ISSN 1988-088X

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http://www.dfaeii.ehu.es

DFAE-II WP Series

2012-16

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Optimal Monetary Policy with Asymmetric Preferences for Output

Optimal monetary policy with asymmetric preferences for output*

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June 21, 2012

Abstract

Using a model of an optimizing monetary authority which has preferences that weigh inflation and unemployment, Ruge-Murcia (2003, 2004) finds empirical evidence that the authority has asymmetric preferences for unemployment. We extend this model to weigh inflation and output and show that the empirical evidence using these series also supports an asymmetric preference hypothesis, only in our case, preferences are asymmetric for output. We also find evidence that the monetary authority targets potential output rather than some higher output level as would be the case in an extended Barro and Gordon (1983) model.

JEL Classification: E31, E52, E61
Keywords: Optimal monetary policy, asymmetric preferences, conditional output volatility

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*Some of this research was supported by the Spanish Ministry of Science and Innovation and the Basque Government, grant numbers ECO2010-16970 and IT-214-07, respectively.

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1 Introduction

The possibility that monetary policy makers may induce an upward bias in inflation was first suggested by Barro and Gordon (1983). They suggested that, because the monetary policy maker is unable to make long term commitments, it is possible that instead they pursue policies which create surprise inflation. This intriguing proposition has been explored in numerous empirical studies including Ireland (1999), Ruge-Murcia (2003, 2004) and others with mixed results. Although Ruge-Murcia (2003, 2004) showed that the Barro and Gordon style inflation bias is not supported by the data, these papers developed a new theory that an inflation bias may arise from asymmetric preferences on the part of the monetary authority. In the Ruge-Murcia model, the inflation bias arises because the monetary authority takes stronger action when unemployment is above the natural rate than when it is below the natural rate.

In this paper, we develop an asymmetric preference model which focuses on an output asymmetry. Such a model is consistent with many important optimal monetary policy papers, including Cukierman (2002), Nobay and Peel (2003) and Walsh (2003), which have a more theoretical emphasis. The structure of our model is similar to the one in Ruge-Murcia (2003, 2004). However, it includes a slightly different trend structure to handle the growing character of the output data.[1]

We find that the monetary authority targets permanent output rather than some higher level of output which would be required in a parallel Barro and Gordon type model in which output is considered instead of unemployment. Furthermore, we find that the preferences of the monetary authority are asymmetric with stronger action taken when output is below its permanent level than when it is above. For this study, we look at two different data periods, including one of the standard periods used in both Ireland (1999) and Ruge-Murcia (2003, 2004) and a second that extends that series up to the second quarter of 2011.

[1] Another approach taken by Surico (2007) also uses output as part of the monetary authorities objective function, but his paper differs from our paper and the Ruge-Mucia (2003, 2004) models in that it focusses on policy rule asymmetries.

2 The Model

The model starts with a common formulation for the short run supply curve given by

$$Y_t = Y_t^p + \alpha(P_t - P_t^e) + \eta_t,$$

where Y_t is observed output at time t, Y_t^p is permanent or potential output at time t, P_t is the price level at time t, P_t^e is the expected price level at time t based on information at time $t-1$ and η_t is a supply disturbance.[2] Adding and subtracting P_{t-1} inside the parenthesis term on the right and rearranging terms gives

$$Y_t = Y_t^p + \alpha(\pi_t - \pi_t^e) + \eta_t, \tag{1}$$

where $\pi_t = P_t - P_{t-1}$ and $\pi_t^e = P_t^e - P_{t-1}$.

Permanent output fluctuates over time in response to a real shock ζ_t according to the autoregressive process

$$\begin{aligned}(1-L)\left[Y_t^p - (1-\delta)t\right] &= \psi - (1-\delta)\left[Y_{t-1}^p - (1-\delta)(t-1)\right] \\ &\quad + \theta(1-L)\left[Y_{t-1}^p - (1-\delta)(t-1)\right] + \zeta_t, \end{aligned} \tag{2}$$

where $-1 < \theta < 1$, $0 < \delta \leq 1$, L is the lag operator and ζ_t is serially uncorrelated and normally distributed with mean zero and standard deviation σ_ζ. As in Ruge-Murcia (2003, 2004) we use δ to capture different types of trend possibilities in the permanent output process. To understand these different trends, rewrite (2) as

$$Y_t^p - Y_{t-1}^p = \psi' + (1-\delta)^2 t - (1-\delta)Y_{t-1}^p + \theta(Y_{t-1}^p - Y_{t-2}^p) + \zeta_t, \tag{3}$$

where $\psi' = \psi + (1-\delta)\left[1 - \theta - (1-\delta)\right]$. This formulation shows that when $\delta = 1$, the model has no deterministic trend, $\psi' = \psi$ and there is a unit root. On the other hand, when $\delta < 1$, there is a deterministic trend and no stochastic trend.[3]

[2] This supply curve can be motivated in a number of ways and standard sources for it can be found in Friedman (1968) and Lucas (1977).

[3] We empirically investigated both the integrated model, where $\delta = 1$, and a trend-stationary model where $\delta < 1$. Results for both models were similar, so only the integrated results are reported below. However, for the sake of replication, we provide some further discussion on how one could replicate our stationary model estimation results.

Actual inflation for the period is then determined as the sum of a policy variable chosen by the monetary authority denoted by i_t and a control error, ε_t, so that

$$\pi_t = i_t + \varepsilon_t, \tag{4}$$

where ε_t is serially uncorrelated and normally distributed with mean zero and standard deviation σ_ε. Define ξ_t to be the 3×1 vector that contains the model's structural shocks at time t. We assume that ξ_t is serially uncorrelated, normally distributed with zero mean, and (possibly) conditionally heteroscedastic:

$$\xi_t | I_{t-1} = \begin{bmatrix} \eta_t \\ \zeta_t \\ \varepsilon_t \end{bmatrix} | I_{t-1} \tilde{} N(0, \Omega_t), \tag{5}$$

where Ω_t is a 3×3 positive-definite variance–covariance matrix. The conditional heteroscedasticity of ξ_t relaxes the more restrictive assumption of constant conditional second moments and captures temporary changes in the volatility of the structural shocks.

The policy maker selects i_t in an effort to minimize a loss function that penalizes variations of output and inflation around target values according to

$$\left(\frac{1}{2}\right)(\pi_t - \pi_t^*)^2 + \left(\frac{\phi}{\gamma^2}\right)(\exp(\gamma(Y_t^* - Y_t)) - \gamma(Y_t^* - Y_t) - 1),$$

where $\gamma \neq 0$ and $\phi > 0$ are preference parameters, and π_t^* and Y_t^* are desired rates of inflation and output, respectively. As in Ireland (1999) and Ruge-Murcia (2003), we assume π_t^* is constant and denote it by π^*. The output level targeted by the central banker is proportional to the permanent value according to

$$Y_t^* = kE_{t-1}Y_t^p, \text{ for } k \geq 1. \tag{6}$$

In this formulation, when $k = 1$, the authority targets permanent output, while for $k > 1$ the authority targets output beyond the permanent level. Substituting (1), (4), and (6) into the objective function gives

$$\min_{i_t} E_{t-1} \left\{ \begin{array}{c} \left(\frac{1}{2}\right)(i_t + \varepsilon_t - \pi_t^*)^2 \\ + \left(\frac{\phi}{\gamma^2}\right) \left(\begin{array}{c} \exp(\gamma(kE_{t-1}Y_t^p - Y_t^p - \alpha(i_t + \varepsilon_t - \pi_t^e) - \eta_t)) \\ -\gamma(kE_{t-1}Y_t^p - Y_t^p - \alpha(i_t + \varepsilon_t - \pi_t^e) - \eta_t) - 1 \end{array} \right) \end{array} \right\}.$$

3 Empirical Results

Solving the optimization problem and linearizing the decision rule gives the following reduced form inflation equation[4]

$$\pi_t = a + bE_{t-1}Y_t + c\sigma^2_{Y,t} + e_t, \tag{7}$$

where a is a constant intercept, $b = \phi\alpha(k-1) \geq 0$, $c = \frac{\phi\alpha\gamma}{2} \gtrless 0$, and e_t is a reduced form disturbance. As in the Ruge-Murcia model, as $\gamma \to 0$ (with $k > 1$) one obtains an inflation-output version of the Barro and Gordon model. So a test of that model is, $H_0 : c = 0$. Also, when $k = 1$ the policy preferences are such that the monetary authority targets permanent output, so a test of this is, $H_0 : b = 0$.

A reduced form for the output equation is also easily derived

$$\begin{aligned} \Delta Y_t &= \psi' + (1-\delta)^2 t - (1-\delta)Y_{t-1} + \theta\Delta Y_{t-1} + \zeta_t + \eta_t + \alpha\varepsilon_t \\ &\quad -\delta\left(\alpha\varepsilon_{t-1} + \eta_{t-1}\right) - \theta(\alpha\Delta\varepsilon_{t-1} + \Delta\eta_{t-1}). \end{aligned} \tag{8}$$

Equations (7) and (8) were estimated jointly using a maximum likelihood procedure. The output conditional variances were estimated first using a $GARCH(1,1)$ model. Since $\sigma^2_{Y,t}$ is identified only if it is not constant, we ran some preliminary tests to see if it is time varying. Table 1 contains the results of various neglected $ARCH$ tests. The first two rows show the results using the original output series. Here the residuals from a four-lag VAR with a time trend were collected. These residuals were then squared and an OLS regression was run on a constant and one to six lags. The last two rows show the results using the standardized residuals from the $GARCH(1,1)$ model. These test statistics have χ^2_q distribution where q is the number of lags. These results show evidence that the original output series does have conditional heteroscedasticity, while the conditional variance series does not.

[4] A mathematical appendix shows how equations (7) and (8) are derived. This appendix is available from the authors upon request.

Table 1. LR tests for neglected $ARCH$

Squared residuals	Sample period	No. of lags					
		1	2	3	4	5	6
Original	1960:1-1999:4	0.60	$5.44^{\dagger}$	5.93	$8.57^{\dagger}$	$9.87^{\dagger}$	8.41
	1960:1-2011:2	1.43	7.40^{*}	8.00^{*}	11.29^{*}	12.51^{*}	$10.97^{\dagger}$
Standardized	1960:1-1999:4	1.04	2.45	2.64	5.47	5.48	5.72
	1960:1-2011:2	0.49	2.59	2.68	5.75	5.83	6.08

Note: We use the convention that tests that are significant at the 10 percent level only have a † while those that are significant at the 5 percent (and 10 percent) level have an $*$.

In estimating the trend-stationary model, we set $\delta = 0.991$, which is the value of the coefficient on the time trend term in a simple regression of output on a constant and a time trend. For the nonstationary model we set $\delta = 1$. Estimation results are almost identical under the two permanent output specifications, so for the sake of brevity, we only report the estimation results for the $ARIMA(1,1,2)$ formulation.[5] The first (second) panel of Table 2 shows the results of the maximum likelihood estimation of the model using the sample period of 1960:1-1999:4 (1960:1-2011:2). The first sample period is one of the data periods used in Ruge-Murcia (2003, 2004) and it is similar to the sample period of 1960:1-1997:2 considered in Ireland (1999). The table is organized so that the first column provides estimates of an output and inflation version of the Barro and Gordon, while the next three columns provide estimates of an asymmetric preference formulation. The first asymmetric preference formulation allows k to vary freely and to possibly have negative values, contrary to the model restriction, while the second asymmetric preference formulation allows k to vary freely in a range greater than 1 and the third case constrains k to equal 1.

Focusing on the first panel, Table 2 shows that whenever k is allowed to vary freely, b takes on a negative, but insignificant, value as Ruge-Murcia (2003) found in one of his estimated specifications using unemployment instead of output. Not surprisingly, constraining k to its theoretical plausible region results in it always moving to its lower bound. Furthermore, using the likelihood ratio test, the null

[5]Estimation results for the $ARIMA(2,0,2)$ formulation are available from the authors upon request.

hypothesis that b equals zero, cannot be rejected. This result is similar to results found using the inflation-unemployment model by Ruge-Murcia (2003, 2004) and implies that policy makers target permanent income rather than some higher level of output. Moreover, Table 2 also allows one to test for the presence of asymmetric preferences over output by testing whether the coefficient of the conditional variance of output, c, is significant. Both the t statistic and the likelihood ratio statistic (the latter takes the value 15.98 using the theoretically consistent model with $k \geq 1$ as the unrestricted model) reject this null at any standard significance level.

The second panel shows the estimation results obtained running the same model, but considering data up through 2011:2. Estimation results are fairly robust across the samples, although b does become significantly negative in this longer sample. Overall, in both samples, we find robust evidence of asymmetric preferences for output on the part of an optimal monetary planner.

Table 2. Estimation results

Sample 1960:1-1999:4

Coefficient	Model			
	Barro and Gordon	Asymmetric with		
		k free	$k \geq 1$	$k = 1$
a	3.90	5.15	2.88	2.88
	(0.20)	(4.39)	(0.33)	(0.33)
b	0.0	-0.26	0.0	
	·	(0.49)	·	
c		1.29	1.33	1.33
		(0.39)	(0.39)	(0.39)
log likelihood	165.29	173.40	173.28	173.28

Sample 1960:1-2011:2

Coefficient	Model			
	Barro and Gordon	Asymmetric with		
		k free	$k \geq 1$	$k = 1$
a o	3.53	14.80	2.68	2.68
	(0.17)	(3.49)	(0.27)	(0.27)
b	0.0	-1.24	0.0	
	·	(0.38)	·	
c		0.91	1.14	1.14
		(0.34)	(0.36)	(0.36)
log likelihood	231.29	245.10	238.90	238.90

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Appendix 1: Solving the planner's optimization problem **(Not intended for publication)**

Taking the derivative with respect to i_t and taking the public's inflation forecast as given yields first order condition

$$E_{t-1}\left\{(\pi_t - \pi^*) + \left(\frac{\phi}{\gamma^2}\right)(-\gamma\alpha\exp(\gamma(kE_{t-1}Y_t^p - Y_t)) + \alpha\gamma)\right\} = 0, \tag{9}$$

or

$$E_{t-1}\pi_t - \pi^* - \left(\frac{\phi\alpha}{\gamma}\right)E_{t-1}\left(\exp(\gamma(kE_{t-1}Y_t^p - Y_t)) - 1\right) = 0. \tag{10}$$

As shown below, the assumption that the structural disturbances are normal implies that, conditional on the information set, output is also normally distributed. Then, $\exp(\gamma(kE_{t-1}Y_t^p - Y_t))$ is distributed log normal. Using the intermediate result

$$E_{t-1}Y_t = E_{t-1}Y_t^p, \tag{11}$$

obtained by taking conditional expectations of both sides of (1) and using the assumption of rational expectations, it is possible to write the mean of this log normal distribution as $\exp\left(\gamma(k-1)E_{t-1}Y_t^p + \frac{\gamma^2\sigma_{Y,t}^2}{2}\right)$. The notation $\sigma_{Y,t}^2$ is the conditional variance of output and is derived below in terms of the elements of ξ_t. Finally, using (4), it is easy to show that

$$\pi_t = \pi^* + \left(\frac{\phi\alpha}{\gamma}\right)\left(\exp\left(\gamma(k-1)E_{t-1}Y_t^p + \frac{\gamma^2\sigma_{Y,t}^2}{2}\right) - 1\right) + \mathbf{A}\xi_t, \tag{12}$$

where $\mathbf{A} = (0, 0, 1)$.[6] Next, using (1), we see

$$[Y_t - E_{t-1}Y_t] = [Y_t^p - E_{t-1}Y_t^p] + [\alpha(\pi_t - \pi_t^e) - E_{t-1}(\alpha(\pi_t - \pi_t^e))] + [\eta_t - E_{t-1}\eta_t].$$

Using (3) and (4) gives

$$Y_t = E_{t-1}Y_t + \mathbf{B}\xi_t,$$

[6] To see this, note that (4) implies

$$[\pi_t - E_{t-1}\pi_t] = [i_t - E_{t-1}i_t] + [\varepsilon_t - E_{t-1}\varepsilon_t],$$

which implies

$$\pi_t = E_{t-1}\pi_t + \varepsilon_t.$$

Next using (10) gives the result.

where $\mathbf{B} = (1, 1, \alpha)$. Next using (11) gives

$$Y_t = E_{t-1}Y_t^p + \mathbf{B}\xi_t. \tag{13}$$

Note that since $E_{t-1}Y_t^p$ is included in the public's information set at time $t-1$ and the linear combination $\mathbf{B}\xi_t$ is normally distributed, so

$$Y_t|I_{t-1}\tilde{}N(E_{t-1}Y_t^p, \sigma^2_{Y,t}) \quad \text{where} \quad Var\,(Y_t|I_{t-1}) \equiv \sigma^2_{Y,t} = \mathbf{B}\mathbf{\Omega}_t\mathbf{B}',$$

as claimed above.

Appendix 2: Estimation results for the ARIMA (2,0,2) formulation **(Not intended for publication)**

Table 2A. Estimation results for the $ARIMA(2,0,2)$ formulation

Sample 1960:1-1999:4

Coefficient	Model			
	Barro and Gordon	Asymmetric with		
		k free	$k \geq 1$	$k = 1$
a	3.90	5.03	2.86	2.86
	(0.20)	(4.47)	(0.32)	(0.32)
b	0.0	-0.25	0.0	
	.	(0.50)	.	
c		1.30	1.34	1.34
		(0.39)	(0.39)	(0.39)
log likelihood	165.67	173.72	173.61	173.61

Sample 1960:1-2011:2

Coefficient	Model			
	Barro and Gordon	Asymmetric with		
		k free	$k \geq 1$	$k = 1$
a	3.53	13.95	2.70	2.70
	(0.17)	(3.48)	(0.27)	(0.27)
b	0.0	-1.26	0.0	
	.	(0.38)	.	
c		0.88	1.12	1.12
		(0.35)	(0.36)	(0.36)
log likelihood	230.26	243.99	237.58	237.59