovations (i.e, it is not significant enough to create a monopoly if only one firm has the new technology), if the goods produced by the industry are complements then the patentee will prefer licensing using ad valorem royalties. Moreover, for the case of substitute goods the superiority of the ad valorem royalties on per-unit royalties depends on the degree of substitutability between goods. By including in the contract an ad valorem royalty the patentee, depending on whether the goods are substitutes or complements, can commit strategically to be less or more aggressive since his licensing revenues become increasing in the price of output of his rival. As a result, licensing may hurt consumers although it always increases social welfare. Lastly, if the innovation is drastic in the sense of Arrow (1962) then the patentee will license the innovation by means of a pure ad valorem royalty and his profits will be equal to those corresponding to a multiproduct monopolist.

The rest of the paper is organized as follows. Section 2 presents the model of patent licensing. Section 3 analyzes the two mechanisms considered, and Section 4

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<sup>3</sup>Hernández-Murillo and Llobet (2006) describe how the optimal contract changes with the degree of product differentiation and show that under the assumptions of their paper, both mechanisms per-unit royalty plus a fixed fee and a share on the value of sales plus a fixed fee implement the same optimal allocation.

concludes with some final remarks.

## 2 The Model

Consider a duopolistic industry, with firms 1 and 2 producing two differentiated products. Each good is produced by only one firm of the industry. The inverse demand function for good  $i$ , which is produced by firm  $i$ , is:

$$P_i = a - q_i - \gamma q_j, \text{ with } i \neq j, i, j = 1, 2, a > 0 \text{ and } \gamma \in [-1, 1],$$

where  $q_i$  and  $q_j$  represent, respectively, the quantities produced by firms  $i$  and  $j$ .<sup>4</sup> Goods may be substitutes, complements or independent of each other depending on  $\gamma$  is positive, negative or zero. Moreover  $\gamma$  is used as a measure of the degree of differentiation between goods when it is positive. One of the firms, which without loss of generality we will assume is firm 1, owns a patent on a cost-reducing innovation. There are no fixed costs of production and the marginal cost of selling licenses is zero. The marginal cost of production for each firm without the innovation is constant and equal to  $c$  ( $c < a$ ) and the innovation reduces the marginal cost of production from  $c$  to 0.<sup>5</sup>

As it is well known an innovation may be drastic or non drastic. In the context analyzed in this paper the innovation will be drastic if  $c > \frac{a(2-\gamma)}{2}$ .

The analysis is modeled as a non-cooperative game in three stages. In the first stage, the patentee sets a two-part tariff contract on a take-it-or-leave-it basis.<sup>6</sup> The set of licensing schemes available to the innovator is the set of all linear combinations of a non-negative upfront fee and a non-negative royalty, per-unit or ad valorem<sup>7</sup>. In the second stage, firm 2 decides whether or not to accept the offer from firm 1. In the last stage, both firms engage in a non-cooperative quantity competition game.

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<sup>4</sup>The inverse demand system may be obtained from the optimization problem of a representative consumer with a utility function with respect goods 1 and 2 equal to  $U(q_1, q_2) = a(q_1 + q_2) - \frac{1}{2}(q_1^2 + 2\gamma q_1 q_2 + q_2^2)$ . In the economy there are a duopolistic sector, a competitive numeraire sector and a continuum of consumers of the same type with a utility function separable and linear in the numeraire good. This model was proposed by Dixit (1979), see Vives (1999) for a review of the specification.

<sup>5</sup>This assumption simplifies the analysis with regard to a situation in which with the innovation the marginal cost of production is positive. However, as we will discuss later, the basic results do not depend on this assumption.

<sup>6</sup>As we will see, in our context the patentee is always interested in selling a license to the other firm.

<sup>7</sup>See Liao and Sen (2005) for the implications of allowing negative fees and royalties.

### 3 Patent Licensing Mechanisms

In this section we proceed to the resolution of the model. First we consider the combination of a per-unit royalty and a fixed fee mechanism in subsection 3.1 and then the combination of an ad valorem royalty and a fixed fee mechanism in subsection 3.2. Finally, we study which mechanism will be preferred by the patentee.

#### 3.1 Per-Unit Royalty plus Fixed Fee Mechanism

We look for the subgame perfect Nash equilibrium of the game. Assume that the patentee charges for the license a non-negative uniform per-unit of production royalty ( $h$ ) and a non-negative fixed fee ( $F$ ). Then, the marginal cost of firm 2 with the license is  $h$ . It is not difficult to show that, in this case, in the third stage of the game the equilibrium production levels for each firm, and for the interior solutions, are  $q_1 = \frac{a(2-\gamma)+\gamma h}{4-\gamma^2}$  and  $q_2 = \frac{a(2-\gamma)-2h}{4-\gamma^2}$ .<sup>8</sup> Firm 2 will buy the license if and only if its profits with the innovation are at least as high as those without the innovation, that is  $\pi_2^n = \frac{(a(2-\gamma)-2c)^2}{(4-\gamma^2)^2}$  if the innovation is non drastic and  $\pi_2^n = 0$  if it is drastic.<sup>9</sup> In the first stage the patentee will choose the contract that maximizes his total revenues, that is the sum of the profits from his own production plus the licensing revenues, taken into account the restrictions given by the second and the third stages of the game. Firm 2 would accept a licensing contract involving a per-unit royalty  $h$  greater than the reduction in the marginal cost induced by the innovation,  $c$ , only if it would be compensated upfront by a negative fee. The reason is that with the license its marginal cost would then be greater with the innovation than without it and its profits would be lower than those without the innovation. Since contracts including a negative fee are ruled out by assumption it is sufficient to consider  $h$  belong to  $[0, c]$ .

Let us denote by  $\pi_2^a$  the profits of firm 2 without discounting the fixed fee paid by the license if it buys a license. Therefore, the patentee solves the following problem:

$$\max_{h,F} hq_2 + (a - q_1 - \gamma q_2) q_1 + F$$

subject to

$$0 \leq h \leq c, \quad F \leq \pi_2^a - \pi_2^n, \quad q_1 = \frac{a(2-\gamma) + \gamma h}{4-\gamma^2}, \quad \text{and} \quad q_2 = \frac{a(2-\gamma) - 2h}{4-\gamma^2}.$$

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<sup>8</sup>These production levels come from the following first order conditions of the maximization problems of both firms:  $a - 2q_1 - \gamma q_2 = 0$  and  $a - 2q_2 - \gamma q_1 - h = 0$ . So, we can see that the licensee behaves less aggressively than in the case in which the per-unit royalty is zero.

<sup>9</sup>The production levels without licensing will be  $q_1 = \frac{a(2-\gamma)+\gamma c}{4-\gamma^2}$  and  $q_2 = \frac{a(2-\gamma)-2c}{4-\gamma^2}$  if the innovation is non drastic and  $q_1 = \frac{a}{2}$  and  $q_2 = 0$  if it is drastic.

Solving this problem we may establish the following proposition:

**Proposition 1.** *The optimal contract  $(F, h)$  from the point of view of the patentee and his profits  $\pi^{pu}$  are such that:*

i) If the innovation is non drastic ( $\frac{2-\gamma}{2} > \frac{c}{a}$ ) and goods are substitutes ( $\gamma > 0$ ) then the optimal two part tariff involves a positive per unit royalty:

i.a) A per unit royalty plus a positive fee where

$$F = \frac{-4c^2(4-3\gamma^2)^2 - 4ac(-2+\gamma)(4-3\gamma^2)^2 - a^2(-2+\gamma)^3\gamma(-8+2\gamma+5\gamma^2)}{(16-16\gamma^2+3\gamma^4)^2}, \quad h = \frac{a\gamma(2-\gamma)^2}{2(4-3\gamma^2)} \quad \text{and}$$

$$\pi^{pu} = \frac{16c^2(-4+3\gamma^2)+16ac(8-4\gamma-6\gamma^2+3\gamma^3)+a^2(-2+\gamma)^2(16-8\gamma^2-4\gamma^3+\gamma^4)}{4(4-3\gamma^2)(-4+\gamma^2)^2} \quad \text{if } \frac{c}{a} > \frac{\gamma(2-\gamma)^2}{2(4-3\gamma^2)} \quad \text{or}$$

i.b) a pure per-unit royalty where

$$F = 0, h = c \text{ and } \pi^{pu} = \frac{c^2(-8+3\gamma^2)+ac(8-4\gamma^2+\gamma^3)+a^2(-2+\gamma)^2}{(-4+\gamma^2)^2} \quad \text{if } \frac{\gamma(2-\gamma)^2}{2(4-3\gamma^2)} \geq \frac{c}{a}.$$

ii) If the innovation is non drastic ( $\frac{2-\gamma}{2} > \frac{c}{a}$ ) and goods are complements or independent ( $\gamma \leq 0$ ) then the patentee sets a fixed fee, such that:

$$F = \frac{4c[a(2-\gamma)-c]}{(-4+\gamma^2)^2}, \quad h = 0 \text{ and } \pi^{pu} = \frac{-4c^2-4ac(-2+\gamma)+a^2(-2+\gamma)^2}{(-4+\gamma^2)^2}.$$

iii) If the innovation is drastic ( $\frac{c}{a} \geq \frac{2-\gamma}{2}$ ) then the optimal two part tariff includes both a fixed fee plus a per-unit royalty, such that:

$$F = \frac{4a^2(-1+\gamma)^2}{(4-3\gamma^2)^2}, \quad h = \frac{a\gamma(2-\gamma)^2}{2(4-3\gamma^2)} \text{ and } \pi^{pu} = \frac{a^2(8-8\gamma+\gamma^2)}{4(4-3\gamma^2)}.$$

As it is known when goods are substitutes the patentee will be, always, interested in including in the contract a positive per-unit royalty. Moreover, in order the



patentee is interested in including both, a positive fixed fee and a positive per-unit royalty, the innovation must be important enough (it will be the case if  $\frac{c}{a} > \frac{\gamma(2-\gamma)^2}{2(4-3\gamma^2)}$ ).

Notice, however, that under this mechanism the royalties will not be used if the goods are complements or independent ( $\gamma \leq 0$ ). A per-unit royalty allows to the patentee to control the marginal cost of production of his rival. When goods are complements the price of the good produced by the patentee increases with the level of production of his rival and as a result he is interested in setting a per-unit royalty equal to zero (indeed the optimal per-unit royalty from the point of view of the patentee would be negative, which is ruled out by assumption).

### 3.2 Ad Valorem Royalty plus Fixed Fee Mechanism

Again we have to solve the game by backward induction. At the third stage, each firm will produce the quantity that maximizes his profits given the ad valorem royalty ( $d$ ) and the fixed fee set in the first stage.

The licensee solves:

$$\max_{q_2} (1-d)(a - q_2 - \gamma q_1)q_2 - F$$

and the patentee solves:

$$\max_{q_1} (a - q_1 - \gamma q_2)q_1 + d(a - q_2 - \gamma q_1)q_2 + F$$

Assuming interior solutions, the first order conditions of these two problems imply:

$$a - \gamma q_1 - 2q_2 = 0 \text{ and } a - 2q_1 - (1+d)\gamma q_2 = 0.$$

Hence, the quantities produced in equilibrium by the two firms are:

$$q_1 = \frac{a(2 - \gamma(1+d))}{4 - \gamma^2(1+d)} \text{ and } q_2 = \frac{a(2 - \gamma)}{4 - \gamma^2(1+d)}.$$

The first order conditions of the maximization problem solved by the patentee in the third stage, show that the use of an ad valorem royalty influences the strategic behaviour of the patentee but does not change the licensee's behaviour. It is obvious that when goods are substitutes (complements) the patentee behaves less (more) aggressively than in the case in which the ad valorem royalty is zero. The patentee is interested in given that it induces a greater price of his rival and, subsequently, the sales of the licensee along with the revenues from licensing the innovation.

At the second stage, firm 2 will buy the license if and only if its profits with the innovation are at least as high as those without the innovation, that is at least as high as  $\pi_2^n$ .

At the first stage the patentee will set the contract that maximizes his total profits subject to the restrictions imposed by the second and third stages of the game. Hence, he will solve the following problem:

$$\max_{d,F} (a - q_1 - \gamma q_2) q_1 + d(a - \gamma q_1 - q_2) q_2 + F$$

subject to

$$q_1 = \frac{a(2 - \gamma(1 + d))}{4 - \gamma^2(1 + d)}, \quad q_2 = \frac{a(2 - \gamma)}{4 - \gamma^2(1 + d)}, \quad \text{and } (1 - d)(a - q_1 - \gamma q_2) q_2 - F \geq \pi_2^n.$$

Solving this problem we may establish the following proposition:

**Proposition 2.** *The optimal contract  $(F, d)$  from the point of view of the patentee and his profits  $\pi^{av}$  are such that:*

i) If the innovation is non drastic ( $\frac{2-\gamma}{2} > \frac{c}{a}$ ) and goods are substitutes ( $\gamma > 0$ ) then the optimal two part tariff contract involves a positive ad valorem royalty:

i.a) An ad valorem royalty plus a positive fee where

$$F = \pi_2^{av} - \pi_2^n, \quad d = \frac{4-4\gamma+\gamma^2}{4-2\gamma-\gamma^2} \quad \text{and} \quad \pi^{av} = \frac{16c^2(-4+3\gamma^2)+16ac(8-4\gamma-6\gamma^2+3\gamma^3)+a^2(-2+\gamma)^2(16-8\gamma^2-4\gamma^3+\gamma^4)}{4(4-3\gamma^2)(-4+\gamma^2)^2}$$

$$\text{if } \frac{c}{a} > \frac{2-\gamma}{2} - \frac{(2-\gamma)(2+\gamma)\sqrt{\gamma(1-\gamma)(4-2\gamma-\gamma^2)}}{2\sqrt{2}(4-3\gamma^2)} \quad \text{or}$$

i.b) a pure ad valorem royalty where

$$F = 0,$$

$$d = \frac{(-4+\gamma^2)(-8c^2\gamma^2-a^2(-2+\gamma)^2(-4+3\gamma^2)+a(-2+\gamma)(-8c\gamma^2+\sqrt{32c^2\gamma^2(-2+\gamma)^2+32ac\gamma^2(4-2\gamma-2\gamma^2+\gamma^3)+a^2(8-4\gamma-6\gamma^2+3\gamma^3)^2})}{2\gamma^4(2c+a(-2+\gamma))^2},$$

$$\text{and } \pi^{av} = (a - q_1 - \gamma q_2) q_1 + d(a - q_2 - \gamma q_1) q_2 \quad \text{if } \frac{2-\gamma}{2} - \frac{(2-\gamma)(2+\gamma)\sqrt{\gamma(1-\gamma)(4-2\gamma-\gamma^2)}}{2\sqrt{2}(4-3\gamma^2)} \geq \frac{c}{a}.$$

ii) If the innovation is non drastic ( $\frac{2-\gamma}{2} > \frac{c}{a}$ ) and goods are complements ( $\gamma \leq 0$ ) then the contract involves a pure ad valorem royalty, identical to the one corresponding to case *i.b*).

iii) If the innovation is drastic ( $\frac{c}{a} \geq \frac{2-\gamma}{2}$ ) then the patentee sets a pure ad valorem royalty:

$$F = 0, d = 1 \text{ and } \pi = \frac{a^2}{2(1+\gamma)}.$$

From the proposition above we may conclude that the patentee is interested in including a positive ad valorem royalty in the licensing contract independently of the kind of goods produced by the industry (substitutes or complements).<sup>10</sup> The reason is that setting a positive ad valorem royalty the patentee may commit to behave less (more) aggressively when goods are substitutes (complements) than in the case in which the ad valorem royalty is zero and he is always interested in it.

Notice also that the patentee will set both a positive ad valorem royalty and a positive fixed fee only if goods are substitutes and the innovation is non drastic but it is important enough (it will be the case if  $\gamma > 0$  and  $\frac{2-\gamma}{2} > \frac{c}{a} \geq \frac{2-\gamma}{2} - \frac{(2-\gamma)(2+\gamma)\sqrt{\gamma(1-\gamma)(4-2\gamma-\gamma^2)}}{2\sqrt{2(4-3\gamma^2)}}$ ). Note that in this case the profits of the firm without license,  $\pi_2^n$ , are low because of the disadvantage in costs.

The optimal contract from the point of view of the patentee, as in the case of the per-unit royalty plus fixed fee, implies that the profits of the licensee will be identical to those corresponding to be a non-licensee  $\pi_2^n$ , so this will be indifferent between both licensing mechanisms.

Lastly, it is important to note that in the case in which the innovation is drastic the patentee will get the profits corresponding to a multiproduct monopolist, that is  $\frac{a^2}{2(1+\gamma)}$ .<sup>11</sup>

<sup>10</sup>It is easy to show that for the case of independent goods ( $\gamma = 0$ ), the patentee is indifferent between a pure ad valorem royalty and a pure fixed fee payment.

<sup>11</sup>A multiproduct monopolist, with a marginal cost equal to zero, would produce of each good a quantity equal to  $\frac{a}{2(1+\gamma)}$ . If the innovation is drastic the profits of firm 2 without the innovation are equal to zero. So, the patentee may set  $d = 1$ , which implies that the profits of the licensee,  $(1-d)(a-q_2-\gamma q_1)q_2$ , will be also zero independently of its production level. So if the patentee produces a quantity  $q_1 = \frac{a}{2(1+\gamma)}$  one of the best responses of the licensee is  $q_2 = \frac{a}{2(1+\gamma)}$  and the best response of the patentee to  $q_2 = \frac{a}{2(1+\gamma)}$  is  $q_1 = \frac{a}{2(1+\gamma)}$ . Notice also that  $d = 1$  implies that the profits of the patentee are equal to the profits of the industry,  $(a-q_1-\gamma q_2)q_1 + (a-q_2-\gamma q_1)q_2$ , and as a result he will obtain the profits of the multiproduct monopolist.

### 3.3 Comparison of Royalty Licensing Mechanisms

By comparing the patentee profits under the two mechanisms just studied we may establish the following Propositions regarding the optimal two-part tariff contract from the point of view of the patentee for drastic and non-drastic innovations (Propositions 3 and 4 respectively).

**Proposition 3.** *In a duopolistic industry that produces two differentiated goods an internal patentee will license a drastic innovation by means of a pure ad valorem royalty and will get the profits of a multiproduct monopolist.*

**Proof:**

It is immediate by comparing the profits under both mechanisms.

■

Erutku and Richelle (2007) for the case of an external patentee and a oligopoly industry consider complex license mechanisms and show that a licensor can obtain revenues equal to the profit that a monopolist endowed with the innovation could make on the market. They conclude, (see page 409) that *"any contract leading to a licensor's benefit equal to the profit a monopoly using the innovation would obtain and to a sum of licensees' reservation profits equal to zero would be equal to our fixed fee plus royalty contracts."* In our analysis the only case where the reservation profits of the licensee may be equal to zero is the one corresponding to a drastic innovation. As we have shown in this situation the patentee is able to get the monopoly profits by using contracts that include ad valorem royalties.

**Proposition 4.** *In a duopolistic industry that produces two differentiated goods an internal patentee will prefer licensing a non-drastic innovation by means of a combination of an ad valorem royalty and a fixed fee rather than by means of a per-unit royalty and a fixed fee if and only if:*

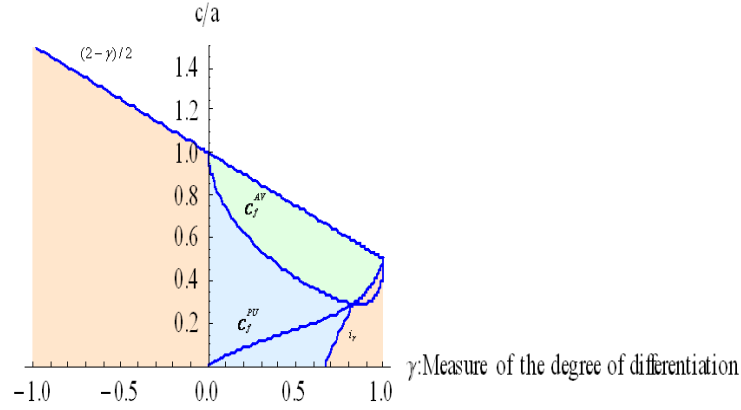
- i) *The goods are complements or*
- ii) *The goods are substitutes and  $\frac{c}{a} < \text{Min}\left\{\frac{\gamma(2-\gamma)^2}{2(4-3\gamma^2)}, \frac{-2+3\gamma}{2\gamma}\right\}$ .*

**Proof:** see Appendix.

■

Figure 1 illustrates the results in Proposition 4. The orange area corresponds to the situations where the optimal contract involves a positive ad valorem royalty. The green area corresponds to the situations where the patentee is indifferent between

both two-part mechanisms, and the blue area corresponds to the cases where the optimal contract includes a positive per-unit royalty.<sup>12</sup>



From Proposition 4, we conclude that from the point of view of an internal patentee it may be optimal to include ad valorem royalties instead of per-unit royalties in the licensing contract.

The inclusion of ad valorem royalties in the licensing contract is *always* optimal from the point of view of the patentee when the goods produced in the duopoly industry are complements. In this case the ad valorem royalties allows to the patentee to strategically commit to behave more aggressively than without royalty. This is not possible using per-unit royalties. A positive per-unit royalty affects only on licensee strategic behaviour making him less aggressive as it increases his marginal cost of production. Given that the price of the product produced by the patentee increases with the quantity produced by his rival the patentee is not interested in setting a positive per-unit royalty.

If goods are substitutes a positive ad valorem royalty allows to the patentee to behave less aggressively and a positive per-unit royalty makes the licensee, as we

<sup>12</sup>In figure 1  $c_f^{AV} = \frac{2-\gamma}{2} - \frac{(2-\gamma)(2+\gamma)\sqrt{\gamma(1-\gamma)(4-2\gamma-\gamma^2)}}{2\sqrt{2}(4-3\gamma^2)}$ ,  $c_f^{PU} = \frac{\gamma(2-\gamma)^2}{2(4-3\gamma^2)}$  and  $i_r = \frac{-2+3\gamma}{2\gamma}$ .

said above, less aggressive. However, under ad valorem royalty the patentee is less aggressive as lower is the degree of differentiation between goods. As a result in order the patentee to prefer the ad valorem royalty over the per-unit royalty it is necessary that the degree of differentiation between goods is low ( $\gamma > 2/3$ ).

Next, we evaluate the effects of the optimal two-part tariff licensing mechanism on consumer surplus and social welfare ( $W(q_1, q_2)$ ), measured as the sum of firms' profits and consumer surplus. That is,  $W(q_1, q_2) = U(q_1, q_2) - c_p q_2$  with  $c_p$  equal to 0 under licensing and equal to  $c$  under no licensing.

As we have shown, for the case of drastic innovations the patentee will license the innovation and will get the profits of a multiproduct monopolist. Even though the market is monopolized consumers are better off and the social welfare is higher as the result of licensing. Social welfare without licensing would be  $\frac{3a^2}{8}$  and it would be  $\frac{3a^2}{4+4\gamma}$  for licensing. Consumers are better off due to, in some way, they have a preference for diversity and under licensing both goods are produced (social welfare does not change when  $\gamma = 1$ ).

By comparing consumer surplus and social welfare for non drastic innovations with and without licensing we may establish the following Proposition.

**Proposition 5.**

(i) *Consumer surplus with licensing may be greater, equal or lower than without licensing. It will be lower if and only if the optimal licensing contract involves the use of an ad valorem royalty.*

(ii) *The social value of the patent is positive due to licensing will always increase social welfare.*

**Proof:** see Appendix.

■

The above proposition implies that licensing will increase social welfare but, in occasions may hurt consumers. Faulí-Oller and Sandonís (2002) show that in a differentiated industry when goods are substitutes and there is Bertrand competition licensing may reduce social welfare. Note that in that case the goods are strategic complements. The goods are also strategic complements in the context considered in our paper for the case of complement goods (Cournot duopoly with  $\gamma < 0$ ). However, in this case licensing will increase social welfare.

## 4 Concluding Remarks

This paper analyzes in a differentiated Cournot duopoly the reasons why an internal patentee may prefer to include ad valorem royalties rather than per-unit royalties in patent licensing contracts. This type of royalty provides the patentee an additional instrument that can capitalize on his strategic behaviour. The patentee strategically commit himself to be less or more aggressive depending on whether the goods are substitutes or complements. As a result the patentee may prefer to use ad valorem royalties, rather than per-unit royalties, although the latter allow him to control the marginal cost of production of his rival. This effect, in turn, implies that consumer surplus may be lower with licensing than without it. This will happen precisely in cases in which the optimal license contract involves the use of an ad valorem royalty. Social welfare is, however, always greater with licensing than without it.

In our analysis we have assumed that with the innovation there are not costs of production. The main result of the paper, i.e., the patentee may prefer ad valorem royalties over per-unit royalties, would maintain if the innovation reduces the marginal cost of production but keep it positive. In that case, the ad valorem royalty affects to the strategic behavior of both, the patentee and the licensee, making them less (more) aggressive than without royalty when goods are substitutes (complements). However, in this context, the per-unit royalty only makes the licensee to be less aggressive given that it does not affect the strategic behavior of the patentee. As a result, this context improves the strategic position of the patentee to obtain higher benefits by means of ad valorem royalties respectively to the use of per-unit royalties.

Lastly, we would like to mention that we have restricted our analysis to Cournot competition. Recently, Colombo and Filippini (2011) conclude, in contrast to our results, that under Bertrand competition when goods are substitutes an internal patentee will prefer to license the innovation to his rival using per-unit royalties instead of ad valorem royalties. They argue that "*... if the equilibrium licensee's quantity is high relatively to the equilibrium licensee's profits (as in Bertrand), a per-unit licensing scheme is preferred to an ad valorem licensing scheme, and vice-versa (as in Cournot).*" It must be mentioned that in that context the ad valorem royalty only affects to the strategic behavior of the licensee who behaves more aggressively than without royalty. However, the per-unit royalty makes both, patentee and licensee, behave more aggressively, and as a result the patentee would prefer to license using per-unit royalties.

*Appendix*

**Proof of Proposition 4**

i) From Proposition 1 we know that if the goods are strategic complements ( $\gamma < 0$ ) then the optimal two-part tariff mechanism, per-unit royalty plus a non-negative fixed fee, implies that the patentee will set a pure fixed fee (the optimal per unit royalty is equal to zero). This solution is a possibility when we consider instead the ad valorem royalty and fixed fee mechanism and we know, from Proposition 2, that if goods are complements then the patentee prefers to set a pure ad valorem royalty, so it is immediate to conclude that in this case the optimal mechanism from the point of view of the patentee is the pure ad valorem royalty.

ii) From Propositions 1 and 2 we know that:

(a) if  $\frac{c}{a} > \text{Max}\left\{\frac{\gamma(2-\gamma)^2}{2(4-3\gamma^2)}, \frac{2-\gamma}{2} - \frac{(2-\gamma)(2+\gamma)\sqrt{\gamma(1-\gamma)(4-2\gamma-\gamma^2)}}{4(-4+\gamma^2)^2(4-3\gamma^2)}\right\}$  the optimal contract includes a positive royalty and a positive fixed fee under both mechanisms;

(b) if  $\frac{\gamma(2-\gamma)^2}{2(4-3\gamma^2)} > \frac{c}{a} > \frac{2-\gamma}{2} - \frac{(2-\gamma)(2+\gamma)\sqrt{\gamma(1-\gamma)(4-2\gamma-\gamma^2)}}{2\sqrt{2}(4-3\gamma^2)}$  the optimal contract will be a pure per-unit royalty under the first mechanism and the combination of fixed fee and ad valorem royalty under the second mechanism, and

(c) if  $\frac{2-\gamma}{2} - \frac{(2-\gamma)(2+\gamma)\sqrt{\gamma(1-\gamma)(4-2\gamma-\gamma^2)}}{2\sqrt{2}(4-3\gamma^2)} > \frac{c}{a} > \frac{\gamma(2-\gamma)^2}{2(4-3\gamma^2)}$  then the optimal contract will be the combination of fixed fee and per-unit royalty in the first case and a pure ad valorem royalty in the second one.

It is easy to show that the profits of the industry are identical under both mechanisms allowing negative fixed fees. If we would not have into account the restriction that the fixed fee cannot be negative then the production levels under the optimal ad valorem royalty ( $d = \frac{-4+4\gamma-\gamma^2}{-4+2\gamma+\gamma^2}$ ) and the optimal per-unit royalty ( $h = \frac{a\gamma(2-\gamma)^2}{2(4-3\gamma^2)}$ ) are such that  $q_i$ , with  $i = 1, 2$ , under one of the mechanisms is equal to  $q_j$ , with  $j \neq i$ , under the other mechanism. In this case, both mechanisms would be indifferent from the point of view of the patentee. So, given that a negative fixed fee is ruled out by assumption, it is clear that the profits of the patentee will be greater under the mechanism where the optimum include both types of payments. More precisely, per-unit royalty plus fixed fee in case (c) and ad valorem royalty plus fixed fee in case (b), being indifferent if both mechanisms imply a positive fixed fee (case (a)).

(d) if  $\text{Min}\left\{\frac{\gamma(2-\gamma)^2}{2(4-3\gamma^2)}, \frac{2-\gamma}{2} - \frac{(2-\gamma)(2+\gamma)\sqrt{\gamma(1-\gamma)(4-2\gamma-\gamma^2)}}{2\sqrt{2}(4-3\gamma^2)}\right\} > \frac{c}{a}$  the optimal contract under both mechanisms implies a fixed fee equal to zero (in other words a pure royalty, per-unit or ad valorem). In this case  $\pi^{av} - \pi^{pu} \geq 0$  if and only if  $\frac{c}{a} \leq \frac{-2+3\gamma}{2\gamma}$ . Therefore,



given that  $\frac{2-\gamma}{2} - \frac{(2-\gamma)(2+\gamma)\sqrt{\gamma(1-\gamma)(4-2\gamma-\gamma^2)}}{2\sqrt{2}(4-3\gamma^2)} < \frac{\gamma(2-\gamma)^2}{2(4-3\gamma^2)} \implies \text{Min}\left[\frac{\gamma(2-\gamma)^2}{2(4-3\gamma^2)}, \frac{-2+3\gamma}{2\gamma}\right] = \frac{\gamma(2-\gamma)^2}{2(4-3\gamma^2)}$ , so Proposition 4 follows.

■

### Proof of Proposition 5:

- If the optimal licensing contract is a pure ad valorem royalty licensing will decrease consumer surplus and increase social welfare.<sup>13</sup> The difference in consumer surplus,  $CS^L - CS^0$ , is negative if  $\frac{c}{a} < \frac{-2+3\gamma}{2\gamma}$ . When goods are substitutes the optimal contract is a pure ad valorem royalty if  $\frac{c}{a} < \frac{-2+3\gamma}{2\gamma}$ ; when goods are complements we know that  $\frac{-2+3\gamma}{2\gamma} > \frac{2-\gamma}{2}$ , which implies  $\frac{c}{a} < \frac{-2+3\gamma}{2\gamma}$  for non drastic innovations. So in both cases  $CS^L - CS^0 < 0$ . However licensing will increase the profits of the industry, and as a result  $W^L - W^0 > 0$ . Proofs are at disposal upon request.

- If the optimal two-part tariff contract includes fixed fee and a positive royalty (per-unit or ad valorem) then consumer surplus with licensing is  $CS^L = \frac{a^2(8-\gamma(4+3\gamma))}{8(4-3\gamma^2)}$ , without licensing  $CS^0 = \frac{2a^2(-2+\gamma)^2(1+\gamma)-2ac(-2+\gamma)^2(1+\gamma)+c^2(4-3\gamma^2)}{2(-4+\gamma^2)^2}$ , and the difference between them  $CS^L - CS^0 = \frac{(a(-2+\gamma)^2(4+3\gamma)+2c(3\gamma^2-4))(2c(4-3\gamma^2)-a\gamma(-2+\gamma)^2)}{8(-4+\gamma^2)^2(4-3\gamma^2)}$ . Given that the innovation is non drastic,  $\frac{c}{a} < \frac{2-\gamma}{2}$ , we have that  $a(-2+\gamma)^2(4+3\gamma)+2c(3\gamma^2-4) > 0$ . So, as  $2c(4-3\gamma^2) - a\gamma(-2+\gamma)^2 \geq 0$  depending on  $\frac{\gamma(2-\gamma)^2}{2(4-3\gamma^2)} \leq \frac{c}{a}$ , we conclude that  $CS^L - CS^0$  will be negative (positive) if the optimal contract includes an ad valorem royalty (a per-unit royalty).

In any case social welfare will increase with licensing. If the optimal contract includes a positive per-unit royalty licensing will increase social welfare due to both, consumer surplus and the profits of the industry, are greater with licensing than without it. If the optimal contract includes an ad valorem royalty licensing will increase social welfare though consumer surplus decreases.<sup>14</sup>

- If the optimal two-part tariff contract is a pure per-unit royalty the royalty is equal to the reduction in the marginal cost of production induced by the innovation.

<sup>13</sup>When the optimal contract is a pure ad valorem royalty we know that

$$\begin{aligned} CS^L - CS^0 &= \\ &= \frac{-a^2(-2+\gamma)(-4+3\gamma^2)(8+\gamma(4+\gamma(-2+\gamma)(5+4\gamma))) + 8\gamma^2c^2(8-10\gamma^2+3\gamma^4) + 16a\gamma^2c(-2+\gamma^2)(4+\gamma(-1+(-3+\gamma)\gamma))}{16\gamma^2(-4+\gamma^2)(-2+\gamma^2)^2} + \\ &+ \frac{(a\gamma^2(-2+\gamma)(5+4\gamma)+4a\gamma+8a)\sqrt{a^2(-2+\gamma)^2(4-3\gamma^2)^2+32a\gamma^2c(-2+\gamma)(-2+\gamma^2)+32\gamma^2c^2(-2+\gamma^2)}}{16\gamma^2(-4+\gamma^2)(-2+\gamma^2)^2} \text{ and} \\ W^L - W^0 &= \frac{a^2(-2+\gamma)(-4+3\gamma^2)(8+\gamma(-12+\gamma(-2+5\gamma))) + 8\gamma^2c^2(8-6\gamma^2+\gamma^4) + 16a\gamma^2c(-2+\gamma^2)(4+\gamma(-3+(-1+\gamma)\gamma))}{16\gamma^2(-4+\gamma^2)(-2+\gamma^2)^2} + \\ &+ \frac{(a\gamma^2(2-5\gamma)+12a\gamma-8a)\sqrt{a^2(-2+\gamma)^2(4-3\gamma^2)^2+32a\gamma^2c(-2+\gamma)(-2+\gamma^2)+32\gamma^2c^2(-2+\gamma^2)}}{16\gamma^2(-4+\gamma^2)(-2+\gamma^2)^2} \end{aligned}$$

<sup>14</sup>In this case  $W^L - W^0 = \frac{a^2(-2+\gamma)^4\gamma(4+\gamma)+8ac(-2+\gamma)^2(3+\gamma)(3\gamma^2-4)+4c^2(48-40\gamma^2+3\gamma^4)}{8(-2+\gamma)^2(2+\gamma)^2(-4+3\gamma^2)} > 0$ .

So, it is clear that under a pure per-unit royalty scheme the production levels are identical to the ones corresponding to the no licensing case, hence so is consumer surplus. Given that the profits of the industry are greater with licensing than without it, it is clear that licensing increases social welfare.

■

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