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Some Fixed Point Theorems of Ćirić Type in Fuzzy Metric Spaces

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Abstract: The main aim of the current paper is the investigation of possibilities for improvements and generalizations contractive condition of Ćirić in the fuzzy metric spaces. Various versions of fuzzy contractive conditions are studied in two directions. First, motivated by recent results, more general contractive conditions in fuzzy metric spaces are achieved and secondly, quasi-contractive type of mappings are investigated in order to obtain fixed point results with a wider class of t-norms.

Keywords: fixed point; fuzzy metric space; t-norm; quasi-contractive mapping

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1. Introduction and Preliminaries

The Banach contraction principle [1] is usually taken as a starting point for many studies in the fixed point theory. The principle is observed in various types of metric spaces, as well as different generalizations of it.

The theory of fuzzy sets [2], with noticeable applications in many sciences [3–7], inspired Kramosil and Michalek [8] to introduce fuzzy metric spaces. Later on, George and Veeramani [9,10] slightly changed its definition and provided a Hausdorff topology for it.

One of the most cited generalizations of the Banach contraction principle in probabilistic metric spaces is by Ćirić [11]. More information about the fuzzy and probabilistic metric spaces, as well as fixed point theory in these spaces, can be found in [12–18].

First, we list basic definitions and propositions about t-norms and fuzzy metric spaces.

Definition 1 (Schweizer and Sklar [19])**.** *A binary operation $T : [0,1] \times [0,1] \to [0,1]$ is called a triangular norm (t-norm) if the following conditions hold:*

(i) $T(a,1) = a,\ a \in [0,1]$,
(ii) $T(a,b) \le T(c,d)$, *whenever* $a \le c$ *and* $b \le d,\ a,b,c,d \in [0,1]$,
(iii) T *is associative and commutative.*

Three basic examples of continuous t-norms are

$$T_{min}(a,b) = \min\{a,b\}, T_P(a,b) = a \cdot b \text{ and } T_L(a,b) = \max\{a+b-1,0\}$$

(minimum, product and Lukasiewicz t-norm, respectively).

Definition 2 ([14])**.** *Let T be a t-norm and $T_n : [0,1] \to [0,1]$, $n \in \mathbb{N}$, be defined in the following way:*

$$T_1(x) = T(x,x),\ T_{n+1}(x) = T(T_n(x),x),\ n \in \mathbb{N},\ x \in [0,1].$$

We say that the T is of H-type if the family $T_n(x)_{n\in\mathbb{N}}$ is equi-continuous at $x = 1$.

A trivial example of t-norm of H-type is T_{min}.
By

$$T^0_{i=1}x_i = 1,\ T^n_{i=1}x_i = T(T^{n-1}_{i=1}x_i, x_n),\ x_1, x_2, \ldots, x_n \in [0,1],$$

t-norm T could be uniquely extended to an n-ary operation [20]. The extension of t-norm T to a countable infinite operation is done as follows:

$$T^\infty_{i=1}x_i = \lim_{n\to\infty} T^n_{i=1}x_i,\ x_n \in [0,1],\ n \in \mathbb{N},$$

where $T^\infty_{i=1}x_i$ exists since the sequence $(T^n_{i=1}x_i)_{n\in\mathbb{N}}$ is non-increasing and bounded from below.

Let $\lim_{n\to\infty} x_n = 1$ and

$$\lim_{n\to\infty} T^\infty_{i=n}x_i = \lim_{n\to\infty} T^\infty_{i=1}x_{n+i} = 1,$$

(see [15,21]). Then,

$$\lim_{n\to\infty} T^\infty_{i=n}x_i = 1 \quad \text{if and only if} \quad \sum_{i=1}^{\infty}(1-x_i) < \infty,$$

for $T = T_L$ and $T = T_P$, while

$$\lim_{n\to\infty} T^\infty_{i=n}x_i = 1 \ \text{ implies } \ \sum_{i=1}^{\infty}(1-x_i) < \infty,$$

for $T \geq T_L$.

Proposition 1 ([15])**.** *Let $\{x_n\}_{n\in\mathbb{N}}$ be a sequence of numbers from $[0,1]$ such that $\lim_{n\to\infty} x_n = 1$ and the t-norm T is of H-type. Then*

$$\lim_{n\to\infty} T^\infty_{i=n}x_i = \lim_{n\to\infty} T^\infty_{i=1}x_{n+i} = 1.$$

Definition 3 (George and Veeramani [9])**.** *A triple (X, M, T) is called a fuzzy metric space if X is a non-empty set, T is a continuous t-norm and $M : X^2 \times (0,\infty) \to (0,1]$ is a fuzzy set satisfying the following conditions:*

(GV1) $M(x,y,t) > 0$,
(GV2) $M(x,y,t) = 1$ *if and only if* $x = y$,
(GV3) $M(x,y,t) = M(y,x,t)$,
(GV4) $M(x,z,t+s) \geq T(M(x,y,t), M(y,z,s))$,
(GV5) $M(x,y,\cdot) : (0,\infty) \to (0,1]$ *is continuous,*

for all $x, y, z \in X$ and $t, s > 0$.

Definition 4 ([9])**.** *Let (X, M, T) be a fuzzy metric space. Then,*

(i) *A sequence* $\{x_n\}_{n\in\mathbb{N}}$ *converge to* $x \in X$ *(i.e.,* $\lim_{n\to+\infty} x_n = x$*), if* $\lim_{n\to+\infty} M(x_n, x, t) = 1$, $t > 0$.
(ii) *A sequence* $\{x_n\}_{n\in\mathbb{N}}$ *is called Cauchy if, for each* $\varepsilon \in (0,1)$ *and* $t > 0$*, there exists* $n_0 \in \mathbb{N}$ *such that* $M(x_n, x_m, t) > 1 - \varepsilon$*, for all* $m, n \geq n_0$.

A fuzzy metric space (X, M, T) is complete if every Cauchy sequence is convergent.

Originally, in [11], a fixed point results in the probabilistic metric spaces with the following generalization of the Banach's contraction principle:

$$F_{Tu,Tv}(qx) \geq \min\{F_{u,v}(x), F_{u,Tu}(x), F_{v,Tv}(x), F_{u,Tv}(2x), F_{v,Tu}(2x)\}, \tag{1}$$

where $x > 0$, are studied. Mappings F which, for some $q \in (0,1)$, satisfies condition (1) are named quasi-contractive mappings. In [11] is used t-norm T such that $T(x,x) \geq x$, $x \in [0,1]$, which means that $T = T_{min}$.

In the first part of the section with the main results, possibilities for further extensions of t-norm in the context of fixed point problems with quasi-contractive mappings in the fuzzy metric spaces are elaborated. Within this observation, the potential for removing the scale 2 in the last two terms of condition (1) is stated.

Let (X, d) be a metric space and mapping $T : X \to X$. Recently, Kumam et al. [22] presented the following generalization contractive condition (1) of Ćirić,

$$d(Tx, Ty) \leq q \max\{d(x,y), d(x,Tx), d(y,Ty), d(x,Ty), d(y,Tx),$$
$$d(T^2x, x), d(T^2x, Tx), d(T^2x, y), d(T^2x, Ty)\},$$

for all $x, y \in X$ and some $q \in [0,1)$. In this case, they called the given condition a generalized quasi-contraction.

In the current paper, we study generalized quasi-contractions in fuzzy metric spaces, the existence and uniqueness of a fixed point are proven and an appropriate example is given.

Definition 5 (Gregori and Sapena [23])**.** *Let* (X, M, T) *be a fuzzy metric space.* $f : X \to X$ *is called a fuzzy contractive mapping if there exists* $k \in (0,1)$ *such that*

$$\left(\frac{1}{M(fx, fy, t)} - 1\right) \leq k\left(\frac{1}{M(x,y,t)} - 1\right), \tag{2}$$

for each $x, y \in X$ *and* $t > 0$*,* k *is called the contractive constant of* f.

Definition 6 (Mihet [24])**.** *Let* Ψ *be the class of all mappings* $\psi : (0,1] \to (0,1]$ *such that* ψ *is continuous, non-decreasing and* $\psi(t) > t$ *for all* $t \in (0,1)$*. Let* $\psi \in \Psi$*. A mapping* $f : X \to X$ *is said to be fuzzy* ψ*-contractive mapping if*

$$M(fx, fy, t) \geq \psi(M(x,y,t)), \tag{3}$$

for all $x, y \in X$ *and* $t > 0$.

Definition 7 (Wardowski [25])**.** *Denoted by* $\mathcal{H}$ *the family of mappings* $\eta : (0,1] \to [0,\infty)$ *satisfying the following two conditions:*

(H1) η *transforms* $(0,1]$ *onto* $[0,\infty)$;
(H2) η *is strictly decreasing.*

Note that *(H1)* and *(H2)* imply that $\eta(1) = 0$.

Definition 8. *Let (X, M, T) be a fuzzy metric space. A mapping $f : X \to X$ is said to be fuzzy $\mathcal{H}$-contractive with respect to $\eta \in \mathcal{H}$ if there exists $k \in (0,1)$ satisfying the following condition*

$$\eta\left(M(fx, fy, t)\right) \leq k\eta(M(x, y, t)), \tag{4}$$

for all $x, y \in X$ and $t > 0$.

Note that for a mapping $\eta \in \mathcal{H}$ of the form $\eta(t) = \frac{1}{t} - 1$, $t \in (0,1]$, Definition 8 reduces to Definition 5.

Remark 1. *It has been shown in [26] that the class of fuzzy $\mathcal{H}$-contractive mappings are included in the class of ψ-contractive mappings.*

Remark 2. *Note that if Definition 3 is allowed to is $M(x, y, t) = 0$ then condition (2) of Gregori and Sapena and condition (3) of Mihet are not correctly defined, which is why condition (GV1) in Definition 3 is important.*

Moreover, if (X, M, T) is a fuzzy metric space then M is a continuous function on $X \times X \times (0, \infty)$ [27], and $M(x, y, \cdot)$ is nondecreasing for all $x, y \in X$, [28].

Proposition 2. *Let (X, M, T) be a fuzzy metric space and let $\eta \in \mathcal{H}$. A sequence $\{x_n\}_{n \in \mathbb{N}}$ in X is Cauchy if and only if, for every $\varepsilon > 0$ and $t > 0$, there exists $n_0 \in \mathbb{N}$ such that*

$$\eta\left(M(x_m, x_n, t)\right) < \varepsilon, \text{ for all } m, n \geq n_0.$$

Proposition 3. *Let (X, M, T) be a fuzzy metric space and let $\eta \in \mathcal{H}$. A sequence $\{x_n\}_{n \in \mathbb{N}}$ in X is convergent to $x \in X$ if and only if,*

$$\lim_{n \to \infty} \eta(M(x_n, x, t)) = 0,$$

for all $t > 0$.

Theorem 1 (Wardowski [25])**.** *Let (X, M, T) be a complete fuzzy metric space and let $f : X \to X$ be a fuzzy $\mathcal{H}$-contractive mapping with respect to $\eta \in \mathcal{H}$ such that*

(a) $T_{n=1}^{k} M(x, fx, t_n) \neq 0$, *for all $x \in X$, $k \in \mathbb{N}$ and any sequence $\{t_n\}_{n \in \mathbb{N}} \subset (0, \infty)$, $t_n \searrow 0$;*
(b) $T(r, s) > 0$ *implies $\eta(T(r, s)) \leq \eta(r) + \eta(s)$, for all $r, s \in \{M(x, fx, t) : x \in X, t > 0\}$;*
(c) $\eta(M(x, fx, t_n) : n \in \mathbb{N}\}$ *is bounded for all $x \in X$ and any sequence $\{t_n\}_{n \in \mathbb{N}} \subset (0, \infty)$, $t_n \searrow 0$.*

Then, f has a unique fixed point $x^* \in X$ and for each $x_0 \in X$, the sequence $\{f^n x_0\}_{n \in \mathbb{N}}$ converges to x^*.

Further, motivated by the contractive condition (1) of Ćirić, in [27] fuzzy $\mathcal{H}$-contractive mappings are generalized and the existence of a fixed point for fuzzy $\mathcal{H}$-quasi-contractive mapping is proven.

Definition 9 ([27])**.** *Let (X, M, T) be a fuzzy metric space. A mapping $f : X \to X$ is said to be fuzzy $\mathcal{H}$-quasi-contractive with respect to $\eta \in \mathcal{H}$ if there exists $k \in (0,1)$, satisfying the following condition:*

$$\begin{aligned} \eta(M(fx, fy, t)) \leq \; & k \max\{\eta(M(x, y, t)), \eta(M(x, fx, t)), \\ & \eta(M(y, fy, t)), \eta(M(x, fy, t)), \eta(M(y, fx, t))\}, \end{aligned} \tag{5}$$

for all $x, y \in X$ and any $t > 0$.

In the last part of the next section fuzzy $\mathcal{H}$-quasi-contractive mappings are generalized in the spirit of generalized quasi-contractions [22] and fixed point result in fuzzy metric spaces is presented. Moreover, the mentioned generalization is confirmed by example.

2. Main Results

In this section, we use the fuzzy metric spaces in the sense of Definition 3 with additional condition $\lim_{t\to\infty} M(x,y,t) = 1,\ x,y \in X$.

To prove the results, we use the following very important lemma:

Lemma 1. *Let $\{x_n\}$ be a sequence in fuzzy metric space (X, M, T). If there exists $q \in (0,1)$ such that*

$$M(x_n, x_{n+1}, t) \geq M(x_{n-1}, x_n, \frac{t}{q}),\ t > 0,\ n \in \mathbb{N}, \tag{6}$$

and

$$\lim_{n\to\infty} T_{i=n}^{\infty} M(x_0, x_1, \frac{1}{\mu^i}) = 1,\ \mu \in (0,1), \tag{7}$$

then $\{x_n\}$ is a Cauchy sequence.

Proof. Let $\sigma \in (q,1)$ and let $t > 0$. Then $\sum_{i=1}^{\infty} \sigma^i < \infty$; therefore, there exists $n_0 = n_0(t)$, such that $\sum_{i=n_0}^{\infty} \sigma^i \leq t$. Clearly, condition (6) implies that

$$M(x_n, x_{n+1}, t) \geq M(x_0, x_1, \frac{t}{q^n}),\ n \in \mathbb{N}.$$

For $n \geq n_0$, $m \in \mathbb{N}$ we have

$$M(x_n, x_{n+m}, t) \geq M(x_n, x_{n+m}, \sum_{i=n}^{\infty} \sigma^i) \geq M(x_n, x_{n+m}, \sum_{i=n}^{n+m-1} \sigma^i)$$

$$\geq \underbrace{T(T(\dots T}_{(m-1)-times}(M(x_n, x_{n+1}, \sigma^n), \dots, M(x_{n+m-1}, x_{n+m}, \sigma^{n+m-1})))$$

$$\geq \underbrace{T(T(\dots T}_{(m-1)-times}(M(x_0, x_1, \frac{\sigma^n}{q^n}), \dots, M(x_0, x_1, \frac{\sigma^{n+m-1}}{q^{n+m-1}}))).$$

Let $\mu = \dfrac{q}{\sigma} \in (0,1)$. Then

$$M(x_n, x_{n+m}, t) \geq T_{i=n}^{n+m-1} M(x_0, x_1, \frac{1}{\mu^i}) \geq T_{i=n}^{\infty} M(x_0, x_1, \frac{1}{\mu^i}),\ n \geq n_0, m \in \mathbb{N}.$$

Now, by (7) follows Definition 4 (ii) and $\{x_n\}$ is Cauchy sequence. $\square$

Our first new result in this section is the following:

Theorem 2. *Let (X, M, T_{min}) be a complete fuzzy metric space and let $f : X \to X$ be a quasi-contractive mapping such that, for some $q \in (0, \frac{1}{2})$:*

$$\begin{aligned} M(fx, fy, t) \quad \geq \quad & \min\{M(x, y, \frac{t}{q}), M(fx, x, \frac{t}{q}), \\ & M(fy, y, \frac{t}{q}), M(x, fy, \frac{t}{q}), M(fx, y, \frac{t}{q})\}, \end{aligned} \tag{8}$$

for all $x, y \in X$ and $t > 0$. Suppose that there exists $x_0 \in X$ such that

$$\lim_{n\to\infty} T_{i=n}^{\infty} M(x_0, fx_0, \frac{1}{\mu^i}) = 1,\ \mu \in (0, 1). \tag{9}$$

Then, f has unique fixed point.

Proof. Let $x_n = fx_{n-1},\ n \in \mathbb{N}$, where initial $x_0 \in X$ satisfied (9). Then, observe (8) with $x = x_{n-1}, y = x_n$:

$$\begin{aligned} & M(x_n, x_{n+1}, t) \geq \min\{M(x_{n-1}, x_n, \frac{t}{q}), M(x_n, x_{n-1}, \frac{t}{q}), M(x_{n+1}, x_n, \frac{t}{q}), \\ & M(x_{n-1}, x_{n+1}, \frac{t}{q}), M(x_n, x_n, \frac{t}{q})\} \\ \geq \quad & \min\{M(x_{n-1}, x_n, \frac{t}{q}), M(x_n, x_{n+1}, \frac{t}{q}), \min\{M(x_{n-1}, x_n, \frac{t}{2q}), \\ & M(x_n, x_{n+1}, \frac{t}{2q})\}, \min\{M(x_{n-1}, x_n, \frac{t}{2q}), M(x_n, x_{n+1}, \frac{t}{2q})\}\}, \end{aligned}$$

$t > 0, n \in \mathbb{N}$. If we suppose that

$$\min\{M(x_{n-1}, x_n, \frac{t}{2q}), M(x_n, x_{n+1}, \frac{t}{2q})\} = M(x_n, x_{n+1}, \frac{t}{2q}),$$

then, using the previous calculations, we get the contradiction

$$M(x_n, x_{n+1}, t) \geq M(x_n, x_{n+1}, \frac{t}{2q}),$$

since $2q < 1$ and $M(x, y, t)$ is increased by t. Thus,

$$M(x_n, x_{n+1}, \frac{t}{2q}) \geq M(x_{n-1}, x_n, \frac{t}{2q}),$$

for all $n \in \mathbb{N}$, and for $q_1 = 2q,\ q_1 \in (0, 1)$:

$$M(x_n, x_{n+1}, t) \geq M(x_{n-1}, x_n, \frac{t}{q_1}),\ t > 0,\ n \in \mathbb{N}.$$

By Lemma 1, it follows that $\{x_n\}$ is Cauchy sequence. Space (X, M, T_{min}) is complete and there exist $x^* \in X$ such that $\lim_{n\to\infty} x_n = x^*$.

If we put $x = x_n, y = x^*$ in (8):

$$M(x_{n+1}, fx^*, t) \geq \min\{M(x_n, x^*, \frac{t}{q}), M(x_{n+1}, x_n, \frac{t}{q}), M(fx^*, x^*, \frac{t}{q}), M(x_n, fx^*, \frac{t}{q}), M(fx, y, \frac{t}{q})\},$$

$n \in \mathbb{N}$, $t > 0$, and take $n \to \infty$ then

$$M(x^*, fx^*, t) \geq M(x^*, fx^*, \frac{t}{q}),\ t > 0,$$

i.e., x^* is the fixed point for f.

Suppose that x^* and y^* are fixed points for f then, by (8):

$$M(fx^*, fy^*, t) \geq \min\{M(x^*, y^*, \frac{t}{q}), M(fx^*, x^*, \frac{t}{q}), M(fy^*, y^*, \frac{t}{q}), M(x^*, fy^*, \frac{t}{q}), M(fx^*, y^*, \frac{t}{q})\},$$

$t > 0$. Then, $M(x^*, y^*, t) \geq M(x^*, y^*, \frac{t}{q})$, $t > 0$, and $x^* = y^*$. □

Remark 3. *Condition (8) is one of the Ćirić's type (1) where scale* 2 *in the last two terms is omitted. This improvement of condition has a narrowing of interval for contractive constant q as a consequence. With small changes in the proof of Theorem 2, it can be shown that for the extension of the interval to* $q \in (0, 1)$, *we need condition*

$$M(fx, fy, t) \geq \min\{M(x, y, \frac{t}{q}), M(fx, x, \frac{t}{q}), M(fy, y, \frac{t}{q}), M(fx, y, \frac{t}{q}), M(x, fy, \frac{2t}{q})\}, \tag{10}$$

for all $x, y \in X$ *and* $t > 0$. *In both observed cases for t-norm is used* T_{min}. *For a wider class of t-norm* $T \geq T_p$, *the condition is slightly weaker, i.e., there exists* $q \in (0, 1)$ *such that*

$$M(fx, fy, t) \geq \min\{M(x, y, \frac{t}{q}), M(fx, x, \frac{t}{q}), M(fy, y, \frac{t}{q}), M(fx, y, \frac{t}{q}), \sqrt{M(x, fy, \frac{2t}{q})}\}, \tag{11}$$

for all $x, y \in X$ *and* $t > 0$. *In a more general case, if T is arbitrary t-norm, we have the following condition: there exist* $q \in (0, 1)$ *such that*

$$M(fx, fy, t) \geq \min\{M(x, y, \frac{t}{q}), M(fx, x, \frac{t}{q}), M(fy, y, \frac{t}{q}), M(fx, y, \frac{t}{q})\}, \tag{12}$$

for all $x, y \in X$ *and* $t > 0$. *However, if we restrict t-norm to H-type additional condition (9) could be omitted, due to Proposition 1.*

Example 1. *Let $X = (0,2)$, $M(x,y,t) = e^{-\frac{|x-y|}{t}}$, $T = T_P$ and*

$$f(x) = \begin{cases} 2-x & x \in (0,1) \\ 1 & x \in [1,2). \end{cases}$$

Case 1. If $x,y \in [1,2)$, then $M(fx,fy,t) = 1, t > 0$ and conditions (11) and (12) are trivially satisfied.
Case 2. If $x \in [1,2)$ and $y \in (0,1)$, then, for $q \geq \frac{1}{2}$, we have

$$M(fx,fy,t) = e^{-\frac{1-y}{t}} \geq e^{-\frac{2q(1-y)}{t}} = M(fy,y,\frac{t}{q}),\ t > 0.$$

Case 3. Analogously as in the previous case for $q \geq \frac{1}{2}$ we have

$$M(fx,fy,t) \geq M(fx,x,\frac{t}{q}),\ x \in (0,1),\ y \in [1,2),\ t > 0.$$

Case 4. If $x,y \in (0,1)$, then, for $q \geq \frac{1}{2}$,

$$M(fx,fy,t) = e^{-\frac{x-y}{t}} \geq e^{-\frac{1-y}{t}} \geq e^{-\frac{2q(1-y)}{t}} = M(fy,y,\frac{t}{q}),\ x > y,\ t > 0,$$

and

$$M(fx,fy,t) \geq M(fx,x,\frac{t}{q}),\ x < y,\ t > 0.$$

Thus, conditions (11) and (12) are satisfied for all $x,y \in X, t > 0$ and by Remark 3 and Theorem 2 follows that $x = 1$ is a unique fixed point for f.

Now, we announce our second new result in the paper.

Theorem 3. *Let (X,M,T) be a complete fuzzy metric space, $T \geq T_P$ and let $f : X \to X$ is a fuzzy generalized quasi-contractive mapping such that for some $q \in (0,1)$:*

$$\begin{aligned} M(fx,fy,t) \quad \geq \quad & \min\{M(x,y,\frac{t}{q}), M(fx,x,\frac{t}{q}), M(fy,y,\frac{t}{q}), \\ & M(fx,y,\frac{t}{q}), \sqrt{M(x,fy,\frac{2t}{q})}, \sqrt{M(f^2x,x,\frac{2t}{q})}, \\ & M(f^2x,fx,\frac{t}{q}), M(f^2x,y,\frac{t}{q}), M(f^2x,fy,\frac{t}{q})\}, \end{aligned} \tag{13}$$

for all $x,y \in X$ and $t > 0$. Suppose that there exists $x_0 \in X$ such that

$$\lim_{n\to\infty} T_{i=n}^{\infty} M(x_0,fx_0,\frac{1}{\mu^i}) = 1,\ \mu \in (0,1). \tag{14}$$

Then, f has a unique fixed point.

Proof. Let $x_0 \in X$ satisfied condition (14) and $x_n = fx_{n-1}$, $n \in \mathbb{N}$. Take $x = x_{n-1}, y = x_n$ in (13)

$$\begin{aligned}
& M(x_n, x_{n+1}, t) \geq \min\{M(x_{n-1}, x_n, \frac{t}{q}), M(x_n, x_{n-1}, \frac{t}{q}), M(x_{n+1}, x_n, \frac{t}{q}), \\
& M(x_n, x_n, \frac{t}{q}), \sqrt{M(x_{n-1}, x_{n+1}, \frac{2t}{q})}, \sqrt{M(x_{n+1}, x_{n-1}, \frac{2t}{q})}, \\
& M(x_{n+1}, x_n, \frac{t}{q}), M(x_{n+1}, x_n, \frac{t}{q}), M(x_{n+1}, x_{n+1}, \frac{t}{q})\} \\
= \; & \min\{M(x_{n-1}, x_n, \frac{t}{q}), M(x_n, x_{n+1}, \frac{t}{q}), \sqrt{M(x_{n-1}, x_{n+1}, \frac{2t}{q})}, 1\} \\
\geq \; & \min\{M(x_{n-1}, x_n, \frac{t}{q}), M(x_n, x_{n+1}, \frac{t}{q}), \sqrt{M(x_{n-1}, x_n, \frac{t}{q}) \cdot M(x_n, x_{n+1}, \frac{t}{q})}\} \\
= \; & \min\{M(x_{n-1}, x_n, \frac{t}{q}), M(x_n, x_{n+1}, \frac{t}{q})\},
\end{aligned}$$

for all $t > 0, n \in \mathbb{N}$. Now, if we suppose that

$$\min\{M(x_{n-1}, x_n, \frac{t}{q}), M(x_n, x_{n+1}, \frac{t}{q})\} = M(x_n, x_{n+1}, \frac{t}{q})$$

contradiction $M(x_n, x_{n+1}, t) \geq M(x_n, x_{n+1}, \frac{t}{q})$ is obtained. Thus,

$$M(x_n, x_{n+1}, t) \geq M(x_{n-1}, x_n, \frac{t}{q}),$$

for all $t > 0, n \in \mathbb{N}$, now by Lema 1, it follows that $\{x_n\}$ is Cauchy sequence. Since (X, M, T) is complete, there exists $x^* \in X$ such that $\lim\limits_{n \to \infty} x_n = x^*$.

Further, let $x = x_n, y = x^*$ in (13):

$$\begin{aligned}
& M(x_{n+1}, fx^*, t) \geq \min\{M(x_n, x^*, \frac{t}{q}), M(x_{n+1}, x_n, \frac{t}{q}), M(fx^*, x^*, \frac{t}{q}), \\
& M(x_{n+1}, x^*, \frac{t}{q}), \sqrt{M(x_n, x_{n+1}, \frac{t}{q}) \cdot M(x_{n+1}, fx^*, \frac{t}{q})}, \sqrt{M(x_{n+2}, x_n, \frac{2t}{q})}, \\
& M(x_{n+2}, x_{n+1}, \frac{t}{q}), M(x_{n+2}, x^*, \frac{t}{q}), M(x_{n+2}, fx^*, \frac{t}{q})\},
\end{aligned}$$

for all $t > 0, n \in \mathbb{N}$. Take $n \to \infty$ in the last relation:

$$\begin{aligned}
M(x^*, fx^*, t) \;\; & \geq \;\; \min\{1, 1, M(fx^*, x^*, \frac{t}{q}), 1, \sqrt{M(x^*, fx^*, \frac{t}{q})}, 1, 1, 1, \\
& \qquad M(x^*, fx^*, \frac{t}{q})\} \\
& = \;\; \min\{1, M(fx^*, x^*, \frac{t}{q}), \sqrt{M(x^*, fx^*, \frac{t}{q})}\} \\
& = \;\; M(fx^*, x^*, \frac{t}{q}),
\end{aligned}$$

for all $t > 0$. Hence, x^* is the fixed point for mapping f.

Suppose that $x^* = fx^*$, $y^* = fy^*$ and $x^* \neq y^*$. Condition (13) with $x = x^*$, $y = y^*$ leads to the contradiction:

$$\begin{aligned} M(x^*,y^*,t) &\geq \min\{M(x^*,y^*,\frac{t}{q}), M(x^*,x^*,\frac{t}{q}), M(y^*,y^*,\frac{t}{q}), M(x^*,y^*,\frac{t}{q}), \\ &\quad \sqrt{M(x^*,x^*,\frac{t}{q}) \cdot M(x^*,y^*,\frac{t}{q})}, \sqrt{M(x^*,x^*,\frac{2t}{q})}, M(x^*,x^*,\frac{t}{q}), \\ &\quad M(x^*,y^*,\frac{t}{q}), M(x^*,y^*,\frac{t}{q})\} \\ &= M(x^*,y^*,\frac{t}{q}), \end{aligned}$$

for all $t > 0$. and x^* is a unique fixed point. □

Remark 4. *Considering the proof of Theorem 3, it is evident that if we replace condition (13) by the following one:*

$$\begin{aligned} M(fx,fy,t) &\geq \min\{M(x,y,\frac{t}{q}), M(fx,x,\frac{t}{q}), M(fy,y,\frac{t}{q}), \\ &\quad M(fx,y,\frac{t}{q}), M(f^2x,fx,\frac{t}{q}), M(f^2x,y,\frac{t}{q}), M(f^2x,fy,\frac{t}{q})\}, \end{aligned} \tag{15}$$

for all $x, y \in X$ and $t > 0$, then condition $T \geq T_P$ could be omitted.

On the other hand, if we restrict t-norm to $T = T_{min}$ instead of (13), we have the stronger condition:

$$\begin{aligned} &M(fx,fy,t) \geq \min\{M(x,y,\frac{t}{q}), M(fx,x,\frac{t}{q}), M(fy,y,\frac{t}{q}), \\ &M(fx,y,\frac{t}{q}), M(x,fy,\frac{2t}{q}), M(f^2x,x,\frac{2t}{q}), M(f^2x,fx,\frac{t}{q}), \\ &M(f^2x,y,\frac{t}{q}), M(f^2x,fy,\frac{t}{q})\}, \end{aligned} \tag{16}$$

for all $x, y \in X$ and $t > 0$.

Example 2. *Let (X, M, T_P) be a fuzzy metric space where $X = \{1, 2, 3, 4, 5\}$ and*

$$M(x,y,t) = \frac{t}{t + d(x,y)}, \; d(x,y) = \begin{cases} 0, & x = y \\ 2, & (x,y) \in \{(1,4),(1,5),(4,1),(5,1)\} \\ 1, & otherwise \end{cases}.$$

Let $f : X \to X$ be defined by

$$f1 = f2 = f3 = 1, \; f4 = 2, \; f5 = 3.$$

Observe that if $x = y$ or $x, y \in \{1, 2, 3\}$, then $M(fx, fy, t) = 1$, $t > 0$ and conditions (11) and (13) are fulfilled.

Let $x = 4, y = 5$. *Then*

$$\frac{t}{t+1} < \min\left\{\frac{t}{t+q}, \frac{t}{t+q}, \frac{t}{t+q}, \frac{t}{t+q}, \sqrt{\frac{t}{t+\frac{q}{2}}}\right\},\ t > 0,$$

for every $q \in (0,1)$, *which is in contradiction with condition (11), while condition (13):*

$$\frac{t}{t+1} \geq \min\left\{\frac{t}{t+q}, \frac{t}{t+q}, \frac{t}{t+q}, \frac{t}{t+q}, \sqrt{\frac{t}{t+\frac{q}{2}}},\right.$$

$$\left.\sqrt{\frac{t}{t+\frac{q}{2}}}, \frac{t}{t+q}, \frac{t}{t+2q}, \frac{t}{t+q}\right\} = \frac{t}{t+2q},\ t > 0,$$

is satisfied for $q \in (\frac{1}{2}, 1)$. *One could check that inequality (13) holds for the rest* $x, y \in X$.

If we keep the definitions for $M(x,y,t), d(x,y)$ *and* $f(x)$ *and take the appropriate* T, *then the same conclusions for conditions (12), (15), (10) and (16) could be obtained, i.e., (12) and (10) failed for* $x = 4, y = 5$, *while (15) and (16) are fulfilled for every* $x, y \in X$.

Example 3. *Let* (X, M, T) *be a fuzzy metric space where* $X = \{1,2,3,4\}$ *and*

$$M(x,y,t) = \frac{t}{t+d(x,y)},\ d(x,y) = \begin{cases} 0, & x = y \\ 1, & (x,y) \in \{(1,2),(3,4)\} \\ 2, & \text{otherwise} \end{cases}.$$

Let $f : X \to X$ *be defined by*

$$f1 = f2 = f3 = 1,\ f4 = 2.$$

If $x = y$ *or* $x, y \in \{1,2,3\}$ *then* $M(fx, fy, t) = 1,\ t > 0$.

Let $x = 3, y = 4$. *Then, for every* $q \in (0,1)$,

$$M(fx, fy, t) = \frac{t}{t+1} < \frac{t}{t+q} = M(x, y, \frac{t}{q}),\ t > 0,$$

and Banach contraction principle is not satisfied. On the other hand, for $x = 3, y = 4$ *and* $q \in [\frac{1}{2}, 1)$:

$$M(fx, fy, t) = \frac{t}{t+1} \geq \frac{t}{t+2q} = M(fx, x, \frac{t}{q}),\ t > 0.$$

Thus, conditions (12), (11) and (10) are satisfied for given values of x and y as well as for all $x, y \in X$.

Finally, we introduce a new type of mapping and prove the corresponding new result in the context of fuzzy metric spaces.

Definition 10. *Let* (X, M, T) *be a fuzzy metric space. A mapping* $f : X \to X$ *is said to be fuzzy generalized* $\mathcal{H}$*-quasi-contractive with respect to* $\eta \in \mathcal{H}$ *if there exists* $q \in (0,1)$ *such that*

$$\begin{aligned} &\eta(M(fx,fy,t)) \leq q \max\{\eta(M(x,y,t)), \eta(M(fx,x,t)), \\ &\eta(M(fy,y,t)), \eta(M(f^2x,fx,t)), \eta(M(fx,y,t)), \eta(M(x,fy,t)), \\ &\eta(M(f^2x,x,t)), \eta(M(f^2x,y,t)), \eta(M(f^2x,fy,t))\}, \end{aligned} \tag{17}$$

for all $x, y \in X$, $t > 0$.

Theorem 4. *Let (X, M, T) be a complete fuzzy metric space and let $f : X \to X$ be a fuzzy generalized $\mathcal{H}$-quasi-contractive mapping with respect to $\eta \in \mathcal{H}$ such that*
(a) $\tau \geq T(r, s)$ implies $\eta(\tau) \leq \eta(r) + \eta(s)$, for all $r, s, \tau \in \{M(f^n x, f^m x, t) : x \in X,\ t > 0,\ n, m \in \mathbb{N}\}$;
(b) $\{\eta(M(x, fx, t_n)) : n \in \mathbb{N}\}$ is bounded for all $x \in X$ and any sequence $\{t_n\} \subseteq (0, \infty)$, $t_n \to 0$.

Then, f has a unique fixed point $x^* \in X$ and for each $x \in X$ the sequence $\{f^n x\}$ converges to x^*.

Proof. Let $A \subseteq X$ and $\delta_t(A) = \sup\{\eta(M(x, y, t)) : x, y \in A\}$. The orbit of f at x is defined by

$$O(x, n) = \{x, fx, \ldots, f^n x\},\ n \in \mathbb{N} \text{ and } O(x, \infty) = \{x, fx, \ldots\}.$$

Take arbitrary $x \in X$, $n \in \mathbb{N}$ and let $i, j \in \{1, 2, \ldots, n-1\}$. By (17), with $x = f^{i-1}x, y = f^{j-1}x$, we have

$$\begin{aligned}
&\eta(M(ff^{i-1}x, ff^{j-1}x, t)) \leq q \max\{\eta(M(f^{i-1}x, f^{j-1}x, t)),\\
&\eta(M(f^i x, f^{i-1}x, t)), \eta(M(f^j x, f^{j-1}x, t)), \eta(M(f^i x, f^{j-1}x, t)),\\
&\eta(M(f^{i+1}x, f^i x, t)), \eta(M(f^{i+1}x, f^i x, t)), \eta(M(f^{i+1}x, f^{j-1}x, t)),\\
&\eta(M(f^{i-1}x, f^j x, t)), \eta(M(f^{i+1}x, f^j x, t))\} \leq q\, \delta_t(O(x, n)),
\end{aligned} \tag{18}$$

for all $t > 0$. Suppose that there exist $i_0, j_0 \in \{2, 3, \ldots, n-1\}$ such that $\delta_t(O(x, n)) = \eta(M(f^{i_0}x, f^{j_0}x, t))$. Then, by (18) with $i = i_0, j = j_0$, it follows that

$$\delta_t(O(x, n)) \leq q\delta_t(O(x, n)),$$

i.e., $\delta_t(O(x, n)) = 0$ and $\eta(M(f^i x, f^j x, t)) = 0$, $i, j \leq n-1$. In particular, $\eta(M(x, fx, t)) = 0$, $M(x, fx, t) = 1$ and x is fixed point for f.

For the case

$$\delta_t(O(x, n)) = \eta(M(x, f^k x, t)),$$

for some $k \leq n$ the proof is analogous with ([27], [Theorem 2.3.]) and $\{x_n\}_{n \in \mathbb{N}}$, $x_n = f^n x$, is a Cauchy sequence. Thus, there exists $x^* \in X$ such that $\lim_{n \to \infty} x_n = x^*$.

By (GV4), condition (a) and (17), with $x = f^n x^*, y = x^*$, for every $\varepsilon > 0$ and $n \in \mathbb{N}$ we have

$$\eta(M(x^*, fx^*, \varepsilon + t)) \leq \eta(M(x^*, f^{n+1}x^*, \varepsilon)) + \eta(M(f^{n+1}x^*, fx^*, t))$$

$$\leq \eta(M(x^*, f^{n+1}x^*, \varepsilon)) + q \max\{\eta(M(f^n x^*, x^*, t)), \eta(M(f^{n+1}x^*, f^n x^*, t)),$$

$$\eta(M(fx^*, x^*, t)), \eta(M(f^{n+1}x^*, x^*, t)), \eta(M(f^n x^*, fx^*, t)), \eta(M(f^{n+2}x^*, f^n x^*, t)),$$

$$\eta(M(f^{n+2}x^*, f^{n+1}x^*, t)), \eta(M(f^{n+2}x^*, x^*, t)), \eta(M(f^{n+2}x^*, fx^*, t))\}.$$

If we take $n \to \infty$ and $\varepsilon \to 0$ in the previous calculation, the next relation is obtained

$$\eta(M(x^*, fx^*, t)) \leq q\eta(M(x^*, fx^*, t))$$

which implies that $\eta(M(x^*, fx^*, t)) = 0$, i.e., $M(x^*, fx^*, t) = 1$ and x^* is a fixed point for f.

Suppose that x^* and y^* are fixed points for f. Then, by (17), with $x = x^*, y = y^*$, we have

$$\eta(M(x^*, y^*, t)) = \eta(M(fx^*, fy^*, t)) \leq q \max\{\eta(M(x^*, y^*, t)), \eta(M(fx^*, x^*, t)),$$

$$\eta(M(fy^*,y^*,t)),\eta(M(fx^*,y^*,t)),\eta(M(x^*,fy^*,t)),\eta(M(f^2x^*,x^*,t)),$$
$$\eta(M(f^2x^*,fx^*,t)),\eta(M(f^2x^*,y^*,t)),\eta(M(f^2x^*,fy^*,t))\} = q\eta(M(x^*,y^*,t)).$$

Thus, $M(x^*,y^*,t) = 1$ and x^* is the unique fixed point for f. $\square$

Example 4. *Let (X, M, T_P) be a fuzzy metric space and $f : X \to X$, where $X, d(x,y)$ and f are the same as in Example 2, while*

$$M(x,y,t) = e^{-\frac{d(x,y)}{t+1}},\ x,y \in X,\ t > 0.$$

Take arbitrary $\eta \in \mathcal{H}$ and let $x = 4$ and $y = 5$. In that case, looking at condition (5), we have the following:

$$\eta(e^{-\frac{1}{t+1}}) \le q\max\left\{\eta(e^{-\frac{1}{t+1}}),\eta(e^{-\frac{1}{t+1}}),\eta(e^{-\frac{1}{t+1}}),\eta(e^{-\frac{1}{t+1}}),\eta(e^{-\frac{1}{t+1}})\right\},\ t > 0,$$

which is not satisfied since $q \in (0,1)$, and f is not fuzzy $\mathcal{H}$-quasi-contractive mapping.

Now, take $\eta(\tau) = -\ln(\tau)$ when, $\eta(M(x,y,t)) = \frac{d(x,y)}{t+1}$, and check condition (17) for $x = 4$ and $y = 5$:

$$\begin{aligned}\frac{1}{t+1} &\le q\max\left\{\frac{1}{t+1},\frac{1}{t+1},\frac{1}{t+1},\frac{1}{t+1},\frac{1}{t+1},\frac{2}{t+1},\frac{1}{t+1},\frac{2}{t+1},\frac{1}{t+1}\right\}\\ &= \frac{2q}{t+1},\end{aligned}$$

for all $t > 0$. Thus, for $q \in [\frac{1}{2},1)$ condition (17) is satisfied. Similarly, it could be shown that (17) holds for all $x,y \in X,\ t > 0$ and f is fuzzy generalized $\mathcal{H}$-quasi-contractive mapping with respect to specified η.

Moreover, conditions (a) and (b) of Theorem 4 hold and $x = 1$ is a unique fixed point for f.

Remark 5. *If, in Theorem 4, we suppose that $f(x) \ge x, x \in X$, then the contractive condition (17) could be replaced by the following one:*

$$\begin{aligned}&\eta(M(fx,fy,t)) \le q\max\{\eta(M(x,y,t)),\eta(M(fx,x,t)),\eta(M(fy,y,t)),\\ &\eta(M(f^2x,fx,t)),\eta(M(fx,y,t)),\eta(M(x,fy,t)),\eta(M(f^2x,x,t)),\\ &\eta(M(f^2x,y,t)),\eta(M(f^2x,fy,t))\},\ x \ge y,\ t > 0\end{aligned}$$

and

$$\begin{aligned}&\eta(M(fx,fy,t)) \le q\max\{\eta(M(x,y,t)),\eta(M(fx,x,t)),\eta(M(fy,y,t)),\\ &\eta(M(f^2y,fy,t)),\eta(M(fx,y,t)),\eta(M(x,fy,t)),\eta(M(f^2y,y,t)),\\ &\eta(M(f^2y,x,t)),\eta(M(f^2y,fx,t))\},\ x < y,\ t > 0.\end{aligned}$$

Then, the proof of Theorem 4 is slightly modified in the part where the existence of the fixed point is proved. Now, we take $x = f^nx^*, y = x^*$ if $x \ge y$ and $x = x^*, y = f^nx^*$ if $x < y$.

Example 5. *Let $X = [0,+\infty)$, $f(x) = 2x$, while $M(x,y,t)$ and $\eta(\tau)$ are the same as in Example 4. It is easy to check that condition (5) is not satisfied. Moreover, in general quasi-contractive conditions are not suitable for functions of type $f(x) = kx, k > 1$.*

The other way, for $q \ge \frac{2}{3}$, we have

$$\eta(M(fx,fy,t)) = \frac{2(x-y)}{t+1} \le \frac{3qx}{t+1} = q\eta(M(f^2x,x,t)),\ x \ge y, t > 0$$

and

$$\eta(M(fx, fy, t)) = \frac{2(y-x)}{t+1} \leq \frac{3qy}{t+1} = q\eta(M(f^2y, y, t)),\ x < y, t > 0,$$

and by Remark 5, we conclude that $x = 0$ *is a unique fixed point for* f.

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