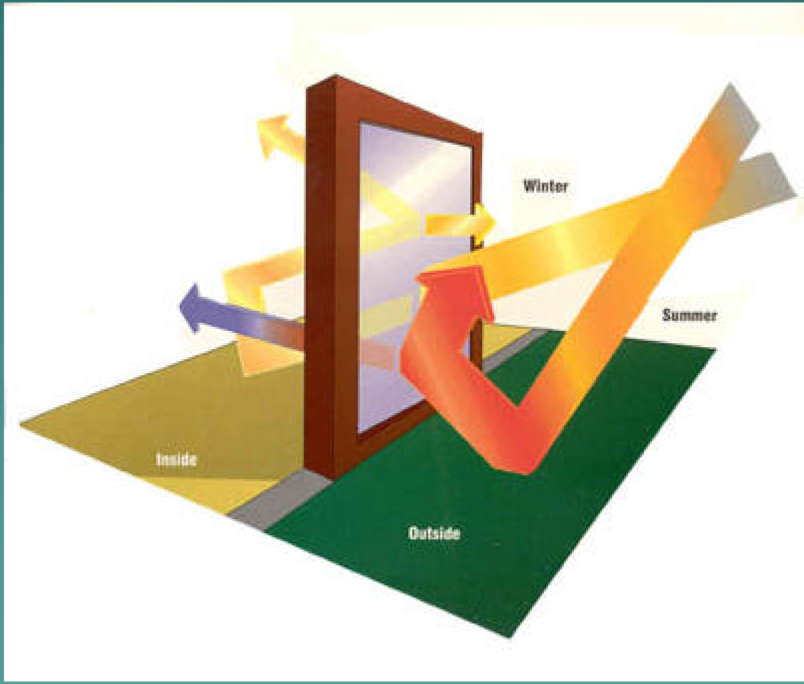
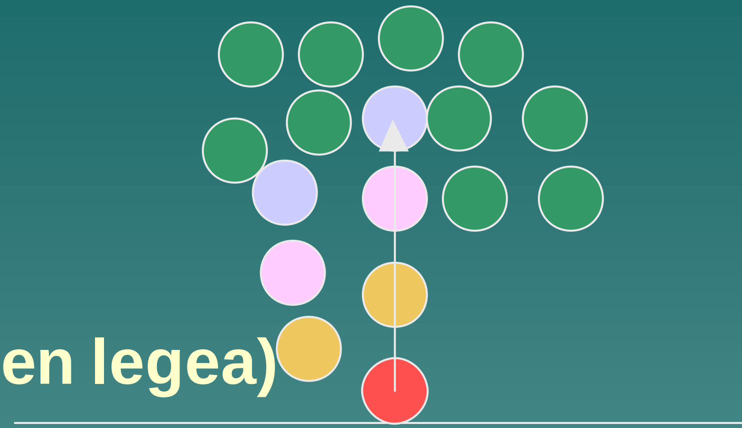
Bero-transmisioko mekanismoak

Egilea: Iñaki Gómez Arriaran



Beroa 3 mekanismoren bitartez transmiti daiteke:

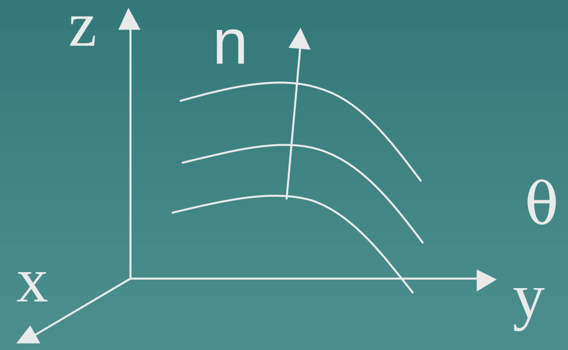


- **KONDUKZIOA (Fourier-en legea)**
- **KONBEKZIOA (Newton-en hozketa-legea)**
- **ERRADIAZIOA (Stefan-Boltzman-en legea)**

KONDUKZIOA

- Tenperatura-eremua ⟹ $\theta = \theta\ (x,y,z,t)$
- Tenperatura-gradientea ⟹ $\text{Grad}\ \theta = (\partial\theta/\partial n)\ \vec{n}$

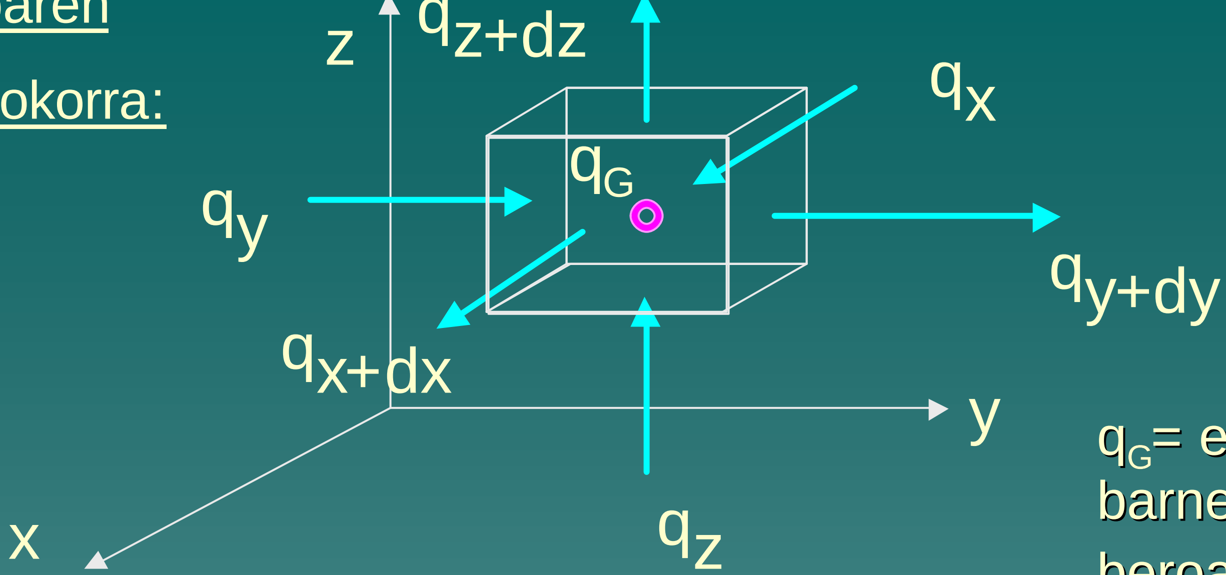
$\text{Grad}\ \theta = \nabla\ \theta = (\partial\theta/\partial x)\ i + (\partial\theta/\partial y)\ j + (\partial\theta/\partial z)\ k$



Fourier-en legea : $q = Q/A = -\ k\ \ (?)\ \nabla\ \theta$

$q = q_x\ i + q_y\ j + q_z\ k = -[\ k_x\ (\theta)\ \nabla\ \theta]\ i - [k_y\ (\theta)\ \nabla\ \theta]\ j - [k_z\ (\theta)\ \nabla\ \theta]\ k$

Kondukzioaren ekuazio orokorra:



q_G= elementuaren barnean garatutako beroa (W/m^3)

Energia-balantzea eginez:

dQsartu + dQgaratu = dQirten + dEmetatu

dQsartu = q_x dydz + q_y dxdz + q_z dxdy

dQgaratu = q_G dV

dQirten = q_{x+dx} dydz + q_{y+dy} dxdz + q_{z+dz} dxdy

dEmetatu = c_p $\partial\theta/\partial t$ dm = ρ dV c_p $\partial\theta/\partial t$

$$q_x\, dydz + q_y\, dxdz + q_z\, dxdy + q_G\, dV = q_{x+dx}\, dydz + q_{y+dy}\, dxdz + q_{z+dz}\, dxdy + \rho\, dV\, c_p\, \partial\theta/\partial$$

Fourier aplikatuz: $q_x = -k(\theta)\partial\theta/\partial x$

Taylor-en seriean garatuz: $q_{x+dx} = qx + (\partial q_x/\partial x)\, dx$

$q_x + [\, \partial(-k(\theta)\partial\theta/\partial x) / \partial x \,]\, dx$

$$q_G\, dV = [\, \partial(-k(\theta)\partial\theta/\partial x) / \partial x \,]\, dx\, dydz + [\, \partial(-k(\theta)\partial\theta/\partial y) / \partial y \,]\, dy\, dxdz + [\, \partial(-k(\theta)\partial\theta/\partial z) / \partial z \,]$$
$$z\, dxdy + \rho\, dV\, c_p\, \partial\theta/\partial t = \nabla[\, -k(\theta)\nabla\theta \,]\, dV + \rho\, dV\, c_p\, \partial\theta/\partial t$$

↓

$$q_G = \nabla[\, -k(\theta)\nabla\theta \,] + \rho\, c_p\, \partial\theta/\partial t$$

Hipotesiak:

- materiale isotropoa ⇒ $K(\theta)_x = K(\theta)_y = K(\theta)_z$
- propietatate fisikoak konstanteak ⇒ $K(\theta) = K = Kte$
- $q_G = kte$

$$k \nabla^2 \theta + q_G = \rho c_p \partial\theta/\partial t$$

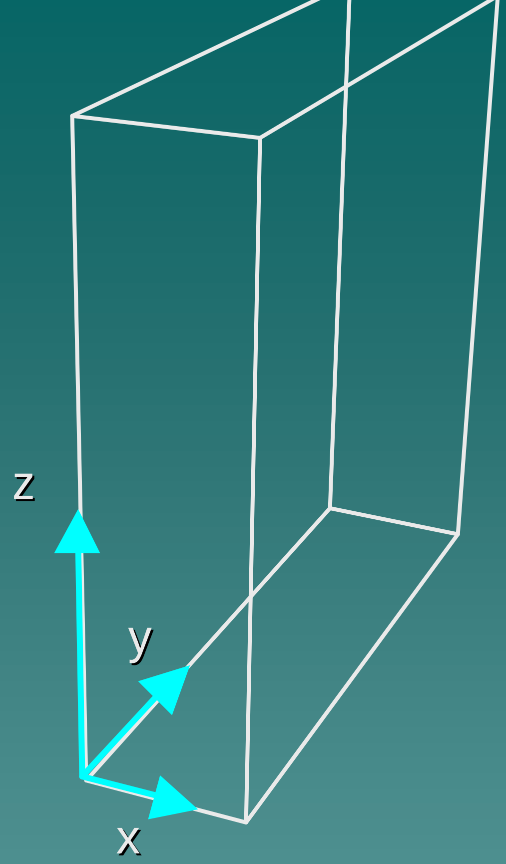
Kondukzioaren ekuazio orokorra

$\nabla^2 \theta$ = laplacetarra:

Koordenatu kartesiarretan ⇒ $\nabla^2 \theta = \partial^2\theta/\partial x^2 + \partial^2\theta/\partial y^2 + \partial^2\theta/\partial z^2$

Koordenatu zilindrikotan ⇒ $\nabla^2 \theta = 1/r\, \partial(r\partial\theta/\partial r)/\partial r + 1/r^2\, \partial^2\theta/\partial\phi^2 + \partial^2\theta/\partial z^2$

Koordenatu esferikotan ⇒ $\nabla^2 \theta = 1/r^2\, \partial(r^2\partial\theta/\partial r)/\partial r + ...$



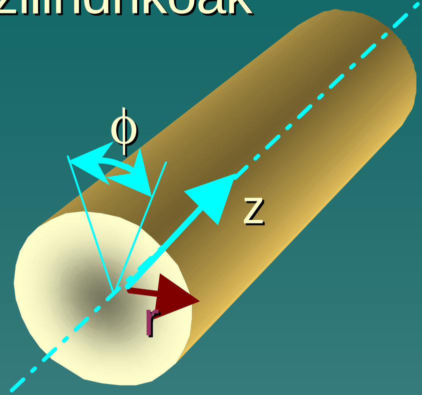
$\nabla^2 \theta$ = laplacetarra:

- koordenatu kartesiarrak ⇒ $\nabla^2 \theta = \partial^2\theta/\partial x^2 + \partial^2\theta/\partial y^2 + \partial^2\theta/\partial z^2$

$\nabla^2 \theta$ = laplacetarra

- koordenatu zilindrikoak

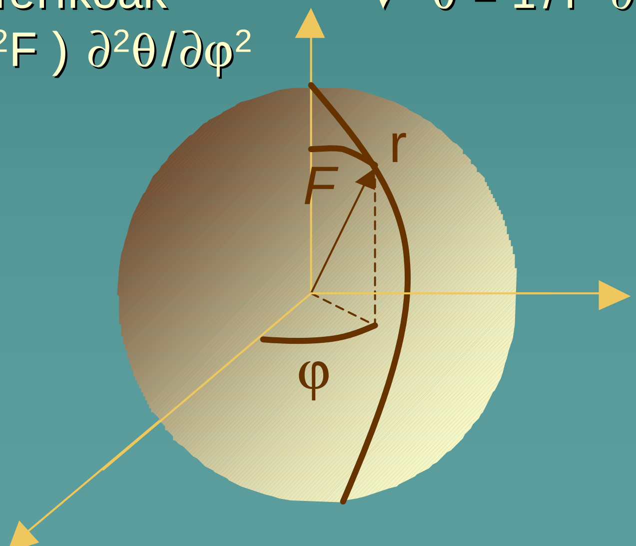
$\nabla^2 \theta = 1/r\, \partial(r\partial\theta/\partial r)/\partial r + 1/r^2\, \partial^2\theta/\partial\phi^2 + \partial^2\theta/\partial z^2$



$\nabla^2 \theta$ = laplacetarra:

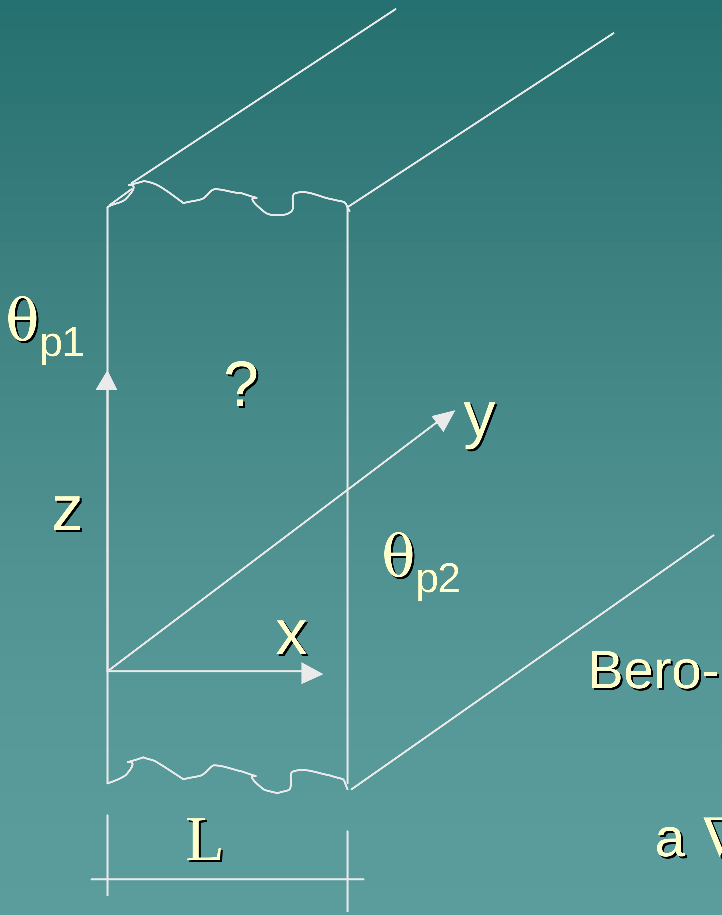
koordenatu esferikoak

$\nabla^2 \theta = 1/r^2\, \partial(r^2\partial\theta/\partial r)/\partial r + 1/(r^2 senF)\, \partial(senF\ \partial\theta/\partial F) + 1/(r^2 sen^2 F)\, \partial^2\theta/\partial\varphi^2$



Bero-garapenik gabeko pareta laua

Kondukzioaren ekuazio orokorra $a \nabla^2 \theta + q_G / \rho\, cp = \partial\theta/\partial t$



Tenperatura-eremua $\theta = \theta\ (x,y,z,t)$

Erregimen egonkorra ➡ $\partial\theta/\partial t = 0$

⬇

$\theta = \theta\ (x,y,z)$

Bero-garapenik gabe ➡ $q_G = 0$

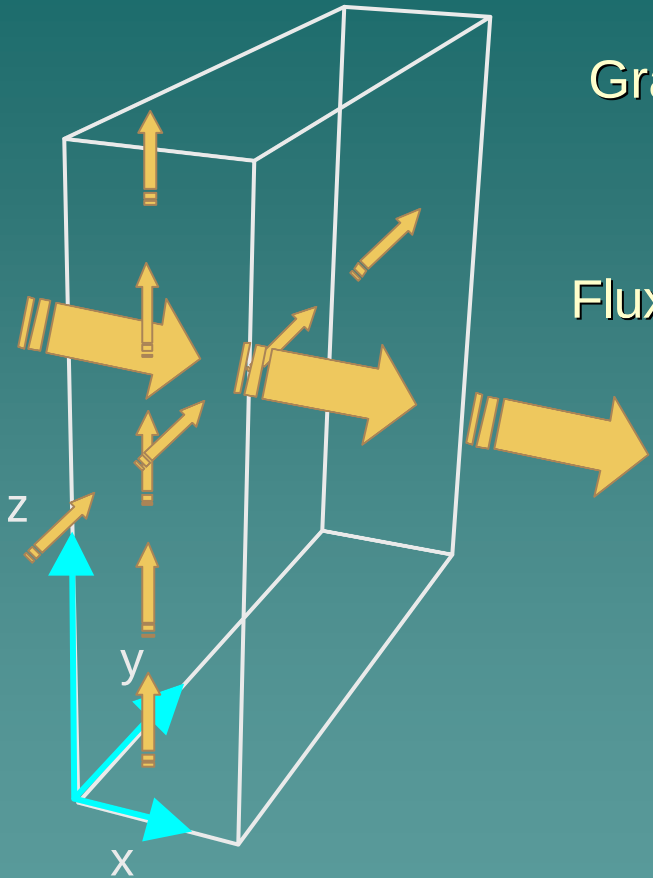
$a \nabla^2 \theta = 0$ ➡ $? \nabla^2 \theta = 0$

Bero-garapenik gabeko pareta laua

$$\text{Grad } \theta = \nabla \theta = (\partial\theta/\partial x)\, \vec{i} + (\partial\theta/\partial y)\, \vec{j} + (\partial\theta/\partial z)\, \vec{k}$$

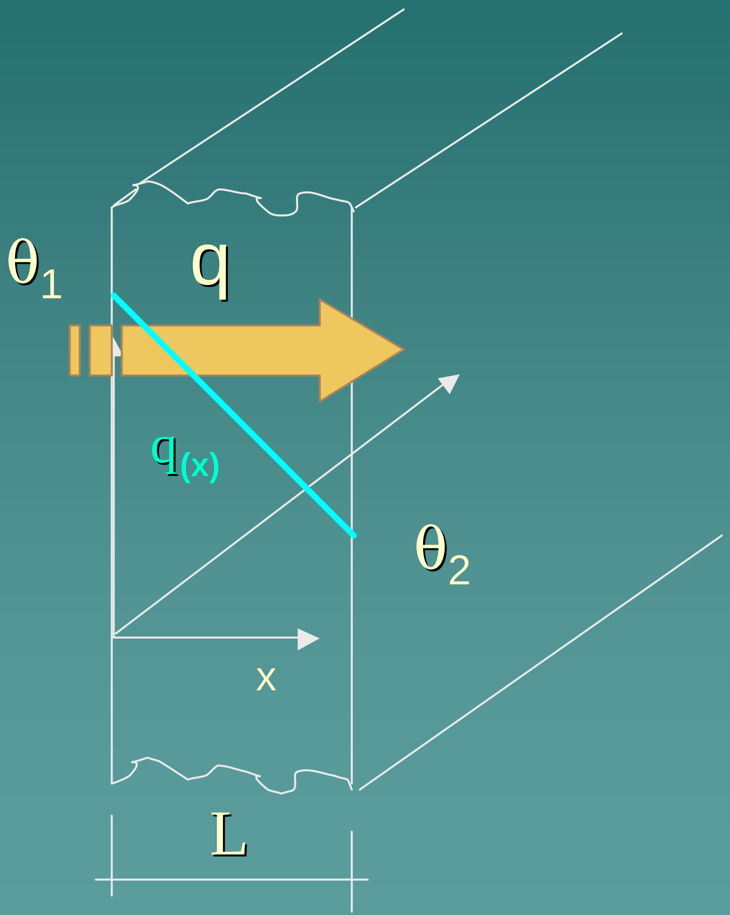
$$\text{Grad } \theta = \nabla \theta = (\partial\theta/\partial x) = d\theta/dx$$

$$\theta = \theta\ (\ x\ )$$



Fluxu dimentsiobakarra

Fluxu dimentsiobakarra



Laplacetarra $\nabla^2 \theta = \partial^2\theta/\partial x^2 = d^2\theta/dx^2$

? $\nabla^2 \theta = 0$

$\nabla^2 \theta = d^2\theta/dx^2 = 0$

$d\theta/dx = C_1$

$\theta(x) = C_1 x + C_2$? Lerro zuzena

C_1 eta C_2 integrazio-konstanteak ingurune-baldintzak aplikatuz askatzen dira:

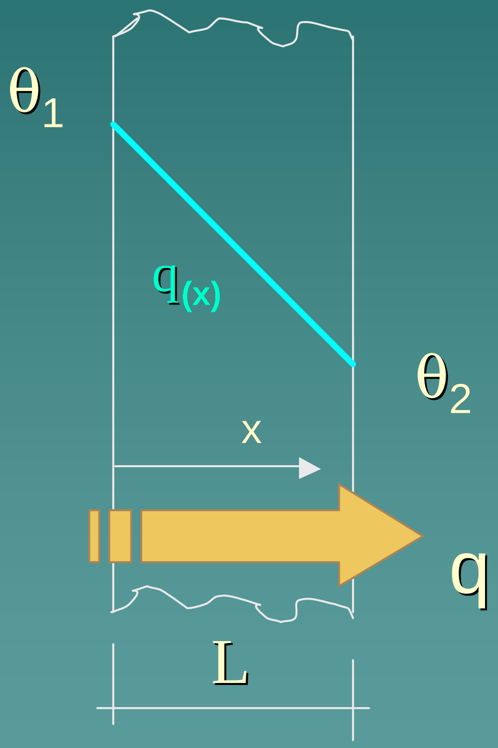
1. ing. bald.: $x = 0$ → $\theta = \theta_1$
2. ing. bald.: $x = L$ → $\theta = \theta_2$

1.i.b.: $\theta_1 = C_1 \cdot 0 + C_2$? $C_2 = \theta_1$

2.i.b.: $\theta_2 = C_1 \cdot L + \theta_1$? $C_1 = (\theta_2 - \theta_1) / L$

Tenperatura-banaketa

$$\theta(x) = \theta_1 + (\theta_2 - \theta_1)\, x / L$$



Fourier-en legea aplikatuz:

$$q_x = - ?\nabla\theta = - ?\, d\theta/dx = - ?\, [\, (\theta_2 - \theta_1) / L \,] = ? / L \cdot (\, \theta_1 - \theta_2 \,) = kte$$

Ohm-en legea

Fourier-en legea

$I = \Delta V_{2-1} / R$

$q = \Delta \theta_{2-1} / (L / ?)$

ΔV_{2-1} = Potentzial elektrikoaren diferentzia

I = Korronte elektrikoa

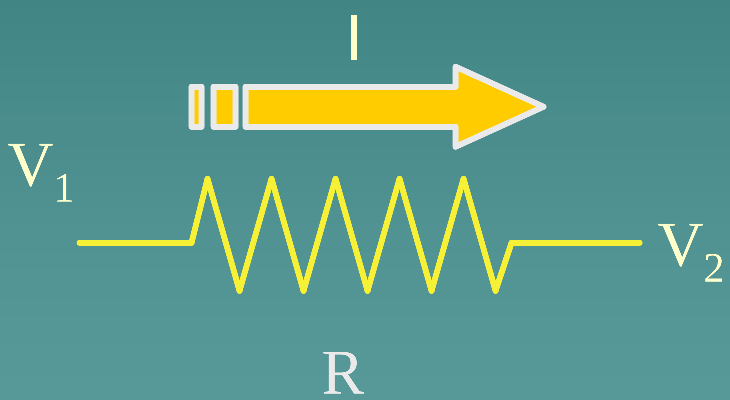
R = erresistentzia elektrikoa

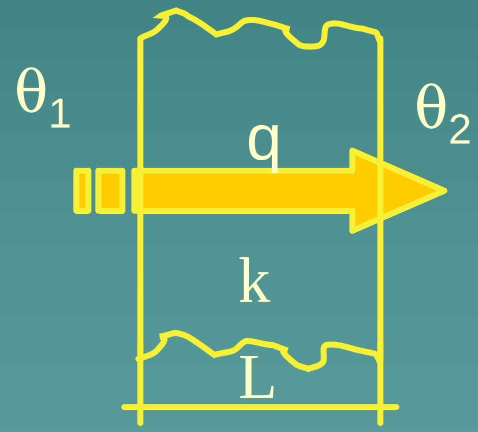
$\Delta \theta_{2-1}$ = Potentzial termikoaren diferentzia

q = Bero-fluxua

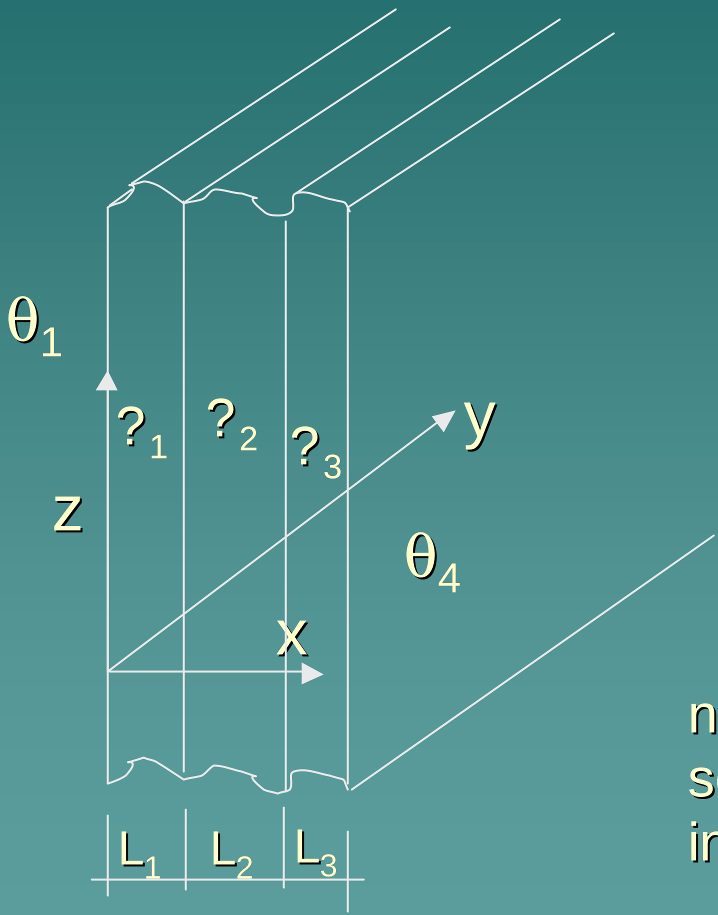
L / ? = erresistentzia termikoa

R (m^2 º C / W) pareta lauaren erresistentzia termikoa





Pareta konposatua



Kondukzioaren ekuazio orokorra, erregimen egonkorrean, fluxu dimentsiobakarra eta bero-garapenik gabe :

$$? \nabla^2 \theta = 0$$

Geruza bakoitzarentzat:

$$?_i \nabla^2 \theta_i = 0$$

$$\nabla^2 \theta_i = d^2\theta_i/dx^2 = 0$$

$$d\theta_i/dx = C_1$$

$$\theta_i(x) = C_1 x + C_2 \; ? \; \text{Recta}$$

n geruzen kasuan, 2n integrazio konstante sortuko dira ($C_1, \ldots, C_{2n}$) eta, askatzeko, 2n ingurune baldintza beharko dira

- 1. mailako 2 ingurune-baldintza:

1. i.b.: $x = 0$ $\theta = \theta_1$

2. i.b.: $x = L_1+L_2+L_3+\ldots L_n$ $\theta = \theta_{n+1}$

- 1. mailako n-1 ingurune-baldintza:

3. i.b.: $x = L_1$ $\theta_1(x) = \theta_2(x)$

.

.

n+1. i.b.: $x = L_1+L_2+L_3+\ldots L_{n-1}$ $\theta_{n-1}(x) = \theta_n(x)$

- 4. mailako n-1 ing. bald.:

n+2. i.b.: $x = L_1$ $q(x)_1 = q(x)_2$

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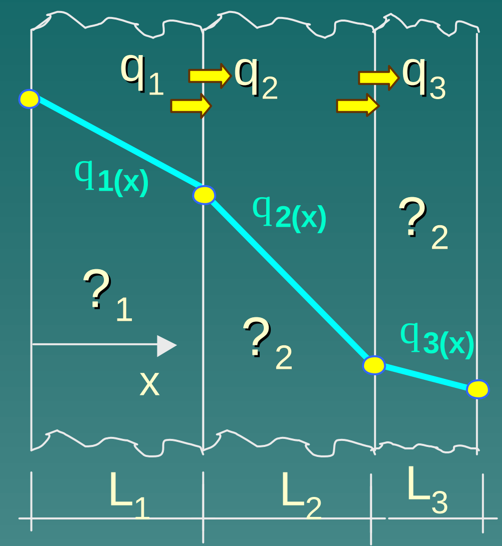
2n. i.b.: $x = L_1+L_2+L_3+\ldots L_{n-1}$ $q(x)_{n-1} = q(x)_n$



2n ekuazio-sistema garatzen da 2n ezezagunekin

Geruza bakoitza banaka aztertuz:

- Fourier aplikatuz 1. geruzan:

$q = -(\theta_2 - \theta_1) / (L_1/?_1)$? $\theta_1 - \theta_2 = q \cdot L_1/ ?_1$

- Fourier aplikatuz 2. geruzan:

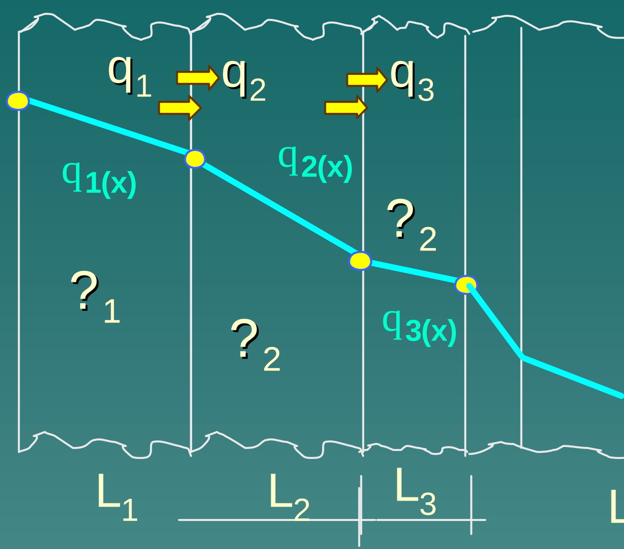
$q = -(\theta_3 - \theta_2) / (L_2/?_2)$? $\theta_2 - \theta_3 = q \cdot L_2/ ?_2$

.

.

- Fourier aplikatuz n. geruzan:

$q = -(\theta_{n+1} - \theta_n) / (L_n/?_n)$? $\theta_n - \theta_{n+1} = q \cdot L_n/ ?_n$

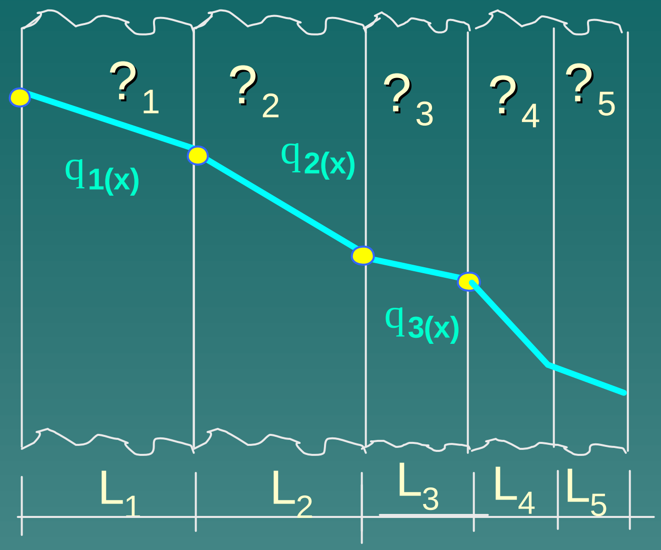


$\theta_1 - \theta_{n+1} = q \cdot (L_1/ ?_1 + L_2/ ?_2 \ldots + L_n/ ?_n)$

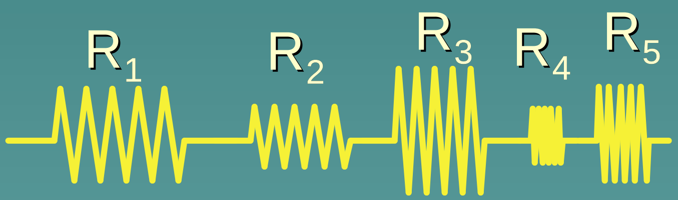
$q = (\theta_1 - \theta_{n+1}) / (L_1/ ?_1 + L_2/ ?_2 + \ldots + L_n/ ?_n)$

R pareta konposatuaren erresistentzia termikoa

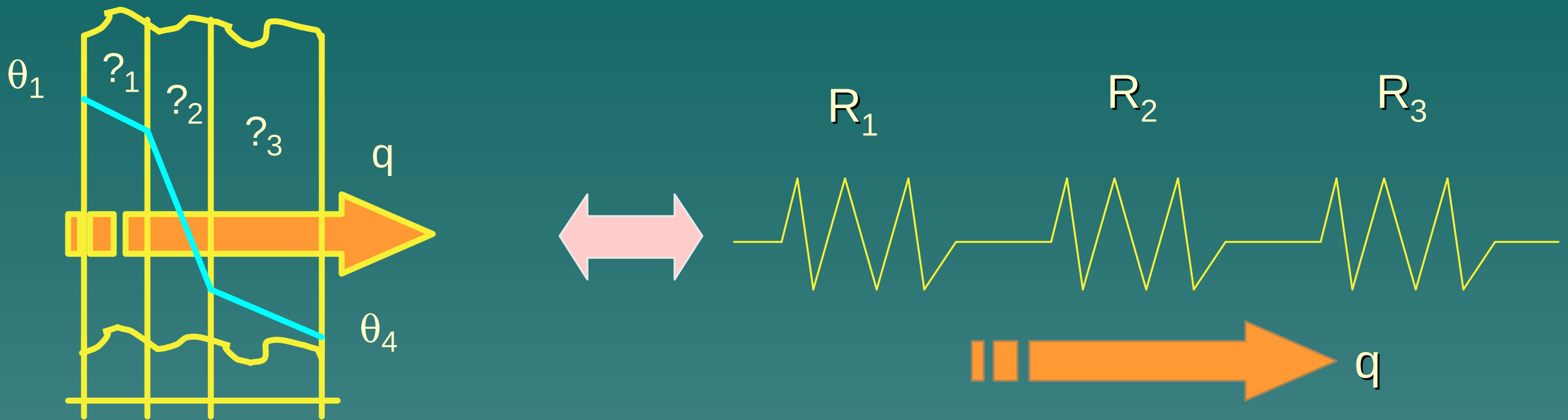
Analogia elektrikoa



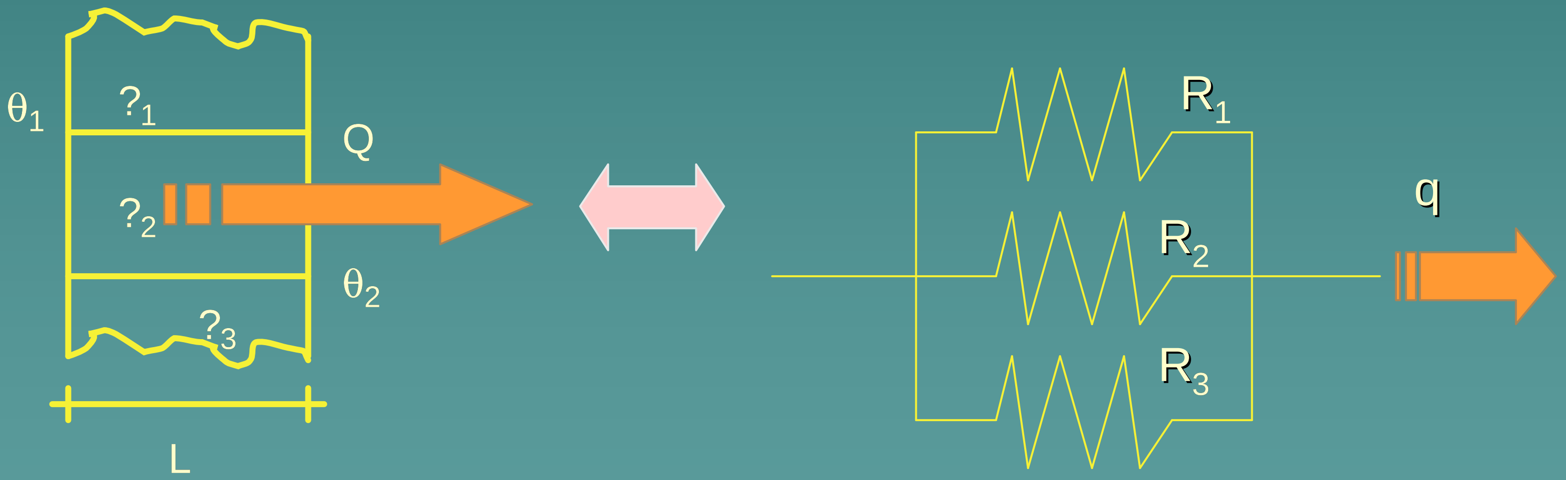
Zirkuito elektriko baliokidea:



Pareta konposatuaren analogia elektrikoa



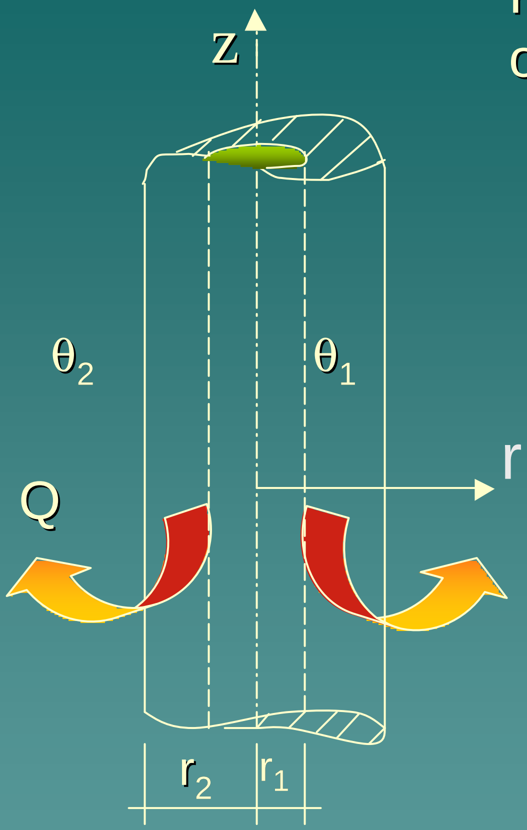
$$q = (\theta_1 - \theta_4) / (R_1 + R_2 + R_3)$$



$$Q = (\theta_1 - \theta_2) \cdot (1/R_1 + 1/R_2 + 1/R_3)$$

$$R_i = L_i / A_i\ ?_i$$

Bero-garapenik gabeko pareta zilindrikoa



Kondukzioaren ekuazio orokorra

$$a \nabla^2 \theta + q_G / \rho \, cp = \partial\theta/\partial t$$

Tenperatura-eremua → $\theta = \theta (r, F , z)$

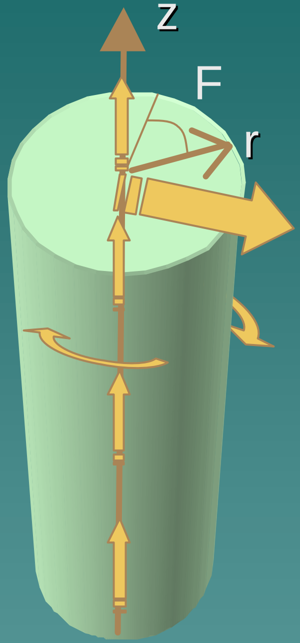
Erregimen egonkorra ⇒ $\partial\theta/\partial t = 0$

⇓

$\theta = \theta (r, F , z)$

Bero-garapenik gabe ⇒ $q_G = 0$

$a \nabla^2 \theta = 0$ ⇒ $? \nabla^2 \theta = 0$



$$\nabla \theta = r(\partial\theta/\partial r) + 1/r\,(\partial\theta/\partial\phi) + (\partial\theta/\partial z)$$

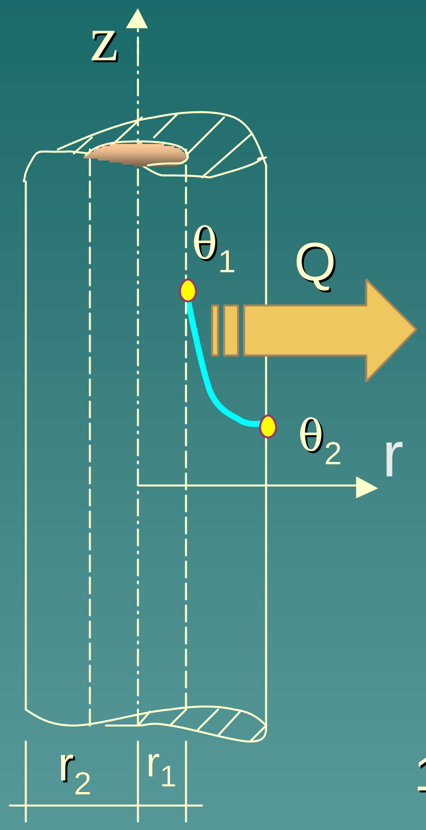
$$\text{Grad}\ \theta = \nabla \theta = r(\partial\theta/\partial r) = r d\theta/dr$$

$$\theta = \theta\,(\,r\,)$$

Fluxu dimentsiobakarra

Fluxu dimentsiobakarra

$$?\nabla^2 \theta = 0$$



Laplacetarra

$$\nabla^2 \theta = 1/r \cdot \partial(r\partial\theta/\partial r)/\partial r = 1/r \cdot d(rd\theta/dr)/dr$$

$$1/r \; d(rd\theta/dr)/dr = 0$$

$$d(rd\theta/dr)/dr = 0$$

$rd\theta/dr = C_1$? $d\theta/dr = C_1/r$

$\theta(r) = C_1 \ln r + C_2$? esponentziala

1.ing. baldintza: $r = r_1$ → $\theta = \theta_1$

2. ing. baldintza : $r = r_2$ → $\theta = \theta_2$

1.i.b. : $\theta_1 = C_1 \ln r_1 + C_2$

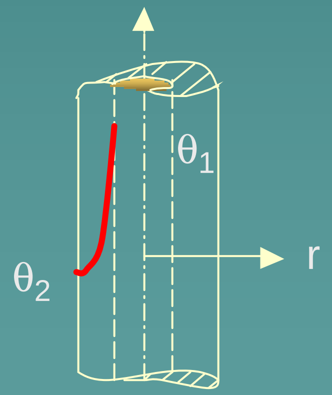
2.i.b. : $\theta_2 = C_1 \ln r_2 + C_2$

$$C_1 = (\theta_1 - \theta_2) / \ln(r_1 / r_2)$$

$$C_2 = \theta_1 - \ln r_1 [(\theta_1 - \theta_2) / \ln(r_1 / r_2)]$$

$$\theta(r) = [(\theta_1 - \theta_2) / \ln(r_1 / r_2)] \ln r + \theta_1 - \ln r_1 [(\theta_1 - \theta_2) / \ln(r_1 / r_2)]$$

$$\theta(r) = [(\theta_1 - \theta_2) \ln(r / r_1) / \ln(r_1 / r_2)] + \theta_1$$



Fourier aplikatuz:

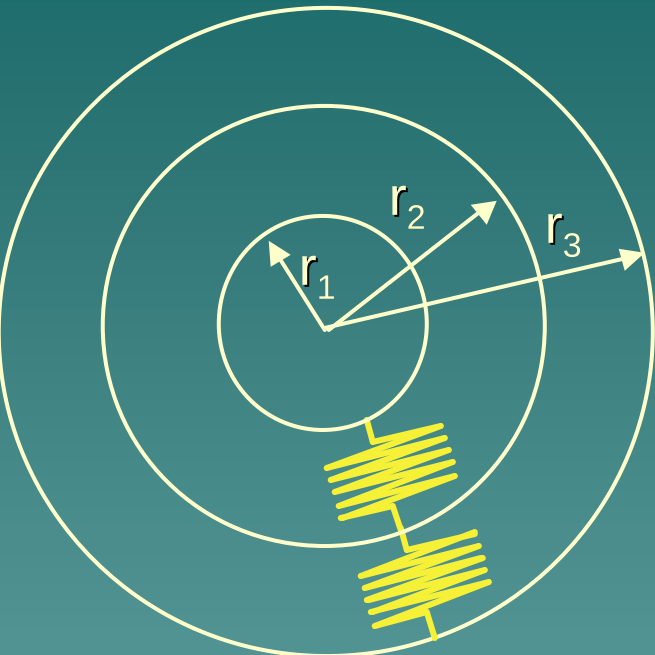
$$Q_r = - ?\ A\nabla\theta = - ?\ A\ d\theta/dr = - ?\ 2\pi\ r\ L \cdot (\ \theta_1 - \theta_2\ ) / \ r \ln (\ r_1 / r_2\ ) =$$

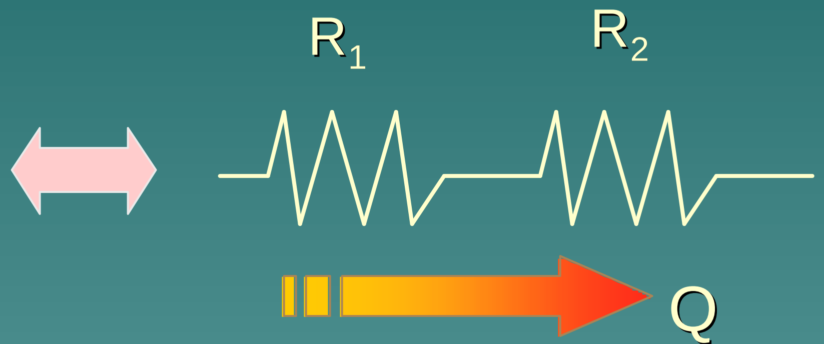
$$Q_r = (\ \theta_1 - \theta_2\ ) / [\ \ln (\ r_2 / r_1\ ) / 2\pi\ ?\ L\ ]$$

R (º C / W) pareta zilindrikoaren errresistentzia termikoa

$$R = \ln (\ r_2 / r_1\ ) / 2\pi\ ?\ L$$

Pareta zilindriko konposatua

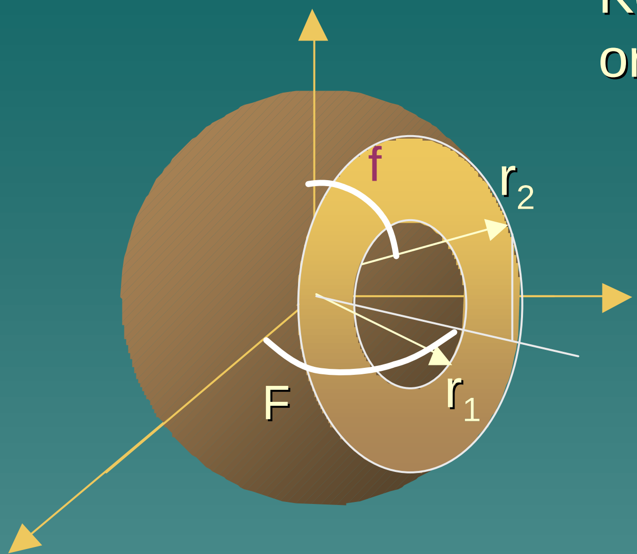




R_1 = ln (r_2 / r_1) / 2π $?_1$ L

R_2 = ln (r_3 / r_2) / 2π $?_2$ L

Bero-garapenik gabeko pareta esferikoa



Kondukzioaren ekuazio orokorra

$$a \nabla^2 \theta + q_G / \rho \, cp = \partial\theta/\partial t$$

Tenperatura-eremua $\theta = \theta\,(r, F, f, t)$

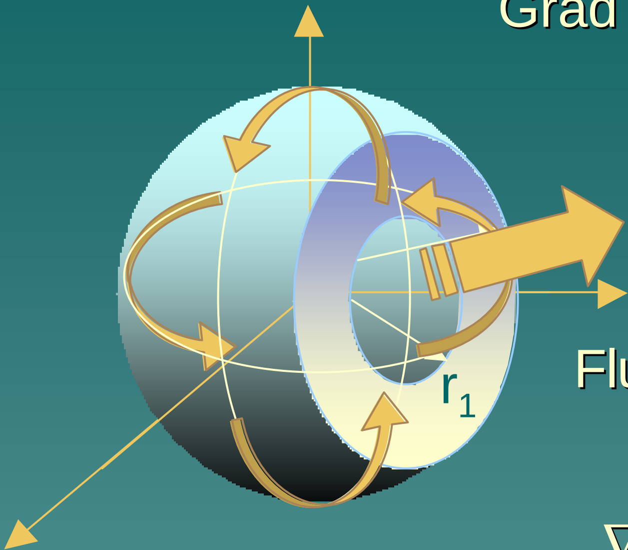
erregimen egonkorra ➡ $\partial\theta/\partial t = 0$

⬇

$$\theta = \theta\,(r, F, f)$$

Bero-garapenik gabe ➡ $q_G = 0$

$a \nabla^2 \theta = 0$ ➡ $? \nabla^2 \theta = 0$



$$\text{Grad}\ \theta = \nabla\ \theta = \partial\theta/\partial r\ +\ 1/r\,\text{senf}\ \ (\partial\theta/\partial F)\ \ +\ 1/r\ (\partial\theta/\partial f\ )$$

$$\text{Grad}\ \theta = \nabla\ \theta = (\partial\theta/\partial r) = d\theta/dr$$

$$\theta = \theta\ (\ r)$$

Fluxu dimentsiokarra

$$\nabla^2\ \theta\ \ = 0 = 1/r^2\ d(r^2 d\theta/dr)/dr$$

$$d(r^2 d\theta/dr)/dr = 0$$

$$r^2 d\theta/dr = C_1 \Rightarrow d\theta/dr = C_1\ /\ r^2$$

$$\theta(r) = C_1\ /\ r + C_2$$

1.i.b.: $r = r_1$ → $\theta = \theta_1$

2. i.b.: $r = r_2$ → $\theta = \theta_2$

1.i.b. : $\theta_1 = C_1 / r_1 + C_2$

2.i.b.: $\theta_2 = C_1 / r_2 + C_2$

→

$C_1 = (\theta_1 - \theta_2) / (1/ r_1 - 1/ r_2)$

$C_2 = \theta_1 - (\theta_1 - \theta_2) / r_1 (1/ r_1 - 1/ r_2)$

$$\theta(r) = C_1 / r + C_2 = (\theta_1 - \theta_2) / r (1/r_1 - 1/r_2) + \theta_1 - (\theta_1 - \theta_2) / r_1 (1/r_1 - 1/r_2) =$$

$$\theta(r) = \theta_1 + (\theta_1 - \theta_2) \cdot [(1/r - 1/r_1) / (1/r_1 - 1/r_2)]$$

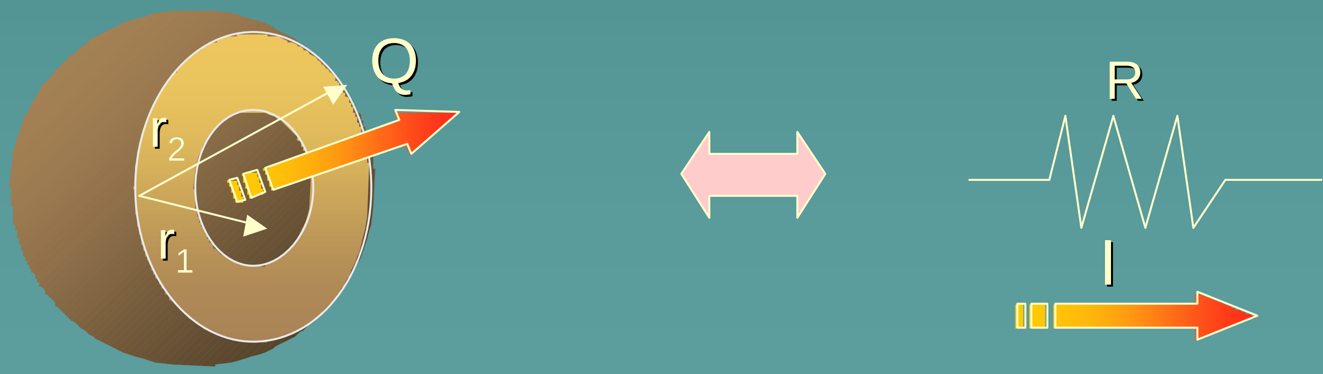
Fourier-en legea:

Q_r = - ? A$\nabla\theta$ = -? A dθ/dr = -? 4π r^2 (θ_1 - θ_2) / r^2(1/r_1 - 1/r_2) =

Q_r = (θ_1 - θ_2) / [(1/r_2 - 1/r_1)/ 4 π ?]

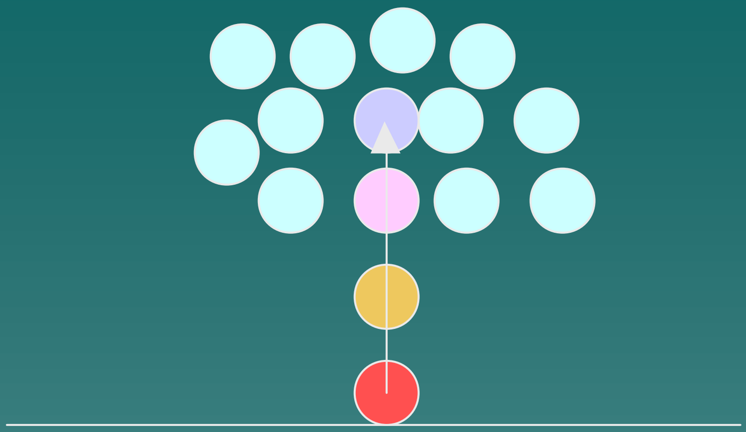
Pareta esferikoaren erresistentzia termikoa :

R = (1/ r_2 - 1/ r_1)/ 4 π ? = (r_2 -r_1) / 4 π ? r_2 r_1



KONBEKZIOA

Fluidoaren molekulen arteko distantzia handia dela eta, kondukzio bidezko bero-transmisioarekiko erresistentzia termikoa handia da.



Molekulen arteko loturak ahulak izanik, bero dagoen molekula fluidoan barne mugi daiteke, berekin batera energia termikoa garraiatu ⇒ bero-transmisioa.

Materiaren garraioaren bitartez gertatzen den bero-transmisioko mekanismo horri KONBEKZIO deritzo.

Konbekzio bidezko bero-transmisioa faktore askoren araberakoa da:

- Jariakinaren abiadura (c)
- Ukipen-azaleraren geometria eta ezaugarriak
- Jariakinaren propietate fisikoak (ρ , μ)
- Solidoaren propietate fisikoak (k , c_p)
- Eta abar.

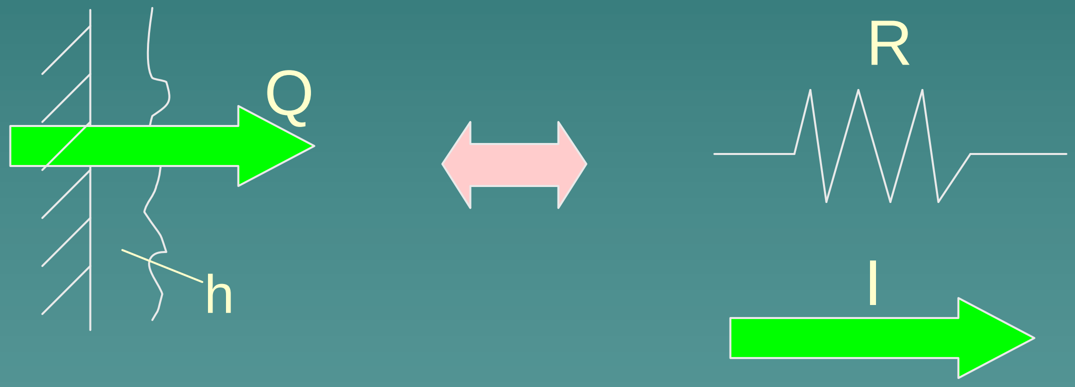
Denak laburbiltzeko, koefiziente bat erabiltzen da: **h** = konbekzio-koefiziente edota pelikula-koefizientea.

h pelikula-koefizientea korrelazio esperimentalen bitartez kalkulatzen da.

Newton-en hozketa-legea: $Q = h\,A\,\Delta\theta$

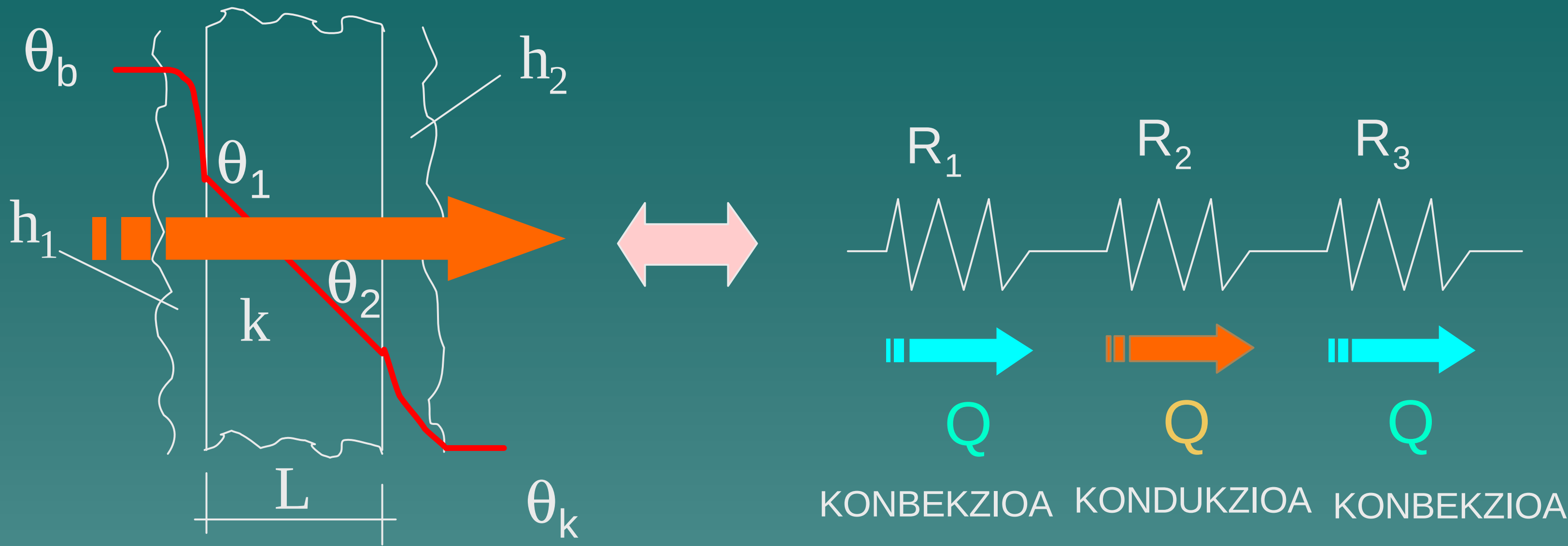
h (W/m^{2}K)

Analogia elektrikoa:



$R = 1 / h\,A$

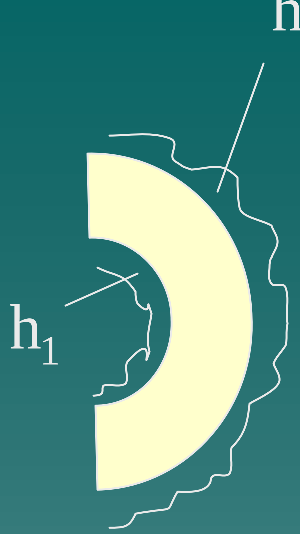
U : Bero-transmisioko koefiziente orokorra

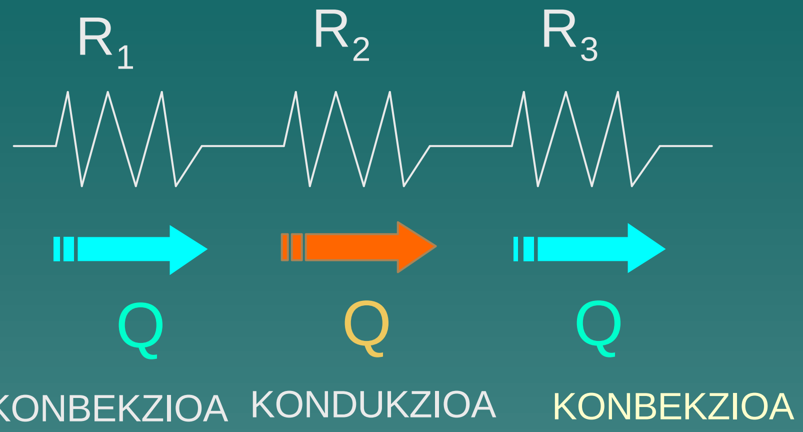


$$R = R_1 + R_2 + R_3 = 1/A \, (1/h_1 + L/k + 1/h_2)$$

$$Q = (\theta_b - \theta_k) / R = A (\theta_b - \theta_k) / [1 / h_1)+ L / k + 1 / h_2]$$

$$U = 1 / [1 / h_1)+ L / k + 1 / h_2]$$ → $$Q = U \, A \, \Delta\theta$$





$$R = R_1 + R_2 + R_3 = 1/2\pi L \ (\ 1/r_1 h_1 + 1/k \ \ln(r_2/r_1) + 1/r_2 h_2$$

$$Q = \ (\theta_b - \theta_k \ ) / R = 2\pi \ r_2 \ L \ (\theta_b - \theta_k \ ) / [\ (r_2 / r_1 \ h_1 \ ) + (\ r_2 / k \ ) \ln(r_2/r_1) + 1 / h_2 \ ]$$

$$Q = U_2 \ A_2 \ \Delta\theta$$

$$U_2 = 1 / [\ (r_2 / r_1 \ h_1 \ ) + (\ r_2 / k \ ) \ln(r_2/r_1) + 1 / h_2 \ ]$$

KONBEKZIO BEHARTUA

Reynolds zenbakia: Jariakinaren inertzia-indarren eta likatsutasun-indarren arteko erlazioa.

$$Re = c\, \Pi / \upsilon$$

Prandtl zenbakia: Jariakinean barne-beroa zein abiaduraz transmititzen den adierazten du.

$$Pr = c_P\, \mu / k$$

Nusselt zenbakia: jariakinaren eta paretaren arteko bero-transmisioa adierazten du.

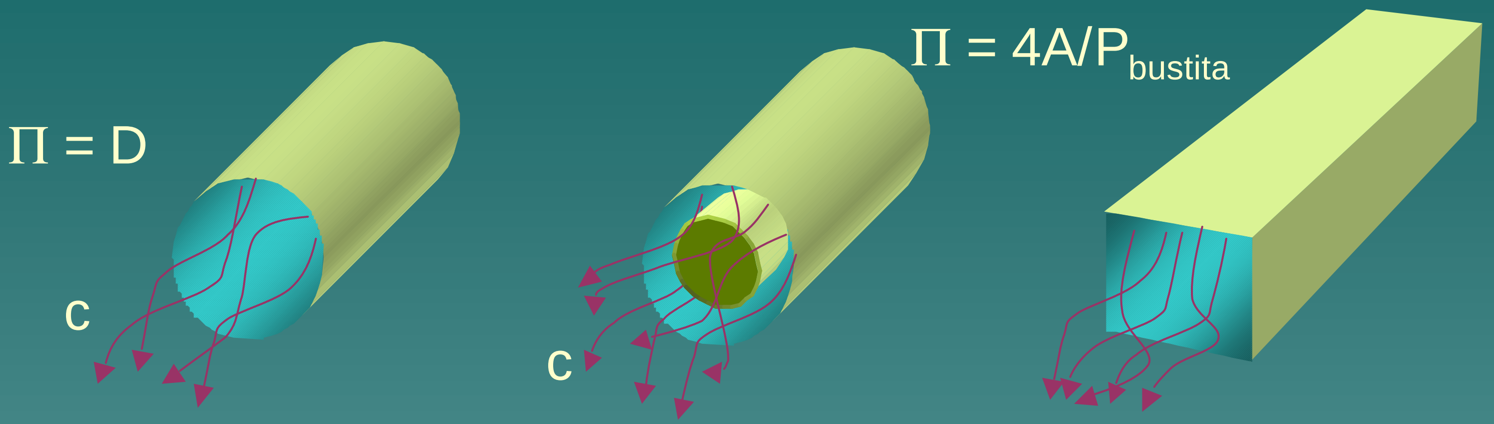
$$Nu = \Pi\, h / k$$

$$Nu = f\,(Re, Pr)$$

Parametro horien arteko erlazioa esperimentalki lortu behar da, saioak eginez.

ZENBAIT KORRELAZIO ESPERIMENTAL

↗ **1.kasua: Tutu baten barnealdeko konbekzioa, jarioa zurrunbilotsua denean.**



Dittus-Boelter → $Nu = 0{,}023\ Re^{0{,}8}\ Pr^{n}$

n=3 hozten bada

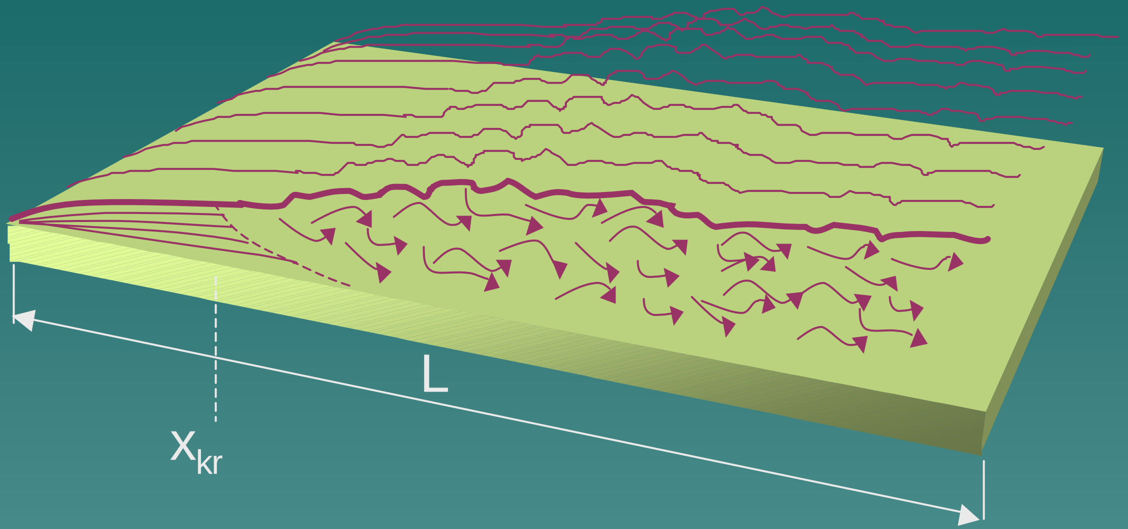
n= 4 berotzen bada

D.B. aplikatzeko baldintzak:

- Re >2100 (zurrunbilotsua)

- parametro adimentsionalak jariakinaren batez besteko tenperaturan

↗ 2.kasua: Gainazal lau batean zeharreko konbekzio behartua.



Parametroak pelikularen batez besteko tenperaturan neurtuak:

$$\theta m = (\theta p + \theta f) / 2$$

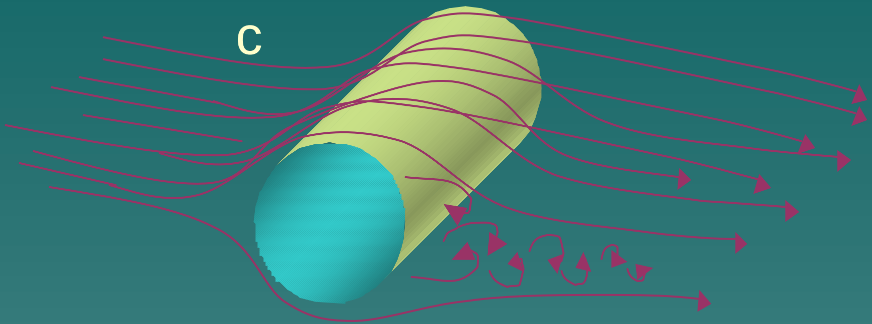
$\Pi = L$

Re < $5 \cdot 10^4 \div 5 \cdot 10^5$ ➡ Fluxu laminarra ➡ $Nu_L = 0{,}664\ Re_L^{1/2}\ Pr^{1/3}$ $L \leq x$

Re > $5 \cdot 10^4 \div 5 \cdot 10^5$ ➡ Fluxu zurrunbilotsua ➡ $Nu = 0{,}036\ Re_L^{0{,}8}\ Pr^{1/3}$ $L >> x_{kr}$

Fluxu mistoa ➡ $Nu = 0{,}036\ Pr^{1/3}\ (Re_L^{0{,}8} - 23.200)$ $L > x_{kr}$

↗ **3.kasua: Zilindro baten kanpoaldeko gainazalarekin gurutzatzen korronte baten konbekzio behartua.**

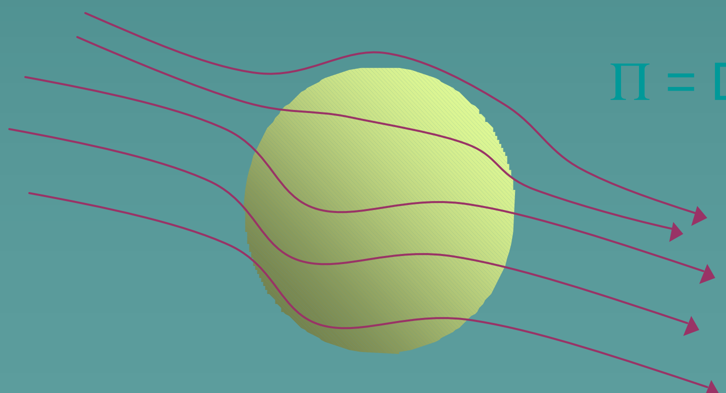


Π = D Parametroak pelikularen batez besteko tenperaturan neurtuak:

Churchill-Bernstein

$$Nu = 0{,}3 + [(0{,}62\ Re^{1/2} Pr^{1/3}) / [1+(0{,}4/Pr)^{2/3}]^{1/4} \cdot [1+(Re/282.000)^{1/2}]]$$

↗ **4.kasua: Esfera baten kanpoaldeko gainazalarekin gurutzatzen den korronte baten konbekzio behartua.**

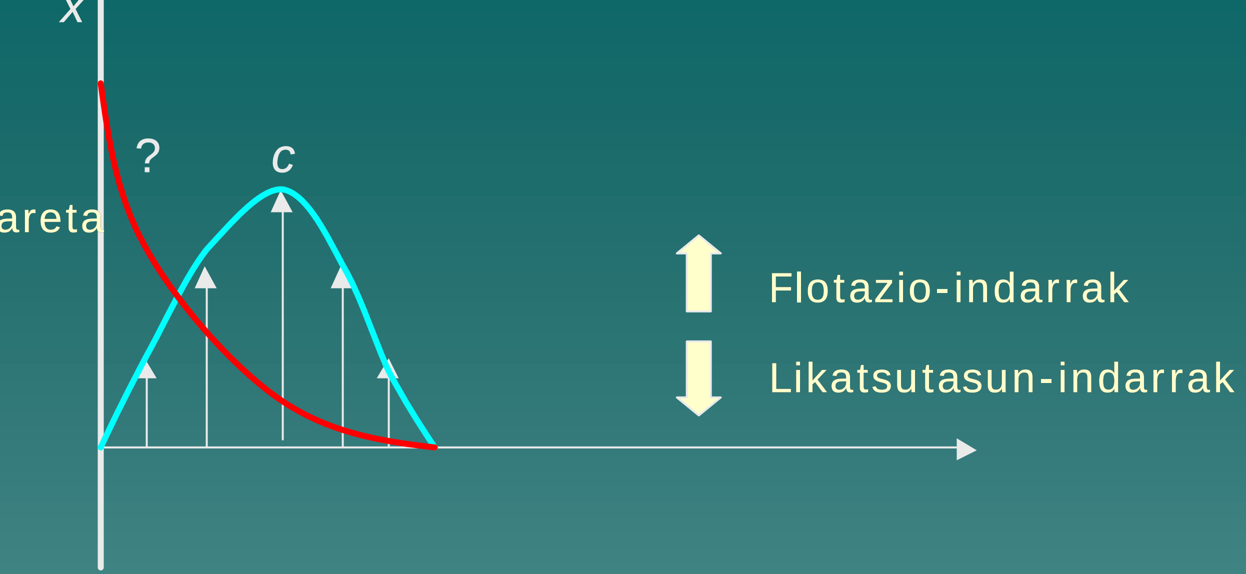


Π = D

Whitaker

$$Nu = 2+(0{,}4Re^{1/2}+0{,}06Re^{2/3})Pr^{0{,}4}$$

KONBEKZIO NATURALA



<u>Grashof zenbakia:</u> Fluidoaren igotze-indarren eta likatsutasun-indarren arteko erlazioa.

$$Gr = g\beta\Delta\theta\Pi^3\rho^2 / \mu^2$$

Gas idealetan : $\beta = 1/T$

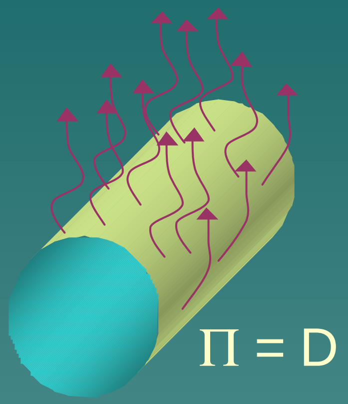
Grashof zenbakia handiagoa den neurrian, handiagoa izango da jariakinaren mugimendu librea

$$Nu = f\ (Gr, Pr)$$

<u>Rayleigh-en zenbakia:</u> $Ra = Gr \cdot Pr$

$Gr \cdot Pr > 10^8$ ➡ jario zurrunbilotsua

↗ 1.kasua: Zilindro horizontal baten kanpoaldeko gainazalarekiko konbekzio naturala.

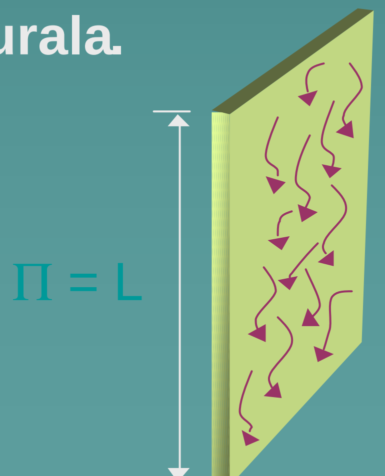


$10^4 < Gr < 10^9$ ➡ $h = 1{,}32 \, [\, (\theta - \theta_f) \, / \, D \,]^{1/4}$

$10^9 < Gr < 10^{12}$ ➡ $h = 1{,}25 \, (\theta - \theta_f)^{1/3}$

θ = gainazalaren tenperatura
θ_f = jariakinaren tenperatura

↗ 2.kasua: Plaka bertikal baten gainazalarekiko konbekzio naturala.



$10^4 < Gr < 10^9$ ➡ $h = 1{,}42 \, [\, (\theta - \theta_f) \, / \, L \,]^{1/4}$

$10^9 < Gr < 10^{12}$ ➡ $h = 1{,}31 \, (\theta - \theta_f)^{1/3}$