ON THE INTERPLAY BETWEEN LOGIC AND PHILOSOPHY: A HISTORICAL PERSPECTIVE

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ABSTRACT

In this historical essay, we examine the reciprocal influences of philosophical doctrines and logic, their interrelations with language, and the place of mathematics in these developments.

Our concern in this essay is the interplay between logic and philosophy. The spectrum of philosophical traditions and topics is wide, ranging from inspirational, aphoristic, and poetic wisdom-searching philosophies to stark anti-metaphysical logical positivism. However, logic has flourished only within those philosophical traditions in which critical discussion and debates played a major role. "Formal logic, so far as we know", writes BOCHEŃSKI (1970, p.10), "originated in two and only two cultural regions: in the West and in India. Elsewhere, e.g., in China, we do occasionally find a method of discussion and a sophistic, but no formal logic in the sense of Aristotle or Dignaga was developed there."

In the discussion that follows, limited essentially to Western thought, an attempt will be made to elucidate how certain philosophical conceptions determine -or at times even block- the development of formal logic, and reciprocally, how formal logic contributes to bringing various philosophical issues into sharper focus. Foremost and inextricably related are problems of language and their interrelations with mathematics

In a diagrammatical way, we conceive of philosophy, logic, and mathematics as a triad, represented by a triangle with logic and mathematics at the base, language at the pivotal center and philosophy at the apex.

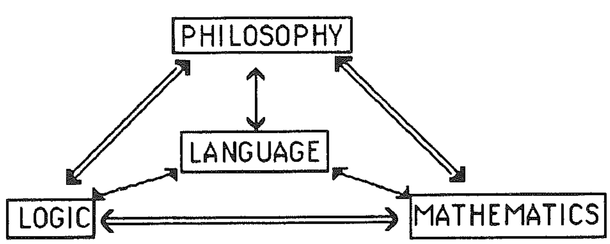

Figure 1

In view of the complexity of the multiple interactions, as indicated by the arrows in Fig.1, we are forced to limit ourselves to some salient points, leaving out

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a discussion of issues within the narrow scope of the philosophy of mathematics. We shall thus concentrate on the following paths:

(1) From philosophy to logic
(2) Between language and logic
(3) From mathematics to logic
(4) From mathematical logic to language and philosophy.

1. Philosophical presuppositions in logic: some case studies

Greek logic grew out of an analysis of effective modes of argumentation and the rational construal of evidence, both in philosophical as well as in legal contexts, and not, as has frequently been claimed, by an analysis of the *principles* of demonstration in geometry. Indeed, as will be discussed below, logic and mathematics developed without significant reciprocal influences until the 19th century, when mathematics was at last ripe to transform and revitalize the science of logic. (The unique place of Leibniz will be discussed later.).

Viewed from a current perspective, logic in the narrow sense deals with principles of argumentation, with an emphasis on canons of valid inferences and proofs. In a larger sense, logic comprises a study of the syntax and semantics of language, propositional attitudes and the like; and extending the boundaries still further, logic borders on epistemology and the methodology of rational inquiry, as for example in MILL(1843), HUSSERL(1900), and POPPER(1935).

From Antiquity to date, philosophers have debated where to fix the boundaries of logic. Even within the narrow confines of logic as the science of valid inferences, *logic is laden with philosophical presuppositions that determine the scope, content, and form of logical inquiries*. Quine's philosophy is a case in point: Quine's strictures and regimentation of ordinary language analysis have their repercussions in his work qua logician. Typical examples are Quine's treatment of quantifiers, his opposition to second-order logic, modal logic and quantum logic, the latter being branded as "deviant" logics. (See QUINE, 1970). Whereas Quine (op. cit.) entitles his book "Philosophy of Logic" (in the singular), HAAK (1978) presents in her "Philosophy of Logi*cs*" a pluralistic conception of logic. We shall subsequently talk about 'logical structures', 'logic embedded in language', etc., though without attempting to offer a precise definition of 'logic' or 'logical', in order to avoid endless regress. (See however the proposals by TARSKI, 1986.) STRAWSON (1952, chapt. 8) speaks of two kinds of logic: "Side by side with the study of formal logic (dealing with entailment-relations), and overlapping with, we have another study: the study of the logical features of ordinary speech." (p.231). We understand 'logic' and 'logical operations' to extend to features of natural language and discourse without further ado.

One of the ancient philosophical/religious problems to have found repercussions in formal logic is the vexed question of fate versus free will, determinism versus indeterminism. Is the future predetermined in some sense or not ? In the parlance of logic, do propositions relating to the future have *at present* a definite truth-value, even if unknown ? Two logical issues emerge. The first is the issue of future contingencies, i.e., whether any contingent proposition about the future has a truth-value prior to the time it refers to. The second issue is the distinction between *necessary* and *contingent* truths. These are distinct logical problems, though they have a common metaphysical origin and were treated technically in a related way by ŁUKASIEWICZ. (See his Collected Works, 1970.).

Let us recall that the principle of bivalence affirms that every proposition receives one and only one of the values 'true' or 'false'. Those who deny the principle of bivalence admit a third truth-value, say 'neuter', and thus uphold a three-valued logic. On the other hand, the logical aspects of necessity and possiblity belong to modal logic The discussion of the logical status of future contingent propositions goes back to Aristotle and is illustrated by his famous example of the seabattle which is to take place tomorrow. (*On Interpr.* 9; 19^a 30). According to most commentators, Aristotle had doubts about the validity of bivalence for future contingent propositions, but his *logical system* is a two-valued logic, and he argues at length for bivalence in *Metaphysics* IV, chapts 7-8. It is conceivable that Aristotle's predicament about future contingencies motivated, in part, his development of a modal logic to account satisfactorily for the intuitive notion of contingency, i.e., that which is possible and not necessary.

Concerning the philosophical debate about the principle of bivalence, RESCHER (1968, pp. 54-55) writes:

> *The acceptance of the principle of bivalence was, in antiquity, closely bound up with the doctrine of determinism. The Epicureans, who were indeterminists, rejected the law of bivalence; the Stoics (and above all Chrysippus) who were rigid determinists, insisted upon it.*
>
> *In medieval times this problem of the truth-status of future contingents was much discussed by logicians, both in the Islamic orbit and in Latin Europe. One school of thought classed such propositions as indeterminate -i.e., neither true nor false. This position did not, however, appeal to the most prominent figures on either the Islamic side (e.g., Alfarabi) nor the Christian (e.g., Aquinas), because it involved theological difficulties- how can there be divine foreknowledge if future-contingent statements are neither true nor false ? A particularly keen debate about those problems took place at the University of Louvain in the 15th century, with Peter de Rivo as the principal advocate of an "indeterminate" truth-value.*

(See also ŁUKASIEWICZ 1970, "On the history of the law of bivalence" pp.176-178).

Modern research on three-valued logic, and in general, multiple-valued logics, began about 1920 by Post and Łukasiewicz, independently. Whereas POST (1921; 1971) was motivated by mathematical considerations, wishing in his multiple-valued logic to create a system that has "the same relation to ordinary logic that geometry in a space of an arbitrary number of dimensions has to Euclidean geometry" (1971, p.266), Łukasiewicz was motivated by philosophical considerations. "The conception of three-valued logic was suggested to Łukasiewicz by certain passages in Aristotle", writes LEJEWSKI (1967, p.105). "By setting up a system of three-valued logic Łukasiewicz hoped to accommodate the traditional laws of modal logic. *He also hoped to overcome philosophical determinism -which he believed was entailed by the acceptance of the bivalence principle- and which he had always found repulsive* (italics, ours). Interesting enough, he modified his views in the course of time and saw no incompatibility between indeterminism and two-valued logic." (Cf. also ŁUKASIEWICZ, 1970, pp.110-128.).

Currently, logicians have come to study multiple-valued logics from a philosophically neutral point of view, without attaching any philosophical import to the systems as such, studying them as *pure calculi*. But then, the very modern conception of *viewing logical systems as uninterpreted calculi* rests on some philosophical presuppositions, namely those of logicism and formalism!

It is instructive to look back and examine the philosophical framework of Aristotelian logic, for it is a paradigm of the intricate interplay between logic and philosophy. We propose to present arguments to support the following claims:

(a) The syllogistic was forged by and geared to Aristotle's theory of knowledge and metaphysics.

(b) There are no indications that Aristotle ever extracted principles of logic from *proof procedures* as employed by Greek mathematicians. Indeed, Aristotle's syllogistic is inadequate for an analysis of the arguments used by Greek geometers, and thus lacks the universality that was subsequently attributed to it.

(Claim (b) is a *technical* point and is not meant disparagingly. For various shades of assessments, see HINTIKKA (1980), CORCORAN (1972), and LEAR (1980).)

Let us look closer at each of these points. Aristotelian logic bears the stamp of *essentialism*, which in one of its forms is expressed by the doctrine that everything that is primarily true of a thing is part of that thing's essence. In Aristotle's words: "It is from 'what a thing is' that syllogisms start." (εκ γαρ του τι συλλογισμοι εισιν. *Met.* 7,9; 1034^a 32). Hence if a property P holds necessarily true of m, it is so by virtue of m's essence. (Post. Anal.,I,6). As a corollary there follows the tenet -which dominated logical thinking until the middle of the last century- that in every true proposition the predicate is 'contained' in the subject. The predominance given by Aristotle and his successors to propositions of the subject-predicate form reflects some grammatical pecularities of Indo-European languages. Also, "that syllogistic rather than the logic of classes was developed at this stage", write the KNEALES (1975, p.40), "is probably to be explained by the fact that Greek, like English, does not have a single expression for each of the class-relationships, so that the class-logic is difficult to develop without an artificial symbolism, a device which did not occur to logicians for many centuries."

Besides essentialism and linguistic factors, Aristotle's theory of *knowledge through causes* has also played a determining role in fashioning his syllogistic. For Aristotle, syllogisms -just like definitions- should provide *causal explanations* for the thing inferred or of the thing defined. Aristotle writes: "Scientific knowledge is judgement about things which are universal and necessary" (1140^b 31). "We think we have scientific knowledge when we know the cause" (94^a 20). "We have to acquire knowledge of the original causes -for we know each thing only when we know its first cause" (983^a 25). "*The premises must be the cause of the conclusion, better known than it and prior to it; its causes, since we possess scientific knowledge of a thing only when we know its cause*". (*Post. Anal.* I,2; 71^b 18-31).

"To sum up, then:" -writes Aristotle- " demonstrative knowledge must be knowledge of a necessary nexus, and therefore must clearly be obained through a necessary middle term; otherwise its possessor will know neither the cause nor the fact that his conclusion is a necessary connection" (75^a 12) To Aristotle, it is through the middle term of a syllogism that causes are grasped -cf. the illuminating and oft discussed example in *Post. Anal.* I, 34; thus, the structure of the syllogism with two premisses and a conclusion, connected by a middle term, is a crucial feature *imposed by Aristotle's epistemology*. Significantly, Aristotle says: "It is clear that every demonstration will proceed through three terms, and no more, unless the same conclusion is established by different pairs of propositions." (41^a 36).

Recall that the first figure has the following form:

M -- P	as in the AAA mood:	All M is P	(major premiss; P major term)
S -- M		All S is M	(minor premiss; M middle term)
S -- P		All S is P	(conclusion; S minor term)
			(S = subject; P = predicate)

(Aristotle's habitual wording is rather "P is predicated of all of M" or "P belongs to all of M", etc.)

As an example, suppose we want to know *why* giraffes chew the cud. If we know that ruminants chew the cud and that giraffes are ruminants, then we can say that giraffes chew the cud *because* they are ruminants. By relating a lower taxon (giraffes) to a higher taxon (ruminants), a causal explanation is furnished. This is what Aristotle thought to produce knowledge through causes. The corresponding syllogism would be as follows:

All ruminants chew the cud	(major premiss)
All giraffes are ruminants	(minor premiss)
All giraffes chew the cud	(conclusion)

In this syllogism, "ruminant" is the middle term which, according to Aristotle, supplies the *causal nexus*.

For the first figure, Aristotle claims: "Of all figures the most scientific is the first. Thus, it is the vehicle of the demonstration of all the mathematical sciences, such as arithmetic, geometry, and optics, and particularly of all sciences that investigate causes... The first is the only figure which enables us to pursue knowledge of the essence of a thing... Clearly, therefore, *the* first figure is the primary condition of knowledge." (*Post. Anal.* I,14) The last cited passage leaves no doubt that Aristotle was *NOT* concerned with the question of how proofs functioned in Greek mathematics. BRUNSCWICG (1912; 1972, chapt.5, § 41), for instance, insists on the biological origin of Aristotelian logic. Moreover, from a technical point of view, besides missing rules of inference like *modus ponens* and universal generalization--the key deductive rules in mathematical proofs, Aristotle's logic does not allow for singular terms in syllogisms, as ŁUKASIEWICZ (1957, pp.1-7) has shown. Yet singular terms occur constantly in Euclidean arguments. (Singular terms contain a name or constant to refer to an individual; if in a geometric demonstration one argues, say, about a property of a given *arbitrary* triangle c, the term containing 'c' is singular. And if 'P' is a predicate to express a property of triangles, universal generalization permits to infer 'for all x, P(x)' from 'P(c)'). Moreover, had Aristotle examined mathematical texts with the aim of extracting their logical machinery, he would have been struck by the ubiquity of *binary relations* such as "greater than", "parallel to", "perpendicular to", etc., all requiring a system of logic containing also *two-place predicates* of the form P(x,y), whereas Aristotelian logic is *monadic*, admitting only predicates of the form P(x) in the subject-predicate mold.

The relationship between Greek mathematics and Greek logic was extensively analyzed by MUELLER (1974). We cite some of his conclusions (p.66):

> *(1) Aristotle's formulation of syllogistic in the fourth century is basically independent of Greek mathematics. There is no evidence that he or his Peripatetic successors did a careful study of mathematical proof.*
>
> *(2) Similarly, the codification of elementary mathematics by Euclid and the rich development of Greek mathematics in the third century are independent of logical theory.*
>
> *(3) Likewise, Stoic propositional logic, investigated most thoroughly by Chrysippus in the third century, shows no real connection with mathematical proof.*

To sum up: We have seen how intimately the form, content and scope of logic are tied from its incipiency to philosophical doctrines and presuppositions. We insisted on the fact that until the last century, there was not any noticeable interaction between logic and mathematics -again, Leibniz apart. We have dwelt at some length on Aristotle in order to furnish textual evidence for our main points. Though Aristotle refers at times to mathematics -notice his speaking of mathematical sciences- to support some general epistemological considerations concerning deductive sciences, Aristotelian and Megarian-Stoic logics did not derive their form and structure through an analysis of proofs in geometry, contrary to what one might suppose The effective interaction between philosophy and logic is tighter than in other domains. Consider, for instance, the philosophy of biology, currently a very active and fertile field. Biologists and philosophers alike debate some of the fundamental problems at the frontiers of biological research. (Cf. RUSE, 1988, for an excellent synopsis and extensive bibliography). Yet actual research in biology is not immediately and directly affected by the debate, though in the long run, a change in direction may result from such conceptual clarifications. The same holds true of the philosophy of physics. On the other hand, to take a well-documented example, Wittgenstein's *philosophical* discussion of 'language games' lead directly to the technical development of gametheoretic semantics by HINTIKKA (1973). This is one of the many instances in which *the flow from philosophy to* *logic is direct and immediate*. But not all is positive, here as elsewhere. Just as interactions in the nervous sytem are mediated by excitatory as well as by inhibitory neurons, so is the interaction between philosophy and logic. Up to now we have talked about "excitatory" pathways. Let us now turn our attention to "inhibitory" interactions, stemming from various prejudicial philosophical doctrines and movements.

From the Renaissance until the infusion of logic by mathematical techniques in the middle of the 19th century, the philosophical climate had become further and further inhospitable -at times even hostile- to the development of logic. "...the rise of humanism and of new interests connected with natural science led gradually to a neglect of formal logic", write the KNEALES (1975, p.246). The opening of new vistas with the rise of physics and the desire to learn from experience favored a shift of interests from *deductive logic*, which seemed barren in producing knowledge, to *inductive logic*, considered an instrument of scientific discovery. This shift is examplified in Bacon's *Novum Organum* (1620), to be opposed to Aristotle's *Organon*. British empiricists, like Locke, were openly hostile to formal logic, thus producing the ire of Leibniz who wrote in a letter to Burnett (26 May 1706): "L'art de demonstrer n'estoit pas son fait", and further, in a letter to Koch (15 July 1715): "Lockius aliique qui spernunt non intelligunt". (Cited by COUTURAT 1901; 1985, footnote p.282). With the laudable emphasis on experience also grew the more problematic tendency to turn to introspection as a source of knowledge, and from there, to psychologism, i.e.,. "the doctrine that logic describes the way we think, and that therefore the laws of logic are the laws of thought." (MUSGRAVE, 1972). Logic became confounded with epistemology and the psychology of thinking. Let us note in passing that George Boole was on slippery ground *only* in the title of his "Investigation of the Laws of Thought" (1854) and in some of the introductory remarks, *not* in the technical developments of his symbolic system. (See MUSGRAVE, 1972, for a detailed discussion). It is understandable why giants like Frege and Husserl found it necessary to argue at such length against psychologism in logic and the foundations of arithmetic. Curiously, not only was psychologism a pernicious blunder in confounding formal logic with the psychology of thinking; it was also *empirically* dead wrong. Recent research in cognitive psychology, spurred by work in artificial intelligence, has shown that people reason by building models of state of affairs, models that work semantically, and not by following rules of inference of formal logic. (See JOHNSON-LAIRD and BYRNE, 1991).

In a more direct way, some empiricists of old and new, from Sextus Empiricus (fl. ca. 200 C.E.) to John Stuart Mill (1806-1873), objected on *philosophical grounds* to some of the fundamental canons of logic. Consider the generally accepted inference in the following stock example: "All men are mortal; Socrates is a man; therefore, Socrates is mortal." Sextus Empiricus maintained in his *Outlines of Pyrrhonism* that in order to ascertain that *all* men are mortal, one has already to know beforehand that each individual, including Socrates, is mortal, hence the quoted syllogism contains a vicious circle. Similarly, Mill held the above syllogism to be circular. Indeed, Mill "rejects the whole of the traditional philosophy of formal logic." (KNEALE, 1975, p.375). Conforming to Mill's extreme form of empiricism, according to which all general propositions are inductive generalizations of particular cases learnt by experience, it is not surprising that Mill wrote that "all inference is from particulars to particulars. General propositions are merely registers of such inferences already made, and short formulae for making more. The major premise of a syllogism, consequently, is a formula of this description; and the conclusion is not an inference drawn *from* the formula, but an inference drawn *according* to the formula; the real logical antecedent or premise being the particular facts from which the general proposition was collected by induction." (MILL, 1843; Book II,chapt.3, p.4).

Mill's influence was considerable and his attack on deductive logic fell on sympathetic ears among British empiricists and continental idealists alike. As we know, the outright philosophical rejection of formal logic did not prevail. Still, an important issue was raised. The very fact that at different epochs some fundamental canons of deductive inferences are rejected by influential philosophers reminds us that the *justification* of rules of inference is open to philosophical debate. But how can the canons of rationality be rationally established without circularity ? Can language guide us ? We shall look closer at these problems in the following section.

2. The logical substratum of language

Logic and grammar are intimately related and their boundaries overlap. In the West and in India, the only cultural regions in which formal logic was developed in antiquity, the study of logic and grammar went hand in hand. Using modern symbolic logic, STAAL (1960) investigated the correlations between language and logic in Indian thought. In his view, the partial similarities between the grammatical categories as studied by Indian logicians and by Aristotle can be attributed to their common Indo-European syntactical background. "Indian logic", writes Staal, "offers striking parallels to Western logic. Its study is interesting not only for its own sake but also for the unexpected light it may throw on the vexed problems of the universality of logical principles and the relation between logical structure and linguistic expression." (STAAL, 1967, p.523). In this perspective, it is particularly instructive to look for a comparison with Chinese thought. Though Chinese philosophy spans a period of over 2500 years and is known for its richness and depth, ancient Chinese philosophers apparently never undertook a *systematic* analysis of deductive reasoning. Rather, one finds sporadic logical writings, dealing on a *semantic* level with *examples* of arguments, and more substantially, with discussions of such topics as the classification and correct use of names. GRAHAM (1967, p.525) attributes the absence of a *formal* logic in ancient China to the distinctive features of the Chinese language in which sentences are compounded by uninflected words organized solely according to word order and the placing of particles. He writes: "Without the inflections that expose the structure of Sanskrit, Greek, or Arabic sentences and encourage the simultaneous growth of grammar and logic, the Chinese sentence, until recently, almost defied analysis; the Chinese have been lexicographers but not grammarians."

Does the example of Chinese disprove the thesis that some basic logic is embedded in all human languages? It does not seem so in view of the multiplicity of arguments and evidence coming from biolinguistics and evolutionary considerations. Rather, an immediate conclusion to be drawn from the foregoing discussion is that the *surface structure* of some languages is more *transparent* with respect to the underlying logic. The syntax of Chinese, being relatively more opaque, may just not lend itself easily to such an analysis. (See however CHENG, 1972). *Syntactical transparency* is certainly a *necessary* condition for using language as an exemplar for abstracting logical laws, but is not by itself a sufficient condition, as the history of logic shows. Semitic languages are by a common consesus of the "transparent" type, yet neither the Babylonians- notwithstanding their major contributions to mathematics- nor the Hebrews have developed a *science of formal logic*. (See GUGGENHEIMER, 1951, however, for principles of deduction in the Talmud.) Obviouly, cultural factors and philosophical interests are indispensable for favoring the development of formal logic.

Much can be said in favor of the thesis affirming that there is a basic universal logical structure embedded in all languages, a structure due to the *common biological roots* of all natural languages. Indeed, we uphold the view that various formal rules of inference can be considered as originating via linguistic abstractions from "wired in" cognitive mechanisms, hence their universally compelling force. The naturalistic epistemology advocated here is consistent with, though not dependent on Chomsky's theory of universal grammar, but is incompatible with and in opposition to both cultural relativism and conventionalism in the philosophy of logic. Similarly, we do not subscribe to the "ordinary-language approach to logic", according to which linguistic usage is the guide and ultimate arbiter in questions of logic. *Language is a mine in whose strata, by a lengthy developmental process, various logical structures have been deposited.* But no surface scratching will reveal the deposits!

"In the mechanisms of language", writes LENNEBERG (1967, p.324), "we find a natural extension of very general principles of organization of behavior which are biologically adapted to a highly specific ethological function." The efficacy, adaptability, and reality-oriented manipulation of physical objects, the 'logic' of coordination between eye and hand, as behavior in general, all have been subjected to natural selection. This is one of the main tenets of evolutionary epistemology (LORENZ, 1973; VOLLMER, 1975; RIEDL, 1984; OESER, 1987; RAV, 1989; WUKETITS, 1990). Currently, many linguists support the theory that articulated language evolved from gestural language, which in turn is closely allied to motor skills and tool-using (HEWES, 1976; BROWN, 1981; BRANDON and HORSTEIN, 1986). In a study of skill systems, particularly rich in data, REYNOLDS (1976, p.162) writes: "The view taken here is that language is a phylogenetic derivative of the skilled motor system. It is a system of communication that requires the skilled motor system not only for its acquisition but for its ordinary expression." Reynolds further discusses the logical operations contained in a skill system, including operations that we gloss linguistically by "If...then..." statements. (op. cit., pp.164-165; see particularly Fig. 9, with the box marked 'logic'). As a matter of fact, one can take any system of natural deduction and convince oneself that all its rules of inference are linguistic glosses of manipulatory actions. Thus, knowing that I put my keys in my pockets and then not finding them in one of them, I safely conclude that they are in the other. Translated into formal terms, we get the rule: "Either the first or the second; but not the second; therefore the first." This is exactly the fifth inference scheme of the Stoic logician Chrysippus! (KNEALE, 1975, p.163). And an evolutionary epistemologist can only rejoice in learning that Chrysippus (279-206 B.C.E.) "himself is reported to have said that his argument from a disjunction with three members was used by a dog when it came to a cross-roads and after sniffing at the entries to two of the new ways went down the third way without sniffing because it knew that its quarry must be there." (KNEALE, 1975, p.167)

Let it be noted in passing that the Stoics were the first to formulate rules of inference and also "the first to make a systematic study of what we now call grammar." (ibid., p.143). Notice also why Euler-Venn diagrams are so helpful in visualizing logical relations. When in doubt, we go from the verbal to the visual/manipulatory, but in phylogenetic evolution, the path went the other way

The connection between motor coordination and logical operations has also been stressed by Piaget and his school. HEWES (1973) discusses at length the similarities between syntax and tool-making and tool-using. In the same direction, REYNOLDS (1983, p.188) gives six characterizations of syntax and concludes: "...only the instrumental manipulation of material objects shares all six." Going down to the neurological level, anatomist GIBSON (1983, p.44) writes: "...the highest constructional levels of both tool use and language are mediated by the same inferior parietal and anterior frontal associative areas." Gibson further mentiones the clinical coupling of apraxia and aphasia. And neurologists JAKOB and MÜLLER (1983, p.247) state: "syntactical organization of verbal language...(is) derived from the serial organization of motor activity." And further: "...our analysis of neostrial function suggests that the syntaxis was derived from the serial organization of motor activity as a result from a close cooperation between forebrain areas and the cerebellum." (p.249). Let us conclude these testimonials with what linguist LIEBERMAN (1983, p.95) had to say at the Paris Symposium (de GROLLIER, 1983): "Certain aspects of linguistic ability, e.g., rule-governed syntax, which at face value would appear to be language-specific also may have a non-linguistic basis... The neural mechanisms of the brain that have evolved for the end of motor control thus can be regarded as an example of Darwinian 'pre-adaptation' for rule-governed grammar."

Let us summarize. Our point of departure at the end of the previous section was the question of how one justifies rules of inference in logic. Though in formal logic, rules of inference are *nowadays* stated axiomatically, their *justification* has often been subjected to philosophical debate. Our intention was to indicate that rules of inference are *codifications* of aspects of the logical deep structure of language, which in turn can be traced to its manipulatory origin. In doing so we are not engaged in a reductionist program, as likewise the biologist is not 'reducing' neurology to embryology by tracing the epigenesis of nervous tissue to the ectoderm. It is our belief, however, that evolutionary epistemology has much to offer by anchoring philosophical debates on scientific findings.

3. Between language and logic

Ultimately, whatever the origin of the logical deep structure of language may be, when language is considered as given, as a *datum*, one cannot just scoop up its logical laws without further ado. Logic, as any science, begins with *theorizing* on the basis of the given, (=*data*), thereby engendering a dynamic process in which other data are discovered or discerned, to serve in turn as a check or stimulus for further building of theoretical frameworks. Ordinary language is given at the onset as a basic datum for *extracting* logical laws, which permit in turn to subject language to a finer analysis. But it must be kept in mind that logic is *prescriptive*, not descriptive of linguistic usage. Furthermore, the logical analysis of language within language, with all due precautions, is subjected to complementarity restrictions which cannot be circumvented (LÖFGREN, 1991)

Historically, the Scholastics brought the analysis of the logical aspects of language and *non-mathematized* logic to their ultimate limits. Logic, to the Schoolmen, was a path to truth, and language their vehicle. The theologians of the eleventh and twelfth century faced the monumental task of unifying into a *coherent* body of doctrine the articles of faith based on Holy Writ, the accumulated dogmatics, and the authorative writings of their predecessors. Here we see again how *philosophical/theological interests promoted and shaped logical inquiries*: "...the *exegetic function of logic*, in the medieval program of education, is perhaps sufficient to account for the metalinguistic method and for its concern with the syntax and semantics of natural language", writes MOODY (1975, p.377). And further: "Medieval logic, like ancient logic, was a quasiempirical formulation of the logical structure of natural language and not, like modern logic, an axiomatic construction of a formal calculus expressed in artificial symbolism. As compared with Aristotelian logic, however, it had two new features. First, *it was formulated metalinguistically throughout*, by means of rules referring to language expressions...Secondly, it included the semantical as well as the syntactical dimensions of language analysis, and was developed in terms of the basic concepts of *denotation, truth, and consequence (or entailment)*." (Italics, ours). With a change in the philosophical climate, the efforts and achievements of medieval logicians ceased to be appreciated or even comprehended. *Take away the appropriate philosophical motivation and that which was once heralded as acumen is suddenly spurned as hair-splitting*! "When logic was revived in the middle of the nineteenth century", write the KNEALES (1975, p.378), "the new vigour came from mathematicians who were familiar with the progress of their own specialty, rather from philosophers who were occupied with the controversies of idealism and empiricism."

4. From mathematics to logic

One of the main threads that runs throughout our analysis of the relationship between logic and philosophy is the argument that until about the middle of the

19th century there was no *technical* interaction between traditional logic and mathematics. As to Indian, Megarian-Stoic, or Scholastic logic, the issue has never been disputed, and we have endeavored to pesent arguments showing that Aristotelian logic could not have been based on an analysis of *proof procedures* as employed in mathematics. Neither for that matter have mathematicians themselves been concerned with such an analysis. At best, as in Euclidean geometry, axioms and postulates were explicitely stated, but the deductive machinery itself was not analyzed; mathematicians for their part did not contribute to logic. On the other hand, throughout the ages, philosophers engaged in logical studies did not consider mathematics *technically* pertinent to their enterprise. Indeed, the mathematics of their days had really nothing to offer in terms of concepts or methods. But the rapid development of mathematics from the 17th century onwards brought about not only a prodigious enrichment in the subject matter itself, but also a growing concern with the legitimacy of the manner in which results were obtained.

The phenomenal transformation of mathematics in the 19th century, both in content and in level of abstraction, set the stage for an intimate interaction between mathematics and logic. (Cf. STEIN, 1988). Whereas philosophical motivation for developing formal logic had been running dry for some centuries, internal developments in mathematics proper now furnished the tools and motivation for infusing logic with a new life. The names of three mathematicians stand out in *initiating* the dual process of mathematization of logic and "logicization" of mathematics: George Boole (1815-1864), Gottlob Frege (1848-1925), and Giuseppe Peano (1858-1932). The new developments had a profound influence on philosophy, contributing largely to the emergence of analytic philosophy in the 20th century. There were also many indirect influences. It is interesting to note, for instance, that Edmund Husserl (1859-1938), a mathematician by training and one-time assistant of Weierstraß, was inspired by the Weierstrassian pursuit of rigor: it was an epoch of "house cleaning" in mathematics, and in that spirit, the quest for a *rigorous methodology in philosophy based on logic* turned out to be a central theme in Husserl's philosophical investigations. As did Frege, Husserl came to reject psychologism in logic and in mathematics; indeed, there are many points of contact and parallelisms between Husserl's *intended* methodology in philosophy and Frege's program to found arithmetic on logical principles. (For the relationship between Frege and Husserl, see MOHANTY, 1982).

We shall briefly look at some of the technical developments in 19th century mathematics which rendered feasible a mathematization of logic, but first a word about Leibniz. As is well known, Leibniz wrote extensively on logic, but these writings were scattered throughout numerous manuscripts, and much of his work in logic remained unknown until the appearance of the study by COUTURAT (1901). Concerning Leibniz, the KNEALES (1975, p.320) write: "That his work in logic had little influence for nearly 200 years after he wrote was due in part to the dominance of other interests, connected with the rise of natural science, but also in part to the defects of his character. He was a universal genius who conceived many projects and made many beginnings but brought little to fruition." Great as the *vision* of Leibniz surely was, his projects and initial undertakings in logic were rather premature: mathematics was not yet ripe for the development of what Bocheński calls a 'mathematical variety of logic'. But toward the middle of the 19th century, two of Leibniz's great projects could be tackled successfully:

(1) The creation of a *calculus ratiocinator*, a symbolic logical calculus.

(2) The development of a *lingua characteristica*, an ideal logical language free of the blemishes of ordinary language.

Boole's algebra of logic materializes the first project: as a symbolic algebra it is part of mathematics; under the Boolean line, *logic becomes part of mathematics*. As to the second project, we turn to Frege. Indeed, Frege's *Begriffsschrift* of 1879 was intentionally conceived as a Leibnizian *lingua characteristica*, an ideal logical language to serve as a basis for arithmetic.In Frege's scheme, *mathematics becomes part of logic*. Two distinct projects (at first sight), two diverse programs, sparking major developments in 20th century philosophy of mathematics.

What were the conceptual developments in 19th century mathematics that furnished the tools and stimulus for bringing the two Leibnizian projects to fruition ? In case of Boole the path is straightforward: Boole's algebraic treatment of logic was an integral part of his investigations in the *calculus of operations*, a branch of mathematics that blossomed in Britain from the beginning of the 19th century. The history and significance of the calculus of operations in the rise of modern algebra have been subject of an extensive study by KOPPELMAN, 1972; a further discussion of the influence of the calculus of operations on the emergence of logicism and formalism can be found in KNOBLOCH,1981. Here we only have space to give some brief indications of these important developments with a view on logic. It all started when in the study of differential equations, the derivative operation was abstracted from its analytic content and treated formally as a symbol. Thus, for instance, expressions like $\log(1 + \frac{d}{dx})$, or in general, $f(\frac{d}{dx})$, were treated formally, giving rise to a *symbolic algebra*, an algebra dealing with non number-designating symbols. Thus, from methods for solving difference and differential equations there emerged a study of abstract, symbolic algebras governed by rules for *manipulating symbols* and admitting diverse interpretations. "Boole's most important work", writes KOPPELMAN (1972, p.197), "was his paper 'On a general method in analysis' published in 1844. This work, which won the Mathematical Medal of the Royal Society, was very significant, both in influencing later workers in the calculus of operations and in shaping Boole's view on the nature of mathematics...The object of Boole's work was to apply symbolic methods to the solution of linear [differential] equations with variable coefficients. In his own words:

> *The object of this paper is to develop a method of analysis, which while it operates with symbols apart from their subject, is nevertheless free from restrictions."*

Encouraged by De Morgan, it was just a small step -for a giant like Boole- to see that his methods extend to logic: the basic propositional operations of conjunction, disjunction and negation can be formally expressed in an algebra whose 'multiplication' is idempotent, i.e., for every x, $x^2 = x$ ($x^2 = xx$ means 'x and x', which amounts to 'x'). Boole clearly saw, contra Aristotle, "that syllogism, conversion, etc., are not the ultimate processes of Logic... Nor is it true in fact that all inference is reducible to the particular forms of syllogism and conversion" (BOOLE, 1854, p.10). Here Boole went quite beyond Leibniz who could not free himself from the Aristotelian and scholastic traditions. (Cf. COUTURAT, 1901, pp.438-441). Moreover, Boole's algebraic method permitted him to show that several arguments which Aristotle had considered valid turned out to be invalid. Mathematization of logic started to bear fruit. But how curious a path, starting from methods for solving differential equations, via the calculus of operations, leading to a realization of Leibniz's *calculus ratiocinator* and beyond!

As with theoretical constructs in any field of learning, mathematical concepts evolve, become modified, extended, generalized, are rendered "purer", more abstract -all as part of new theoretical developments. For example, one of the key concepts in mathematics, that of a *function*, has gone through such a transformation since the 17th century. The study of trigonometric series in the 19th century acted as a catalyst to numerous conceptual refinements, foremost among which was the need to clarify and generalize the notion of a function. In a paper on Fourier series, published in 1837, Dirichlet (1805-1859) set forth the essentially modern definition of a *function* as an *arbitrary pairing* The new, more abstract notion of a function, further elaborated by Riemann (1826-1866), subsequently became central to many new developments in mid and late nineteenth century mathematics. Frege, who completed his doctoral studies in mathematics in 1873 at the University of Gottingen must certainly have been familiar with the work of Dirichlet and Riemann, two former Gottingen professors. It seems very likely that the studious young Frege also came to know Dedekind's rigorous construction of the real number system, published in 1873. Imbued with knowledge and the prevailing spirit of conceptual analyses in mathematics, Frege turned to the problem of a rigorous construction of the natural numbers and their arithmatic, still wanting of a solution. As Frege wrote many years later: "With mathematics I began. There seemed to me a most urgent need for better foundations of that science...The logical imperfections of language was an obstacle for such investigations. Thus, I came from mathematics to logic." (Transl. by SLUGA, 1980, p.42). The then 'modern' concept of a function turned out to be a pivotal technical tool in Frege's *Begriffsschrift* of 1879. By decomposing propositions into *function* and *argument(s)* rather than into *subject* and *predicate*, Frege was led to invent quantification theory, an innovation that marks 1879 as a landmark in the history of logic. Because of the technical novelty, Frege devotes all of 9 of the *Begriffsschrift* to explicate and illustrate the general concept of a function. (For a detailed analysis, see BAKER and HACKER, 1984).

In the preface, Frege reminds us "that logic has hitherto always followed ordinary language and grammar too closely." In creating an idealized formal language, a *lingua characteristica*, Frege opened new vistas in philosophy, as he was quite aware of when he wrote in the preface to the *Begriffsschrift*:

> *If it is one of the tasks of philosophy to break the domination of the word over the human spirit by laying bare the misconceptions that through the use of*

> *language often almost unavoidably arise concerning the relations between concepts and by freeing thought from that with which only the means of expression of ordinary language, constituted as they are, saddle it, then my ideography, further developed for these purposes, can become a useful tool for the philosopher.*

Frege's hope was not in vain: for not only did he found quantification theory in the *Begriffsschrift* and thereby revolutionized logic, but subsequently Frege also provided the logical framework for a constructive *philosophy of language*. Upon the foundaions laid by Frege, his successors erected twentieth century *analytic philosophy*.

We shall not dwell on the intricacies of the post-Fregean developments of logic through the ever increasing employment of mathematical methods, giving rise to mathematical logic as we know it today. (For an overview, see the Handbook edited by BARWISE, 1977). Let us just stress that not only has logic been profoundly transformed through mathematization, but also reciprocally, mathematics itself has been deeply enriched by this process of crossfertilization. Thus, for instance, *real* algebraic geometry owes its very existence to model theory, a branch of mathematical logic. (For details, see SINACEUR, 1991). As to technological applications, not that we consider computer programming and all that goes with it the *summum bonum* of mankind, but given its current importance, one ought perhaps be reminded that it is all based on theoretical developments in mathematical logic dating back to the 1930s, particularly recursion theory, whose origins in turn are the philosophical foundational problems of pure mathematics.

After having gone their separate paths for over two millennia, logic and mathematics are henceforth fruitfully conjugated, bringing the great vision of Leibniz to its splendrous fulfillment.

5. From mathematical logic to language and philosophy

The rise of natural science and the subsequent orientation of empiricist philosophers, as already mentioned, led to a neglect and at times to a disdain of formal logic. Thus, it surely is one of the more remarkable developments in twentieth century philosophy that not only has empiricism ceased to be at odds with logic, but a *united* philosophical orientation emerged under the banner of *logical empiricism*. As one of its eminent spokesmen, the American philosopher Charles MORRIS (1938, p.66), put it: "The union of formal logic and empiricism is linked with the development of symbolic or mathematical logic...This modern version of formal logic developed in the hands of philosophic rationalists who were themselves mathematicians. It arose out of the cross-fertilization of the medieval approach to logic in terms of a general theory of signs and the methods of modern mathematics, a union which is first significantly made by Leibniz." Indeed, the Schoolmen were keen in analyzing the logical structure of language and erected a remarquable metalinguistic theory of logic and language. But more powerful tools were required beyond just sharp categorizations *within* natural language. For further progress, a fulcrum from the outside, an *external foothold* was needed in order to forge ahead.

And this is where *mathematical* logic comes in by providing an external leverage in the form of an *artificial symbolic language*.

Throughout the history of philosophy, East and West, discussions of language have always taken place. However, as of the last century, the emergence of new interests and a simultaneous development of mathematical logic resulted in raising the issue of language to the status of *the* philosophical topic in twentieth century analytic philosophy. Among the contributing factors were the interests enkindled by 18-19th century comparative philology, the rapid development of the science of linguistics in which the school of Ferdinand de Saussure marks the turning point, and a general philosophical concern with "critique of language". Notable is the line of language critics from Alexander Bryan Johnson (1786-1867), through Felix Mauthner (1849-1923), and all the way to Ludwig Wittgenstein (1889-1951). Though each in his way considered the critique of language as the most important task of philosophy, their path led eventually to a *mysticism of silence*, (proclaimed, but not quite observed...). That *negative* line did not prevail, however. Equipped with deep knowledge of physics and mathematics, Frege's former doctoral student, Rudolf Carnap (1891-1970) undertook a *positive* critical reconstruction of the language of philosophy: a program of "logicism" for philosophy in the manner of the Frege-Peano-Russell logicism in mathematics, based on mathematical logic. This "reconstruction", signaled by the very title *Aufbau* (CARNAP,1928), set out to create a *scientific philosophy* in which "the individual no longer undertakes to erect in one bold stroke an entire system of philosophy. Rather, each works at his special place within one unified science...As soon as philosophers are willing to follow a scientific course (in the strict sense), they will not be able to avoid using this penetrating and efficient method, [i.e., mathematical logic], for the *clarification of concepts and the purification of concepts.*" (CARNAP, 1928; 1967, p.XVI; our italics). A more radical thesis followed in CARNAP, 1937 (p.XIII): "*Philosophy is to be replaced by the logic of science* -that is to say, by the logical analysis of the concepts and sentences of the sciences, *for the logic of science is nothing other than the logical syntax of the language of science*" (Carnap's italics).

On the technical side, mathematical logic provided the tools for *constructing* artificial logical languages, with rigorously formulated syntax and inference rules, in which the relationship between syntax, semantics and pragmatics could be studied with precision. Linguistics itself has been deeply influenced by the use of concepts and techniques of mathematical logic. For instance, Chomsky's theory of generative grammar has a recursion-theoretic underpinning, and in some of the journal articles, as in CHOMSKY (1961), mathematical methods are explicitly employed. *With the advent of mathematical logic, language, logic, and philosophy are henceforth intimately welded together.*

Though the radical theses of the logical positivists belong to the past, under the impact of technical devices derived from logic, the philosophical focus has shifted from solving problems *about the world* to problems *about the language through which we speak about the world.* A more rigorous, science-like methodology moved into the foreground, with two major consequences: (1) The splitting of philosophy into 'philosophies of', such as philosophy of art, philosophy of history, philosophy of religion, philosophy of science, etc. (2) The emergence of a technical study by methods of mathematical logic of topics having particular philosophical interest -a field now known as *philosophical logic*. (See the four-volume Handbook edited by GABBAY and GUENTHNER, 1983-89). "The application of techniques of modern logic to the study of philosophical issues", writes RESCHER (1968, p.333), "is not a matter of borrowing the finished end-products of one field for use in the investigation of problems in another field, as is frequently the case in applied mathematics. It is not in this sense of an employment of certain accomplished results that I speak of 'applied logic', but in the sense of borrowing from logic certain of its tools, i.e., concepts, formalization techniques, methods of inference, etc. The keystone to the method is the concept of *formalization*: the construction of a formal framework whose concepts are more sharply defined and whose logical interrelationships are more explicitly articulated...The gain in exactness and precision will -it is hoped- aid in diagnosing the exact sources of problems and difficulties, and afford instrumentalities for their resolution."

Critical discussion lies at the heart of philosophy, and under the impact of the great advances in mathematical logic, with its focus on language, many traditional issues in philosophy have been brought into sharper focus. Ours is in-*deed* the *Age of Analysis*. But just as not every scientific field can or ought be forced into a mathematical mold, neither can or ought all of philosophy be enlaced by mathematical logic. In the mansion of *philo-sophia* there are many dwellings, with ample space to accommodate each of the critical, the speculative, as well as the poetic-inspirational philosophies.

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