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Some new exact solutions of (4+1)-dimensional Davey–Stewartson-Kadomtsev–Petviashvili equation

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ABSTRACT

Exact solutions of nonlinear equations have got formidable attraction of researchers because these solutions demonstrate the physical behaviour of a model. In this paper, we focus on extracting some new exact solutions of a (4+1)-dimensional Davey–Stewartson-Kadomtsev–Petviashvili (DSKP) equation. To find new travelling wave solutions of the DSKP equation, we use $(\frac{\mathscr{G}'}{\mathscr{G}'+\mathscr{G}+\mathbb{A}})$-expansion technique. The obtained solutions are in the form of the exponential and trigonometric functions. We obtain different kinds of waves solutions for specific values of parameters. We simulate the achieved solutions in 3D and 2D plots.

Introduction

A partial differential equation (PDE) with time t as one of the independent variables is typically referred to as an evolution equation (EE). Two basic examples of evolution equations are the wave equation and the heat equation, which describe the vibration of a string or heat conduction, respectively. But there are a lot of naturally occurring nonlinear evolution equations in applied sciences and engineering that need to be studied. In nonlinear systems research, exact solutions to nonlinear EEs are crucial because these solutions can efficiently explain a variety of actual phenomena, including oscillations, solitons, and dispersion with a finite speed.

Researchers have used many analytical procedure to extract exact solutions of nonlinear EEs. Alam et al. investigated travelling solutions of nonlinear EEs using $\frac{G'}{G}$ expansion approach [1]. Roshid studied new exact solutions of a shallow water nonlinear equations using MSE technique [2]. Saifullah et al. demonstrated interaction solutions for a perturbed KdV equation using Hirota bilinear approach [3]. Hossieni et al. investigated soliton solutions of chiral schrodinger equation using MK technique [4]. Using symmetry analysis method, Kumar et al. studied multisoliton solutions of a Boussinesq equation [5]. Biswas et al. demonstrated soliton solution of KdV–Caudrey–Dodd–Gibbon Equation using modified F-expansion technique [6].

In the literature, researchers have analysed some well known NEEs by using different analytical techniques. For instance, Wang investigated waves solutions of NEEs such as: Kadomtsov–Petviashvili–Benjamin–Bona–Mahony equation [7], Camassa–Holm-Kadomtsev–Petviashvili equation [8], fractional wave equation [9], fractal Riemann wave equation [10], and short water wave equation [11], by using different techniques. The authors studied soliton solution of NEEs by using Jacobi elliptic function method [12,13]. Ali et al. investigated waves dynamics of coupled-Higgs equation via ϕ^6-expansion approach [14]. Ali et al. produced some soliton solutions of Bogoyavlenskii–Kadomtsev–Petviashvili using Hirot method [15]. We list some other NEEs in [16–18].

In this research paper, we consider DSKP equation as:

$$4\mathcal{V}_{xt} - \mathcal{V}_{xxxy} + \mathcal{V}_{xyyy} + 12\mathcal{V}_x\mathcal{V}_y + 12\mathcal{V}\mathcal{V}_{xy} - 6\mathcal{V}_{zw} = 0. \tag{1}$$

This Fokas equation (1) represents a 4th-order nonlinear EEs in four spatial and temporal coordinates. The well-known Greek mathematician A. S. Fokas initially obtained Eq. (1) by expanding the integrable KP equation and the DS equation, which are the two basic nonlinear evolution equations. It is possible to utilize this Fokas equation to represent both non-elastic and elastic interactions between internal waves [19,20]. In light of this, it is possible to use the higher-dimensional Fokas equation to model a variety of intricate nonlinear phenomena, including shallow-water waves, plasma physics, and many more. Eq. (1) has been investigated by using modified simple equation method [21]. The authors have studied kink, bell-shaped soliton, cuspon and some other solitons of the Eq. (1). In this paper, we use another expansion method which we called $(\frac{\mathscr{G}'}{\mathscr{G}'+\mathscr{G}+\mathbb{A}})$- method. This method is rarely used for analysis of nonlinear EEs. Only few works are available in the literature [22,23].

The primary goal of the proposed study is to derive the new solutions in closed form of Eq. (1) utilizing the $(\frac{\mathscr{G}'}{\mathscr{G}'+\mathscr{G}+\mathbb{A}})$-expansion

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technique. In this context, the travelling wave solutions might be expressed as $(\frac{\mathscr{G}'}{\mathscr{G}'+\mathscr{G}+\mathbb{A}})$ or $\mathscr{G}=\mathscr{G}(\xi)$, which satisfies the $\mathscr{G}''(\xi)+\mathrm{H}_1\mathscr{G}'(\xi)+\mathscr{U}\mathscr{G}(\xi)+\mathrm{K}$ where $\mathrm{H}_1, \mathscr{U}$ and K are constants and $\mathscr{G}=\frac{d\mathscr{G}}{d\xi}$.

We organize the paper as: Section "Solution strategy of the $(\frac{\mathscr{G}'}{\mathscr{G}'+\mathscr{G}+\mathbb{A}})$- expansion method" provides the general strategy of solution by using the proposed method. The application of the proposed strategy is given in Section "New exact solutions of the (4 + 1)-dimensional DSKP equation". The graphical analysis of the obtained solutions is provided in Section "Numerical Simulations". Section "Conclusion" deals with the conclusion of the manuscript.

Solution strategy of the $(\frac{\mathscr{G}'}{\mathscr{G}'+\mathscr{G}+\mathbb{A}})$ - expansion method

The detail strategy are as follows:

Consider any non-linear PDE with $\mathscr{M}=\mathcal{V}(x,y,z,w,t)$ in five independent variables x, y, z, w and t as,

$$\mathcal{R}\left(\mathcal{V},\mathcal{V}_x,\mathcal{V}_y,\mathcal{V}_t,\mathcal{V}_{xx},\mathcal{V}_{xy},\mathcal{V}_{xt},\mathcal{V}_{yy},\ldots\right)=0. \tag{2}$$

The proposed method is described in the following steps:

Step-1: Let the solution of (2) is considered as:

$$\mathcal{V}(x,y,z,w,t)=\mathscr{M}(\xi), \tag{3}$$

where $\xi=nx+my+kw+rw-pt$ such that n,m,k,r and p are constants.

Now substituting Eq. (3) in Eq. (2), this gives:

$$\mathcal{R}(\mathscr{M},\mathscr{M}',\mathscr{M}'',\ldots)=0, \tag{4}$$

where, $\mathscr{M}'=\frac{d\mathscr{M}}{d\xi}, \mathscr{M}''=\frac{d^2\mathscr{M}}{d\xi^2},\ldots$

Step-2: For the proposed approach, we consider the solution of Eq. (4) as:

$$\psi(\xi)=\sum_{i=0}^{r}a_i(\frac{\mathscr{G}'}{\mathscr{G}'+\mathscr{G}+\mathbb{A}})^i, \tag{5}$$

where r provides the polynomial's degree, which has to be find out by utilizing the homogeneous balancing principle (HBP) and the coefficients of $(\frac{\mathscr{G}'}{\mathscr{G}'+\mathscr{G}+\mathbb{A}})^i$ $a_i(i\,:\,0\le i\le r)$; are evaluated by utilizing system of algebraic equations generated from the proposed method. Also, $\mathscr{G}=\mathscr{G}(\xi)$ satisfies:

$$\mathscr{G}''+\mathscr{U}\mathscr{G}'+\mathrm{K}\mathscr{G}+\mathrm{H}_1\mathrm{K}=0, \tag{6}$$

where, $a_i(i\,:\,0\le i\le p)$, $\mathrm{H}_1,\mathscr{U},\mathrm{K}$ represents real numbers. By utilizing the values of H_1, $\mathscr{U}$, K, the ODE (6) is solved.

Step-3: Now substituting Eq. (5) into Eq. (4). The coefficients of $(\frac{\mathscr{G}'}{\mathscr{G}'+\mathscr{G}+\mathbb{A}})$ may be collected and equating them to zero, which we will solve for the variables a_i,η and $\mathrm{H}_1,\mathscr{U},\mathrm{K}$.

New exact solutions of the (4 + 1)-dimensional DSKP equation

In this portion, we give application of the suggested method. We find some new solitary wave solution by using the developed strategy. Now, we apply the proposed technique to figure out new exact solutions of the Eq. (1). Recall the Eq. (1) as

$$4\mathcal{V}_{xt}-\mathcal{V}_{xxxy}+\mathcal{V}_{xyyy}+12\mathcal{V}_x\mathcal{V}_y+12\mathcal{V}\mathcal{V}_{xy}-6\mathcal{V}_{zw}=0.$$

Let $\mathcal{V}(x,y,z,w,t)=\mathscr{M}(\xi)$ where $\xi=nx+my+kw+rw-pt$.

According to the above transformation, we have

$$\mathcal{V}_{tx}=-pn\mathscr{M}'';\mathcal{V}_{xxxy}=n^3m\mathscr{M}'''';\mathcal{V}_{xyyy}=nm^3\mathscr{M}'''';\mathcal{V}_x\mathcal{V}_y=nm\mathscr{M}'^2;$$

$$\mathcal{V}\mathcal{V}_{xy}=nm\mathcal{U}\mathcal{U}'';\mathcal{V}_{zw}=rk\mathscr{M}''.$$

Plugging these values in Eq. (1), we get

$$(nm^3-n^3m)\mathscr{M}''''-(4np+6rk)\mathscr{M}''+12nm\mathscr{M}'^2+12nm\mathcal{U}\mathcal{U}''=0. \tag{7}$$

Let $\eta=nm^3-n^3m$. and $q=4np+6rk$. Then, the above ODE becomes:

$$\eta\mathscr{M}''''-q\mathscr{M}''+12nm\mathscr{M}'^2+12nm\mathcal{U}\mathcal{U}''=0. \tag{8}$$

Integrate the Eq. (8) two times, we obtain

$$\eta\mathscr{M}''+6nm\mathscr{M}^2-q\mathscr{M}=0. \tag{9}$$

With the aid of HBP, we have $p=2$. So, the solution of the above equation becomes:

$$\mathcal{V}(\xi)=a_0+a_1(\frac{\mathscr{G}'}{\mathscr{G}'+\mathscr{G}+\mathbb{A}})+a_2(\frac{\mathscr{G}'}{\mathscr{G}'+\mathscr{G}+\mathbb{A}})^2. \tag{10}$$

Using Eq. (10) into Eq. (9) and collect the coefficients of $(\frac{\mathscr{G}'}{\mathscr{G}'+\mathscr{G}+\mathbb{A}})$ and equate them to zero, we have:

$$\begin{cases}6nma_0^2+\eta\mathscr{U}\mathrm{K}a_1-2\eta\mathrm{K}^2a_1+2\eta\mathrm{K}^2a_2-qa_0=0,\\ \eta\mathscr{U}^2a_1+2\eta\mathrm{K}a_1-6\eta\mathscr{U}\mathrm{K}a_1+6\eta\mathrm{K}^2a_1+6\eta\mathscr{U}\mathrm{K}a_2-12\eta\mathrm{K}^2a_2\\ +12nma_0a_1-qa_1=0,\\ 3\eta\mathscr{U}a_1-3\eta\mathscr{U}^2a_1-6\eta\mathrm{K}a_1+9\eta\mathscr{U}\mathrm{K}a_1-6\eta\mathrm{K}^2a_1+4\eta\mathscr{U}^2a_2+8\eta\mathrm{K}a_2\\ -24\eta\mathscr{U}\mathrm{K}a_2+24\eta\mathrm{K}^2a_2+\\ 6nna_1^2+12nma_0a_2-qa_2=0,\\ 2\eta a_1-4\eta\mathscr{U}a_1+2\eta\mathscr{U}^2a_1+4\eta\mathrm{K}a_1-4\eta\mathscr{U}\mathrm{K}a_1+2\eta\mathrm{K}^2a_1+10\eta\mathscr{U}a_2\\ -10\eta\mathscr{U}^2a_2-20\eta\mathrm{K}a_2+30\eta\mathscr{U}\mathrm{K}a_2-\\ 20\eta\mathrm{K}^2a_2+12nma_1a_2=0,\\ 6\eta a_2-12\eta\mathscr{U}a_2+6\eta B^2a_2+12\eta\mathrm{K}a_2-12\eta\mathscr{U}\mathrm{K}a_2+6\eta\mathrm{K}^2a_2+6nma_2^2=0.\end{cases} \tag{11}$$

Solving Eq. (11), the following sets are achieved:

$Set-1$: For this set, we get

$$q=(-nm^3+n^3m)(\mathscr{U}^2-4\mathrm{K}),a_0=-(m^2-n^2)(\mathrm{K}^2+\mathrm{K}-\mathscr{U}\mathrm{K}),$$

$a_1=(m^2-n^2)(\mathscr{U}^2-3\mathcal{B}\mathrm{K}-\mathscr{U}+2\mathrm{K}^2+2\mathrm{K}), a_2=\frac{-m^2+n^2}{6}(\mathscr{U}^2-2\mathscr{U}\mathrm{K}-2\mathscr{U}+\mathrm{K}^2+2\mathrm{K}+1)$. For this set, we consider two cases:

CASE#1: $(\mathcal{D}=\mathscr{U}^2-4\mathrm{K}>0)$

$$\begin{aligned}\psi_1(\xi)=&-(m^2-n^2)(\mathrm{K}^2+\mathrm{K}-\mathscr{U}\mathrm{K})+(m^2-n^2)\\ &\times(\mathscr{U}^2-3\mathcal{B}\mathrm{K}-\mathscr{U}+2\mathrm{K}^2+2\mathrm{K})\\ &\times\left[1+\frac{2\mathrm{K}_1+2\mathrm{K}_2e^{\sqrt{\mathcal{F}}\xi}}{\mathrm{K}_1(-2+\mathscr{U}+\sqrt{\mathcal{F}})+\mathrm{K}_2(-2+\mathscr{U}-\sqrt{\mathcal{F}})e^{\sqrt{\mathcal{F}}\xi}}\right]\\ &\times\frac{-m^2+n^2}{6}(\mathscr{U}^2-2\mathscr{U}\mathrm{K}\\ &-2\mathscr{U}+\mathrm{K}^2+2\mathrm{K}+1)\\ &\times\left[1+\frac{2\mathrm{K}_1+2\mathrm{K}_2e^{\sqrt{\mathcal{F}}\xi}}{\mathrm{K}_1(-2+\mathscr{U}+\sqrt{\mathcal{F}})+\mathrm{K}_2(-2+\mathscr{U}-\sqrt{\mathcal{F}})e^{\sqrt{\mathcal{F}}\xi}}\right]^2.\end{aligned}$$

CASE#2: $(\mathcal{F}=\mathscr{U}^2-4\mathrm{K}<0)$

$$\begin{aligned}&\psi_2(\xi)=-(m^2-n^2)(\mathrm{K}^2+\mathrm{K}-\mathscr{U}\mathrm{K})+(m^2-n^2)\\ &\times(\mathscr{U}^2-3\mathcal{B}\mathrm{K}-\mathscr{U}+2\mathrm{K}^2+2\mathrm{K})\\ &\times\left[\frac{(\upsilon\mathrm{K}_1-\sqrt{-\mathcal{F}}\mathrm{K}_2)\cos(\frac{\sqrt{-\mathcal{F}}\xi}{2})+(\mathscr{U}\mathrm{K}_2+\sqrt{-\mathcal{F}}\mathrm{K}_1)\sin(\frac{\sqrt{-\mathcal{F}}\xi}{2})}{(\mathscr{U}-2)\mathrm{K}_1-\sqrt{-\mathcal{F}}\mathrm{K}_2\cos(\frac{\sqrt{-\mathcal{F}}\xi}{2}+(\mathscr{U}-2)\mathrm{K}_2+\sqrt{-\mathcal{F}}\mathrm{K}_1)\sin(\frac{\sqrt{-\mathcal{F}}\xi}{2})}\right]\\ &+(\frac{-m^2+n^2}{6})(\mathscr{U}^2-2\mathscr{U}\mathrm{K}\\ &-2\mathscr{U}+\mathrm{K}^2+2\mathrm{K}+1)\\ &\times\left[\frac{(\mathscr{U}\mathrm{K}_1-\sqrt{-\mathcal{F}}\mathrm{K}_2)\cos(\frac{\sqrt{-\mathcal{F}}\xi}{2})+(\mathscr{U}\mathrm{K}_2+\sqrt{-\mathcal{F}}\mathrm{K}_1)\sin(\frac{\sqrt{-\mathcal{F}}\xi}{2})}{(\mathscr{U}-2)\mathrm{K}_1-\sqrt{-\mathcal{F}}\mathrm{K}_2\cos(\frac{\sqrt{-\mathcal{F}}\xi}{2}+(\mathscr{U}-2)\mathrm{K}_2+\sqrt{-\mathcal{F}}\mathrm{K}_1)\sin(\frac{\sqrt{-\mathcal{F}}\xi}{2})}\right]^2.\end{aligned}$$

$Set-2$: For this set, we have $q=(nm^3-n^3m)(\mathscr{U}^2-4\mathrm{K})$, $a_0=(\frac{-m^2+n^2}{6})(\mathscr{U}^2-6\mathscr{U}\mathrm{K}+6\mathrm{K}^2+2\mathrm{K})$,

$a_1=(m^2-n^2)(\mathscr{U}^2-3\mathcal{B}\mathrm{K}-\mathscr{U}+2\mathrm{K}^2+2\mathrm{K})$, $a_2=\frac{-m^2+n^2}{6}(\mathscr{U}^2-2\mathscr{U}\mathrm{K}-2\mathscr{U}+\mathrm{K}^2+2\mathrm{K}+1)$. For this set, we take two cases:

CASE#1: $(\mathcal{D}=\mathscr{U}^2-4\mathrm{K}>0)$

$$\begin{aligned}\psi_3(\xi)=&(\frac{-m^2+n^2}{6})(\mathscr{U}^2-6\mathscr{U}\mathrm{K}+6\mathrm{K}^2+2\mathrm{K})+(m^2-n^2)\\ &\times(\mathscr{U}^2-3\mathcal{B}\mathrm{K}-\mathscr{U}+2\mathrm{K}^2+2\mathrm{K})\end{aligned}$$

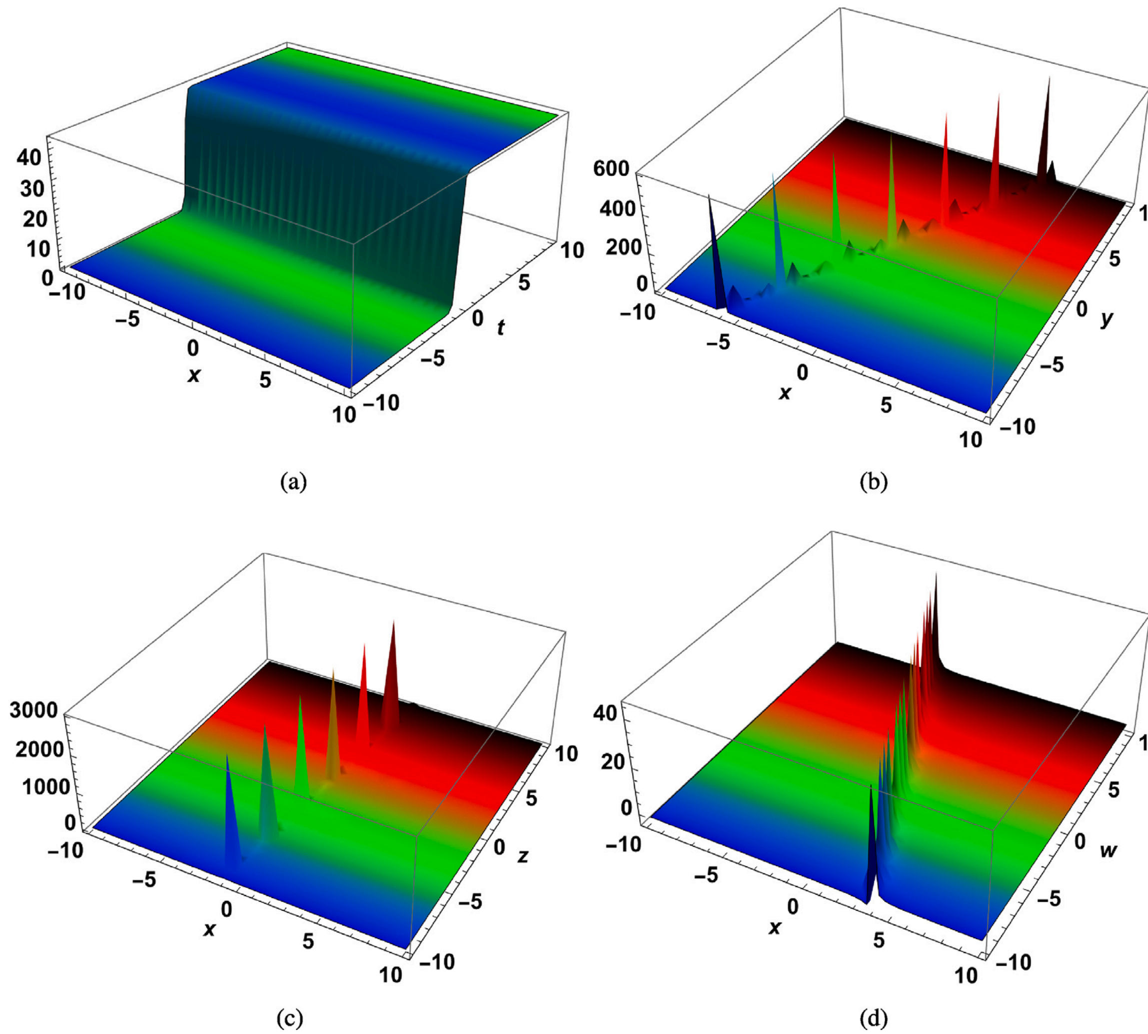

Fig. 1. The dynamics of the solution ψ_1.

$$\times\left[1+\frac{2\mathrm{K}_1+2\mathrm{K}_2e^{\sqrt{\mathcal{F}}\xi}}{\mathrm{K}_1(-2+\mathcal{U}+\sqrt{\mathcal{F}})+\mathrm{K}_2(-2+\mathcal{U}-\sqrt{\mathcal{F}})e^{\sqrt{\mathcal{F}}\xi}}\right]$$
$$+(\tfrac{-m^2+n^2}{6})(\mathcal{U}^2-2\mathcal{U}\mathrm{K}$$
$$-2\mathcal{U}+\mathrm{K}^2+2\mathrm{K}+1)$$
$$\times\left[1+\frac{2C_1+2C_2e^{\sqrt{\mathcal{F}}\xi}}{\mathrm{K}_1(-2+\mathcal{U}+\sqrt{\mathcal{F}})+\mathrm{K}_2(-2+\mathcal{U}-\sqrt{\mathcal{F}})e^{\sqrt{\mathcal{F}}\xi}}\right]^2.$$

CASE#2: ($\mathcal{D}=\mathcal{U}^2-4\mathrm{K}<0$)

$$\psi_4(\xi)=(\tfrac{-m^2+n^2}{6})(\mathcal{U}^2-6\mathcal{U}\mathrm{K}+6\mathrm{K}^2+2\mathrm{K})+(m^2-n^2)$$
$$\times(\mathcal{U}^2-3\mathcal{B}\mathrm{K}-\mathcal{U}+2\mathrm{K}^2+2\mathrm{K})$$
$$\times\left[\frac{(\mathcal{U}\mathrm{K}_1-\sqrt{-\mathcal{F}}\mathrm{K}_2)\cos(\frac{\sqrt{-\mathcal{F}}\xi}{2})+(\mathcal{U}\mathrm{K}_2+\sqrt{-\mathcal{F}}\mathrm{K}_1)\sin(\frac{\sqrt{-\mathcal{F}}\xi}{2})}{(\mathcal{U}-2)\mathrm{K}_1-\sqrt{-\mathcal{F}}\mathrm{K}_2\cos(\frac{\sqrt{-\mathcal{F}}\xi}{2}+(\mathcal{U}-2)\mathrm{K}_2+\sqrt{-\mathcal{F}}\mathrm{K}_1)\sin(\frac{\sqrt{-\mathcal{F}}\xi}{2})}\right]$$
$$+(\tfrac{-m^2+n^2}{6})(\mathcal{U}^2-2\mathcal{U}\mathrm{K}$$
$$-2\mathcal{U}+\mathrm{K}^2+2\mathrm{K}+1)$$
$$\times\left[\frac{(\mathcal{U}\mathrm{K}_1-\sqrt{-\mathcal{F}}\mathrm{K}_2)\cos(\frac{\sqrt{-\mathcal{F}}\xi}{2})+(\mathcal{U}\mathrm{K}_2+\sqrt{-\mathcal{F}}\mathrm{K}_1)\sin(\frac{\sqrt{-\mathcal{F}}\xi}{2})}{(\mathcal{U}-2)\mathrm{K}_1-\sqrt{-\mathcal{F}}\mathrm{K}_2\cos(\frac{\sqrt{-\mathcal{F}}\xi}{2}+(\mathcal{U}-2)\mathrm{K}_2+\sqrt{-\mathcal{F}}\mathrm{K}_1)\sin(\frac{\sqrt{-\mathcal{F}}\xi}{2})}\right]^2.$$

Numerical simulations

In this part, we graphically present the derived solutions in 3D and 2D. The first solution of the considered DSKP equation is presented in Fig. 1. The solution is represented for specific values of parameters. For the Fig. 1(a), we take $y=1, z=1, w=1, n=-0.06, m=-4.7, p=5, \mathrm{K}=0.3, \mathrm{K}_1=-4, \mathcal{K}_2=2, \mathcal{V}=2, k=2, r=2$. For this case, we have $\mathcal{D}>0$. So, the solution is real. Moreover, the Fig. 1(a) displays the kink behaviour of the obtained solution in the $x--t$ plane. The Fig. 1(b) provides single strip soliton solutions in x–y plane for the specific values of the parameter such as: $z=1, w=1, t=0.1, n=-0.9, m=0.5, p=0.9, \mathrm{K}=0.1, \mathrm{K}_1=-1, \mathrm{K}_2=0.3, \mathcal{V}=-2, k=0.1, r=-0.3$. The Fig. 1(c) portrays the singular soliton solutions in the $x--z$ plane for the specific values of the parameter such as: $y=1, w=1, t=0.1, n=-0.9, m=0.5, p=0.3, \mathrm{K}=0.1, \mathrm{K}_1=-1, \mathrm{K}_2=0.3, \mathcal{V}=-2, k=0.1, r=-0.3$. The Fig. 1(d) provides the singular soliton solutions in the $x--w$ plane for the same values of the parameters.

The second solution is defined for $\mathcal{D}<0$. We present the second solution in Fig. 2. It represents multistrip soliton solutions for various values of parameters. The Figs. 2(a) and 2(b) gives two strip soliton solutions for the specific values of parameters: $y=1, z=1, w=1.1, n=-0.1, m=1, p=0.1, \mathrm{K}=6, \mathrm{K}_1=3, \mathrm{K}_2=0.8, \mathcal{V}=0.3, k=-7, r=0.3$. The

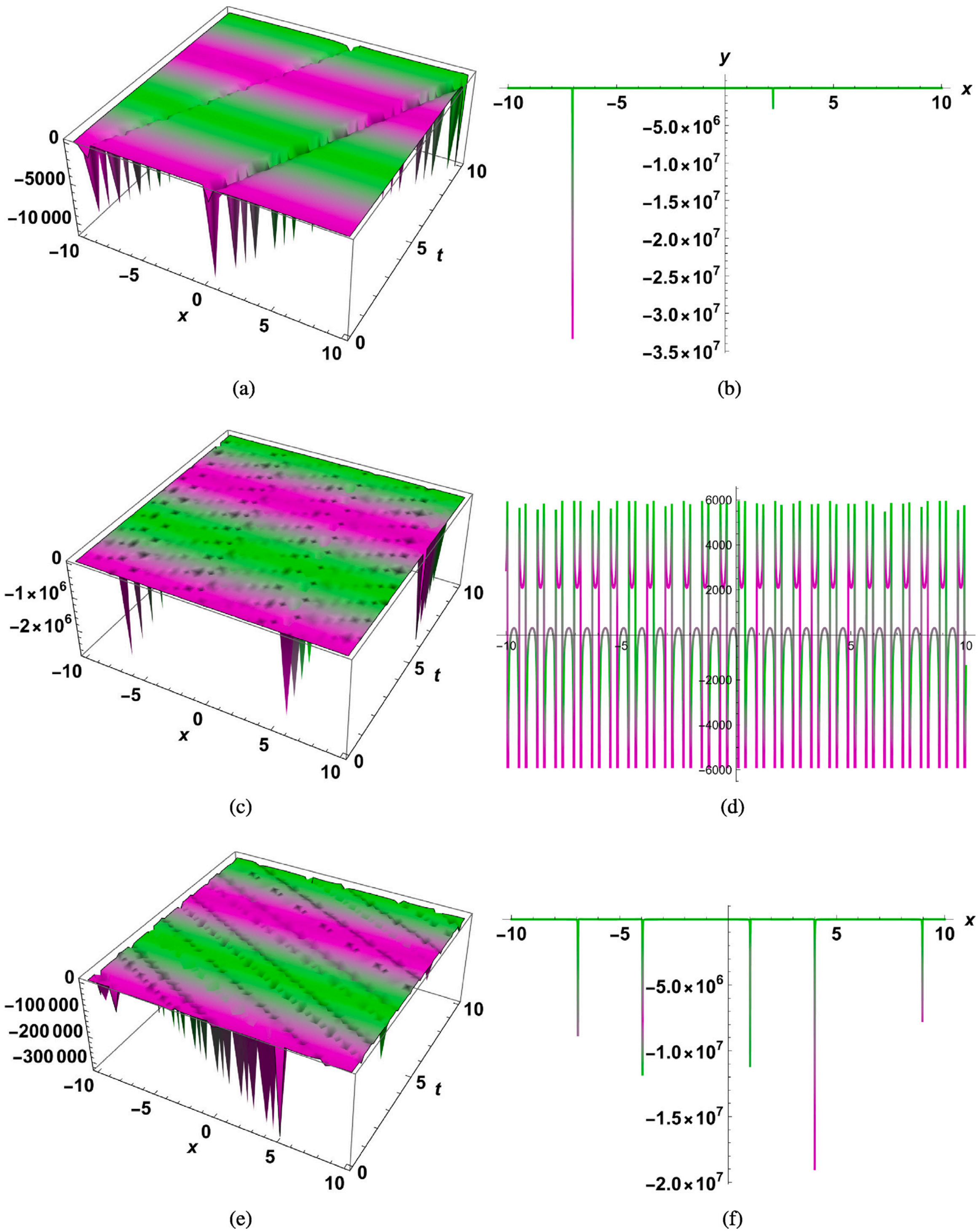

Fig. 2. The dynamics of the solution ψ_2.

Figs. 2(c)–2(f) shows multi-strip soliton solutions for specific values of parameters.

The third solution is defined for $\mathcal{D} > 0$. The graph of the solution is depicted in Fig. 3 for various values of parameters. The Figs. 3(a) and 3(b) show the kink behaviour for parameter values: $y = 1, z = 1, w = 3, n = 3, m = 2, p = 1, \mathrm{K} = 1, \mathrm{K}_1 = -2, \mathrm{K}_2 = 2, \mathcal{U} = 4, k = 0.8, r = 0.7$. The Figs. 3(c)–3(d) show the dark soliton solution for $p = 2$, and $\mathrm{K} = 2$. he Figs. 3(e)–3(f) show the bright soliton solution for $p = 2$, and $\mathrm{K} = 3$.

The last exact solution is valid for $\mathcal{D} < 0$. The solution is presented in Fig. 4. The graphs represents periodic waves solutions in different planes. The Fig. 4(a) show travelling wave solution for $y = -2, z = 1, w = 1.1, n = 2, m = 1, p = 1.2, \mathrm{K} = 0.3, \mathrm{K}_1 = 0.3, \mathrm{K}_2 = 6, \mathcal{U} = 3, k = 0.5, r = -2$. The Fig. 4(b) displays the convex travelling solution for same values of the parameters in $x--z$ plane. The Figs. 4(c) and 4(d) show the convex type travelling waves solutions in the $x--y$ and $x--w$ plane.

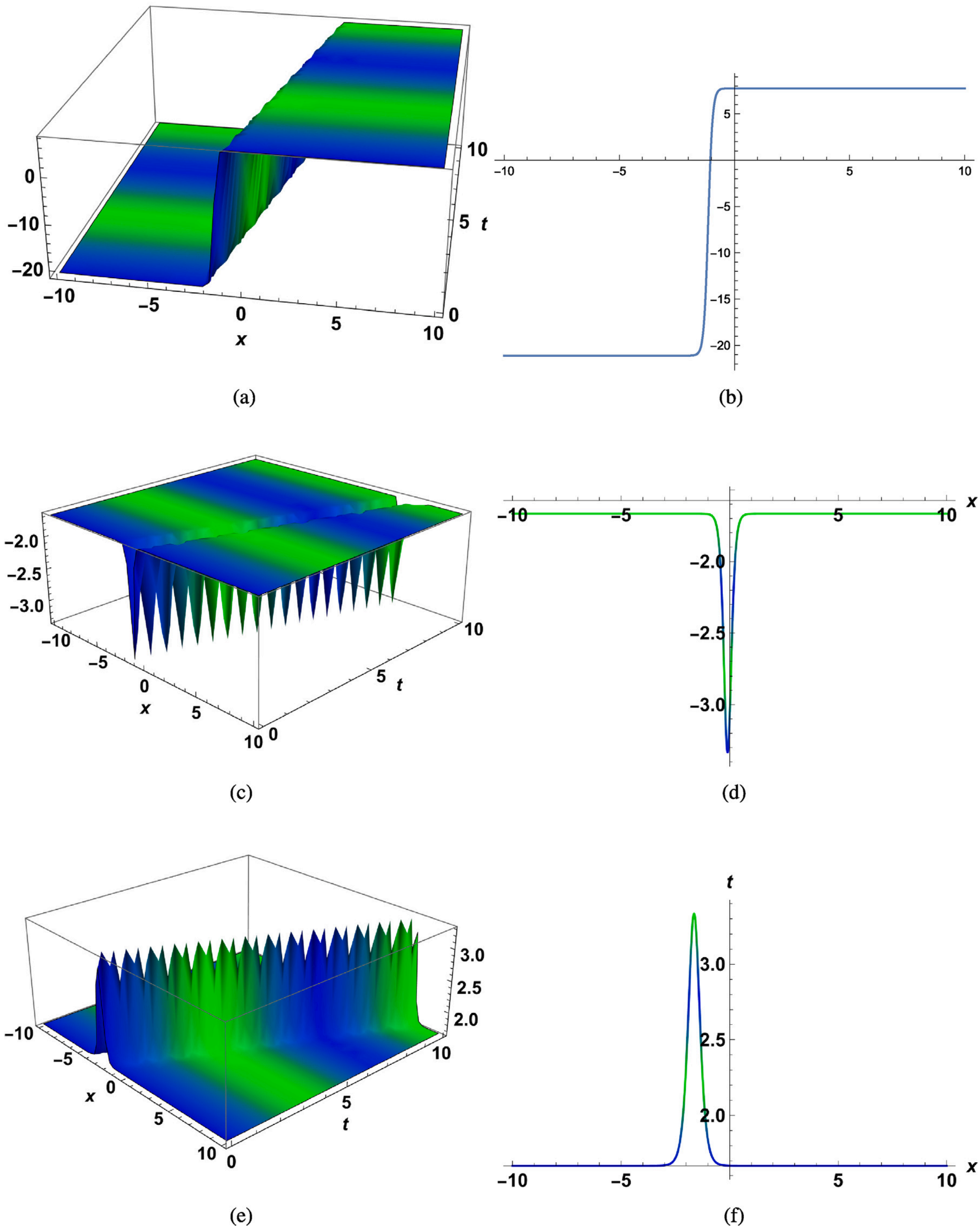

Fig. 3. The dynamics of the solution ψ_3.

Conclusion

In this paper, the analytical study of (4 + 1)-dimensional DSKP equation has been carried out. The considered equation has been solved to extract some new travelling waves solutions. The newly proposed $(\frac{\mathscr{G}'}{\mathscr{G}'+\mathscr{G}+\mathbb{A}})$-expansion technique has been used for the investigation of new exact solutions. The obtained solution have not been studied for the considered equation in the literature. The solutions have been depicted via Mathematica software to study their nature and behaviour for different values of parameters. We have observed different waves structure such kink, singular solitons, and travelling waves. In near future, we will use the proposed method to solve more NEEs equations of higher dimensions.

CRediT authorship contribution statement

Israr Ahmad: Methodology, Writing – original draft. **Abdul Jalil:** Methodology, Formal Analysis, Investigation, Writing – original draft.

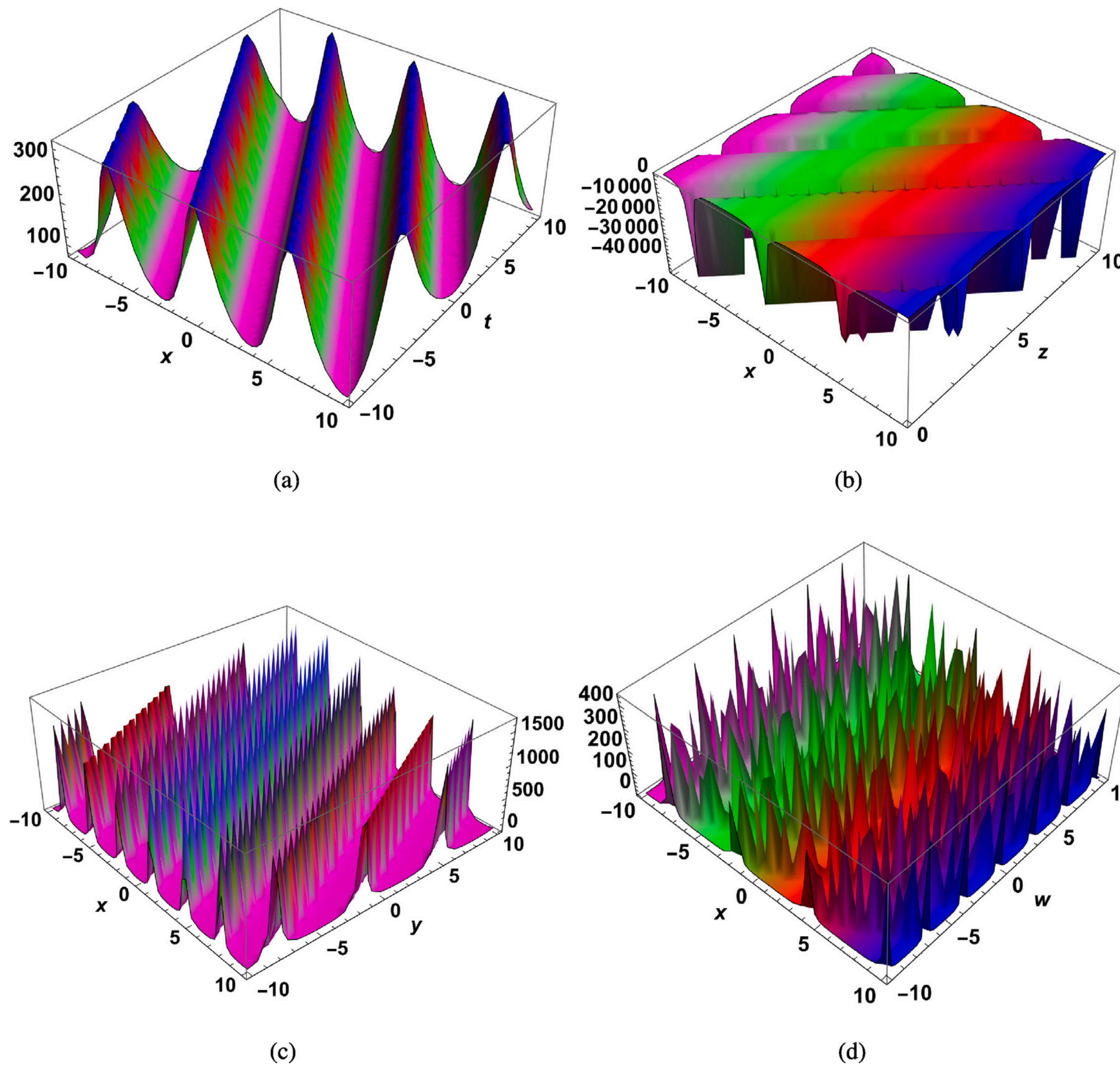

Fig. 4. The dynamics of the solution ψ_4.

Aman Ullah: Validation, Formal Analysis, Investigation, Writing – review & editing. **Shabir Ahmad:** Conceptualization, Writing – original draft, Writing – review & editing. **Manuel De la Sen:** Validation, Funding acquisition.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

No data was used for the research described in the article.

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