Optimal Design of a Pump-Hydro Energy Storage System

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Abstract – Energy storage systems are a step forward for renewable energy generation. These systems cover energy shortages at peak demand by storing energy generated at times of low demand. Reversible pumping systems are the perfect solution for energy generation regulation with respect to demand. This article discusses the optimal design of a reversible pumping station in terms of power input/output and mass of water capacity, as well as its way of generating/consuming electricity from the grid with a Model Predictive Control. In this design two cases are analyzed; an ideal case where there are no losses in the plant other than the performance of the equipment, and another case where friction losses exist depending on the design of the pipes.

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ACRONYMS

1. INTRODUCTION

Renewable energies have been gaining importance in recent years due to climate change and the desire to decarbonize the planet. This type of energy comes from nature, being an inexhaustible and totally clean source (it does not create polluting emissions). Renewable energies can come from wind, sun, water movement, etc. But what happens when there is no wind, or it is night? The generation of this type of energy may not be continuous making it unreliable. This is why energy storage is a key factor in being able to rely on renewable energy and decarbonize the planet. Such storage consists of storing surplus generated energy at times of low demand and transferring it to the grid when the generated energy is not sufficient at times of high demand.

The renewable energy studied in this article is hydroelectric, which is based on transforming the potential energy of water into electric energy. Conventional hydroelectric plants have two categories; Run-of-river hydropower and reservoir hydropower. The run-of-river hydropower plants use the velocity of the flow of a river through a bypass tunnel to reach the turbine. There is little or no storage in this type of plants. Instead, reservoir hydropower plants use a reservoir such as a lake (natural) or a dam (built) to accumulate and direct water to the turbines through forced pipes. These two types of hydropower plants generate electricity continuously and according to water conditions, without considering changes in the electricity demand of the grid. For this reason, there is a third type of power plant, which is analyzed in this paper. The pumping or reversible plants work with two reservoirs at different heights as shown in Figure 1.

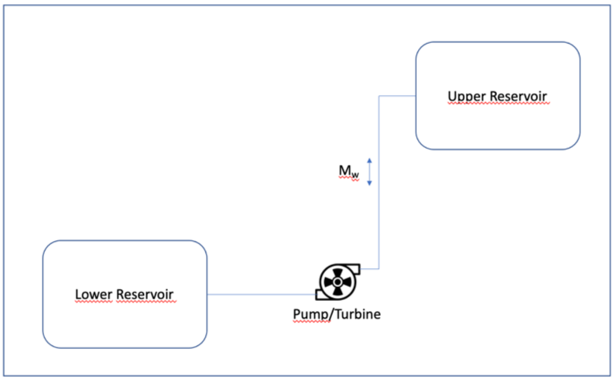


Figure 1 Simplified process diagram of reversible hydropower

This type of plant pumps water from the lower reservoir to the upper reservoir at times of low electrical demand from the network and turbine from the upper reservoir to the lower at times of high demand, thus stabilizing the electricity generation. The aim of these plants is to maximize profits by buying electricity at a lower price and selling it at a higher price. The price of electricity varies at all times and is unpredictable in the long run with accuracy. There are, as previously mentioned, moments of low demand in which the price of electricity usually decreases and, on the contrary, moments of high demand in which this price increases. In addition, the price of electricity also depends on the energy available at the time and the cost to these generations of energy production. For this article, 9 consecutive days are analyzed in which the price of electricity is known for each hour, thus making an estimate as close as possible to reality, which does not know the next value of energy but can approximate it thanks to the previous price patterns and the energies available in each moment. Figure shows the energy price variation over the previously mentioned 9 days.

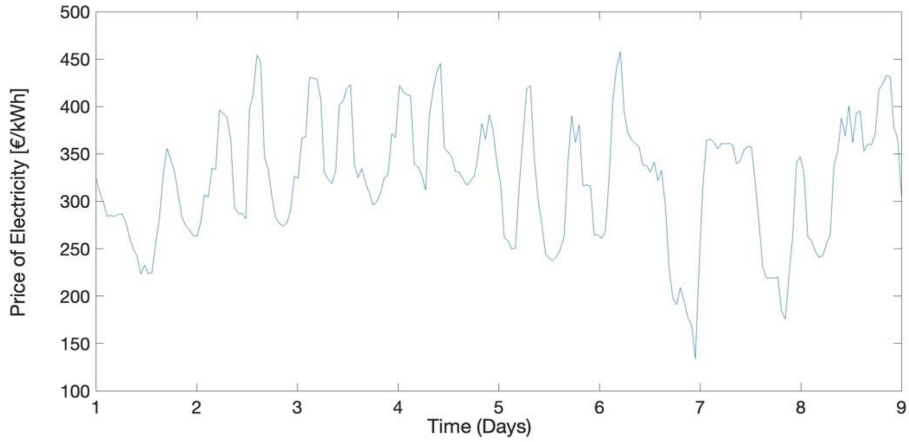


Figure 2 Price of the electricity during 9 random days

One way to approach this issue is through optimization by MATLAB, in which the algorithm decides the optimal design of the power plant and when to generate or consume electrical energy. The following section highlights how to maximize this benefit through LP.

2. Modeling of Pumped Hydro

As shown in Figure 1, a reversible pumping station consists of two water reservoirs, one upper and one lower, one or more power units and the pipes that derive the fluid. Clearly, the more water mass capacity and power, the greater the benefit will be. However, there are limits to the installation of this type. Power finds its limit in the existing capacities on the market. The capacity of the reservoir is found in the characteristics of the place where the plant is to be installed, as well as the conduits that transport the fluid, in this case water.

In this article, two cases will be analyzed. The operation of the plant in the ideal case, in which the only loss of energy comes from the performances of the power unit, and the case with losses in the transport of the fluid due to friction between this and the conduits.

2.1 Ideal Case

In the ideal case, the movement of the mass of water transforms potential energy into mechanical energy or vice versa depending on whether the plant is generating or consuming electrical energy almost 100% of its total, as the power unit will have mechanical and electrical performance. The relation between the mass of water in the upper reservoir and the power generated or consumed could be summed up by the following equation:

$$P = \rho g Q H = \dot{m} g H \ [W] \tag{1}$$

Equation (1), performance of the unit included, determines the power obtained in a hydroelectric plant as a function of the flow *Q* and the head *H* of the installation. If that equation is represented in mass flow terms (kg/s), we obtain the following equation (2), equal to (1)

$$\frac{dM}{dt} = \alpha P \qquad (2)$$

Where $\alpha = \frac{1}{g \cdot H}\ [\left(\frac{s^2}{m^2}\right)]$ is a constant factor. This symbolizes the potential energy contained in the mass of water that can be converted into electrical energy when passing through a downstream turbine and vice versa if pumped upstream. This relation can be observed more easily if we convert the equation to discrete time, in which it can be observed that the mass of water that is in the upper reservoir plus the power generated or consumed at the same time determines the amount of mass of water that is obtained at the subsequent instant. Convert (2) to discrete time:

$$M_w(i+1) = M_w(i) + bP(i) \qquad (3)$$

Where $b = \alpha \cdot \Delta t = \frac{\Delta t}{g \cdot H}$ and i is the actual time of the process. In this case, If the power P is a negative value, the amount of water in the upper reservoir will decrease due to the water being turbinated, so the negative value for the power corresponds to electricity generation. Instead, if the power is positive, the water in the upper reservoir will increase, because the water is being pumped, so a positive value for the power corresponds to the pumping of water.
Consider a plant with P, M_w and C_e, where these are the nominal power generated/consumed, the mass of water in the upper reservoir and the cost of the electricity respectively, and the following design boundary conditions:

$$P_{min} < P(i) < P_{max} \qquad (4)$$
$$0 < M_w(i) < M_{wMax} \qquad (5)$$

Where i represents the time for each step. If $C_e(i)$ is the energy value for each time, the cost of the energy during period i is $C_e(i)P(i)\Delta t = C_e(i)P_c(i)\Delta t - C_e(i)P_G(i)\Delta t$ then the operating costs will be:

$$Operating\ Costs = \sum_{i=1}^{N} C_e(i) * P(i) \qquad (6)$$

As the consumed power is positive and the generated is negative, the negative value of the operating costs will make the revenue. However, if the case is analyzed for several years, the money obtained in the future will worth less than the money today. Because of inflation, the value of the dollar or the euro today is worth more than the dollar of tomorrow, a week from now and 20 years from now. For that reason, to analyze the profit obtained over a time n contrasted with the capital cost of the installation (which is not affected by the inflation since the investment is made in the beginning) there must be a factor that estimates the value of the revenue during the years of study, being this factor the following:

$$PV_f = 365 * n * \frac{1}{r_i}[1 - \frac{1}{(1+r_i)^n}] \quad (7)$$

Where PV_f is the present value of the increase in revenue, r_i is the annual interest rate and *n* is the project horizon. Therefore, the net present value (NPV) in the studied *n* years will be:

$$NPV = PV_f * Operating\ Costs \quad (8)$$

Moreover, the necessary capital cost for the construction of the reversible hydroelectric plant should also be considered. According to *IRENA* (International Renewable Energy Agency) and *Renewable First*, the capital cost follows a curve, where the small hydropower systems are disproportionately expensive, due to a fix cost for every hydropower plant, and from 25-50 MW and above the curve levels off. In addition, the construction of the water reservoir will be more expensive as the capacity of the reservoir increases. So the total capital cost will depend on the installed capacity and the required water capacity. For this case, it has been estimated that the curves of this cost form a surface as shown in the figure and that it is governed by the following equation:

$$Capital\ Costs = a * P_N^{0.6} + b * M_w^{0.6} + c\ [\$] \quad (9)$$

Where *a* and *b* are two estimated variables and *c* represent the fix costs for every hydropower plant.

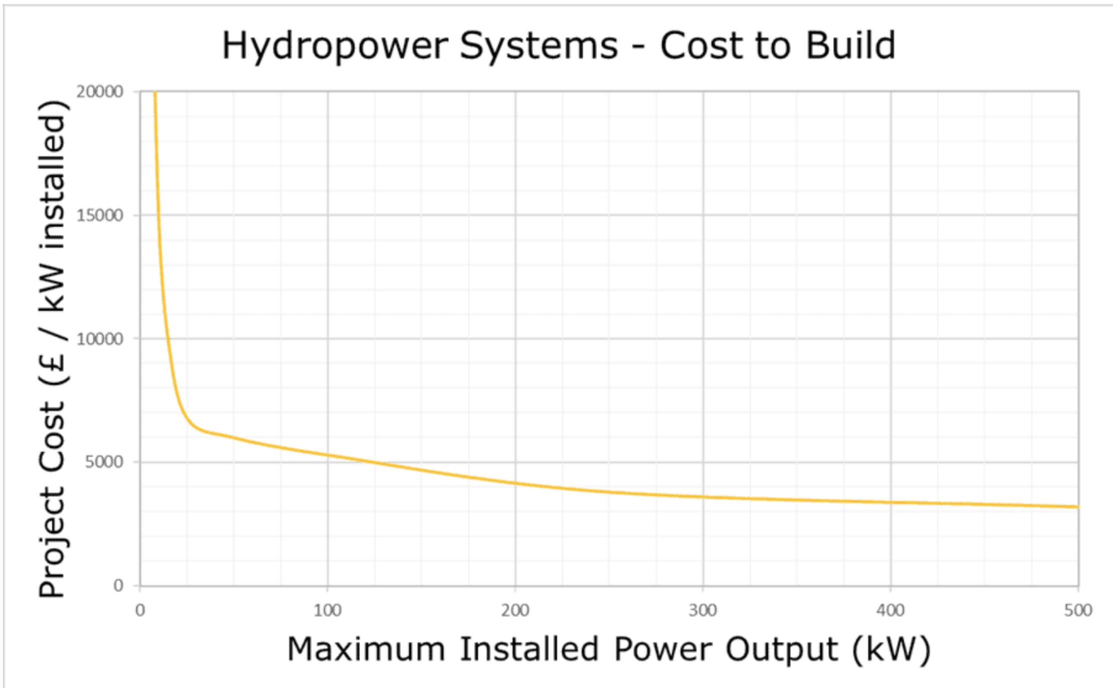


Figure 3 Capital Cost of Hydropower Plants (Source: RenewableFirst)

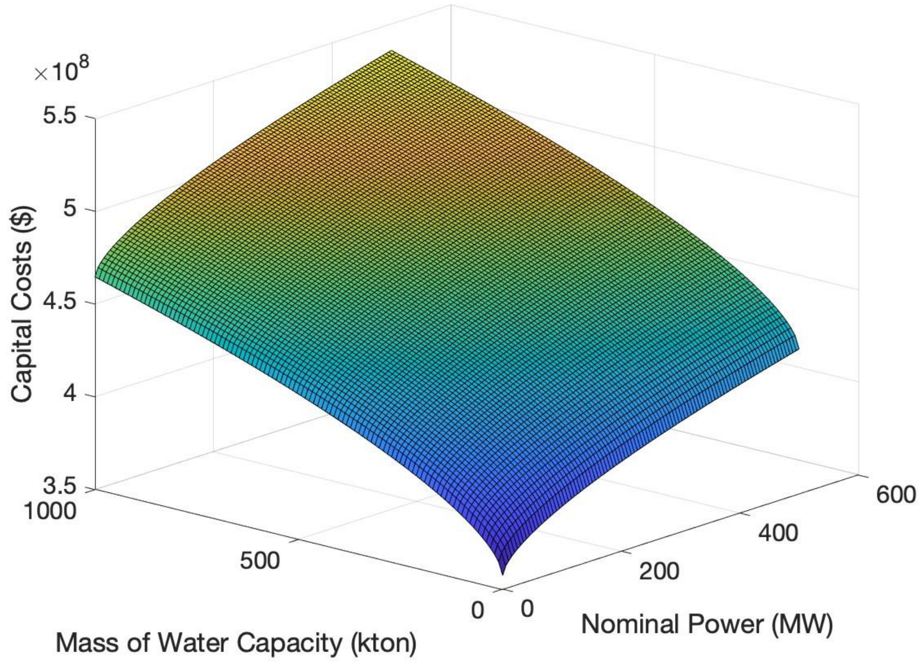


Figure 4 Capital Costs depending on Mass capacity and Power

Thus, in addition to analyze when to generate and turbine in order to maximize the revenue obtained, the total cost should be minimized within the number of years *n* of study. The total cost of the installation will be the sum of the initial capital cost plus the operational costs in that time *n*. Therefore, it will be studied in a future section how to minimize these total costs.

$$Total\ Cost = Project\ Cost\ \left[\frac{\$}{kW}\right] * P[MW] + NPV[\$] \qquad (10)$$

2.2. Modeling of Pump Hydro with Losses.

As mentioned in the previous section, the only energy loss in its conversion is that related to plant equipment performance. However, in the real case, this is not so. There are certain energy losses in the pipes related to friction between the fluid and the duct material. These losses are governed by the Darcy-Weisbach equation (for circular forced pipes) where H_f are the head loss for each pipe:

$$H_f = f\,\frac{LV^2}{2Dg} = f\,\frac{LQ^2}{2A^2Dg} = f\,\frac{8LQ^2}{\pi^2D^5g} = f\,\frac{8L\dot{m}^2}{\pi^2D^5\rho^2g}\,[m] \qquad (11)$$

As can be seen in the equation (11), friction losses will vary depending on the flow rate or fluid velocity, the type of material used and the pipe diameter, the latter being the most decisive factor. Taking these losses into account, it is necessary to reformulate the equation (2) of the previous section, where the real mass of water in movement will vary due to the power loss. It is worth mentioning that in the following equation the losses due to the performances of the equipment are implied as in the previous section.

$$\frac{dM}{dt} = \alpha P_I = \alpha(P_{elec} - P_L) \tag{12}$$

Where the real power P_{elec}, in terms of the ideal power P_I is:

$$P_{elec} = P_I + P_L = \dot{m}gH + N\left(\frac{\dot{m}}{N}\right)gH_f$$

$$P_{elec} = \dot{m}gH + \dot{m}g\left(f\frac{8L(\frac{\dot{m}}{N})^2}{\pi^2 D^5 \rho^2 g}\right) = \dot{m}gh + \dot{m}^3 g^3 H^3\left(f\frac{8L}{\pi^2 D^5 \rho^2 g^3 H^3 N^2}\right)$$

$$P_{elec} = P_I + P_I^3\beta \tag{13}$$

where $\beta = f(\frac{8L}{\pi^2 D^5 \rho^2 g^3 H^3 N^2})$. If the equation (13) is analyzed, the friction losses depend mainly in the diameter *D* of the pipes and the number *N* of them. Gravity, ρ water density, and π are constant, and the length of the pipes *L* and the head *H* depend on the location of the plant, fix values for this example. It makes sense that friction losses depend on the diameter and number of pipes. The greater the diameter or the greater number of pipes, with the same mass flow, the lower the fluid velocities, which causes less friction between the water and the pipe material. In particular, these losses depend on the diameter, as can be seen in the equation (13), since it is elevated to the 5th, while the number of tubes is squared. Of course, if this were the case, all hydroelectric plants would base the design on the largest diameter and largest number of pipes. However, there are limitations. Pipe diameters are standardized and installed within a range of values, outside that range there are no pipes of that diameter either for physical reasons, risk prevention, or economic limit. In addition, digging a hole for a very large diameter can be very costly or impossible to perform. On the other hand, excavation for several pipes will be limited by the physical conditions of the site and the economic cost of its construction as well. For these reasons, a compromise must be found between the capital costs of the installation in relation to the construction of the pipes according to their diameter and the number of them, and long-term operational costs due to energy loss in friction losses.

As in the previous section, the equation (12) may be easier to understand if it is converted to discrete time and it does not vary. In this equation it is again seen what the variation in the water mass of the upper reservoir is, however, it is necessary to include an equation in relation to friction losses. In this case, an additional constraint is added which relates the electrical energy sold/purchased with the actual energy that moves the mass of water, said constraint being (13). In this case as well, if the power *P* is a negative value, the amount of water in the upper reservoir will decrease due to the water being turbinated, so the negative value for the power corresponds to electricity generation. Instead, if the power is positive, the water in the upper reservoir will increase, because the water is being pumped, so a positive value for the power

corresponds to the pumping of water. However, the mass of water that is turbined or pumped for that time *i* varies due to the term added by friction losses.
Consider the same plant with *P, M_w* and C_e, and the design boundary conditions (3)-(4). If $C_e(i)$ is the energy value for each time will be the same, the cost of the energy during period *i* is $C_e(i)P(i)\Delta t = C_e(i)P_c(i)\Delta t$-$C_e(i)P_G(i)\Delta t$ then the operating costs will be also the same as in equation (6).

3. Review of Model Predictive Control

The basic idea behind MPC is to utilize a dynamic model to make predictions about future outcomes (including potential constraint violations). Based on these predicted outcomes, the manipulated variable is selected. To highlight the predictive aspect of MPC, two-time indices are utilized. The index *k* represents actual time, while the index *i* denotes predictive time. Specifically, $x_{i|k}, i = k \dots k + N - 1$ is the sequence of state predictions, indexed by *i*, but determined at the current time *k*. Thus, a linear predictive model can be compactly stated as

$$x_{i+1k} = f_d\left(x_{i|k}, u_{i|k}\right) \quad (14)$$

$$q_{i|k} = h_d(x_{i|k}, u_{i|k}) \quad (15)$$

$$q^{min} \le q_{i|k} \le q^{max} \quad (16)$$

$$x_{k|k} = \hat{x}_k \quad (17)$$

The parameter $\hat{x}_k$ is the estimate of the state, x_k, which is governed by the actual process:

$$x_{k+1} = f_d(x_k, u_k), k = 0 \dots \quad (18)$$

Equation (18) suggests that we should not expect the predicted trajectory to be exactly the same as that of the actual process. In addition to likely being nonlinear, we expect the process to be excited by disturbances. In the unlikely event of having perfect measurements, the state estimate, $\hat{x}_k$, can be replaced by the true state, x_k. However, in spite of all the potential errors, we should expect the estimate of (17) to be reasonably close to the actual state.

Given this predictive model, the first objective is to select a sequence, $u_{i|k}, i = k \dots k + N - 1$, such the constraints of (16) are satisfied. Since it is likely that more than one sequence $u_{i|k}$ is capable of satisfying these constraints, the selection process is cast as a staged optimization problem with an objective function typically equal to that of the LQOC.

$$\phi(\hat{x}_k) = \min_{u_{i|k}, u_{i+1|k}, \cdots, u_{i+N-1|k}} \{\textstyle\sum_{i=k}^{k+N-1}(x_{i|k}^* Q x_{i|k} + u_{i|k}^* R u_{i|k})\} \quad (19)$$

Subject to (14)-(17). Once an optimal sequence of predicted inputs, $u_{i|k}$, has been calculated, the controller does something that seems counterintuitive — the MPC algorithm utilizes only the first value, $u_{k|k}$, of the optimal sequence $u_{i|k}, i =$

$k \dots k+N-1$. That is, the actual manipulated variable at time *k* is set equal to the first step of the prediction: $u_k = u_{k|k}$. While this approach may seem like a waste of computational effort, the other elements of $u_{i|k}$, will play an important role, especially if a constraint violation is imminent. The primary reason for implementing only the first time-step is to introduce feedback into the algorithm. Specifically, as measurements about the process become available (during each time-step), one will find that the actual state is not where the predictive model predicted — recall Equation (18). Thus, the appropriate action is to replace the initial condition of problem (19), namely $x_{k|k}$, with the current estimate of the state, $\hat{x}_k$. In summary, the receding-horizon algorithm of MPC is as follows: At $k = 0, u_{0|0}$ is determined by (19), and $u_0$is set to this value. Then, at the next time step (*k = 1*) the initial condition of the MPC calculation, $x_{1|1}$, is set equal to the estimated value of the state, $\hat{x}_1$. Then, the process is repeated by setting $u_1$equal to $u_{1|1}$The basic idea is that we know that the predicted trajectory, $x_{i|k}$, contains errors. So, before starting to solve Problem (19), we update the initial condition, $x_{k|k}$, with an estimate based on the latest measurement information, which is $\hat{x}_k$.

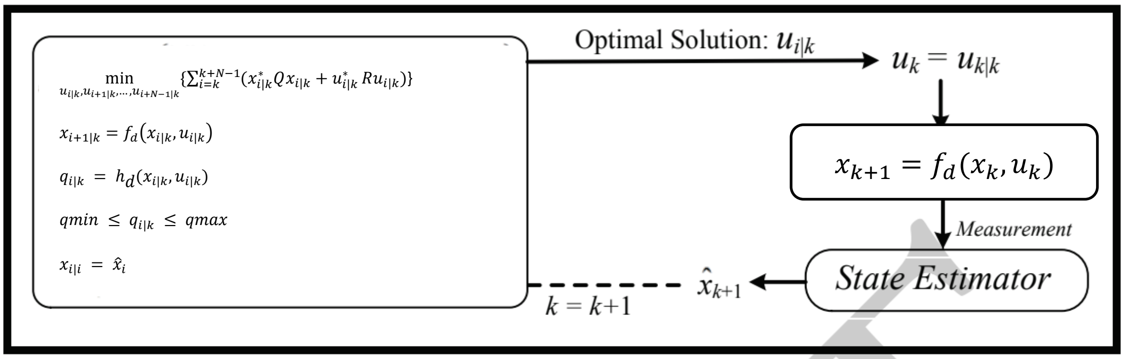


Figure 5 Illustration of feedback aspect of MPC (Source: 4)

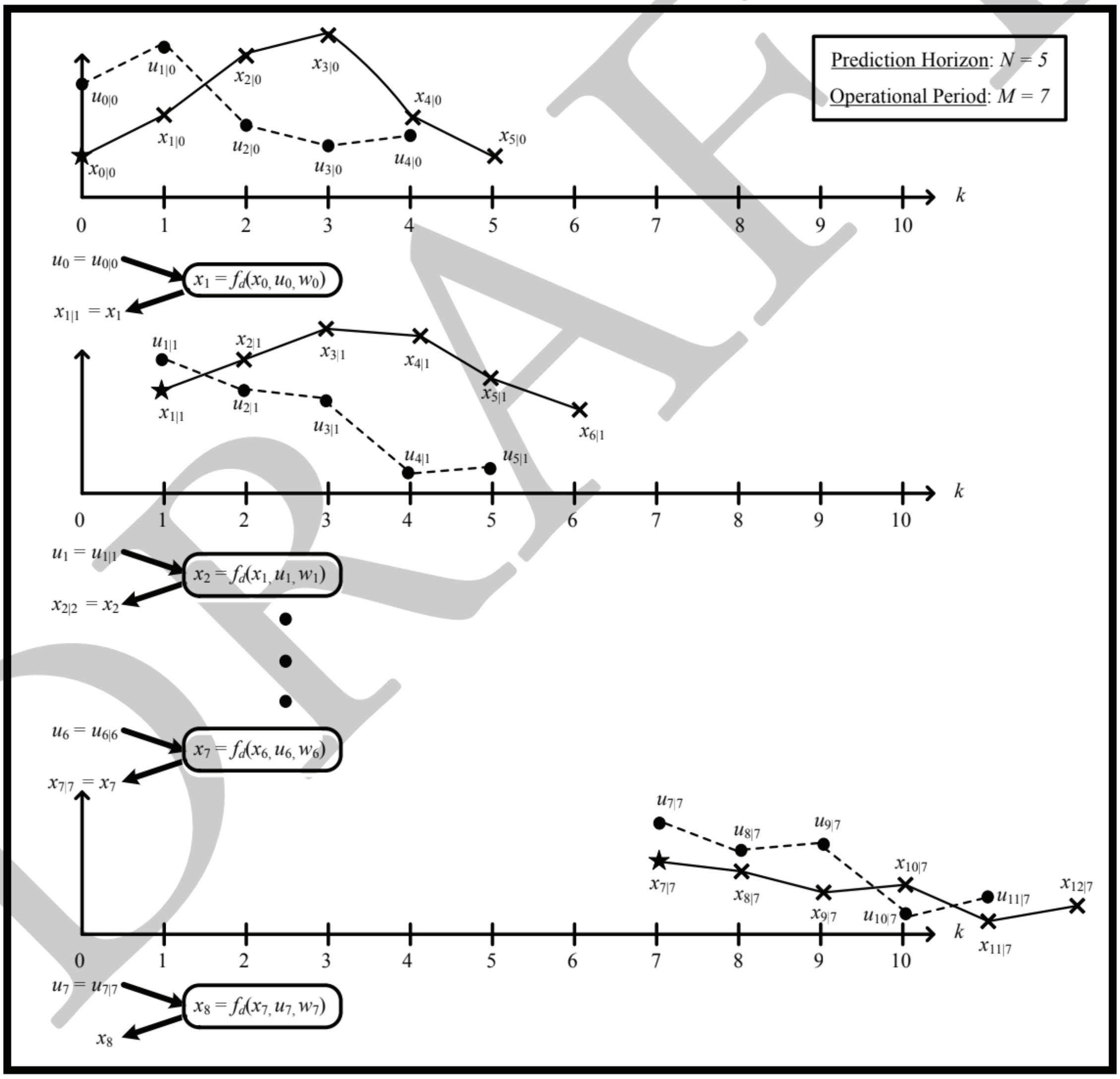


Figure 6 Illustration of receding-horizon aspect of MPC (full state information assumed)(Source:4)

A second reason to apply only the first time-step of the prediction is to reduce computational effort. Consider a scenario in which the process of interest is to be run for a long period of time. Furthermore, assume the model is perfect and all future disturbances are known (i.e., there is no discrepancy between the realized state and the predictions). In this case, it would seem reasonable to implement the entire open-loop policy generated at the first-time step, $u_{i|0}$. However, if the time period is very large and the number of state and manipulated variables is large, then the optimization problem one would need to solve would be enormous – potentially intractable. The alternative is to decouple the prediction time period from the operational time period. In this case, the prediction time indicates how much of the future the controller will consider, which could be significantly smaller than the operational period. Then, at each time step, when time index, *k*, is increased, to *k+1,* the final point of the prediction is also be increased by one. Typically, the prediction time period is denoted as the prediction horizon (to identify icebergs), travel toward the horizon (hopefully avoiding icebergs), but will never reach the horizon. This analogy illustrates a possible origin of the receding-horizon (or rolling-horizon) terminology commonly used to describe MPC.

4. Application of MPC to Pumped Hydro

As in section 2, the application of MPC is analyzed for an ideal case with no losses apart of the lost energy due to the performance of the power unit and a case where friction losses are included. These two cases are compared in terms of energetic and economic efficiency.

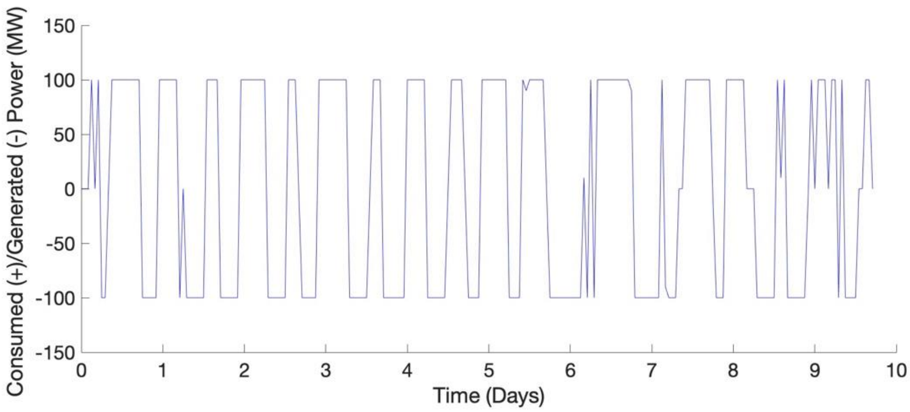
4.1. Application of MPC to Pumped Hydro

Considered the previous section, it can easily be solved using a standard LP solver where the energy value for each time times the power consumed (+) or generated (-) must minimize:

$$\min_{M_w(k|i),P(k|i)} \{\textstyle\sum_{k=i}^{i+N-1} C_e(k|i)P(k|i)\} \tag{20}$$

Subject to the previous conditions (3)-(5) and $k = i \ldots i - N - 1$. In this case, the sample time is Δt = 1 hr, then $b = \Delta t/(g \cdot h)$ = 0.9174 kg of water/MW.
This can clearly be solved using a standard LP solver.

Example 1: The following conditions were assumed for this case. P_{max} = 100 MW, $P_{min} = -P_{max}$, M_{wMax} = 1000 kton, H = 400 m and g = 9,81 m/s^2. The energy values are taken from the public data of the REE (Spanish Electric Network). It is assumed that the upper reservoir starts empty and simulates a 24-hour prediction horizon.



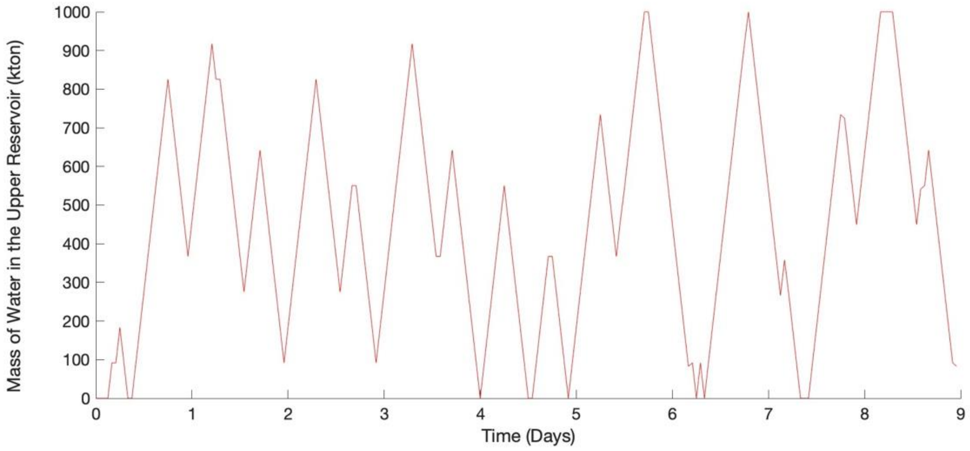
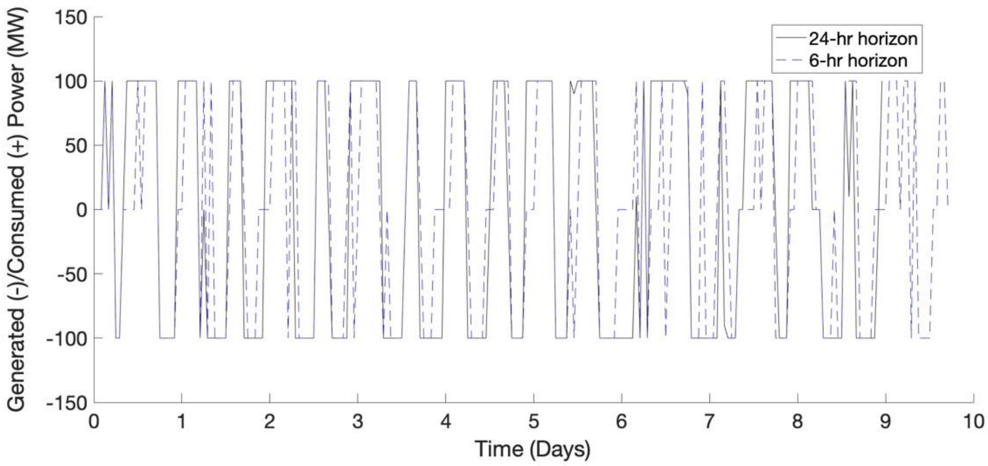


Figure 7 Closed-loop simulation of EMPC using a 24 hr prediction horizon

The top plot of the figure 2 indicates the energy prices used for this example. The next two plots (figure 7) show the optimized energy generated (-) or consumed (+) and the mass of water in the upper reservoir during the analyzed 9 days.

In the 6-hour horizon case (figure 8) the MPC loses its good sense. In this case the controller tends to reach the maximum value of the power more frequently than needed. This occurs due to the short horizon established so the revenue tends to maximize in the short term. However, in the 24-hour horizon case, as the energy values are known for a longer period, the controller would decide to storage more mass of water in order to maximize the revenue in this longer term. So it clearly can be seen that the 24-hr horizon seems more accurate than a shorter one, being closer to reality.



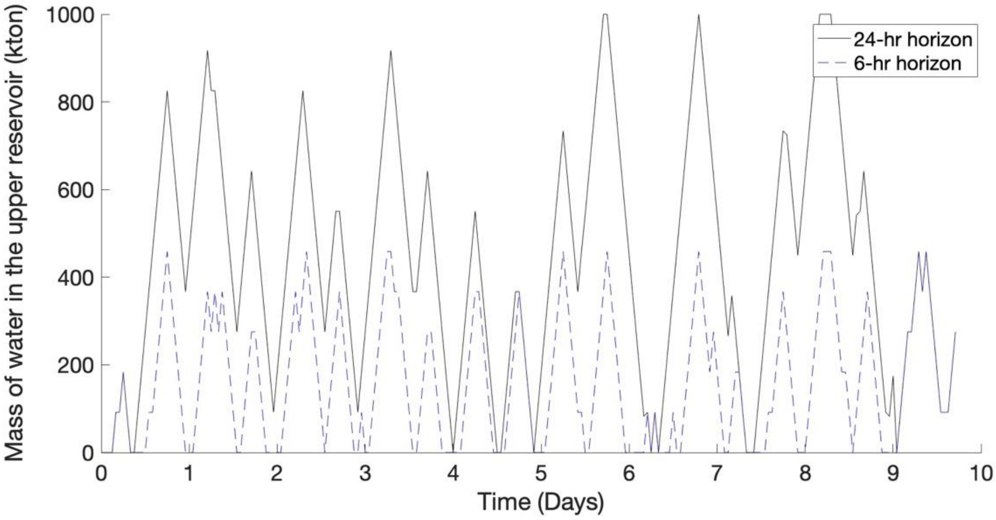


Figure 8 Comparison of EMPC using different prediction horizon

4.2. Application of MPC to Pumped Hydro with Losses

In this case, the application of the MPC is the same as in the ideal case, with a small change. Due to friction losses, a constraint is added in the simulation to then return to optimization and continue with the process. It can easily be solved using a standard LP solver as well where the energy value for each step times the power consumed (+) or generated (-) must minimize using the same equation (20). Only, in this section when arriving at the simulation the constraint (13) is added.

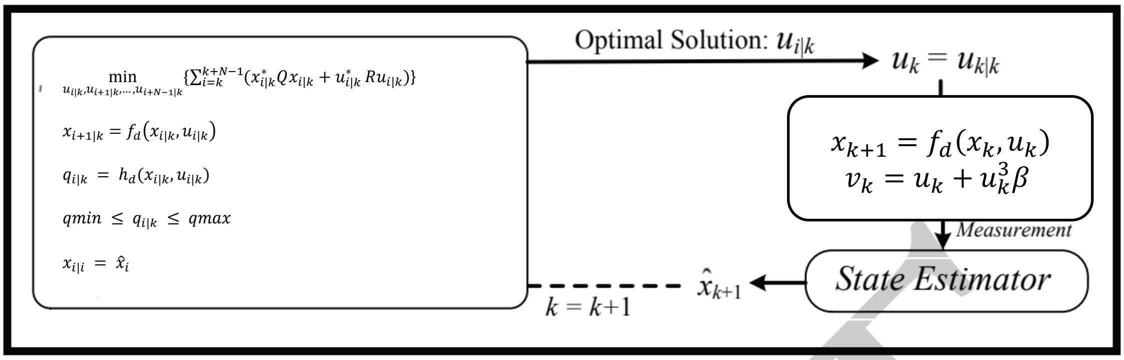


Figure 9 Illustration of feedback aspect of MPC with Losses (Source: 4).

Therefore, as the optimization problem to be solved is the same as in the previous case, the distribution of the water movement in the upper reservoir and the ideal power will be the same as in figure 7. On the other hand, the electrical energy to buy/sell will be different depending on the friction losses, the diameter and the number of pipes.

Using **Example 1**, under the same starting conditions, a diameter of 0.5 meters and 2 pipes has been assumed. It is also assumed a 24-hr horizon since it has been seen that this is more accurate. In figure 10 it can be seen the difference between the ideal power, which is the one that moves the mass of water at each time, and the electrical power for or from the power grid, if generating or pumping respectively.

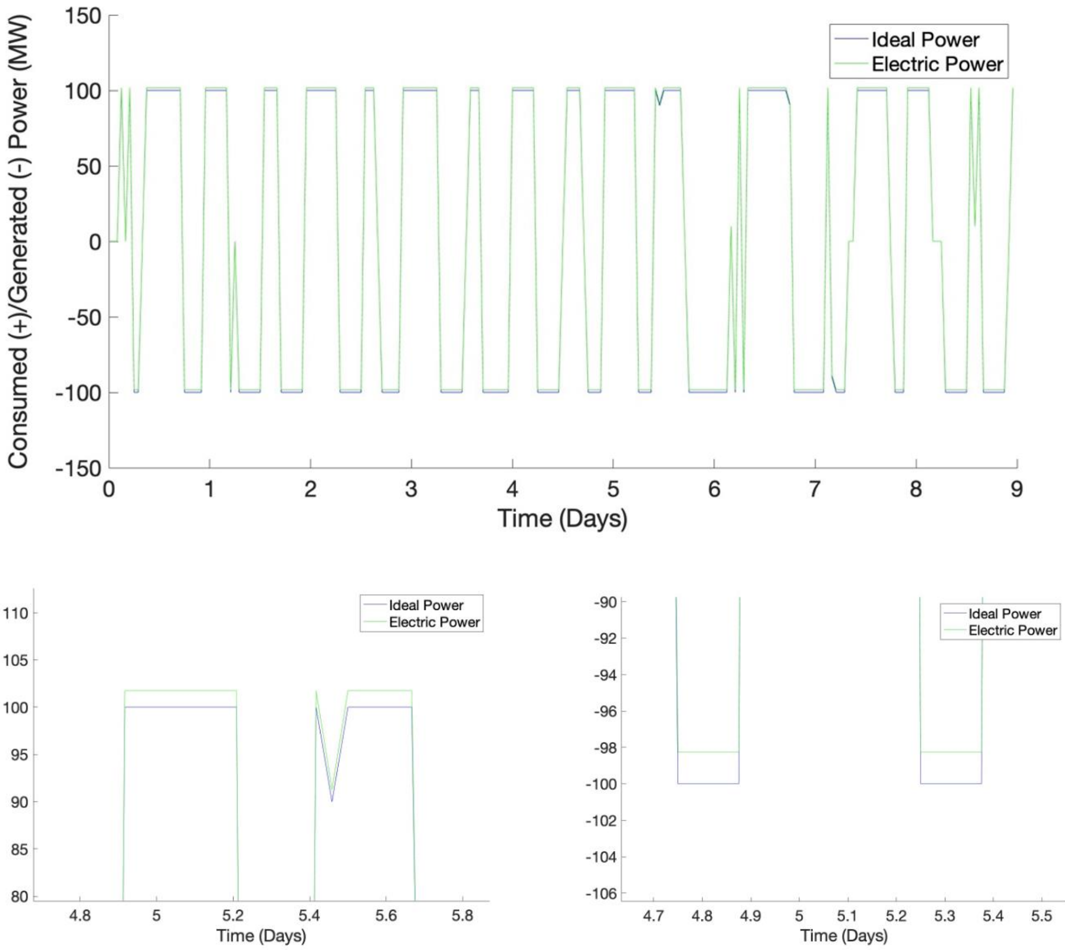


Figure 10 Comparison between Ideal Power and Electric Power

The difference due to friction losses can be seen in the graphs. When the plant is pumping water, the necessary electrical power from the grid is greater than the power that actually moves the mass of water. Otherwise, when the plant is generating moving water trough de turbine, the electricity generated is less than ideal due to the same reason.

5. Revenue and Design of the Plant

5.1. Revenue and Design of the Plant

In order to choose the size of the plant, the revenue, the power and the mass of water should be analyzed. For the previous example, P_N = 100 MW and M_{wMax} = 1000 kton, to make a R = 960.000 €. But what will be the maximum revenue for different P_N and M_{wMax}? If the same problem is calculated with different values for these variables, the graph in the figure 11 is obtained.

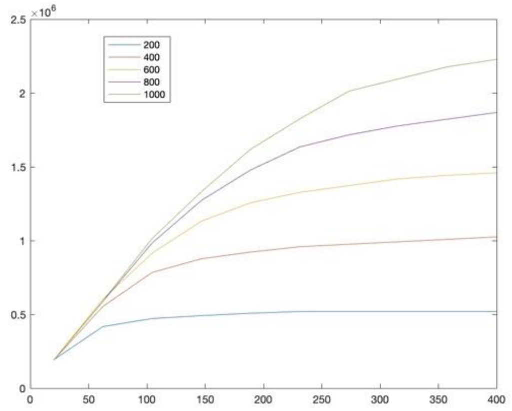


Figure 11 Revenue for each Output Power and Mass of Water Capacity

For this example, the nominal power takes the values from 20 to 400 MW and the maximum mass of water the values from 200 to 1000 ktons. For each curve, it seems to reach a nominal power when the revenue levels off and the increase slows. If this would be the case, the selected power and mass of water capacity would be the biggest values in order to get the maximum revenue. However, this is limited by the capital cost needed for the installation. Therefore, the ratio between the capital cost of the plant installation and the benefit obtained for a water capacity and a given power must be optimized.

$$\min_{P_G(i), M_{wMax}(i)} \{Capital\ Cost + Operating\ Costs * PV_f\} \tag{21}$$

In this case, the annual interest rate (r_i) is established in 7% for a period of 20 years. By minimizing problem (21), the optimal values for power *P* and water mass capacity in the upper reservoir *Mw* are achieved for which the benefit in relation to the initial cost of capital is maximized in that time period *n.*

5.2. Revenue and Design of the Plant with Losses

The revenue obtained and the design of the plant in case of friction losses is carried out in the same way as in equation (21). Different revenue curves are obtained for different power and water mass capacities and compared on the basis of initial capital costs. However, at the time of plant design, such friction losses add two additional factors to be analyzed, the diameter and the number of pipes in the plant, as discussed in section 2.2. In this section it was studied how the influence of losses depends mainly on the diameter and, to a lesser extent, the number of pipes installed. In case of selecting a very small diameter or the wrong number of pipes, the influence on the total revenue of the plant can be very noticeable, which could increase the operating costs in the long term and make this plant unprofitable. Also, in case of selecting the plant with the lowest losses due to friction possible, the cost of installation can increase considerably without the influence of losses with a smaller diameter or fewer pipes being noticeable. In figure 12 can be seen how, with a smaller diameter (0.5 m), the profit decreases by 45.94%, something unacceptable. In contrast, for a diameter of 1 meter the same profit decreases by 1.43%, which compared to 0.04% obtained with the diameter of 2 meters is practically negligible when comparing the cost of installing a pipe of 2 meters with a pipe of 1 meter of diameter.

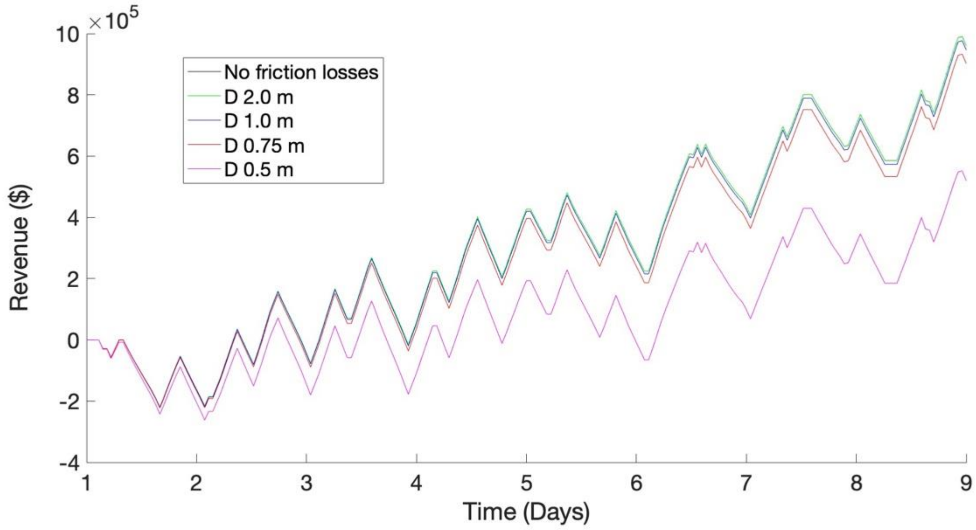


Figure 12 Comparison in the revenue obtained for different diameters (N=1)

In the opposite case, if it is analyzed for a given diameter (for the following figure D=1m), as the number of pipes increases, and being the mass flow constant, the loss due to friction decreases and the influence in the revenue obtained is less. In figure 13, for N=1, the revenue obtained decreases in a 45.94% and for N=2 a 11.48%, while for N=4 and N=5 it is less than a 3%. Again, the operating costs should be analyzed in case of choosing the smallest number of tubes, as well as the installation costs in case of choosing a larger number, being the lower operational costs in the long term. Comparing the two graphs, it can be seen that the variation in the length of the diameter has a greater influence on the lost profit than the choice of the number of tubes. Therefore, a compromise should be reached regarding these two variables depending on the physical conditions of the site and the estimated costs, both initial installation and operating costs in a time *n.*

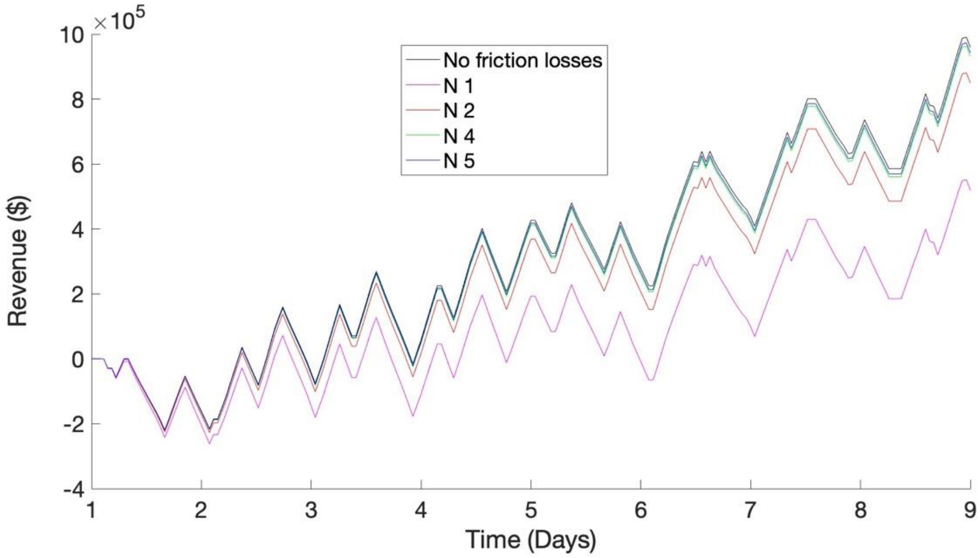


Figure 13 Comparison in the revenue obtained for different number of pipes (D=1m)

6. Conclusion

In this work, the notion of EMPC has been introduced to maximize revenue from a pump hydro energy storage system. Ignorance of the future price of electricity makes the intervals of generating or consuming electricity in this type of plants cause losses in the revenue created. This problem has been evaluated with different horizons estimating electricity prices over a 9-day period. For short time horizons, the MPC attempts to maximize the revenue for that time interval by considering that it does not continue from the end of that horizon. For this reason, in the case of pump-hydro energy storage plants, the intervals of generating or consuming electricity are shorter than for larger horizons. However, when this time horizon widens, the MPC sees more beyond, storing a greater mass of water in the upper reservoir at times of lower electricity prices to generate more energy at times of higher prices, thus approaching, with estimated price data, a maximum revenue.

On the other hand, the maximization of this revenue also depends on the capital costs involved in the installation of the plant, which depend on the selected nominal power and capacity of the upper reservoir. Each power and capacity combination obtains a maximum profit curve obtained by the MPC and another curve with the initial capital cost of the plant. So, another optimization problem arises in which based on those two curves for each combination mentioned, the net final profit is maximal (the inflation factor being considered as well). As mentioned, electricity prices, although these are real for certain days already past, symbolize an estimate of future prices for this case, which can approach reality since the pattern of prices is usually constant. Also, the capital cost curve varies according to nominal power and capacity, and equation (9) is an estimate based on the data found, which may vary for each specific case.

When applied to an actual case, account should also be taken of energy losses in pipes, elbows, etc. called friction losses. This type of loss depends essentially on the velocity of the fluid and the material used in the installation, as they arise from the friction between the movement of said fluid and the pipe. At the same time, the velocity of the fluid depends on a number of factors. As can be seen in the Darcy-Weisbach equation (11), this velocity, and, consequently, the friction losses depend mainly on the diameter of the pipe and the number of pipes. In addition, losses should also consider the length of the pipes and the friction factor between the pipe and the fluid. For the same reason, a correct selection of the design of the pipes is necessary. It has been seen that, for a poor selection of diameter or number of pipes, the influence on the revenue obtained, or the losses originated can be significant. On the other hand, a selection of these parameters that minimize friction losses to the maximum can lead to a very high operating cost. Thus, a third optimization problem arises, which analyzes, for each selection of design in the pipes, the initial cost of the installation and the influence of the operational cost over the years in which the investment is to be recovered. In this case, and as discussed in section 4.2, the losses are not included in the optimization process, but a constraint is added in the simulation which analyzes the influence of these losses. The most correct way to operate would be to include this constraint in the MPC optimization process, however, as it is a non-linear constraint, the computational cost is very high, and it has been seen that with a correct design selection, the influence of such losses on total profit is not significant to a large extent.

So, as can be seen, there are 3 optimization processes. First and foremost, the distribution of electricity generation and consumption to maximize the revenue obtained. The second, the optimization of capital cost curves and plant operating costs in a time period *n* with a correct selection of nominal power and mass capacity in the upper reservoir. And third, the optimal design of pipelines to minimize both initial installation costs and operational costs in that time *n* due to friction losses.

Finally, although all these estimations have a logical basis, different ones may be made in each case for electricity prices, installation costs, etc. Each case should be analyzed in detail.

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